

**Optimization and estimation study of manpower  
planning models**

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## SUMMARY

An attempt is made in this thesis to study the optimization and estimation of stochastic models of manpower systems incorporating one or more of the following aspects:

- (i) recruitment, (ii) promotion, (iii) training and (iv) wastage

The thesis contains eight chapters and brief summaries of which are given below:

Chapter 1 is introductory in nature and gives review of the literature. The techniques used in the analysis of manpower systems are also provided.

Two models are studied in chapter 2. Model 1 is the extension of Ragavendra (1991) where maintainability of grade is considered. In model 2 we give importance to efficiency and skills of the employees by allowing multiple promotions. That is, an employee has been promoted to the next higher grade due to seniority and efficiency, whereas he is promoted to other higher grades due to efficiency only. The promotional probabilities and recruitment vectors and cut-off levels of seniority and efficiency for promotions are found.

The study of intermittently busy manpower system is studied in chapter 3. By identifying the important random variables, busy and lean periods, the amount of crisis has been obtained in the stationary case. The asymptotic confidence limits are also obtained for the crisis. A non-Markovain model is also studied by assuming that some of the distributions are arbitrary. Various system measures have been obtained using the correlated alternating renewal process.

In chapter 4, an attempt is made to analyze impact of category and grade dependent promotion probabilities on the grade structure of hierarchical manpower systems. To be specific, we consider a multi-grade manpower system in which each grade is subdivided into several categories according to length of service in that grade. The last category of each lower grade consists of persons who have completed a specified



period of service in that grade and do not get promotion. An employee in a lower grade is eligible for promotion to the most junior category of the next higher grade and the probability of promotion is dependent on the grade and category of the employee. Un-promoted employee of a category of a lower grade will move to the next higher category of the grade in the next unit of time until he reaches the last category of the grade from where he is either promoted or leaves the system. The unit of time may be taken as a year. The movement of an employee from one category to another category is called transition. New entrants to the system are allowed in the lowest category of the lowest grade. Wastages are allowed from any category of any grade and no demotions take place. The probability distribution of the state of the system is derived. The recurrence relation for the moments of the grade sizes is derived and the expected time to reach the top-most grade by a new entrant in the lowest grade is found. A numerical example is provided to highlight the impact of category and grade dependency on the grade structure of a particular organization.

Analysis of optimal promotion policy using queuing approach is studied in chapter 5. Queuing approach is used for the first time in Manpower systems. Various system measures have been studied and cost analysis is also studied. Numerical example illustrates the results obtained.

The wastage and attrition rates in various manpower categories of higher educational institutions are analyzed using life tables technique in chapter 6. It is justified that persons with higher qualifications can get better jobs. Numerical example is shown to illustrate the results obtained.

In some situations the optimal period of training for the newly recruited persons should be found out. In such cases the cost of training should be considered and incorporated into the model in order to obtain the optimal period of training. This aspect has been introduced in chapter 7.

In chapter 8, voluntary retirement schemes have been introduced so as to reduce the grade size to avoid surplus staff strength. The optimal time intervals for introducing such schemes have been worked out taking cost into considerations.

In brief, the present work is an attempt to provide the optimal policy for recruitment, training, promotion and wastage in manpower planning models, with special provisions such as intermittently busy, efficiency and seniority, introduction of queuing approach, attrition in manpower systems etc.