

Appendix

Here we give a more detailed description of some of the packages that are mentioned in Chapter 3. We describe what these packages do and state the user friendliness where possible. The packages are listed alphabetically.

- (1) **Addition and subtraction of story problems.** The problems in this package are presented in groups of five. In the first drill, the learner is asked to enter two numbers. The operation is provided. The learner then enters the answer and the proper label. In the next level of difficulty, the learner types in a word to complete the question and then computes and enters the answer. As the difficulty level increases, the learner is given fewer and fewer aids.
- (2) **Algebra blaster.** Each topic involves three levels of work for learners. The first level is called 'study the steps' in which two examples are worked out in detail with an accompanying explanation of how the problem should be solved. For example, the rule of adding signed numbers may be explained. By using the arrow keys, learners may go back and review the previous step as many times as necessary. The second level is called 'build your skill'. When learners choose this level, they may tell the computer whether they wish to start at level 1 or where they left off. In this level, a prompt is available for learners who require assistance. If the problem is missed twice, the correct answer is provided. Learners may choose to retake any missed items at the end of the sequence. The third level, called 'solve it' is the most difficult. There are no prompts available at this level, but there are certain hints. The type of cursor tells learners whether a sign, numeral, variable, exponent or a fraction is expected. Blanks are provided so that learners know how many digits the answer should contain.
- (3) **Algebra drill and practice.** This package also offers drill and practice in equations and inequalities involving absolute values, simplifying expressions with integral exponents, factorising linear and quadratic expressions, finding roots of quadratic equations, x -intercepts and the vertex of parabolic

graphs, and solving inequality.

- (4) **Algebra and function plotter.** This package allows learners to use the computer's graphics to aid their understanding of the effects of changing various parameters of a function, such as reversing the sign of a leading coefficient [32, p66].
- (5) **Beginning mathematics-word problems.** This package makes good use of graphics, and aims to motivate learners and to provide positive reinforcement for correct answers. The problems are all based on the theme of gold miners making their way to the West in search of gold. The addition/subtraction disk has a series of six units that build in progression. Each unit consists of five problems for learners to solve. The multiplication/division disk has six units that parallel the addition/subtraction ones exactly. The disks provide a wide variety of ways to practice solving word problems. Since learners can start at a simple level, they can enjoy success and not be frustrated. The last section of each disk allows learners to make up their own problems.
- (6) **Build.** The programme starts with one cube already drawn on the screen. Learners build pictures by instructing the computer to place a brick next to the brick last placed. The position of the new brick can be to the right, to the left, up or down and in or out. One activity is to investigate the various different ways of instructing the computer to draw an eight-brick cube [4, p80].
- (7) **Calculus-Pad.** This is a superior plotting software package which learners can use to learn operations on functions in a regular calculus course. It is designed so that learners can do experiments with functions quickly to increase their understanding.
- (8) **CAMI maths.** CAMI is a drill and practice package that randomly generates a multiple of examples for learners to work through. CAMI is not a multimedia package as it does not make use of sound and animation.

The only graphics are graphs and diagrams that are directly relevant to the question on the screen at that moment. It does not have an adaptive mechanism to adjust the difficulty level. CAMI frees learners from dependence on the instructor, and it makes it possible for learners to share ideas and work co-operatively in solving problems. Since the role of the package is to promote mastery of a sharply focused syllabus and set of techniques, it is errorless in that each problem has only one correct solution. The motivation in the package comes from getting the correct answer. This is marked with a green tick, and a word of praise. The package builds confidence and mastery of the fundamental mathematical techniques, reducing fear and feelings of alienation [24, p20].

- (9) **Darts.** Balloons appear on the number line and learners guess where the balloons are by typing in numbers. Each time they guess an arrow shoots to the specified position. The arrow pops the balloon when the guess is correct [20, p289].
- (10) **Derive.** Derive offers symbolic algebra and powerful graphics. It is an excellent tool for teaching and learning mathematics. Derive eliminates the drudgery of performing long and tedious mathematical calculations. While Derive takes the burden of doing algorithmic parts of solving a problem, learners can concentrate on the mathematical meaning of concepts. Instead of teaching and learning boring technical skills, instructors and learners can concentrate on the exciting and useful techniques of problem solving. Derive algebraically simplifies, expands and factors expressions. Equations and systems of non-linear polynomial equations can easily be solved, yielding their real and complex solutions [23, p1].
- (11) **Differential equations graphics package.** This package includes programmes for linear first and second-order differential equations, Euler approximation, graphical comparisons of Euler, tangent fields, logistics differential equations, Frobenius series, and Fourier series. The linear first and second-order programmes allow construction of non-zero right-hand-

side from a menu of components. All other programmes have this feature, and allow the learner to enter the desired expression directly. The Euler approximation programme allows equations of the form $y' = f(y)$ and plots several approximations for different numbers of intervals. The tangent field programme plots trajectories for initial positions entered from the keyboard. The Frobenius programme allows input of the first 5 terms and plots 5 approximations. The Fourier programme plots 3 approximations [28, p70].

(12) **Discrete mathematics.** This package is user-friendly and requires no knowledge of programming. According to Smith [28, p44], there are eight routines in this package. These are:

- **Truth table.** This routine generates truth tables for logical statements entered by the learner. These tables have up to four variables and a number of logical connectives and(\wedge), or(\vee), not(\neg), implies(\Rightarrow), and if and only if(\Leftrightarrow). Learners can enter a single statement and get its truth table or enter two statements to get a truth table for each, and a statement as to whether one logically implies the other.
- **Venn diagrams.** This routine enables learners to enter up to three set names and to request venn diagrams of intersections, unions, set differences and complements. Learners can also enter the number of elements in each of the sets, and request that the venn diagram shows the corresponding number of elements in some combinations of the sets.
- **Counting.** This routine enables learners to calculate combinations, permutations, partitions (the number of ways of dividing n things into cells of specified sizes), boxes (the number of ways of arranging n identical objects in j boxes) and the Pascal triangle.
- **Recursive.** This routine allows learners to define a function recursively and request the value $f(n)$ for any natural number n for which the function is properly defined.

- **Graphs.** This routine contains options for drawing graphs and digraphs specified by learners' input of vertices and edges.
 - **Binary.** This routine provides a graphic demonstration of a binary search and a binary tree for the learner's list of words.
 - **Sorting.** This routine demonstrates bubble sort, demonstration sort, and a quick sort for a random list of numbers.
 - **Equation.** This routine solves systems of linear equations with up to eight unknowns.
- (13) **Equations.** This package is designed for introductory algebra courses when learners are learning to solve linear equations of the form $AX + B = C$. Instructions are sequenced so that learners practice steps, in the proper order to solve equations. Since problems are randomly generated with A , B and C chosen as nonzero integers, a wide variety of problems are available. After each equation is presented, the learner must choose whether to add or subtract the same number from each side of the equation. After simplifying both sides, the learner then chooses whether to multiply or divide each by the same number. The operations of multiplication and division are interchangeable to the extent that the learner may choose to divide, say by 2 or multiply by $\frac{1}{2}$, in order to finish solving a problem.
- (14) **Explorer metros.** In this package, learners explore a space colony and encounter situations requiring them to choose one of three alternatives based on a given metric length, mass capacity or temperature. Learners must complete the journey within a time limit. One particularly useful feature of this package is the modification option which allows instructors or learners the opportunity to redesign the encounters and the material covered. The package varies the elements of the simulation each time it is run so that individuals or small groups will enjoy playing the game several times. Use of graphics and on-screen documentation are also a plus [20, p289].

- (15) **EZQ**. This package has very good graphic capability. Functions are built up by filling in blocks that take the exponential of a number or multiply two results or integrate a result. Depending on the amount of memory available, 32 to 96 blocks can be used at one time. Learners need to think about the proper sequence of blocks to solve a given problem. The sequence of blocks can be saved on a disk and used later. The package can be used by the instructor to work out a problem. The instructor can save the problem on a disk, and either show it in class or give the disk to learners [28, p70].
- (16) **Function graph plotter**. This is another graph-drawing package that provides similar facilities and that can be used in similar ways as **Graph**. It draws graphs quicker. It also has a useful facility that enables learners to input a function of the form $y = 2x + a$, for example, and then draw a set of graphs by continually incrementing the value of a [4, p62].
- (17) **Furs, nomad, sumeria, voyageur**. Learners attempt varying tactics, for example, different numbers of furs to be sold to maximise profits. In **nomad**, learners apply some geometry skills to read a map. In **voyageur**, learners make decisions on what to take on their imaginary voyage depending on the cost and practicality. **Sumeria** deals with starving people and for this reason some instructors are reluctant to use it [32, p106].
- (18) **Geometry concepts**. This is a four-part package.
- **Identification game**. This part provides practice in identifying and spelling geometric terms. A figure appears on the screen after the learner correctly identifies it. A screen of information about figures is displayed. Points are awarded based on the number of incorrect guesses the learner makes in naming the figure.
 - **Data-retrieval utility**. This part allows the learner to select a figure from the mathematical data bank. A screen of information about the figure is then presented.

- **Quiz machine.** In this part, learners are asked a series of multiple-choice questions.
 - **Constructions.** This part presents five basic constructions in an easy-to-follow fashion. Learners may work along with the constructions step by step or watch the sequence for review. The following constructions are presented:
 - perpendicular bisector of a segment.
 - angle bisector.
 - perpendicular to a line through a point not on the line.
 - perpendicular to a line through a point on the line, and
 - inscribed square.
- (19) **Geometric constructor.** This package allows learners in college-level geometry to perform a number of elementary constructions using points, lines, circles, bisectors, perpendiculars and parallels. Putting these concepts together enable learners to create every possible Euclidean construction. This package includes nine conic-plotting options which allow learners to accumulate as many lines, circles and conics on the screen as they wish.
- (20) **Geometric supposer.** Learners specify the procedure at first. For example, give me triangle ABC . Bisect AB , and label the midpoint as D . Bisect BC and label the midpoint as E . As learners type in instructions, the constructions they have defined appear on the screen. Now that they have defined their constructions, they have the whole construction accessible as a procedure. They can ask for a new triangle ABC , and the supposer will repeat learners' constructions. This permits learners to test conjectures on a new figure quickly [28, p3 and 26, p415].
- (21) **Graph.** This package has an option which enables the instructor to enter the equation of the graph to be drawn without displaying the equation on

the screen. The instructor can enter an equation, get the programme to draw the graph, and then invite learners to suggest the equation of the graph being displayed. Alternatively, the instructor can use the option in which the computer selects an equation of the graph to be drawn. In this case, both the instructor and learners share the problem of having to decide which graph is being displayed [4, p60].

(22) **Graph-calc.** Learners can manipulate graphs and equations in much the same way they would manipulate numbers with a calculator. The package consists of three components:

- **Graph.** In this component, learners graph functions in rectangular coordinate, parametric equations, equations in polar coordinates, a function and its derivative, and a function and its integral. Any number of curves may be plotted on the same graph for the sake of comparison.
- **Conic.** In this component, learners graph conic sections and compute pertinent data such as foci, vertices, and asymptotes. Two curves may be graphed simultaneously.
- **Checker.** This component enables learners to check steps in evaluating expressions, solving equations, solving identities, and finding derivatives and antiderivatives. Evaluated integrals may also be checked. This component does not work out the problem for the learner, but helps to isolate the step where the error was made. With this feature, learners are encouraged to make up problems on their own for extra practice, because they are then able to check the accuracy of their work [32, p105].

(23) **Green globs.** This game combines fantasy, challenge and curiosity in an intrinsically mathematical activity. Learners are challenged to destroy thirteen green globs displayed in a coordinate grid. Learners write equations to create graphs that pass through the globs, thereby obliterating them. Points are scored for hitting the most globs with the smallest number of equations. The computer stores the ten highest scores for the game along

with the series of equations that the record holders wrote to achieve the scores. Curious challengers may recall and display records of the record setting games to improve their own winning strategies [20, p58].

- (24) **Guesstimation.** Learners are asked to estimate distances on a number line, order numbers, and use negative numbers. A wide variety of problems allow learners to practice with integers, fractions, decimals, and absolute values. Learners are encouraged to make an estimate before any actual measurement is made.
- (25) **Gyro graphics.** This package allows learners to input their own functions and then creates the graph in seconds, complete with labeled axes and automatic scaling. It also allows learners to automatically add tangent and acceleration vectors to space curves and tangent planes to surfaces.
- (26) **Integer.** Learners may choose two to nine practice problems. A number line is available by typing ‘ N ’, and a marker on the number line can be moved back and forth using ‘ $<$ ’ and ‘ $>$ ’ keys. Typing ‘ R ’ allows learners to review multiplication and division rules. After two incorrect responses, the computer shows the correct solution [32, p106].
- (27) **Integer fast facts.** This game-like drill package has three versions. In the solo version, the learner gets ten problems per round. Learners may play over and over again while trying to improve their scores. The team version may be played by teams of two to five learners. The team score is the total of the individual scores to encourage learners to co-operate with one another to achieve higher point totals. In the competition version, learners compete against one another. In any of the three versions, learners may choose addition, subtraction, multiplication, division or assorted problems. Learners can repeat the same type of problems or change to another type at the end of the round. The game is a good motivator since the computer keeps track of both the learner’s highest score and most recent score for as long as the same kinds of problems are repeated.

- (28) **Laser math.** When the learner enters the answer to the problem presented, and then presses the 'F' key, the computer fires at a phantom cruiser. A correct response destroys the enemy, but an incorrect response results in losing a token to the enemy's torpedo attack. Learners earn points and are rewarded with promotions to higher ranks as their scores increase. All answers to problems range from -5 to $+5$. There are separate units on addition, subtraction and multiplication [32, p108].
- (29) **Lepack.** The package includes algorithms for linear equations, eigenvalues and applications, and has facilities for dealing with arrays. It was initially used to support the teaching of computational mathematics. Its use has now been extended to second year engineering mathematics level [15, p2].
- (30) **Locus.** This package can be used to present ideas to a class about loci or about situations involving loci. Using this package, learners can specify one or more fixed objects to be drawn on the screen with reference to which loci are to be drawn. Each fixed object is either a point or a line segment. For instance, if there is just one fixed object, consisting of one point, the locus drawn will be a circle [4, p60].
- (31) **Maclogic.** The rationale for this package is to ensure familiarity with first-order logic. It covers proof theory. It does not implement specialised methods such as truth tables and venn diagrams [10, p3].
- (32) **Macmath.** This package allows the learner of the Euler, improved Euler or fourth-order Runge-Kutta methods to solve first-order equations for one, two or three-dimensional systems of differential equations. Other routines find eigenvalues and eigenvectors for matrices by the Jacobian method, locate and identify bifurcation points for autonomous planar systems, and compute partial sums of Fourier series [28, p69].
- (33) **Macsyma.** This is a powerful mathematical software package. It contains facilities for algebra, linear algebra, calculus, numerical analysis, on-line help system, and versatile graphics. It includes 1,300 executable demonstrations, easily accessible at many points in the help system. Recent

mathematical improvements include enhanced speed in solving linear and algebraic equations, stochastic mathematics and better evaluation of special functions. This package can be used to differentiate, integrate, solve equations of the form $f(x) = 0$, solve systems of linear equations, and perform matrix manipulations [17, p1].

- (34) **Magic grid.** Learners must solve nine different equations selected from a tic-tac-toe-like grid. Each equation requires information from a previously solved equation. Help is given in a friendly manner each time an incorrect response is returned. The answer is supplied after two wrong choices. When Magic grid is completed, a summary of equations solved, with and without help, is provided, as well as the opportunity to do another Magic grid [20, p295].
- (35) **Mastering mathematics.** This package consists of seven content disks and three utility disks. Titles of the content disks suggest the subject matter covered in each. Problems are sequenced according to difficulty levels and may involve operations with up to three digits. Concepts of regrouping and working with real numbers are gradually introduced. One advantage of this package is that instructions on the various disks are very similar, so learners do not waste a lot of time becoming familiar with a new set of commands each time a new disk is used. Concepts are also built in logical progression. Since each content disk has a management option, the instructor can turn graphics on or off, set up the printer, change speeds to review programmes, and review learners' performance. The three utility disks consist of diagnostic and management systems and a worksheet generator. The diagnostic disk enables instructors to test learners on concepts covered in the seven content disks, to establish mastery levels, and to select the number of problems to be given on diagnostic tests. The worksheet generator allows the instructor to create worksheets covering some or all of the material on content disks. The management utility permits the instructor to monitor the progress of learners on each of the content disks.

- (36) **Mathematica.** This package contains a large number of built-in functions. It includes a collection of standard add-on packages that define many additional functions in areas such as algebra, calculus, graphics and discrete mathematics. Mathematica provides many functions for working with polynomials, and standard add-on algebra packages extend that capability. Mathematica has extensive built-in support for calculus including integration, differentiation, differential equation solving and limits. Calculus packages extend this capability by providing Fourier and Laplace integral transforms. Discrete mathematics is concerned with enumerable mathematical structures, such as are studied in combinatorics and graph theory. Graphical packages offer log, polar, and three-dimensional plots [25, p1].
- (37) **Math invaders.** The basic aim is for learners to destroy space invaders. By typing a correct answer and touching the space bar, the learner fires at the enemy. If the answer is incorrect, the enemy continues to move forward. There are never more than three enemies approaching at one time. Learner motivation is further enhanced by promotion to various ranks. The speed factor setting allows learners to compete with themselves as they try to improve the speed with which they can recall the basic facts. Learners who miss the same problem three times are given the answer by the computer. Learners must then touch the space bar and re-enter the correct answer to help in learning the fact [32, p115].
- (38) **Math pac.** Major topics include the arithmetic of algebraic expressions, factorising quadratics, solving linear and quadratic equations, graphing, and solution of linear and quadratic problems. The package can be used as an independent tutorial for learners whose algebra skills are barriers to success in other courses.
- (39) **Mathpert.** This programme is designed to eliminate road blocks by actively helping learners to solve any problem correctly. It allows learners to focus on the correct strategy of problem solving. Mathpert is available in three forms: Module 1 algebra only, module 2 includes algebra and pre-calculus,

and module 3 which is the complete package of algebra, pre-calculus and calculus. Algebra topics include linear functions, systems of linear equations, polynomials and polynomial functions. Pre-calculus topics include logarithmic and exponential functions, complex numbers, trigonometry and matrices. Calculus topics include limits, differentiation and integration.

- (40) **Math 246 programmes.** According to Smith [28, p71], this package consists of a menu driven selection of 4 programmes. These are:
- **Directional fields.** This programme allows a choice from a menu of five differential equations and plots the direction field. Learners can then sketch a trajectory with the mouse. The real trajectory is plotted through the chosen point.
 - **Damping.** This programme allows learners to pick values for the damping and the magnitude and frequency of the forcing function for a vibrating string which is shown while the graph is drawn.
 - **Systems.** This programme solves systems of the form $\frac{dx}{dt} = ax + by$, $\frac{dy}{dt} = cx + dy$. The eigenvalues are computed and the tangent fields and trajectories are drawn.
 - **Phase plane.** This programme allows learners to pick a list of differential equations and plot the $x - t$, $y - t$ and $x - y$ graphs.
- (41) **Matlab.** Matlab stands for matrix laboratory. Matlab is an interactive system in which the basic data element is a matrix that does not require dimensioning. This feature allows learners to solve many numerical problems easily. It has a routine that solves a system of linear equations. The main drawback of this package is that it gives only the final solution. It does not give the intermediate steps, which instructors are interested in for the purpose of demonstrating the Gaussian elimination procedure. The package is however user-friendly [35 and 36].
- (42) **Matrix calculator.** Learners begin by entering matrices into the programme and giving them specific names. Then, when prompted by the

programme, learners simply type in the operation they wish to perform and the names of the matrices involved. Since the programme works only with integers and rational numbers, all answers are reported as exact. The package allows learners to perform the following operations: addition, multiplication, row operations, reduction to reduced row-echelon form, and scalar multiplication. Learners can also find the inverse, determinant, transpose, adjoint, characteristic polynomials, eigenvalues and vectors of matrices.

- (43) **Matrix master.** This package consists of two disks and a short manual which enable instructors to demonstrate or learners to perform the following operations on matrices: input, store or retrieve, change individual entries, insert or delete a row or a column, elementary row operations, elementary column operations, sums and averages, transpose, determinants, inverses, add, subtract, multiply, juxtapose horizontally, vertically or diagonally, evaluate polynomials at matrices, characteristic polynomials, eigenvalues, rank, solve linear problems on equations.
- (44) **MATT graphical display package and differential equation solver.** In this package, problems are solved by entering instructions. The package can solve first-order differential equations and systems of first-order differential equations. It can be used to solve higher-order equations by converting them to systems of first-order equations [28, p69].
- (45) **Mind over matter.** One section of this package involves determining which letter is missing from an alphabetical list. At the easiest level, only one letter is omitted. At more difficult levels, several letters are left out. Learners may choose to have letters given in alphabetical order or in reverse alphabetical order. Another section of the package presents a numerical sequence and asks the learner for the missing number or numbers. The numerical sequence may be in ascending or descending order. The beginning problems involve counting and skip counting while the advanced ones challenge learners with obscure patterns.
- (46) **Minus mission.** A blob of green slime drops towards a robot at the bottom

of the screen. Learners try to find the slime by entering the answer to a subtraction fact and firing at the slime. Successful hits are recorded in a large slime at the top of the screen. Misses are also recorded. If the slime reaches the robot without being destroyed, the robot disintegrates. Learners may choose from several options at the beginning of the game. There are nine skill levels, and the run time may be varied from one to nine minutes [20, p54].

- (47) **Monte-Carlo.** This method was developed for the design of early nuclear reactors in order to calculate the area of an irregular shape. The method consists of placing an irregular shape on a suitable grid and throwing darts at random so that an estimate of the area of the shape is given by $[(\text{number of darts landing on shape}) / (\text{total number of darts thrown})] \times (\text{total area of grid})$. The more darts that are thrown the better this estimate will be [5, p2].
- (48) **Moves.** The programme draws axes on the screen and displays a flag. The learner can specify the transformation to be used and the flag can then be transformed. It can be used to discuss properties of different transformation with learners. For example, if different translations are presented on the screen, learners can observe that a translation just moves the flag a bit, that the size and shape of the flag do not change and that the flag is still the same way up. This programme is more useful when the instructor is in control of the situation and is guiding the course of learning. It encourages questions and observations from learners. It is very effective in connection with learners' work that involves drawing and transforming shapes using squared paper [4, p41].
- (49) **Multiploy.** This is an arcade-type game in which learners try to shoot down problem ships before their base is destroyed. The game may be played with addition, subtraction, multiplication, or division problems. There are three levels of difficulty. Learners may choose to have problem ships drop at normal speed or at 'lightening speed'. The game lasts until all the problem

ships have been destroyed or until the answer base is destroyed. Learners may change levels of difficulty or operations to be practised at the end of the game. The highest score is retained for each level and operation until the computer is turned off [32. p125].

- (50) **Number forms.** This package is for exploring and learning mathematics. Learners can move a number line on the screen and pull off from that number line a series of blocks, 'number forms', which display the number. Learners can then add, subtract, multiply, or divide numbers as the blocks visually perform the same operations. Number forms, thus, provide the bridge between concrete manipulatives and abstract symbols [20. p293].
- (51) **Number munchers.** This game allows learners to practice basic arithmetic skills by munching numbers while avoiding the evil troggles. This game can be played by two learners as a team. One may concentrate on the screen and call out the answer to the other who acts as a typist. For a second round the team may exchange positions. In this way learners practice both mathematics and keyboarding.
- (52) **Ordinary differential equations.** This package is designed for use with the book "Ordinary differential equations with numerical techniques." It is a set of programmes that implement 17 numerical methods to solve differential equations. These include Euler, improved Euler, Runge-Kutta for first and second-order equations, Milne, Hamming predictor-corrector methods, Bessel methods, 3 series methods and 3 methods for systems of equations. When a method is selected, the learner is asked to enter a differential equation. The programme graphs the direction field and the solution of an initial value problem. It can overlay graphs of solutions to the same differential equation with different initial conditions [28. p70].
- (53) **Phaser.** This programme is packaged with a 224-page book titled "Differential and difference equations through computer environments." It has over 60 differential and difference equations built in, each of which has parameters that may be set. Just about any equation learners might wish to

look at is already available, but learners can define some equations. Among the plots available are direction field, trajectories, and phase planes. Three-dimensional plots are available for simultaneous equations, and these graphs can be changed by rotation about any axis [28, p69].

- (54) **Plot pak.** This package includes a 26-page instructor's manual and 200 function files so that learners or instructors can quickly recall properly scaled and labelled graphs. Learners can study topics with visual support from powerful, but easy to use, graphic software.
- (55) **Quick-Graph.** This package is intended to be used either for in-class demonstrations or by an individual learner. After the learner selects a class of functions, an expression containing cells for parameter will appear at the bottom of the screen. Values for the parameters may be entered into these cells and the resulting function graphed. Since the graph remains on the screen while values are entered, the learner can readily observe the effect of changing a particular parameter.
- (56) **Reduce.** The capabilities of this package include expansion and ordering of polynomials and rational functions, calculations with symbolic matrices, analytic differentiation and integration, and many more. The main aim is to support calculations that are not feasible by hand [30, p1].
- (57) **Sell apples, sell plants, sell Lemonade.** In each of the three packages, learners have a business which sells one of these products. Learners must try to maximise gains and minimise losses by balancing various factors. Teams of learners can be used to make decisions. They soon learn that decisions are not as simple as they seem. For example, weather, advertising, and the price of the item must be considered. Learners learn that decisions have consequences. They begin to appreciate real-life applications for mathematics [32, p30]
- (58) **SERGO.** SERGO is an Afrikaans acronym for Centre for Computer Aided Instruction (Sentrum vir Rekenaarsgesteunde Onderrig). SERGO authors

are university mathematicians. The package ensures that learners will not be allowed to proceed to a new concept or technique if they have not proved that they have mastered the work. This is how a proof is given: the learner is confronted with a question which must be answered. If answered correctly, the next question is posed. If it is answered incorrectly, the learner is given another opportunity to answer the question. A correct answer to the second attempt allows the learner to proceed. If the question is again answered incorrectly, the correct answer is displayed on the screen, and before the learner is permitted to attempt the next question, he must correctly answer ten questions, in a row, similar to the one answered incorrectly. The system does not replace the instructor. It serves as a support to normal teaching [2, p279 and 11].

- (59) **SIGAD**. This computer graphics system has been developed to help in the design of three-dimensional objects such as houses and cars. Sigad is an interactive graphics system for learning three-dimensional geometry and for learning how to visualise in three dimensions. It allows learners to define, for example, a plane relative to three points or relative to a point and a line or relative to two lines. It provides learners with information about positions of current figures. It also helps learners to learn how three-dimensional drawings can be represented. It is a very useful tool for visualising drawings which involve points, vectors, segments, lines and planes [33, p129].
- (60) **Singos**. When the programme starts, a circle with a horizontal radius is displayed, and a box in the top-left corner of the screen indicates that the value of the angle is currently zero. If the learner enters an angle in degrees, the radius rotates slowly until it reaches a position corresponding to that angle. As it rotates, the number in the box indicating the angle changes continuously from zero to the angle specified. In this way, learners can gain familiarity with the way in which different positions of the radius arm represent different positive or negative angles. This programme provides a computerised set of trigonometric tables. It has the advantage that the value of the function is accompanied by a visual explanation. It can be used

to discuss signs of trigonometrical functions of angles that are not acute. For example, to demonstrate the fact that as an angle close to zero changes, its sine changes faster than its cosine [4. p44].

- (61) **Solve.** The learner enters a linear equation which is displayed on the screen. The learner can then transform the equation by performing the same operation on the expression on either side of the equal sign. Thus, this programme carries out manipulation for the learner. This leaves instructors and learners free to concentrate on the difficult task of deciding what the appropriate next step in solving the equation should be and the time available for experimenting with suggestions made by learners. Such experimenting may give learners considerable insight into the reasons why one operation is more appropriate than another as the next step in solving the equation. Learners can also discover whether their suggestions are appropriate or inappropriate by seeing the computer carrying them out [4. p45].
- (62) **Solving quadratic equations.** The screen design, sequence and pedagogy of this package has been rated highly by learners. The package has the following features: Each step towards solving quadratic equations is built as a concept. For each concept there is an explanation screen and a sequence of questions leading to the solution of the problem. Learners may either choose to select the problem generated by the computer or define their own problem. At each step learners get feedback and assistance if responses are not correct.
- (63) **Songwriter.** This package helps learners to learn and understand music by composing and editing music scores. The package plays music in tune through the external speaker as the screen scrolls the notes like a player piano. Learners are able to adjust the musical notes by adding, subtracting, multiplying, or dividing. The package has been carefully designed to be easy to use and to be fun to play [20. p295].
- (64) **South dakota.** This package calls for learners to use mathematical skills to make farm management decisions. Learners must decide which crops to

plant and in what quantities in order to make a maximum profit. Learners also choose whether to hire extra workers at an additional cost in order to increase production.

- (65) **Spring and string.** This package consists of specialised programmes for two applications of differential equations. Both programmes have very good graphics. The spring programme allows the input of values for the mass, damping constant, spring constant, magnitude and period of a sine wave forcing function, and initial position and velocity. The programme displays the analytic solution and shows the spring moving up and down while the graph is traced out. The string programme has three options. The first option displays the partial sums of the Fourier series solution for one of five problems at a specified time. The second displays the Fourier series solution at various times. Learners control the number of terms used and the number of points displayed. The third option allows the d'Alembert solution [28, p71].
- (66) **Stuckybear word problems.** In this package, each correct answer is followed by a short animation sequence in which the tally of correct responses is increased by one. In one of the cartoons the drips of an ice-cream cone activates the correct response counter. The combination of extrinsic reinforcement (from the animated packages) and intrinsic reinforcement (from solving interesting problems) motivates learners to work at the solution of word problems.
- (67) **Survival math.** This package is a set of four simulations. All of these simulations are practical, well designed and challenging. The activities are an excellent way to measure learners' abilities to use their arithmetic skills and to work on tasks as a group. Because of the on-screen documentation and the worksheets provided, the first two simulations can be attempted by capable learners without much assistance. The latter two, however, will require instructor's organisation and coaching. Parameters are changed each time packages are run so that they can be used several times.

- (68) **TK solver.** Learners using this software begin by listing different elements of the problem on different screens. One is for rules, one for variables, one for dimensional relationship units and a number of others for specialised applications. Once these screens have been filled out, learners can ask the programme to solve the problem for any variable. The package sorts through equations provided by the learner to determine which are appropriate and in which order they must be used to provide the variable value needed to solve the next equation [20, p71].
- (69) **Turtle graphics.** Turtle graphics is a graphic component that is found in LOGO, a computer language. Turtle graphics allows the learner to move a cursor, often called a turtle, around the screen. The geometry based on this tool is called turtle geometry. The turtle can respond to a few simple commands: Forward moves the turtle, in the direction it is facing, a number of units. Right rotates it in its place, clockwise moves it some number of degrees. Back and left cause opposite movements. The number that goes with a command to specify how much to move is called a command input. In describing the effects of these operations, we say that forward and back change the turtle's position. That is the point on the plane where the turtle is located. Right and left change the turtle's heading. That is the direction in which the turtle is facing. The turtle can leave a trace of the places it has been. This is controlled by the commands pen-up and pen-down. When the pen is down, the turtle draws lines [1, p3 and 20, p96].
- (70) **Visicalc.** This is a software for numerical analysis. It permits instructors and learners to explore problems while employing traditional mathematical skills and concepts. Instructors can introduce this software in topics such as functions, equations and general problem solving. The disadvantage of this software is that learners arrive at answers without even seeing the formulae that provide those answers. It is as if learners are not solving the problem, but the software is [20, p70].
- (71) **Wizard.** In this package, learners choose input numbers and the computer

provides output numbers. Whenever learners have tried enough input numbers to think they know the pattern, they may make a guess. The computer then gives four numbers as input numbers and has learners predict output numbers. Learners who answer correctly are rewarded with a treasure and allowed to continue the treasure hunt. Learners who respond incorrectly are allowed to try additional input numbers or a different problem. Trying a different problem allows learners, frustrated by a particular problem, to move on [32, p47].

(72) **Word-problem tutor.** The package is unique in that word problems are presented in two parts. Learners are required to choose the correct operation required for the problem. After the correct response, they are asked to complete the computation. After two incorrect answers, the computer gives learners the correct response with explanation. One strength of this package is that it provides a mixture of problems involving each of the four basic operations so that they can decide which operation is correct for each situation. Each disk contains eighty questions and has four levels of difficulty. Branching moves learners to higher or lower levels or suggests they repeat the lesson depending on the score attained [32, p114].