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FURTHER REFINEMENTS AND A NEW EFFICIENT SOLUTION
OF A NOVEL MODEL FOR PREDICTING INDOOR CLIMATE

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FURTHER REFINEMENTS AND A NEW
EFFICIENT SOLUTION OF A NOVEL MODEL
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CHRISTOFFEL LOMBARD

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Promotors:

Prof. E.H. Mathews

Dr. J.D. Wentzel

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ABSTRACT

In the first two chapters of this thesis, the novel method, developed by Mathews and Richards¹, for predicting the thermal performance of buildings is introduced. The further enhancement and theoretical clarification of this method is the objective of this thesis. The method is based on a very simple thermo-flow network which models only the most important aspects of heat-flow in buildings. While Mathews and Richards based their network on analysis of the primary aspects of heat-flow in buildings, this thesis derives the simplified model by reduction from a comprehensive model. In this way, the assumptions and limitations is illuminated and the theoretical foundation of the method can be established. As a result of the investigation, a new simplified model with certain theoretical benefits is suggested. In later chapters, the method is extended and refined. Also, a new calculation procedure for finding solutions of the model is presented. In particular the method is extended to include multi-zone heat-flow, structural storage- and variable thermal systems. The new solution method is powerful, simple and efficient. This thesis contributes to the establishment of a viable tool for thermal analysis of buildings.

¹See reference [10] on page 13.

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Appendix A – Synopses

Appendix B – Papers to which this study contributed.

C. Lombard, E.H. Mathews – Efficient Steady State Solution of a Time Variable RC Network, for Building Thermal Analysis.

E.H. Mathews, P.G. Rousseau, P.G. Richards, C. Lombard – A Procedure to Estimate the Effective Heat Storage Capability of a Building.

SUMMARY of the CHAPTERS

In the **first chapter** of this thesis an overview of the current status and future prospects of building thermal analysis is given. Within this perspective; the objectives, scope and contributions of this study is presented.

In the **second chapter**, the origins of the method of building thermal analysis of Mathews and Richards, it's current status and future prospects are discussed, with emphasis on the basic assumptions on which the method is based. The simplified mathematical model of heat flow in buildings, which forms the foundation, is derived from a more comprehensive theory. The original derivation of the method by Mathews and his co-workers was based on apt reasoning about the critical aspects of heat-flow in buildings. It was not derived from first principles. In this thesis, it is attempted to derive this model by reduction from a more extensive and refined model. A relatively simple structure is used in the discussion, in an attempt to reduce the many complications of building thermo-flow to the essentials. The most important aspect of this discussion is the modelling of the storage effect of the massive structures of the building, since this aspect was previously most heavily based on empirical considerations. The validity of the various simplifying assumptions are investigated. Some possible refinements of the model are also pointed out.

In **chapter three**, the model is extended to include structural storage. The practical application of structural storage systems in buildings is first briefly touched. It is shown that the usual definition of the convective coefficient, for heat transfer from and to flat surfaces, can not be applied since the thermal model of Mathews and Richards makes no provision for surface temperatures. This problem is easily circumvented by defining the

convective coefficient with respect to the bulk wall temperature. With this definition of the heat transfer coefficient the inclusion of structural storage in the model is straightforward. This chapter concludes with a demonstration of the viability of structural cooling in two actual buildings, an office block and a shop.

In **chapter four**, a new method for extending the thermal model to cater for inter-zone heat flow is proposed. The main idea is to first obtain the thermal response of each individual zone, under the assumption that partitions between zones are adiabatic. This can be done with the present method of analysis without any alterations. In a second phase of the calculation, the various zones are combined, the actual heat flow between zones are determined and the final temperatures or loads in each zone are evaluated. This procedure forms a very natural extension of the present single zone method. It can be easily implemented in the case of time-invariant thermo-systems, but it requires the inversion of a complex matrix, the size of which is given by the number of zones. The method is verified by comparing measured temperatures in a two-zone experimental setup with predicted values.

In the introductory sections of **chapter five**, the solution of the thermal network for buildings is discussed from a systems point of view. In particular, it is shown that the various forcing functions can be effectively combined into one. This leads to considerable simplification in the solution of the network and facilitates easy introduction of new forcing functions. In this chapter, a new method of solution for the equivalent thermal network is given. This new method extends the applicability of the model to time variable ventilation of buildings, time variable shell conductance etc. The new calculation method is very efficient and simple. It is based on a first order numerical algorithm but it can be easily extended to higher order numerical methods. The periodicity of the parameters and forcing functions are exploited to derive a closed form solution for the initial value, so that the lengthy period of integration, which is usually

required to get rid of initial transients, is circumvented. In addition to the numerical method, various other methods for solving the time variable network are discussed. In particular, Fourier methods are also treated in some depth and an alternative method, based on Fourier series expansions, is discussed.

The last chapter, **chapter six**, concludes by summarizing the main results–, suggestions for the future–, and utility of this thesis.

NOTATION, ABBREVIATIONS and NOMENCLATURE

Notation

Generally, the following conventions are adhered to, although exceptions exist:

- (...) equation numbers
- [...] references (given at the end of each chapter), or units in SI system

A decimal point is used and not a comma.

Variables are written in *italics*.

Boldface symbols indicate a vector or a matrix.

Operators are written in *script* font.

Often used symbols [SI units]

- C Thermal capacitance [kJ/K]
- H System transfer function.
- h Heat transfer coefficients in [W/m²·K].
- q Heat flow in [W], sometimes per unit area [W/m²].
- Q Building loads in [W].
- R Thermal resistance [K/W].
- T Usually temperature [°C], sometimes the period of a periodic function [h].
- t Time [h].
- x A spatial dimension [m], or system input function.
- y System output function.

A list of symbols is provided at the end of each chapter.

Abbreviations

- ASHRAE American Society of Heating, Refrigerating and Air-Conditioning Engineers.
- CENT Centre for Experimental and Numerical Thermo-Flow.
- CIBS Chartered Institute Building Services.

CSIR	Council for Scientific and Industrial Research.
HVAC	Heating Ventilating and Air-Conditioning.
LGI	Laboratory for Advanced Engineering.
RC	Resistor-Capacitor lumped model.
RCR	Lumped section consisting of 2 resistors and a capacitor as in figure 2.14 page 67, also called T-section.
TTC	Thermal Time-Constant.
TTTCB	Total Thermal Time-Constant.
UP	University of Pretoria.

Nomenclature

cascade matrix	A two-port (see below) model as in figure 2.12 page 64. Also called a cascade matrix.
load	Heating or cooling load to be supplied by a thermal control system, e.g. air-conditioning cooling load.
model of Mathews and Richards	Refers to the model presented by Mathews and Richards in reference [10] of chapter 2.
network	A graphic model of thermo-flow in buildings (thermal network), usually in terms of thermal resistances and capacitances.
shell	The structural part of a building which separates interior and exterior.
thermal response	The temperature in a building and heat flow into-and out of a building.
transmission matrix	See 'cascade matrix' above.
T-section	See 'RCR' above, also RCR T-sections, RCR-section etc.
two-port	A black box model of a particular structure with two boundaries or ports, e.g. a wall where the two flat surfaces form the boundaries. A two-port model

	usually employs a 2x2 matrix as a mathematical model of the structure.
partition	Refers to a partition between two thermal zones in a building, as opposed to a wall which divides interior and exterior.
zone	A region in building which is thermally well connected so that the temperature in the region will always be uniform.