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CHAPTER 1: INTRODUCTION

1.1. BACKGROUND

Heat generating devices, such as high power electronic equipments and heat exchangers are widely applicable in engineering fields such as the automobile industry, power system, heating and air conditioning, chemical engineering, electronic chip cooling, and in the aerospace and nuclear energy sectors. Heat generation can cause overheating problems and thermal stresses and may leads to system failure. The removal of heat from these devices has been a critical challenge to thermal design engineers and researchers. Heat generating devices are designed in such a way as to optimise the structural geometry by packing and arranging array of cooling channels into given and available volume constraint without exceeding the allowable temperature limit specified by the manufacturers. For example, in the design of an electronic package, it is desirable to pack as many of the electronic chips as possible into in a fixed volume. This translates into the maximisation of heat transfer density or the minimisation of overall global thermal resistance, which is a measure of the thermal performance of the cooling devices.

Heat transfer in heat generating devices occurs by conjugate heat conduction and forced convection. Heat conduction is the transfer of thermal energy from more energetic particles to less energetic counterparts. This is largely influenced by the thermal conductivity of the material. Heat transfer by convection is made possible by the movement of fluid molecules. When this movement is facilitated by external forces such as fans and pumps, the term, ‘forced convection’ is used. The convective heat transfer is influenced by the heat transfer coefficient. The performance of cooling system is measured by its thermal resistance *R*, which can be expressed for conductive and convective heat transfer as shown in Equations (1.1) and (1.2) respectively:

$$R_{\text{cond}} = \frac{1}{kA} \quad (1.1)$$

$$R_{\text{conv}} = \frac{1}{hA} \quad (1.2)$$

where *h* and *k* are the convective heat transfer coefficient and thermal conductivity respectively. These factors greatly influence the overall performance of the system as also expressed in equation (1.3):

$$Nu = \frac{h\xi}{k} \quad (1.3)$$

where ξ is a characteristic length scale which could be external or internal length scale

However, in modern heat transfer, the new trend for thermal performance is shape and geometric optimisation. Constructal theory and design [1, 2] have emerged as an evolutionary design philosophy for developing flow architectures that offer greater flow access and system performance. The constructal method - constructal design - is based on the principle of objective and constraints. The approach is summarised by the constructal law [1, 2]: *For a finite-size system to persist in time (to live), it must* *evolve in such a way that it provides easier access to the imposed (global) currents that flow through it*.

In all problems of Constructal design, especially, engineering analyses, the characteristic dimension ξ or A of the configuration (shape, size) is unknown prior to optimisation process and this must be determined optimally for the overall thermal performance of the system. But, from equation (1.1) – (1.3) to minimise thermal resistance or maximise thermal conductance, ξ needs to be increased subject to the global volume constraint. However, we cannot keep increasing this characteristic dimension indefinitely because the global volume constraint is fixed. Constructal theory [2] suggests that we must find the correct optimal values of ξ or cross-section area that provide the direction of greater flow access or less global flow resistances in order to minimise global thermal resistance. That is, the number of the lengths necessary to draw the domain minus number of constraints gives the number of degrees of freedom.

The application of this evolutionary design approach to the discovery of internal heat exchanger started with Bejan and Sciubba [3]. These researchers obtained the design rule for spacing an array of parallel plates to channels so that the heat transfer density of a volume filled with heat generating components was maximum. The spacing was determined by using the intersection of asymptotes method.

Bejan and his co-researchers later applied the theory to the conductive cooling of electronics with internal heat generation and other convective heat transfer optimisations [4-11]. These studies played a significant and important role in the extension and application of Constructal theory and design to problems in engineering, other branches of science and even in the humanities [12, 13]. In addition, Constructal theory has been employed to describe deterministically the generation of shapes in nature [1].

From Equations (1.1) to (1.3), it is obvious that the geometry (size and shape) of the cooling structure plays a crucial role in the performance of its cooling channels. Other factors that influence the cooling performance of a heat-generating device include materials selected. In case constructal law (in relation to thermal design and management), geometry in terms of size and shape is the most important factor that the designers and engineers can control. In general, for optimum thermal performance, multivariable optimisations of the various geometric parameters of the cooling structures must be considered. The fact is that the impact of a single geometric parameter cannot be generalised without considering its consequence on the other parameters. For example, increasing the channel aspect ratio will generally improve the overall thermal performance of the system for a fixed-flow rate, [14, 15].

Analytical solutions for the optimisation of cooling channels always prove to excel in the search for solutions to specific design problems [16-18]. Although the analytical optimisation may be limited and not perfect due to the various assumptions made when using this procedure, it remains a powerful method.

The advent of computational fluid dynamics (CFD) simulations has made numerical optimisation more interesting and robust in designing near-optimal solutions for various applications of cooling devices [19-21]. Coupled with the advent of supercomputers over the last few decades, CFD has proven to give more accurate predictions for flow velocity, temperature and various thermodynamic properties.

Furthermore, optimisations by experimentation [22-29] are performed in heat transfer analyses to investigate the effects of various geometric parameters on the thermal performance of heat exchangers and cooling devices. However, experimental optimisation is very expensive and time consuming compared to analytical and numerical optimisations. Unlike numerical studies, many design variables cannot be varied simultaneously to produce the global optimisation that gives the desired objective function in experimental studies. Another unique advantage of numerical analysis over experimental analysis is that it provides more quantitative insight into the flow and heat transfer process, especially at the micro-scale level. It is often difficult to use conventional measurement techniques with the micro-scale channels to extract data despite the validity of the conventional governing differential equations for fluid flow and heat transfer analyses and continuum flow assumption in the micro-scale devices.

Coupling CFD with the mathematical algorithm for optimisation has proved to be capable of producing optimal designs within reasonable computational times. Modern high-speed computers have made it possible to automate the optimisation process by integrating and coupling the CFD software package with the optimisation algorithm for the optimal modification of various design parameters. Hence, numerical optimisation is a systematic way of searching for an optimal design based on certain specified criteria. The optimisation algorithm could be integrated into numerical modelling, which would enable optimal design to be achieved for the overall thermal performance of the system.

1.2. MOTIVATION

The Constructal theory by Bejan [1, 2, 30] which is aimed at the optimisation of shapes and structure has been the major inspiration for this research. This is because the advent of compact high density components required the investigation of novel and modern techniques for removing heat from heat generating devices for an optimal performance at minimised cost. The constructal theory provided the ideas on how to optimise all the constraints.

1.3. JUSTIFICATION (THE NEED FOR THIS STUDY)

From the literature, the optimisation of the global performance of heat-exchanging devices is still a difficult task because of different design parameters involved. There is a need for further and extensive research into improving cooling channel materials so as to ensure high performance, more efficient, more accurate, long lasting and low cost heat exchanging. Since the design variables are mutually interdependent parameters, they cannot be optimised individually to achieve the near-optimal solution of the global performance.

Bejan *et al*. [31] argue that "when a system consists of several components, the overall system should be optimised, since optimisation of components individually does not guarantee an optimum overall system". This argument is supported by Ordonez and Bejan [32] who suggested that an entire system can be conceived from the beginning as a system designed to perform certain global objectives optimally, rather than as an assembly of already existing parts. Therefore, we believe that there is a need to introduce a mathematical algorithm that can be coupled with CFD in the optimisation process in order to achieve a near-optimal solution of the global performance.

Most of the currently applied methods are time-consuming, expensive and do not achieve a near-optimal solution. The advance in technology improvement and especially the advent of CFD has brought about great improvement, which has made it possible to analyse of difficult design variables with a satisfactory design. However, the approach is to assume that there must be optimal design variables at which the system will perform best. Thus, a multidimensional and robust gradient-based optimisation algorithm that does not require an explicit line search is introduced and incorporated into the finite volume solver and grid (geometry and mesh) generation package. Its aim is to search and identify the optimal design variables at which the system will perform optimally. The combination of CFD and mathematical optimisation algorithms can produce unexpected improvement in the design optimisation process.

The advantage of our mathematical optimisation algorithm – which will be discussed in detail later – is that it can be coupled with the CFD simulation and grid generation packages in a MATLAB environment. This is done in such a way that it captures the numerical objective function in the simulation that is not available analytically, and searches for and identifies the optimal design variables that correspond to the objectives function.

1.4. AIM OF THE PRESENT RESEARCH

The aim of the current research is to carry out theoretical and numerical optimisation studies on conjugate heat transfer in cooling channels with variable cross-sections based on constructal theory and design. Also, we propose, develop and implement a design optimisation methodology that is founded on mathematical gradient-based techniques that will allow the optimisation of a peak temperature profile for the entire system. This will be achieved by coupling computational fluid dynamics and a mathematical algorithm. The design parameters become design variables and the optimisation process with respect with these design variables is automatically taken into account by the optimisation algorithm. The approximated peak temperature profile obtained from the computational fluid dynamics simulation is used in the optimisation process for the computation of an objective function.

1.5. OBJECTIVES OF THE PRESENT RESEARCH

The objective of this study is to geometrically optimise the cooling structure in such a way that the global thermal resistance or peak temperature between the volume and the cooling fluid is minimised.

1.6. SCOPE OF THE STUDY

In this thesis, an optimal design approach is employed to computationally and efficiently optimise the heat transfer capabilities of cooling channels of different cross-sectional shapes by means of theoretical analysis, computational fluid dynamics and numerical optimisation algorithm. Five different cross-sectional shapes of cooling channels are introduced, namely cylindrical, square, isosceles, equilateral triangular and rectangular cooling channels.

Our models can be used for the entire laminar flow range, materials of different thermal conductivities and any fluid having Prandtl numbers ≥ 0.71. However, water is used throughout the research as cooling fluid. Also, the fluid flow and heat transfer are steady-state, three-dimensional, conjugate and constant thermo-physical properties.

The models are designed for micro-scale devices because of recent developments in large-scale Micro-Electro Mechanical Systems (MEMS) with low-cost and small-space advantages, as well as high heat dissipation ability (e.g compact heat exchangers and micro-channel heat sinks). However, the models are also applicable to all the heat exchange devices at that mini- and macro-scale level, because convectional governing equations are used (also valid for micro, mini and macro-scale devices)

This research takes a comprehensive look at the optimisation of heat transfer such as peak temperature (global thermal resistance) at the hotspot of heat-generating devices that experience heating and need to be cooled and the cooling channels. All the design variables are subjected to various constraints which are numerically approximated by the automated optimisation algorithm.

1.7. RESEARCH METHODOLOGY

The objectives of this research were accomplished by both numerical and theoretical analysis. The analytical solution was based on the intersection of asymptotes method and scale analysis. The numerical simulations that were guided by the analytical solutions give a comprehensive explanation of the global thermal behaviour of the problem. The numerical method involves the modelling and discretisation of the computational domain, the solving of conjugate heat transfer by using necessary governing equations, and the processing of results. An automated mathematical optimisation algorithm that uses numerically approximated functions is employed by coupling it to the commercial Computational fluid dynamic software – FLUENT – to search the optimal design variables that minimised thermal resistance. The detailed procedures of both numerical and theoretical analysis are discussed in the subsequent chapters.

1.8. MATERIAL SELECTION

Material selection is a very important part of the optimised design of cooling devices design as it provides a good balance between thermal properties, weight and material cost. For example, diamonds prove to be best suited for any cooling devices design but their use in heat-exchanging devices design is highly impractical from an economic point of view. Materials such as aluminium and copper dominate the materials commonly used in heat exchangers (recently silicon was introduced for use in micro-channel heat sinks), as they provide a good balance of the thermal conductivity-to-density ratio. The effect of material properties on the minimum thermal resistance will be shown in the later part of the thesis.

1.9. ORGANISATION OF THE THESIS

This thesis consists of ten chapters and each chapter is divided into sections and subsections. These provide a detailed description of the subject matter and make for easy reading and referencing. The chapters of the thesis are itemised below:

- Chapter One introduces the constructal theory and presents the motivation, justification and background of the study.

- Chapter Two provides literature reviews on the subject of Constructal theory with the focus on engineering applications especially heat transfer analysis.

- Chapter Three contains a review of the numerical model employed for the analysis, which is explained with the computational domain used for the simulations of cooling channels and vascularised material. The mass, momentum and energy conservation equations governing the transport of mass and heat are discussed. The iterative method of coupling these governing differential equations is also shown.

- Chapter Four deals with the subject of numerical optimisation and focuses on the operation of the DYNAMIC-Q algorithm. The underlying principles and governing equations of the optimisation algorithm are discussed.

- Chapter Five presents and develops a theoretical solution to the optimal channel geometry of parallel channels of different cross-sectional shapes that penetrate and cool a volume with uniformly distributed internal heat generation. This is based on an application of the intersection of asymptotes method and scale analysis to provide proof of the existence of an optimal geometry that minimises the peak temperature and global thermal resistance in a heat generation volume with an array of channels of different shapes. The geometric configurations of the cooling channels optimised are cylindrical, square, rectangular and triangular (isosceles right and equilateral).

- Chapter Six investigates further analytical solutions by implementing the numerical modelling and mathematical optimisation methodology developed in Chapters Three and Four. These are aimed at simulating and optimising the geometric configurations of conjugate heat transfer in cooling channels with different cross-sectional shapes and uniformly distributed internal heat generation. The steps involved in coupling the optimisation algorithm with FLUENT (a commercial computational fluid dynamic software) are also shown.

- Chapter Seven develops (numerically and analytically) the geometric optimisation of parallel cooling channels in the forced convection of a vascularised material with the localised self-cooling property subjected to a heat flux on one side in such a way that the peak temperature is minimised at every point in the solid body. The analytical solution is also based on the application of the intersection of asymptotes method and scale analysis to provide an optimal geometry that minimises the peak temperature and global thermal resistance. The effect of material properties on the minimum thermal resistance and optimised internal configuration is also shown.

- Chapter Eight presents a three-dimensional geometric optimisation of conjugate cooling channels in forced convection and internal heat generation within the solid, for an array of circular cooling channels and different flow orientations based on constructal theory. Three flow orientations were studied: the study started with an array of channels with parallel flow; followed by an array of channels in which the flow of every second row is in counter direction, and lastly an array of channels in which the flow in every channel is opposite to the previous channel. The configuration and flow orientation were optimised in such a way that the peak temperature and global thermal resistance were minimised, subject to the constraint of fixed global volume of solid material.

- Chapter Nine provides a general summary of the findings of the study. It also presents the conclusions and contributions, as well as recommendations for future work.

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CHAPTER 2: LITERATURE REVIEW

2.1. INTRODUCTION

This chapter deals with the literature review relevant to this thesis and gives an insight into the constructal theory and design and the effect of geometry on the heat transfer capabilities of cooling channels of heat-generating devices. Continuous literature studies and reviews were carried out during the research work.

2.2. CONSTRUCTAL THEORY

The application of this evolutionary design approach to the discovery of internal heat exchanger started with Bejan and Sciubba [3], who proposed the design rule for the spacing of an array of parallel plates to channels in such a way that the heat transfer density of a volume filled with heat-generating components was maximal. The spacing was determined by using intersection of asymptotes method. This philosophy was applied to all the facets of flow system design, from biology and physics, to engineering and social organisation [33-45].

In nature, water always takes the path of least flow resistance when of navigating in the river basin [33]. Thermodynamically, every system exhibits a level of imperfection due to entropy generation which leads to the degradation of performance of the system [34]. However, a system must adjust itself to operate maximally by optimising the process and geometric configuration of the system to reduce the entropy.

In medicine [35], this physical law can also be applied to the treatment of cancer. The spreading of cancer can be controlled by maintaining the temperature field of the unaffected tissues in the neighbourhood of the turmoil below the temperature at which the cancer virus can survive. In the business world [36,37], constructal theory shows that transportation costs can be minimised by optimising the transportation routes of goods and products from one area to another in a dendritic form so as to travel shortest and easiest distance.

In heat transfer [38], the peak temperature must be minimised at every hot spot of a system. This promotes better thermal performance and avoids thermal stress by optimising shape and geometry.

In fluid mechanics [39], Bejan proposed that the flows pressure drop can be minimised by optimising the internal flow architecture of different ducts with variable cross-sections and by using constructal theory.

In academia [40-41], the constructal law was used to optimise the hierarchal rankings of universities in the global flow of knowledge. Also in military defence, the constructal law was used to provide insight into and information on the optimisation of warfare tactics and strategy [42].

In sport industry [43- 45], constructal law has been used to optimised and predict the fastest men/women in athletics and swimming.

Other area of the applications of constructal law in the humanities, natural sciences and social sciences can also be found in [46-50].

The recent comment by Meyer [51] on Bejan and Lorente's work [52] on constructal theory and those of other researchers [53-59] shows that the application of constructal law in human life, nature and all fields of educational design is a wide road to future progress.

In this thesis our focus is on the first engineering application of constructal theory, which is the optimisation shape and structure for heat transfer and fluid flow [60-71]. The advantage of constructal law in the engineering field is that the flow architecture is not assumed in advance, but it is the consequence of allowing the structure to morph [72]. The applications of this theory have been reviewed recently by Bejan and Lorente [12], They argue that under certain global constraints, the best architecture of a flow system can be archived as the one that gives little global flow resistances, or allows high global flow access. In other words, the shapes of the channels and unit structure that is subject to global constraint are allowed to morph. The development of heat exchangers and multi-scale architecture by constructal theory was also, reviewed by Reis [73] and Fan and Luo [74].

2.3. HEAT TRANSFER IN COOLING CHANNELS

2.3.1. Theoretical analysis

Bau [75], provided an analytical model solution that minimises thermal resistance by reducing maximum heat surface temperature and temperature gradient. He did this by optimising the cross-sectional dimension of the rectangular conduit of micro heat exchangers in terms of uniform and non-uniform width as a function of the axial coordinate of the conduit. The analytical model for a non-uniform conduit proved to be more effective in reducing the maximum surface temperature compared to the model with a uniform cross section of the conduits.

Yilmaz *et al.* [76] analytically studied the optimum shape and dimensions for convective heat transfer of laminar flow at constant wall temperatures for ducts with parallel plate, circular, square and equilateral triangle geometries. Approximate equations were derived in the form of maximum dimensionless heat flux and optimum dimensionless hydraulic diameter in terms of the duct shape factors and the Prandlt number (*Pr*).

Yang *et al* [77] presented a thermal optimization of a stack of printed circuit board using an entropy generation minimisation (EGM) method. They provided a dimensionless optimal channel spacing correlation in terms of the Reynolds number (Re) and height-to-width ratio.

Bejan and Fautrelle [78] maximised the heat transfer density in a multiscale structure filled by multiple length scale plates that generate heat. They inserted additional parallel plates and optimised the spacing in the flow structure.

Knight *et al*. [79-81] presented the governing equations for fluid dynamic and heat transfer in a generalised, dimensionless form, along with a geometrical relationship that can be used to determine the dimensions of a rectangular micro-channel heat sink that result in the minimisation of thermal resistance and they later performed an experiment to verify and validate their analytical optimisation scheme.

Muzychka [17] used this theory and Bejan's intersection of asymptotes method to present an analytical optimisation of circular and non-circular cooling channel geometries. He also studied and analysed the optimisation of microtube heat sinks and heat exchangers for maximum thermal heat transfer by using a multiscale design approach based on Constructal theory [18]. In his analysis, he was able to show that through the use of interstitial microtubes, the maximum heat transfer rate density for an array of circular tubes could be increased.

Bejan and Fautrelle [68] maximised the heat transfer density in a multiscale structure filled by multiple length scale plates that generate heat. They inserted additional parallel plates and optimised the spacing in the flow structure.

Knupp et al. [82] presented an analytical approach towards conjugated conduction-convection heat transfer problems by proposing a single domain formulation for modeling both the fluid stream and the channel wall regions. This made use of coefficients represented as space variable functions with abrupt transitions occurring at the fluid-wall interface region.

2.3.2. Numerical analysis

Kim and Anand [83] performed a two-dimensional numerical simulation of laminar developing flow and heat transfer between a series of parallel plates with surface-mounted discrete heat sources.

Ryu *et al.* [84] conducted a three-dimensional numerical analysis and presented an optimisation techniques called random search techniques to investigate the effect of thermal entrance on the thermal performance of a micro-channel heat sink and to predict the optimal design variables that minimise the thermal resistance of microchannel. They noted that thermal entrance effects could not be ignored especially when the working fluid was water, due to the thin thermal boundary layer in the developing region.

Fedorov and Viskanta [85] as well as Qu and Mudawar [86] used conventional Navier – Stokes and energy equations to analyse (numerically) a three-dimensional fluid flow and conjugate heat transfer characteristic in a rectangular silicon micro-channel heat sink, using water as coolant. The effect of the Reynolds number and thermal conductivity of the solid substrate were also considered. Their results in terms of frictional coefficient and thermal resistant as a function of the Reynolds number were compared and found to be in good agreement the with analytical solution of Shah and London [87] and the experimental solution of Kawano *et al*. [88]. Their models showed that the forced convective water-cooled micro-channnel heat sink has superior potential for application in the thermal management of the electronics package.

Ambatipudi and Rahman [89] performed numerical simulations for conjugate heat transfer in micro-channel heat sinks. The channel depth, channel width, number of channels and flow rate were used as design variables. They found that the performance of a heat sink can be improved by increasing the number of channels in the heat sink and the flow rate through the heat sink.

Chen [90] further investigated this influence mainly by studying the aspect ratio and effective thermal conductivity in forced convection heat transfer within a micro-channel. As the aspect ratio increased, fluid temperature and the overall Nusselt number increased. However, the influence of the channel aspect ratio on the temperature of the solid was minimal.

Da Silva *et al.* [91, 92] optimised the space allocation on a surface and wall occupied by discrete heat sources with a given heat generation rate by natural and forced convection. They used the constructal theory to minimise the temperature of the hot spot on the wall.

Bello-Ochende and Bejan [93] studied and numerically extended the initial work of Bejan and Fautrelle [68], based on the concept of constructal theory. Also, Bello-Ochende *et al.* [94-97] conducted a three-dimensional optimisation of heat sinks and cooling channels with heat flux. They used scale analysis and the intersection of asymptotes method (based on constructal theory) to investigate and predict the design and optimisation of the geometric configurations of the cooling channels.

Furthermore, Bello-Ochende *et al*. [98] numerically reported a new design concept to improve the constructal design for increasing the heat transfer rate density by using wrinkled entrance regions in square ducts with laminar-forced convection. The numerical simulations results showed the effects of the dimensionless pressure drop on the optimised configurations, as well as an enhanced heat transfer rate density. Bello-Ochende *et al.* [99] optimised the internal configurations of rectangular micro-channels heat sink subjected to high heat flux. Other works of Bello-Ochende and his co-researchers using Constructal theory in engineering application can be found in [100-103].

Rajkumar, *et al.* [104] presented a numerical solution of laminar conjugate natural convection in a vertical channel containing a short planar heat generating element by using a hybrid finite element/finite volume method together with a restricted domain approach. They also provided a correlation for dimensionless temperature as a function of dimensionless volumetric heat generation.

Mohammed *et al.* [105] conducted numerical simulations to solve the three-dimensional steady and conjugate heat transfer governing equations by using the Finite-Volume Method (FVM) to evaluate the effect of studies for different channel shapes on the performance of MCHS with the same cross-section.

Rocha *et al.* [106] and Biserni *et al.* [107] applied the constructal theory to optimise the geometry of C- and H-shaped cavities respectively that intrude into a solid conducting wall in order to minimise the thermal resistance between the solid and the cavities. Also, Biserni *et al*. [108] minimised the thermal resistance between the solid and the cavities by optimising the geometry of inverted fin shapes that also intrude into a solid conducting wall.

Lorenzini and Rocha [109] and, as well as Lorenzini *et al.* [110] used the constructal theory to minimise the thermal resistance between the solid and the cavities by optimising the geometry of isothermal cavities that evolve from T- and Y-shape of a solid conducting wall. Also, Lorenzini et al. [111- 113] optimised the complex assemblies of fins and cavity spaces. They used constructal theory and design to minimise the thermal resistance between the solid conducting wall and the cavities by increasing the degrees of freedom that can lead to improvements in the performance of flow systems.

Salimpour *et al.* [114] studied numerically the optimal scale channels of circular and non-circular configurations to achieve maximum heat transfer density.

Matos *et al.* [115, 116] performed a numerical and experimental analysis of laminar-forced convective heat transfer. Air was used as an external cooling fluid. They conducted a two-dimensional numerical analysis for the geometric optimisation of staggered circular and elliptic tubes [115]. Here, they provided an optimum spacing between rows of tubes in such a way that a maximum rate of heat transfer between the constraint volume and the cooling fluid (free stream) was obtained. As stated, they conducted a three-dimensional numerical and experimental analysis [116] and presented geometric optimisation for staggered arrangements as well as for finned circular and elliptic tubes heat exchangers that were subjected to a fixed-volume constraint so as to maximise the volumetric heat transfer density. In their analysis, they considered three degrees of freedom – (the spacing between rows of tubes, the eccentricity of the tubes, and the fin-to-fin spacing) – as design constraints. The global optimal results with respect to these three degrees of freedom were established and expressed in a dimensionless form. Their result showed that the elliptical tubes have higher global performance.

Ordonez [117] conducted a two-dimensional heat transfer analysis in a heat-generated volume with cylindrical cooling channels and air as the working fluid. In their, Dirker and Meyer [118] investigated a three-dimensional numerical simulation. They considered the optimal thermal behaviour and performance of embedded internal cooling layers in a rectangular heat-generating volume of an electronic structure under two thermal boundary conditions by the use of externally mounted heat sinks. Their results showed that the thermal performance of an embedded layered technique strongly depends on the fraction of the volume occupied by cooling layers and the ratio of the thermal conductivities of the cooling layers and the heat-generating layers. Their technique – when compared with a traditional planar conduction approach – was found to better peak temperatures, because the traditional approach neglected layer thickness

Reis *et al*. [119] optimised the internal configurations of parallel plate and cylindrical channels using Constructal theory to understand the morphology of particle agglomeration and the design of air-cleaning devices.

In fuel cell [120 - 122], Constructal law was used to optimise the internal architecture and external structure of a Proton exchange membrane (PEM) and a single solid oxide (SO) fuel cell so that the power density was maximised.

Obayopo *et al*. [123] studied a three-dimensional numerical optimisation of the effect of operating and internal design parameters of PEM fuel cell performance that would maximise the current density of the fuel cell. They showed that the maximised current density increases as the mass flow rate of the reactant gas increases at an optimal channel depth. Obayopo *et al*. [124] also investigated the effect of pin fins of small hydraulic diameter transversely arranged along the internal flow channel on the reactant gas distribution and pressure drop characteristics of the fuel cell performance. There existed an optimal pin fin clearance ratio, which offered minimum pumping power requirements and that maximum fuel cell performance.

2.4. VASCULARISED SOLID WITH COOLING CHANNELS

Recent advancements in the constructal theory have led to another cooling concept – vascularised material – getting a relative amount of attention. Material with the property of self-healing and self-cooling is becoming increasingly promising in heat transfer analysis [125-130]. The development of vascularisation of materials indicates flow architectures that conduct and circulate fluids at every point within the solid body. This solid body (slab) may be performing or experiencing mechanical functions such as mechanical loading, sensing and morphing. The self-cooling ability of vascularised material to bathe volumetrically at every point of a solid body gave rise to the concept of – ‘smart materials’.

In smart materials, constructal theory and design [1,2] helps with the vascularisation of the smart material structure by morphing the flow architecture configuration to provide easier and greater access of flow through it.

Kim *et al.* [130] theoretically and numerically analysed vascularised materials with heating from one side and coolant forced from the other side for parallel plates and cylindrical channel configurations. They attempted to find the channel configurations that minimised the non-uniform temperature distribution of a vascularised solid body.

Cho *et al.* [131] numerically investigated the flow and thermal behaviour of vascular cooling plate for the volumetric bathing of the smart structures.

Constructal theory applications on the vascularisation revolution of smart materials can also be found in open literature [132-137].

Wang *et al*. [133] investigated systematically optimised channel diameters, the shapes of the loops formed by the closest channels, the shapes of the vascularised slabs of two-dimensional bodies (by employing multi-scale grids) and tree-shaped flow structures that will justify the designs for smart materials with self-healing and self-cooling functionality. Also, Kim *et al.* [134] numerically proposed the use of tree-shaped flow architectures to vascularise a solid body and prevent the overheating that would be caused by intense side heating.

Kim *et al*. [135] also showed the emergence of vascular design in three dimensions of smart material. They focused on a mass flow network to understand how to define and refine vascular designs for a particular application by minimising the total system pressure loss as a measure of the overall goodness of a network design.

Wang *et al.* [136] numerically considered the use of vasculatures to evaluate the volumetric cooling performance of slabs with embedded flow architectures consisting of grids and radial channels. The results showed that grids have lower global flow resistance than radial designs while local junction losses are important.

The constructal theory for optimisation of several components and systems, as well as of components in many engineering applications (such as in the area of energy and power sectors, civil engineering, industrial air-conditioning system, chemical engineering nano-fluid designs) has been extensively discussed and documented in the literature [138-181].

2.5. BEJAN NUMBER

Prior to 1992, a lot of dimensionless numbers named after researchers based on their finding and discovering , such as the Reynolds number (*Re*), which has to do with viscous flow in forced convection, the Rayleigh number (*Ra*) applies to natural convection, and the Prandlt number (*Pr*), – that deals with convective heat transfer.

However, Bejan and Sciubba [3] came up with a new dimensionless number called the dimensionless pressure difference number in an attempt to optimise the spacing of an array of parallel plate to channels so that the heat transfer density of a volume filled with heat generating components is maximised by using the method of intersection of asymptotes method. Bhattacharjee and Grosshandler [182], and Petrescu[183] later called this dimensionless pressure difference number as Bejan number (*Be*).

The Bejan number (*Be*) plays the same role in forced convection that the Rayleigh number (*Ra*) plays in natural convection [184-185]. *Be* is used by many authors [17-19, 64-70] to describe the driving force in forced convective heat transfer.

2.6. FLOW ORIENTATION IN CONJUGATE COOLING CHANNELS

The growing constructal theory literature focuses on convective heat transfer analysis, but does consider the effects of flow orientation. However, Ma *et al.* [186] experimentally investigated the flow resistance and forced convective heat transfer influence for flow orientation in a packed channel that experience heating at the bottom. Wang *et al*. [187] carried out a numerical investigation to study the effect of the orientation of a heat sink on the thermal performance of a PCM-based cooling system.

Other research on the effect of orientation can be seen in [188-191]. Huang *et al*. [188] carried out experimental analysis on natural convection heat transfer for square pin fin heat sinks subject to the influence of orientation. They found out that the downward facing orientation yields the lowest heat transfer coefficient compare to upward and sideward facing orientations. However, the heat transfer coefficients for upward and sideward facing orientations are of comparable magnitude. Zhang *et al*.[189] performed experimental and theoretical studies of the effects of orientation on flow boiling critical heat flux (CHF). The results showed that the interfacial lift-off model is very effective at capturing the overall dependence of CHF on orientation. P. Dutta, and S. Dutta [190] experimentally investigated the heat transfer and frictional loss behavior of turbulent flow in a rectangular channel with isoflux heating from the upper surface for different sizes, positions, and orientations of inclined baffles attached to the heated surface. It was found that both average and local Nusselt numbers are significantly dependent on the baffle plate orientation and the Nusselt number ratio decreases as the plate is placed at a more streamlined position.

2.7. MATHEMATICAL OPTIMISATION ALGORITHM

The history of mathematical optimisation dates back to the 1940s when it was first used as steepest descent for solving very simple problems where functions of many variables are considered [192]. Different types of algorithms have been developed to solve the optimisation problem. These include the genetic algorithm, multiplier methods, surrogate, annealing simulation, Powel algorithm and sequential quadratic programming [192 - 198].

Foli *et al*. [196] analytically and numerically studied geometric optimisation of micro-heat exchangers by using a multi-objective genetic algorithm to maximize the heat transfer rate and minimizing the pressure drop. Husain and Kim [197] numerically performed a multi-objective performance optimisation of microchannel heat sinks by applying surrogate analysis and an evolutionary algorithm to obtain global Pareto-optimal solutions that gave minimum thermal resistance.

Baodong *et al*. [198] numerically presented a multi-objective optimisation design of a micro-channel heat sink by using an adaptive genetic algorithm to simultaneously minimise thermal resistance and pressure drop.

2.8. CONCLUSION

This chapter referred to some of the available literature on constructal theory and design. It also, provide information on the geometric optimisation of cooling channels and various optimisation techniques used in the past to optimally design heat generating devices . The bulk of the published work includes theoretical analysis, and numerical modelling, which are used to generate optimal correlations with regard to the thermal performance of different heat generating devices.

This research provides novel method of optimising the thermal structure and cooling channel flow architecture by theoretical and numerical analyses coupled with optimisation algorithm with the aim of at further improving and enhancing the performance of thermal system.

The self-healing and self-cooling ability of vascularised material to bathe volumetrically at every point of a solid body is becoming increasingly promising in heat transfer analysis. Constructal theory ideally helps in the vascularisation of the smart material structure by morphing the flow architecture configuration to provide easier and greater access of flow through it. The works on vascularisation of the smart material structure found in literature have only been done on flat plates and cylindrical channel configurations. This work extended the knowledge by considering square configuration with the use of a mathematical optimisation algorithm.

Again, the growing constructal theory in literature does not consider the effects of flow orientation on convective heat transfer analysis. Our research on effects of flow orientation is new and novel, especially in cases where an array of channels which in flow of the every second row is in counter flow with its neighbours; and an array of channels in which flow in all the channels are counter flow to one another.

Also, based on the literature, most of the algorithms used in the past did not handled the effect of a numerical noise function during simulation or its effect on the gradient-based optimisation algorithms due to the influence of changes of grid and convergence problem during iterations. This could lead to inaccurate solutions. The ability of our mathematical optimisation algorithm to be coupled with simulation software and to capture data directly from the simulation and its ability to overcome the numerical noise made our solution unique optimal design.