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GEOMETRIC OPTIMISATION OF CONJUGATE HEAT TRANSFER IN COOLING CHANNELS WITH DIFFERENT CROSS-SECTIONAL SHAPES

by

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ABSTRACT

Title : Geometric Optimisation of Conjugate Heat Transfer in Cooling Channels with Different Cross-sectional Shapes

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In modern heat transfer, shape and geometric optimisation are new considerations in the evaluation of thermal performance. In this research, we employed constructal theory and design to present three-dimensional theoretical and numerical solutions of conjugate forced convection heat transfer in heat generating devices with cooling channels of different cross-sectional shapes.

In recent times, geometric configurations of cooling channel have been found to play an important role in thermal performance. Therefore, an efficient ways of optimally designing these cooling channels shapes is required.

Experimentation has been extensively used in the past to understand the behaviour of heat removals from devices. In this research, the shapes of the cooling channels and the configurations of heat-generating devices were analytically and numerically *studied to minimise thermal resistance and thus illustrate cooling performance under various design conditions.*

The cooling channels of five different cross-sectional shapes were studied: Circular, square, rectangular, isosceles right triangular and equilateral triangular. They were uniformly packed and arranged to form larger constructs.

The theoretical analysis is presented and developed using the intersection of asymptotes method. This proves the existence of an optimal geometry of parallel channels of different cross-sectional shapes that penetrate and cool a volume with uniformly distributed internal heat generation and heat flux, thus minimising the global thermal resistance.

A three-dimensional finite volume-based numerical model was used to analyse the heat transfer characteristics of the cross-sectional shapes of various cooling channels. The numerical computational fluid dynamics (CFD) package recently provided a more cost-effective and less time-consuming means of achieving the same objective. However, in order to achieve optimal design solutions using CFD, the thermal designers have to be well experienced and carry out a number of trial-and-error simulations. Unfortunately, this can not always guarantee an accurate optimal design solution. In this thesis a mathematical optimisation algorithm (a leapfrog optimisation program and DYNAMIC-Q algorithm) coupled with numerical CFD was employed and incorporated into the finite volume solver, –FLUENT, and grid (geometry and mesh) generation package, – GAMBIT to search and identify the optimal design variables at which the system would perform optimally for greater efficiency and better accuracy. The algorithm was also specifically designed to handle *constraint problems where the objective and constraint functions were expensive to evaluate.*

The automated process was applied to different design cases of cooling channels shapes. These cooling channels were embedded in a highly conductive solid and the peak temperature was minimised.

The trend and performance of all the cooling channel shapes cases studied were compared analytically and numerically. It was concluded that an optimal design can be achieved with a combination of CFD and mathematical optimisation.

Furthermore, a geometric optimisation of cooling channels in the forced convection of a vascularised material (with a localised self-cooling property subjected to a heat flux) was also considered. A square configuration was studied with different porosities. Analytical and numerical solutions were provided. This gradient-based optimisation algorithm coupled with CFD was used to determine numerically the optimal geometry that gave the lowest thermal resistance. This optimiser adequately handled the numerical objective function obtained from numerical simulations of the fluid flow and heat transfer.

The numerical results obtained were in good agreement with results obtained in the approximate solutions based on scale analyses at optimal geometry dimensions. The approximate dimensionless global thermal resistance predicted the trend obtained in the numerical results. This shows that there were unique optimal design variables (geometries) for a given applied dimensionless pressure number for fixed porosity.

The results also showed that the material property had a significant influence on the performance of the cooling channel.

Therefore, when designing the cooling structure of vascularised material, the internal and external geometries of the structure, material properties and pump power requirements would be very important parameters to be considered in achieving efficient and optimal designs for the best performance.

Finally, this research investigated a three-dimensional geometric optimisation of conjugate cooling channels in forced convection with an internal heat generation within the solid for an array of cooling channels. Three different flow orientations based on constructal theory were studied numerically- firstly, an array of channels with parallel flow; secondly, an array of channels in which flow of every second row was in a counter direction and finally, an array of channels in which the flow direction in every channel was opposite to that of previous channel. The geometric configurations and flow orientations were optimised in such a way that the peak temperature was minimised subject to the constraint of fixed global volume of solid material. The optimisation algorithm coupled with CFD was also used to determine numerically the optimal geometry that gave the lowest thermal resistance.

The use of the optimisation algorithm coupled with the computational fluid dynamics package; render the numerical results more robust with respect to the selection of *optimal structure geometries, internal configurations of the flow channels and dimensionless pressure difference.*

Keywords: *Geometric configurations, computational fluid dynamics, mathematical optimisation, thermal conductivity, constraints, laminar flow, forced convection, optimal geometry, peak temperature, constructal theory, thermal resistance, Dynamic-Q, flow orientation*

DEDICATION

This thesis is dedicated to:

The Almighty God whose mercy triumphs over judgement – James 2:13

and

The memory of my late Mother – Felicia Olakoyejo

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NOMENCLATURES

A	Area, m^2
$\boldsymbol{A}$	Hessian matrix of the objective function
A_c	Cross sectional area of the channel, m^2
A_s	Cross sectional area of the structure, m^2
Be	Bejan number
$\boldsymbol{B}_i$	Hessian matrix of the inequality function
$CF-2$	Counter-flow row
$CF-3$	Counter-flow channel
$\boldsymbol{C}_j$	Hessian matrix of the equality function
C_P	Specific heat at constant pressure, J/kg K
Cyl	Cylindrical configuration
a, b, c	Diagonals of Hessian matrices ***A, B***, ***C***
d_h	Hydraulic diameter, m
D	Substantial derivative
$E\text{-}T$	Equilateral triangle
$f(\mathbf{x})$	Objective function
$f(\mathbf{x})$	Objective approximate function
g	Gravity
$g_i(\mathbf{x})$	*i*-th inequality constraint function
$g_i(\mathbf{x})$	*i*-th inequality constraint approximate function

G	Computational domain width
h	Elemental height , m
h	Enthalpy, J.kg^{-1}
h	Heat transfer coefficient, W.m^{-2}K^{-1}
h_c	Channel height , m
$h_j(\boldsymbol{x})$	*j*-th equality constraint function
$h_j(\boldsymbol{x})$	*j*-th equality constraint approximate function
H	Structure height, m
i	Mesh iteration index
$\boldsymbol{I}$	Identity matrix
$I–T$	Isosceles right triangle
k	Thermal conductivity, W/mK
k_r	Conductivity ratio
L	Axial length, m
N	Number of channels
n	Normal
P	Pressure, kPa
P_c	Perimeter of the channel
P_o	Poiseuille number
$PF – 1$	Parallel – flow
Pr	Prandtl number
$P[k]$	Successive sub-problem

$p(\mathbf{x})$	Penalty function
$\dot{q}''$	heat flux, W/m^2
q_s'''	Internal heat generation density, W/m^3
$\dot{q}$	Heat transfer rate, W
$\tilde{Q}$	Heat transfer, W
$\square^n$	*n*-dimensional real space
R	Dimensionless thermal resistance
Re	Reynolds number
R^2	Coefficient of correlation
s	Channel spacing, m
Sqr	Square configuration
T	Temperature, $^{\circ}$C
$\tilde{T}_{\max}$	Dimensionless maximum temperature, $\left(\tilde{T}_{\max} = \dfrac{T - T_{in}}{q'''v^{2/3}/k_f}\right)$
$\vec{u}$	Velocity vector, m/s
V	Global structure volume, m^3
v_c	Channel volume, m^3
v_{el}	Elemental volume, m^3
W	Structure width, m
w	Elemental width, m
x, y, z	Cartesian coordinates, m
$\mathbf{x}^*$	Design variables

$\boldsymbol{x}^{k}$	Design points
m,n,l,k,r	Positive integer

Greek symbols

α	Thermal diffusivity, m^2/s
β	Penalty function parameter for equality constraint
μ	Viscosity, kg/m.s
ν	Kinematics viscosity, m^2/s
ρ	Density, kg/m^3
∂	Differential or Derivative
∞	Far extreme end, free stream
ϕ	Porosity
Δ	Difference
∇	Differential operator or gradient function
τ	Shear stress, Pa
γ	Convergence criterion
γ	Penalty function parameter for objective constraint
δ	Kronecker delta function
δ	Move limit
ε	Value tolerance
λ	Vexing coefficient
ξ	Characteristic length scale

Φ	Dissipation function
ρ	Penalty function parameter
μ	Large positive value
Ω	Dimensionless temperature difference

Subscripts

$\sqcup$	Dimensionless
in	Inlet
l	Large
opt	Optimum
s	Solid
sm	Small
r	Ratio
0	Initial
1	Phase 1
$1,2,3,4$	Design variable number
ave	Average
$best$	Best
c	Channel
f	Fluid
f	Function
h	Hydraulic

$inlet$	Inlet
L	Length
max	Maximum
min	Minimum
$norm$	Normalised
i,j,k,l,n	Positive integers
opt	Optimum
$solid$	Solid
s	Surface
w	Wall
x	Step size
∞	Free stream

Superscripts

T	Transpose
k	Positive integer

PUBLICATIONS IN JOURNALS, BOOKS AND CONFERENCE PROCEEDINGS

The following articles, book chapter and conference papers were produced during this research.

1. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, "Mathematical optimisation of laminar forced convection heat transfer through a vascularised solid with square channels", *International Journal of Heat and Mass Transfer*, Vol. 55, pp. 2402-2411, 2012. **(Published)**

2. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer; "Constructal conjugate cooling channels with internal heat generation", *International Journal of Heat and Mass Transfer*, Vol. 55, pp. 4385-4396, 2012. **(Published)**

3. T. Bello-Ochende, **O.T. Olakoyejo** and J.P Meyer, Chapter 11, "Constructal Design of Rectangular Conjugate Channels" **Published in the book, "*Constructal Law and the Unifying Principle of Design*",** L.A.O Rocha, S. Lorente and A. Bejan, eds., pp. 177-194, Springer Publishers, New York, 2012. **(Published)**

4. J.P Meyer**, O.T. Olakoyejo**, and T. Bello-Ochende; "Constructal optimisation of conjugate triangular cooling channels with internal heat generation",

International communication of Heat and Mass Transfer, Vol. 39, pp. 1093 - 1100, 2012. **(Published)**.

5. T. Bello-Ochende, **O.T. Olakoyejo**, and J.P Meyer; "Constructal flow orientation in conjugate cooling channels with internal heat generation", *International Journal of Heat and Mass Transfer*, Vol. 57, pp. 241 - 249, 2013. **(Published)**.

6. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, "Optimisation of circular cooling channels with internal heat generation", *Proceedings of the 7th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics*, Antalya, Turkey, pp. 1345-1350, 19-21 July 2010. **(Presented)**

7. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, "Geometric Optimisation of Forced Convection In Cooling Channels With Internal Heat Generation *Proceedings of the 14th International Heat Transfer Conference*, Washington D.C, USA, pp. 1345-1350, 8 -13 August 2010. **(Presented)**

8. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, "Geometric optimisation of forced convection in a vascularised material", *Proceedings of the 8th International Conference on Heat Transfer, Fluid Mechacs and Thermodynamics*, Pointe Aux Piments, Mauritius, pp. 38 - 43, 11-13 July, 2011 (**Presented and awarded best paper of the session**).

9. **O.T. Olakoyejo**, T. Bello-Ochende and J.P. Meyer, “Constructal optimisation of rectangular conjugate cooling channels for minimum thermal resistance”, *Proceedings of the Constructal Law Conference*, 01-02 December, 2011, Porto Alegre, Universidade Federal do Rio Grande do Sul, Brazil. **(Presented)**

10. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, “Optimisation of conjugate triangular cooling channels with internal heat generation”, *9th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics, Malta*, 16 -18 July, 2012. **(Presented)**

11. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, “Flow orientation in conjugate cooling channels with internal heat generation”, *9th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics, Malta*, July 16 – 18, 2012. **(Presented).**