# 3. MODEL DEVELOPMENT

## 3.1 INTRODUCTION

This chapter describes the mathematics used to develop energy conversion models for all relevant components of the water reticulation system in a deep-level gold mine. In keeping with the input output methodology, models need to be developed to a low enough level for them to have a single output. Most of the developed models have a "friction head" input, and for this reason the "Darcy" model will be dealt with before actual models for physical components.

### 3.1.1 Fluid-Dynamic Background

The fundamental law that governs any system of fluid flow is Bernoulli's Theorem [31, p. 85], which states that in any system of fluid flow, the total energy in the system at any two points is the same provided that the energy is neither given to nor extracted from the system. When energy losses due to friction and other causes play a role, the theorem makes provision in quantifying the losses as well as energy that gets added to the system in an equation known as Bernoulli's equation [32, pp. 410 – 423]:

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+h_1+h_w=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+h_2+h_f+h_{misc-losses} \tag{3.1}$$

Bernoulli's Equation quantifies all variables in a fluid flow system in terms of equivalent heads that get calculated and equated. Pressure for example is equal to the product of density ($\rho$), the gravitational constant ($g$) and water head ($h$). By making $h$ the subject of the formula, the first term of Bernoulli's equation is found. The terms of the equation represent (from left to right), pressure head before the process, velocity head before the process, natural head before the process, energy added to the system represented by head, pressure head after the process, velocity head after the process, natural head after the process, friction head calculated using the Darcy formula described below and other miscellaneous losses which may be incurred in isolated cases.

## 3.2 FRICTION HEAD

Friction head is a mathematical means of representing losses incurred in a fluid-dynamic system from physical friction of the fluid within a pipe. These losses are called the system's "friction head", due to the fact that they are represented as an equivalent natural head that could be added to the system to have the same effect. This is in keeping with the Bernoulli Theorem, which equates "head" values.

In order to calculate the friction head of a system, the Darcy formula needs to be used. This formula is such a fundamental part of these systems, it can be seen as a model on it's own which serves as in input to the pumping model defined later. The friction head (Darcy) model is [32, p. 440]:

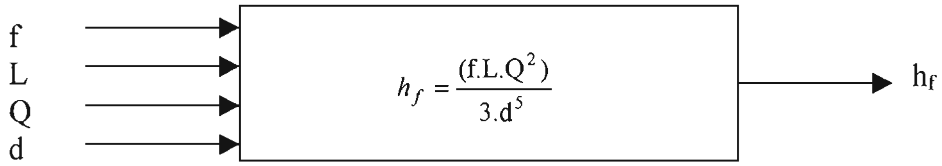

Figure 3.1: Model of Friction Head in Fluid Systems

From this model, one can see that influences on the value of the friction head are the length of the pipe, the flow rate through the pipe and the diameter of the pipe. $f$ represents a constant, dependant on the material of the pipe, which usually needs to be determined experimentally. However applications for which this model will be used are limited to smooth steel pipes used in mining applications. $f$ thus remains constant and can be approximated to 0.02 [33, p. 1271].

## 3.3 PUMPS

The most fundamental model that needs to be developed in any water-reticulation system is for pumping. In order to generate a pump model it is necessary to examine the function that pumps perform in processes such as the water reticulation system of deep-level mines.

In essence pumps are moving water from one level of the mine to a higher one. This means that the pumps are adding potential energy to the water and so should be seen as devices, which are adding energy to the fluid-dynamic system in which they are present.

The first step in the process of developing an energy conversion model for pumps is to examine which of the heads in the Bernoulli equation will have an influence on the power which needs to be injected into the system. In doing this, a number of approximations can be made.

The first approximation that can be made from the Bernoulli equation is the ignoring of the pressure head. This can be done because the pressure heads in the Bernoulli equation represent the pressure of the fluid before it enters the process and the pressure of the fluid after the process. In other words, referring to equation 1, $p_1$ would be the pressure of the water in a dam in the mine before it gets pumped. This means that for example, if the dam is 30m deep and the column of water that the pump is pumping up the shaft is 600m high, then the pressure at the outlet of the dam represents a mere 5% of the pressure due to the natural head. The pressure at the outlet at the upper level, $p_2$ would reduce this value even more, although the pressure at the outlet will not have any head to increase it above atmospheric pressure, seeing that the outlets are generally placed at the top of the dam.

The next approximation that can be made is the omission of the velocity head. This is because the head will not change at all between the two conditions that the system is working. In typical water reticulation systems of deep-level mines, the cross-sectional area of the pipe remains the same from the suction at the lower dam to the outlet at the higher dam. This implies that for a constant value of mass movement, there must be a constant velocity and so the velocity head does not change.

Miscellaneous losses would include added losses in systems that may for example have a number of pipe bends and orifices. These losses will not be included in the pumping model due to the fact that most mining applications have mainly straight, vertical columns of water to pump against and so have most of their losses in pure frictional losses. This does not mean that these losses should be forgotten about. Some systems may have many different piping configurations and would then need to have the associated losses taken into account. It is not possible to list the losses associated with all the different components that may be found in water reticulation systems and so it would be necessary to research the losses associated with the different devices present in systems where they appear. An example of the head loss that would be present at the widening of a pipe would be calculated using the following formula [34, p. 174]:

$$h_{loss} = \frac{(v_1 - v_2)^2}{2g} \tag{3.2}$$

Where $v_1$ is the velocity in the narrower part of the pipe and $v_2$ is the velocity in the wider part of the pipe.

There are thus 2 fundamental heads which play a role in the power that needs to be added to the system by any pumps, namely the natural head, $h$ and the friction head $h_f$. In order to relate these to the power that needs to be injected by the pump, we need to define power in a fluid system. The power delivered by a pump in fluid system in defined as [35, p. 80]:

$$Power = Q.P \tag{3.3}$$

This simple relationship tells us that the power is equal to the product of the flow rate and the pressure. Defining pressure will complete the model of a pump. The fundamental definition of pressure is [36, p. 3.1/5]:

$$P = \rho.g.h \tag{3.4}$$

Including natural and friction head in $h$ in equation 4, substituting equation 4 in equation 3 and including a factor for the efficiency of the pump leaves us with a rather robust model of a pump which can be used in the water reticulation systems of deep-level mines:

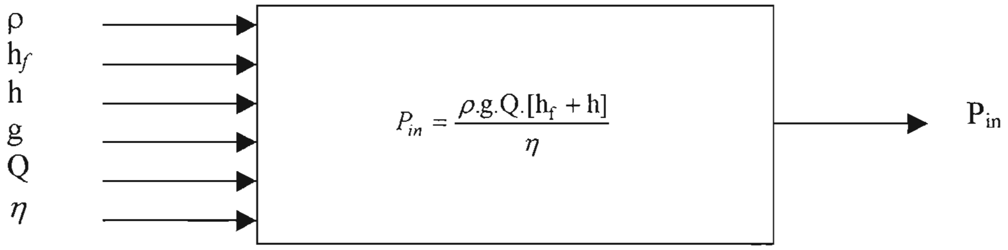

Figure 3.2: Pump Model

The model above requires density, pump efficiency, flow rate, friction head and natural head as inputs. If all of these are not available but delivery pressure, flow rate and efficiency are, then the following model can be used per pump:

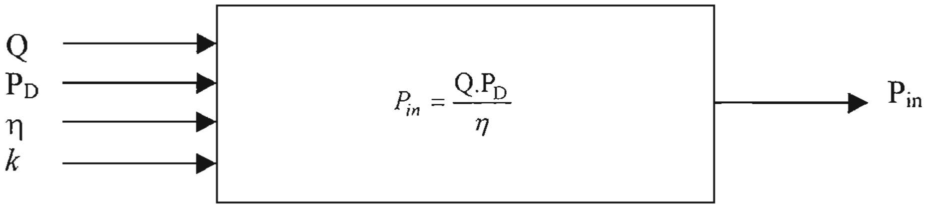

Figure 3.3: Alternative Pump Model

## 3.4 MULTIPLE PUMPS IN ONE LINE

In configurations where multiple pumps are used to pump water up a single pipe in a mineshaft, one must be aware of the fact that even if identical pumps are used, the increase in flow rate is not proportional to the increased number of pumps. The reason for this is that the flow rate increases with more pumps; friction losses are proportional to the square of the flow rate as shown in figure 3.1. Thus a higher flow rate means a much higher friction head.

From experimental data, it has been determined that the increased flow rate is a bounded exponential relationship. This means that the power used by the system also does not increase linearly with flow rate when more pumps are added.

If more than one pump is present in a single pipeline, and one needs to model the power used in such a system, the flow rate needs to be measured and reduced to the flow rate of a single pump using an equation of the form of equation 3.5. The values of $A$ and $k$ must be determined experimentally by running one and then two pumps alone to generate two separate sets of test figures. Once the equivalent flow rate of a single pump has been determined, the power of a single pump can be found using the pump model and then multiplied by the number of pumps to find the total power used.

$$Flow = A(1 - e^{-k.(pumps)}) \tag{3.5}$$

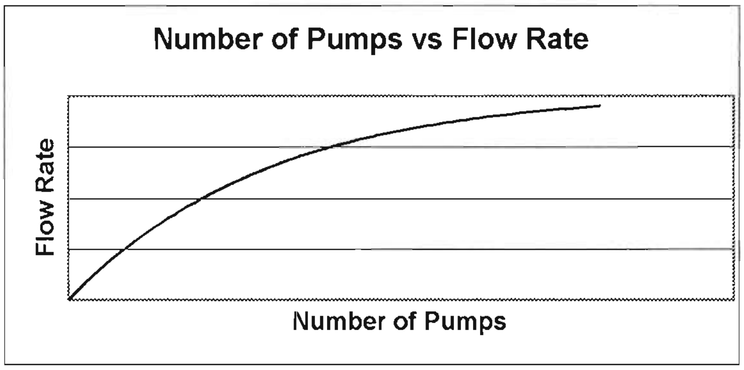

Figure 3.4: Illustration of bounded exponential effect on flow rate when adding parallel pumps in a single line

## 3.5 TURBINES

Energy recovery is very important in any water reticulation system. The easiest way of performing energy recovery is by using a turbine. The power, which is regained by a turbine, can then be used to either directly drive a pump or first be converted to electrical energy using a generator and then driving an electrical pump. If the power is converted into electrical energy, it does not only have to be used for pumps, but can be utilised in a number of ways.

Turbines are very similar to pumps in the sense that they convert power in fluids to mechanical and then electrical power, where pumps convert electrical and mechanical power into power within a fluid. It thus makes sense that the same factors influence the turbine model as the pump model. The system is still a fluid system for which the Bernoulli equation must hold. Only in this case the energy is not being added to the system, but removed as an additional loss.

The same simplifications hold for the generation of a turbine model as for the generation of a pump model. Velocity and pressure heads can be ignored. The value for $f$ can still be assumed to be 0.02. In this configuration however, the heads cannot be added. The pump model requires the natural head to be added to the friction head. For turbines the turbine is using the natural head to generate power and the friction head is still a negative influence to the available power. For this reason the friction head needs to be subtracted from the natural head and the efficiency factor needs to be moved to the numerator of the model as the power obtained from the turbine will be less than the power available in the fluid system.

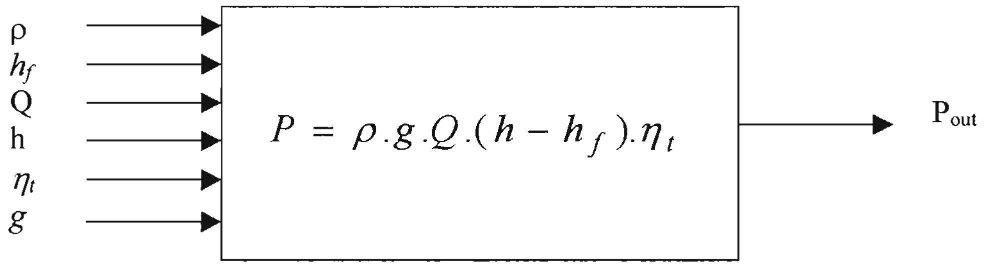

Figure 3.5: Turbine Model

If the turbine is directly coupled to a pump, then the power produced by this model can simply be used as the pump input power. If the turbine however drives a generator, the generator will have a single input and output, both being power, and therefore merely add an efficiency factor to the output power of the turbine, as follows:

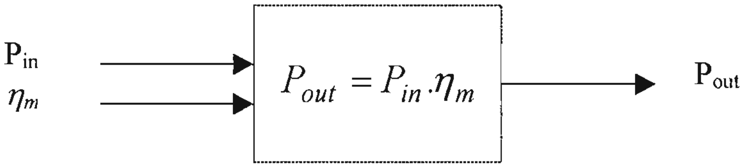

Figure 3.6: Motor Model

Baker-Duly, Ramsden and Mackay [37, p. 384], mention that part of the potential energy of the water passing down the shaft is in thermal form. They continue that this means that an added benefit of using energy recovery systems such as turbines is that it reduces the energy required to cool the water in the fridge plant of the mine.

## 3.6 THREE CHAMBER PIPE FEEDER SYSTEM

The 3 Chamber Pipe Feeder System (3CPF), is not very widespread due to its complexity and cost. This system goes a step further from the directly coupled turbine-pump configuration, in that it actually uses the head of chilled water passing down the shaft to balance the head of warm water which needs to be pumped up the shaft, using the U-tube principle [13, p. 422]. At present the only fully-functional 3CPF system in South Africa is used in the Tshepong mine in the Freestate. The system consists of 3 high-pressure chambers located on 45 level of the mine, each having a capacity of 16 kl. In a process of sequential pressurising and depressurising, the chambers are filled with the warm water that needs to be drained to the surface. This warm water then gets pressurised to 13,7 MPa, the chilled water, which is at the same pressure due to its head, is allowed to flow down the shaft and into the chamber. This displaces the hot water to the surface with the help of two 800 kW pumps, which are used to overcome friction and other losses. At the same time the other chambers are being filling with warm water and emptying chilled water at low pressure

[38, pp. 10 – 20]. One of the pumps is on surface and the other on 45 level of the mine, at the same level as the underground hot and chilled water dams.

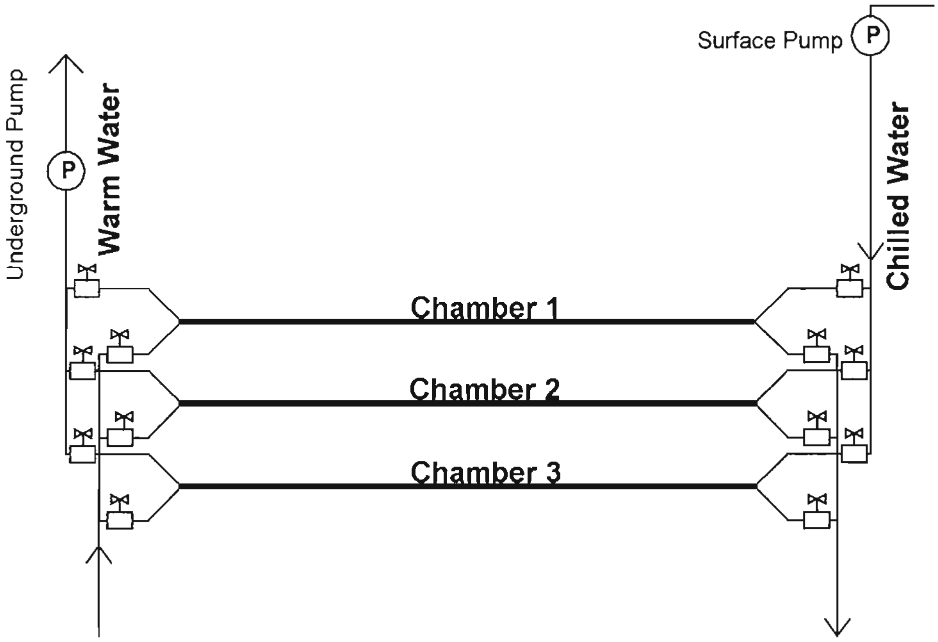

Figure 3.7: Three Chamber Pipe Feeder System

The head of chilled water is used to balance the head of warm water that needs to be drained up the shaft. Relatively small pumps are used to overcome friction and other losses in the system. These pumps can be modelled using the standard pump model shown earlier.

In order to model the 3CPF, it must be realised that it uses potential energy in the downward chilled water column to balance the warm water head. This prevents the need for most of the energy in conventional pumping systems, namely to overcome the head, $h$. The purpose of the pumps are to overcome the friction head. The same simplifications hold as for normal pumps regarding being able to ignore pressure head and assuming $f$ in the Darcy equation to be 0.02. The pumps are also there to overcome all other losses such as the imperfect efficiency of the entire system, due to amongst others the pressurising, depressurising and valve switching. The model for the pumps would thus be:

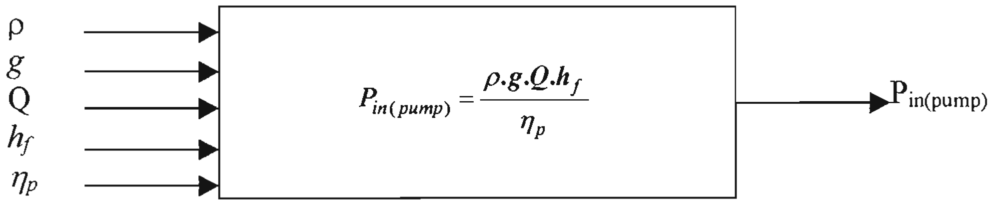

Figure 3.8: 3CPF Pump Model

Note the absence of the natural head, *h* in the pumping power model. The friction head can be calculated using the friction head (Darcy) model, it must be remembered that the downward and upward pipe lengths must be added; the heads balance out but total friction and other losses still need to be taken into account.

In general, one would expect more than one pump per 3CPF due to the size and associated flow of the system. In a system such as the one installed at Tshepong mine, two pumps are in place, one on surface pumping chilled water down and the other on the same level as where the 3CPF is installed – in this case 45 level of the mine. These pumps are thus not in parallel, the way pumps are generally added to increase flow rate, but in series. This means that the total flow of multiple pumps cannot be calculated using the bounded exponential model described earlier for multiple pumps. In this case, the pumps must be treated individually and the total load shared proportionally between them. In the Tshepong example, this means that the downward friction head gets used as an input to the model that will be used for the surface pump and the upward friction head gets used for the 45 level pump.

The 3CPF model however does not consist of only friction heads and pumps. It has a number of other losses associated with its operation. The causes of these losses include the switching and the many bends found in the 3CPF system. All such additional losses can collectively be quantified in an efficiency factor for the 3CPF as follows:

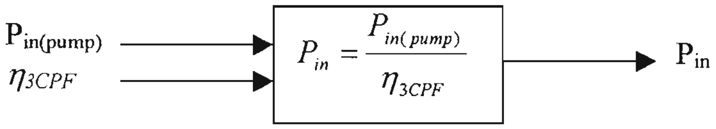

Figure 3.9: 3CPF Model

The model shown above is very useful in keeping with the modular methodology. This however may not always be possible. In fact, in systems such as the one installed at Tshepong Mine, available data is not precise and defined enough to be able to precisely determine all the variables of the pump models. This complicates properly distinguishing between the efficiencies of the pumps and the efficiencies of the 3CPF system. For this reason, it simplifies circumstances by simply including the losses of the 3CPF in the pump efficiencies.

## 3.7 STORAGE / BUFFER MODEL

In any system that is being modelled, it is important to take careful note of all constraints present. Before any models can be implemented successfully, it is important to verify that they will be valid for the entire period that they are proposed to operate.

The main constraints that can be mathematically represented in the modelling of the water reticulation system of a deep-level gold mine are the dam levels. The system component models all operate with the assumption that there is enough available water from their source and that they are not overfilling any dams.

Summing the inflow, outflow and initial level of all storage devices and ensuring that the results obtained are within set tolerances before calculating the energy usage, ensures that the models will in fact produce reliable, valid results.

Roos [3, p. 121] quantifies this in a model he developed for any general storage process. The following is a slightly modified version of Roos's model:

$$\boxed{Contents = \sum \int InflowRate - \sum \int OutflowRate + InitialContents}$$

Figure 3.10: Storage / Buffer Model

This model is general enough for it to be able to be continuously implemented. It is important to ensure that the integration period used is a small enough interval for the buffer not to exceed its limits during that time. Using the small enough interval in the model, means that the model will need to be repeated a number of times to completely verify the other models for the full duration of their operation. The summations in the model are in case a storage device has more than one input and output.

Giles and Wunderlich [39, pp. 40 – 45], point out that storage devices such as dams have more inputs and outputs than initially meet the eye. In addition to the conventional inlets and outlets to dams, one must keep track of other factors such as ground water being added as well as seepage losses occurring from the dams.

Giles and Wunderland continue to stress the importance of the storage model as it is not only a constraint which must be adhered to, but if utilised correctly, can form a reasonable cost saving tool. When energy recovery systems such as turbines are in place, stored water can be used to add energy to the water reticulation system in times of high demand and so reduce total energy costs.