#### **REFERENCES**

- Abalu, G., and Hassan, R. (1999). Agricultural productivity and natural resource use in Southern Africa. *Food Policy*, 23(6): 477 490.
- Abebe, F. (1988). Need for soil survey studies. In: Desta Beyene (eds.), *Soil Science Research in Ethiopia*: A Review. Proceedings of the First Soil Science Research Review Workshop 11-14 February 1986, Addis Ababa, Ethiopia.
- Adal, Y. (2000). Preliminary assessment of the impacts of the Post-Derg land policy on agricultural production: a case study of land redistribution in two communities in North Western Ethiopia. p.173 190. In: Workneh Negatu, Legesse Dadi, Abebe Haile Gabriel and Solomon Bellete (eds.), *Institutions for Rural Development*, Proceedings of the 4th Annual Conference of Agricultural Economics Society of Ethiopia, 23-24 November 1999, Addis Ababa.
- Adal, Y. (2003). Problems and prospects of rural land policy in Ethiopia. p.49–62. In: Workneh Negatu, Legesse Dadi and Abebe Haile Gabriel (eds.), *Agricultural Policy in Ethiopia's Economic Development: Scope, Issues and Prospects*, Proceedings of the 6th Annual Conference of Agricultural Economics Society of Ethiopia, 30 31 August 2002, Addis Ababa.
- Adesina, A.A., and Baidu-Forson, J. (1995). Farmers' perceptions and adoption of new agricultural technology: evidence from analysis in Burkina Faso and Guinea, West Africa. *Journal of Agricultural Economics*, 13:1-9.
- Admassie, A., and Heidhues, F. (1996). Estimation of technical efficiency of smallholder farmers in the central highlands of Ethiopia. *Ethiopian Journal of Agricultural Economics*, 1(1): 18 37.
- Adugna, T., and Demeke, M. (2000). Institutional reforms and sustainable input supply and distribution in Ethiopia. p.125 156. In: Workneh Negatu, Legesse Dadi, Abebe Haile Gabriel and Solomon Bellete (eds.), *Institutions for Rural Development*,

- Proceedings of the 4th Annual Conference of Agricultural Economics Society of Ethiopia, 23-24 November 1999, Addis Ababa.
- Alene, A.D., Poonyth, D., and Hassan, R.M. (2000). Determinants of the adoption and intensity of use of improved maize varieties in central highlands of Ethiopia. A Tobit analysis. *Agrekon*, 39(4): 633 643.
- Baidu-Forson, J. (1999). Factors influencing adoption of land-enhancing technology in the Sahel: lessons from a case study in Niger. *Journal of Agricultural Economics*, 20: 231-239.
- Bamire, A.S., Fabiyi, Y.L., and Manyong, V.M. (2002). Adoption pattern of fertilizer technology among farmers in the ecological zones of southwestern Nigeria: a Tobit analysis. *Australian Journal of Agricultural Research*, 53: 901-910.
- Barbier B. and Bergeron, G. (1999). Impact of policy interventions on land management in Honduras: results of a bio-economic model. *Agricultural Systems*, 60: 1 16.
- Barbier, B. (1998). Induced innovation and land degradation: results from bioeconomic model of a village in West Africa. *Agricultural Economics*, 19: 15-25.
- Barbier, E. B. (1990). The farm-level economics of soil conservation: the uplands of Java. *Land Economics*, 66(2): 199 - 211.
- Barbier, E.B. (1995). The economics of soil erosion: theory, methodology and examples. http://www.eepsea.org/publications/specialp2/ACF2B4.html; p.1 - 23 (Accessed on July 12, 2004).
- Barbier, E.B., and Burgess, J.C. (1992). Agricultural pricing and environmental degradation. Background paper for World Development Report 1992. Washington D.C: World Bank.
- Barret, S. (1991). Optimal soil erosion conservation and the reform of agricultural pricing policies. *Journal of Development Economics*, 36:167 187.

- Bayu, A. (2003). Food security in Ethiopia: review of policy, strategy and program. p.25 40. In Workneh Negatu, Legesse Dadi and Abebe Haile Gabriel (eds.), *Agricultural Policy in Ethiopia's Economic Development: Scope, Issues and Prospects*, Proceedings of the 6th Annual Conference of Agricultural Economics Society of Ethiopia, 30-31 August 2002, Addis Ababa.
- Beasley, D.B., Huggins, L.F., and Munke, E.J. (1980). ANSWERS a model for watershed planning. Trans Am Soc. *Agric Eng*, 23: 938 944.
- Bekele, W. (2003). Economics of soil and water conservation: theory and application to subsistence farming in the eastern Ethiopian Highlands. Doctoral thesis, Swedish University of Agricultural Sciences.
- Bekele, W., and Drake, L. (2003). Soil and water conservation decision behaviour of subsistence farmers in the Eastern Highlands of Ethiopia: a case study of the Hunde-Lafto area. *Ecological economics*, 46: 437 451.
- Bishop, J. (1995). The economics of soil degradation: an illustration of the change in productivity approach to valuation in Mali and Malawi. *Discussion LEEC Paper DP* 95 02IIED, Environmental Economics Program.
- Bishop, J., and Allen, J. (1989). The on-site costs of soil erosion in Mali. *Working paper No.* 21, The World Bank, Environmental Department, Washington DC.
- Bojo, J. (1992). Cost-benefit analysis of soil and water conservation projects: a review of 20 empirical studies. p.195 205. In: Tato, K., and Hurni, H. (Eds.). *Soil Conservation for Survival*, Soil and Water Conservation Society (SWCS) in cooperation with International Soil Conservation Organization (ISCO) and World Association of Soil and Water Conservation (WASWC).
- Bojo, J. (1996). The costs of land degradation in sub-Saharan Africa. *Ecological Economics*, 16:161 173.

- Bojo, J. and Cassells, D. (1995). Land degradation and rehabilitation in Ethiopia: a reassessment. *AFTES Working Paper No. 17*, The World Bank: Washington, DC.
- Brekke, A.K., Iversen, V., and Aune, J.B. (1999). Tanzania's soil wealth. *Environment and Development*, 4 (3): 333 356.
- Bulte, E., and Soest D. van. (1999). A note on soil depth, failing markets and agricultural pricing. *Journal of Development Economics*, 58: 245 254.
- Burt, O.R. (1981). Farm level impact of soil conservation in the Palouse Area of the Northwest. *American Journal Agricultural Economics*, 63: 83 92.
- Byerlee, D., and Hesse de Polanco, E. (1986). Farmers' stepwise adoption of technological packages: evidence from the Mexican Altiplano. *American Journal of Agricultural Economics*, August (1986): 519 527.
- Cárcamo, J.A., Alwang, J., and Norton, G.W. (1994). On-site economic evaluation of soil conservation practices in Honduras. *Agricultural Economics*, 11:257 269.
- Caswell, M., Fuglie, K., Ingram, C., Jans, S., and Kascak, C. (2001). Adoption of agricultural production practices. *Report No.* 792, Resource Economics Division, Economic Research Service, U.S. Department of Agriculture.
- Central Statistical Authority (CSA). (1999). The Federal Democratic Republic of Ethiopia. Statistical Abstract for 1998. CSA, Addis Ababa.
- Central Statistical Authority (CSA). (2004). Federal Democratic Republic of Ethiopia, Statistical Abstract for 2002, Addis Ababa.
- Chiang, A. C. (1992). Elements of dynamic optimization. McGraw-Hill, Inc., USA.
- Clark, H.R. (1992). The supply of non-degraded agricultural land. *Australian Journal of Agricultural Economics*, 36(1): 31 56.

- Clay, D., Reardon, T., and Kangasniemi, J. (1998). Sustainable intensification in the highland tropics: Rwandan farmers' investments in land conservation and soil fertility. *Economic Development and Cultural Change*, 42(2): 351 377.
- Croppenstedt, A., and Demeke, M. (1996). Determinants of adoption and levels of demand for fertilizer for cereal growing farmers in Ethiopia. *Working Paper Series, WPS/96-3*, Centre for the Study of African Economies, C.S.A.E Publishing, University of Oxford.
- Croppenstedt, A., and Demeke, M. (1997). An empirical study of cereal crop production and technical efficiency of private farmers in Ethiopia: a mixed fixed-random coefficient approach. *Applied Economics*, 29: 1217-1226.
- Croppenstedt, A., Demeke, M., and Meschi, M.M. (2003). Technology adoption in the presence of constraints: the case of fertilizer demand in Ethiopia. *Review of Development Economics*, 7(1): 58 70.
- Dadi, L., Burton, M., and Ozanne, A. (2001). Adoption and intensity of fertilizer and herbicide use in the Central Highlands of Ethiopia. *Agrekon*, 40(3): 316 333.
- De Jager, A., Nandwa, S.M., Okoth, P.F. (1998). Monitoring nutrient flows and economic performance in African Farming System (NUTMON) concept and Methodology. Agriculture, Ecosystem, Environment and Environment, 71: 37-48.
- Demeke, M., Ali, S., and Thomas, S.J. (1997). Promoting fertilizer use in Ethiopia: the implications of improving grain market performance, input market efficiency, and farm management. *Working Paper 5*, Ministry of Economic Development and Cooperation: Grain Market Research Project, Addis Ababa, Ethiopia.
- Dimara, E., and Skuras, D. (2003). Adoption of agricultural innovations as a two-stage partial observability process. *Agricultural Economics*, 28: 187 196.
- Dorfman, J.H. (1996). Modeling multiple adoption decisions in a joint framework. *American Journal of Agricultural Economics*, 78: 547 557.

- Eaton, D. (1996). The economics of soil erosion: a model of farm decision-making. *Discussion Paper DP 96–01*, International Institute for Environment and Development, Environmental Economics Program.
- Elies, E. (2002). Farmers' perceptions of soil fertility change and management. SOS-Sahel, Institute for Sustainable Development, Addis Ababa.
- Elwell, H.A., and Stocking, M.A. (1982). Developing a simple yet practical method of soilloss estimation. *Tropical Agriculture*, 59(1): 43 48.
- Erkossa, T., Selamyihun, K., Takalign, M., and Mesfin, A. (1999). Effect of land preparation methods on runoff and soil loss on Vertisols at Ginchi (Ethiopia). *Ethiopian Journal of Natural Resources*, 2: 1 15.
- Ersado, L., Amacher, G., and Alwang, J. (2004). Productivity and land enhancing technologies in Northern Ethiopia: health, public investments, and sequential adoption. *American Journal of Agricultural Economics*, 86 (2): 321-331.
- Ervin, C.A., and Ervin, D.E. (1982). Factors affecting the use of soil conservation practices: Hypotheses, evidence, and policy implications. *Land Economics*, 58(3): 277 292.
- FAO. (1986). Ethiopian Highlands Reclamation Study. Final Report, Volume I.
- FAO. (1998). FAO Production Yearbook. Rome, Italy.
- FAO. (1999). Integrated soil management for sustainable agriculture and food security in Southern and East Africa. Proceedings of the Expert Consultation, Harare, Zimbabwe, 8-12 December 1997. Food and Agricultural Organization of the United Nations, Rome.
- FAO. 1988. Fertilizer yearbook, Rome.
- FDRE. (2000). Ethiopian Interim Poverty Reduction Strategy Paper, 2000/01-2002/03, Addis Ababa, Ethiopia.

- Feder, G., Richard, E.J, and David, Z. (1985). Adoption of agricultural innovations in developing countries: a survey. *Economic development and cultural change*, 33: 255-298.
- Franzel, S., Dadi, L., Colburn, F., and Degu, G. (1992). Grain-marketing policies and peasant production. p.212-226. In: Franzel, S. and Houten, van H. (eds). *Research with Farmers: Lessons from Ethiopia*. C.A.B International, Wallingford, UK.
- Fufa, B., and Hassan, R.M. (2003). Stochastic maize production technology and production risk analysis in Dadar district, East Ethiopia. *Agrekon*, 42(2): 116-128.
- Gavian, S., and Ehui, S. (1999). Measuring the production efficiency of alternative land tenure contracts in a mixed crop-livestock system in Ethiopia. *Agricultural Economics*, 20: 37-49.
- Gebre Michael, Y. (1999). The use, maintenance and development of soil and water conservation measures by small-scale farming households in different agro-climatic zones of northern Shewa and southern Wello, Ethiopia. *Research Report 44*, Center for Development and Environment, University of Berne, Switzerland in association with The Ministry of Agriculture, Ethiopia.
- Gebremedhin, B., and Swinton, S.M. (2003). Investment in soil conservation in northern Ethiopia: the role of land tenure security and public programs, *Agricultural Economics*, 29: 69-84.
- Gelan, A. (2002). Trade liberalization and urban-rural linkages: a CGE analysis for Ethiopia. *Journal of Policy Modeling*, 24(7-8): 707-738.
- Ghadim, A. K. A., and Pannell, D.J. (1999). A conceptual framework of adoption of an agricultural innovation. *Agricultural Economics*, 21:145-154.
- Goetz, R.U. (1997). Diversification in agricultural production: a dynamic model of optimal cropping to manage soil erosion. *American Journal of Agricultural Economics*, 79: 341-356.

- Green, W. H. (2000). Econometric Analysis, 4th Ed., New Jersey: Prentice-Hall, 2000.
- Grepperud, S. (1996). Population pressure and land degradation: the case of Ethiopia. Journal of Environmental Economics and Management, 30:18-33.
- Grepperud, S. (1997a). Soil conservation as an investment in land. *Journal of Development Economics*, 54:455-467.
- Grepperued, S. (1997b). Poverty, land degradation and climatic uncertainty. *Oxford Economic Paper*, 49(4): 586-603.
- Haile Gabriel, A. (2003). Challenges of achieving agricultural development in Ethiopia: a macro-policy perspective. p.11–24. In: Workneh Negatu, Legesse Dadi and Abebe Haile Gabriel (eds.), *Agricultural Policy in Ethiopia's Economic Development:* Scope, Issues and Prospects, Proceedings of the 6th Annual Conference of Agricultural Economics Society of Ethiopia, 30-31 August 2002, Addis Ababa.
- Hassan, R. M. (1996). Planting strategies of maize farmers in Kenya: a simultaneous equations analysis in the presence of discrete dependent variables. *Agricultural Economics*, 15: 137-149.
- Hassan, R.M., Kiarie, N., Mugo, N., Robin, O. and Laboso, A. (1998a). Adoption and performance of maize in Kenya. In: Hassan, R.M. (ed.) *Maize Technology Development and Transfer: A GIS Approach to Research Planning in Kenya*. CAB international, London.
- Hassan, R.M., Onyango, R., and Rutto, J.K. (1998b). Determinants of fertilizer use and the gap between farmers' maize yield and potential yields in Kenya. In: Hassan, R.M. (ed.) Maize Technology Development and Transfer: A GIS Approach to Research Planning in Kenya. CAB international, London.
- Hassena, M., Hassan, F., Mwangi, W., and Kassa, B. (2000). Factors influencing technical efficiency of barley production in Asasa district of southeastern Ethiopia. *Ethiopian Journal of Agricultural Economics*, 4(1&2): 1-21.

- Hediger, W. (2003). Sustainable farm income in the presence of soil erosion: an agricultural Hartwick rule. *Ecological Economics*, 45: 221-236.
- Heinrich, C.J., and Wenger, J. B. (2002). The economic contribution of James J. Heckman and Daniel L. McFadden. Review of Political Economy 14 (1): 69-89.
- Herath, P.H.M.U., and Takeyah, H. (2003). Factors determining intercropping by rubber smallholders in Sri Lanka: a logit analysis. *Agricultural Economics*, 29:159-168.
- Hoag, D.L. (1998). The intertemporal impact of soil erosion on non-uniform soil profiles: a new direction in analyzing erosion impacts. *Agricultural systems*, 56(4): 415-429.
- Holden, S., and Shiferaw, B. (2004). Land degradation, drought and food insecurity in a less favoured area in the Ethiopian highlands: a bio-economic model with market imperfections. *Agricultural Economics*, 30: 31-49.
- Holden, S., Shiferaw, B., and Wik, M. (1998). Poverty, market imperfections and time preferences: of relevance for environmental policy? *Environment and Development Economics*, 3: 105-130.
- Hopkins, J.W., Lal, R., Wiebe, K.D., and Tweeten, L.G. (2001). Dynamic economic management of soil erosion, nutrient depletion and productivity in the North Central USA. *Land Degradation & Development*, 12: 305 318.
- Hurni, H. (1985). Erosio-Prouctivity-Conservatin systems in Ethiopia. In: Sentis, I.P. (eds.), *Soil conservation and productivity*. Proceedings of IV international conference on soil conservation, Venezuela.
- Hurni, H. (1993). Land degradation, famines and resource scenarios in Ethiopia. In: Pimental (ed.), World Soil Erosion and Conservation, Cambridge, Cambridge University Press.
- Hussain, I., Olson, K.R., Wander, M.M., and Karlen, D.B. (1999). Adoption of soil quality indices and application to three tillage systems in southern Illinois. *Soil and Tillage Research*, 50: 237-249.

- Igodan, C.O., Ohoji, P.E., and Ekpare, J. A. (1990). Factors associated with the adoption of recommended practices for maize production in the Lake Basin of Nigeria. Agricultural Administration and Extension, 29(2): 149-156.
- Jutzi, S. C., and Mohammed-Saleem, M.A. (1992). Improved productivity on highland Vertisols: the oxen-drawn broadbed maker. p.97-108. In: Franzel, S. and Houten, van H. (eds). Research with Farmers: Lessons from Ethiopia, C.A.B International, Wallingford, UK.
- Kappel, R. (1996). Economic analysis of soil conservation in Ethiopia: issues and research perspective. *Research Report 35*, Soil Conservation Research Program, University of Berne, Switzerland in association with The Ministry of Agriculture, Ethiopia.
- Katchova, A. L., and Miranda, M.J. (2004). Two-step econometric estimation of farm characteristics affecting marketing contract decisions. *American Journal of Agricultural Economics*, 86 (1): 88-102.
- Kazianga, H., and Masters, W.A. (2002). Investing in soils: field bunds and micro-catchments in Burkina Faso. *Environment and Development Economics*, 7: 571-591.
- Kebede, Y, Gunjal, K., and Coffin, G. (1990). Adoption of new technologies in Ethiopian agriculture: the case of Tegulet-Bulga District, Shewa Province. *Agricultural Economics*, 4: 27-43.
- Kidanu, S. (2003). Using Eucalyptus for soil and water conservation on the highland Vertisols of Ethiopia. *Tropical Resource Management Papers*, Wageningen University and Research Center, Department of Environmental Science and Water Conservation Group, the Netherlands.
- Kidanu, S., Tanner, D.G., and Mamo, T. (1999). Effect of nitrogen fertilizer applied to teff on the yield and N response of succeeding teff and durum wheat on a highland Vertisol. *African Crop Science Journal*, 7(1): 35-46.

- Knisel, W.G. (1980). CREAMS: A field scale model for chemicals, runoff and erosion from agricultural management systems. USDA.
- Kruger, H. J., Fantaw, B., Gebre Michael, Y., and Kajela, K. (1996). Creating an inventory of indigenous SWC measures in Ethiopia. In: Reij, C., Scoones, I., and Toulmin, C. (eds). Sustaining the Soil, International Institute for Environment and Development, Earthscan Publications Limited, London.
- Kruseman, G., and Bade, J. (1998). Agrarian policies for sustainable land use: bio-economic modeling to assess effectiveness of policy instruments. *Agricultural Systems*, 58(3): 465-481.
- LaFrance, J. (1992). Do increased commodity prices lead to more or less soil degradation? Australian Journal of Agricultural Economics, 36:57-82.
- Lal, R. (1988). Soil degradation and the future of agriculture in sub-Saharan Africa. *Journal of Soil and Water Conservation*, 43:445-451.
- Lal, R. (1993). Tillage effects on soil degradation, soil resilience, soil quality and sustainability. *Soil and Tillage Research*, 27:1-8.
- Lal, R. (1994). Soil erosion by wind and water: problems and prospects. p.1-9. In: Lal, R. (eds.). Soil Erosion Research Method, The Soil and Water Conservation Society and St. Lucie Press, Second Edition.
- Lal, R. (1995). Trends in world agriculture land use: potential and constraints. p.521-536. In: Lal, R., and Stewart, B. A. (eds.) Soil Management: Experimental Basis for Sustainability and Environmental Quality. Paper presented at a workshop held June 26-18, 1993, Ohio State University, UP.
- Lal, R., and Elliot, (1994). Erodibility and Erosivity. p.181-208. In: Lal, R. (ed.). *Soil Erosion Research Methods*, The soil and Water Cosnervation Society and St. Lucie Press, Second Edition.

- Lapar, M. L. A., and Pandey, S. (1999). Adoption of soil conservation: the case of Philippine uplands. *Agricultural Economics*, 21(3): 241-256.
- Larson, W.E., and Pierce, E.J. (1994). The dynamics of soil quality as a measure of sustainable management. P.37-52. In: Doran, J.W., Coleman, D.G., Bezdick, D.F.,
  Stewart, B.A. (Eds.). *Defining Soil Quality for Sustainable Environment*, First Ed.,
  Pub. No.35, Soil Science Society of America, Soil Science America, Madison.
- Leathers, H.D., and Smale, M. (1991.) A Bayesian approach to explaining sequential adoption of components of a technological package. *American Journal of Agricultural Economics*, 73:734-742.
- Leonard, D., and Long, N.V. (1992). *Optimal control theory and static optimisation in economics*. Cambridge University Press, UK.
- Lin, J.Y. (1991). Education and innovation adoption in agriculture: evidence from hybrid rice in China. *American Journal of Agricultural Economics*, 73(3): 713-723.
- Lin, T., and Schmidt, P. (1984). A test of the Tobit specification against an alternative suggested by Cragg. *The Review of Economics and Statistics*, 66(1): 174-177.
- Long, J. S. (1997). Regression models for categorical and limited dependent variables: advanced quantitative techniques in the social science, Series 7, SAGE Publications: Thousand Oaks, California.
- Magleby, R. (2002). Agricultural resurgences and environmental indicators: soil management and conservation, http://www.ers.usda.gov/publications/aeri/ah722/ari4\_2/soilmgmt.pdf
- Magrath, W., and Arens, P. (1989). The costs of soil erosion on Java: a natural resource accounting approach. *Environmental Department Working Paper No.18*, The World Bank, Washington, DC.
- Makken, F. (1993). Nutrient supply and distribution of country level: case studies of Malawi and Ethiopia. p.165-233. In: Reuler H. van, and Prins, W.H. (eds.). *The role of plant*

- nutrients for sustainable food production in Sub-Saharan Africa, Dutch Association of Fertilizer Producers, Ponsen & Looijen, Wageningen, The Netherlands.
- Mamo, T., Astatke, A., Sirvastava, K.L., and Debaba, A. (eds.). (1992). Improved management of Vertisols for sustainable crop-livestock production in the Ethiopian highlands: synthesis report 1986-92. Technical Committee of Joint Vertisol Project, Addis Ababa, Ethiopia.
- Maxwell, D., and Wiebe, K. (1998). Land tenure and food security: a review of concepts, evidence and methods. *LTC Research Paper 129*, Land Tenure Center, University of Wisconsin-Madison.
- McConnell, K.E. (1983). An economic model of soil conservation. *American Journal of Agricultural Economics*, 65: 83-89.
- MEDaC. (1999). Survey of the Ethiopian Economy: review of post-reform development (1992/93-1997/98), Addis Ababa, Ethiopia.
- Mekuria, M., Steven, F., and Beyene, H. (1992). Farming systems research in Ethiopia: evolution, development and organization. p.28-39. In: Franzel, S., and Houten., van H. (eds.), *Research with Farmers: Lessons from Ethiopia*, C.A.B. International, Wallingford, UK.
- Mengistu, A. (2003). Forage resource profiles of Ethiopia. http://www.fao.org/WAICENT/FAOINFO/AGRICULT/AGP/AGPC/doc/Counprof/E thiopia/Ethiopia.htm
- MFD. (1957). First five-year plan. Imperial Ethiopian Government, Addis Ababa, Ethiopia.
- MFD. (1962). Second five-year plan. Imperial Ethiopian Government, Addis Ababa, Ethiopia.
- Miranowski, J.A. (1984). Impacts of productivity loss on crop production and management in a dynamic economic model. *American Journal of Agricultural Economics*, 66: 62-71.

- Murphy, H.F. (1963). Fertility and other data on some Ethiopian soils. *Exp. Stat. Bull. No.11*, Ethiopian College of Agriculture and Mechanical Arts, Dire Dawa, Ethiopia, 511 pp.
- Nakhumwa, T.O. (2004). Dynamic costs of soil degradation and determinants of adoption of soil conservation technologies by smallholder farmers in Malawi. PhD Thesis, Department of Agricultural Economics, Extension and Rural Development, Faculty of Natural and Agricultural Sciences, University of Pretoria, South Africa.
- Nakhumwa, T.O., and Hassan, R.M. (2003). The adoption of soil conservation technologies by smallholder farmers in Malawi: A selective Tobit analysis. *Agrekon*, 42(3): 271-284.
- Nearing, M.A., Lane, L.J., and Lopes, V. L. (1994). Modeling soil erosion. p.127-156. In: Lal, R. (ed.), *Soil Erosion Research Methods*, The soil and Water Conservation Society and St. Lucie Press, Second Edition.
- Negatu, W., and Parickh, A. (1999). The impact of perception and other factors on the adoption of agricultural technology in the Moret and Jiru districts of Ethiopia. *Agricultural Economics*, 21: 205-216.
- Norris, E. P., and Batie, S.S. (1987). Virginia farmers' soil conservation decisions: an application of Tobit analysis. *Southern Journal of Agricultural Economics*, 19(1): 89-97.
- Ofori, C.S. (1995). Need for long-term soil management research in Africa. p.485-497. In: Lal, R., and Stewart, B. A. (eds.), *Soil Management: Experimental Basis for Sustainability and Environmental Quality*. Papers presented at a workshop held June 26-18, 1993, Ohio State University, USA.
- Okumu, B.N., Jabbar, M.A., Colman, D., Rusell, N., Mohamed Saleem, M.A., and Pender, J. (2003). Technology and policy impacts on economic performance, nutrient flows and soil erosion at watershed level: the case of Ginchi in Ethiopia. Memo.

- Pagiola, S. (1996). Price policy and returns to soil conservation in semi-arid Kenya. Environment and Resource Economics, 8: 255-271.
- Pagiola, S. (1999). The global environmental benefit of land degradation control on agricultural land. *World Bank Environmental Paper No. 16*, The World Bank, Washington D.C.
- Pender, J., Gebremehin, B., and Ehui, S. (2001). Strategies for sustainable agricultural development in the Ethiopian highlands. *American Journal of Agricultural Economics*, 83:1231-1240.
- Pender, J.P., and Kerr, J. M. (1998). Determinants of farmers' indigenous soil and water conservation investments in semi-arid India. *Agricultural Economics*, 19:113-125.
- Pierce, F.J., and Lal, R. (1994). Monitoring the impact of soil erosion on crop productivity. In: Lal, R. (eds.), *Soil Erosion Research Method*, The Soil and Water Conservation Society and St. Lucie Press, Second Edition.
- Pindyck, R.S., and Rubinfeld, D.L. (1997). *Econometric models and economic forecasts*, 4th ed. McGraw-Hill International Editions: New York.
- Pryanishnikov, I., and Katarina, Z. (2003). Multinomial logit models for the Australian labor market. *Australian Journal of Statistics*, 4: 267-282.
- Rahmato, D. (1984). Agrarian reform in Ethiopia. Two-Tier Land Tenure System and Sustainable Economic Development, Uppsala: SIAS.
- Rahmato, D. (1994). Land policy in Ethiopia at crossroads. In: *Rahmato*, *D. (ed.). Land tenure and land policy in Ethiopia after the Derg*, Norway: Reprocentralen AVH.
- Rahmato, D. (2004). Searching for tenure security? The land system and new policy initiatives in Ethiopia. *FSS Discussion Paper No. 12*, Forum for Social Studies, Addis Ababa.

- Reardon, T., and Vosti, S.A. (1995). Links between rural poverty and the environment in developing countries: asset categories and investment poverty. *World Development*, 23(9): 1495-1506.
- Regassa, S. (2001). The economics of managing land resources towards sustainability in the highlands of Ethiopia. PhD Thesis.
- Renard, K.G., Laflen, J.M, Foster, G.R., and McCool, D.K. (1994).. The revised soil loss equation. p.105-124. In: Lal, R. (eds.), *Soil Erosion Research Methods*, The soil and Water Conservation Society and St. Lucie Press. USA. Second Edition.
- Rose, W.C. (1994). Research progress on soil erosion processes and a basis for soil conservation practices. The revised soil loss equation. p.159-178. In: Lal, R. (eds.). *Soil Erosion Research Methods*, The soil and Water Conservation Society and St. Lucie Press. USA, Second Edition.
- Ruben, R., Moll, H., and Kuyvenhoven, A. (1998). Integrating agricultural research and policy analysis: analytical framework and policy applications for bio-economic modeling. *Agricultural Systems*, 58(3): 331-349.
- Saliba, B.C. (1985). Soil productivity and farmer's erosion control incentives-a dynamic modeling approach. *Western Journal of Agricultural Economics*, 10: 354 364.
- Sanchez, P. (2000). Tropical soil and crop management: Key issues for policymakers. In: Raising Agricultural Productivity in the Tropics: Biophysical challenges for Technology and Policy, 16 17 October 2000. Center for International Development at Harvard University. http://www2.cid.harvard.edu/cidbiotech/ag/PedroSanchez.pdf
- Sanchez, P.A., Shepherd, K. D., Soule, M.J., Place, F.M., Buresh, R.J., Izac, A-M.N., Mokwunye, A.U., Kwesiga, F.R., Ndiritu, C.G., and Woomer, P.L. (1997). Soil fertility replenishment in Africa: an investment in natural resource capital. In: Buresh, R.J., Sanchez, P.A., and Calhoun, F. (eds.), *Replenishing Soil Fertility in Africa, SSSA Special Publication Number 51*, Soil Science Society of America, USA.

- Schulze, R.E. (1995). Hydrology and agrohydrology. A text to accompany the ARCU 3.00 agrohydrologyical modeling system. Department of Agricultural Engineering. University of Natal, Pietermaritzburg, South Africa.
- SCRP. (1996). *Data base report (1982-1993), Series IV*. Andit Tid Research Unit, University of Berne, Switzerland in association with The Ministry of Agriculture, Ethiopia.
- Shaffer, M.J., Schumacher, T.E. and Ego, C.L. (1994). Long-term effects of erosion and climate interactions on corn yield. *Journal of Soil and Water Conservation*, 49:272-275.
- Shane, M., Roe, T., Teigen, L., and Gopinath, M. (1997). World food insecurity: a policy dilemma, Economic Research Service/USDA.
- Shiferaw, B., and Holden, S. (1998). Resource degradation and adoption of land conservation technologies in the Ethiopian highlands: a case study in Andit Tid, North Shewa. *Agricultural Economics*, 18:233-247.
- Shiferaw, B., and Holden, S. (1999). Soil erosion and smallholders' conservation decisions in the highlands of Ethiopia. *World Development*, 27(4): 739-752.
- Shiferaw, B., and Holden, S. (2000). Policy instruments for sustainable land management: the case of highland smallholders in Ethiopia. *Agricultural Economics*, 22:217-232.
- Shiferaw, B., and Holden, S. (2001). Farm-level benefits to investment for mitigating land degradation: empirical evidence from Ethiopia. *Environment and Development Economics*, 6: 335-358.
- Shively, G.E. (2001). Poverty, consumption risk and soil conservation. *Journal of Development Economics*, 65: 267 290.
- Sonneveld, B.G.J.S., and Keyzer, M.A. (2003). Land under pressure: soil conservation concerns and opportunities for Ethiopia. *Land degradation & Development*, 14:5-23.

- Soule, M.J., Abebayehu, T., and Keith D. W. (2000). Land tenure and the adoption of conservation practices. *American Journal of Agricultural Economics*, 82(4): 993-1005.
- StataCorp. (2003). Stata base reference manual. Volume 4, S Z, release 8. Collage Station, TX: Stata Corporation.
- Stocking, M., and Tengberg, A. (1999). Erosion induced loss in soil productivity and its impacts on agricultural production and food security. In: Nabhan, H., Mashali, A.M., and Mermut, A.R. (eds.), *Integrated soil management for sustainable agriculture and food security in Southern and Eastern Africa*, Proceedings of the Expert Consultation, Harare, Zimbabwe, December 8-12, 1997, FAO/AGRITEX, Food and Agricultural Organization of the United Nations, Rome.
- Stocking, M.A. (1984). Erosion and soil productivity: a review. *Food and Agricultural Organization Consultants'*, *Working Paper No.1*, Soil Conservation Program, Land and Water Development Division, Food and Agricultural Organization, Rome, Italy.
- Stocking, M.A. (2003). Tropical soils and food security: the next 50 years. State of the planet, Science, Vol. 302, http://www. Sciencemag.org.
- Stoorvogel, J.J., and Smaling, E.M.A. (1990). Assessment of soil nutrient depletion in sub-Saharan Africa, 1983-2000. *Report 28*, DLO Winand starring Center for integrated land, soil and water research (CSC-DLO), Wageningen, Netherlands.
- Stroud, A., and Mekuria, M. (1992). Ethiopia's agricultural sector: an overview. p.9-27. In: Franzel, S., and Houten, van H. (eds.), *Research with Farmers: Lessons from Ethiopia*, C.A.B International, and Wallingford, UK.
- Sutcliffe, J.P. (1993). Economic assessment of land degradation in the Ethiopian Highlands: a case study. National Conservation Strategy Secretariat, Ministry of Planning and Economic Development, Transitional Government of Ethiopia, Addis Ababa.

- Tadesse, B., Admassie, A., and Croppenstdent, A. (2000). The impact of agricultural extension on farm productivity. *Ethiopian Journal of Agricultural Economics*, 4(1&2): 23-50.
- Tanner, D.G., Gorfu, A., Zewdie, L., and Taa, A. (1992). Developing technologies to improve soil fertility, weed control and wheat varieties. p.158-170. In: Franzel, S., and Houten, van H. (eds.), *Research with Farmers: Lessons from Ethiopia*, C.A.B International, Wallingford, UK.
- Tanner, D.G., Taa, A., and Girma, K. (1999). Determination of economic optimum fertilizer rates using discrete and continuous analytical methods. In: The Tenth Regional Wheat Workshop for Eastern, Central and Southern Africa, 273-297. Addis Ababa: CIMMYT.
- Teklu, T., and Lemi, A. (2004). Factors affecting entry and intensity in informal rental land markets in Southern Ethiopian highlands. *Agricultural Economics*, 30: 117-128
- Tiffen, M., Mortimore, M., and Gichuki, F. (1994). *More people, less erosion: environmental recovery in Kenya*, John Wily & Sons Ltd.
- Vella, F. (1998). Estimating models with sample selection bias: a survey. *Journal of Human Resources*, 33(1): 127-169.
- Waktola, A. (1980). Assessment of diffusion and adoption of agricultural technologies in Chilallo. *Ethiopian Journal of Agricultural Sciences*, 2(2): 51-68.
- White, H. (1980). A heteroscendasticity-consistent covariance matrix estimator and a direct test for heteroscendasticity. *Econometrica*, 48: 817–838.
- Wiebe, K. (1997). Resources, sustainability, and food security. *Food Security Assessment/GFA-9*, Economic Research Service/USDA.
- Wiebe, K. (2003). Linking land quality, agricultural productivity, and food security. Resource Economics Division, Economic Research Service, U.S. Department of Agriculture, (AER 823). http://www.ers.usda.gov/publications/aer823/

- Williams, J.R., Jones, C.A. and Dyke, P.T. (1984). A modeling approach to determining the relationship between erosion and soil productivity. Trans. ASAE: 129-44.
- Winship, C., and Mare, R.D. (1992). Models for sample selection bias. *Annual Review of Sociology*, 18:327-350.
- World Bank. (1994). World development reports, Oxford University Press.
- World Bank. (2004). Ethiopia at a glance. The World Bank Group: http://www.worldbank.org/data/ (accessed on 30 June 2004).
- Wu, J., and Babcock, B. A. (1998). The choice of tillage, rotation, and soil testing practices: economic and environmental implications. *American Journal of Agricultural Economics*, 80: 494-511.
- Yanggen, D., Kelly, V., Reardon, T., and Naseem, A. (1998). Incentives for fertilizer use in Sub-Saharan Africa: a review of empirical evidence or fertilizer response and profitability. *MSU International Development Working Paper No. 70*, Department of Agricultural Economics and Department of Economics, Michigan State University, East Lansing.
- Yirga, C., Alemayehu, F. and Sinebo, W. (1998). Barley-livestock production systems in Ethiopia: An overview. PP 1-10. In: Yirga, C., Alemayehu, F. and Sinebo, W. (eds.), *Barley-based farming systems in the highlands of Ethiopia*, EARO, Addis Ababa, Ethiopia.
- Yirga, C., and Hassena, M. (2001). Crop-livestock farming systems in the highlands of Ethiopia: smallholder farmers' management practices and constraints. p.145-165. In: Wall, P.C (ed.), Wheat and Weeds: Food and Feed, Proceedings of Two Stakeholder Workshops. Santa Cruz, Bolivia: CIMMYT.
- Yirga, C., Shapiro, B.I., and Demeke, M. (1996). Factors influencing adoption of new wheat technologies in Wolemera and Addis Alem areas of Ethiopia. *Ethiopian Journal of Agricultural Economics*, 1(1): 63-84.

- Young, R.A., Onstad, C.A., Bosch, D.D., Anderson, W.P. (1989). AGNPS: A Non-Point Source model for evaluating agricultural watersheds. *Journal of Soil and Water Conservation*, 44(2): 4522-4561.
- Zegeye, T., and Habtewold, D. (1995). Food security and situation analysis. In: Demeke, M., Amha, W., Ehui, S., Zegeye, T. (eds.), *Food Security, Nutrition and Poverty Alleviation in Ethiopia*, Addis Ababa.
- Zeleke, G. (2003). Resource use and poverty in the Ethiopian highlands. P.51-62. In: Amede, T. (ed.), *Natural Resource Degradation and Environmental Concerns in the Amhara National Reginal State: Impact on Food Security*, Proceedings of the Natural Resource Management Conference, July 24-26, 2002, Bahir Dar, Ethiopia.
- Zeller, M., Aliou, D., and Charles, M. (1998). Market access by smallholder farmers in Malawi: implications for technology adoption, agricultural productivity and crop income. *Agricultural Economics*, 19: 219-229.

#### **APPENDICES**

# Appendix I. Current Value Hamiltonian version of the soil nutrient mining control problem

The current value Hamiltonian of the nutrient mining problem is given by:

$$\begin{split} & H_{C}(F, L_{Y}, L_{S}, K_{Y}, \eta) = Pf(L_{Y}, K_{Y}, N) - \left(W_{F}F + W_{L}L_{Y} + W_{S}L_{S} + W_{K}K_{Y}\right) \\ & + \eta[G(F) - D(Y) + M(Z, L_{S}, Y)] \end{split} \tag{1.1}$$

Where: 
$$\eta = \rho^{\dot{\alpha}} \mu$$
 (1.2)

The FOC for this system:

$$\frac{\partial H_C}{\partial F} = -W_F + \eta \frac{\partial G}{\partial F} = 0 \Rightarrow W_F = \eta G_F \tag{1.3}$$

$$\frac{\partial H_C}{\partial L_v} = P \frac{\partial f}{\partial L_v} - W_L - \eta \frac{\partial D}{\partial L_v} + \eta \frac{\partial M}{\partial L_v} = 0$$
 (1.4)

$$\frac{\partial H_C}{\partial K_V} = P \frac{\partial f}{\partial K_V} - W_K - \eta \frac{\partial D}{\partial K_V} + \eta \frac{\partial M}{\partial K_V} = 0$$
 (1.5)

$$\frac{\partial H_C}{\partial L_S} = -W_S + \eta \frac{\partial M}{\partial L_S} = 0 \tag{1.6}$$

$$\dot{\eta} = \delta \eta - \frac{\partial H_C}{\partial N} = \delta \eta - P \frac{\partial f}{\partial N} + \eta (\frac{\partial D}{\partial N} - \frac{\partial M}{\partial N})$$
(1.7)

$$\frac{\partial H_c}{\partial \eta} = N = G(F) - D(Y) + M(Z, L_s, Y)$$
(1.8)

Assuming a steady state where  $\eta = N = 0$ , the first order conditions shown in equations (1.1 through 1.8) could be restated as follows:

$$\eta = \frac{W_F}{G_F} \tag{1.3b}$$

$$\eta = \frac{Pf_{L_{y}} - W_{L}}{D_{L_{y}} - M_{L_{y}}} \tag{1.4b}$$

$$\eta = \frac{Pf_{K_Y} - W_K}{D_{K_Y} - M_{K_Y}} \tag{1.5b}$$

$$\eta = \frac{W_{S}}{M_{L_{S}}} \tag{1.6b}$$

$$\eta = \frac{Pf_N}{[(D_N - M_N) + \delta]} \tag{1.7b}$$

$$G(F) = D(Y) - M(Z, L_{\varsigma}, Y)$$

$$\tag{1.8b}$$

Equation (1.8b) above states that at steady state, the net nutrient depletion through crop harvest, erosion and natural processes are matched by external nutrients added to the soil. Further combining equations 1.3b with each of 1.4b through 1.7b, the following equations are derived:

$$\frac{W_F}{G_F} = \frac{Pf_N}{[(D_N - M_N) + \delta]} \tag{1.9}$$

$$\frac{Pf_{L_{\gamma}} - W_{L}}{D_{L_{\gamma}} - M_{L_{\gamma}}} = \frac{Pf_{N}}{[(D_{N} - M_{N}) + \delta]}$$
(1.10)

$$\frac{Pf_{K_{Y}} - W_{K}}{D_{K_{Y}} - M_{K_{Y}}} = \frac{Pf_{N}}{[(D_{N} - M_{N}) + \delta]}$$
(1.11)

$$\frac{W_{S}}{M_{L_{S}}} = \frac{Pf_{N}}{[(D_{N} - M_{N}) + \delta]}$$
 (1.12)

Eliminating, common terms the following fundamental equation is derived:

$$\eta = \frac{W_F}{G_F} = \frac{Pf_N}{[(D_N - M_N) + \delta]} = \frac{Pf_{L_Y} - W_L}{D_{L_Y} - M_{L_Y}} = \frac{Pf_{K_Y} - W_{K_Y}}{D_{K_Y} - M_{K_Y}} = \frac{W_{L_s}}{M_{L_s}}$$
(1.13)

# Appendix II. Summary of specified functions and functional relationships used in the empirical soil degradation control model

The reduced form solutions of the control model for both scenarios (soil nutrient mining only) as well as (soil nutrient mining and physical soil degradation) are based on the following empirical specifications:

1. The production function:

$$Y = AL_Y^b K_Y^c SD^d N^g (2.1)$$

- 2. The aggregate soil regeneration and decay function (H)
  - 2.1. Relationship between soil conservation effort and erosion damage

$$E_{t} = \boldsymbol{z}e^{\alpha L_{S}} \tag{2.2}$$

2.2. Contribution of canopy to soil decay

$$J = \phi(1 - e^{-\nu Y}) \tag{2.3}$$

Accordingly combinning equations (2.2 and 2.3), we have

$$h = E_t - J = \pi e^{-\alpha L_S} - \phi (1 - e^{-\nu Y})$$
 (2.4)

The natural rate of soil regeneration, Z, is constant. The aggregate soil regeneration and decay function,  $H\left(Z,L_{S},Y\right)$  is given by

$$H = Z - h = Z - E + J$$

$$H = Z - \tau e^{-\alpha L_S} + \phi (1 - e^{-\nu Y})$$
 (2.5)

- 3. The nutrient regeneration and damage function
  - 3.1. The nutrient augmentation function, G(F)

$$G(F) = \beta_1 F \tag{2.6}$$

3.2. Nutrient depletion due to biomass removal

$$D(Y) = \beta_2 Y \tag{2.7}$$

3.3. Nutrient regeneration and damage due to erosion and natural processes

$$M = \beta_3 [Z - \tau e^{-\alpha L_S} + \phi (1 - e^{-\nu Y})]$$
 (2.8)

Accordingly the aggregate nutrient regeneration and damage function is given by

$$N = G(F) - D(Y) + M(Z, L_s, Y)$$
(2.9)

$$N = \beta_1 F - \beta_2 Y + \beta_3 [Z - \tau e^{-\alpha L_S} + \phi (1 - e^{-\nu Y})]$$
 (2.10)

Reduced form solutions for the optimality conditions derived in appendices III, IV and V are based on the following functional relationships.

The production function is specified as:

$$f(L_Y, K_Y, SD, N) = Y = AL_Y^b K_Y^c SD^d N^g$$
(2.11)

Accordingly, the respective marginal products (partial derivatives with respect to its arguments) are given by:

$$\frac{\partial f}{\partial L_{Y}} = f_{L_{Y}} = bAL_{Y}^{b-1}K_{Y}^{c}SD^{d}N^{g} = \frac{bY}{L_{Y}}$$
(2.12)

$$\frac{\partial f}{\partial K_{v}} = f_{K_{v}} = cAL_{y}^{b} K_{y}^{c-1} SD^{d} N^{g} = \frac{cY}{K_{v}}$$
(2.13)

$$\frac{\partial f}{\partial SD} = f_{SD} = dAL_Y^b K_Y^c SD^{d-1} N^g = \frac{dY}{SD}$$
(2.14)

$$\frac{\partial f}{\partial N} = f_N = gAL_Y^b K_Y^c SD^d N^{g-1} = \frac{gY}{N}$$
(2.15)

The respective partial derivatives of the soil regeneration and damage function are given by:

$$\frac{\partial H}{\partial L_{Y}} = H_{L_{Y}} = \phi v e^{-vY} \frac{\partial f}{\partial L_{Y}} = \varphi \frac{bY}{L_{Y}}$$
(2.16)

Where:  $\phi v e^{-vY} = \varphi$ 

$$\frac{\partial H}{\partial K_{v}} = H_{K_{v}} = \phi v e^{-vY} \frac{\partial f}{\partial K_{v}} = \varphi \frac{cY}{K_{v}}$$
(2.17)

$$\frac{\partial H}{\partial SD} = H_{SD} = \phi v e^{-vY} \frac{\partial f}{\partial SD} = \varphi \frac{dY}{SD}$$
 (2.18)

$$\frac{\partial H}{\partial N} = H_N = \phi v e^{-vY} \frac{\partial f}{\partial N} = \varphi \frac{gY}{N}$$
 (2.19)

$$\frac{\partial H}{\partial L_s} = H_{L_s} = \tau \alpha e^{-\alpha L_s} \tag{2.20}$$

Partial derivatives of the nutrient regeneration and damage function with respect to its arguments:

$$\frac{\partial M}{\partial L_{Y}} = M_{L_{Y}} = \beta_{3} \phi_{V} e^{-\nu Y} \frac{\partial f}{\partial L_{Y}} = \beta_{3} \varphi \frac{bY}{L_{Y}}$$
(2.21)

$$\frac{\partial M}{\partial K_Y} = M_{K_Y} = \beta_3 \phi v e^{-vY} \frac{\partial f}{\partial K_Y} = \beta_3 \phi \frac{cY}{K_Y}$$
(2.22)

$$\frac{\partial M}{\partial SD} = M_{SD} = \beta_3 \phi v e^{-vY} \frac{\partial f}{\partial SD} = \beta_3 \varphi \frac{dY}{SD}$$
 (2.23)

$$\frac{\partial M}{\partial N} = M_N = \beta_3 \phi v e^{-vY} \frac{\partial f}{\partial N} = \beta_3 \varphi \frac{gY}{N}$$
 (2.24)

$$\frac{\partial M}{\partial L_s} = M_{L_s} = \beta_3 \tau \alpha e^{-\alpha L_s} \tag{2.25}$$

Partial derivatives of the nutrient depletion function due to biomass removal, D(Y), with respect to its arguments:

$$\frac{\partial D}{\partial L_{\gamma}} = D_{L_{\gamma}} = \beta_2 \frac{\partial f}{\partial L_{\gamma}} = \beta_2 \frac{bY}{L_{\gamma}}$$
(2.26)

$$\frac{\partial D}{\partial K_{Y}} = D_{K_{Y}} = \beta_{2} \frac{\partial f}{\partial K_{Y}} = \beta_{2} \frac{cY}{K_{Y}}$$
(2.27)

$$\frac{\partial D}{\partial SD} = D_{SD} = \beta_2 \frac{\partial f}{\partial SD} = \beta_2 \frac{dY}{SD}$$
 (2.28)

$$\frac{\partial D}{\partial N} = D_N = \beta_2 \frac{\partial f}{\partial N} = \beta_2 \frac{gY}{N}$$
 (2.59)

Partial derivative of the nutrient augmentation function:

$$\frac{\partial G}{\partial F} = \beta_1 \tag{2.30}$$

Accordingly,

$$\frac{H_N}{H_{I_N}} = \frac{gL_Y}{bN} \tag{2.31}$$

$$\frac{H_N}{H_{K_Y}} = \frac{gK_Y}{cN} \tag{2.32}$$

$$\frac{H_N}{H_{L_s}} = \frac{\varphi g Y}{\tau \alpha e^{-\alpha L_s} N} \tag{2.33}$$

$$D_{L_{Y}} - M_{L_{Y}} = \frac{bY}{L_{V}} (\beta_{2} - \beta_{3} \phi v e^{-vY}) = \frac{bY}{L_{V}} \xi$$
(2.34)

Where:  $\beta_2 - \beta_3 \phi v e^{-vY} = \xi$ 

$$D_{K_{Y}} - M_{K_{Y}} = \frac{cY}{K_{Y}} \left( \beta_{2} - \beta_{3} \phi_{V} e^{-vY} \right) = \frac{cY}{K_{Y}} \xi$$
 (2.35)

$$D_{SD} - M_{SD} = \frac{dY}{SD} \left( \beta_2 - \beta_3 \phi_V e^{-\nu Y} \right) = \frac{dY}{SD} \xi \tag{2.36}$$

$$D_{N} - M_{N} = \frac{gY}{N} (\beta_{2} - \beta_{3} \phi v e^{-vY}) = \frac{gY}{N} \xi$$
 (2.37)

Given the above formulations, the optimal solutions derived from the first order conditions for the soil-mining scenario are specified below:

$$\frac{W_F}{G_F} = \frac{W_F}{\beta_1} \tag{2.38}$$

$$\frac{Pf_{L_{Y}} - W_{L}}{D_{L_{Y}} - M_{L_{Y}}} = \frac{PbY - W_{L}L_{Y}}{bY\xi}$$
(2.39)

$$\frac{Pf_{K_{Y}} - W_{K}}{D_{K_{Y}} - M_{K_{Y}}} = \frac{PcY - W_{K}K_{Y}}{cY\xi}$$
(2.40)

$$\frac{W_{L_S}}{M_{L_S}} = \frac{W_S}{\beta_3 \tau \alpha e^{-\alpha L_S}} \tag{2.41}$$

$$\frac{Pf_N}{[(D_N - M_N) + \delta]} = \frac{PgY}{gY\xi + \delta N}$$
(2.42)

$$G(F) = D(Y) - M(Z, L_S, Y) \Rightarrow \beta_1 F = \beta_2 Y - \beta_3 \left[ Z - \tau e^{-\alpha L_S} + \phi (1 - e^{-\nu Y}) \right]$$
 (2.43)

# Appendix III. Derivation of the reduced form solutions for the choice variables $(L_Y,\,L_S,\,K_Y\,\,and\,\,F)$ and the optimal nutrient stock (N) for the soil-mining scenario

Equating equation (2.39) with equation (2.40)

$$\frac{PbY - W_L L_Y}{bY\xi} = \frac{PcY - W_K K_Y}{cY\xi}$$

$$c(PbY - W_L L_Y) = b(PcY - W_K K_Y)$$

$$cW_L L_Y = bW_K K_Y$$

$$L_{Y} = \frac{bW_{K}K_{Y}}{cW_{I}} \tag{3.1a}$$

$$K_{Y} = \frac{cW_{L}L_{Y}}{bW_{K}} \tag{3.2a}$$

Equating equation (2.38) with equation (2.39) to solve for N and assuming  $\beta_1 = 1$ :

$$W_F = \frac{PbY - W_L L_Y}{bY\xi}$$

$$W_F b Y \xi = P b Y - W_L L_Y$$

$$W_L L_Y = PbY - W_F bY \xi$$

$$W_L L_Y = bY(P - W_F \xi)$$

$$\frac{bY}{W_L L_Y} = \frac{b(AL_Y^{b-1}K_Y^c N^g)}{W_L} = \frac{1}{(P - W_F \xi)}$$
(3.3)

Substituting equation (3.2a) into equation (3.3):

$$\left(\frac{b}{W_L}\right) A L_{\gamma}^{b-1} \left(\frac{cW_L}{W_K b}\right)^c L_{\gamma}^c N^g = \frac{1}{(P - W_F \xi)}$$

$$AL_{\gamma}^{(b+c-1)} \left(\frac{b}{W_{L}}\right)^{1-c} \left(\frac{c}{W_{K}}\right)^{c} N^{g} = \frac{1}{(P - W_{F}\xi)}$$

$$L_{Y} = \left(\frac{b}{W_{L}}\right)^{\frac{c-1}{b+c-1}} \left(\frac{c}{W_{K}}\right)^{\frac{-c}{b+c-1}} \left(\frac{1}{AN^{g}(P-W_{F}\xi)}\right)^{\frac{1}{b+c-1}}$$
(3.1b)

Substituting equation (3.1b) back to equation (3.2a)

$$K_{Y} = \left(\frac{c}{W_{K}}\right) \left(\frac{b}{W_{L}}\right)^{-1} \left[\left(\frac{b}{W_{L}}\right)^{\frac{c-1}{b+c-1}} \left(\frac{c}{W_{K}}\right)^{\frac{-c}{b+c-1}} \left(\frac{1}{AN^{g}(P-W_{F}\xi)}\right)^{\frac{1}{b+c-1}}\right]$$

$$K_{Y} = \left(\frac{b}{W_{L}}\right)^{\frac{-b}{b+c-1}} \left(\frac{c}{W_{K}}\right)^{\frac{b-1}{b+c-1}} \left(\frac{1}{AN^{g}(P-W_{F}\xi)}\right)^{\frac{1}{b+c-1}}$$
(3.2b)

Equating equation (2.38) with equation (2.42) to solve for N:

$$W_{F} = \frac{PgY}{gY\xi + \delta N}$$

$$W_F g Y \xi + W_F \delta N = P g Y$$

$$W_F \delta N = PgY - W_F gY \xi$$

$$gY(P-W_{E}\xi)=W_{E}\delta N$$

$$\frac{Y}{N} = AL_Y^b K_Y^c N^{g-1} = \frac{W_F \delta}{g(P - W_F \xi)}$$
 (3.4a)

Considering the LHS and substituting equations (3.1b and 3.2b)

$$AL_{Y}^{b} = A \left(\frac{b}{W_{L}}\right)^{\frac{bc-b}{b+c-1}} \left(\frac{c}{W_{K}}\right)^{\frac{-bc}{b+c-1}} \left(\frac{1}{AN^{g}(P-W_{F}\xi)}\right)^{\frac{b}{b+c-1}}$$

$$K_{Y}^{c}N^{g-1} = \left(\frac{b}{W_{L}}\right)^{\frac{-bc}{b+c-1}} \left(\frac{c}{W_{K}}\right)^{\frac{cb-c}{b+c-1}} \left(\frac{1}{AN^{g}(P-W_{F}\xi)}\right)^{\frac{c}{b+c-1}} N^{g-1}$$

Pulling the components of the LHS together and equating with the RHS

$$\left[ A^{-1} \left( \frac{b}{W_L} \right)^{-b} \left( \frac{c}{W_K} \right)^{-c} \left( \frac{1}{N^g (P - W_F \xi)} \right)^{b+c} \right]^{\frac{1}{b+c-1}} N^{g-1} = \frac{W_F \delta}{g (P - W_F \xi)}$$

$$N^{\frac{-g(b+c)}{b+c-1}}(N^{g-1}) = \left[A\left(\frac{c}{W_K}\right)^c \left(\frac{b}{W_L}\right)^b \left(P - W_F \xi\right)^{b+c}\right]^{\frac{1}{b+c-1}} \left(\frac{W_F \delta}{g(P - W_F \xi)}\right)$$

$$N^{\frac{1-g-c-b}{b+c-1}} = \left[A\left(\frac{c}{W_K}\right)^c \left(\frac{b}{W_L}\right)^b\right]^{\frac{1}{b+c-1}} \left(\frac{W_F \delta}{g}\right) (P - W_F \xi)^{\frac{1}{b+c-1}}$$

Let: 
$$1 - g - c - b = \varpi$$
,

Accordingly, substituting the above expression and solving for N:

$$N^{\frac{\varnothing}{b+c-1}} = A^{\frac{1}{b+c-1}} \left(\frac{c}{W_K}\right)^{\frac{c}{b+c-1}} \left(\frac{b}{W_L}\right)^{\frac{b}{b+c-1}} \left(\frac{W_F \delta}{g}\right) (P - W_F \xi)^{\frac{1}{b+c-1}}$$

$$N^* = A^{\frac{1}{\overline{\sigma}}} \left(\frac{c}{W_K}\right)^{\frac{c}{\overline{\sigma}}} \left(\frac{b}{W_L}\right)^{\frac{b}{\overline{\sigma}}} \left(\frac{W_F \delta}{g}\right)^{\frac{b+c-1}{\overline{\sigma}}} (P - W_F \xi)^{\frac{1}{\overline{\sigma}}}$$
(3.4b)

Substituting the values of N from equation (3.4b) into equation (3.1b) to solve for the optimal value of  $L_Y$ :

From equation (3.1b), we have

$$L_{Y} = \left(\frac{b}{W_{L}}\right)^{\frac{c-1}{b+c-1}} \left(\frac{c}{W_{K}}\right)^{\frac{-c}{b+c-1}} \left(\frac{1}{AN^{g}(P-W_{F}\xi)}\right)^{\frac{1}{b+c-1}}$$

Substituting equation (3.4b) to the above and solving for  $L_Y$ :

$$L_{Y} = \left(\frac{c}{W_{K}}\right)^{\frac{-c}{b+c-1}} \left(\frac{b}{W_{L}}\right)^{\frac{c-1}{b+c-1}} A^{\frac{-1}{b+c-1}} \left(P - W_{F} \xi\right)^{\frac{-1}{b+c-1}} \left[A^{\frac{1}{\varpi}} \left(\frac{c}{W_{K}}\right)^{\frac{c}{\varpi}} \left(\frac{b}{W_{L}}\right)^{\frac{b}{\varpi}} \left(\frac{W_{F} \delta}{g}\right)^{\frac{b+c-1}{\varpi}} \left(P - W_{F} \xi\right)^{\frac{-1}{\varpi}} \left(\frac{c}{b+c-1}\right)^{\frac{-2}{\varpi}} \left(\frac{b}{W_{L}}\right)^{\frac{b}{\varpi}} \left(\frac{W_{F} \delta}{g}\right)^{\frac{b+c-1}{\varpi}} \left(\frac{c}{b+c-1}\right)^{\frac{-2}{\varpi}} \left(\frac{b}{W_{L}}\right)^{\frac{-2}{\varpi}} \left(\frac{b}{W_{L}}\right)^{\frac{b}{\varpi}} \left(\frac{w_{F} \delta}{g}\right)^{\frac{-2}{\varpi}} \left(\frac{b}{W_{L}}\right)^{\frac{-2}{\varpi}} \left(\frac{b}{W_{L}}$$

$$L_{Y}^{*} = A^{\frac{1}{\overline{\sigma}}} \left(\frac{c}{W_{K}}\right)^{\frac{c}{\overline{\sigma}}} \left(\frac{b}{W_{L}}\right)^{\frac{\overline{\sigma}(c-1)-bg}{\overline{\sigma}(b+c-1)}} \left(\frac{W_{F}\delta}{g}\right)^{\frac{-g}{\overline{\sigma}}} \left(P - W_{F}\xi\right)^{\frac{1}{\overline{\sigma}}}$$
(3.1c)

Substituting the values of N from equation (3.4b) to equation (3.2b) to solve for the optimal value of  $K_Y$ :

From equation (3.2b), we have

$$K_{Y} = \left(\frac{b}{W_{L}}\right)^{\frac{-b}{b+c-1}} \left(\frac{c}{W_{K}}\right)^{\frac{b-1}{b+c-1}} \left(\frac{1}{AN^{g}(P-W_{F}\xi)}\right)^{\frac{1}{b+c-1}}$$

Substituting equation (3.4b) to the above and solving for  $K_Y$ :

$$\mathbf{K}_{\mathbf{Y}} = \left(\frac{c}{W_{K}}\right)^{\frac{b-1}{b+c-1}} \left(\frac{b}{W_{L}}\right)^{\frac{-b}{b+c-1}} A^{\frac{-1}{b+c-1}} \left(P - W_{F} \xi\right)^{\frac{-1}{b+c-1}} \left[A^{\frac{1}{\omega}} \left(\frac{c}{W_{K}}\right)^{\frac{c}{\varpi}} \left(\frac{b}{W_{L}}\right)^{\frac{b}{\varpi}} \left(\frac{W_{F} \delta}{g}\right)^{\frac{b+c-1}{\varpi}} \left(P - W_{F} \xi\right)^{\frac{1}{\varpi}} \right]^{\frac{-g}{b+c-1}} \left[A^{\frac{1}{\omega}} \left(\frac{c}{W_{K}}\right)^{\frac{c}{\varpi}} \left(\frac{b}{W_{L}}\right)^{\frac{b}{\varpi}} \left(\frac{W_{F} \delta}{g}\right)^{\frac{b+c-1}{\varpi}} \left(P - W_{F} \xi\right)^{\frac{1}{\varpi}} \right]^{\frac{-g}{b+c-1}} \left(P - W_{F} \xi\right)^{\frac{1}{\omega}} \left(\frac{c}{W_{K}}\right)^{\frac{1}{\omega}} \left(\frac{c}{W_{K}}\right)^{\frac{b}{\varpi}} \left(\frac{c}{W_{K}}\right)^{\frac{b}{\varpi}} \left(\frac{c}{W_{K}}\right)^{\frac{1}{\omega}} \left(\frac{c}{W_{K}}\right)^{\frac{1}{\omega$$

$$K_{Y}^{*} = A^{\frac{1}{\omega}} \left(\frac{c}{W_{K}}\right)^{\frac{\varpi(b-1)-cg}{\varpi(b+c-1)}} \left(\frac{b}{W_{L}}\right)^{\frac{b}{\varpi}} \left(\frac{W_{F}\delta}{g}\right)^{\frac{-g}{\varpi}} (P - W_{F}\xi)^{\frac{1}{\varpi}}$$
(3.2c)

Equating equation (2.38) with equation (2.41) to solve for L<sub>S</sub>:

$$W_F = \frac{W_S}{\beta_3 \tau \alpha e^{-\alpha L_S}}$$

$$W_F \beta_3 \tau \alpha e^{-\alpha L_S} = W_S$$

$$e^{-\alpha L_{S}} = \frac{W_{S}}{W_{F} \beta_{3} \tau \alpha}$$

$$e^{\alpha L_{S}} = \frac{W_{F} \beta_{3} \tau \alpha}{W_{S}}$$

$$\alpha L_{S} = \ln \left( \frac{W_{F} \beta_{3} \tau \alpha}{W_{S}} \right)$$

$$L_{S}^{*} = \left( \frac{1}{\alpha} \right) \ln \left( \frac{W_{F} \beta_{3} \tau \alpha}{W_{S}} \right)$$
(3.5)

Given the optimal values for  $L_Y$ ,  $K_Y$  and N above, the optimal output at a desirable steady state is given by:

$$Y^* = AL_Y^{*b} K_Y^{*c} N^{*g}$$
 (3.6)

Where :  $L_Y^*$ ,  $K_Y^*$ ,  $N^*$  are given by equations (3.1c, 3.2c and 3.4b), respectively.

Solving equation (2.43) provides the steady state optimal fertilizer use as follows

$$\beta_{1}F = \beta_{2}Y - \beta_{3} \left[ Z - \tau e^{-\alpha L_{S}} + \phi (1 - e^{-\nu Y}) \right]$$

$$F^{*} = \left\{ \beta_{2}Y^{*} - \beta_{3} \left[ Z - \tau e^{-\alpha L_{S}^{*}} + \phi (1 - e^{-\nu Y}) \right] \right\} / \beta_{1}$$
(3.7)

Where :  $Y^*$  and  $L_S^*$  are given by equations (3.6 and 3.5), respectively.

### University of Pretoria etd – Tizale, C Y (2007) Appendix IV. Current Value Hamiltonian Version and Derivation of Reduced form Solutions for the problem of physical soil degradation and nutrient mining (scenario II)

The current value Hamiltonian of this scenario is given by:

$$\Pi_{C}(F, L_{Y}, L_{S}, K_{Y}, SD, N, \psi, \eta) = Pf(L_{Y}, K_{Y}, SD, N) - (W_{F}F + W_{L}L_{Y} + W_{S}L_{S} + W_{K}K_{Y}) + \psi[H(Z, L_{S}, Y)] + \eta[G(F) - D(Y) + M(Z, L_{S}, Y)]$$
(4.1)

Where: 
$$\psi = e^{\delta} \lambda$$
 and  $\eta = e^{\delta} \mu$  (4.2)

The FOC for this system:

$$\frac{\partial \Pi_C}{\partial F} = -W_F + \eta \frac{\partial G}{\partial F} = 0 \tag{4.3}$$

$$\frac{\partial \Pi_C}{\partial L_Y} = P \frac{\partial f}{\partial L_Y} - W_L + \psi \frac{\partial H}{\partial L_Y} - \eta \frac{\partial D}{\partial L_Y} + \eta \frac{\partial M}{\partial L_Y} = 0 \tag{4.4}$$

$$\frac{\partial \Pi_{C}}{\partial K_{Y}} = P \frac{\partial f}{\partial K_{Y}} - W_{K} + \psi \frac{\partial H}{\partial K_{Y}} - \eta \frac{\partial D}{\partial K_{Y}} + \eta \frac{\partial M}{\partial K_{Y}} = 0 \tag{4.5}$$

$$\frac{\partial \Pi_C}{\partial L_c} = -W_L + \psi \frac{\partial H}{\partial L_c} + \eta \frac{\partial M}{\partial L_c} = 0 \tag{4.6}$$

$$\dot{\psi} = \delta \psi - \frac{\partial \Pi_C}{\partial SD} = \delta \psi - P \frac{\partial f}{\partial SD} - \psi \frac{\partial H}{\partial SD} + \eta (\frac{\partial D}{\partial SD} - \frac{\partial M}{\partial SD})$$
(4.7)

$$\dot{\eta} = \delta \eta - \frac{\partial \Pi_C}{\partial N} = \delta \eta - P \frac{\partial f}{\partial N} - \psi \frac{\partial H}{\partial N} + \eta (\frac{\partial D}{\partial N} - \frac{\partial M}{\partial N})$$
(4.8)

$$\frac{\partial \Pi_c}{\partial w} = \vec{SD} = Z - E + J \tag{4.9}$$

$$\frac{\partial \Pi_c}{\partial \eta} = \stackrel{\bullet}{N} = G(F) - D(Y) + M(Z, L_S, Y) \tag{4.10}$$

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In a steady state the rate of change of the resource stock and hence the implicit prices are zero. That is  $\psi = \dot{\eta} = \dot{N} = \dot{SD} = 0$ .

The first order conditions could, therefore, be restated as follows:

$$\eta = \frac{W_F}{G_F} \tag{4.3b}$$

$$\eta = \frac{(Pf_{L_{Y}} - W_{L}) + \psi H_{L_{y}}}{D_{L_{y}} - M_{L_{y}}}$$
(4.4b)

$$\eta = \frac{(Pf_{K_Y} - W_K) + \psi H_{K_Y}}{D_{K_Y} - M_{K_Y}}$$
(4.5b)

$$\eta = \frac{W_s - \psi H_{L_s}}{M_{L_s}} \tag{4.6b}$$

$$\eta = \frac{Pf_{SD} + \psi (H_{SD} - \delta)}{D_{SD} - M_{SD}}$$
 (4.7b)

$$\eta = \frac{Pf_N + \psi H_N}{D_N - M_N + \delta} \tag{4.8b}$$

$$Z = E - J \tag{4.9b}$$

$$G(F) = D(Y) - M(Z, L_s, Y)$$
(4.10b)

Equation (4.9b) above states that at the steady state the net rate of natural soil regeneration (Z) is exactly matched by the net soil loss due to erosion and cultivation (E-J). Analogously, equation (4.10b) describes that the sum of nutrients lost through crop harvest, damage function D(Y) and net nutrient gains/losses though M are matched by the nutrients added to the soil through the nutrient augmentation function G(F).

University of Pretoria etd – Tizale, C Y (2007) Combining equations 4.3b with each of 4.4b through 4.8b and eliminating  $\eta$ , the following equations are derived:

$$\psi = \frac{(W_F/G_F)(D_{L_Y} - M_{L_Y}) - (Pf_{L_Y} - W_L)}{H_{L_Y}}$$
(4.11)

$$\psi = \frac{-[(W_F/G_F)M_{L_S} - W_S]}{H_{L_S}}$$
(4.12)

$$\psi = \frac{(W_F/G_F)(D_{K_Y} - M_{K_Y}) - (Pf_{K_Y} - W_K)}{H_{K_Y}}$$
(4.13)

$$\psi = \frac{(W_F/G_F)(D_{SD} - M_{SD}) - Pf_{SD}}{H_{SD} - \delta}$$
(4.14)

$$\psi = \frac{(W_F/G_F)(D_N - M_N + \delta) - Pf_N}{H_N}$$
(4.15)

Further combining equations (4.11 through 4.13) first with Equations (4.14) and then with equation (4.15), the following equations are derived.

$$Pf_{N} + \frac{H_{N}}{H_{L_{Y}}} \left[ \left( \frac{W_{F}}{G_{F}} \right) (D_{L_{Y}} - M_{L_{Y}}) - (Pf_{L_{Y}} - W_{L}) \right] = \frac{W_{F}}{G_{F}} (\delta + D_{N} - M_{N})$$
(4.16)

$$Pf_{N} + \frac{H_{N}}{H_{K_{V}}} \left[ \left( \frac{W_{F}}{G_{F}} \right) (D_{K} - M_{K}) - (Pf_{K} - W_{K}) \right] = \frac{W_{F}}{G_{F}} (\delta + D_{N} - M_{N})$$
(4.17)

$$Pf_{N} + \frac{H_{N}}{H_{L_{S}}} \left[ (W_{S} - M_{L_{S}}) \left( \frac{W_{F}}{G_{F}} \right) \right] = \frac{W_{F}}{G_{F}} \left( \delta + D_{N} - M_{N} \right)$$

$$(4.18)$$

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$$Pf_{SD} + \left(\frac{H_{SD} - \delta}{H_{L_Y}}\right) \left[\left(\frac{W_F}{G_F}\right)(D_{L_Y} - M_{L_Y}) - (Pf_{L_Y} - W_L)\right] = \frac{W_F}{G_F}(D_{SD} - M_{SD})$$
(4.19)

$$Pf_{SD} + \left(\frac{H_{SD} - \delta}{H_{K_{Y}}}\right) \left[\left(\frac{W_{F}}{G_{F}}\right)(D_{K} - M_{K}) - (Pf_{K} - W_{K})\right] = \frac{W_{F}}{G_{F}}(D_{SD} - M_{SD})$$
(4.20)

$$Pf_{SD} + \left(\frac{H_{SD} - \delta}{H_{L_s}}\right) \left[ (W_S - M_{L_s}) \left(\frac{W_F}{G_F}\right) \right] = \frac{W_F}{G_F} \left( D_{SD} - M_{SD} \right)$$
(4.21)

The above equations along with equations (4.9b) and (4.10b) form a system of eight equations with eight unknowns and could be solved simultaneously for optimal steady state values of the four choice variables (F,  $L_Y$ ,  $K_Y$ ,  $L_S$ ), the optimal resource stocks (SD and N) and the respective dynamic prices ( $\Psi$  and  $\eta$ ).

Given the above formulations and the functional relationships given in Appendix II equations (4.16 - 4.21) and (4.9b and 4.10b) are specified as follows:

$$PgY + \frac{g}{h} [W_F bY \xi - PbY + W_L L_Y)] = W_F (\partial V + gY \xi)$$

$$\tag{4.22}$$

$$PgY + \frac{g}{c} \left[ W_F cY \xi - PcY + W_K K_Y \right] = W_F \left( \delta N + gY \xi \right)$$

$$\tag{4.23}$$

$$PgY + \frac{\varphi gY}{\tau \alpha e^{-\alpha L_{s}}} \left[ W_{s} - \beta_{3} \tau \alpha e^{-\alpha L_{s}} \right] = W_{F} \left( \delta N + gY \xi \right)$$

$$(4.24)$$

$$PdY + \left(\frac{\varphi dY - \delta SD}{\varphi bY}\right) \left[W_F bY \xi - PbY + W_L L_{L_Y}\right] = W_F dY \xi \tag{4.25}$$

$$PdY + \left(\frac{\varphi dY - \delta SD}{\varphi cY}\right) \left[W_F cY \xi - PcY + W_K K_{Y}\right] = W_F dY \xi \tag{4.26}$$

$$PdY + \left(\frac{\varphi dY - \delta SD}{\tau \alpha e^{-\alpha L_{S}}}\right) \left[W_{S} - \beta_{3} \tau \alpha e^{-\alpha L_{S}}\right] = W_{F} dY \xi \tag{4.27}$$

$$Z = \tau e^{-\alpha L_s} - \phi (1 - e^{-\nu Y}) \tag{4.28}$$

$$\beta_1 F = \beta_2 Y - \beta_3 [Z - \tau e^{-\alpha L_S} + \phi (1 - e^{\nu Y})] \tag{4.29}$$

Using the above formulations, the reduced form solutions for the choice variables ( $L_Y$ ,  $L_S$ ,  $K_Y$  and F) and the optimal nutrient stocks (N and SD) are solved as follows:

Using equation (4.22) to solve for  $L_Y$ :

$$PgY + \left(\frac{g}{b}\right)(W_FbY\xi - PbY + W_LL_Y) = W_F\delta N + W_FgY\varepsilon$$

$$PgY + gW_{\scriptscriptstyle F}Y\xi - gPY + \frac{g}{b}W_{\scriptscriptstyle L}L_{\scriptscriptstyle Y} = W_{\scriptscriptstyle F}\delta\!N + W_{\scriptscriptstyle F}gY\varepsilon$$

$$\frac{g}{h}W_L L_Y = W_F \delta N$$

$$L_{Y} = \frac{W_{F} \delta b}{g W_{L}} N = \left(\frac{W_{F} \delta}{g}\right) \left(\frac{b}{W_{L}}\right) N \tag{4.30a}$$

Using equation (4.23) to solve for  $K_Y$ :

$$PgY + \left(\frac{g}{c}\right)(W_F cY \xi - PcY + W_K K_Y) = W_F \delta N + W_F gY \varepsilon$$

$$PgY + gW_{\scriptscriptstyle F}Y\xi - gPY + \frac{g}{c}W_{\scriptscriptstyle K}K_{\scriptscriptstyle Y} = W_{\scriptscriptstyle F}\delta\!N + W_{\scriptscriptstyle F}gY\varepsilon$$

$$\frac{g}{c}W_{K}K_{Y} = W_{F}\delta N$$

$$K_{Y} = \frac{W_{F} \delta c}{g W_{K}} N = \left(\frac{W_{F} \delta}{g}\right) \left(\frac{c}{W_{K}}\right) N \tag{4.31a}$$

## University of Pretoria etd – Tizale, C Y (2007) Using equation (4.25) to solve for SD:

$$PdY + \left(\frac{\varphi dY - \partial SD}{\varphi bY}\right) (W_F bY \xi - PbY + W_L L_Y) = W_F dY \varepsilon$$

 $\varphi b Y P dY + \varphi dY W_F b Y \xi - \varphi dY P bY + \varphi dY W_L L_Y - \delta SDW_F b Y \xi + \delta SDP bY - \delta SDW_L L_Y = W_F dY \xi \varphi bY$ 

$$\varphi dYW_LL_Y - \delta SDW_FbY\xi + \delta SDPbY - \delta SDW_LL_Y = 0$$

$$\varphi dYW_L L_Y - \delta SDW_F bY \xi + \delta SDP bY = \delta SDW_L L_Y$$

$$Y(\varphi dW_{L_{V}} - \delta SDW_{E}b\xi + \delta SDPb) = \delta SDW_{L_{V}}$$

$$Y = \frac{\delta SDW_L L_Y}{(\varphi dW_L L_Y - \delta SDW_F b\xi + \delta SDPb)}$$

$$\frac{Y}{L_v} = \frac{\delta SDW_L}{b \delta SD(P - W_E \xi) + \varphi dW_t L_v}$$
(4.32)

Equating equations (4.25 and 4.27) and solving

$$PdY + \left(\frac{\varphi dY - \delta SD}{\varphi bY}\right) \left(W_F bY \xi - PbY + W_L L_Y\right) = PdY + \left(\frac{\varphi dY - \delta SD}{\gamma \alpha e^{-\alpha L_S}}\right) \left(W_L - \beta_3 \gamma \alpha e^{-\alpha L_S}\right)$$

$$W_{F}bY\xi - PbY + W_{L}L_{Y} = \frac{\varphi bY}{\gamma \alpha e^{-\alpha L_{S}}} \left(W_{L} - \beta_{3} \gamma \alpha e^{-\alpha L_{S}}\right)$$

$$W_F bY \xi - PbY + W_L L_Y = \frac{\varphi bY W_L e^{\alpha L_S}}{\gamma \alpha} - \beta_3 \varphi bY$$

$$\frac{\varphi b Y W_L e^{\alpha L_S}}{\gamma \alpha} - \beta_3 \varphi b Y - W_F b Y \xi + P b Y = W_L L_Y$$

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$$bY \left( \frac{\varphi W_L e^{\alpha L_S}}{\gamma \alpha} - \beta_3 \varphi - W_F \xi + P \right) = W_L L_Y$$

$$\frac{Y}{L_{Y}} = \frac{W_{L}}{b \left[ \varphi \left( \frac{W_{L} e^{\alpha L_{S}}}{\gamma \alpha} - \beta_{3} \right) + (P - W_{F} \xi) \right]}$$

Let: 
$$\varphi \left( \frac{W_L e^{\alpha L_S}}{\gamma \alpha} - \beta_3 \right) = \zeta$$
, hence

$$\frac{Y}{L_Y} = \frac{W_L}{b(\zeta + P - W_F \xi)} \tag{4.33}$$

Further equating equation (4.32 and 4.33)

$$\frac{\partial SDW_L}{b \, \partial SD(P - W_F \xi) + \varphi dW_L L_V} = \frac{W_L}{b(\zeta + P - W_F \xi)}$$

$$\delta SDb(\zeta + P - W_F \xi) = b \delta SD(P - W_F \xi) + \varphi dW_L L_Y$$

$$\delta SDb(\zeta + P - W_F \xi) - b \delta SD(P - W_F \xi) = \varphi dW_L L_Y$$

$$\delta SDb(\zeta + P - W_F \xi - P + W_F \xi) = \varphi dW_L L_Y$$

$$\delta SDb\zeta = \varphi dW_L L_Y$$

$$SD = \left(\frac{\varphi d}{\delta \zeta}\right) \left(\frac{W_L}{h}\right) L_{\gamma} \tag{4.34a}$$

Substituting equations (4.30a) into the above equation (4.34a)

$$SD = \left(\frac{\varphi d}{\delta \zeta}\right) \left(\frac{W_L}{b}\right) \left(\frac{W_F \delta}{g}\right) \left(\frac{b}{W_L}\right) N$$

$$SD = \left(\frac{\varphi d}{\delta \zeta}\right) \left(\frac{W_F \delta}{g}\right) N \tag{4.34b}$$

Solving for N using equations (4.30a, 4.31a, 4.33 and 4.34b)

From equation (4.33), we have

$$\frac{Y}{L_{Y}} = \frac{W_{L}}{b(\zeta + P - W_{F}\xi)}$$

Y has been specified as:  $Y = AL_Y^b K_Y^c N^g SD^g$ 

Hence, 
$$\frac{Y}{L_Y} = AL_Y^{b-1}K_Y^cN^gSD^g = \left(\frac{W_L}{b}\right)\left(\frac{1}{\zeta + P - W_F\xi}\right)$$

Substituting equations (4.30a, 4.31a, and 4.34b) for L<sub>Y</sub>, K<sub>Y</sub> and SD, respectively, into the above equation and solving for N:

$$A\left[\left(\frac{W_{F}\delta}{g}\right)\left(\frac{b}{W_{L}}\right)N\right]^{b-1}\left[\left(\frac{W_{F}\delta}{g}\right)\left(\frac{c}{W_{K}}\right)N\right]^{c}\left[\left(\frac{\varphi d}{\delta \zeta}\right)\left(\frac{W_{F}\delta}{g}\right)N\right]^{d}\left(N\right)^{g} = \left(\frac{W_{L}}{b}\right)\left(\frac{1}{\zeta + P - W_{F}\xi}\right)$$

$$A \left(\frac{W_F \delta}{g}\right)^{d+c+b-1} \left(\frac{b}{W_L}\right)^{\text{b-1}} \left(\frac{c}{W_K}\right)^{\text{c}} \left(\frac{\varphi d}{\delta \zeta}\right)^{\text{d}} N^{g+d+c+b-1} = \left(\frac{W_L}{b}\right) \left(\frac{1}{\zeta + P - W_F \xi}\right)$$

Let:  $g + d + c + b - 1 = \theta$ , Substituting and solving for N:

$$N^{\theta} = A^{-1} \left( \frac{W_F \delta}{g} \right)^{-(d+c+b-1)} \left( \frac{b}{W_L} \right)^{-b} \left( \frac{c}{W_K} \right)^{-c} \left( \frac{\varphi d}{\delta \zeta} \right)^{-d} \left( \frac{1}{\zeta + P - W_F \xi} \right)$$

$$N^* = A^{\frac{-1}{\theta}} \left( \frac{W_F \delta}{g} \right)^{-\left(\frac{d+c+b-1}{\theta}\right)} \left( \frac{b}{W_L} \right)^{\frac{-b}{\theta}} \left( \frac{c}{W_K} \right)^{\frac{-c}{\theta}} \left( \frac{\varphi d}{\delta \zeta} \right)^{\frac{-d}{\theta}} \left( \frac{1}{\zeta + P - W_F \xi} \right)^{\frac{1}{\theta}}$$
(4.35)

Substituting the value of N from equation (4.35) into equation (4.30a) to solve for L<sub>Y</sub>:

From equation (4.30a), we have,

$$L_{Y} = \left(\frac{W_{F}\delta}{g}\right) \left(\frac{b}{W_{L}}\right) N$$

Substituting equation (4.35) into the above equation and solving for  $L_Y$ :

$$L_{Y} = \left(\frac{W_{F}\delta}{g}\right) \left(\frac{b}{W_{L}}\right) \left[A^{\frac{-1}{\theta}} \left(\frac{W_{F}\delta}{g}\right)^{-\left(\frac{d+c+b-1}{\theta}\right)} \left(\frac{b}{W_{L}}\right)^{\frac{-b}{\theta}} \left(\frac{c}{W_{K}}\right)^{\frac{-c}{\theta}} \left(\frac{\varphi d}{\delta \zeta}\right)^{\frac{-d}{\theta}} \left(\frac{1}{\zeta + P - W_{F}\xi}\right)^{\frac{1}{\theta}}\right]$$

$$L_{Y}^{*} = A^{\frac{-1}{\theta}} \left( \frac{W_{F} \delta}{g} \right)^{\frac{g}{\theta}} \left( \frac{b}{W_{L}} \right)^{\frac{g+d+c-1}{\theta}} \left( \frac{c}{W_{K}} \right)^{\frac{-c}{\theta}} \left( \frac{\varphi d}{\delta \zeta} \right)^{\frac{-d}{\theta}} \left( \frac{1}{\zeta + P - W_{F} \xi} \right)^{\frac{1}{\theta}}$$
(4.30b)

Likewise substituting the value of N from equation (4.35) into equation (4.31a) to solve for  $K_Y$ :

From equation (4.31a), we have,

$$K_{Y} = \left(\frac{W_{F}\delta}{g}\right)\left(\frac{c}{W_{K}}\right)N$$

Substituting equation (4.34) into the above equation

$$K_{Y} = \left(\frac{W_{F}\delta}{g}\right) \left(\frac{c}{W_{K}}\right) \left[A^{\frac{-1}{\theta}} \left(\frac{W_{F}\delta}{g}\right)^{-\left(\frac{d+c+b-1}{\theta}\right)} \left(\frac{b}{W_{L}}\right)^{\frac{-b}{\theta}} \left(\frac{c}{W_{K}}\right)^{\frac{-c}{\theta}} \left(\frac{\varphi d}{\delta \zeta}\right)^{\frac{-d}{\theta}} \left(\frac{1}{\zeta + P - W_{F}\xi}\right)^{\frac{1}{\theta}}\right]$$

$$K_{\gamma}^{*} = A^{\frac{-1}{\theta}} \left( \frac{W_{F} \delta}{g} \right)^{\frac{g}{\theta}} \left( \frac{b}{W_{L}} \right)^{\frac{-b}{\theta}} \left( \frac{c}{W_{K}} \right)^{\frac{g+d+b-1}{\theta}} \left( \frac{\varphi d}{\delta \zeta} \right)^{\frac{-d}{\theta}} \left( \frac{1}{\zeta + P - W_{F} \xi} \right)^{\frac{1}{\theta}}$$
(4.31b)

Substituting the value of N from equation (4.35) into equation (4.34b) to solve for the optimal value of SD:

From equation (4.34b), we have,

$$SD = \left(\frac{\varphi d}{\delta}\right) \left(\frac{1}{\zeta}\right) \left(\frac{W_F \delta}{g}\right) N$$

Substituting equation (4.34) into the above equation

$$SD = \left(\frac{\varphi d}{\delta \zeta}\right) \left(\frac{W_F \delta}{g}\right) A^{\frac{-1}{\theta}} \left(\frac{W_F \delta}{g}\right)^{-\left(\frac{d+c+b-1}{\theta}\right)} \left(\frac{b}{W_L}\right)^{\frac{-b}{\theta}} \left(\frac{c}{W_K}\right)^{\frac{-c}{\theta}} \left(\frac{\varphi d}{\delta \zeta}\right)^{\frac{-d}{\theta}} \left(\frac{1}{\zeta + P - W_F \xi}\right)^{\frac{1}{\theta}}$$

$$SD^* = A^{\frac{-1}{\theta}} \left( \frac{W_F \delta}{g} \right)^{\frac{g}{\theta}} \left( \frac{b}{W_L} \right)^{\frac{-b}{\theta}} \left( \frac{c}{W_K} \right)^{\frac{-c}{\theta}} \left( \frac{\varphi d}{\delta \zeta} \right)^{\frac{g+c+b-1}{\theta}} \left( \frac{1}{\zeta + P - W_F \xi} \right)^{\frac{1}{\theta}}$$
(4.34c)

Solving for the optimal value of  $L_S$  using equations (4.24 and 4.27)

From equation (4.27)

$$PdY + \left(\frac{\varphi dY - \partial SD}{\gamma \alpha e^{-\alpha L_S}}\right) \left(W_S - \beta_3 \gamma \alpha e^{-\alpha L_S}\right) = W_F dY \varepsilon$$

$$PdY + (\varphi dY - \delta SD) \left( \frac{W_S e^{\alpha L_S}}{\gamma \alpha} - \beta_3 \right) = W_F dY \varepsilon$$

$$PdY + \frac{\varphi dYW_{S}e^{\alpha L_{S}}}{\gamma \alpha} - \varphi dY\beta_{3} - \frac{\delta SDW_{S}e^{\alpha L_{S}}}{\gamma \alpha} + \delta SD\beta_{3} = W_{F}dY\varepsilon$$

$$PdY + \frac{\varphi dYW_{S}e^{\alpha L_{S}}}{\gamma \alpha} - \varphi dY\beta_{3} - W_{F}dY\varepsilon = \frac{\delta SDW_{S}e^{\alpha L_{S}}}{\gamma \alpha} - \delta SD\beta_{3}$$

$$dY \left( P + \frac{\varphi W_{S} e^{\alpha L_{S}}}{\gamma \alpha} - \varphi \beta_{3} - W_{F} \varepsilon \right) = \delta SD \left( \frac{W_{S} e^{\alpha L_{S}}}{\gamma \alpha} - \beta_{3} \right)$$

$$Y = \frac{\delta SD\left(\frac{W_{S}e^{\alpha L_{S}}}{\gamma \alpha} - \beta_{3}\right)}{d\left(P + \frac{\varphi W_{S}e^{\alpha L_{S}}}{\gamma \alpha} - \varphi \beta_{3} - W_{F}\varepsilon\right)} = \left(\frac{\delta SD}{d}\right) \left[\frac{\left(\frac{W_{S}e^{\alpha L_{S}}}{\gamma \alpha} - \beta_{3}\right)}{\varphi\left(\frac{W_{S}e^{\alpha L_{S}}}{\gamma \alpha} - \beta_{3}\right) + \left(P - W_{F}\varepsilon\right)}\right]$$

Since :  $\varphi\left(\frac{W_S e^{\alpha L_S}}{\gamma \alpha} - \beta_3\right) = \zeta$ , the above equation could be rewritten as

$$Y = \left(\frac{\delta}{d}\right) \frac{\left(\frac{W_s e^{\alpha L_s}}{\gamma \alpha} - \beta_3\right)}{\left(\zeta + P - W_{\scriptscriptstyle E} \varepsilon\right)} SD \tag{4.36}$$

Similarly using equation (4.24)

$$PgY + \left(\frac{\varphi gY}{\gamma \alpha e^{-\alpha L_S}}\right) \left(W_S - \beta_3 \gamma \alpha e^{-\alpha L_S}\right) = W_F(\delta N + gY\varepsilon)$$

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$$PgY + \frac{\varphi gYW_S}{\gamma \alpha e^{-\alpha L_S}} - \beta_3 \varphi gY = W_F \delta N + W_F gY \varepsilon$$

$$PgY + \frac{\varphi gYW_S}{\gamma \alpha e^{-\alpha L_S}} - \beta_3 \varphi gY - W_F gY \varepsilon = W_F \delta N$$

$$gY\left(P + \frac{\varphi W_S}{\gamma \alpha e^{-cL_S}} - \beta_3 \varphi - W_F \varepsilon\right) = W_F \delta N$$

$$Y = \frac{W_F \delta N}{g \left(P + \frac{\varphi W_S}{\gamma \alpha e^{-\alpha L_S}} - \beta_3 \varphi - W_F \varepsilon\right)} = \left(\frac{W_F \delta}{g}\right) \left[\frac{1}{\varphi \left(\frac{\varphi W_S}{\gamma \alpha e^{-\alpha L_S}} - \beta_3\right) + \left(P - W_F \varepsilon\right)}\right] N$$

$$Y = \left(\frac{W_F \delta}{g}\right) \left(\frac{1}{\zeta + P - W_F \varepsilon}\right) N \tag{4.37}$$

Equating equations (4.36 and 4.37) to solve for L<sub>S</sub>:

$$\left(\frac{\delta}{d}\right) \frac{\left(\frac{W_{S}e^{\alpha L_{S}}}{\gamma \alpha} - \beta_{3}\right)}{\left(\zeta + P - W_{F}\varepsilon\right)} SD = \left(\frac{W_{F}\delta}{g}\right) \left(\frac{1}{\zeta + P - W_{F}\varepsilon}\right) N$$

$$\frac{W_{S}e^{\alpha L_{S}}}{\gamma \alpha} - \beta_{3} = \left(\frac{W_{F}\delta}{g}\right) \left(\frac{d}{\delta}\right) \frac{N^{*}}{SD^{*}}$$

Substituting equations (4.35 and 4.34c) for  $N^*$  and  $SD^*$  and solving

$$\frac{W_{S}e^{\alpha L_{S}}}{\gamma\alpha} - \beta_{3} = \frac{\zeta}{\varphi}$$

$$e^{\alpha L_{\rm S}} = \frac{\gamma \alpha}{W_{\rm S}} \left( \frac{\zeta}{\varphi} + \beta_{\rm 3} \right)$$

$$\alpha L_{S} = \ln \left[ \frac{\gamma \alpha}{W_{S}} \left( \frac{\zeta}{\varphi} + \beta_{3} \right) \right]$$

$$L_S^* = \left(\frac{1}{\alpha}\right) \ln \left[\frac{\gamma \alpha}{W_S} \left(\frac{\zeta}{\varphi} + \beta_3\right)\right] \tag{4.38}$$

Given the optimal values for  $L_Y$ ,  $K_Y$ , N and SD above, the optimal output at a desirable steady state is given by:

$$Y^* = AL_y^{*b} K_y^{*c} N^{*g} SD^{*d} (4.39)$$

Where :  $L_Y^*$ ,  $K_Y^*$ ,  $N^*$  and  $SD^*$  are given by equations (4.30b, 4.31b, 4.35 and 4.34c), respectively

Solving equation (4.29) provides the steady state optimal fertilizer use:

$$\beta_{1}F = \beta_{2}Y - \beta_{3} \left[ Z - \tau e^{-\alpha L_{S}} + \phi (1 - e^{-\nu Y}) \right]$$

$$F^{*} = \left\{ \beta_{2}Y^{*} - \beta_{3} \left[ Z - \tau e^{-\alpha L_{S}^{*}} + \phi (1 - e^{-\nu Y}) \right] \right\} / \beta_{1}$$
(4.40)

Where :  $Y^*$  and  $L_S^*$  are given by equations (4.39 and 4.38), respectively.

#### Appendix V. Derivation of static optimal solutions

$$Max\Pi(F, L_Y, L_S, K_Y) = Pf(L_Y, K_Y, F) - W_F F - W_L L_Y - W_K K_Y$$
 (5.1)

The FOC for this system:

$$\frac{\partial \Pi}{\partial F} = P \frac{\partial f}{\partial F} - W_F = 0 \implies \frac{PgY}{F} = W_F \tag{5.2}$$

$$\frac{\partial \Pi}{\partial L_{v}} = P \frac{\partial f}{\partial L_{v}} - W_{L} = 0 \implies \frac{PbY}{L_{v}} = W_{L}$$
(5.3)

$$\frac{\partial \Pi}{\partial K_{Y}} = P \frac{\partial f}{\partial K_{Y}} - W_{K} = 0 \Rightarrow \frac{PcY}{K_{Y}} = W_{K}$$
 (5.4)

Equations (5.2 to 5.4) could be solved simultaneously for the optimal values of  $L_Y$ ,  $K_Y$  and F as follows.

Combining equations (5.2 and 5.3),

$$\frac{PgY}{F} * \frac{L_Y}{PbY} = \frac{W_F}{W_L}$$

$$L_{Y} = \frac{W_{F}}{g} \frac{b}{W_{L}} F \tag{5.5}$$

Combining equations 5.2 and 5.4

$$\frac{PgY}{F} * \frac{K_Y}{PcY} = \frac{W_F}{W_K}$$

$$K_{Y} = \frac{W_{F}}{g} \frac{c}{W_{K}} F \tag{5.6}$$

Combining equations 5.3 and 5.4

$$\frac{PbY}{L_{v}} * \frac{K_{Y}}{PcY} = \frac{W_{L}}{W_{K}}$$

$$K_{Y} = \frac{W_{L}}{b} \frac{c}{W_{\kappa}} L_{Y} \tag{5.7}$$

Solving for  $L_Y$  using equation 5.2:

$$\frac{PgY}{F_{v}} = W_{F}$$

Since,  $Y = AL_Y^b K_Y^c F^g$ , the above can be written as:

$$AL_Y^b K_Y^c F^{g-1} = \frac{W_F}{gP}$$

Substituting equations (5.5 and 5.6) into the above expression

$$A\left(\frac{W_F}{g}\frac{b}{W_L}F\right)^b \left(\frac{W_F}{g}\frac{c}{W_K}F\right)^c F^{g-1} = \frac{W_F}{gP}$$

$$F^{g+c+b-1} \left(\frac{W_F}{g}\right)^{c+b} \left(\frac{b}{W_I}\right)^b \left(\frac{c}{W_K}\right)^c = \frac{W_F}{PAg}$$

Let  $1 - g - c - b = \varpi$ , then  $g + c + b - 1 = -\varpi$ 

Substituting and solving for F:

$$F^{-\varpi} = \left(\frac{W_F}{g}\right)^{1-c-b} \left(\frac{b}{W_L}\right)^{-b} \left(\frac{c}{W_K}\right)^{-c} \left(\frac{1}{PA}\right)$$

$$F^* = \left(\frac{1}{PA}\right)^{-\frac{1}{\varpi}} \left(\frac{b}{W_L}\right)^{\frac{b}{\varpi}} \left(\frac{c}{W_K}\right)^{\frac{c}{\varpi}} \left(\frac{W_F}{g}\right)^{\frac{c+b-1}{\varpi}}$$
(5.8)

## University of Pretoria etd – Tizale, C Y (2007) Solving for $L_Y$ using equation (5.5):

$$L_{Y} = \frac{W_{F}}{g} \frac{b}{W_{I}} F$$

Substituting equation (5.8) into the above expression and solving for L<sub>Y</sub>:

$$L_{Y} = \frac{W_{F}}{g} \frac{b}{W_{L}} \left(\frac{W_{F}}{g}\right)^{\frac{c+b-1}{\varpi}} \left(\frac{b}{W_{L}}\right)^{\frac{b}{\varpi}} \left(\frac{c}{W_{K}}\right)^{\frac{c}{\varpi}} \left(\frac{1}{PA}\right)^{-\frac{1}{\varpi}}$$

$$L_{y}^{*} = \left(\frac{1}{PA}\right)^{-\frac{1}{\sigma}} \left(\frac{b}{W_{L}}\right)^{\frac{1-g-c}{\sigma}} \left(\frac{c}{W_{K}}\right)^{\frac{c}{\sigma}} \left(\frac{W_{F}}{g}\right)^{-\frac{g}{\sigma}}$$
(5.9)

Solving for  $L_K$  using equation (5.7),

$$K_{Y} = \frac{W_{L}}{b} \frac{c}{W_{K}} L_{Y}$$

Substituting equation (5.9) into the above expression

$$K_{Y} = \frac{W_{L}}{b} \frac{c}{W_{K}} \left(\frac{1}{PA}\right)^{-\frac{1}{\varpi}} \left(\frac{W_{F}}{g}\right)^{-\frac{g}{\varpi}} \left(\frac{b}{W_{L}}\right)^{\frac{1-g-c}{\varpi}} \left(\frac{c}{W_{K}}\right)^{\frac{c}{\varpi}}$$

$$K_{y}^{*} = \left(\frac{1}{PA}\right)^{-\frac{1}{\overline{\sigma}}} \left(\frac{b}{W_{L}}\right)^{\frac{b}{\overline{\sigma}}} \left(\frac{c}{W_{K}}\right)^{\frac{1-g-b}{\overline{\sigma}}} \left(\frac{W_{F}}{g}\right)^{-\frac{g}{\overline{\sigma}}}$$
(5.10)

Appendix VI. Soil loss for two plot categories estimated using the USLE modified for Ethiopia

Conservation			Rainfall	Soil	Slop	Slop		Managem	
structure			erosivity	erodibility	length	gradient	Land cover	ent factor	Soil loss
	Crop type	Plot category	R	K	L	S	С	P	(Ton/ha)
No	Tef	Uplands	430.2	0.25	2.1	1.78	0.25	0.75	75.38
No	Tef	Bottomlands	430.2	0.15	3.5	0.4	0.25	0.75	16.94
Yes	Tef	Uplands	430.2	0.25	0.6	1.78	0.25	0.9	25.84
Yes	Tef	Bottomlands	430.2	0.15	1.2	0.4	0.25	0.9	6.97
No	Other cereals	Uplands	430.2	0.25	2.1	1.78	0.18	0.75	54.27
No	Other cereals	Bottomlands	430.2	0.15	3.5	0.4	0.18	0.75	12.20
Yes	Other cereals	Uplands	430.2	0.25	0.6	1.78	0.18	0.9	18.61
Yes	Other cereals	Bottomlands	430.2	0.15	1.2	0.4	0.18	0.9	5.02
No	Pulses	Uplands	430.2	0.25	2.1	1.78	0.15	0.75	45.23
No	Pulses	Bottomlands	430.2	0.15	3.5	0.4	0.15	0.75	10.16
Yes	Pulses	Uplands	430.2	0.25	0.6	1.78	0.15	0.9	15.51
Yes	Pulses	Bottomlands	430.2	0.15	1.2	0.4	0.15	0.9	4.18

Source: Shiferaw and Holden (1999)

# Appendix VII. Parameter estimates of the multinomial logit soil fertility adoption model, Central highlands of Ethiopia, 2003

	Seasonal fa	llowing			Animal	manure			Inorganic	fertilizer
		onal fallowing or Crop rotations Animal manure (AM)		associated with wither SF		Inorganic fertilizers		associated with		
	(LG)	otations	alone	c (Awi)	or LR		Inorganic fertilizers (IF) alone		SF, LR or MR	
Explanatory	(20)		urone		or Ext		(II ) urone		SI, ER OI IVII	
Variables	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
Constant	-3.6566***	0.000	-1.4414***	0.003	-3.3788***	0.000	-2.8492***	0.000	-3.9765***	0.000
Education	0.0327	0.378	0.1124***	0.042	0.0897	0.130	0.0324	0.634	0.1371***	0.007
Off-farm income	0.1462	0.476	-0.1372	0.609	-0.1457	0.698	0.5326	0.216	0.4205	0.167
Livestock	-0.0137	0.582	0.0295	0.305	0.0136	0.747	0.0741*	0.099	0.0944***	0.004
Plot size	2.3760***	0.000	0.1526	0.797	1.5744*	0.066	3.0165***	0.000	2.8956***	0.000
No. of plots	0.0365	0.349	-0.1194***	0.002	-0.0951*	0.085	-0.0070	0.935	-0.1786***	0.001
Plot distant	-0.0016	0.724	-0.1075***	0.000	-0.1086***	0.002	-0.0036	0.633	0.0040	0.432
Light	-0.1013	0.596	0.6588***	0.007	1.4831***	0.000	0.8180**	0.019	0.3381	0.261
Medium	-0.1109	0.627	0.1745	0.517	0.9849**	0.038	0.7563*	0.060	0.5645*	0.055
Sever	0.3095	0.262	-0.0326	0.932	0.8488	0.113	1.0768**	0.026	0.7690**	0.015
Tenure	0.0666	0.705	1.2011***	0.001	0.7503*	0.087	-0.4493	0.214	0.6156**	0.030
Credit	0.5557**	0.029	$0.4815^*$	0.075	-0.1725	0.686	1.4017***	0.000	2.0172***	0.000
Extension	0.0094	0.981	0.2558	0.624	0.9767	0.145	1.1393*	0.063	$0.9797^*$	0.052
Agro-ecology	3.1615***	0.000	0.9560***	0.001	1.6719***	0.001	-1.3625***	0.000	1.0375***	0.001
Kossi	-1.2191**	0.049	1.3675***	0.001	-0.0151	0.984	-0.8746	0.183	-0.6484	0.198
Diagnostics										
No. Observations	1411									
Wald Chi-Square	771.08***									
Log pseeudo	-1810.0929									
likelihood										
Pseudo R-Square	0.2314									

<sup>\*\*\*, \*\*, \*=</sup> Significant at 1%, 5% and 10% probability level, respectively

## Appendix VIII. Coefficient estimates of the multinomial logit soil conservation adoption model, Central highlands of Ethiopia, 2003

Variable	Cut-off drainage	(golenta)	Stone and soil bunds		
	Coefficient	P-level	Coefficient	P-level	
	***		***		
Constant	-3.3845***	0.000	-7.5696 <sup>***</sup>	0.000	
Education	0.0552	0.358	0.1192**	0.019	
Plot area	-0.2998	0.302	0.5288	0.226	
No. of plots	-0.0196	0.816	-0.0721	0.206	
Plot distance	0.0079	0.263	-0.0027	0.640	
Tenure	0.4967	0.163	0.3503	0.242	
Livestock	0.0671	0.193	-0.0109	0.725	
Off-farm income	-0.0814	0.854	-0.6829***	0.009	
Extension	-0.2043	0.826	0.3214	0.577	
Credit	-0.6764	0.169	-0.4494	0.224	
Plot slope	0.2570	0.300	1.6694***	0.000	
Soil degradation					
Sever	1.4714***	0.002	2.7903***	0.000	
Medium	1.9818***	0.000	2.9154***	0.000	
Light	1.9555****	0.000	2.9223***	0.000	
Assistance	1.9876*	0.056	2.7938***	0.000	
District	-1.2241**	0.043	4.2534***	0.000	
Diagnostics					
No. Observations	97		309		
Wall Chi-Square	270.03***				
Pseudo Chi-Square	0.4017				

<sup>\*\*\*, \*\*, \*=</sup> Significant at 1%, 5% and 10% probability levels, respectively

Appendix IX. Parameter estimates of the Tobit adoption model for the intensity of inorganic fertilizer use (kg/ha), Central highlands of

	 •	$\Delta \alpha$	$\boldsymbol{\alpha}$
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Variable		Marginal effects					
			Adoption	P-level	Expected	P-	
	Coefficient	P-level	(index)		use (kg/ha)	level	
Constant	-116.3897***	0.000	N.A.	N.A.	N.A.	N.A.	
Education <sup>1</sup>	9.2427***	0.000	$0.0210^{***}$	0.000	2.0126***	0.000	
Off-farm income <sup>2</sup>	29.0357**	0.025	0.0689**	0.030	$6.5489^{**}$	0.029	
Livestock <sup>3</sup>	6.8069***	0.000	0.0154***	0.000	1.4822***	0.000	
Plot size <sup>4</sup>	49.9898***	0.002	0.1133***	0.002	10.8850***	0.002	
No. of plots	-10.4497***	0.000	-0.0237***	0.000	-2.2754***	0.000	
Plot distant <sup>5</sup>	$0.4583^*$	0.094	$0.0010^{**}$	0.093	$0.0998^*$	0.093	
Severity of soil							
degradation <sup>6</sup>	* * * * * * *						
Light	21.8407*	0.097	0.0509	0.106	4.8545	0.104	
Medium	38.0750***	0.004	0.0927***	0.007	8.7697***	0.007	
Sever	39.4582 <sup>**</sup>	0.013	0.0983**	0.022	9.2578**	0.021	
Tenure <sup>7</sup>	17.9643	0.165	0.0391	0.146	3.8033	0.153	
Credit <sup>8</sup>	99.6655***	0.000	0.2419***	0.000	23.2970***	0.000	
Extension <sup>9</sup>	74.3334***	0.000	0.2017***	0.000	19.0840***	0.000	
Agro-ecology <sup>10</sup>	-51.9767***	0.000	-0.1281***	0.000	-12.1180***	0.000	
SFM used previous year <sup>11</sup>							
Legume							
rotations	-26.8184**	0.025	-0.0610	0.024	-5.8586**	0.025	
Manure	-119.2528***	0.000	-0.2190	0.000	-23.1081***	0.000	
Fallow	33.1012**	0.034	0.0806	0.046	7.6161**	0.044	
District*Bund <sup>12</sup>	-20.5157	0.196	-0.0448	0.177	-4.3600	0.184	
Diagnostics							
No.							
Observations	1293						
LR Chi-Square	492.44***	0.000					

<sup>\*\*\*, \*\*, \*=</sup> Significant at 1%, 5% and 10% probability levels, respectively; N.A.=Not applicable; <sup>1</sup>Number of years; <sup>2</sup>Dummy variable, 1 denoting participation in off- farm activities; <sup>3</sup>Tropical Livestock Unit

(TLU); <sup>4</sup>hectares; <sup>5</sup>Minutes walked from residence; <sup>6</sup>comparison category is plots perceived not having shown any form of soil degradation; <sup>7</sup>dummy variable, 1 denoting PA allotted plots, 0 otherwise; <sup>8</sup>dummy variable, 1 denoting access to institutional credit; <sup>9</sup>dummy variable, 1 representing access to government extension; <sup>10</sup>dummy variable, 1 referring to upper highlands; <sup>11</sup>dummy variables with 1 indicating use of the respective practices. <sup>12</sup>dummy variable with 1 indicating plots with stone/soil bunds in Debre Berihan district.