



**ANALYSIS OF SPATIALLY DISTRIBUTED  
ADAPTIVE ANTENNA ARRAY SYSTEMS  
IN CELLULAR NETWORKS**

by

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Submitted in partial fulfillment of the requirements for the degree

**Philosophiae Doctor**

in the

Faculty of

Engineering, Built Environment and Information Technology

UNIVERSITY OF PRETORIA

PRETORIA

2004



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## SUMMARY

The spatially distributed adaptive array is defined and analyzed. It is applied to both time division multiple access (TDMA) and code division multiple access (CDMA) cellular networks to improve the outage probability at either the base station or mobiles.

In a TDMA network, the distributed array consists of three sub-arrays at alternate corners of a hexagonal cell. It is shown analytically that the SINR of combined beamforming of the distributed sub-arrays is greater than or equal to the SINR of independent beamforming of the sub-arrays. Closed form solutions are derived for estimating the BER performance of Rayleigh fading mobile signals received at a distributed adaptive array with combined beamforming of the sub-arrays. The simulated TDMA uplink outage probability of multiple same-cell co-channel users in a fading environment is compared between conventional, spatially distributed arrays with independent beamforming of the sub-arrays and combined beamforming of the sub-arrays. The effect of the antenna element spacing, number of elements and angular spread is also investigated. Spatially distributed arrays are formed in a CDMA network on the downlink with arrays in multi-way soft handoff with the mobiles. The outage probability performance of combined beamforming of the arrays in handoff is compared to independent beamforming of the arrays as well as to conventional sectorized antennas.

The range between mobiles and distributed sub-arrays in the case of a spatially distributed array can be larger than between conventional center cell arrays and mobiles. Therefore, the effect of interference on the range increase relative to an omni antenna of adaptive and phased arrays in a multipath environment for both narrowband and wideband spread spectrum systems is investigated. An analytical model for predicting the asymptotic range limitation of phased arrays when the angular spread exceeds the array beamwidth is derived.

**KEYWORDS:** Cellular systems, base stations, spatially distributed adaptive antenna arrays, capacity increase.

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## SAMEVATTING

'n Ruimtelike verspreide aanpasbare antenne samestelling vir sellulêre toepassings word gedefinieer en geanaliseer. Die konsep word vir beide tyd divisie meervoudige toegang (*TDMA*) asook kode divisie meervoudige toegang (*CDMA*) sellulêre netwerke gebruik ten einde die waarskynlikheid vir 'n seinonderbreking by beide die basisstasie en die selfoon te verminder.

In *TDMA* netwerke word drie sub-antenne samestellings by die alternatiewe hoeke van 'n heksagonale sel geplaas. Die analitiese sein-tot-steuring en ruisverhouding (*SINR*) moontlik met gekombineerde bundelvorming van die sub-antenne samestellings is groter of gelyk aan die *SINR* moontlik met individuele bundelforming van die sub-antenne samestellings. Geslote-vorm uitdrukkings vir die benaderde data fout tempo (*BER*) van die seine wat ontvang word deur so 'n samestelling vanaf 'n selfoon in 'n Rayleigh multipad omgewing word afgelei. 'n Vergelyking deur middel van simulasies word getref van die waarskynlikheid vir 'n seinonderbreking tussen gekombineerde en individuele bundelvorming van die samestelling vir die geval van etlike selfoongebruikers wat deur dieselfde kanaal en in dieselfde sel bedien word. Die invloed van inter-element spasiëring, aantal elemente en multipad hoekverspreiding word ook ondersoek. Die verspreide aanpasbare samestelling word ook in die basisstasie-selfoon skakel van 'n *CDMA* netwerk waar meer as een samestelling in sagte oorhandiging (*soft handoff*) met die selfoon gebruikers is, gebruik. Die waarskynlikheid van 'n seinonderbreking vir gekombineerde bundelvorming, individuele bundelvorming en konvensionele sector-antennes wat in sagte oorhandiging is, word vergelyk.

Die effek van steuringseine op die afstandsbepanking van aanpasbare en fase-gerigte samestellings relatief tot omni-direksionele antennes word vir beide noue- en wye-band stelsels ondersoek. 'n Analitiese model word vir die asimptotiese afstandsbepanking van fase-gerigte samestellings in die geval waar die multipad hoekverspreiding wyer is as die antenne bundelwyde afgelei.

**SLEUTELWOORDE:** Sellulêre netwerke, basisstasies, ruimtelik verspreide aanpasbare antenne samestellings, kapasiteitsverhoging.

## ACKNOWLEDGEMENTS

I would like to express my gratitude to all those who gave me the opportunity to complete this thesis.

First of all, I wish to thank my supervisor Prof. Wimpie Odendaal and co-supervisor Prof. Johan Joubert for taking me on as one of their PhD students and giving me the chance to do this PhD at the University of Pretoria. I wish to thank them for their leadership and advice with the technical content of the thesis, thesis document layout and contents as well technical papers. This thesis was done on a part-time basis in Canada. Thank you for working with me under these unusual circumstances.

I am also grateful to my supervisor for his motivation throughout the duration of this thesis as well as for arranging the NRF bursaries for me to be able to attend the conferences and for the yearly university fees. Thanks for your encouragement and friendship over all these years.

I would like to thank the National Research Foundation for their financial support in the form of bursaries granted to me for several years.

I greatly love and appreciate my parents, brother and sister. My parents supported me through my Bachelors engineering degree at the University of Pretoria, which ultimately allowed me to do this PhD. I will always be thankful to them for laying a solid foundation for my life.

Lastly, and not the least, I would like to thank my wonderful wife Angelika and beautiful son Ethan for their love. Thanks so much for the time you gave to me to do this, for coping with me over this time and for supporting and motivating me to finish this enormous task. I have the deepest love for both of you.



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**LIST OF ABBREVIATIONS**

<b>Abbreviation</b>	<b>Meaning</b>
1D	One dimensional
1xRTT	CDMA standard for 1x CDMA bandwidth with voice and data
ACF	Autocorrelation function
2D	Two dimensional
BER	Bit error rate
BF	Beamformer or beamforming
BTS	Base station
C/I	Carrier to interference ratio
CDF	Cumulative distribution function
CDMA	Code division multiple access
CIR	Carrier to interference ratio
Comb	Combined
CDVCC	Color code used in TDMA timeslots
dB	Decibel
deg	Degree
DMI	Direct matrix inversion
DOA	Direction of arrival
erfc	Complimentary error function
FFT	Fast Fourier transform
GSM	Global system for mobile communications
Ind	Independent
INR	Interference plus noise ratio
LMS	Least mean squares
MHz	Megahertz
MRC	Maximum ratio combiner
MS	Mobile station
PDF	Probability distribution function
PSK	Phase-shift keying
RAKE	Receiver used in CDMA systems to rack multipath components



<b>Abbreviation</b>	<b>Meaning</b>
RF	Radio frequency
RLS	Recursive least squares
SDMA	Space division multiple access
SINR	Signal to interference plus noise ratio
SIR	Signal to interference ratio
SNR	Signal to noise ratio
TDMA	Time division multiple access
ULA	Uniform linear array
UMTS	Universal standard for mobile telecommunications
Walsh	Orthogonal codes used in CDMA systems
WCDMA	Wideband code division multiple access
wvl	Wavelength

## MATHEMATICAL NOTATIONS

The symbols used in the thesis formulations together with their associated meanings are given in Table 1.

**Table 1: Symbols that are used the thesis with their associated meaning**

Symbol	Meaning
$A$	Array vector
$A_{z_d,d}$	Sector $z_d$ array vector transmitting to mobile $d$
$\ddot{A}_{(\cap z_d),d}$	Combined beamforming array vector for arrays $(\cap z_d)$ that mobile $d$ is in handoff with
$a(\psi_{z,z_a,d,k})$	function of angle of arrival relative to array boresight of $k$ -th component of the scattered signal $d$ located in cell $z$
$  $	Absolute of the value or determinant of matrix
$b$	User transmit signal bits
$c_d$	Spreading code of mobile $d$
$CIR_o$	Carrier to interference ratio protection ratio
$C/I$	Carrier to interference ratio
$()^*$	Complex conjugate
$()^H$	Complex conjugate transpose
$()_{\cap(\kappa)}$	Concatenation or combination of sub-array signals
$d$	User or mobile number
$D$	Number of users
$\Delta$	Array element spacing
$\delta$	Dirac-delta
$\varepsilon_d$	Difference in amplitude between the array output and reference signal
$E\{\}$	Average
$F(m,\psi)$	Array element $m$ pattern in direction $\psi$
$G$	Path gain
$G_{z_d,d}$	Path gain between sector $z_d$ and mobile $d$
$G_{z_{d_{des}},d_{des}}$	Path gain between sector $z_{d_{des}}$ and mobile $d_{des}$
$\ddot{G}_{(\cap z_d),d}$	Combined beamforming path loss between the sectors $(\cap z_d)$ and the



Symbol	Meaning
	mobile $d_{des}$
$G_T$	Transmit antenna gain
$G_R$	Receive antenna gain
$\Gamma$	Signal to interference plus noise ratio
$\Gamma_n$	Interference to noise ratio
$\gamma$	Pathloss exponent
$h_{z,za,d}$	RF propagation vector channel transfer function between the $d$ -th mobile in cell $z$ and array in cell $z_a$
$H$	Conjugate transpose
$\eta$	Instantaneous SINR
$I$	Identity matrix
$\mathcal{S}_d$	Voice activity factor of mobile $d$
$\varphi_{z,za,d,k}$	Phase between mobile $d$ in cell $z$ to array in cell $z_a$ via scatterer $k$
$k$	Scatterer number
$K$	Number of scatterers
$\kappa$	Sub-array number
$\lambda$	Carrier wavelength
$\lambda_m$	$m$ -th eigenvalue
$L$	Number of resolvable multipath components (or fingers) in a RAKE receiver
$\ell$	Resolvable multipath component or finger in a RAKE receiver
$m$	Array element number
$M$	Number of array elements
$\mu$	Convergence constant
$\mu_w$	Constraint on weight vector to have unity response in desired signal direction
$N$	Number of samples over which average is taken
$n_s$	Thermal noise vector
$n$	Symbol number
$n_s(t, m)$	Additive white noise at array element $m$
$n_{d_{des}}$	Noise power at the desired mobile
$p(t)$	Transmit pulse shape



Symbol	Meaning
$P_{AOA}(\psi)$	Probability of a signal arriving at an angle $\psi$
$p^{des}$	Desired signal power
$p^{int}$	Interference power
$p^{int+noise}$	Interference plus noise power
$P_d$	Power of signal d
$p(\ )$	Probability density function
$q$	Symbol number
$Q$	Total Number of symbols
$R$	Cell radius
$\hat{\mathbf{R}}_{nn}$	Interference plus noise covariance matrix averaged over short term (multipath) fading
$\mathbf{R}_{nn}$	Interference plus noise covariance matrix
$\ddot{\mathbf{R}}_{nn}$	Interference plus noise covariance matrix of combined distributed array averaged over short term (multipath) fading
$\ddot{\mathbf{R}}_{nn}$	Interference plus noise covariance matrix of combined distributed array
$r_{d,z}^c$	The distance between mobile d in cell z and the boresight of the array
$r_{max}^s$	Scatterer maximum radius
$r_{z,za,d,k}^t$	Total distance between mobile d in cell z and array in cell za via scatterer k
$r_{z,d,k}^e$	Distance between mobile d in cell z and scatterer k
$r_{z,za,d,k}^s$	Distance between scatterer k and array in cell za for mobile d in cell z
$ref_d$	Reference signal
$\rho$	Slow fading amplitude
$S$	Baseband signal amplitude
$\sigma_N^2$	Thermal noise power
$\sigma$	Standard deviation
$\sigma_{as}$	Angular spread standard deviation
$\sigma_{sf}$	Standard deviation of the slow fading PDF
$T_c$	Chip period

Symbol	Meaning
$T$	Matrix transpose
$t_{HO}$	Handoff threshold
$\tau_{z,za,d,k}$	Delay between mobile $d$ in cell $z$ to array in cell $z_a$ via scatterer $k$
$\tau_{z_d,d}$	Propagation delay for signals between sector $z_d$ and mobile $d$
$\ddot{\tau}_{(\cap z_d),d}$	Combined beamforming propagation delay for arrays $(\cap z_d)$ that mobile $d$ is in handoff with
$U_d$	Propagation vector of signal $d$ at the array elements
$\ddot{U}_d$	Propagation vector of signal $d$ at the elements of the combined distributed array
$U_{des}$	Propagation vector of desired signal at the array elements
$U_{d_{des}}$	Propagation vector of desired signal at the array elements
$U_{Qd}$	Propagation vector of interference signal $d$ at the array elements
$v$	Eigenvector
$W$	Weight vector
$\hat{W}$	Approximation of the weight vector
$\ddot{W}_{(\cap z_d),d}$	Combined beamforming weight vector for arrays $(\cap z_d)$ that mobile $d$ is in handoff with
$\xi_{z,d,k}$	Amplitude of $k$ -th component of the scattered signal $d$ in cell $z$
$X(m)$	Received signal at each array element
$X^{dB}$	Received signal in dB
$\aleph$	Power loss of multipath components due to antenna azimuth taper
$X$	Received signal vector across the array
$\ddot{X}$	Received signal vector across the combined distributed array
$X_\kappa$	Received signal vector across the sub-array $\kappa$
$X_Q$	Interference received signal vector across the array
$Y_{\kappa,d}$	Output of sub-array $\kappa$ optimized for user $d$
$\psi$	Angle between array boresight and mobile
$\Psi$	Characteristic function
$\Psi_{d,z,k}$	Incidence angle of the $k$ -th scattered component of signal $d$ in cell $z$





Symbol	Meaning
$\psi_{d,z}^c$	Incidence angle between the array boresight and the signal d in cell z
$z_d$	Sector number that the mobile d is in soft-handoff with
$z_{d_{oc}}$	Handoff sector number of mobile d in a sector not containing the desired mobile
$z$	Cell Number
$Z$	Number of cells /Rake receiver output
$\cap z_d$	Combination of sectors that the mobile d is in handoff with