CHAPTER 5: PRELIMINARY AND PILOT STUDIES

The results of this study are presented in Chapters 5, 6, 7 and 8. The presentation is done in three different phases. Chapter 5 presents Phase I being the preliminary study in July 2005 as Part 1; and the pilot study in October 2006 as Part 2. Chapter 6 presents Phase II, being the main results from the developed instrument in April 2007 (the Questionnaire 1st run) and in October 2007 and in April 2008 (both as Questionnaire 2nd run) and the analysis of the students' responses from the 2007 August mathematics N6 examinations scripts (examination response analysis and detailed selected written examination responses). Chapter 7 presents the correlations of the elements from the questionnaire runs in Chapter 6, while Chapter 8 presents Phase III: the classroom observations and the interview with one student. The data collected are described qualitatively and quantitatively where possible. The qualitative data are presented in terms of students' written responses which were marked and classified into five different skill factors as discussed in Chapter 3 as the conceptual framework of this study, narratives (verbal or written) and tables. The quantitative data are presented in terms of tables, diagrams and graphs. In Chapter 4, the mode of data collection was discussed for both the qualitative and the quantitative data that are presented in Chapters 5, 6, 7 and 8. The interpretation of the data presented and analysed in these chapters is done in Chapter 9 and all the phases are consolidated. It is positioned within the conceptual framework of this study which was discussed in Chapter 3 and related to previous studies done, discussed in Chapter 2.

Table 5.1 indicates a schematic process that will be followed in the presentation of the results.

Table 5.1: Schematic process in the presentation and analysis of the results

Chapter 5: Phase I	Chapter 6: Phase II	Chapter 7: Correlations	Chapter 8: Phase III
Part 1: Preliminary study	Investigation 1: Questionnaire 1st run in April 2007 (37 respondents) and Investigation 2: Questionnaire 2nd run in October 2007 (122 respondents) and April 2008 (54 respondents).	Correlating the elements from the questionnaire runs and the examination analysis from Phase II.	Investigation 5: Classroom observations (±40 students) and Investigation 6: An interview with a former N6 student.
Part 2 : Pilot study	Investigation 3: Examination analysis (151 respondents) and Investigation 4: Detailed examination responses (7 respondents).	Correlating the elements from the examination analysis from Phase II.	

5.1 PART 1: PRELIMINARY STUDY IN JULY 2005

The preliminary study was conducted at College A to investigate students' difficulties through the interplay between the visual and algebraic skills when learning volumes of solids of revolution (VSOR) through visualisation using Mathematica. Mathematica was used to illustrate (visually) how a given region may be rotated to formulate a solid of revolution (through graphics and animations), with an attempt to make the concept concrete for the students. Students' written responses, as they translated from a *visual* representation to an *algebraic* representation (from the *diagram* to the correct *formula* for computing volume) after the correct rotation of the selected strip, were explored. What was investigated here was that after identifying the rectangular strip that approximates the bounded region as Δx or as Δy, are the students able to rotate the selected rectangular strip and to use it to generate the formula for volume (be it disc, washer or shell), which was then evaluated. At this stage of the research, there was less focus on the evaluation of volume, the focus was on how the students translate from the drawn graphs to the algebraic formula for volume, which involves rotating the strip correctly and using the correct formula for volume from the given graphs.

Two tests (Test 1 and Test 2) were written (refer to Appendix 1B). In some questions, the graphs were given, while in other questions only equations were given where the students were expected to first draw the graphs. Test 1 was written after a verbal instruction presented by their lecturer (chalk and talk) and Test 2 was written after a visual instruction presented by the researcher using Mathematica. The two tests were not at the same level of difficulty. The graphs in Test 1 were much easier than those in Test 2, except for Question 2(a), which was the same in both tests. However, as mentioned earlier, the focus of the tests was on students' written responses, as they translated from a *visual* representation to an *algebraic* representation (from the *diagram* to the correct *formula* for computing volume) after the correct rotation of the selected strip, and not on how the students draw graphs as it was anticipated that the students were competent in drawing graphs. The assumption was that if students are able to translate from a *visual* representation to an *algebraic* representation when given a region bounded by more difficult graphs after being taught using Mathematica, then Mathematica would be regarded as having an effect on the enhancing of learning rotations visually and enhancing students' imagination skills from what was demonstrated to new situations, even with the difficult graphs.

Fifteen students participated in the preliminary study. In the section that follows, seven students' written responses are presented. Responses for only seven students out of the fifteen students (who participated in the study) are presented since these seven students wrote both tests. Test 1 had four questions, resulting in 28 responses (7×4), while the Test 2 had six questions, resulting in 42 responses (7×6). The rest of the students were excluded because some of them wrote Test 1 only or Test 2 only.

The results for the preliminary study are presented in three stages. Firstly the overall responses for Test 1 and Test 2 are presented in tabular form in Table 5.2 to reveal the emerging patterns from the responses. Secondly, selected individual students' written responses are presented, described and interpreted giving examples where possible. The existing trends between the visual skills and the algebraic skills are displayed. Finally, Table 5.3 is used to show students' competencies in drawing graphs, in questions where graphs were not given.

5.1.1 The results from the seven students

In the section that follows, responses for Test 1 and Test 2 are presented in tabular form. The focus is on how the strip is rotated and used to come up with the formula for volume. Even if the strip was drawn incorrectly, I focused on how the strip was interpreted further in relation to how it is rotated and used to select the correct formula for volume.

The students' responses are classified in the following categories:

- Rotating the strip correctly and using the correct formula (*able and able*)
- Rotating the strip correctly but using the incorrect formula (*able but unable*)
- Rotating the strip incorrectly but using the correct formula (*unable but able*)
- Rotating the strip incorrectly and using the incorrect formula (*unable and unable*)

Since this research focuses on students' difficulties with the VSOR, it was necessary to investigate the interplay between the visual (from the drawn graphs) and algebraic (using the selected strip to come up with the correct formula for volume) with the emphasis on the correct rotation of the strip selected and the solid of revolution generated and the selection of the correct formula. The focus was on the first steps of the calculation only, where the students were to show the correct method, whether disc, washer or shell. So even if a student made a mistake, the student is regarded as *able and able*, as long as he/she managed to rotate the strip correctly as well as using the correct formula for volume.

5.1.1.1 Overall responses

In Table 5.2 the four categories are denoted as follows: able and able is denoted by *aa* with the sum of all the *aa* students per question denoted by Σ*aa*, for both the Test 1 and Test 2. The same applies to the other three categories, able but unable; unable but able and unable and unable denoted by *au*, *ua* and *uu* respectively, with the total number of students as Σ*au*, Σ*ua* and Σ*uu* respectively. The categories for responses from both tests are presented, with the sum of the total responses given in bold. The individual responses given for the seven students are denoted as S1, S2, S3, S4, S5, S6 and S7, while NG denotes questions without drawn graphs. In Table 5.2, the strip that is easy to work with upon rotation by the *x*-axis or the *y*-axis is identified, as well as the formula to be used, whether disc (D), washer (W) or shell (Sh).

Table 5.2: Classification of students' written responses

	Test 1 results					**Test 2 results**						
	Q1a	Q1b-**NG**	Q2a	Q2b-**NG**	**Σ**	Q1a	Q1b	Q1c-**NG**	Q2a	Q2b	Q2c-**NG**	**Σ**
Strip Rotation Formula	Δ*x* R*x* D	Δ*x* R*x* D	Δ*x* R*y* Sh	Δ*y* R*y* D		Δ*x* R*x* W	Δ*x* R*x* D	Δ*y* R*x* Sh	Δ*x* R*y* Sh	Δ*x* R*y* Sh	Δ*y* R*y* W	
S1	aa	aa	aa	au		aa	aa	au	au	au	au	
S2	ua	ua	ua	uu		uu	ua	uu	uu	uu	uu	
S3	aa	ua	uu	au		uu	ua	uu	ua	uu	uu	
S4	ua	uu	au	uu		au	au	uu	au	uu	au	
S5	aa	uu	au	uu		au	aa	au	aa	au	au	
S6	aa	aa	ua	aa		au	aa	uu	au	au	uu	
S7	aa	ua	uu	uu		uu	uu	uu	au	au	uu	
Σaa	5	2	1	1	**9**	1	3	0	1	0	0	**5**
Σau	0	0	2	2	**4**	3	1	2	**4**	**4**	3	**17**
Σua	2	**3**	2	0	**7**	0	2	0	1	0	0	**3**
Σuu	0	2	2	**4**	**8**	3	1	**5**	1	3	**4**	**17**
					28							**42**

From Table 5.2, Test 1 results indicate that students performed better in Question 1(a) involving the rotation of the region bounded by the drawn graph of $y = \cos x$ about the *x*- axis in comparison with other questions. Five students were able to rotate the strip correctly and used the correct formula, where a disc method was appropriate. Question 1(b) and Question 2(b) required that the students start by drawing the graphs of $y = x^2$ and $x = 3$; and the first quadrant area of $x^2 + y^2 = 9$ respectively. The rotation for the region bounded by the graphs for Question 1(b) was about the *x*-axis, resulting in a disc, whereas for Question 2(b), rotation was about the *y*-axis, also resulting in a disc. From the drawn graphs for Question 2(b), more than half of the students (4) were unable to rotate the strip correctly and to use the correct formula for volume. For Question 1(b), only 3 students were unable to rotate correctly but used the correct formula for volume. For Question 2(a), where the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ was to be rotated about the y-axis, seemingly students encountered problems since most of them used a Δx strip but could not rotate it accordingly to give rise to a shell. Students were seen to use the washer method even if a Δx strip which was supposed to be rotated about the *y*-axis, was drawn. In most instances the students gave the formula for volume without drawing the strip.

The conclusions that could be drawn from Test 1 are that students avoid using a Δy strip and cannot rotate properly if rotation is about the *y*-axis. The students find it easy to work with a Δx strip when rotated about the *x*-axis, resulting in a disc or a washer. The number of responses where students were able to rotate the strip correctly and used the correct formula (9 out of 28) was more or less the same as the number of responses where students were unable to rotate the strip correctly and unable to use the correct formula (8 out of 48). However, the Σua total of 7 out of 28 responses revealed that students were unable to rotate the strip correctly, but were able to use the correct formula based on the strip they selected. The lowest number of responses, was Σau with 4 out of 28 responses where students were able to rotate the strip correctly but unable to use the correct formula for volume. In general the performance was not very good since only 9 out of 28 (32%) of the responses were fully correct.

In Test 2 after instruction in Mathematica, students had more difficulties with Questions 1(c) and 2(c) with the highest of 5 students and 4 students respectively unable to rotate the strip correctly and to use the correct formula for volume. Question 1(c) required that they start by drawing the graph of $y = x^2 + 1$, $y = 2$ and $y = 4$ which most students could not draw, while for Question 2(c) they had to draw the graph of $y^2 = 4x$ and $y = 2x - 4$ before calculating the volume. For both questions, a Δy strip was appropriate, with a shell resulting after rotation of the graphs in Question 1(c) about the *x*-axis and a washer resulting after rotation for the graphs in Question 2(c) about the *y*-axis. For Questions 2(a) that was similar to Test 1 and Question 2(b), many students (4), were able to rotate the strip correctly but were unable to select the correct formula for volume from the rotated strip. Both questions required rotation of about the *y*-axis and the use of the shell method where a Δx strip was mostly appropriate. The students in this case were unable to translate the visual graph to the algebraic formula for volume.

The responses for Question 2(a) in both tests which required rotation of the region bounded by the drawn graphs of $y = \sqrt{x}$ and $y = x^2$ about the *y*-axis were interesting. The performance in Test 1 for Question 2(a) was better than the performance in Test 2 even though the students were doing this question for the second time. In Test 1 two students were able to rotate correctly, but failed to come up with the correct formula for volume, with only one student (S1) able to rotate correctly and coming up with the correct formula for volume. In Test 2 four students were able to rotate correctly, but failed to come up with the correct formula for volume, with only one student (S5) able to rotate correctly and coming up with the correct formula for volume. Student S1 who was able to rotate correctly, and able to come up with the correct formula for volume in Test 1 as a shell without substituting the equations of the graphs, was now only able to rotate correctly, but did not write down any formula to calculate the volume in Test 2. This student only calculated the point of intersection of the two graphs.

Even though the performance for Question 2(a) was better in Test 1 than in Test 2, some students improved in the way in which they rotated the strip. In addition to the three students who were able to rotate in both tests, two students S6 and S7 who were unable to rotate in Test 1 were now able to rotate after instruction using Mathematica in Test 2.

Test 2 results were remarkably different and worse than in Test 1, indicating Σau as the highest number of 17 and Σuu as the highest number of 17. The 17 Σau responses were of students who were able to rotate the strip correctly, but unable to use the correct formula and the 17 Σuu were responses where students were unable to rotate the strip correctly and unable to use the correct formula. A very low number of responses (5) revealed that the students were able to rotate the strip correctly and to use the correct formula. Even if the performance in Test 2 was bad, 17 out of 42 responses (40%) indicated that the students were now able to rotate correctly even if they failed to come up with the formula for volume compared to 4 out of 28 (14%) of the responses in Test 1.

I wanted to analyse these categories further to explore the students' written responses. One example of students' responses for the four questions from Test 1 and one example from the six questions in Test 2 were then analysed further. Question 2(a) from Test 1 was analysed further because it was the same in both tests. It was used to reveal students' ability to translate from the visual strip to the algebraic equation, even if the selected strip was rotated incorrectly. Question 1(c) from Test 2 was analysed further because that is where many students were unable to rotate the strip correctly and unable to come up with the correct formula for volume, which was the category revealing the highest sum of 17 (17 Σuu).

5.1.1.2 Individual responses

- **Test 1 results for S6**

In Figure 5.1 a written response for Question 2(a) from Test 1 for category *ua* is presented. In this question Figure 5.1 shows the response for S6 where the strip was correct, the rotation was *incorrect* but the formula for volume used was *correct* (unable but able) based on the rotated strip.

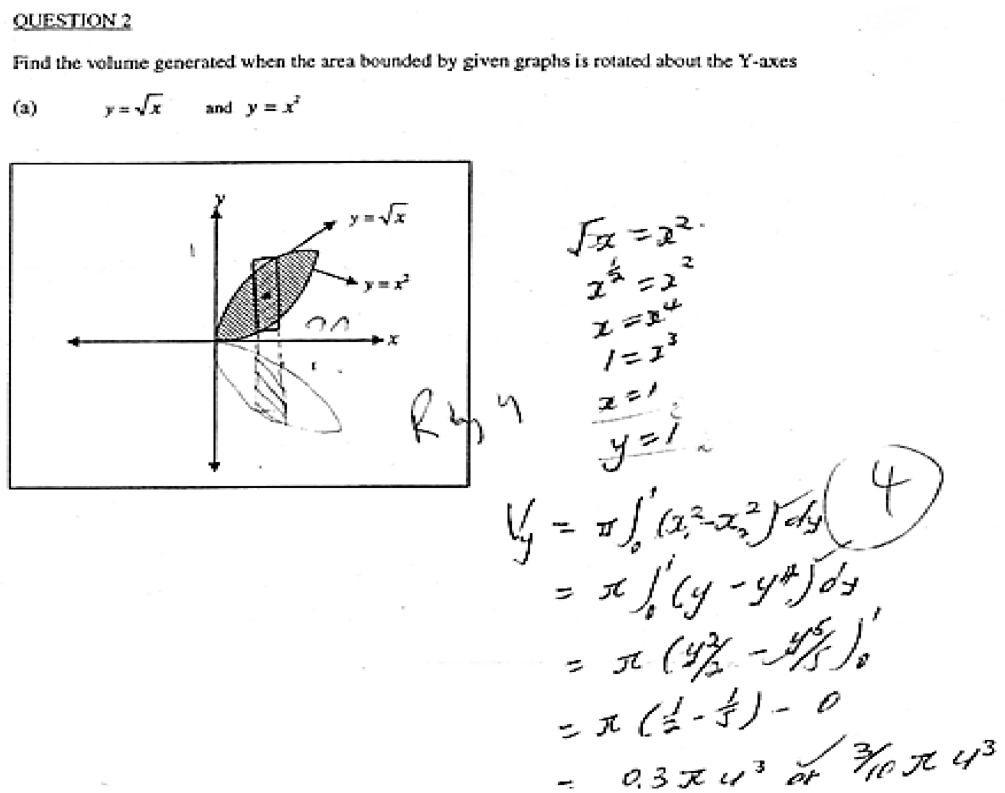


Figure 5.1: S6 interpreting a Δ*x* strip as a Δ*y* strip

This student (and some of the other students) drew the strip correctly but rotated incorrectly about the *x*-axis instead of the *y*-axis, hence ended up with the washer method. The washer method used was not translated correctly from the drawn strip, since the student used a Δ*y* in the formula, even though a Δ*x* was drawn on the diagram. The student referred to the washer method as $\pi\int_a^b (x_1^2 - x_2^2)\,dy$ instead of $\int_a^b (y_1^2 - y_2^2)\,dx$, failing to translate from the given graph, where a Δ*x* strip was used. In many instances, even if the correct strip was selected, some students failed to translate from the drawn graph to the correct formula for volume. The students in most cases were able to do the calculations correctly.

- **Test 2 results for S1**

In Figure 5.2 a written response to Question 1 (c) for element *au* is presented.

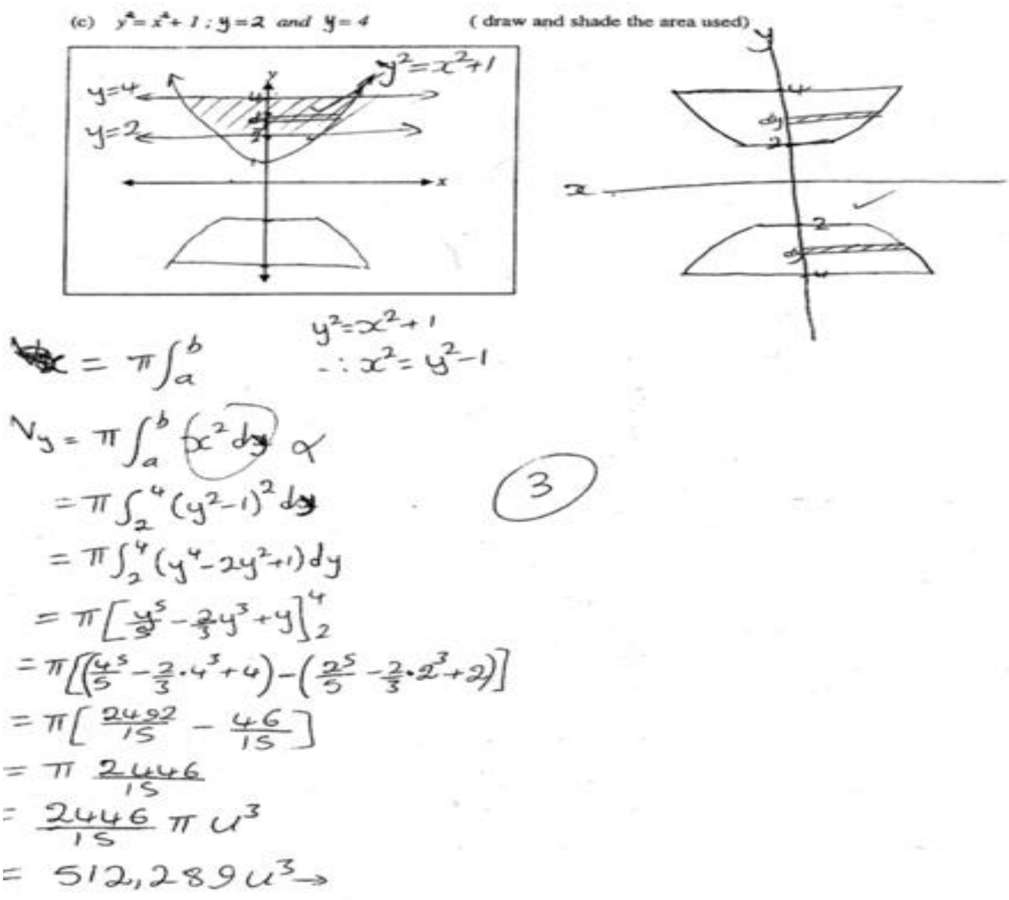


Figure 5.2: S1 written response

Question 1(c), shown in Figure 5.2 was very difficult for the majority of the students. The rest of the students could not draw the correct graphs, nor draw the strip. Only one student (S1) out of the seven managed to draw the correct graph, rotate correctly but used an incorrect formula when translating from visual to algebraic representation (able but unable). The student used the disc method, upon rotation about the *x*-axis instead of using the shell method, even though a correct strip after rotation was drawn below the *x*-axis, without showing that it was shell. Another mistake was substituting the x^2 for the disc method with $(y^2-1)^2$, instead of y^2-1 without a square. However, the steps that followed in manipulation of the incorrect method used were correct, with correct limits used. With the standard marking of the N6 examinations, this student will not be given any marks for the correct manipulation after the substitution with $(y^2-1)^2$ instead of y^2-1, even though all steps in calculating the volume are fully correct including the integration techniques used.

5.1.1.3 Graphing skills

Another aspect that was seen to be important in the preliminary study was the way in which students drew graphs. Table 5.3 gives a summary of how students drew graphs, in combination with the data presented in Table 5.2 for *aa*, *au*, *ua* and *uu* categories. In Table 5.3, F refers to fully correct, A refers to almost correct, P refers to partially correct, I refers to incorrect and N refers to not drawn. The ΣF; ΣA; ΣP; ΣI; and ΣN. are also given.

Table 5.3 The graphs drawn

	Test 1			Test 2		
	Q1b	Q2b	Sum	Q1c	Q2c	Σ
	NG	**NG**		**NG**	**NG**	
S1	Faa	Fau		Fau	Fau	
S2	Pua	Auu		Iuu	Nuu	
S3	Nua	Fau		Iuu	Nuu	
S4	Fuu	Auu		Nuu	Fau	
S5	Puu	Auu		Aau	Fau	
S6	Faa	Faa		Iuu	Auu	
S7	Iua	Fuu		Iuu	Puu	
ΣF	3	4	**7**	1	3	**4**
ΣA	0	3	**3**	1	1	**2**
ΣP	2	0	**2**	0	1	**1**
ΣI	1	0	**1**	4	0	**4**
ΣN	1	0	**1**	1	2	**3**

The results reveal that the students were most successful in drawing the graphs for Test 1 with 7 fully correct responses and 3 almost correct responses out of 14 responses. The 3 almost correct responses involved the first quadrant area of the circle $x^2 + y^2 = 9$, where the students drew a full circle. Four students drew this graph fully correct and there were no students who struggled with this graph. The graphs in Test 2 were difficult for the students. The graph that seemed most problematic to draw was a hyperbola with two straight lines, Question 1(c), $y^2 = x^2 + 1$; between $y = 2$ and $y = 4$, with the highest number of four incorrect responses. Only one student managed to draw this graph fully correctly.

5.1.2 Discussion of the results

The results of this study reveal that a significant number of students were able to identify the proper method used to compute the required volume (by rotating the region bounded by the given graphs) but tend to abandon the drawn graphs when they had to calculate algebraically or to select the correct formula. Some students were able to do what was expected by rotating the strip correctly but failed to make proper connections between the visual representation and the algebraic manipulations. The students performed better in Test 1 than in Test 2. The conclusion to be made is that Mathematica did not assist the students in improving visualisation in learning the content, or that perhaps Test 2 was more difficult. The lesson within Mathematica might have confused these students as they could not recall what they saw during the lesson since Mathematica was used for two consecutive days only. Another reason might be the fact that the students could not go back to the computer lab and practice for revision, since Mathematica was only used as a demonstration tool in class for illustration.

5.1.3 Conclusions

The focus of the preliminary study was to investigate students' difficulties with VSOR through investigating the interplay between the visual and algebraic skills with the emphasis on the correct rotation of the graph from the selected strip and being in a position to use the correct formula through visualisation using Mathematica**.** The results reveal that after visual illustrations using Mathematica, a significant number of students were able to rotate the strip correctly for the region bounded by the given graphs, but tended to abandon the drawn graphs when they had to calculate algebraically, as they chose the incorrect formula. As a result they failed to use the rotated strip for the selection of the formula for volume. Mathematica in this study only became useful during the lesson as was evident from the students' comments. When working on their own, most of the students could not recall what they had learnt through Mathematica. Not all students improved in the way in which they rotated the strip. Even if the strip was rotated correctly, the students could not identify the new shape after rotation of the strip as a disc washer or shell, particularly because the new shape was not drawn. The results of this experiment revealed that even if most students were able to select the correct strip, it was found that when coming to rotating the selected strip, students prefer the method that results in a disc or a washer method, hence ending up rotating the strip incorrectly. Most students avoided using the shell method. Mathematica is regarded as having little effect if any in the enhancing of learning rotations visually and enhancing students' imagination skills from what was demonstrated for new situations, especially with the difficult graphs.

The number of questions in Test 1 (four questions) and Test 2 (six questions) and the number of participants in the preliminary study (seven participants) were too limited to allow for transferability of the results to other settings. The results of the pilot study below, with more questions, testing different aspects of VSOR and more participants, are presented to allow for transferability to other settings.

5.2 PART 2: THE PILOT STUDY IN OCTOBER 2006

The pilot for this study was done at three different FET colleges using the 21-item instrument. The colleges were College A with 15 students, College B with 29 students and College C with 10 students. All students participated in the study voluntarily. At College C the lessons were also observed for six days before administering the 21-item instrument.

5.2.1 Lesson observations at College C

Students were taught areas and volumes for six days. The lecturer was the author of the textbook that the students used. The examples, exercises and short test questions were taken from the textbook. At the beginning of every lesson, students were given blank papers to write a short test based on the work done the previous day. The students worked individually for about 30 minutes and then the papers they wrote on were given to other students to mark. The lecturer presented the marked solutions on the board in the form of a lesson involving the students actively. The lecturer then continued to introduce the next section, also involving the students actively. Thereafter students were given classwork to do, which they extended as homework to prepare for the short test the following lesson. In the end the lecturer recorded the marks for the short test and the students papers were returned to them. Most of the students were performing very well even though the lecturer was complaining about their efforts. One is not sure whether the good performance was based on the fact that the questions were familiar to the students or whether they did the questions beforehand, since the questions were selected from the textbook, or whether the students knew their work very well. The examples of written responses given below are from College C since the students were observed being taught.

5.2.2 The results for the 21-item questionnaire

The results of the 21 questions are discussed per element using tables. Each element is discussed individually. The total scores obtained by the students are summarised in terms of raw scores and percentages for a particular question under a particular element and discussed. The participants were 15 students from College A, 29 students from College B and 10 students from College C. It is clearly indicated for every question in the tables how many students responded *correctly*, *partially correct*, *incorrectly* or *not done*. A *correct* response would be if everything is correct, a partially *correct* response would be where part of the solution is correct, an incorrect response would be where everything is incorrect, and not done would be where the student left a blank space. Examples of students' written responses are scanned in from College C only, where lessons were observed. For some questions one or two examples are given of either partially correct or incorrect responses. The percentage of *acceptably correct* responses (sum of correct and partially correct responses) is given in Table 5.4.

5.2.2.1 Responses for Element 1: Translation from algebraic to visual (2D)

Table 5.4: Responses for Element 1

Questions	College	Correct	Partially Correct	Incorrect	Not Done
1A: Represent $x^2 + y^2 \le 9$ by a picture.	A	10	4	0	1
	B	25	2	1	1
	C	8	1	0	1
	ALL	43	7	1	3
	%	80 %	13 %	2 %	6 %
1B: Represent $\int_0^1 (x - x^2)\, dx$ by a picture.	A	4	0	9	2
	B	12	3	6	8
	C	3	0	3	4
	ALL	19	3	18	14
	%	35 %	6 %	33 %	26 %
Acceptably correct responses %		**67%**			

From Table 5.4 we see that the performance in Question 1A was very good, 80% correctly drawn graph as compared to 35% in Question 1B, with a fairly high percentage 33% and 26% respectively for those students who drew incorrect graphs or did not respond to the question. For Question 1A, an ordinary graph (a circle), had to be drawn, whereas for Question 1B a graph involving the definite integral was to be drawn. For Question 1B, seemingly the students did not recognise the possibility of two different graphs (straight line and a parabola) or one graph (a parabola) for the given interval. In this element we investigated students' skills in translating from an algebraic expression to visual graphs. The students were expected to draw a graph from the given algebraic expression. One can therefore assume that for Question 1A the students were able to translate from algebraic to visual whereas for Question 1B it was problematic. Figure 5.3 and 5.4 give two examples of written responses for Question 1B revealing the *incorrect responses*.

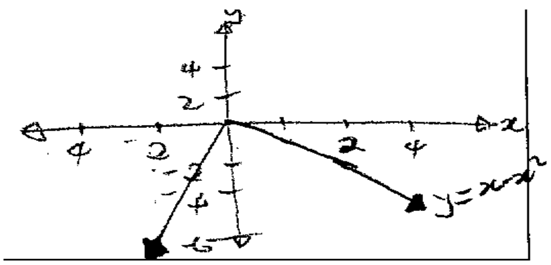


Figure 5.3: Straight lines as parabolas

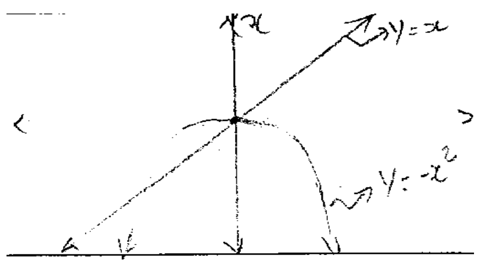
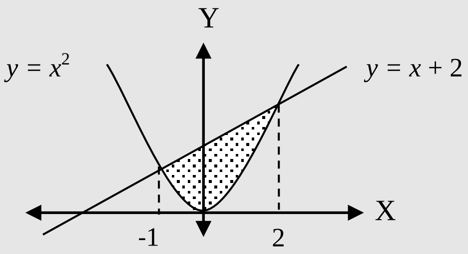
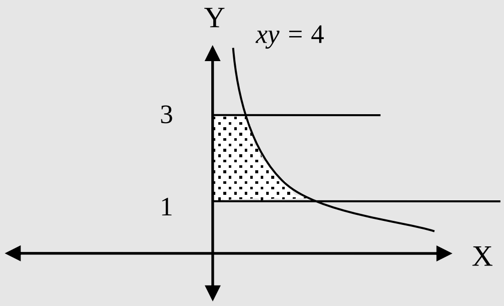


Figure 5.4: A parabola without limits

5.2.2.2 Responses for Element 2: Translation from visual to algebraic (2D)

Table 5.5: Responses for Element 2

Questions	College	Correct	Partially Correct	Incorrect	Not Done
2A: Give the formula for the area of the shaded region.	A	3	5	6	1
	B	8	19	2	0
	C	3	7	0	0
	ALL	14	31	8	1
	%	26 %	57 %	15 %	2 %
2B: Give a formula for the area of the shaded region.	A	2	6	6	1
	B	6	13	10	0
	C	3	3	3	1
	ALL	11	22	19	2
	%	20 %	41 %	35 %	4 %
Acceptably correct responses %		**72%**			

For Question 2A and 2B the graphs were given and students were expected to give the formula that describes what was drawn. Most of the students did the questions partially correct, 57% in Question 2A and 41% in Question 2B. The partial correct response to this question was when the formula for area was given without substituting with the given graphs as $\int_{-1}^{2} (y_1 - y_2)\, dx$ for Question 2A and $\int_{1}^{3} (x_1 - x_2)\, dy$ for Question 2B.

In this element students worked from visual to algebraic. Seemingly the question was not that clear to the students as they showed the first step only. One is therefore not sure whether the students were going to substitute the graphs correctly or not as they did not proceed to the next step, except the fact that the first step was correct. One can therefore not make any claims or assumptions. In other instances students did not use the integral sign. The responses to Question 2 reveal that the questions were not clear to students since many students did not continue to do the substitution after using the formula for area.

Discussion on Element 1 and Element 2

If one compares Elements 1 and 2 it is evident that the large number of students got the *correct response* in Element 1, 80% for Question 1A and 35% for Question 1B, whereas for Element 2 the percentages were 26% and 20% respectively for Questions 2A and 2B. But overall, when considering the proportion of acceptably correct responses in Tables 5.4 and 5.5, many students (72%) were able to translate from visual to algebraic in 2D, while 67% were able to translate from algebraic to visual in 2D. This means that performance was good in translation from algebraic to visual and from visual to algebraic in 2D.

5.2.2.3 Responses for Element 3: Translation from algebraic to visual (3D)

Table 5.6: Responses for Element 3

Questions	College	Correct	Partially Correct	Incorrect	Not Done
3A:Draw the 3-D solid of which the volume is given by $V=\pi\int_0^1 (1-x^2)^2\ dx$.	A	0	6	5	4
	B	0	5	12	12
	C	0	3	4	3
	ALL	0	14	21	19
	%	0%	26%	39%	35%
3B:Draw the 3-D solid of which the volume is given by $V=2\pi\int_0^1 x(1-x^2)\ dx$.	A	5	1	5	4
	B	4	1	10	14
	C	1	3	3	3
	ALL	10	5	18	21
	%	19%	9%	33%	39%
Acceptably correct responses %		**27%**			

From Table 5.6 the performance for Questions 3A and 3B was fairly poor. Many students (39%) got the answer incorrect and 35% did not respond to Question 3A, whereas 33% got the answer incorrect and 39% did not respond to Question 3B. Both questions required that students draw graphs in three dimensions, with Question 3A deriving from the washer method and Question 3B deriving from the shell method. For both questions the graphs that were drawn by students were in two-dimensions, mostly incorrect parabolas. For Question 3A no student managed to draw the correct graph. The students were not aware that they had to draw the parabola $1-x^2$ on the interval [0,1], which represents half a parabola when rotated about the *x*-axis. For Question 3B a fair number of students (19%) managed to draw the graph. The students were not aware that they had to draw the parabola $1-x^2$, on the interval [0,1], which represents a parabola when rotated about the *x*-axis, where the *x* next to $1-x^2$ is part of the formula for the shell method and need not be drawn.

The conclusion that could be made from Element 3 is that a large number of students (more than 70%) are unable to translate from an algebraic equation to a visual diagram in three-dimensions. The students are also unable to recognise that the formula given derives from the disc method (Question 3A) from the πr^2 of the given equation from the circle and the other formula from the shell method (Question 3B) from the $2\pi r$ as the surface area of the cylinder.

Examples of students' *incorrect responses* are shown in Figures 5.5 and 5.6.

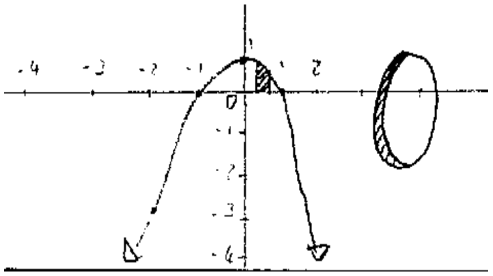


Figure 5.5: A disc and a parabola

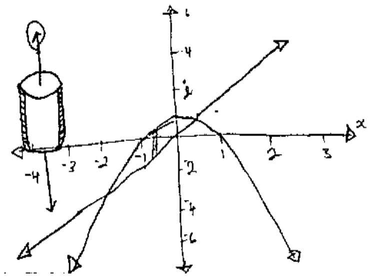
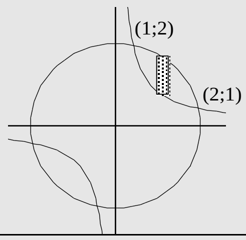
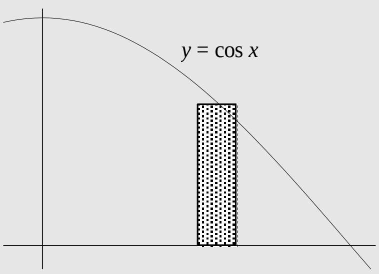


Figure 5.6: A shell and a parabola

5.2.2.4 Responses for Element 4: Translation from visual to algebraic (3D)

Table 5.7: Responses for Element 4

Questions	College	Correct	Partially Correct	Incorrect	Not Done
4A: Below the 1st quadrant region bounded by graphs of $x^2 + y^2 = 5$ and $xy = 2$ is selected using the given strip. Give the formula for the volume generated if this region is rotated **about the *x*-axis.** Do not calculate the volume.	A	3	9	3	0
	B	8	16	5	0
	C	1	4	5	0
	ALL	12	29	13	0
	%	22%	54%	24%	0%
4B: Below the region bounded by the graph of $y = \cos x$, the *x*-axis and the *y*-axis is selected by the given strip. Give the formula for the volume generated when this region is rotated **about the *y*-axis**. Do not calculate the volume.	A	1	7	6	1
	B	0	3	25	1
	C	0	7	3	0
	ALL	1	17	34	2
	%	2%	31%	63%	4%
Acceptably correct responses %		**55%**			

As indicated in Table 5.7 for Questions 4A and 4B, different response patterns were found. For Question 4A, most students (54%) had the answer partially correct. The partially correct response was if the student managed to give the correct formula of the washer as $\pi\int_1^2 (y_1^2 - y_2^2)\,dx$, which can also be classified as incorrect as one is not sure if the students were going to substitute correctly using the given equations of the graphs. If the partial responses were to be classified as incorrect, the highest percentage would be 78% for the incorrect response. Few students (22%) gave the correct response. For Question 4B the highest number of students (63%) got the answer incorrect. 31% of the students got the answer partially correct, while only 2% got the correct answer. The partial response for Question 4B was when the equation given was that of a shell as $2\pi\int_0^{\frac{\pi}{2}} xy\,dx$. If we also consider the partial correct response as incorrect the total number for the incorrect responses would be 94%. The incorrect responses given included instances where limits for integration were incorrect, for example the limits were given as between 0 and 1 or not given at all. In other instances, the formula given did not represent a shell method. It at times represented a disc or a washer. In general the performance was not good. The students were not able to translate from visual to algebraic in three-dimensions, since a low percentage got the correct answer.

Discussion on Element 3 and Element 4

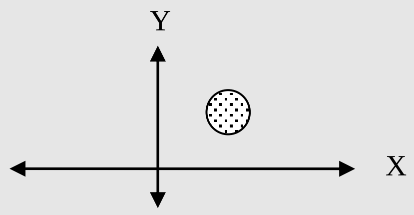
A large number of students (more than 70%), were unable to translate the given equations for volume to diagrams (from algebraic representation to visual representation). The students performed better when translating the visual diagram to algebraic equations when a washer method was required and failed (more than 60%) when they had to use a shell method.

Considering the proportion of acceptably correct responses in Tables 5.6 and 5.7, only 27% of the students were able to translate from algebraic to visual in 3D, regarded as performance that is not satisfactory, while 55% of the students were able to translate from visual to algebraic in 3D, regarded as satisfactory performance. Overall, the students were struggling to translate from algebraic to visual, than from visual to algebraic in 3D.

The average of proportion of acceptably correct responses as 55.25%, from Elements 1, 2, 3 and 4 (72%, 67%, 55% and 27%) from Tables 5.4, 5.5, 5.6 and 5.7, reveal that overall, performance was satisfactory in translation between algebraic and visual in 2D and in 3D.

5.2.2.5 Responses for Element 5: Translation from 2D to 3D

Table 5.8: Responses for Element 5

Questions	College	Correct	Partially Correct	Incorrect	Not Done
5A: Draw the solid that will be formed if a line with a positive gradient passing through the origin is rotated about the ***x*-axis**, where $x \in [0,3]$.	A	2	2	8	3
	B	0	8	21	0
	C	1	1	6	2
	ALL	3	11	35	5
	%	6%	20%	65%	9%
5B: What solid do you get if you rotate the circle below about the ***y*-axis**?	A	0	0	9	6
	B	0	0	17	12
	C	3	0	6	1
	ALL	3	0	32	19
	%	6%	0%	59%	35%
Acceptably correct responses %		**16%**			

In this element the questions required that the students had to analyse critically. More insight was needed in order to deal with these questions properly. Students had to imagine and to draw or explain their solutions. As seen in Table 5.8, for Questions 5A and 5B, the same small percentage (6%) of students got the answer correct and the majority of students responded incorrectly (65% and 59% respectively). It seemed as if most of the students did not know what a line with positive gradient means. For some of those who drew a correct line, y intervals were used instead of the given x intervals. The students moreover did not recognise that the given circle for Question 5B was not lying on any of the axis. Examples of the *incorrect responses* for Question 5A are given in figures 5.7 and 5.8, where a line with a negative gradient with the y-intercept as 3 and a solid similar to an ellipsoid were drawn.

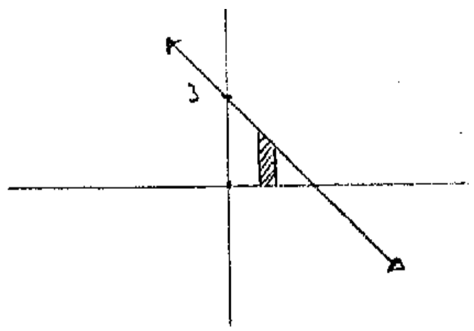


Figure 5.7: A line representing a solid

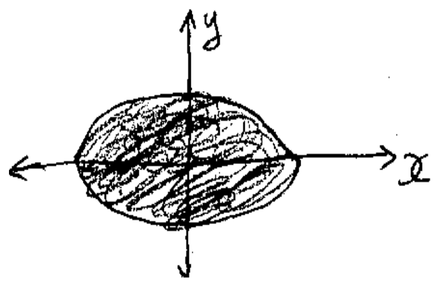
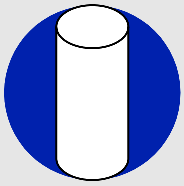


Figure 5.8: An ellipsoid

5.2.2.6 Responses for Element 6: Translation from 3D to 2D

Table 5.9: Responses for Element 6

Questions	College	Correct	Partially Correct	Incorrect	Not Done
6A: Discuss how a hemisphere is generated as a solid of revolution.	A	0	0	4	11
	B	0	0	2	27
	C	0	0	3	7
	ALL	0	0	9	45
	%	0%	0%	17%	83%
6B: A hole with radius 2cm is drilled through the centre of the sphere of radius 5 as in the picture. Describe the curves that are rotated to generate this solid. 	A	0	1	4	10
	B	0	4	5	20
	C	0	2	2	6
	ALL	0	7	11	36
	%	0%	13%	20%	63%
Acceptably correct responses %		**7%**			

As displayed in Table 5.9, Questions 6A and 6B seemed to be quite difficult since none of the students got the answer correct and most of the students did not respond to the questions at all (83% and 63% respectively). Question 6A fared the worst with no single student even getting it partially correct. The problem might have been that both questions were given as word problems even though for Question 6B a diagram was given. The students did not only fail to translate from 3D to 2D, but they also failed to analyse and comprehend the questions.

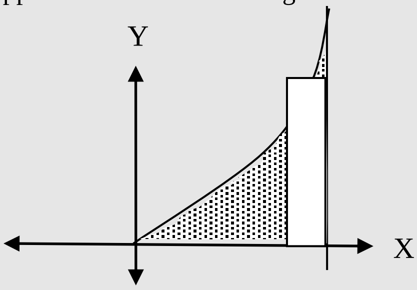
Discussion on Element 5 and Element 6

From the questions that were asked by the students as they were writing while I was walking around, it was clear that the students had problems in interpreting and understanding the terminology used in these elements. The question that some students asked based on Element 5 was that they did not know what a *solid of revolution* was. For Element 6, is seemed as if students did not know what a *hemisphere* was. In general, the students failed to give the correct response, whether a diagram was given or not. In fact, the performance was lower if the diagram was given when they had to translate from 2D to 3D and much lower if the diagram was not given when they had to translate from 3D to 2D. The high percentage of incorrect responses and no responses (between 74% and 100%) in total, alludes to that. It seems as if the majority of the students had no idea what the questions entailed. The students failed to translate from 2D to 3D and from 3D to 2D.

The average percentage of the acceptably correct responses (11.5%) from table 5.8 as 16% and from Table 5.9 as 7% reveals that the performance in translating between 2D and 3D was poor.

5.2.2.7 **Responses for Element 7**: Translation from continuous to discrete (visual 2D)

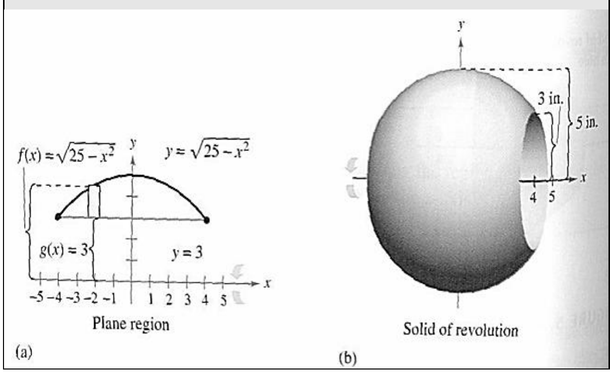
Table 5.10: Responses for Element 7

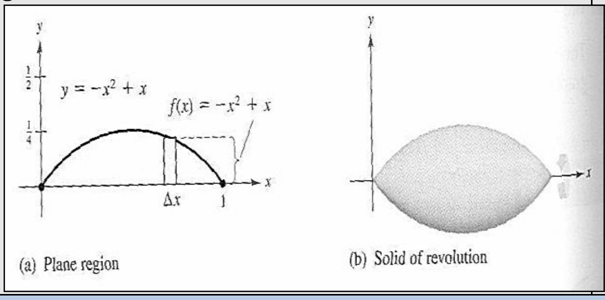
Questions	College	Correct	Partially Correct	Incorrect	Not Done
7A: Sketch three additional rectangles (similar to the given rectangle) so that the total area of the rectangles approximates the shaded region.	A	2	3	5	5
	B	1	7	8	13
	C	0	5	2	3
	ALL	3	15	15	21
	%	6%	28%	28%	39%
Acceptably correct responses %		**34%**			

Question 7 in Figure 5.10 involved the approximation of the area under the curve using *rectangles*. The students were expected to translate to discrete, what was given as continuous. A high number of students (39%) did not respond to the question, with only 6% responding correctly. A large number of students failed to approximate the area with 28% partially correct responses and the same number of incorrect responses. Some students drew rectangles but there were spaces between them. These students failed to translate from continuous to discrete in 2D.

5.2.2.8 **Responses for Element 8**: Translation from continuous to discrete (visual 3D)

Table 5.11: Responses for Element 8

Questions	College	Correct	Partially Correct	Incorrect	Not Done
8A: When the graph below is rotated, the solid on the right is generated. Show how you would cut the solid in appropriate shapes (discs, washers or shells) to approximate the volume of the solid.	A	0	3	5	7
	B	0	7	7	15
	C	0	6	2	2
	ALL	0	16	14	24
	%	0%	30%	26%	44%

Questions	College	Correct	Partially Correct	Incorrect	Not Done
8B: When the graph below is rotated, the solid on the right is generated. Discuss how you would cut it to generate either discs, washers or shells	A	4	2	3	6
	B	0	7	12	10
	C	1	2	4	3
	ALL	5	11	19	19
	%	9%	20%	35%	35%
Acceptably correct responses %		**29.5%**			

In Table 5.11, Question 8A involved approximation of the volume using washers, while Question 8B used discs. Seventy per cent of the students failed to approximate the volume with no correct responses for Question 8A and 9% of the correct responses for Question 8B. In this question the students were expected to translate from a continuous representation to a discrete representation. Apparently they did not succeed, based on what was evident from the students' responses. Some students only drew the slices (washers or discs) without the order that would give rise to the given solid of revolution. The responses from the students reveal that most of them were confused or they had no idea what the question was about. The students did not make any connections about the whole figure and breaking it down into smaller pieces that would still resemble the original diagram. These students failed to translate from continuous to discrete (3D). In Figures 5.9 and 5.10, one example of a partially correct response for Question 8A and an *incorrect response* for Question 8B is given.

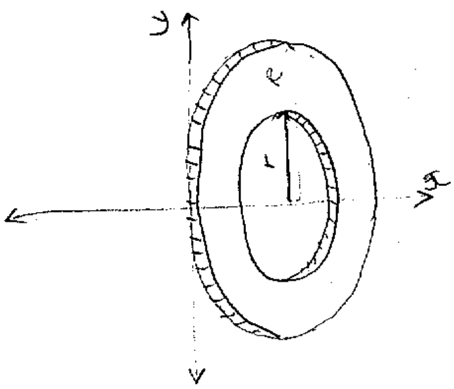


Figure 5.9: A cross-section of a washer

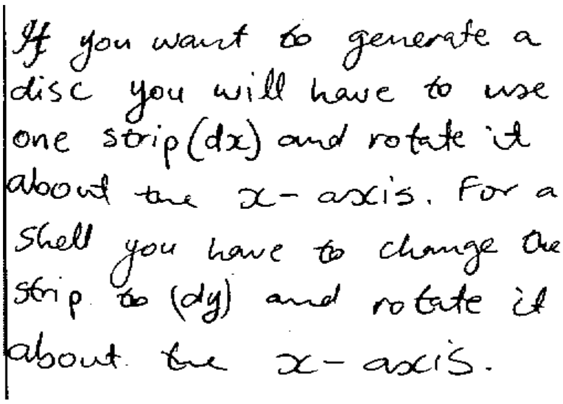


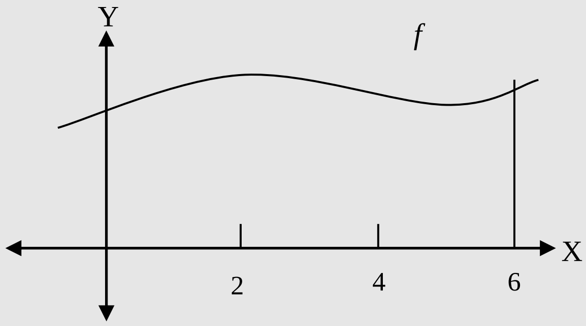
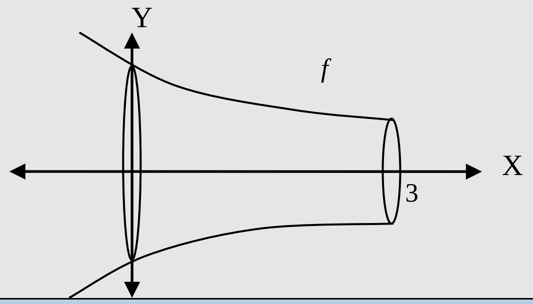
Figure 5.10: Misconceptions about the strips

Discussion on Element 7 and Element 8

In Questions 7, 8A and 8B, the questions were given including the diagrams, with Question 7 focusing on area and Question 8 focusing on volume. Most students failed to approximate the area and the volume from the given 2D and 3D diagrams. They had to translate from continuous representation to discrete representation. The responses from the students reveal that most of the students were confused since they had no idea how to do the approximations, irrespective of the problem given in 2D or in 3D. Students did not make any connections about the whole figure and breaking it down into smaller pieces that would still resemble the original diagram (moving from continuous to discrete representations).

5.2.2.9 Responses for Element 9: Translation from discrete to continuous and continuous to discrete (algebraic) in 2D and 3D

Table 5.12: Responses for Element 9

Questions	College	Correct	Partially Correct	Incorrect	Not Done
9 A: Show what the following represent with a sketch. $2f(0)+2f(2)+2f(4)$ 	A	1	0	4	10
	B	0	1	7	21
	C	0	0	4	6
	ALL	1	1	15	37
	%	2%	2%	28%	69%
9B: If the volume of the given solid of revolution is approximated by discs, sketch the discs that would give the volume. $\pi(f(0))^2+\pi(f(1))^2+\pi(f(2))^2$ 	A	0	1	7	7
	B	0	2	6	21
	C	0	3	5	2
	ALL	0	6	18	30
	%	0%	11%	33%	56%
Acceptably correct responses %		**7.5%**			

In Question 9A and 9B an expression and a diagram were given. Question 9A involves a 2D diagram and Question 9B involves a 3D diagram. The majority of the students did not respond to this question, 69% for Question 9A and 56% for Question 9B, and many of the other students got the answer incorrect, 28% and 33% respectively. Only one student (2%) out of the 54 students produced a correct response for Question 9A and none of the students gave a correct response for Question 9B. The students were unable to relate the given expressions to rectangles and discs respectively. The expressions given were not related to the graphs drawn. In this element students failed to translate from discrete to continuous and from continuous to discrete. The results here also reveal that most of the students had no idea about what the question required.

Discussion on Elements 7, 8 and 9

Questions 7, 8A and 8B, 9A and 9B are about the translation between discrete and continuous representations, involving area and volume. The questions given all included diagrams, with Question 7 and Question 9A focusing on area and Question 8 and Question 9B focusing on volume. Approximately 90% of the students failed to approximate the area and the volume, more so if given in a form of an area formula or a formula for volume as it was the case in Questions 9A and 9B, where the students had to translate from discrete to continuous and from continuous to discrete. The responses from the students reveal that most of the students were confused or they had no idea what these formulae in Question 9 were all about, they could not recognise an area formula or a disc formula. Translation between 2D and 3D was also problematic, with no connections made in relation to the given diagrams.

The proportion of acceptably correct responses in Table 5.10 as 34% for translation from continuous to discrete (visually) in 2D and in Table 5.11 as 29.5% translation from continuous to discrete (visually) in 3D, both reveal that performance is not satisfactory. For translation from discrete to continuous and continuous to discrete (algebraic) in 2D and 3D, the performance was poor, with only 7.5% of the responses acceptably correct as shown in Table 5.12. Overall, the average for the proportion of acceptably correct responses (19.5%) in translation from continuous to discrete (visually) in 2D and in 3D and translation from discrete to continuous and continuous to discrete (algebraic) in 2D and 3D, reveal that the performance in these elements was poor.

5.2.2.10 Responses for Element 10: General manipulation skills

Table 5.13: Responses for Element 10

Questions	College	Correct	Partially Correct	Incorrect	Not Done
10A: Calculate $\int_0^1 \pi(1-x^2)^2\,dx$	A	9	3	3	0
	B	19	3	7	0
	C	6	1	2	1
	ALL	34	7	12	1
	%	63%	13%	22%	2%
10B: Calculate $\int_0^1 2\pi\,x(1-\sin x)\,dx$	A	0	2	12	1
	B	2	5	21	1
	C	1	0	8	1
	ALL	3	7	41	3
	%	6%	13%	76%	6%
Acceptably correct responses %		**47.5%**			

Questions in Table 5.13 are similar to some of the questions in the mathematics N6 examination paper, involving a definite integral. Question 10A involved basic rules for integration, whereas Question 10B involved integration by parts. The highest number of students (63%) got the correct answer for Question 10A while the highest number (76%) failed to give the correct response for Question 10B. The students failed to use integration by parts properly. Those who tried to use it got confused along the way. Question 10A involved direct integration and pure manipulation skills.

The proportion of acceptably correct responses, given as 47.5% in Table 5.13, reveals that the students' performance in the case of general manipulation skills was satisfactory.

In Figure 5.11, one example is given for Question 10B of an *incorrect response*.

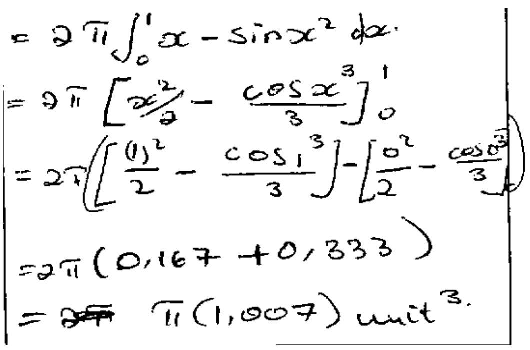


Figure 5.11: Errors with integration rules

5.2.2.11 Responses for Element 11: Consolidation and general level of cognitive development

Table 5.14: Responses for Element 11

Questions	College	Correct	Partially Correct	Incorrect	Not Done
11A: Given the graphs of $y = \sin x$ and $y = 1$ (i) Draw the graphs and shade the area bounded by the graphs and $x = 0$	A	12	0	3	0
	B	15	2	12	0
	C	7	0	3	0
	ALL	34	2	18	0
	%	63%	4%	33%	0%
(ii) Show the rotated region about the ***y*-axis** and the strip used	A	4	0	10	1
	B	3	0	24	2
	C	6	0	3	1
	ALL	13	0	37	4
	%	24%	0%	69%	7%
(iii) Write down a formula to find the volume when the region between $y = \sin x$ and $y = 1$ is rotated about the ***y*-axis**.	A	0	5	8	2
	B	0	1	27	1
	C	1	8	0	1
	ALL	1	14	35	4
	%	2%	26%	65%	7%
11B: Use integration methods to derive the formula of a volume of a cone of radius *r* and height *h*.	A	0	0	8	7
	B	0	0	13	16
	C	0	0	7	3
	ALL	0	0	28	26
	%	0%	0%	52%	48%
Acceptably correct responses %		**29.8%**			

Questions in Table 5.14 are also similar to the question on application of VSOR in the mathematics N6 examination paper, where the students are expected to start by drawing the graph(s), indicate the representative strip that they would use and to calculate the area of the bounded region or the volume generated when this region is rotated about the *x*-axis or about the *y*-axis. Question 11A is subdivided into three subquestions, whereas Question 11B involved one question only. For Question 11 A (i), the majority of the students (63%) managed to draw correct graphs, while about 33% failed. Most students were able to draw the graphs given and those who failed to draw the proper graphs did not draw the line $y = 1$ as well; they only drew the graph of $y = \sin x$. For Question 11A (ii), 69% of the students were unable to show the rotated region about the ***y*-axis** from the strip used, whereas about 65% failed to give the correct formula to calculate volume for Question 11A (iii). For Question 11B, it seems as if the students did not have any clue as to what the question entailed. None of the students got the answer correct or partially correct. Nearly half of the students gave an incorrect response and about half did not respond to the question, 52% and 48% respectively. The performance in consolidation and general level of cognitive development was not satisfactory, with 29.8% of the responses shown in Table 5.14 as nearly correct.

For Element 11, the students' performance raises questions as to how students cope with the final N6 examinations. Even if the students had some capabilities in drawing graphs, they were seen to struggle to interpret them. The implication here is that if the students fail to carry out one step, then the chances are that they might fail to give the correct solution for the questions that follow if the questions are not independent. The students cannot cognitively cope with the section of VSOR.

The difficulties that the student had in Element 11 were evident from the fact that the majority of students failed in Elements 5 and 6 (translation between 2D and 3D) involving three-dimensional thinking and Elements 7, 8 and 9 (translation between continuous and discrete representations in 2D and 3D), where the performance was poor. For an example, in Question 11A, even though a large number of students (63%) were able to draw the correct graphs, 69% failed to show the rotated graph about the *y*-axis. Some of the reasons for failure were that the representative strip selected was not correct or rotated incorrectly.

Examples are given in Figures 5.12 and 5.13 for Question 11A. The first example shows how the graph was drawn correctly but not interpreted correctly from the Δ*y* strip, while the second example shows how the graph was drawn *incorrectly* and the *partially correct* formula for volume was given from the formula sheet without being adapted to the drawn graphs. In both cases, the equations were not interpreted further.

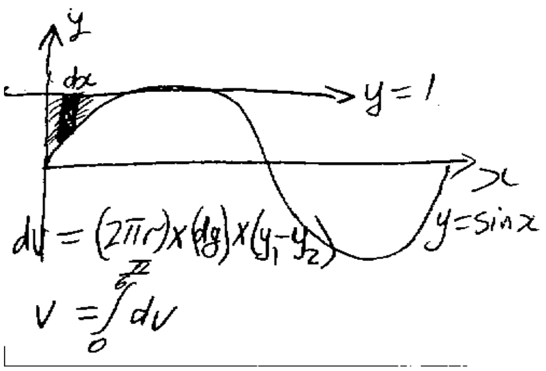


Figure 5.12: Incorrect limits used

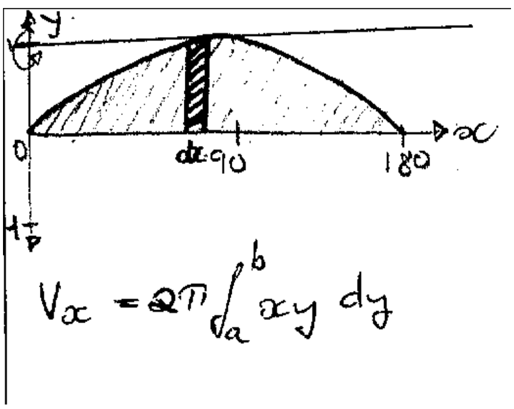


Figure 5.13: Incorrect region shaded

5.3 CONCLUSIONS FROM THE RESULTS

The results of this study reveal that, even though in general the performance in the pilot study was not satisfactory (32.7%), there were instances where students' performance was satisfactory and good. The conclusions that could be made from the proportion of acceptably correct responses in all the 11 elements are that the students are struggling with VSOR. At times they manage to show some competency, but mostly they seem to give solutions that are incorrect. For most of the elements the performance was not satisfactory. The students were unable to solve problems that involved translation between two-dimensions and in three-dimensions (poor performance). In particular the students failed to select the representative strip and to interpret graphs (poor performance). The poor performance in both Skill factors II and III, might have led to the very poor performance in VSOR. The results from students at the College C, where I observed lessons for six days were poor. Even though these students appeared to be performing well in class, taught by the author of the N6 textbook, their performance using the 21-item questionnaire was nonetheless the same as that of other students who were not observed, at times even outclassed by them, given from the way in which they responded to the questions: correctly, partially correct, incorrectly or not done.

The results for both the preliminary study and the pilot study reveal that even though the students have some capabilities in drawing some graphs, they cannot interpret them properly. For the pilot study it was further revealed that students are finding it difficult to solve problems where they need to translate between two and three dimensions and those involving the selection of the representative strip as well as translating between the algebraic and the visual representations in 3D. However, since it was evident that some questions from the pilot study were not clear to the students, the instrument was modified and used for main data collection in Phase II that is presented in Chapter 6.