CHAPTER 3: CONCEPTUAL FRAMEWORK

Having discussed the background, defined the research question and discussed the literature for this study, this chapter aims at establishing the conceptual framework (my own model based on my experience) and the theoretical orientation (the work of others) that frames this study. The different modes of representations (visual/graphical; algebraic/symbolical and numerical) that affect the learning of Volumes of Solids of Revolution (VSOR) are explored and used to develop the five skill factors of knowledge that constitute the learning of VSOR as the framework for this study. The five skill factors are: (I) Graphing skills and translation between visual graphs and algebraic equations/expressions (both in 2D and in 3D); (II) translation between 2D and 3D diagrams; (III) translation between continuous and discrete representations; (IV) general manipulation skills and (V) consolidation and general level of cognitive development. It is identified whether the different skill factors require procedural and/or conceptual knowledge. Finally the conceptual framework is partially positioned within other related frameworks.

3.1 THE THREE MODES OF REPRESENTATIONS

A conceptual framework is a system of concepts, assumptions, expectations, beliefs and theories that supports and informs research (Maxwell, 2005, p. 33). The conceptual framework of this study is rooted in the following representation of knowledge:

Visual/graphical – where students' interpretations are analysed from the graphs, diagrams (both in 2D and in 3D), or other forms of pictorial illustration they produce.

Algebraic/symbolical – where students' interpretations of the visual/graphical are analysed from the equations/expressions, symbols and notations they use.

Numerical – where students' interpretations are analysed from the calculations they use (points of intersection, intercepts with the axes and other important points) when drawing graphs, computations and manipulation of the given integrals (using equations/expressions and symbols) to calculate area and volume, and their further applications.

3.2 MY CONCEPTUAL FRAMEWORK INVOLVING THE FIVE SKILL FACTORS

This research focuses on students' difficulties involving VSOR. In learning about VSOR, students are expected to sketch graphs, shade the region bounded by the graphs, show the representative strip for the shaded region, rotate the graphs (focusing on the shaded region) and calculate the volume generated. In so doing the students are expected to use visualisation as a tool for learning in order to translate the visual graphs to algebraic equations and to do the manipulations that follow. Based on these premises and also relating to literature, a theoretical framework was developed.

The theoretical framework is based on five skill factors given in Table 3.1. The five skill factors were developed from the analysis of the section based on VSOR from the N6 textbooks, to determine which skills students need for competency. Each skill factor is categorised according to elements (that clarify the skill factor in detail), as shown in Table 3.1 below, 11 elements in total. In order to investigate the difficulties, students' written and verbal interpretations were analysed in line with the five skill factors. The use of writing in this study helped the researcher monitor the students' conceptual understanding level required for VSOR. According to McDermott and Hand (2010, p. 519), writing in the science classroom is viewed as a communication tool as well as an epistemological tool to develop conceptual understanding.

Table 3.1: The five skill factors

	I Graphing skills and translating between visual graphs and algebraic equations/expressions in 2D and 3D	II Three-dimensional thinking	III Moving between discrete and continuous representations	IV General manipulation skills	V Consolidation and general level of cognitive development, incorporating Skill factors I, II,III and IV
Skill Factors					
Elements	*1*: *Graphing skills* *2*: *Algebraic to Visual (2D)*. *3*: *Visual to Algebraic (2D)*. *4*: *Algebraic to Visual (3D)*. *5*: *Visual to Algebraic (3D)*.	*6*: *2D to 3D*. *7*: *3D to 2D*	*8*: *Continuous to discrete (Visual 2D and 3D)* *9*: *Discrete to continuous and continuous to discrete (Algebraic)*	*10*: *General manipulation skills*	*11*: *Consolidation and general level of cognitive development*

The five skill factors and elements are now discussed individually in order to motivate the framework for this study, with the subsequent elements under each skill factor.

3.2.1 Skill Factor I: Graphing skills and translating between visual graphs and algebraic equations/expressions in 2D and 3D

Skill factor I involves visual learning and consists of Elements 1, 2, 3, 4 and 5. Visual learning involves the learning process whereby students can make sense of what they can visualise. The assumption is that if students learn visually they can reflect on pictures and diagrams mentally, on paper or with technological tools. In calculating VSOR, students are expected to visualise the area bounded by the drawn graphs between certain values (along the y-axis) and between certain values (along the x-axis) that serve as boundaries/limits for integration.

Elements 1, 2 and 4: Graphing skills and translating algebraic equations/expressions to visual graphs in 2D and 3D

Element 1 refers to the skills required when students are given the equations of one or more graphs that they have to draw, or in words. In Elements 2 and 4 the equations/expressions are given to represent a 2D or a 3D diagram or in the form of an integral formula. In drawing those graphs the students must show the intercepts with the x-axis and the y-axis and other important points including the parameters, turning points, points of inflection and the points of intersection if any.

Elements 3 and 5: Translating visual graphs to algebraic equations/expressions in 2D and 3D

Students translate the drawn graph(s) (given in 2D or 3D) to the algebraic formula in order to compute the area (skill required in Element 3) and the volume of the rotated area (skill required in Element 5). In so doing, they are translating between the visual graphs and the algebraic equations/expressions, which involve the use of equations or formulae in order to calculate the volume of the solid generated, after rotation of the shaded area about the x-axis or the y-axis. Students are expected to demonstrate what they have visualised or imagined from the shaded area by translating that into correct equations, in order to further calculate the value of the integral that represents the area or the volume of the shaded area upon rotation. Using integration the volume can be calculated by using the correct formula related to the selected strip (the Δx or the Δy strip), which results in a different method upon rotation, being the disc/washer or shell method. The Δx and the Δy represent the width of each selected rectangular strip. Depending on the selected strip, the students also need to calculate the necessary parameters if not given as coordinates, representing the x-value and the y-value. The substitution to the algebraic formula involves all three representations. As students do the

substitution, they are at the same time translating from the visual graphs to the algebraic equations. The general manipulation skills are also used after selection of the formula for area or volume resulting from the translation from the graphical representation to the algebraic representation. Students are also working in the visual/graphical representation as they use the selected strip (Δx or Δy) during the substitution of the different graphs in terms of top graph minus bottom graph or right graph minus left graph which can be done from interpreting the drawn graph for calculating area or volume.

The area and the volume to be calculated are integrals. Hence area could be expressed as $A = \int_a^b y\,dx$ and the volume of the solid could be given as $V = \int_a^b A(x)\,dx$. The formulae respectively if the radius is y and the strip width is Δx (or dx), are as follows:

$$V = \pi \int_a^b y^2\,dx \; ; \; V = \pi \int_a^b (y_1^2 - y_2^2)\,dx \quad \text{and} \quad V = 2\pi \int_a^b xy\,dx$$

The first formula represents the disc method, the second represents the washer method and the last one represents the shell method. In order to use the correct formula, the student must relate to the drawn graph to the correct strip.

Depending on how the students are taught or how they prefer to learn, students may tend to portray some kind of preferences and capabilities. In one question from the FET National examination paper, students were asked to *calculate the volume described* which refers to a drawn graph. In this question students were required to translate the visual graphs to the algebraic equations, with an emphasis on graphs being the starting point in translating to algebraic from what one sees as the formula for the disc, washer and shell methods respectively. The numerical representation is also evident when students use the FTC after integration for evaluating the definite integral. In calculating area and volume, students may in some instances guess the correct formula without the correct reasoning.

In learning about VSOR students make mental pictures as they imagine rotations for disc, washer or shell methods respectively. The mental pictures are referred to by Dreyfus (1995) as *concept images*. Visualisation is a key component in mathematical problem-solving (Deliyianni et al., 2009). Christou et al. (2008, p. 2) and Gutiérrez (1996, p. 9) clarifies visualisation as integrated by four main elements: mental images, external representations, processes of visualisation, and abilities of visualisation. Thornton (2001, p. 251) argues that

visual thinking should be an integral part of students' mathematical experiences. He argues that visualisation plays a significant role in developing *algebraic* understanding (an important aspect to be explored in this study) and that it is also seen as valuing a variety of learning styles as well as providing a powerful problem-solving tool. According to Thornton, "powerful algebraic thinking arises when students attach meaning to variables and visualise the relationship in a number of different ways" (Thornton, 2001, p. 252). In this study I investigate those relationships, with the main focus on the development of algebraic thinking as students translate the visual (rotation of graphs after selection of an appropriate strip) to the algebraic manipulations (of equations) as they compute the volume using integration.

Under the Skill factor I, all three modes of representations overlap. As students draw graphs, they use general manipulation skills to calculate the intercepts with the axis and other important points; hence they operate in the numerical representation. At the same time they are translating between the given equations/symbols and the visual/graphical representation.

3.2.2 Skill Factor II: Three-dimensional thinking

Skill factor II also involves translation from 2D diagrams to 3D diagrams (Element 6) and translation from 3D diagrams to 2D diagrams (Element 7). In learning of VSOR students draw graphs, giving rise to two-dimensional shapes. The two-dimensional shapes are given in terms of the region within the given parameters, which upon rotation result in three-dimensional objects. The strip drawn approximates the area within the given parameters. The drawn strip for the area selected (in 2D) is used to calculate area from integration. If the selected area is rotated, students should use integration to compute the volume generated using the disc, washer or shell methods. In order to compute the volume generated as a result of rotating the region bounded by those graphs, students are expected to work in one dimension to identify the points that serve as parameters to these graphs. Students are then expected to relate (transfer) to prior knowledge regarding Riemann[2] sums when working in two-dimensions to compute the generated volume in three-dimensions as a solid of revolution.

In generating a solid of revolution, the students have to argue that when a 2D object (e.g. a segment, a circle, a square, a triangle, a sinusoidal curve or a free shape curve) is rotated in 3D around a vertical axis it can generate a variety of 3D rotational objects (Christou et al.,

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[2] Using a number of rectangular strips (slicing vertically or horizontally) to calculate the area bounded by curves and summing them up using integration.

2008, p. 5). If the students fail to make such connections, it may be because their mental schemes do not recognise what they see. This may be due to their *internal representations* which conflict with the *external representations* (Knuth, 2000). For example, a student may not have the necessary tools (preknowledge or cognitive skills) to deal with the data presented by the external representation (diagram/graph) and internalise it. With Skill factor II, as it was with Skill factor I, all three modes of representations overlap. As students draw graphs or diagrams (in 2D or 3D), they use general manipulation skills to calculate the intercepts with the axis and other important points, hence they operate in the numerical representation. At the same time they are translating between the given equations/symbols and the visual/graphical representation, both in 2D and in 3D.

3.2.3 Skill Factor III: Moving between continuous and discrete representation

Skill factor III involves only visual/graphical representation. It focuses on Elements 8, where translation is from continuous to discrete representations involving 2D and 3D diagrams and Element 9 where translation is from discrete to continuous representation and from continuous to discrete representation involving algebraic expressions. After drawing the graphs or when interpreting the drawn graphs, the students are expected to draw the representative strip (Δx or Δy) that would be used to compute the area or the volume from the shaded region bounded by the graphs, with or without using the Riemann sums. They are expected to see the shaded region and the volume generated as a result of rotating this region as being *continuous* and not as *discrete* isolated parts in order to use integration to compute the area and volume generated. In this study moving between discrete and continuous representation is possible when the shaded region bounded by graphs is approximated from the Riemann sums for area into thin rectangular strips which are summed to give an approximation of the area, as well as sliced into thin discs or washers or approximated with nested shells. Three rectangles are used in Figure 3.1 to demonstrate Riemann sums for the bounded area.
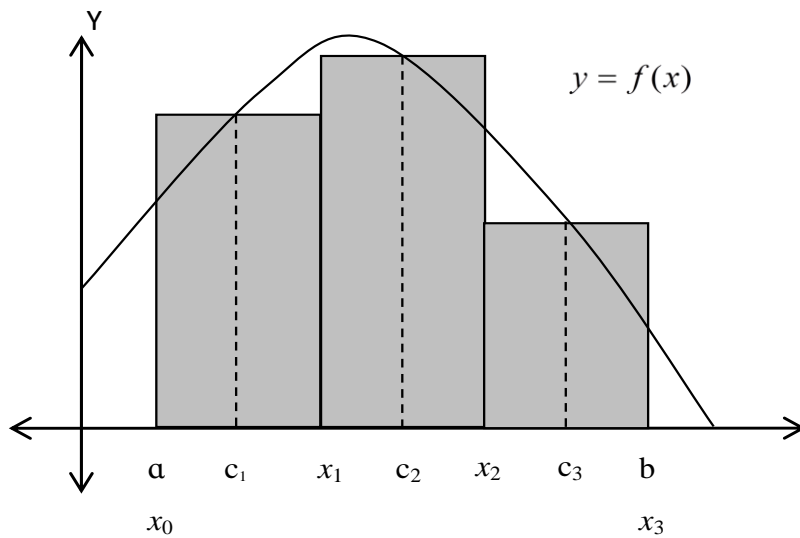
Figure 3.1: The Riemann sum

The area of the shaded region in Figure 3.1 above is an approximation of the area below the graph of $y = f(x)$ given as follows:

$$\int_a^b f(x)\, dx \approx f(c_1)(x_1 - x_0) + f(c_2)(x_2 - x_1) + f(c_3)(x_3 - x_2)$$

Students are expected to be in a position to use the widths of these rectangles as Δx (or Δy if roles of x and y are switched), which is represented on the diagram above as $x_1 - x_0$ and $x_2 - x_1$ and $x_3 - x_2$ and their given heights $f(c_1)$, $f(c_2)$ and $f(c_3)$ respectively to compute the area, depending on the number of rectangular strips selected. The more the number of rectangular strips within a given area are used, the better the approximation of the shaded area by summing the areas of those rectangular strips.

If the area for the region to be calculated is identified and the correct strip is drawn (where only one strip is required), students may be in the position to compute the volume generated as this region is rotated. Students are not in all cases required to sketch the exact solid of revolution that is generated, but they are expected to sketch the rotated strip that represents the method that will be used as either the *disc/washer* or *shell*. With Skill factor III, the visual/graphical representation is used, as students draw the selected strip and use it to approximate the area or the volume.

3.2.4 Skill Factor IV: General manipulation skills

General manipulation skills regarding Element 10 fall under the numerical representation as well as the algebraic/symbolic representation, where different equations/expressions are solved, including integration techniques. If the integral equation/ formula is given, the students are expected to compute the integral from the given equation or to calculate area or the volume of the given definite integral with respect to x or with respect to y. The numerical representation is also evident when students use the FTC after integration for evaluating the definite integral.

Within the numerical representation and the algebraic/symbolic representation under Element 10, procedural knowledge is involved. When using procedural knowledge, calculations done are based on the rules and algorithms used in learning VSOR. General manipulation skills used while calculating the value of the integral to find the area or volume generated can be regarded as being procedural since it involves applications of rules and algorithms. Finding the value of the integral does not only involve general manipulation skills, but require proper knowledge of integration rules. The numerical representations in this case involve the way in which the students do calculations and general manipulations. How do they solve problems and how do they perform during the process? How do they use general manipulation skills in solving problems involving calculation of the necessary points of intersection of the graphs, calculating the intercepts of the graphs with the axes and other important points? How do they use general manipulation skills in solving problems involving area and volume from integration and using integration techniques? What are their successes or failures during the manipulation process?

3.2.5 Skill Factor V: Consolidation and general level of cognitive development

This skill factor only involves Element 11, with the focus on the cognitive demands of the content learnt. The level of cognitive development may be affected by aspects such as the nature of difficulty of the content learnt as well as the time taken to learn a particular content.

Students' cognitive abilities and time constraints

Regarding learning about VSOR, I wanted to investigate whether failure is a result of the subject being of too high level of difficulty. Is the topic of VSOR maybe too high in terms of the students' cognitive abilities? If a new concept that is to be learnt is cognitively high for

the student's internal representation to comprehend, it is argued that students normally fail to make sense of such a concept or understand it conceptually (Tall, 1991).

Learning aspects that are above students' cognitive level becomes accessible if the students are given enough time to deal with such new concepts. Unfortunately that is not always possible with the FET College students due to the volume of work that needs to be completed within ten weeks, thus affecting the pace at which learning takes place. Eisenberg (1991, p. 148) argues that the abstraction of the new mathematical knowledge and the pace with which it is presented often becomes the downfall of many students. He further argues that in most cases the instructor had already internalised the topic, but this is not the case with the students who normally struggle to make sense of the new knowledge to be learnt.

It is argued that

> As meaningful learning proceeds, new concept meanings are integrated into our cognitive structure to a greater or lesser extent, depending on how much effort we make to seek this integration, and on the quantity and quality of our existing, relevant cognitive structure (Novak, 2002, p. 552).

It is therefore the responsibility of both the lecturer and the student to ensure that meaningful learning occurs as they negotiate and re-negotiate meaning during the learning process. The way in which the lecturer integrates the content knowledge and the pedagogical knowledge during the teaching process also impact tremendously on meaningful and in-depth knowledge.

One does not know how different instructors approach the VSOR section. The question is do they start with simple graphs, like for an example rotation of a circle or a straight line before using the complicated graphs or do they just introduce the section without much order (haphazardly)? Do the instructors help the students to understand the relationship between area and volume and rotations in general? To what extent is their prior knowledge taken into consideration? What is actually happening in the classroom? It is anticipated that students will be taught traditionally and via technology with the emphasis on integrating the three representations in relation to the 5 skill factors discussed above.

3.3 THE THREE MODES OF REPRESENTATIONS AND THE LEVEL OF COGNITIVE DEVELOPMENT

In teaching calculus teachers use graphical, symbolic and numerical representations (Tall, 1996; Habre & Abboud, 2006). As is the case in my study, the use of graphical, symbolic and numerical representations will be required both in teaching and in learning of VSOR. According to Amoah and Laridon (2004, p. 6), the use of multiple representations is expected to increase students' understanding, even though students struggle to move comfortably among the different representations. To improve students' performance in calculus, it is necessary that teaching focuses also on concepts, not only the techniques. Concepts should be introduced graphically, algebraically and numerically (Serhan, 2006). This study suggests that practices of teaching calculus concepts should change to achieve a comprehensive concept image of the derivative concept that includes all the different representations. Students' concept images can be enriched if instruction is aimed at helping students to acquire the ability to visualise mathematical concepts (Harel et al., 2006, p. 149).

In his study Cheng (1999) investigated the critical role that representations have on conceptual learning in complex scientific and mathematical domains. Cheng (1999, p. 115) argues that approaches to conceptual learning should ensure that concepts are organised in a specific order. Cheng (1999, p. 116) writes: "building the conceptual network clearly does not occur by simply transmitting the knowledge of the domain to the learner", but that consideration must be given to ''the role of the external representation used for the domain'' as well as to "the role of individual concepts". By external representations he refers to charts, graphs, diagrams, equations, tables and all sorts of formal and semi-formal notations and by individual concepts he refers to schemas, sets of related propositions or groups of rules (Cheng, 1999, p. 116). With the use of computers in learning (Cheng, 1999, p. 117) argues, "there appears to have been no dramatic improvement in conceptual learning because programs typically support just a few of the processes". In this study the use of computers will only be addressed during the preliminary phase. The focus will be on the teaching and learning involving the five skill factors of knowledge identified, also focusing on conceptual knowledge and procedural knowledge.

3.4 PROCEDURAL AND CONCEPTUAL KNOWLEDGE

In teaching and learning of VSOR, students are expected to use procedural knowledge (involving algorithmic use) as well as conceptual knowledge (involving cognitive abilities and critical thinking), which complement one another. Students' cognitive abilities in VSOR are measured in the way in which students are capable of solving problems that translate from conceptual knowledge to procedural knowledge and vice versa and integration of conceptual knowledge to procedural knowledge.

In Haapasalo's (2003) terms conceptual knowledge is

> Knowledge of and a skilful drive along particular networks, the elements of which can be concepts, rules (algorithms, procedure, etc) and even solved problems (a solved problem may introduce a new concept or rule) given in various representation forms (Haapasalo, 2003, p. 3).

While procedural knowledge is

> dynamic and successful utilisation of particular rules, algorithms or procedures within relevant representation forms. This usually requires not only the knowledge of the object being utilised, but also the knowledge of format and syntax for the representational system(s) expressing them (Haapasalo, 2003, p. 4).

Engelbrecht et al. (2005) pointed out that along the process of learning, conceptual knowledge that is repeatedly taught might end up being procedural knowledge, in that students might not be thinking about what they are doing when presented with repeated problems, since the problems might have been done many times in class. In this study, I observed which aspects were learnt procedurally, and which ones were learnt conceptually.

According to Rittle-Johnson and Koedinger (2005, p. 317), students need to develop conceptual knowledge in a domain that can be flexibly applied to new tasks. They further argue that visual representations such as pictures and diagrams are one potential scaffold for eliciting conceptual knowledge and facilitating integration. In that way, students can instead of rote learning of rules, justify their knowledge from what they see. In the learning of VSOR, the visualisation of the graphical representation and the translation to algebraic can be regarded as the conceptual learning since it involves critical thinking to enable the student to use a particular method. For the different given graphs, the region bounded may be different; hence one cannot procedurally proceed without proper conceptual understanding of what is being visualised. The students must engage with the drawn graph, analyse what region need to be rotated, what parameters are given and how the selected region must be rotated.

If students in this study possess the cognitive abilities, they should be in a position to succeed in problems that require the use of procedural knowledge and in a problem that has a conceptual base. In that regard, students will succeed in solving problems involving *level of cognitive development*, as it is required under the Skill factor V. Interpreting graphs and diagrams and translation from visual to algebraic or other forms of translations would also not be problematic.

Below, a VSOR model is proposed where the extent to which the skill factors require conceptual knowledge and / procedural knowledge or both are shown.

3.5 PROCEDURAL AND CONCEPTUAL KNOWLEDGE WITHIN THE FIVE SKILL FACTORS

The VSOR model as a concept mapping is presented by the researcher to show where the five skill factors fit, in relation to conceptual understanding and procedural understanding.

3.5.1 The VSOR model

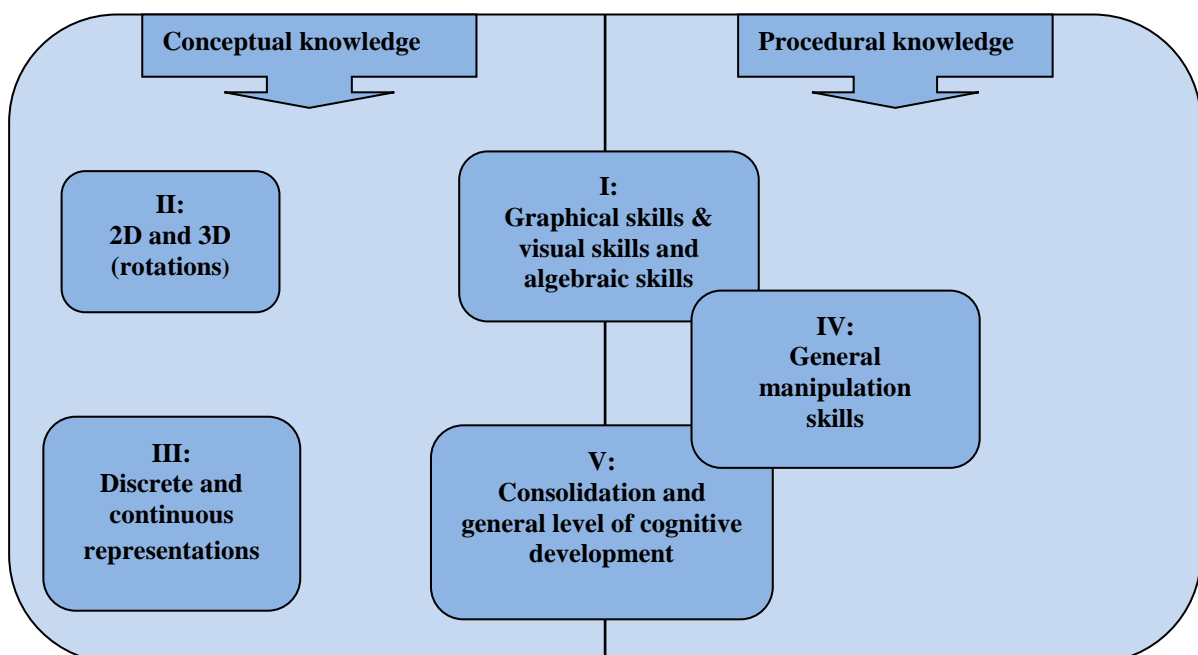The VSOR model is presented in Figure 3.2 showing all five skill factors.

Figure 3.2: The VSOR model

Any skill factor can be performed to measure different aspects of competency in VSOR, whether procedural or conceptual or both, without any order. One does not have to always start by drawing a graph in order to calculate the area or the volume. The instrument designed is such that one can start anywhere. For example, there are cases where a graph (diagram) is given and students are asked to interpret it, either by coming up with the formula for area or volume; to represent on it the Riemann sums or the disc, washer or shell; or to translate it between 3D and 2D algebraically or in a form of a diagram. The VSOR model suggests that all the different skill factors individually affect the learning of VSOR, with Skill factor IV being incorporated in Skill factors I and V.

The model further shows that different factors of knowledge require different skills. For example, **Skill factor I** involves drawing graphs (translating from an algebraic equation to a visual graph), which requires the use of procedural skills while interpreting the drawn graphs (translating from the visual graph to an algebraic equation) requires conceptual skills. **Skill factor II** involves rotating 2D diagrams that result in 3D diagrams and interpreting a given problem from 3D to 2D, thus requiring conceptual skills. **Skill factor III** involves selection of the correct representative strip and interpreting an equation that relates a continuous graph to discrete form, thus requiring conceptual skills. **Skill factor IV** involves general manipulation skills where the calculations depend on algorithmic usage, which is procedural in nature, while Skill factor **V** involves level of cognitive development where a student is able to succeed in all first four skill factors which involve both procedural and conceptual skills.

The above VSOR model is adapted in this study focusing on the individual components of the model and the way in which they affect the learning of VSOR. Any explicit relationships that the elements from the five skill factors may have on each other were also investigated through correlations. For an example, do students who fail to draw graphs always fail to translate from 2D to 3D or fail to exhibit general manipulation skills and vice versa?

The VSOR model discussed above is further incorporated within Bernstein's (1996) theories of *knowledge transmission* and *knowledge acquisition* and Kilpatrick, Swafford and Findell (2001) five strands of *mathematical proficiency* as a theoretical framework.

3.6. RELATED FRAMEWORKS

In dealing with students' difficulties, it is necessary to study students' thinking processes and how these hamper or enhance learning. Do these students possess what is necessary for learning to take place or is learning just not possible? Using students' written and verbal interpretations, one can investigate their ways of thinking.

The way in which the students construct knowledge, interpret and make sense of what they have learnt about VSOR, is located within the two theoretical frameworks below, by Bernstein (1996) and Kilpatrick et al. (2001). The students' ways of learning is discussed and located within Bernstein's (1996) rules of knowledge acquisition as well as within the five strands of mathematical proficiency of Kilpatrick et al. (2001), while teaching practices are discussed using Bernstein's (1996) rules of knowledge transmission. The VSOR model is discussed in each case where relationships are possible for each framework.

3.6.1 Bernstein's framework

The other theoretical framework that is used in this study is that of Bernstein (1996) involving knowledge transmission and acquisition. Knowledge transmission relates to the *teaching process* while knowledge acquisition refers to the *learning process*. In this study the two processes are explored, with the main focus being on how learning takes place. In the process of learning, knowledge acquisition occurs when students are able to interpret the question and to give the correct answer. Bernstein (1996) refers to that process as involving the recognition and the realisation rules. He refers to the *recognition rules* as the means by which 'individuals are able to recognise the speciality of the context that they are in' (Bernstein 1996, p. 31) during a learning situation, while the *realisation rules* allow the production of the 'legitimate text' in giving the correct answer. If one considers what is happening in the classroom, the recognition rules enable the necessary realisations, while the realisation rules determine how meaning is being put together and made public (Bernstein, 1996, p. 32). In terms of this study the recognition and the realisation rules are related to the students' ability to link their internal representations (mental image) properly with the external representation (visualising and interpreting the graphs correctly) in volumes of solids of revolution. The ability to recognise and realise in a learning context, using procedural knowledge flexibly may be influenced by the way in which instruction occurred (knowledge transmission) or what the students believe mathematical knowledge to be.

The way in which the students construct knowledge, interpret and make sense of what they have learnt in class is located within Bernstein's (1996) rules of knowledge acquisition as our theoretical framework. Since one is dealing with students' difficulties, it is necessary to study students' thinking processes and how they impact on their ways of learning. Using students' written and verbal interpretations, one can investigate their ways of thinking. Under Skill factors I, II, III and V, are the students able to interpret the drawn graph(s) correctly? Or are they able to translate the given equations/expressions to graphs/diagrams or correct calculations. If they are able to, we say that they recognised the drawn graph (from its characteristics) and were as well able to realise, by translating the drawn graph to the correct equation for area or volume.

The ability to recognise and realise in a learning context, is also possible with problems that require the use of procedural knowledge such as Skill factors, I, IV and V. Are students able to recognise what is given and solve it accordingly. How is the integral sign interpreted? Are the students able to use general manipulation skills to solve problems and to get correct points that can be used to draw graphs like, for an example, the points of intersection?

Using conceptual knowledge and procedural knowledge flexibly in order to recognise and realise what students have learnt, may be influenced by the way in which instruction occurred, knowledge transmission according to Bernstein (1996), or what the students believe mathematical knowledge to be. In this study students are asked to sketch graphs, interpret graphs, interpret the drawn graphs, calculate from a given equation or even justify how a certain graph could be drawn. In so doing, the way in which the students recognise and realise is interpreted according to Bernstein's framework verbally and in written form.

3.6.2 Kilpatrick's et al. framework

Using Kilpatrick's et al. (2001) five strands of Mathematical Proficiency (MP), students' verbal interpretations were explored and scaffolding was used during group interactions. The way in which the students construct knowledge, interpret and make sense of what they have learnt is located within the theoretical framework involving the five strands of Mathematical Proficiency (MP), listed below. These five strands are not independent, they are interwoven and interdependent in the development of proficiency in mathematics (Kilpatrick et al., 2001, p. 116). MP is used to explain what is believed to be necessary for anyone to learn mathematics successfully. They argue that the way in which "learners represent and connect pieces of knowledge is a key factor in whether they will understand it deeply and can use it in

problem-solving" (Kilpatrick et al., 2001, p. 117). The fact that the five strands are interwoven relates to the way in which students connect the pieces of knowledge in problem-solving situations. They also highlight the importance of the central role of mental representations in enhancing learning with understanding as opposed to memorisation.

The five strands are discussed as follows:

- *Conceptual understanding* involves comprehension of mathematical concepts, operations and relations (understand).
- *Procedural fluency* involves skill in carrying out procedures flexibly, accurately and appropriately (compute).
- *Strategic competence* involves ability to formulate, represent, and solve mathematical problems (solve).
- *Adaptive reasoning* involves capacity for logical thought, reflection, explanation and justification (reason).
- *Productive disposition* involves habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (attitudes).

The five strands are discussed below from the point of view of (Kilpatrick et al., 2001)

a) *Conceptual understanding* refers to an integrated and functional grasp of mathematical ideas. They argue that students with conceptual understanding know more than isolated facts and methods as they understand why a mathematical idea is important as well as its use in the context relevant to it. They further argue that with conceptual understanding one is able to represent mathematical situations in different ways as well as knowing how different representations can be useful for different purposes. With conceptual understanding students may discuss the similarities and differences of representations and the way in which they connect. These connections are found to be useful if related concepts and methods are related appropriately. The argue that

> when students have acquired conceptual understanding in an area of mathematics, they see the connections among concepts and procedures and can give arguments to explain why some facts are consequences of others. They gain confidence, which then provides a base from which they can move to another level of understanding (Kilpatrick et al., 2001, p. 119).

b) *Procedural fluency* refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently. When

students study algorithms as general procedures, they can gain insight into the fact that mathematics is well structured, that is highly organised and filled with patterns. They argue that a certain level of skill is required to learn many mathematical concepts with understanding and that using procedures can help and develop that understanding. According to Boaler (1997), knowledge that cannot be used flexibly is said to be inert.

If students learn procedures that they do not understand, they will fail to use them in new or different contexts when solving activities. They will also fail to understand the reasons underlying the applications of such procedures. If no emphasis is made on procedural fluency, students would have trouble deepening their understanding of mathematical ideas or solving mathematics problems. The problem with learning incorrect procedures is that the incorrect procedures make it difficult for them to learn correct ones.

c) *Strategic competence* refers to the ability to formulate mathematical problems, represent them and solve them. Strategic competence is evident if students build mental images of the essential components of a problem during problem-solving situations. In so doing, students should be able to generate a mental representation like diagrams and equations/expressions that capture the core mathematical elements of the question, whereby the students are able to detect mathematical relationships in the given problem. Strategic competence can be used in both routine where a known procedure is reproduced and used, and non-routine tasks where one does not immediately know the procedure but has to invent some rule or reconstruct. The way in which challenging mathematical problems are solved depends on the ability to carry out procedures readily.

d) *Adaptive reasoning* refers to the capacity to think logically about the relationships among concepts and situations. It involves informal explanation and justification when making conclusions given reasons for assumptions or conclusions made. With adaptive reasoning students are given the opportunity to use new concepts and procedures to explain and justify by relating them to already known concepts and procedures, hence adapting the old to the new. With strategic competence, students draw on their strategic competence to formulate and represent a problem using heuristic approaches that may provide a solution strategy leading to adaptive reasoning where a student will be determining whether an appropriate procedure is used in solving a problem. When solving the problem strategic competence is used, but if a student is not satisfied with the solution plan, adaptive reasoning is used to change the plan to another method that will be suitable by reasoning and justification.

e) *Productive disposition* refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. If students see themselves as capable of learning mathematics and using it to solve problems, they become able to develop further their procedural fluency or their adaptive reasoning abilities. Educational success in mathematics can also be affected by one's disposition. Failure to develop productive disposition is seen when students avoid challenging mathematics courses. Students who have developed a productive disposition are found to be confident in their knowledge and ability, with perception of mathematics as both reasonable and intelligible. These students have a belief that with appropriate effort and experience, they can learn.

Mathematical Proficiency in that regard goes beyond being able to understand, compute, solve and reason, but also involves one's attitude towards mathematics. Since these five strands are interwoven, they influence each other. For example, conceptual understanding and procedural fluency continually interact. As one gains conceptual understanding one will be able to compute, as a result of being able to use the correct and relevant procedures flexibly, irrespective of whether the problem at hand is new or challenging. If conceptual understanding is achieved, new understanding may develop. It is believed that to become mathematically proficient, students need to spend sustained periods of time doing mathematics that involve solving problems, reasoning, developing understanding, practising skills and building connections between previous knowledge and new knowledge. Problems in this study will require the use of the first four strands relating to content, with the last strand captured during the interviews.

Within the VSOR model, students are asked to sketch graphs and draw diagrams, interpret graphs, come up with the correct equations and use them to calculate from the given graphs, calculate from a given equation or even justify how a certain graph or diagram could be drawn both in 2D and in 3D. The five skill factors can be located within this model. The first strand involves conceptual understanding which is possible under the Skill factors I, II, III and V when students solve problems that are conceptual in nature. The second strand involves procedural fluency which is possible under Skill factors I, IV and V, where the problems require the use of general manipulation skills and use of rules. The third strand involves strategic competence which is possible under Skill factor I and IV where general manipulation skills are involved. The fourth strand, adaptive reasoning, is applicable in all the five skill factors where logical thought, justification and reflection are required. The fifth

strand, productive disposition, involves attitude towards mathematics that can be evident from the way students behave in class during the observations and from the interview conducted with one previous student.

3.7 CONCLUSION

In this chapter, the conceptual framework has been developed based on the 5 skill factors. The 5 skill factors include graphing skills and translating between visual graphs and algebraic equations/expressions; three-dimensional thinking, moving between continuous and discrete representations, general manipulation skills and consolidation and general level of cognitive development. The 5 skill factors were as well categorised as requiring procedural knowledge or conceptual knowledge or both and used towards the design of the VSOR model for this study. Other factors affecting the teaching and learning of VSOR in general, including students' thinking processes and the role of representations in learning were also discussed. The designed model opts for the interrelation between the five factors affecting VSOR and its 11 elements. The conceptual framework has also been located within the related theoretical framework and the work of others, focussing on the five skill factors. The theoretical frameworks discussed above relate to how students learn and how they go about showing that learning has occurred. The thinking processes are evident from their written and verbal interpretations. Bernstein's framework also extends to how knowledge is transmitted (teaching). The five strands of mathematical proficiency in mathematics by Kilpatrick et al. (2001) are used to explain successful learning in mathematics from the way in which the students represent and connect knowledge.

The chapter that follows presents a discussion on the research design and methodology for this study. Issues pertaining to ethical considerations governing this research are also discussed.

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

This chapter outlines the research design and methodology regarding the investigation of learning difficulties involving volumes of solids of revolution (VSOR). In Section 4.1 the research strategy, involving the research methods and the sampling procedures is discussed for both qualitative and quantitative approaches. In Section 4.2 the mode of data collection and analysis including the instruments used for data collection in three different phases are discussed. Phase I involves the preliminary study and the pilot study, Phase II involves four different investigations, while Phase III involves two different investigations. The validity and reliability of the study are discussed in Section 4.3 and 4.4, respectively, for both quantitative and qualitative methods. In Section 4.5 the way in which generalisation of this study was done is discussed. The ethical considerations are discussed in Section 4.6. The delineation and limitations of the study are discussed in Section 4.7 and Section 4.8, respectively to shed light on what this research could or could not achieve and where it was restricted. The summary for this chapter is done in Section 4.9.

4.1 RESEARCH STRATEGY

4.1.1 Research methods

The research design for this study includes a number of strategies. This research is *empirical* since it focuses on data collection through observation and evidence (Bassey, 2003; Blaikie, 2003; Cohen et al, 2001) and *applied* since it is aimed to answer questions based on programs and organisations (Mason & Bramble, 1989) to produce recommendations in relation to organisational practices and change (Denscombe, 2002). This research is also *interpretive*, focussing on interpreting students' actual written and verbal interpretations and *descriptive*, since it reports information on the frequency or the extent at which something happens (Mertler, 2006). The research is *comparative* since the data collected from six different investigations are compared. This research also involves *correlation methods*. In correlational studies a researcher is interested in knowing whether variations in one trait correspond with variation in another (Mason & Bramble, 1989, p. 43), by finding a statistical relationship between variables (Brown & Dowling, 2001; Mertler, 2006). In this study, the correlation between variables was found using *scatter plots* as well as *Pearson's product moment* and *Kendall's tau* correlation coefficients.

Both qualitative and quantitative approaches are used. In the qualitative approach, I collect data through observation of what people do or say and interpret data as it occurs in the natural setting of the participants and explain it without numbers (Blaikie, 2003; Gelo, Braakmann, & Benetka, 2008; Mertler, 2006; Taylor-Powell & Renner, 2003). The qualitative approach is about what was said or done, while the quantitative approach is about numbers and ratings used (Blaikie, 2003; Gelo et al., 2008; Mertler, 2006). In this study tables, graphs and any form of statistical analysis will be used for the quantitative data. While quantitative researchers are interested in whether and to what extent variance in x causes variance in y, qualitative researchers are interested in finding out how x plays a role in explaining change in y and why (Maxwell, 2005, p. 23).

Maxwell (2010) does not support the idea that a qualitative approach is about words (verbally or written) and a quantitative approach is about numbers. He believes that using numbers in qualitative research is quasi-statistics, which correlates variables. According to Maxwell, with quasi-statistics, conclusions of qualitative studies have implicit quantitative components. He uses the terms "variance theory" and "process theory". Variance theory on the one hand

> "deals with variables and the correlations among them; it is based on an analysis of the contribution of differences in values of particular variables to differences in other variables. The comparison of conditions or groups in which a presumed causal factor takes different values, while other factors are held constant or statistically controlled, is central to this approach to understanding and explanation and tends to be associated with research that employs experimental or correlational designs, quantitative measurement, and statistical analysis (Maxwell, 2010, p. 477).

Process theory on the other hand

> "deals with *events* and the processes that connect them; its approach to understanding relies on an analysis of the *processes* by which some events influence others. It relies much more on a local analysis of particular individuals, events, or settings than on establishing general conclusions and addresses "how" and "why" questions, rather than simply "whether" and "to what extent." This aspect of qualitative research has been widely discussed in the methodological literature but has rarely been given prominence in works on the philosophical assumptions of qualitative research" (Maxwell, 2010, p. 477).

As a qualitative researcher, I adhere to the conventionalist view, that knowledge is constructed symbolically, and as a quantitative researcher, I adhere to the positivist view that "order exists among elements and phenomenon … regardless of whether humans are conscious of order" (Mason & Bramble, 1989, p. 36). A conventionalist view is similar to the interpretative view and the constructivist view that there is no reality out there (Bassey, 2003), but it needs to be constructed in a social environment. The conventionalist or the interpretive or the constructivist researcher believes in finding meaning from what is being observed and also believes that the world is viewed differently depending on the observer and avoids general statements (Bassey, 2003; Cohen et al., 2001 & Denscombe, 2002). Positivists

make discoveries about realities of human actions and express it as factual statements also with expectations that other researchers handling similar data must come up with the same conclusions that they found (Bassey, 2003, p. 42), they also observe patterns in the social world empirically in order to explain it (Denscombe, 2002). The interpretive view focuses on qualitative methods interested in narrative data (verbal or written) while the positivists view focuses on quantitative methods interested in numerical data (Brown & Dowling, 2001; Denscombe, 2002; Mertler, 2006; Teddlie & Tashakkori, 2009). In this research both methods are used as they complement one another.

Using both qualitative and quantitative approaches is regarded as a *mixed method* approach (Bazeley, 2009; Creswell & Tashakkori, 2007; Christ, 2007; Morgan, 2007; Gelo et al., 2008; Hall & Howard, 2008; Hancock & Algozzines, 2006; Mertler, 2006; Teddlie & Tashakkori, 2009). The use of purely qualitative methods or purely quantitative methods can be overcome by integrating the two methods. Morgan (2007) refers to integration of qualitative and quantitative methods as a pragmatic approach. As it is the case in my study, "a strong mixed methods study starts with a strong mixed methods research question or objective" (Tashakkori & Creswell, 2007, p. 207) involving a 'why' research question. The research question for this study: **Why do students have difficulty when learning about Volumes of Solids of Revolution?** can be addressed from a mixed methods approach (MMA). Students' performance on how they approached the problem can be analysed qualitatively relating to what written responses they actually produced and quantitatively as to which questions they did better in. It is also important to ensure that the writing up of MMA findings is integrated when reporting on the research done (Bryman, 2007).

My research involves *action research* since it is aimed at improving ways of teaching, learning and assessing VSOR. The important aspects that are neglected or not emphasised when teaching, learning and assessing VSOR are made public. Action research leads to innovation and change, but not necessarily to generalisation of the results to other settings (Cohen et al., 2001; Mason & Bramble 1989). Action research enables the researcher to understand the current practice, evaluate it and change it (Bassey, 2003; Mertler, 2006). In the data collected, action researchers qualitatively analyse data inductively (as they start from the observation of phenomena in order to build up theories about those phenomena) analysing patterns and similarities and quantitatively analyse data deductively (as they observe specific phenomena on the base of specific theories of reference) using descriptive statistics or inferential statistics (Mertler, 2006; Gelo et al., 2008). Analysing data both inductively and

deductively is referred to by Cohen et al. (2001, p. 4) as the *inductive-deductive* approach, which is referred to by Morgan (2007) as *abduction*, where results of the study can only be transferable, not generalised. This action research used multiple case studies during the data collection process in order to triangulate the data.

During a case study a researcher can use multiple sources of information (Cresswell, 2007; Mertler, 2006; Teddli & Tashakkori, 2009). In this study, the different sources of information included administration of different tests, direct observations, an interview and documentary analysis in order to determine the quality of events in the participants' natural setting as well as testing the theory designed in the conceptual framework presented in Chapter 3. Case studies address a particular event studied in its natural context to get a rich description of the event from a participant point of view (Gelo et al., 2008; Hancock & Algozzine, 2006). As a case study, I looked in-depth at individual student's written responses in class from the given tests and during examinations and observed group work in the classroom on how teaching and learning took place.

Case studies are seen as intensive investigations of the factors that contribute to characteristics of the case (Mason & Bramble, 1989, p. 40) as well as collecting sufficient data (Bassey, 2003) during the research process. Bassey (2003, p. 65) further advises that if a case study is conducted, the researcher will be able to:

- explore the significant features of the case;
- create plausible features of what is found;
- test for trustworthiness of this interpretation;
- construct a worthwhile argument or a story;
- relate the argument or the story to any relevant research in the literature;
- convey convincingly to an audience this argument or story;
- provide an audit trial by which other researchers may validate or challenge the findings, or construct alternative arguments.

After the case study investigations, correlations were also used to determine the association of the different elements from the students' performance. In this study I wanted to correlate variables (x and y) in order to determine any association between the variables as well as its direction and magnitude (Cohen et al., 2001). A correlation is a measure of the linear association between variables (Field, 2005). A scatterplot was used to determine such an

association as well as its direction and its magnitude (*r*). A scatter plot was used to display any association between the given variables from students' performance. Scatter plots can be useful in helping one understand how, and to what extent the variables are related (Myers, & Well, 2003, p. 40).

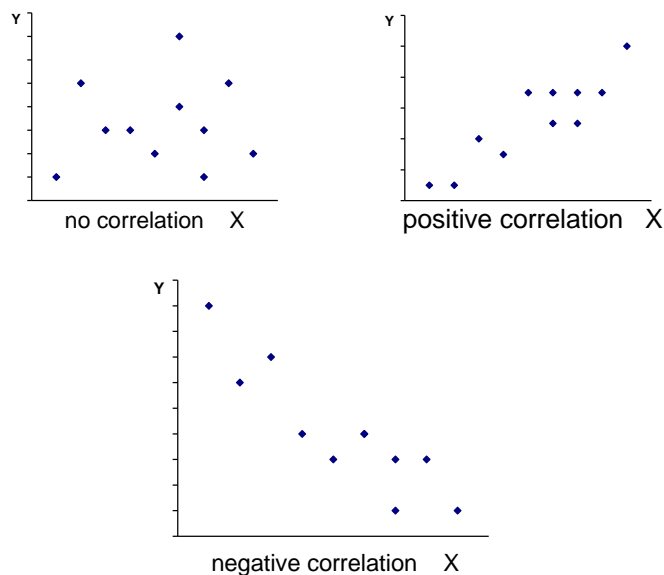Examples of correlations using a scatterplot are given in Figure 4.1.

Figure 4.1: Examples of scatter plots (Adapted from Willemse, 2004, p. 116)

A correlation coefficient *r*, range between the values -1 and 1, which is $-1 \leq r \leq 1$. The closer the correlation coefficient is to -1 or 1, the more the linear association between the two variables. If *r* is close to 0, there is little or no linear association between the two variables, the variables compared do not show any related pattern and the points are scattered around, not forming anything like a straight line. When the slope of the scattered points is positive, the *r*-value is positive and when it is negative, the *r*-value is negative. The sign of *r* indicates the direction of the association between the variables *x* and *y*. The strength of the correlation is not dependent on direction, that is *r* = 0.84 and *r* = -0.84 are equal in strength. The value of the coefficient reflects the strength of the correlation; a correlation of -0.84 is stronger than a correlation of 0.23 and the correlation of -0.44 is weaker than a correlation of 0.67.

However, a correlation of 0.23 for a sample of 100 students and for a sample of 2000 students is interpreted differently. In order to determine the different interpretations of such correlations, we use the level of significance of the correlations. The level of significance of the correlations refers to the confidence that one has about the conclusion based on the

association of the elements being correlated by using the null hypothesis represented as H_o which is either rejected or not rejected based on the p-value (Fields, 2005; Cohen et al., 2001). A p-value according to Keller (2005, p. 333) is a "test of probability of observing a test static at least as extreme as the one computed given that the null hypothesis is true".

According to Cohen et al. (2001, p. 194-295), if an association occurs 95 times out of a 100 observations, then we can say with some confidence that there is less than 5% probability that an occurrence happened by chance, reported as ($p < 0.05$) for a statistically significant correlation. If the association occurs 99 times out of a 100 observations, then we can say with some confidence that there is less than 1% probability that an occurrence happened by chance, reported as ($p < 0.01$). In both cases ($p < 0.05$ and $p < 0.01$), the null hypothesis is rejected and we can conclude that there is a statistically significant association between the elements correlated. According to Field (2005), in showing the level of significance, $p < 0.01$ is represented by two asterisks (**), $p < 0.05$ with one asterisk (*) and $p > 0.05$ by no asterisk.

In Table 4.1, Keller (2005, p. 335) describes the p-value in terms of the rejection region. For an example, if the p value is less than 0.01, there is overwhelming evidence (highly significant) to reject the null hypothesis.

Table 4.1: The p-value table

p-value	Evidence	Interpretation	Conclusion
$p < 0.01$	Overwhelming	Highly significant	Reject H_o
$0.01 < p < 0.05$	Strong	Significant	Reject H_o
$0.05 < p < 0.1$	Weak	Not significant	Fail to reject H_o
$p > 0.1$	None	No evidence	Fail to reject H_o

After having determined the correlation using a scatter plot and finding the association between the variables, as well as the direction of that association and the magnitude, I determined the level of significance for those correlations using Pearson's correlation coefficient (r) and Kendall's tau correlation coefficient (τ) (Field, 2005). For the test statistic to be valid, Pearson's correlation coefficient, a parametric static was used to correlate the marks obtained in Question 5 to the whole examination paper since the marks obtained were numerical (Field, 2005, p. 125). I presumed that there might be an association between the variables, but did not predict any direction between the associations of such variables; as a result, a null hypothesis was set up. Kendall's tau correlation coefficient is a non-parametric statistic that was used since the data was ranked. Kendall's tau correlation coefficient in

particular, was used since some of the data had tied ranks (Field, 2005). The coefficient of determination (R^2) is also determined to account for the amount of variability in one variable that is explained by the other (Fields, 2005), since other factors might have been involved.

The research design that was followed in this research is based on the three models discussed below, the *interactive model;* the *mixed method* model and *Gowin's Vee* model. The interactive model is mainly qualitative while the mixed method model integrates the qualitative and the quantitative approaches, and the Gowin's Vee integrates the conceptual framework and the methodology of this study.

The interactive model is the qualitative research design that follows Maxwell's (2005) model.
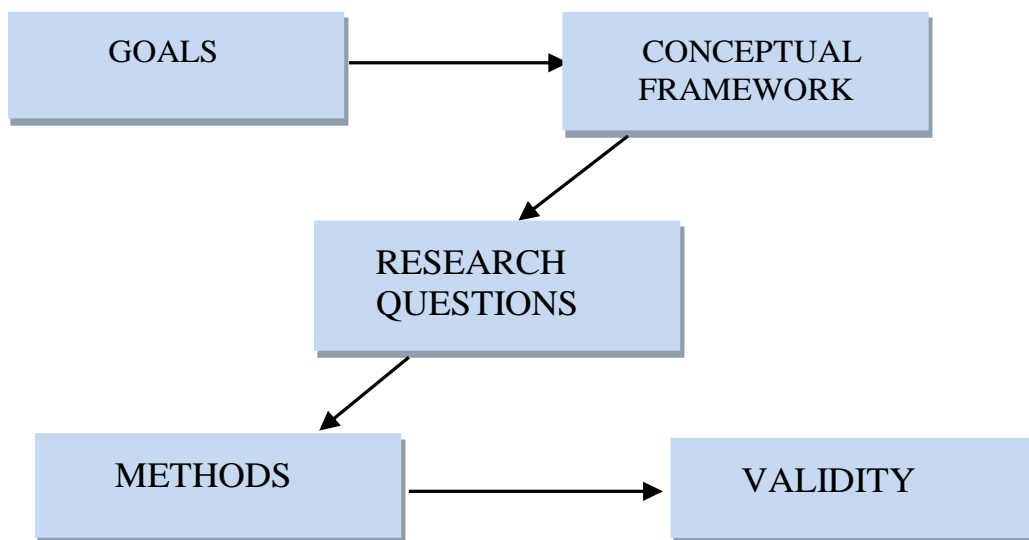

Figure 4.2: The interactive model of research design (Adapted from Maxwell, 2005, p. 11)

The model shown in Figure 4.2 has five components: *Goals*, c*onceptual framework*, *research questions, methods and validity* (starting from the goals and ending up with issues of validity).

- The goals for this study include outlining challenges faced by students when learning VSOR. In particular, what are their learning difficulties? The significance of this study includes why is it important to research this area, which factors contribute towards improving learning and improved teaching and assessment practices. The goals include what practices the researcher wants to inform or change, for example, teaching or assessment and who will benefit from those changes.

- The conceptual framework for this research refers to specially designed elements in VSOR that I wish to build my research on as well as other theories that may be influential.

- The main research question for this study is: **Why do students have difficulty when learning about Volumes of Solids of Revolution?** The research question and subquestions are classified as the hub of the model as they connect all other components of the model and inform all other components, shown in Figure 4.2.

- The methods used are informed by the type of data that I want to collect, who the participants are, how I collected such data and how the analysis was done.

- The validity relates to the correctness of the results obtained, why other people should believe them, including interpretations that other people might have.

The strength of qualitative research according to Maxwell (2005) derives primarily from its inductive approach, focussing on what people do or say and the meanings they bring about as well as understanding their context to explain their behaviour.

With the mixed method research design (refer to Figure 4.3), the qualitative data collection involved written responses through tests and examinations; documentary analysis; classroom observations and an interview, in Phases I, II and III, while quantitative data collection involved assigning *rank scores* to the students' written responses in tests and examinations in Phase II. In Phase I the preliminary study was conducted through Test 1 and Test 2 and the pilot study was administered after content analysis of the textbooks and the examinations. In Phase II, the main data collection for this study was conducted through six *investigations*. Investigation 1 involved the administration of the 23-item instrument (questionnaire), called the Questionnaire 1st run whereas Investigation 2 involved the Questionnaire 2nd run in a different trimester including an analysis of the examination results (Investigation 3) and detailed written responses (Investigation 4) by students. The results from the questionnaire runs and the examination responses are as well compared. In Phase III classroom observations (Investigation 5) and an interview (Investigation 6) with a former N6 student were conducted. Even if a mixed method design is used, initially, the intention of the research was mainly qualitative, based on the interpretation and meaning of the data collected (Cresswell, 2009:4). The inclusion of quantitative methods is in order to make statistical inferences where necessary and to validate the qualitative data collected, where variables are related to one another (Cresswell, 2009:4).
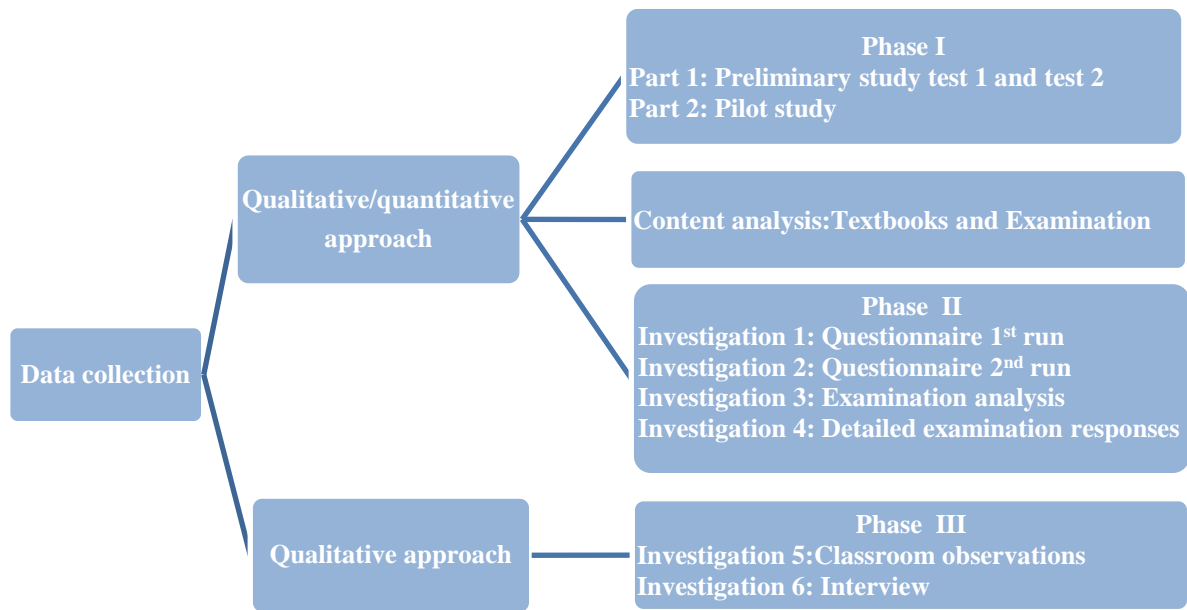
Data collection

Qualitative/quantitative approach

Qualitative approach

Phase I
Part 1: Preliminary study test 1 and test 2
Part 2: Pilot study

Content analysis: Textbooks and Examination

Phase II
Investigation 1: Questionnaire 1st run
Investigation 2: Questionnaire 2nd run
Investigation 3: Examination analysis
Investigation 4: Detailed examination responses

Phase III
Investigation 5: Classroom observations
Investigation 6: Interview

Figure 4.3: The mixed method research design model

Conceptual

Methodological

Interaction

Philosophy
The researcher knows that the students are having difficulties with VSOR?

Why do students have difficulties when learning about VSOR?

Value Claims:
Are difficulties as a result of the nature of the content, ways of learning, teaching or assessment?

Theory:
Students observed to construct.
They construct and reconstruct.
Evidence of cognitive conflict.

How competent are the students with questions in skill factors I, II, III, IV?

Knowledge Claims: From interpretations of why the difficulties?

How competent are the students in skill factor V?

Transformations:
(Qualitative + quantitative) Data re-organized, re-arranged, recorded.
Show meaningful relationships using tables, charts, graphs, statistics, narratives, written interpretations etc

Constructs:
Show relationships between concepts as stepping stones to knowing from correlations of the 11 elements.

Concepts:
Elements 1,2,3,4 up to 11 in VSOR
Eg Graphing skills; algebraic to visual (2D); general manipulation skills etc.

Raw data:
Data collected from the events/objects studied (Tests, observations, interview)

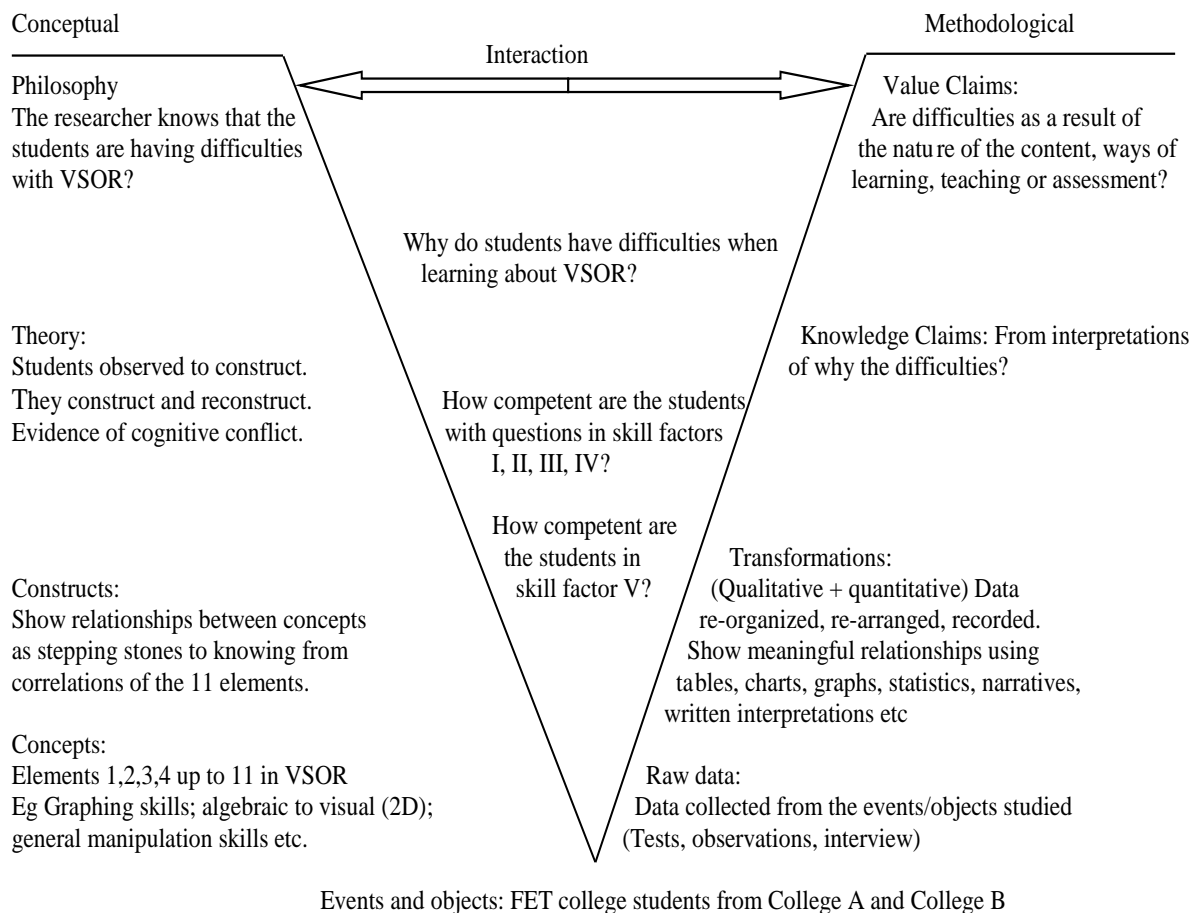Events and objects: FET college students from College A and College B

Figure 4.4: Gowin's knowledge Vee (Novak & Gowin, 1984)

In this research the V model starts from the bottom, with events or objects to be studied. In this case the objects to be studied are FET college students taking mathematics at N6 level. The conceptual framework follows upwards on the left while the methods follow upwards on the right. On the left the *concepts* to be learnt are the elements established throughout the three phases of data collection up to the 11 elements established from the 23-item instruments for learning VSOR. The *constructs* refer to how the relationships between these concepts are shown as a way of revealing how learning takes place. The theory involves constructivism as students construct knowledge. The philosophy relates to what the researcher knows about learning VSOR and what is guiding the enquiry in order to answer the research question. On the right, raw data are collected, rearranged and interpreted to establish how students perform in different elements. Data are interpreted (qualitatively and quantitatively) in order to answer the research question. Finally the value of knowledge found is established, be it in terms of validity and reliability and the trustworthiness of the results. The inside part of the Vee diagram involves the research question: Why do students have difficulties when learning about VSOR as the interaction of the conceptual ideas and the methodology. The other aspects that follow inside the Vee shape include how students address questions based on the Elements 1 up to 10 and how that relates to the performance with Element 11. The interaction of the left (with the order: concepts, constructs, theory and philosophy) and the right (with the order: raw data, transformations, knowledge claims, value claims), from the bottom of the Vee to the top, is an attempt to answer the research questions of this study, inside the Vee.

4.1.2: The research sample

The participants for this study are students from three FET colleges (aged 17 and more) enrolled for N6 mathematics. The colleges used are, College A in a township with students coming predominately from rural areas; College B and College C, both in industrial areas with students coming predominately from urban areas. All three colleges are in the Gauteng province. The number of students enrolled per trimester for N6 mathematics in each of these colleges is ± 40 for College A, ± 140 for College B and ± 70 for College C. Initially, all three colleges were used for the pilot study. For the main study, due to poor participation from College C, only the two remaining colleges (College A and B) were used, using different students from the pilot study.

The data collected for this study from the sample is presented in three different phases as Phase I (the preliminary and the pilot studies), Phase II (the main study as four investigations) and Phase III (the main study as two investigations), discussed in detail below.

Phase I: Data was collected through the preliminary and the pilot studies.

Part 1: For the preliminary study in July 2005, fifteen mathematics N6 students from one class from College A and their lecturer participated in this study, with only seven final responses, for those students who wrote all tests.

Part 2: For the pilot study in October 2006, three different FET colleges, College A; College B and College C, were sampled where finally only 15; 29 and 10 students respectively participated in the study.

Phase II: The main data collection was done in four investigations. All classes from College A and College B were sampled, using different students from the pilot study.

Investigation 1: Questionnaire 1st run, done in April 2007, with 37 final responses (17 students from College A and 20 students from College B).

Investigation 2: Questionnaire 2nd run, done in October 2007 with 122 students (30 students from College A and 92 students from College B) and in April 2008 with 54 students (15 students from College A and 39 students from College B) respectively.

Investigation 3: Examination analysis of the August 2007 mathematic N6 examinations results, done in November 2007 with 151 students (25 students from College A and 126 students from College B).

Investigation 4: A detailed examination analysis of the students' written responses from College A only (seven students).

Phase III: Classroom observations and an interview.

Investigation 5: Classroom observations from College A with ± 40 students in October 2007. One focus group with eight students was observed during the classroom observations.

Investigation 6: An interview with one former student from College A.

The colleges were selected purposively for convenience (using the nearest colleges), since they were all accessible to the researcher and all were willing to participate (Cohen et al., 2001; Gelo et al., 2008; Maxwell, 2005) in terms of proximity. Purposive sampling is associated with qualitative approaches (Teddlie & Tashakkori, 2009). In purposive sampling,

some members of the broader population will definitely be excluded and others will definitely be included (Cohen et al., 2001). In this study only N6 students taking mathematics were chosen.

In this study more than one college was used to increase the sample size of this study and not to compare the students' performance from the colleges used. All students from the sampled colleges participated in this study.

4.2. DATA COLLECTION AND ANALYSIS

4.2.1 Phase I: Data collection process and analysis

In this research I used paper-and-pencil tests as a measurement technique to assess students' performance. The data collection process and analysis of this study, done in three different phases (Phase I, II and III) is discussed in that order.

4.2.1.1 Part 1: The preliminary study (July 2005)

- **Data collection process**

The preliminary study was done as an attempt to improve my own teaching relating to VSOR. After observations that many students were experiencing difficulty with this section, I introduced teaching of VSOR though technology, using Mathematica to aid students with visualisation of the rotations (for the disc, washer and shell methods). I also wanted to share my experience involving teaching VSOR with other lecturers.

Data for the preliminary study was collected in July 2005. A class of 15 students were participants in this study. Students were taught VSOR for four periods of 80 minutes each, focussing on calculating areas and volumes only. In the first two periods their lecturer taught them in a traditional verbal way (using chalk and talk), and then Test 1 was administered. In the last two periods, two days later, the students were taught by the researcher with the aid of a Mathematica through visualisation and verbalisation, where visualisation was the main method, with verbalisation being used for clarification of ideas and to highlight conceptual understanding portrayed visually with Mathematica with the main emphasise on rotations of the selected rectangular strip to show how a disc, a washer and a shell are formed. The animations and the graphics were displayed via a data projector. Students did not have access to computers. The intention was that after the lesson the students would be in a position to

draw the graph if it is not given, select the bounded area and the correct strip as well as to illustrate the correct method for rotation (disc/washer/shell) to calculate volume. After the lesson, Test 2 was administered. The use of Mathematica was explored by investigating the way in which students responded after being taught, relating to the performance level in two tests.

The students were given four questions in Test 1 and six questions in Test 2 (refer to Appendix 1B). The questions in Test 1 and Test 2 were discussed with their lecturer to ensure compliance with the required level as well as the level of difficulty. In the questions designed by the researcher, graphs were not given in two questions in both Test 1 and Test 2, but in the rest of the questions graphs were given. The students responded in writing in both Test 1 and Test 2. Only seven out of the 15 students wrote both Test 1 and Test 2. Some students were excluded because they wrote Test 1 only or Test 2 only whereas others wrote Test 1 and Test 2 but did not receive instruction via Mathematica. Students' names were written at the back of each test paper so that I could identify who wrote Test 1, Test 2 or both. Whether the names were correct or not was not important. What was important was that the students wrote the same name throughout the tests. During Test 2 students were also asked to indicate whether they were taught using Mathematica or not, also at the back of their written responses from the test paper. All the students who were present during the Mathematica demonstration lesson were asked to give written comments about how Mathematica impacted on their visual and algebraic thinking and whether they had benefited from the program or not.

- **Data analysis**

For the analysis of the data, the seven students' written responses were marked and discussed with their lecturer and an expert (the researcher) validating the analysis and the interpretations before the scripts were given back to the students. The written responses for only seven students from Test 1 and from Test 2 were therefore analysed and discussed in this study to give the actual students' interpretations. The written responses were analysed qualitatively for students' interpretations and summarised in tables in terms of comparing the students' visual and algebraic abilities. Students' actual written responses are also presented and analysed to identify any response patterns. One example of students' written responses for questions that reveal some interesting trends is used to show what students actually did. Students' comments on the lesson presented using Mathematica were also analysed and discussed in relation to the two teaching methods used.

- **Conclusion**

The low number of questions used in the preliminary study (ten) and the few students (seven) used make claims questionable as to how valid and reliable they are. Generalisation of the results was also not possible. I therefore decided to design more questions also focussing on all aspects that influence learning VSOR in terms of all skill factors of knowledge involved and using a bigger sample. That was possible after revising the whole VSOR content from textbooks and previous question papers in order to design an instrument that covers the main aspects that affect learning of VSOR. In the preliminary study, only four aspects were looked at. The aspects involved how students draw graphs, how they select the strip and rotate it, how they translate from the drawn graphs to the algebraic equations and to a lesser extent how they use general manipulation skills. The preliminary study did give an indication of what the focus of the research should be and how to strengthen it.

4.2.1.2 Part 2: The pilot study (October 2006)

- **Data collection process**

The data for the pilot study was collected in October 2006 at College A; College B and College C, using the 21-item instrument (refer to Appendix 2A) as a written test. The 21-item instrument was subcategorisedinto 11 different *elements* on VSOR with a maximum of two questions per element. In addition to the written test, I observed the lessons for five days at College C (with ± 23 students), before the 21-item instrument was administered on the sixth day (with 10 students), after their lecturer covered the section on areas and volumes. The lessons were approximately 80 minutes long.

Table 4.2: 11 elements from the 21-item instrument

Elements
1.   Algebraic to visual (2D)
2.   Visual to algebraic (2D)
3.   Algebraic to visual (3D)
4.   Visual to algebraic (3D)
5.   2D to 3D
6.   3D to 2D
7.   Continuous to discrete (Visual 2D)
8.   Continuous to discrete (Visual 3D)
9.   Discrete to continuous and continuous to discrete (Algebraic)
10.   General manipulation skills
11.   Consolidation and general level of cognitive development

The questions from the 21-item instrument that were given to students were randomised without being arranged according to the designed elements so that questions from the same element were not recognised by the students. Spaces were provided on the question paper

where students had to write down their responses, to ensure that the students do not remove the question papers since the questions were to be used in the next phases. The designed questions were discussed and verified with experts to ensure that proper standards were maintained throughout. The students responded to the questions individually in a class test setting with the researcher moving around to handle questions and to help when students needed clarification. The reason for using the pilot was that it could be used to assess the likelihood of errors in the test (Viswanathan, 2005), before conducting the main study. The pilot was also useful in enabling the researcher to find out if questions were clear and not ambiguous and if enough time was given to finish writing the test.

- **Data analysis**

The students' written responses were marked and then reorganised in the 11 elements for further analysis and interpretation. The results are presented in tables and the total scores are summarised in terms of the raw scores and the percentage for how many students responded to a particular question falling under a particular element. It is clearly indicated for each and every question how many students responded *correctly* (C), *partially correct* (PC), *incorrect* (I) or *not done* (ND). A response was done correctly if everything is correct, a partially correct response would be where part of the solution would be given (what is deemed legitimate by the researcher), an incorrect response would be where nothing is correct and not done would be where the student had left a blank space. The performance in questions within each element was compared, and the performances from the 11 elements were also compared. Table 4.8 (p. 108), which is discussed in the main study was adapted, where the performance levels were based on the total percentage from the raw scores, for both correct and partially correct responses, regarded as *acceptably correct* responses. One or two examples of students' written responses are given for selected questions (where interesting trends are found) for partially correct responses and incorrect responses only, since the study is on learning difficulties.

After conducting the pilot study, I realised that most of the students were unable to finish writing the test because it was too long. According to Cohen et al. (2001), the length of the test may have an influence on students' performance. In preparation for the main data collection instrument, the test was therefore broken up into three different sections (to ensure that students finish writing), Section A (Test 1), Section B (Test 2), and Section C (Test 3) where questions were randomly assigned to sections.
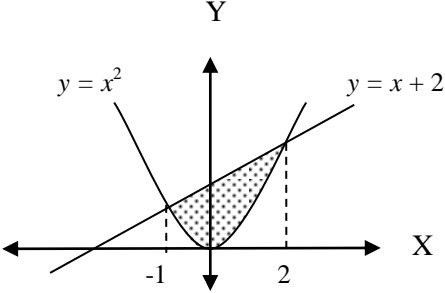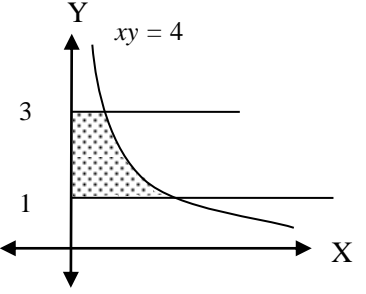
4.2.2 Phase II: The main study

Phase II of the main study was done in four separate investigations, Investigation 1 Investigation 2, Investigation 3 and Investigation 4.

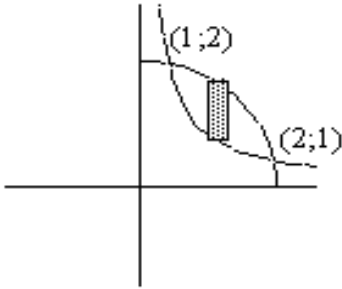### 4.2.2.1 Investigation 1: April 2007 as the Questionnaire 1st run

From the results of the pilot study it was evident that some questions were not clear to students. This was picked up from the responses given by students in such questions. Some questions from the pilot instrument were changed and modified, some were replaced while two more questions were added and an instrument with 23 questions under 11 different *elements* on VSOR was designed (refer to Appendix 3A). There is a maximum of two questions per element, except Element 10 with three questions. The 11 elements are categorised into five *skill factors*. The main instrument for data collection, the 23- item instrument is given Table 4.3.

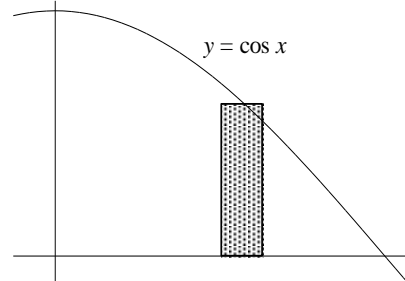**Table 4.3: Classification of questions under the 11 elements**

1. Graphing Skills	
1 A: Draw a line with a positive gradient passing through the origin for $x \in [0,3]$	1 B: Sketch the graphs and shade the first quadrant area bounded by $x^2 - y^2 = 9$ and $x = 5$
2. Algebraic equations/expressions ⟶ Visual graphs (2D)	
2A: Represent $x^2 + y^2 \leq 9$ by a picture.	2B: Sketch the area represented by $\int_0^1 (x - x^2)\, dx$
3. Visual graphs ⟶ Algebraic equations (2D)	
3A: Substitute the equations of the given graphs in a suitable formula to represent the area of the shaded region.	3B: Substitute the equations of the given graphs in a suitable formula to represent the area of the shaded region.
4. Algebraic equations ⟶ Visual graphs (3D)	
4A: Draw the 3-D solid of which the volume is given by $V = \pi \int_0^1 (1 - x)^2\, dx$ and show the representative strip.	4B: Draw the 3-D solid of which the volume is given by $V = 2\pi \int_0^1 x(1 - x^2)\, dx$ and show the representative strip.

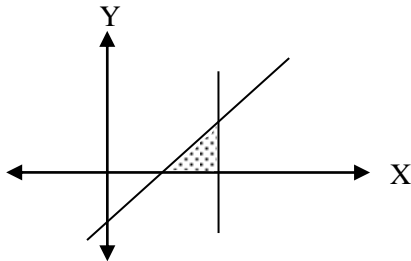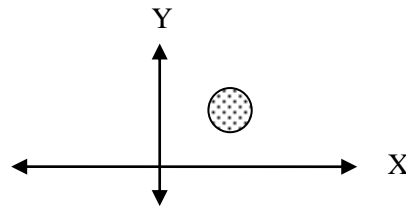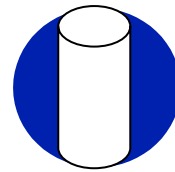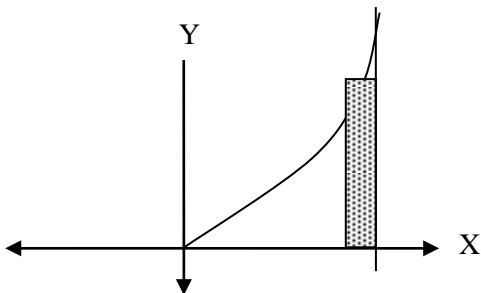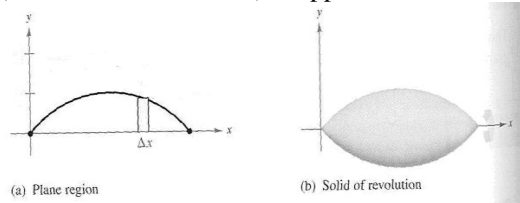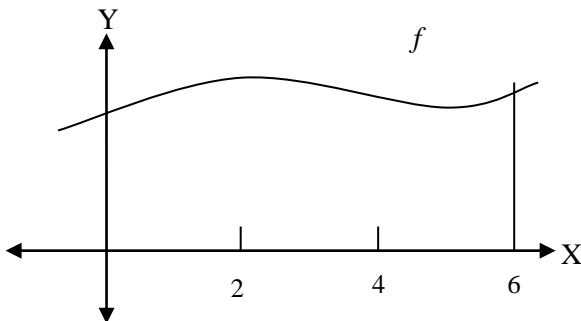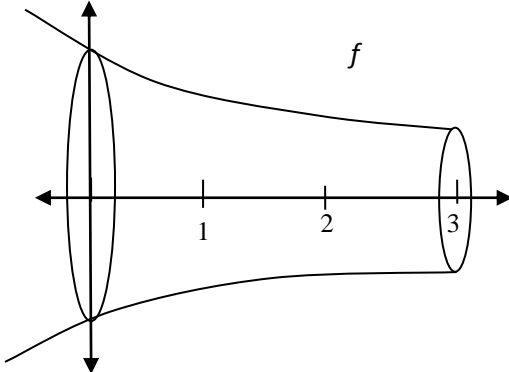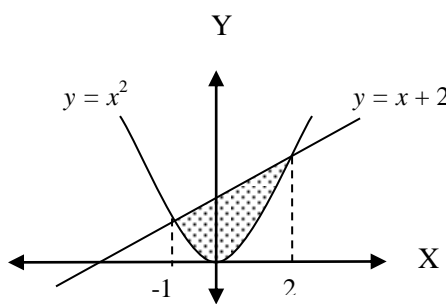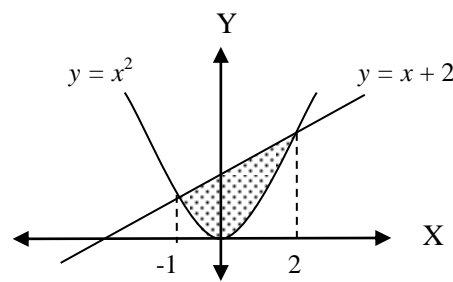| 5. Visual graphs ⟶ Algebraic equations (3D)	
5A: The figure below represents the first quadrant area bounded by the graphs of $x^2 + y^2 = 5$ and $xy = 2$. Using the selected strip, substitute the equations of the given graphs in a suitable formula to represent the volume generated if the selected area is rotated **about the *x*-axis.** Do not calculate the volume. .	5B: The figure below represents the area bounded by the graphs of $y = \cos x$, the *x*-axis and the *y*-axis. Using the selected strip, substitute the equations of the given graphs in a suitable formula to represent the volume generated when this area is rotated **about the *y*-axis.** Do not calculate the volume.
6. 2D ⟶ 3D	
6A: Draw the 3-dimensional solid that is generated when the shaded area below is rotated about the ***x*-axis.**	6 B: Draw a 3-dimensional solid that will be generated if you rotate the circle below about the ***y*-axis.**
7. 3D ⟶ 2D	
7 A: Sketch a graph that will generate half a sphere when rotated about the ***y*-** axis.	7 B: A hole is drilled through the centre of the sphere as in the picture. Sketch the graphs that were rotated to generate the solid as in the picture below.
8. Continuous ⟶ discrete (Visual) 2D and 3D	
8 A: Sketch three additional rectangular strips (similar to the given rectangle) so that the total area of the rectangles approximates the area under the graph.	8B: When the plane region (a) on the left is rotated, the 3-dimensional solid of revolution (b) on the right is generated. Show using diagrams how you would cut the solid of revolution (b) in appropriate shapes (discs, shells or washers) to approximate its volume. (a) Plane region (b) Solid of revolution

9. Discrete ⟶ continuous and continuous ⟶ discrete (Algebraic)	
9 A: Show in terms of rectangles what the following represent with a sketch: $2f(0) + 2f(2) + 2f(4)$ 	9 B: If the volume of the given solid of revolution is approximated by discs, sketch the discs that would give the volume: $\pi(f(0))^2 + \pi(f(1))^2 + \pi(f(2))^2$
10. General manipulation skills	

10 A: Calculate the point of Intersection $4x^2 + 9y^2 = 36$ and $2x + 3y = 6$	10B : Calculate $\displaystyle\int_0^1 \pi(1-x^2)^2\, dx$	10 C: Calculate $\displaystyle\int_0^1 2\pi\, x(1-\sin x)\, dx$

11.Consolidation and general level of cognitive development	
11 A: Given: $y = \sin x$ where $x \in \left[0, \dfrac{\pi}{2}\right]$ and $y = 1$ (i) Sketch the graphs and shade the area bounded by the graphs and $x = 0$ (ii) Show the rotated area about the **y-axis** and the representative strip to be used to calculate the volume generated. (iii) Calculate the volume generated when this area is rotated **about the y-axis.**	11 B: Use integration methods to show that the volume of a cone of radius r and height h is given by $\dfrac{1}{3}\pi r^2 h$.

In Table 4.4 examples of the questions that were changed from the pilot study (on the left) to the main study (on the right) are given as follows:

(a)　　Question 2A was modified to be Question 3A.

Table 4.4: Question 2A modified to be Question 3A

2. Visual ⟶ Algebraic (2D)	3. Visual ⟶ Algebraic (2D)
2A. Give the formula for the area of the shaded region. 	3A: Substitute the equations of the given graphs in a suitable formula to represent the area of the shaded region.

For Question 2A students were asked to give the formula for the shaded region. Some students responded by giving the formula for area as $\int_{-1}^{2} (y_1 - y_2)\, dx$ without continuing to substitute the given graphs for y_1 and for y_2. Apparently the question was not that clear to the students as they showed the first step only. I expected that they would give the formula for area by substituting with the equations for the given graphs. The question was modified to make explicit that the equations for the given graphs must be substituted as in Question 3A.

(b) Question 3A was modified and made easier to be Question 4A. The original question seemed more difficult for the students.

Table 4.5: Question 3A modified to be Question 4A

Algebraic ⟶ Visual (3D)	Algebraic ⟶ Visual (3D)
3A: Draw the 3-D solid of which the volume is given by $V = \pi \int_{0}^{1} (1 - x^2)^2 \; dx$	4A: Draw the 3-D solid of which the volume is given by $V = \pi \int_{0}^{1} (1 - x)^2 \; dx$ and show the representative strip.

(c) Another element, the graphing skills, was added by changing one question from 2D ⟶ 3D to graphing skills, since it seemed difficult for the students. The question was as follows:

Table 4.6: Question 5A modified to be Question 1A

5A: Draw the solid that will be formed if a line with a positive gradient passing through the origin is rotated about the **x-axis**, where $x \in [0,3]$	1 A: Draw a line with a positive gradient passing through the origin for $x \in [0,3]$

(d) Another example, Question 11B in Table 4.7, was also modified, where the formula to be found was now given

Table 4.7: Modified Question 11B

11. Consolidation and general level of cognitive development	
11 B: Use integration methods to derive the formula of a volume of a cone of radius r and height h.	11 B: Use integration methods to show that the volume of a cone of radius r and height h *is* given by $\frac{1}{3}\pi r^2 h$.

In this question students were asked to derive the formula of volume of a cone with radius r and height h using integration. From the inappropriate responses that the students made, it was evident that the students had no clue about what the question was asking for. The question was therefore modified to be Question 11B where the formula for volume that they had to derive was given (stated in the question).

- **Data collection process**

Data was collected over the period of one week during the first trimester (April) of 2007 using the adapted 23-item instrument given above as the Questionnaire 1st run.

The questions that were given to the students were randomised and split into three different shorter tests (Section A, B and C) that were written in three consecutive days without being arranged into elements so that similar elements were not recognised by the students. There were eight questions in Section A, eight in Section B and seven in Section C, without any order or preference. As in the pilot study spaces were provided on the question paper where students had to write down their responses, to ensure that the students do not remain with the questions since the questions papers were to be used again. The students at College A and College B responded to the questions individually.

- **Data analysis**

There were 37 responses overall for this period (the Questionnaire 1st run) for those students who wrote all three tests, thus students who did not write all the tests were excluded. The data analysis was done qualitatively and quantitatively. The students' responses were marked and were coded according to *ranking* as follows: FC:4 if the answer is *Fully Correct*; AC:3 if the answer is *Almost Correct*; TU:2 if there were some *traces of understanding*; NU:1 if there was *no indication of understanding* and ND;0 if the there was no attempt in answering the question, hence *not done*. It is highlighted that: "Codes or elements are tags or labels for allocating units of meaning to the descriptive or inferential information compiled during a study" (Basit, 2003, p. 144).

For the qualitative part of the data, students' written responses were shown per question for different individuals. A summary of written responses for all the students are given in every question from the 23 questions according to the different rankings. Examples of the actual solutions for the written responses are given from the selected students for some of the 11 elements (one or two examples) under the five factors of knowledge for the responses showing traces of understanding and no indication of understanding responses only. The written responses provide a better justification of what was done and more clarity.

For the quantitative analysis, the marked responses were reorganised under the elements for further analysis and presented in tables (using raw scores and percentages) and *multiple bar graphs*. Multiple bar graphs are a result of the use of two or more bar graphs, which are

grouped together in each category (Willemse, 2004). In this study multiple bar graphs are used since there are five rank scores (FC, AC, TU, NU and ND) grouped together under 23 categories, being the 23 questions.

It is indicated using tables and multiple bar graphs, how the students' responses were classified as fully correct, almost correct, showing traces of understanding, no understanding and not done per question (comparing the questions) and the percentage thereof. The 11 elements were also compared under the five skill factors of knowledge and classifying the skill factors in terms of requiring conceptual or procedural skills, or both.

The shapes of the multiple bar graphs are discussed in terms of the *symmetry* and s*kewness* of the *distribution* in relation to the position of the mode on the bar graphs, being where the number of responses is the highest. Data are symmetric (normally distributed), if when a vertical line is drawn in the centre of the distribution, the two sides of the distribution are identical in shape and size; skewed if the mode is to the far left, with the data *skewed to the right*, or to the far right, with the data *skewed to the left* of the distribution and bi-modal if the data have two modes which are not necessarily equal in height (Keller, 2008).

If the data are symmetric (normally distributed), we conclude that the number of responses that are correct are equal to the number of responses that are incorrect; if the data are *positively skewed*, we conclude that most responses are correct and if the data are *negatively skewed*, we conclude that most responses are not correct. For the data that have two modes (bi-modal), the position of the mode is the one that determines whether there are more correct responses or few correct responses.

After the presentation of data involving the comparison of the different elements and the skill factors, the total number of responses per question in the Questionnaire 1st run (from all 37 respondents), showing fully correct and almost correct responses are added and discussed in terms of percentages from the raw scores. It is also indicated how students performed overall in individual questions per element and for each of the 11 elements.

The *performance criteria* used (set by the researcher) as in Table 4.8 (adapted from 2008 DoE Assessment standards) is used as follows: If the proportion of *acceptably correct* responses (the sum of fully correct and almost correct responses) for individual questions, per element and per skill factor) is in the interval [0, 20), the performance is regarded as *poor*, if

it is in the interval [20, 40), it is regarded as *not satisfactory*, if it is in the interval [40, 60), it is regarded as being *satisfactory* , if it is in the interval [60, 75), it is regarded as *good*, while performance in the interval [75, 100] is regarded as excellent.

Table 4.8 Criteria for performance level

Range in %	Description of performance
[0, 20)	Poor
[20, 40)	Not satisfactory
[40, 60)	Satisfactory
[60, 75)	Good
[75, 100]	Excellent

The sections where less than 40% of the students get acceptably correct responses (performance that is poor and not satisfactory) are regarded by the researcher as critical areas and issues for concern, since the majority of the students experience difficulty in such sections. These areas are highlighted as sections where students lack the necessary skills, resulting in poor performance in learning VSOR due to their incompetency. The incompetency may be due to the fact that the sections are difficult for the students as they require high cognitive abilities, which they do not possess or because the students are not properly taught or not taught at all.

After comparing the performance in the 11 elements in terms of the percentages of acceptably correct responses in the Questionnaire 1st run, performance in the 11 elements is compared in terms of the five skill factors. The same performance criteria as in Table 4.8 are used. Finally, individual student's responses are added and compared for the whole instrument for all response categories. All fully correct responses, almost correct responses, responses showing traces of understanding, responses showing no understanding and where there were no responses are added separately per category and compared. It is shown what percentage all the fully correct responses are in relation to other categories. In total since there are 23 questions and 37 individual responses per question, there are 851 total responses from the Questionnaire 1st run.

After comparing the performance in the skill factors in terms of percentages, performance in the five skill factors were classified and compared in terms of conceptual or procedural skills and both in terms of tables and multiple bar graphs, based on the performance in terms of

percentage. The comparison is done in order to determine how students performed in elements that require conceptual or procedural skills and both. It is determined in which skill factor the students performed poorly or excellently and also how they performed, be it good, satisfactory or not satisfactory.

As highlighted before, Skill factor I is composed of Elements 1;2;3;4 and 5 . Skill factor II composed of Elements 6 and 7; Skill factor III composed of Elements 8 and 9; Skill factor IV composed of Element 10 while Skill factor V is composed of Element 11 questions. The skill factors are classified as being composed of questions that require procedural skills, conceptual skills and both (Table 4.9) as discussed in Chapter 3, and given here again.

Table 4.9: Classification of skill factors

	I: Procedural and Conceptual Graphing skills and translating between visual graphs and algebraic equations/expressions in 2D and 3D	**II: Conceptual** Three-dimensional thinking	**III: Conceptual** Moving between discrete and continuous representations	**IV: Procedural** General manipulation skills	**V: Procedural and Conceptual** Consolidation and general level of cognitive development incorporating some elements in Skill factors I, II,III and IV
Skill Factors					
Elements	*1*: *Graphing skills* *2*: *Algebraic to Visual* (*2D*). *3*: *Visual to Algebraic* (*2D*). *4*: *Algebraic to Visual* (*3D*). *5*: *Visual to Algebraic* (*3D*).	*6*: *2D to 3D.* *7*: *3D to 2D*	*8*: *Continuous to discrete* (*Visual 2D and 3D*) *9*: *Discrete to continuous and continuous to discrete* (*Algebraic*)	*10*: *General manipulation skills*	*11*: *Consolidation and general level of cognitive development*

4.2.2.2 Investigation 2: October 2007 and April 2008 as the Questionnaire 2nd run

In Investigation 2, the 23-item instrument was administered for the second time with a different group of students using only Test 1 and Test 2 with 8 questions for each test from 122 respondents (in October 2007) and Test 3 with 7 questions from 54 respondents (in April 2008), called the Questionnaire 2nd run.

- **Data collection**

The process of data collection was the same as in the Questionnaire 1st run. Test 1 and Test 2 from the 23-item instrument were administered to 30 students from College A and 92 students from College B after the classroom observations (discussed in Chapter 8). Overall 122 students responded to the 16 questions individually. For Test 3 15 students from College A and 39 students from College B responded to the questions individually.

- **Data analysis**

The process for data analysis for the 23 questions involved marking and interpretation of data using tables and multiple bar graphs as in the Questionnaire 1st run. The process of data analysis was the same as in the Questionnaire 1st run. Examples of the responses that revealed interesting interpretations are discussed. The results for the Questionnaire 2nd run are compared to the results for the Questionnaire 1st run to establish if there were any trends.

4.2.2.3 Investigation 3: Analysis of the 151 examination scripts for August 2007 examinations

Before doing the analysis of the 151 scripts, the question involving VSOR (Question 5) from the August 2007 question paper was analysed according to the five skill factors used in the 23-item instrument (refer to appendix 6 for the questions as well as a detailed memorandum of the question). Out of the 11 elements from the main instrument only five elements were examined in Question 5. Question 5 had subquestions, from which the five elements were identified. Question 5 contributes 40% to the overall examination and seems very difficult for students.

- **Data collection**

The data were collected from 25 students from College A and 126 students from College B. In total 151 examination scripts were analysed, obtained from the National Department of Education for students who wrote the August 2007 examinations, focussing on the question based on VSOR only. These students were not participants in any of the questionnaire runs.

- **Data analysis**

The analysis for the 151 examination scripts is descriptive and inferential. I re-marked the 151 examination scripts in line with the classifications FC:4; AC:3; TU:2; NU:1 and ND:0 used in the main instrument of this research and average ranking per subquestion falling under the same element were calculated. The data were summarised in tables and multiple bar graphs to display the level of performance per element in percentage, under each element and compared. After comparing the elements in terms of the rank scores, the elements in Question 5 were correlated using the Kendal tau correlation coefficient.

After comparing the elements in terms of the rank scores, the actual marks obtained in Question 5 were correlated with the actual marks obtained in the whole examination paper. Analysis of the marks obtained in Question 5 and the whole examination paper is conducted

by using histograms, scatter plots, and correlation coefficients Pearson for numerical data. The marks that the students obtained in Question 5 (out of 40%) are correlated to the marks they obtained in the whole examination (out of 100%). Since this question contributes 40 per cent of the whole examination paper, the correlation coefficient was calculated to determine how the marks obtained in this question correlate with the marks that the students obtain overall. These results are displayed in four different quadrants to show this association.

4.2.2.4 Investigation 4: Detailed examination analysis

To obtain actual written responses of the August 2007 examination paper, the seven students' written responses from the group of eight students that was observed in class in November 2007 were analysed and discussed qualitatively. These students wrote the same examination question paper that was written by the 151 students in class as a formal test individually. One student out of these eight students did not write the test. The results were interpreted qualitatively after marking in line with the rank scores used in the questionnaire runs. Examples of the actual written responses are displayed under each element where possible.

4.2.2.5 Correlating the elements

Correlations for the 11 elements were done in terms of the average of the rankings from the questions under each element, called the *Average Ranking for Individual Responses per Element* (ARIRE). Correlations were calculated by comparing elements (from the average rankings) to determine association between the elements and the level of significance of those correlations. The correlations were used to determine whether students' responses within questions in one element correlate with other elements. Performance in all 11 elements was correlated with the Questionnaire 1st run, four for the Questionnaire 2nd run and five for the examination analysis with the 151 students. The correlations were therefore done only if all questions under each element were given to students to respond to. The average rank for elements from the questionnaire runs and the examination were then correlated to determine the level of significance using the Kendall tau correlation coefficient. For the Questionnaire 1st run all elements were correlated, since students were given all questions under each element to respond to. For the Questionnaire 2nd run only four elements (1; 4; 7 and 10) were correlated since students were given all questions from those elements. The elements correlated were therefore for the 122 students only. For the 54 students no correlations were done since the students were given only one question from each element. The correlation from the 151 examinations responses were for five elements only since the analysis of the examination questions resulted in five elements only.

4.2.3 Phase III

In addition to data collected in Phase II, the data collection process in Phase III (October 2007) involved classroom observations and the interview with a previous N6 student as the fifth and the sixth investigations respectively.

- **Data collection**

4.2.3.1 Investigation 5: Classroom observations

I observed and documented how students were being taught VSOR in their natural setting for five days in terms of addressing the 11 elements under the five skill factors. A video recorder was used in observing the lessons, with the main focus on the lecturer. I wanted to find out how students are taught and what preknowledge students had regarding all the different types of graphs. I also wanted to find out how students were assessed regarding different graphs, how they drew them and how they shaded the region bounded by these graphs. I wanted to investigate how the rectangular strip was selected on the shaded region and rotated about a given axis, what the strip would result in upon rotation and sometimes the diagram of the possible new solid and how they compute the area and volume generated. After the lesson, I identified one group comprising eight students (from Investigation 5) randomly to document the actual written responses. They agreed to be observed for five days. The group selected was assessed in writing from what the lecturer did in class during the observations. I was involved in observations as a participant observer, scaffolding during the group interactions. The group members were also assessed individually through a test (from the August 2007 examination paper) and the 23-item instrument on separate days.

4.2.3.2 Investigation 6: Student interview

I did not initially plan to interview students. After the first classroom observation, one former N6 student approached me, wanting to share her experiences regarding VSOR. The student was interviewed immediately using a video recorder. Even if interviews are time-consuming, they provide rich data. The interview was based on the student's impressions about the ways of learning and assessment of VSOR. The interview was open-ended and lasted for about 15 minutes. The interview focused on learning difficulties with VSOR in relation to Skill factor V. Excerpts from the interviews were transcribed, analysed and reported on.

- **Data analysis**

(i) Observations

The five lessons observed were transcribed. The lecturer's ways of presenting the lesson and the relationship with the students were analysed and interpreted, with the focus on the way in which the content learnt was introduced, the use of procedural knowledge and conceptual knowledge, the level of difficulty of the content and the assessment strategies. The chosen group's written work during the five days of observation was analysed and interpreted. Extracts of the students' written responses are presented in Chapter 6. The chosen group's written work done individually during the last classroom observation (as a test) was marked first by their lecturer before I could analyse and interpret it.

(ii) Student interview

The interview data with the previous N6 student was transcribed and analysed. The interview transcripts were discussed and interpreted to reveal what impression this student had about how VSOR is being taught, learnt and assessed.

4.2.4 Final remarks

In this study the tests and examinations were used to assess the students in writing. As McDermott and Hand (2010, p. 519) wrote: "Written composition provides a record of thought that can be read by an outside audience, as well as the author". Tests in this study were analysed qualitatively by looking at what students exactly wrote as well as looking at patterns, and quantitatively looking at how many students responded in which ways in different elements used as well as finding correlations.

4.3 VALIDITY

Validity is the degree to which the data collected measures accurately what it is supposed to measure (Mason & Bramble, 1989; Mertler, 2006). In this section the validity of the assessments used (Test 1 and Test 2, the 21-item instrument, the 23-item instrument, the August 2007 examination) in the data collection process; their interpretation and their analysis are discussed. The validity of the classroom observations and the interviews are also discussed as well as threats for validity.

4.3.1 Validity in tests

Validity concerns the accuracy of the questions asked, the data collected and the explanations offered (Denscombe, 2002, p. 100). During Test 1 and Test 2 (Phase 1), I tried to control threats to internal validity (the relationship between cause and effect), by making use of the same students when Test 1 and Test 2 were conducted to avoid different abilities and in case some students withdrew or were absent; making sure that Test 1 and Test 2 are conducted two days apart to avoid maturation (Brown & Dowling, 2001); making sure that the questions in Test 1 are not the same as those in Test 2 to avoid familiarity, except one question. For the analysis of the data, students' written responses were marked and discussed with their lecturer and with an expert. This was done in order to validate the analysis and the interpretations before the scripts were given back to the students.

There are three types of validity: *content validity*, which is the degree to which the test items represent the domain of the property being measured (where subject matter tested is relevant), *construct validity*, which is the degree of the relationship between the measure and the construct being measured (where performance on the test is fairly explained by particular appropriate constructs or concepts) and *criterion-related validity*, which is the ability of the test to predict or estimate a criterion by correlating it with other tests (Cohen et al., 2001; Mason & Bramble, 1989).

With regard to content validity in this study, VSOR was identified and the aim was to measure students' difficulties with it. The questions in Test 1 and the Test 2 were discussed with their lecturer to ensure compliance with the required level of the syllabus in terms of the content tested. Students' written responses were as well marked and discussed with their lecturer before the scripts were given back to the students. Questions in the main 21-item and the 23-item instrument were validated through scrutiny by experts in the field of VSOR. Experts were used to make sure that the elements that I defined were correct and that the marking memorandum was also correct. Subjective judgement of content was done to ensure that the items make sense (Viswanathan, 2005). This is also called member validation with the informed people (Denscombe, 2002).

The validity of the study was promoted since a statistician and various experts (including the supervisors and the researcher) in the field of VSOR were consulted throughout the study. In that way, the instrument used for data collection was carefully designed. The researcher in this case is more knowledgeable about the domain for the task to ensure that all aspects

relating to content to be tested are covered (Nitko, 2004). It was therefore anticipated that the questions designed for the students will enable me to get valid data through their written responses. Since the tests given to the students and the programme used (on visualising solids of revolution) focussed on the problematic aspect of their N6 syllabus, an assumption made was that the students were serious about their studies involving VSOR. The students would feel that being part of the research would benefit them since they would be able to achieve their goal of scoring higher on VSOR in their N6 examinations. As for the August 2007 examinations, content validity was ensured by the National DoE since experts in that level (from Umalusi) were used to assure the quality of the content that was tested. Throughout all the three phases the students' successes or failures were validated by their written responses. All the responses were marked, analysed and summarised, and discussed with experts in the field to validate the interpretations and the analysis.

Another method of ensuring validity was that I conducted all the tests and that the students were given the same tests with the same instructions and the same length of time to complete the tests under the same conditions (Nitko, 2004, p. 385). The results are also valid since the test was given to the mathematics N6 students after completing the section on VSOR in all colleges sampled for this study.

In construct validity VSOR is designed to measure the construct it was designed to measure (Viswanathan, 2005), which is *drawing graphs,* solving problems that involve *general manipulation skills, cognitive skills, reasoning* among others. To ensure that construct validity is supported, I compared the results of the designed tests with other studies done elsewhere (in Chapter 6). If there are similar trends, then the results of the designed tests are convergent (to ensure internal consistency) with the results from other studies (Mason & Bramble, 1989 and Viswanathan, 2005), provided the results from those tests that comparison is applied for, are also valid.

Finally, criterion-related validity in this study was done in terms of finding the relationship of the test and the students' difficulties in different elements by administering the pilot study and restructuring the questions. The responses in different elements were also compared.

To ensure validity of the results, the analysis of students' written responses is "descriptive, interpretive and theoretical" (Maxwell, 1992, p. 284-285). The main aim is to avoid false claims. In that way, an expert or any other person would see the students' written responses

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presented as factual and allowing possibilities for verification if the need arises. For the quantitative part of the data, validity was improved through appropriate instrumentation (the 23-item instrument and the examination paper administered) and the statistical treatment of data (Cohen et al., 2001, p. 105). The Pearson and Kendall measures of correlations are used (with advise from a statistician) to ensure validity of the results and the claims that are made.

4.3.2 Validity in observations and interviews

The data that were collected for the observation was 'rich data' since the video recorder was used, and same data were collected more than once (Maxwell, 2005), to ensure validity of the claims made. By using the video recorder I was able to observe more than what could have been observed without it, and having to remember or document all that was observed since the video can be rewound (Brown & Dowling, 2001). The use of interviews in this research was another way in which valid data were collected. Interviews allowed me to probe further in order to get clarifications of what was not clear (Brown & Dowling, 2001).

4.3.3 Threats for validity

There are two threats for validity, researcher bias and reactivity, and the effect that I have on individuals studied (Maxwell, 2005). I controlled for bias by ensuring that the elements used were designed from the N6 examination question paper that the students write at the end of the year, and from the VSOR content in general, not from my existing theory. In relation to controlling reactivity, I tried to keep the respondents relaxed and encouraged them to view the data collection period as another learning stage.

4.3.4 Validity for the claims made

The validity and credibility for the whole data collection phase and analysis was controlled through triangulation, where different methods of data collection (observations, tests, examinations and interviews) and analysis (Brown & Dowling, 2001; Denscombe, 2002; Kimchi, Polivka & Stevenson, 1991; Mertler, 2006; Teddlie & Tashakkori, 2009,) for the same item were used as indicated in the three phases with different respondents to ensure that the results were trustworthy (Cohen et al., 2001; Mertler, 2006; Schumacher & McMillan, 1993). In this research using a mixed method approach also led to triangulation as the use of both methods (qualitative and quantitative) complements one another (Creswell, 2007; Robson, 2002). The results were also verified by collecting data more than once using the 23-item instrument. Data triangulation in this research was ensured by collecting data using

different students, during different times (three different trimesters) also in different phases, referred to by Denzin (1978) as person and time triangulation from his three types of data triangulation. In data triangulation the results are valid if similar findings were found from the different students (person), who wrote the same assessments during different periods (times) in different social settings (space). However, in this study, space triangulation was not done since the results of the students from the different colleges were not compared. As mentioned earlier, more than one college was used to increase the sample size, and not for comparative purposes.

4.4 RELIABILITY

Reliability involves the consistency, dependability or stability of the results or a coding process, and if the test is repeated or used many times by a different researcher; the same results are achieved (Bassey, 2003; Brown & Dowling, 2001; Cohen et al., 2001; Denscombe, 2002; Mason & Bramble, 1989; Mertler, 2006). According to Cohen et al. (2001, p. 117), reliability is concerned with precision and accuracy. Reliability was ensured by making available the instructions and the solutions for the tests as clear as possible so that they could be used by another person for marking the test. To ensure reliability, it was necessary that the tests be administered more than once (for the main data collection with the 23-item instrument) so that the reliability could be established from the proportion of individuals who consistently met the set criteria for the test. It was also important for me to make sure that the respondents did not get copies of the tests by providing blank spaces on the instrument, since the tests were conducted more than once to ensure that all the respondents would see the instrument for the first time. The students who were repeating the course were also excluded for the test. That was verified with the lecturer. Such procedures were necessary to ensure the reliability of the responses.

Reliability of the instrument used in the main data collection process was also ensured by administering the 21-item instrument as a pilot study and reviewing, modifying, as well as adding some questions depending on the responses given by the students to avoid ambiguity, in designing the 23-item instrument. Reliability of the tests used was ensured by making sure that more questions were used in the main data collection instrument (23 items) as shorter tests are less reliable (Nitko, 2004). Caution must be taken in designing tests that accurately

measure students' performance by ensuring that they are valid, reliable and unbiased (Zucker, 2003).

For the classroom observation and the interview, member checking was not done after the data collection process since the video recorder was used to collect accurate data. Reliability was ensured by making use of two observers to review the video recorder for analysis of the interpretations in order to code the same sequence of events (Brown & Dowling, 2001, p. 53). This is also called 'inter-rate reliability' (Jacobs, Kawanaka, & Stigler, 1999, p. 720).

4.5 GENERALISATION

Generalisation involves making conclusions about the selected sample or element of things on the basis of information drawn from particular examples or instances from the sample or from the element of things studied (Denscombe, 2002). The results of this study cannot be generalised to other settings, since the sample comprised of two colleges only for the data collection phase and only three colleges with very few students for the pilot study. I can rather infer how the findings might relate to different situations by transferring the results of this study to other similar settings. Qualitative researchers talk about transferability (a process in which the researcher and the readers infer how the findings might relate to other situations) rather than generalizability (Denscombe, 2002, p. 149).

4.6 ETHICAL CONSIDERATION

Ethics relate to the rights and the interests of the participants in the research. For ethical considerations, students and lectures were given consent forms for willingness to participate in the study and this also ensured that the results will be treated with confidentiality without their names and their institutions' names being revealed elsewhere and during publishing of the results (Bassey, 2003; Cohen et al., 2001). The participants were also reassured that their faces would never be revealed from the video recordings. The students should be non-traceable unless they give consent (Cohen et al., 2001, p. 335). Permission was also granted from the National DoE.

In this research the respondents were not paid. They participated voluntarily. In that way, there would not be any possibility of research bias in terms of responses that the students

would give compared to if they were paid. Instead students were encouraged to participate by emphasising on how the designed instrument was relevant to their syllabus. They were also encouraged to develop trust based on my background and in-depth knowledge of VSOR. The respondents were assured that they would not be harmed during their participation, and that the research was solely based on improving their knowledge on VSOR.

For ethical considerations in this research, I ensured that there was respect for democracy, for truth and for persons (Bassey, 2003, p. 73) during data collection, analysis and reporting. Care was taken that:

> researchers should be committed to discovering and reporting things as faithfully and as honestly as possible, without allowing their investigations to be influenced by considerations other than what is the truth of the matter (Denscombe, 2002, p. 177).

Since observations were used and I had contact with the participants, it was also important that I made the participant feel at ease.

4.7 DELINEATION OF THE STUDY

The main study (Phase II and III) focussed on only two FET colleges, where only mathematics N6 students were sampled. The learning difficulties were not explored on the whole content studied for the N6 curriculum. Only a section that constitutes 40 per cent of the mathematics N6 syllabus, VSOR was used. The research was based on students' written responses interpretations, with less focus on their verbal responses interpretations.

4.8 LIMITATIONS OF THE STUDY

The results of this study could not be generalised to all the FET colleges in South Africa, because of the sample size. Only two FET colleges were used for the main study. The colleges sampled for this study were those accessible to the researcher and were limited to one township college with black students only and one industrial area college with predominantly black students. Even if all students at these colleges were sampled, some were either absent in other phases of the data collection process or did not complete all the questions asked. The other limitations for this study are that only one class was observed and that the students who completed the questionnaire from the two colleges were not the same students whose written responses were analysed in the 151 examination scripts. However all

students from this colleges were used, for the questionnaires, observations and examination analysis.

In addition the research was conducted a week or two prior to the examinations, which may have been stressful to some students or lectures like this could be time-consuming for the lecturers who did not complete the required syllabus. Some students were therefore absent on the day of data collection.

4.9 SUMMARY

In the above discussion I attempted to present and clarify the research design and methods used in this study in order to do research on students' difficulties involving VSOR. The research strategy used involved the MMA for data collection, analysis and reporting, where multiple case studies were used to ensure triangulation of the results. Mathematics N6 college students were sampled for this study. The research is interpretive and descriptive, following the interpretive and the positivists' view. As action research it aims to lead to innovation and change, but not necessarily to generalisation of the results to other settings (Cohen et al., 2001; Mason & Bramble 1989).

The data collected was analysed using MMA and reported, with qualitative approach as the dominant method. The issues regarding the validity and reliability of the study were discussed for both qualitative and quantitative methods. The issue of validity and credibility for the whole data collection phase and analysis was controlled through triangulation. The use of tests as the main mode for data collection was discussed in-depth. The ways in which the results could be transferable in qualitative research and generalised in quantitative research were also discussed. Ethical considerations were discussed in relation to respect towards participants and confidentiality. The delineation of the study clarified the focus of the study while the limitations of the study were associated with the small sample used, the exclusion of other racial groups and the time constraints when teaching N6 mathematics.