

**Learning difficulties involving volumes of solids
of revolution: A comparative study of
engineering students at two colleges of Further
Education and Training in South Africa**

by

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DECLARATION

I, the undersigned, declare that the thesis which I hereby submit for the degree Philosophiae Doctor to the University of Pretoria contains my own, independent work and has not previously been submitted by me for any degree at this or any other tertiary institution.

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Date:

ABSTRACT

This study investigates learning difficulties involving volumes of solids of revolution (VSOR) at two FET colleges in Gauteng province, in South Africa. The research question for this study was: **Why do students have difficulty when learning about volumes of solids of revolution?** In order to answer the research question five skill factors were identified as the conceptual framework, subdivided into 11 elements. The five skill factors are: I. Graphing skills and translating between visual graphs and algebraic equations/expressions, II. Three-dimensional thinking, III. Moving between discrete and continuous representations, IV. General manipulation skills and V. Consolidation and general level of cognitive development.

Before collecting the main data for this study, a preliminary study and a pilot study were conducted. The data for the main study were then collected in six different investigations. The investigations consisted of two runs of a questionnaire, classroom observations, examination analysis; detailed examination responses and an interview with one student.

The results from the questionnaire runs as well as the pilot study reveal that students performed poorly in tasks involving three-dimensional thinking (Skill factor II), moving between discrete and continuous representations (Skill factor III), and consolidation and general level of cognitive development (Skill factor V). Students' performance was satisfactory in tasks involving graphing skills and translating between visual graphs and algebraic equations/expressions (Skill factor I) and general manipulation skills (Skill factor IV). Students were also more competent in solving problems that involved procedural skills than those that required conceptual skills. The challenges that students were faced with in class, evident from the classroom observations allude to the fact that the topic of VSOR is difficult to teach and to learn.

It is recommended that VSOR be taught and assessed more conceptually in line with the five skill factors; that curriculum developers must communicate with other stakeholders like industries and other institutions of higher learning and that the Department of Education must provide adequate training for these teachers and liaise with industry in this regard. It is also recommended that the suitability of this topic for the particular cohort of students be reconsidered as it appears to be of too high cognitive demand.

DEDICATION

This study is dedicated to my family for walking this long path with me. I dedicate this study to my husband Majagaodwa Mbokane, my daughter Mmamonkwe, my son Umalusi, my mother Mmamonkwe Mofolo and in soul my father Malebye Mofolo who inspired me throughout this journey. Perseverance is what kept me going. I can now sing and praise GOD, who lifted me up when I was tripping.

“Praise the LORD with the harp; make melody to Him with an instrument of ten strings. Sing to Him a new song; play skilfully with a shout of joy”.

PSALM 33: 2-3

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LIST OF ACRONYMS

ABET - Adult Basic Education and Training
ACE - Advanced Certificate in Education
ARIRE - Average Ranking for Individual Responses per Element
CAS - Computer Algebra System
DoE - Department of Education
DoL - Department of Labour
FET - Further Education and Training
FTC - Fundamental Theorem of Calculus
GET - General Education and Training
HE - Higher Education
MMA - Mixed methods approach
NC(V) - National Certificate (Vocational)
NQF - National Qualification Framework
SAQA - South African Qualifications Authority
TIMSS - Third International Mathematics and Science Study
VSOR - Volumes of solids of revolution (VSOR)
ZPD - Zone of Proximal Development
2D - Two-dimensional
3D - Three-dimensional