CHAPTER 4. PHASE SHIFTER SENSITIVITY ANALYSIS

4.1. Introduction

In this chapter the practicality of the proposed novel class of phase shifter is investigated. Equations will be derived to enable a designer to assign tolerances to various element parameters in order to yield a component that satisfies certain specifications. On the other hand the requirements of a fabrication technology suitable for achieving a given circuit performance may be specified. A sensitivity analysis is the best way to examine the trade-offs between specifications and tolerances.

An analysis of the effect of changes in the values of various element parameters on the component performance parameters is called a sensitivity analysis of the component. The sensitivities of the elements comprising the phase shifter yield information as to how design parameter tolerances affect the electrical specifications of the elements. Linking this information with the sensitivity analysis of the phase shifter, the sensitivity of the phase shifter performance parameters with respect to the design parameters is obtained [1]. First the error parameters and transfer functions of the phase shifter will be defined. Then the effect of the error parameters on the component performance, classified as amplitude and phase errors, is investigated. General guidelines will be presented to provide a feel for the sensitivity of certain performance parameters with respect to tolerance errors.

4.2. Error Parameters and Transfer Functions

A convenient way to do a sensitivity analysis is to model the component, in this case the phase shifter, as a combination of elements (here the splitter, impedance transformers and coupler). The sensitivity analysis may be simplified by modelling each element as an ideal element, with an additional multiplicative error voltage element, to represent any tolerance or another non-ideal electrical behaviour due to material or manufacturing tolerance. These error voltage elements, in general, will consist of amplitude and phase components. It will be shown that these error elements can be cascaded to represent the general case of four phase shifter error elements.

The error voltage elements of the splitters, tapers, and couplers may be called E_{sp} E_{ti} and E_{ci} respectively. The subscript will be used to distinguish between different splitters, tapers and couplers, to maintain generality. As seen in Figure 25, these errors may be cascaded to form four generic error elements in the phase shifter.

$$E_i = E_{si} \cdot E_{ti} \cdot E_{ci} \quad i = 1, 2, 3, 4.$$
 (4.1)

where these error vectors may be defined as

$$E_{ki} = |E_{ki}| e^{j\theta ki}$$
 $k = s, t, c$. (4.2)

$$\therefore E_i = |E_i| e^{i\theta i}$$
 (4.3)

The amplitude factors may be cascaded in multiplication $|E_i| = |E_{si}| \cdot |E_{ti}| \cdot |E_{ci}|$, (4.4)

and the phase angles may be added in summation $\theta_i = \theta_{si} + \theta_{ti} + \theta_{ci}$. (4.5)

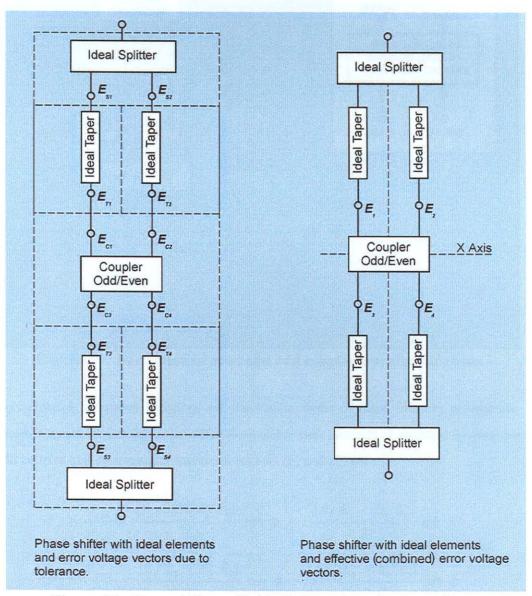


Figure 25: Phase shifter with ideal elements and error voltage

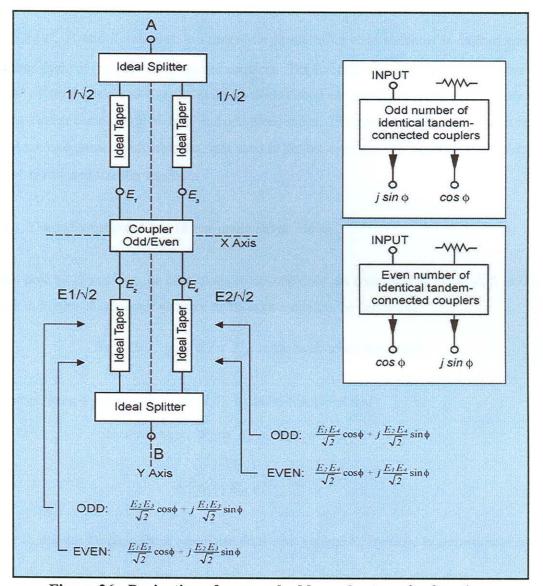


Figure 26: Derivation of even and odd coupler transfer functions

The performance parameter functions are derived in terms of these element parameters. The performance parameter functions also differ when odd or even numbers of cascaded couplers are used. The odd coupler and even coupler transfer functions (F_o and F_e) are

$$F_o = \frac{B_{odd}}{A_{odd}} = \frac{[E_1 \ E_4 + E_2 \ E_3]}{2} \cos \phi + j \frac{[E_1 \ E_3 + E_2 \ E_4]}{2} \sin \phi , \qquad (4.6)$$

$$F_{e} = \frac{B_{even}}{A_{even}} = \frac{[E_{1} \ E_{3} + E_{2} \ E_{4}]}{2} \cos \phi + j \frac{[E_{1} \ E_{4} + E_{2} \ E_{3}]}{2} \sin \phi . \tag{4.7}$$

where E_1 , E_2 , E_3 and E_4 represent the composite phase shifter error elements as defined and ϕ the coupling angle of the tandem-connected couplers. The errors can be seen as a combination of two "states" of error, analogous to even- and odd-mode circuit analyses, where any general state can be seen as a linear combination of even- and odd-mode states. The even- and odd-mode circuit analyses should not be confused with the subscripts used to denote even and odd numbers of couplers in the above- mentioned transfer functions.

4.2.1. Y-axis Symmetry State (Even-mode Error State)

It will now be shown that the even-mode error state does not contribute to phase shift errors. As shown in Figure 25, the error voltages in the even-mode can be derived as

$$E_1 = E_2$$
, and $E_3 = E_4$ in the even-mode error state. (4.8)

Therefore, from equations (4.6) and (4.7), it can be concluded that

$$F_o = F_e = E_1 E_3 e^{j\phi} , \qquad (4.9)$$

$$= |E_{I}| |E_{3}| e^{j(\theta I + \theta 3)f} . e^{j\phi} . (4.10)$$

where θ_i denotes the phase error associated with error voltage E_i . By close examination of equation (4.10), the following conclusions can be made regarding the even-mode error state:

- The gain distortion equals the cascaded losses of the elements comprising the phase shifter.
- The phase distortion, presumed to be a linear function of frequency, can be eliminated by the addition or subtraction of a small length of line in the reference line. This is only true if the phase distortion is caused typically by a small distance offset
- It can therefore be concluded that the state of Y-axis symmetry does not contribute to phase shifter errors, and only to small losses.

4.2.2. X-axis Symmetry State (Odd-mode Error State)

It will now be shown that the odd-mode error state causes complex amplitude and phase errors. As shown in Figure 25, the error voltages in the odd-mode can be defined as

$$E_1 = E_3$$
, and $E_2 = E_4$ in the odd-mode error state.

Therefore, from equations (4.6) and (4.7), it can be concluded that

$$F_o = E_1 E_2 \cos \phi + j \frac{1}{2} [E_1^2 + E_2^2] \sin \phi , \qquad (4.11)$$

$$F_e = \frac{1}{2} \left[E_1^2 + E_2^2 \right] \cos \phi + j E_1 E_2 \sin \phi . \tag{4.12}$$

By close examination of equations (4.11) and (4.12) it is clear that x-axis imbalance errors create complex phase and amplitude errors which need to be thoroughly investigated.

4.3. Derivation of Error Functions

Since it was demonstrated in paragraph 4.2.1 that even-mode imbalance has no effect on the phase shift response, only the odd mode error functions need to be derived. For this purpose an odd-mode amplitude imbalance of δ dB and a phase imbalance of θ degrees are assumed. The voltage imbalance factor can be defined as

$$D = 10^{\delta/20} . (4.13)$$

Let

$$E_I = \sqrt{P_I} e^{j\theta/2} \quad , \tag{4.14}$$

and

$$E_2 = \sqrt{P_2} \, e^{-j\theta/2} \quad . \tag{4.15}$$

be the error voltages, with P_1 and P_2 representing power factors. The power imbalance and energy conservation relations are specified as

$$P_1/P_2 = D^2$$
, (4.16)

$$P_1 + P_2 = 2 . (4.17)$$

Solving for P_1 and P_2 in terms of D yields

$$P_1 = \frac{2D^2}{D^2 + 1} \quad , \tag{4.18}$$

$$P_2 = \frac{2}{D^2 + 1} \quad . \tag{4.19}$$

Substituting in equations (4.14) and (4.15) yields

$$E_1 = \sqrt{\frac{2 D^2}{D^2 + 1}} e^{j\theta/2} , \qquad (4.20)$$

$$E_2 = \sqrt{\frac{2}{D^2 + 1}} e^{-j\theta/2} . {(4.21)}$$

Equations (4.20) and (4.21) can be written in the more convenient form

$$E_I = \sqrt{2} \sin \left(\arctan D\right) e^{j\theta/2} \quad , \tag{4.22}$$

$$E_2 = \sqrt{2} \cos \left(\arctan D\right) e^{-j\theta/2} \quad . \tag{4.23}$$

It can be shown that

$$E_1 E_2 = \sin(2 \arctan D) = S_D$$
, (4.24)

and

$$\frac{E_1^2 + E_2^2}{2} = \cos \theta - j \sin \theta \cos (2 \arctan D)$$
 (4.25)

$$= \cos \theta - jC_D \sin \theta , \qquad (4.26)$$

where

$$C_D = \cos (2 \arctan D) \quad . \tag{4.27}$$

The odd and even coupler transfer functions associated with the odd-mode error state are therefore

$$F_o(\phi, \theta, \delta) = (S_D \cos \phi + C_D \sin \phi \sin \theta) + j \cos \theta \sin \phi$$
, (4.28)

$$F_e(\phi, \theta, \delta) = \cos \theta \cos \phi + j (S_D \sin \phi - C_D \sin \theta \cos \phi).$$
 (4.29)

The important phase shifter performance parameters, namely phase shift and gain response, associated with imbalance errors δ dB and θ degrees, can now be defined.

$$P_{\theta}(\phi, \theta, \delta) = |S_{21}|_{\theta}^{2} = \cos^{2}\theta \sin^{2}\phi + (S_{D}\cos\phi + C_{D}\sin\phi\sin\theta)^{2}$$
, (4.30)

$$P_e(\phi, \theta, \delta) = |S_{21}|_e^2 = \cos^2 \theta \cos^2 \phi + (S_D \sin \phi - C_D \cos \phi \sin \theta)^2$$
, (4.31)

$$\phi_o (\phi, \theta, \delta) = arg (S_{21_0}) = \arctan \left[\frac{\tan \phi \cos \theta}{S_D + C_D \tan \phi \sin \theta} \right],$$
(4.32)

$$\phi_e (\phi, \theta, \delta) = arg (S_{21e}) = \arctan \left[\frac{S_D \tan \phi - C_D \sin \theta}{\cos \theta} \right] .$$
(4.33)

From the phase shift, the phase shift errors can be derived as

$$\Psi_o (\phi, \theta, \delta) = \phi - \phi_o \tag{4.34}$$

$$= \arctan \left[\frac{C_D \tan^2 \phi \sin \theta + \tan \phi (S_D - \cos \theta)}{\tan^2 \phi \cos \theta + C_D \tan \phi \sin \theta + S_D} \right], \qquad (4.35)$$

$$\Psi_e (\phi, \theta, \delta) = \phi - \phi_e \tag{4.36}$$

$$= \arctan \left[\frac{\tan \phi (\cos \theta - S_D) + C_D \sin \theta}{S_D \tan^2 \phi - C_D \sin \theta \tan \phi + \cos \theta} \right] . \tag{4.37}$$

Expressions (4.30), (4.31), (4.35) and (4.37) can be used to evaluate the phase shifter sensitivity with respect to element changes, in the odd-mode error state, referred to as imbalance. The remainder of this chapter will deal with the tracking requirement of elements comprising the phase shifter, using the above results.

4.4. Sensitivity Analysis Results

4.4.1. Amplitude Errors

For amplitude-only errors, we set $\theta = 0$ in the phase shifter performance parameter functions as derived in the previous section. Amplitude errors due to amplitude imbalance, for an odd and even number of tandem-connected couplers, are calculated as

$$P_{4a}(\phi, \delta) = S_D^2 + C_D^2 \sin^2 \phi$$
, (4.38)

and

$$P_{Ae}(\phi, \delta) = 1 - C_D^2 \sin^2 \phi$$
 (4.39)

Phase shift errors due to amplitude imbalance, for an odd and even number of tandem-connected couplers, are as follows:

 $\Psi_{Ao} (\phi, \delta) = \arctan \left[\frac{\tan \phi (S_D - 1)}{\tan^2 \phi + S_D} \right] , \qquad (4.40)$

and

$$\Psi_{Ae} (\phi, \delta) = \arctan \left[\frac{\tan \phi (1 - S_D)}{S_D \tan^2 \phi + 1} \right] . \tag{4.41}$$

To find the phase shift range over which the phase shifter is least sensitive to amplitude imbalance we derive the un-normalized sensitivities of the performance parameters with respect to phase:

a) In the odd number of tandem-connected coupler case, amplitude sensitivity can be calculated by solving the following derivative:

$$\frac{\partial}{\partial \phi} P_{A,o} (\phi, \delta) = 0 \tag{4.42}$$

Most sensitive to amplitude imbalance:

$$\phi = \pm 180^{\circ} n \qquad n = 0, 1, 2... \tag{4.44}$$

Least sensitive to amplitude imbalance:

$$\phi = \pm (2n + 1) \cdot 90^{\circ} \qquad n = 0, 1, 2... \tag{4.45}$$

b) In the even number of tandem-connected coupler case, amplitude sensitivity can be calculated by solving the following derivative:

$$\frac{\partial}{\partial \Phi} P_{A,e} (\Phi, \delta) = 0 \qquad \qquad : \Phi = \pm 90^{\circ} \cdot n \qquad n = 0, 1, 2... \tag{4.46}$$

Most sensitive to amplitude imbalance:

$$\phi = \pm (2n + 1) \cdot 90^{\circ} \quad n = 0, 1, 2... \tag{4.47}$$

Least sensitive to amplitude imbalance:

$$\Phi = \pm 180^{\circ} \cdot n \qquad n = 0, 1, 2... \tag{4.48}$$

c) In the odd and even number of tandem-connected coupler cases, the two derivative equations yield the same solution for phase sensitivity:

$$\frac{\partial}{\partial \Phi} \Psi_{A,o} (\Phi, \delta) = 0 \quad , \tag{4.49}$$

and

$$\frac{\partial}{\partial \phi} \Psi_{A,e} (\phi, \delta) = 0 \quad . \tag{4.50}$$

$$\therefore \ \phi = \pm \ 45^{\circ} \cdot n \qquad n = 0, 1, 2... \tag{4.51}$$

Most sensitive to amplitude imbalance:

$$\Phi = \pm (2n + 1) \cdot 45^{\circ} \qquad n = 0, 1, 2... \tag{4.52}$$

Least sensitive to amplitude imbalance:

$$\phi = \pm 90^{\circ} \cdot n \qquad n = 0, 1, 2...$$
 (4.53)

d) In a worst case analysis, to determine maximum amplitude imbalance errors we observe that maximum amplitude errors occur when $\sin \phi = 0$ in the odd number of tandem-connected coupler case, and $\sin \phi = 1$ in the even coupler number case.

$$P_{A, MAX}(\delta) = P_{A, o MAX}(\delta) = P_{A, e MAX}(\delta) = S_D^2$$
 (4.54)

Maximum phase shift errors occur when $\tan \phi = 1$ in the case of both even and odd number of tandem connected couplers.

$$|\Psi_{A, MAX}(\delta)| = |\Psi_{A, e MAX}(\delta)| = |\Psi_{A, o MAX}(\delta)| = |\arctan\left[\frac{S_D - 1}{S_D + 1}\right]| \qquad (4.55)$$

From Figure 27, which shows the graphs of amplitude errors, it is clear that these errors do not severely affect the phase shifter performance. In the odd number of tandem-connected coupler case, these errors affect the phase shifter gain most at low values of ϕ , while in the even coupler number case, the phase shifter gain is more sensitive to these errors for phase shift values closer to 90°.

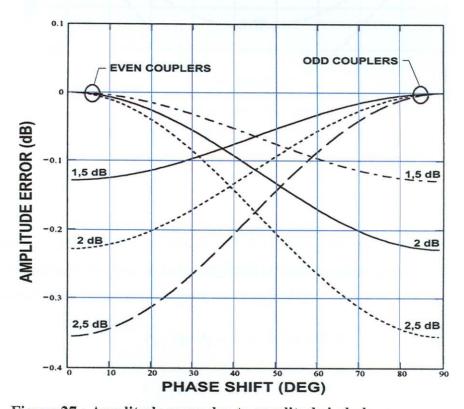


Figure 27: Amplitude error due to amplitude imbalance

Values of amplitude imbalance as high as 2.5 dB cause a phase shifter insertion loss of only 0.35 dB. The phase shift is most sensitive for amplitude imbalance at phase shift values of 45°, but a 2.5 dB imbalance causes a phase shift error of only 1.2°. From the results as displayed in Figure 28, it is clear that the phase shifter performance parameters are insensitive to amplitude imbalance errors.

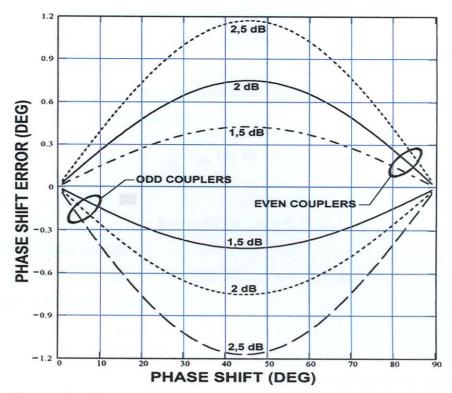


Figure 28: Phase shift error due to amplitude imbalance

4.4.2. Phase Errors

To analyse phase errors, we set $C_D = 0$ and $S_D = 1$ ($\delta = 0$) in the phase shifter performance parameter functions, derived in the previous section. Amplitude errors due to phase imbalance, for an odd and even number of tandem-connected couplers, are calculated as

$$P_{P,o}(\phi, \theta) = 1 - \sin^2 \phi \sin^2 \theta , \qquad (4.56)$$

and

$$P_{P,e} (\phi, \theta) = 1 - \cos^2 \phi, \sin^2 \theta . \qquad (4.57)$$

Phase shift errors due to phase imbalance, for an odd and even number of tandem-connected couplers, are as

$$\Psi_{P,o}(\phi, \theta) = \arctan \left[\frac{\tan \phi (1 - \cos \theta)}{\tan^2 \phi \cos \theta + 1} \right],$$
 (4.58)

and

$$\Psi_{P,e} (\phi, \theta) = \arctan \left[\frac{\tan \phi \cdot (\cos \theta - 1)}{\tan^2 \phi + \cos \theta} \right]$$
 (4.59)

To find the phase shift range over which the phase shifter is least sensitive to phase imbalance errors we derive the un-normalized sensitivities of the performance parameters with respect to phase:

a) In the odd number of tandem-connected coupler case, amplitude sensitivity can be calculated by solving the following derivative:

$$\frac{\partial}{\partial \Phi} P_{P,o} (\Phi, \Theta) = 0 \tag{4.60}$$

$$\therefore \Phi = \pm 90^{\circ} n \quad n = 0, 1, 2...$$
 (4.61)

Most sensitive to phase imbalance:

$$\phi = \pm (2n + 1) 90^{\circ} \qquad n = 0, 1, 2... \tag{4.62}$$

Least sensitive to phase imbalance:

$$\phi = \pm 180^{\circ} \cdot n \qquad n = 0, 1, 2... \tag{4.63}$$

b) In the even number of tandem-connected coupler case, amplitude sensitivity can be calculated by solving the following derivative:

$$\frac{\partial}{\partial \Phi} P_{P, e} (\Phi, \theta) = 0 \tag{4.64}$$

$$\therefore \ \phi = \pm \ 90^{\circ} \ n \quad n = 0, 1, 2...$$
 (4.65)

Least sensitive to phase imbalance:

$$\phi = \pm (2n + 1) 90^{\circ} \qquad n = 0, 1, 2... \tag{4.66}$$

Most sensitive to phase imbalance:

$$\phi = \pm 180^{\circ} \cdot n \qquad n = 0, 1, 2... \tag{4.67}$$

b) In the odd and even number of tandem-connected coupler case, the two derivative equations yield the same solution for phase sensitivity:

$$\frac{\partial}{\partial \phi} \Psi_{P,o} (\phi, \theta) = 0 \quad , \tag{4.68}$$

and

$$\frac{\partial}{\partial \phi} \Psi_{P,e} (\phi, \theta) = 0 . \tag{4.69}$$

$$\therefore \ \varphi = \pm \ n \cdot 45^{\circ} \quad n = 0, 1, 2... \tag{4.70}$$

Most sensitive to phase imbalance:

$$\Phi = \pm (2n + 1) \cdot 45^{\circ} n \qquad n = 0, 1, 2... \tag{4.71}$$

Least sensitive to phase imbalance:

$$\Phi = \pm n \cdot 90^{\circ} \qquad n = 0, 1, 2... \tag{4.72}$$

In a worst case analysis, to determine maximum phase imbalance errors we observe that maximum amplitude errors occur when $\sin \phi = 1$ in the odd number of tandem connected coupler case, and $\cos \phi = 1$ in the even coupler number case.

$$P_{P, MAX}(\theta) = P_{P, 0 MAX}(\theta) = P_{P, 0 MAX}(\theta) = \cos^2 \theta \tag{4.73}$$

Maximum phase shift errors occurs when $\tan \phi = 1$ for both even and odd number of tandem connected couplers.

$$\mid \Psi_{P MAX} \left(\theta \right) \mid = \mid \Psi_{P,0 MAX} \left(\theta \right) \mid = \mid \Psi_{P,e MAX} \left(\theta \right) \mid ,$$

$$= |\arctan\left[\frac{1 - \cos\theta}{1 + \cos\theta}\right]| = |\arctan\left[\tan^2\left(\frac{\theta}{2}\right)\right]| . \tag{4.74}$$

From the graphs of phase errors in Figure 29, it is clear that these errors severely affect the phase shifter performance. In the odd number of tandem-connected coupler case, these errors affect the phase shifter gain most at phase shift values close to 90°. The even number of tandem-connected coupler case, the phase shifter gain is more sensitive to these errors for low phase shift values. Phase imbalance errors of 45° cause a 3 dB loss. The phase shift shown in Figure 30 is most sensitive for phase imbalance at phase shift value of 45°, where a 45° imbalance causes a 10° phase shift error. From these results, it is clear that the phase shifter performance parameters are quite sensitive to phase imbalance errors. Application of symmetry to phase shifters is therefore a very important measure to enhance the performance of ultra-wideband devices.

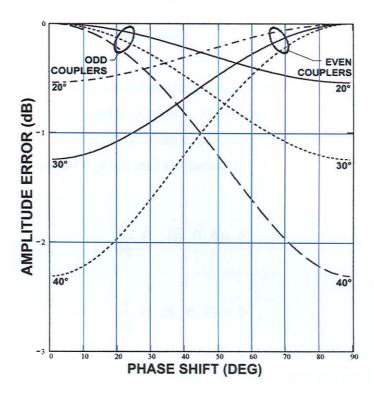


Figure 29: Amplitude error due to phase imbalance

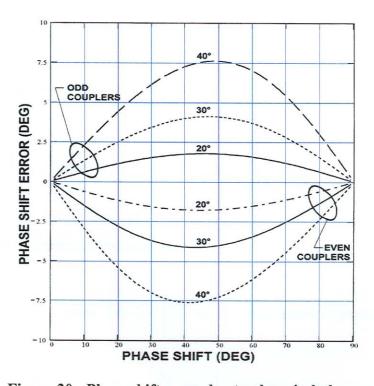


Figure 30: Phase shift error due to phase imbalance

4.4.3. Combined Amplitude and Phase Errors

The next step is to derive the un-normalized sensitivity of phase shifter loss error with respect to phase shift, for combined amplitude and phase imbalance. This is used to derive expressions for maximum loss distortion, and to find the phase shift values where losses is most sensitive to these errors. The derivatives of loss with respect to phase shift are derived as

$$\frac{\partial}{\partial \phi} P_o \quad (\phi, \ \theta, \ \delta) = 0 \quad , \tag{4.75}$$

and

$$\frac{\partial}{\partial \phi} P_e (\phi, \theta, \delta) = 0 . \tag{4.76}$$

The solution of these derivatives yields the phase shift values least and most sensitive to both amplitude and phase imbalance. The derivatives yield the same solutions:

$$\tan^{2} \phi + \left[\frac{S_{D}^{2} - C_{D}^{2} \sin^{2} \theta - \cos^{2} \theta}{S_{D} C_{D} \sin \theta} \right] \tan \phi - 1 = 0$$
 (4.77)

Two solutions are found when solving equation (4.77):

$$\tan \phi = \left[\frac{S_D}{C_D} \sin \theta\right]^{-1} \quad \text{or} \quad -\left[\frac{S_D}{C_D} \sin \theta\right] .$$
(4.80)

For the odd number of tandem-connected coupler case:

$$P_{o, MAX}(\theta, \delta) = 1$$
 when $\tan \phi = \left[\frac{S_D}{C_D} \sin \theta\right]^{-1}$ (unsensitive to phase imbalance) (4.81)

=
$$S_D^2 \cos^2 \theta$$
 when $\tan \phi = -\left[\frac{S_D}{C_D} \sin \theta\right]$ (sensitive to phase imbalance) (4.82)

For the even coupler case:

$$P_{e MAX}(\theta, \delta) = 1 \text{ when tan } \phi = -\left[\frac{S_D}{C_D} \sin \theta\right] \text{ (completely unsensitive)},$$
 (4.83)

=
$$S_D^2 \cos^2 \theta$$
 when $\tan \phi = \left[\frac{S_D}{C_D} \sin \theta \right]^{-1}$ (most sensitive). (4.84)

From the graphs of maximum loss distortion in Figures 31 and 32, it is clear that even for severe amplitude imbalance errors, the curve does not change significantly. Depending on odd or even number of tandem-connected couplers, the error curve shifts slightly left or right. This confirms that these phase shifters are relatively insensitive to amplitude distortion.

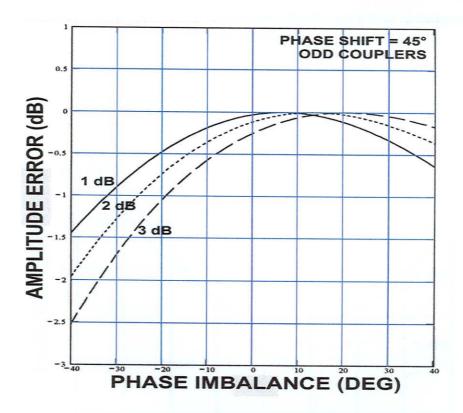


Figure 31: Amplitude error due to amplitude and phase imbalance - odd couplers

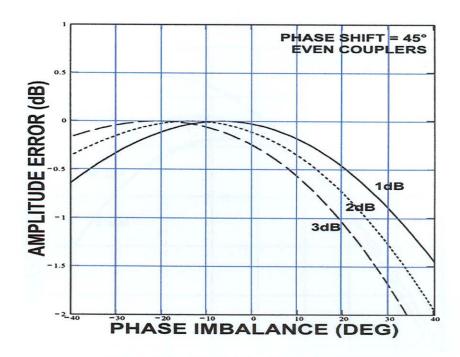


Figure 32 : Amplitude error due to amplitude and phase imbalance - even couplers

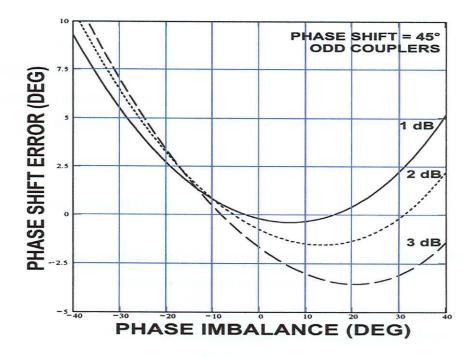


Figure 33: Phase shift error due to amplitude and phase imbalance - odd couplers

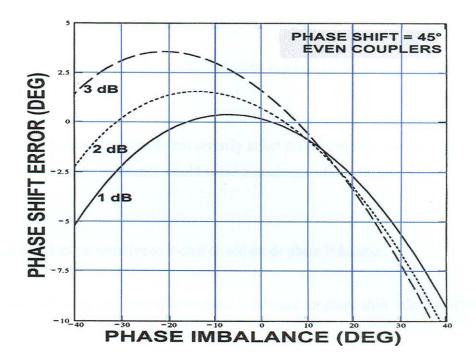


Figure 34: Phase shift error due to amplitude and phase imbalance - even couplers

In Figures 33 and 34 graphs of phase shift error due to amplitude and phase imbalance can be seen. The error curves of the odd and even number of tandem-connected couplers form mirror pairs around the zero phase shift error line. As a parasitic element can even introduce a 20° phase imbalance at some frequencies, it is clear that large phase shift errors can be caused. The phase shifter is, therefore, sensitive to phase imbalance errors.

4.5. Conclusion and General Guidelines

To avoid phase shifter configurations which are sensitive to tolerance variations, the following guidelines, derived using the sensitivity analyses, are suggested:

• X-axis or even-mode asymmetry does not contribute to phase shift errors. Phase shifter losses are equal to the cascade of the amplitude errors:

- For phase shift values below 45°, an odd number of tandem-connected couplers is recommended.
- For phase shift values above 45°, an even number of tandem-connected couplers is recommended.
- Amplitude imbalance errors do not severely affect the performance parameters of the phase shifter. A 2.5 dB imbalance would cause a maximum of 0.4 dB loss and 1.2° phase shift error.
- Phase shifters are sensitive to Y-axis or odd-mode phase imbalance.
- Phase shifters are most sensitive to phase imbalance for phase shift values of 45°. A 45° imbalance cause a 10° phase shift error.

References

[1] K. C. Gupta, R. Garg, and R. Chadha, "Computer aided design of microwave circuits", Artech House, Massachusetts, 1981.