On some aspects of non-stationary time series

by

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CONTENTS

CHAPTER 1

SUMMARY AND LITERATURE REVIEW

1.1 Outline and Summary

This study consists of eight chapters. Testing for a unit root in a process reveals whether a series needs to differenced before analysis. In Chapter 2, the concept of non-stationary and non-invertibility in univariate time series will be defined. The Dickey-Fuller tests for unit roots in a process will be thoroughly discussed.

Chapter 3 reviews the Phillips-Perron and some other tests for unit roots. In Chapter 4, the issue of obtaining the degree of differencing *d*, in a non-stationary time series will be discussed. Fourier transforms is a pre-requisite for spectral analysis. Chapter 5 deals with spectral analysis of time-dependent non-stationary time series with the Fourier analysis forming the basis of the discussion. Results will then be extended to the bivariate case. Thorough discussion will be done on the estimations of evolutionary co-spectrum, evolutionary quadrature spectrum, among others. In order to learn more about our unit root test statistics with regards to their long-run performance, we have cconsidered Monte Carlo studies on three commonly used unit root test procedures, namely, the Augmented Dickey-Fuller (ADF), the Phillips-Perron (PP), and the Reversed Dickey-Fuller Unit Root (RDFUR) test statistics in Chapter 6. Chapters 7 and 8 present practical illustrations of all the methodologies. A summary of results and notes on further research are given in Chapter 9. All analyses would be carried out using one of four statistical software packages namely SAS, Minitab, Matlab, and EViews.

1.2 Survey of the Literature

Considerable progress has been made in the analysis of time series under the assumption the series involved is stationary. Broadly speaking, a time series is considered to be stationary if its mean and variance are constant over time and the value of covariance between two time points depends only on the distance or lag between the two time points and not on the actual time at which the covariance is estimated. Stationarity has always been the bedrock in practical analysis of time series. It suffices to say that abandoning the assumption of stationarity may lead to misleading postulation of models. However, in practice, most time series show non-stationary behaviour, i.e. in vague terms they do not oscillate or fluctuate about a constant mean.

An informal way of handling non-stationarity is by taking an appropriate number of differences and fitting a model to the differenced series. The degree of differencing required can be determined by visual inspection of the sample autocorrelation function. A formal statistical test for the need to difference further if one assumes that at most one more differencing is needed for stationarity have been proposed by Dickey and Fuller (1979), among others.

More intuitively, a series is stationary if all the roots of the autoregressive polynomial equation lie outside the unit circle. A root that is equal to one in absolute value is referred to as a unit root. A time series having a unit root is said to be integrated of order one. In this case, the series must be differenced once to induce stationarity. Testing for the presence of a unit root is quite problematic in the analysis of time series. In this test it is assumed that the series are serially correlated and for that matter tests for the presence of a unit root takes into consideration serial dependence.

Several tests have been proposed by many authors to handle the issue of testing for the presence of unit roots in time series. Notable among these authors are Dickey and Fuller (1979) and Phillips and Perron (1988) who, respectively, proposed the Dickey-Fuller and the Phillips-Perron unit root tests.

The testing of a unit root in a time series has strong implications with the economic theory and its interpretations. Research conducted by Nelson and Plosser (1982) and Perron (1990) revealed that most macroeconomic time series exhibit some kind of stochastic non-stationarity and thus concluded that the total variability of a time series is explained in greater part by permanent shocks. Such shocks (sudden changes) are noticed from a quick glance at the graph of the series. They may appear so big and sudden compared to the variability exhibited over the rest of the periods.

For stationary time series, the spectrum is a popular concept that has long since proved its worth. Spectral analysis is a highly developed procedure for the analysis of a time series. It is widely adapted to the interpretation of economic and biomedical time series, among others. Time series modelled in these fields are studied with the help of periodograms, i.e. studying the spectral features of the stationary series. Spectral analysis, as the name goes, is the fundamental tool used to study the cyclical behaviour of time series.

For stationary time series, on the other hand, it is right to comment that although quite an amount of work has been reported, the bulk of it has not been conclusive. It seems reasonable to conclude that there is no definition that is entirely satisfactory for processes that are neither asymptotically stationary nor almost periodic in their features (Lyones, 1968). In fact, Lyones (1968) has listed all the requirements for a spectral function of a process and concluded that when a process is not-stationary, there does not seem to exist a spectral function satisfying all the requirements.

The Fourier transform is a substantial tool in most fields of science. In statistics it has proved to be a special tool when it comes to the analysis of time-invariant systems. It is an approach used to model the fluctuation of time series in terms of sinusoidal behaviour at various frequencies. In fact, Brillinger (1981) even described spectral analysis as an extension of Fourier transform.

The basic tool for spectral analysis is the periodogram. It is the task of the periodogram to create the basis for the estimation of the spectrum. With the spectrum any periodicities that may be hidden in the series are unveiled. The spectral analysis of non-stationary processes has, however, attracted the attention of few authors. Notable among them are Priestley (1965), Abdrabbo and Priestley (1967), and Adak (1998). In this study, we shall pay attention to the work of Priestley (1965, 1967, 1968) and Granger and Hatanaka (1964) regarding the spectral analysis of time-dependent time series, particularly processes with slowly varying spectra. Priestley's spectrum has been shown to be useful for linear prediction, filtering and a test for stationarity.

The concept of repeated sampling forms the basis of most statistical inference, for instance, parameter estimators and test statistics. One way to driving home this concept is through a Monte Carlo study - a study that involves hundreds, thousands, or even millions of times more calculation than usually done. The Monte Carlo study gives us a deeper insight about the long-run performance of these statistical inference. Simply put, investigating the finite-sample properties of estimators and test statistics using Monte Carlo simulations allows us to interpret our statistical results with confidence.

1.3 Methodology

1.3.1 Time Series

Methodologies will be illustrated using monthly data on nominal exchange rate of the South African rand to the U.S. dollar indexed at 1990=100, percentage *Eskom* yields on loan stock, consumer price index (CPI) for South Africa at 1995 prices, and number of Gold shares - all traded on the Johannesburg Stock Exchange (JSE). *Eskom* yields comprise observations from January 1990 to June 1999; Gold shares, from January 1990 to April 1999; and consumer price index, from January 1994 to October 1999.

1.3.2 Data Source

Both data sets used to illustrate the methodologies are from the official bulletin published quarterly by the South African Reserve Bank (SARB) and Statistics South Africa. They are used in this dissertation with kind permission.

1.4 Importance of the Study

The primary objective of this study is to compare the size and power of three commonly used unit root test statistics using Monte Carlo simulations. The test statistics include the Augmented Dickey-Fuller (ADF), the Phillips-Perron (PP), and the Reversed Dickey-Fuller Unit Root (RDFUR). We also consider linking the theory and the applications of some aspects of non-stationary univariate time series and to show how valuable they are when it comes to postulating an appropriate model for a given time series. For instance, a unit root test in a given non-stationary time series may reveal that non-stationarity is driven either by a linear trend or random walk with drift. In this case, stationarity may be induced by detrending, and differencing if the series is driven by random walk with trend.

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CHAPTER 2

UNIT ROOT TESTS FOR NON-STATIONARITY

2.1 Introduction

A univariate time series $\{X_t\}$ is *covariance stationary* if neither the mean μ_t nor the autocovariances γ_{kt} depend on time $t = 1,2,3,\ldots,T$. In this case

$$E(X_t) = \mu \quad , \tag{2.1a}$$

$$\text{var}(X_t) = E(X_t - \mu)^2 = \sigma^2 \tag{2.1b}$$

$$\gamma_k = E\left[(X_t - \mu)(X_{t-k} - \mu)\right]. \tag{2.1c}$$

If the series fails to satisfy one or more of the conditions for stationarity given in (2.1a) and (2.1b), $\{X_t\}$ is described as a *non-stationary* series. In the analysis of time series, the most common requirement is the assumption of stationarity. However, in practice most series data are non-stationary. When a time series is found to be non-stationary, an appropriate number of differencing operations are usually required to transform the series to a stationary series.

For the non-stationary series $\{X_t : t = 1,2,3,\ldots,T\}$ the general autoregressive-integrated-moving average, ARIMA (*p, d, q*) process is given by

$$\Phi(B)(1 - B)^d X_t = C + \theta(B)\varepsilon_t, \tag{2.2}$$

or

$$\Phi(B)\nabla^d X_t = C + \theta(B)\varepsilon_t, \tag{2.3}$$

where C is a constant, $\nabla^d X_t$ is the *d*th difference of X_t and $\{\varepsilon_t\} \sim WN(0,\sigma^2)$ represents a white noise process with mean 0 and variance σ^2. A sequence, $\{\varepsilon_t\}$, of uncorrelated random variables from a fixed distribution is said to be a *white noise* process if it satisfies the following conditions

$$E(\varepsilon_t) = 0,$$

$$\text{var}(\varepsilon_t) = \sigma^2,$$

and

$$\text{cov}(\varepsilon_t, \varepsilon_{t-k}) = 0 \text{ for all } k \neq 0.$$

The operators $\Phi(B)$ and $\theta(B)$ are respectively defined as

$$\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p \qquad \theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q \quad .$$

If $d = 0$, (2.2) or (2.3) is referred to as an ARMA (p, q) process. Setting $(1-B)^d X_t = Y_t$ (2.3) becomes

$$\Phi(B) Y_t = C + \theta(B)\varepsilon_t \quad . \tag{2.4}$$

Using the backshift operator $B^k X_t = X_{t-k}$ and the fact that $E(X_t) = \mu$, taking expectations on both sides of (2.4) shows that

$$C = \mu - \phi_1\mu - \phi_2\mu - \ldots\ldots - \phi_p\mu \tag{2.5}$$

Substituting (2.5) into (2.4) simplifies to give

$$Y_t - \mu = \phi_1\left(Y_{t-1} - \mu\right) + \phi_2\left(Y_{t-2} - \mu\right) + \ldots + \phi_p\left(Y_{t-p} - \mu\right) + \varepsilon_t - \theta_1\varepsilon_{t-1} - \ldots\theta_q\varepsilon_{t-q},$$

$$\Rightarrow \qquad Y_t - \mu = \sum_{i=1}^{p} \phi_i\left(Y_{t-i} - \mu\right) + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}. \tag{2.6}$$

or

$$Y_t = C + \sum_{i=1}^{p} \phi_i Y_{t-i} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}, \tag{2.7}$$

If X_t follows a deterministic time-trend t, μ is given by

$$\mu_t = \sum_{m=0}^{d} \beta_m t^m, \qquad \text{(a polynomial function of time)} \tag{2.8}$$

we have

$$X_t - \sum_{m=0}^{d} \beta_m t^m = \sum_{i=1}^{p}\left[\phi_i\left(X_{t-i} - \sum_{m=0}^{d} \beta_m (t-i)^m\right)\right] + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}. \tag{2.9a}$$

$$\Rightarrow \qquad Y_t = \sum_{i=1}^{p} \phi_i Y_{t-i} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}\,, \qquad (2.9.b)$$

where $Y_t = X_t - \mu_t$ .Equation (2.2) is stationary if the roots $z_i\,(i = 1,2,\ldots\ldots, p)$ of the equation

$$1 - \phi_1 z - \phi_2 z^2 - \ldots - \phi_p z^p = 0\,, \qquad (2.10)$$

all lie outside the unit circle. If one or more of the roots are unity, the assumption of stationarity is inappropriate. The stationarity assumption does not apply when it comes to testing the random walk hypothesis and testing the first difference hypotheses. This is our subject of discussion in this chapter - unit root tests for stationarity.

The organization of this chapter is as follows. Since estimations of parameters of time series are usually obtained by the method of maximum likelihood, we briefly touch this topic in Section 2.2. Section 2.3 discusses some unit root tests due to Dickey and Fuller (1979) when the process is AR(p). In Section 2.4, we briefly discuss the root test in an MA(1). Section 2.5 discusses one approach due to Phillips and Perron (1988). In Section 2.6, a numerical example is used to illustrate the methods.

2.2 Maximum Likelihood Estimations

Maximum likelihood and other estimators possess properties that can pose problems for estimation when a root of the process is close to unity.

2.2.1 Maximum Likelihood Estimation for the AR(1) Process

From (2.7), the AR(p) process is given by

$$Y_t = \sum_{i=1}^{p} \phi_i Y_{t-i} + \varepsilon_t. \qquad (2.11)$$

where $Y_t = X_t - \mu$. Setting $p = 1$ and $\phi_1 = \rho$, then for $t = 1, 2, \ldots\ldots\ldots, T$, (2.11) becomes

$$Y_t = \rho Y_{t-1} + \varepsilon_t, \qquad \text{with} \quad Y_o = 0. \qquad (2.12)$$

If we assume that ε_t is independdently distributed as $N(0,\sigma^2)$, then the conditional likelihood function L given $Y_o = 0$ is

$$L = \left(2\pi\sigma^2\right)^{-T/2} \exp\left\{\frac{-1}{2\sigma^2}\sum_{t=1}^{T} \varepsilon_t^2\right\}$$

$$\Rightarrow \qquad L = \left(2\pi\sigma^2\right)^{-T/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{t=1}^{T} \left(Y_t - \rho Y_{t-1}\right)^2\right\} \tag{2.13}$$

Taking the logarithm of (2.13) we obtain

$$\ln L = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\left(Y_t - \rho Y_{t-1}\right)^2 \tag{2.14}$$

Maximizing (2.14) with respect to ρ and σ^2, the results are exactly the same as the OLS estimates:

$$\frac{\partial}{\partial\rho}\ln L = \frac{1}{\sigma^2}\sum_{t=1}^{T}[(Y_t - \rho Y_{t-1})Y_{t-1}] = 0, \tag{2.15a}$$

$$\Rightarrow \qquad \hat{\rho} = \frac{\sum_{t=1}^{T} Y_t Y_{t-1}}{\sum_{t=1}^{T} Y_{t-1}^2} \tag{2.15b}$$

which is asymptotically distributed as $N\left(\rho, \frac{1-\rho^2}{T}\right)$. Similarly,

$$\frac{\partial}{\partial\sigma^2}\ln L = -\frac{T}{2\sigma^2} + \frac{1}{2\left(\sigma^2\right)^2}\sum_{t=1}^{T}\left(Y_t - \rho Y_{t-1}\right)^2 = 0, \tag{2.16a}$$

or

$$\Rightarrow \qquad \hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}(Y_t - \hat{\rho}Y_{t-1})^2. \tag{2.16b}$$

2.3 Unit Roots in Autoregressive Processes

A unit root in the polynomial $\Phi(z) = 1 - \phi_1 z - \phi_2 z^2 - \ldots - \phi_p z^p$ suggests that a series should be differenced before fitting the ARMA model.

2.3.1 Testing for a Unit Root in an AR(1) Process with Mean Zero

This section derives the asymptotic distribution for the test statistic for the AR(1) process

$$Y_t = \rho Y_{t-1} + \varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0,\sigma^2).$$

We have shown in (2.15b) that

$$\hat{\rho} = \frac{\sum_{t=1}^{T} Y_t Y_{t-1}}{\sum_{t=1}^{T} Y_{t-1}^2}. \tag{2.15b}$$

The likelihood ratio test of the hypothesis

$$H_0: \rho = 1 \tag{2.17}$$

is

$$\hat{\tau} = \frac{\hat{\rho} - 1}{\hat{S}e(\hat{\rho})}, \tag{2.18}$$

where

$$\hat{S}e(\hat{\rho}) = \sqrt{\frac{\sum_{t=1}^{T} \varepsilon_t^2 \big/ T}{\sum_{t=1}^{T} Y_{t-2}^2}} = \sqrt{\frac{\sum_{t=1}^{T} (Y_t - \hat{\rho} Y_{t-1})^2}{T \sum_{t=1}^{T} Y_{t-2}^2}}. \tag{2.19}$$

Substituting the expression $Y_t = \rho Y_{t-1} + \varepsilon_t$ into (2.15b) yields

$$\hat{\rho} = \frac{\sum_{t=1}^{T} (\rho Y_{t-1} + \varepsilon_t) Y_{t-1}}{\sum_{t=1}^{T} Y_{t-1}^2} = \frac{\rho \sum_{t=1}^{T} Y_{t-1}^2 + \sum_{t=1}^{T} Y_{t-1}\varepsilon_t}{\sum_{t=1}^{T} Y_{t-1}^2} = \rho + \frac{\sum_{t=1}^{T} Y_{t-1}\varepsilon_t}{\sum_{t=1}^{T} Y_{t-1}^2}$$

and hence
$$\hat{\rho} - \rho = \frac{\sum_{t=1}^{T} Y_{t-1}\varepsilon_t}{\sum_{t=1}^{T} Y_{t-1}^2}. \tag{2.20}$$

Under the null hypothesis $H_o\!:\rho = 1$, (2.20) and (2.12) are respectively given by

$$\hat{\rho} - 1 = \frac{\sum_{t=1}^{T} Y_{t-1}\varepsilon_t}{\sum_{t=1}^{T} Y_{t-1}^2}. \tag{2.21}$$

and
$$Y_t = Y_{t-1} + \varepsilon_t = \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \ldots + \varepsilon_1, \quad \text{with } Y_0 = 0. \tag{2.22}$$

The mean $E(Y_t)$ of (2.22) is thus

$$E(Y_t) = E(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \ldots + \varepsilon_1)$$
$$= E(\varepsilon_t) + E(\varepsilon_{t-1}) + E(\varepsilon_{t-2}) + \ldots + E(\varepsilon_1)$$

$$\Rightarrow \quad E(Y_t) = 0. \tag{2.23}$$

The variance $\mathrm{var}(Y_t)$ of (2.23) is

$$\mathrm{var}(Y_t) = \mathrm{var}(\varepsilon_t) + \mathrm{var}(\varepsilon_{t-1}) + \mathrm{var}(\varepsilon_{t-2}) + \ldots + \mathrm{var}(\varepsilon_1)$$
$$= \sigma^2 + \sigma^2 + \sigma^2 + \ldots + \sigma^2 \quad (t \text{ times})$$

and hence
$$\mathrm{var}(Y_t) = t\sigma^2. \tag{2.24}$$

Thus, if writing $\varepsilon_t \sim N(0,\sigma^2)$, we can also write

$$Y_t \sim N(0,\sigma^2 t) \quad \text{and} \quad \left(\frac{Y_t}{\sigma\sqrt{t}}\right) \sim N(0,1), \tag{2.25}$$

and hence
$$Y_{t-1} \sim N\left[0,\sigma^2(t-1)\right]. \tag{2.26}$$

Furthermore,

$$Y_t^2 = (Y_{t-1} + \varepsilon_t)^2 = Y_{t-1}^2 + 2Y_{t-1}\varepsilon_t + \varepsilon_t^2 \quad \Rightarrow \quad Y_{t-1}\varepsilon_t = \tfrac{1}{2}\left\{Y_t^2 - Y_{t-1}^2 - \varepsilon_t^2\right\}$$

$$\Rightarrow \quad \sum_{t=1}^{T} Y_{t-1}\varepsilon_t = \tfrac{1}{2}\sum_{t=1}^{T}\left(Y_t^2 - Y_{t-1}^2 - \varepsilon_t^2\right) = \tfrac{1}{2}\sum_{t=1}^{T}\left(Y_t^2 - Y_{t-1}^2\right) - \tfrac{1}{2}\sum_{t=1}^{n}\varepsilon_t^2$$

$$= \tfrac{1}{2}(Y_T^2 - Y_0^2) - \tfrac{1}{2}\sum \varepsilon_t^2 ,$$

$$\Rightarrow \quad \sum_{t=1}^{T} Y_{t-1}\varepsilon_t = \tfrac{1}{2}Y_T^2 - \tfrac{1}{2}\sum_{t=1}^{T}\varepsilon_t^2 , \tag{2.27}$$

since it is assumed that $Y_0 = 0$. Multiplying both sides of equation (2.27) by $(1/_{\sigma^2 T})$ yields

$$\tfrac{1}{\sigma^2 T}\sum_{t=1}^{T} Y_{t-1}\varepsilon_t = \tfrac{1}{2}\left(\frac{Y_T}{\sigma\sqrt{T}}\right)^2 - \left(\frac{1}{2\sigma^2}\right)\left(\frac{1}{T}\right)\sum_{t=1}^{T}\varepsilon_t^2$$

$$\tfrac{1}{\sigma^2 T}\sum_{t=1}^{T} Y_{t-1}\varepsilon_t = \tfrac{1}{2}\left[\left(\frac{Y_T}{\sigma\sqrt{T}}\right)^2 - \tfrac{1}{\sigma^2}\sum_{t=1}^{T}\tfrac{\varepsilon_t^2}{T}\right] \tag{2.28}$$

From (2.25), we can write

$$\left(\tfrac{Y_T}{\sigma\sqrt{T}}\right) \sim N(0,1), \quad \text{and so its square} \quad \left(\tfrac{Y_T}{\sigma\sqrt{T}}\right)^2 \sim \chi^2(1). \tag{2.29}$$

Also, since $\sum_{t=1}^{T}\varepsilon_t^2$ is the sum of T identically and independently distributed random variables, each with mean σ^2, we have

$$\tfrac{1}{T}\sum_{t=1}^{T}\varepsilon_t^2 \rightarrow \tfrac{1}{T}(T\sigma^2) = \sigma^2. \tag{2.30}$$

Employing the expressions in (2.29) and (2.30), equation (2.28) becomes

$$\left(\tfrac{1}{\sigma^2 T}\right)\sum Y_{t-1}\varepsilon_t \rightarrow \tfrac{1}{2}\left[\chi^2(1) - 1\right]. \tag{2.31}$$

Finally, from equation (2.26),

$$E\left(\sum_{t=1}^{T} Y_{t-1}^2\right) = \sum_{t=1}^{T} E(Y_{t-1}^2) = \sum_{t=1}^{T} \sigma^2 (t-1)$$

$$= \sigma^2 (0 + 1 + 2 + 3 + \ldots + T)$$

$$\Rightarrow \qquad E\left(\sum_{t=1}^{T} Y_{t-1}^2\right) = \sigma^2 \times S_T = \frac{\sigma^2}{2} T(T-1), \qquad (2.32)$$

where $S_T = \frac{T}{2}\left[a + (T-1)d\right] = \frac{T(T-1)}{2}$ is the arithmetic series with first term $a = 0$ and common difference $d = 1$. From (2.49) we have

$$\tfrac{1}{T^2} E\left(\sum_{t=1}^{T} Y_{t-1}^2\right) = \tfrac{\sigma^2}{2} \cdot \tfrac{1}{T^2} (T^2 - T) = \tfrac{\sigma^2}{2} (1 - \tfrac{1}{T})$$

$$\Rightarrow \qquad \tfrac{1}{T^2} \sum_{t=1}^{T} Y_{t-1}^2 \longrightarrow \tfrac{\sigma^2}{2} (1 - \tfrac{1}{T}). \qquad (2.33)$$

Using (2.31) and (2.33) implies

$$T(\hat{\rho} - 1) = \frac{(1/T^2) \sum_{t=1}^{T} Y_{t-1} \varepsilon_t}{(1/T^2) \sum_{t=1}^{T} Y_{t-1}^2} \to \frac{\frac{1}{2}\left[\chi^2(1) - 1\right]}{\frac{1}{2}\sigma^2 (1 - 1/T)} = \frac{\left[\chi^2(1) - 1\right]}{\sigma^2 (1 - 1/T)}. \qquad (2.34)$$

Equation (2.34) shows that the asymptotic distribution of $(\hat{\rho} - 1)$ is neither normally distributed nor has the standard t-distribution. We therefore conclude that even if the model is in fact true, the test statistic in testing for significance does not have even asymptotically the t-distribution or normal distribution on which we shall base our conclusions. By contrast, in the standard case the t-ratio behaves asymptotically like a unit normal. The difference between the asymptotic behaviour of the two cases makes it evidently clear that it will be unsatisfactory to rely for our inference on the tabulations given for the standard case. This calls for special t-ratio based on the asymptotic behaviour given in (2.34). Based on the asymptotic behaviour of $(\hat{\rho} - 1)$ given in (2.34), Dickey and Fuller (1979) employed the Monte Carlo method to simulate values for some finite-sample tests. Tests based on these tabulations are referred to as Dickey-Fuller tests.

2.3.2 Testing for a Unit Root in the AR(1) Process with a Constant Term

The AR(1) process is given by

$$X_t = C + \phi X_{t-1} + \varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0,\sigma^2). \tag{2.35}$$

Subtracting X_{t-1} from both sides of (2.35) and setting $\phi_1 = \rho$, we obtain

$$X_t - X_{t-1} = C + (\rho - 1)X_{t-1} + \varepsilon_t \quad \Rightarrow \quad Z_t = C + (\rho - 1)X_{t-1} + \varepsilon_t, \tag{2.36}$$

where $Z_t = X_t - X_{t-1}$, and $C = \mu(1-\rho)$. If $\rho = 1$, (2.36) describes a random walk, which implies non-stationarity. Thus, testing the hypothesis that $\rho = 1$ is of great importance because it corresponds to the hypothesis that it is appropriate to transform the original series X_t by differencing. Additionally, (2.36) suggests the OLS regression of Z_t on (1) and X_{t-1}. The OLS model has 2 parameters, namely C and $(\rho - 1)$, and hence, the estimated standard error of $(\hat{\rho} - 1)$ in the OLS estimation is

$$\hat{S}e(\hat{\rho} - 1) = \sqrt{\frac{\sum_{t=2}^{T} \varepsilon_t^2 \big/ (T-2)}{\sum_{t=2}^{T} (X_{t-1} - \mu)^2}} = \sqrt{\frac{\sum_{t=2}^{T} \left[Z_t - \hat{C} - (\hat{\rho} - 1)X_{t-1}\right]^2}{(T-2)\left(\sum_{t=2}^{T} (X_{t-1} - \mu)^2\right)}}, \tag{2.37}$$

The likelihood ratio test of the hypothesis

$$H_o: \rho = 1, \tag{2.38}$$

with a set of tables of the percentiles [see Appendix F(a)] for the limiting distribution as $T \to \infty$ derived by Dickey and Fuller (1979) is given by

$$\hat{\tau} = \frac{\hat{\rho} - 1}{\hat{S}e(\hat{\rho} - 1)}. \tag{2.39}$$

2.3.3 Testing for a Unit Root in the AR(p) Process

Extending the results for the AR(1) process with $\rho = 1$ to the general AR(p) process given by

$$X_t = C + \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t$$

or

$$X_t = \mu\left(1 - \sum_{i=1}^{p} \phi_i\right) + \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t. \qquad (2.40)$$

Let $\rho = \sum_{i=1}^{p} \phi_i$ and , $\alpha_j = -\sum_{i=j+1}^{p} \phi_i$, $j = 1,2,\ldots\ldots,p-1$. Then (2.40) becomes

$$X_t = \mu(1-\rho) + \rho X_{t-1} + \sum_{j=1}^{p-1} \alpha_j\left(X_{t-j} - X_{t-j-1}\right) + \varepsilon_t, \text{ with } X_o = 0. \qquad (2.41)^{2.2}$$

Employing the notation $Z_{t-j} = X_{t-j} - X_{t-j-1}$, (2.41) becomes

$$X_t - X_{t-1} = \mu(1-\rho) + (\rho - 1)X_{t-1} + \sum_{j=1}^{p-1} \alpha_j\left(X_{t-j} - X_{t-j-1}\right) + \varepsilon_t$$

$$Z_t = \delta + (\rho - 1)X_{t-1} + \sum_{j=1}^{p-1} \alpha_j Z_{t-j} + \varepsilon_t, \qquad (2.42)$$

where $\delta = \mu(1-\rho) = \mu\left(1 - \sum_{i=1}^{p} \phi_i\right)$. By a similar argument as in the case of AR(1), (2.42) suggests the OLS regression of Z_t on (1), $X_{t-1}, Z_{t-1}, Z_{t-2}, \ldots, Z_{t-p+1}$ The Augmented Dickey-Fuller (1981) $\hat{\tau}$ statistic for testing the hypothesis that $\rho = 1$ applies the same way as in the case of an AR(1) process.

Example 2.1: Consider the AR(2) process given by

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0, \sigma^2). \qquad (2.43)$$

([2.2] See Chapter Appendix 2.1 for full derivation)

If X_t is stationary, the roots, $z = \{z_1, z_2\}$, of the characteristic equation

$$1 - \phi_1 z - \phi_2 z^2 = 0, \qquad (2.44)$$

must all lie outside the unit circle. This means that the parameters ϕ_1 and ϕ_2 must lie in the triangular region

$$\phi_2 + \phi_1 < 1, \quad \phi_2 - \phi_1 < 1, \text{ and } -1 < \phi_2 < 1. \qquad (2.45)$$

In this case $\rho = \sum_{i=1}^{2} \phi_i = \phi_1 + \phi_2$, and $\alpha_1 = -\sum_{i=1+1}^{2} \phi_i = -\sum_{i=2}^{2} \phi_i = -\phi_2$. Equation (2.45) thus becomes

$$Z_t = \delta + [(\phi_1 + \phi_2) - 1]X_{t-1} + \alpha_1(X_{t-1} - X_{t-2}) + \varepsilon_t$$

$$\Rightarrow \qquad Z_t = \delta + [(\phi_1 + \phi_2) - 1]X_{t-1} - \phi_2(X_{t-1} - X_{t-2}) + \varepsilon_t. \qquad (2.46)$$

From the fact that $\rho = \phi_1 + \phi_2 < 1$ implies testing the hypothesis that $\rho = 1$ is equivalent to testing for the presence of a unit root in the AR(2) process. Equation (2.46) suggests regressing of $Z_t = X_t - X_{t-1}$ on a constant, X_{t-1} and Z_{t-1}. The augmented Dickey-Fuller $\hat{\tau}$ statistic for testing the hypothesis that $\rho = 1$ applies the same way as in the case of an AR(1) process.

2.3.4 Testing for a Unit Root in the AR(1) Process With a Linear Trend

If X_t is an AR(1) process with a linear trend, then by (2.9a) the process becomes

$$X_t - (\beta_o + \beta_1 t) = \phi_1[X_{t-1} - (\beta_o + \beta_1(t-1))] + \varepsilon_t,$$

$$\Rightarrow \qquad X_t = (\beta_o - \phi_1\beta_o + \phi_1\beta_1) + (\beta_1 - \phi_1\beta_1)t + \phi_1 X_{t-1} + \varepsilon_t, \qquad (2.47)$$

where $X_o = 0$. Subtracting X_{t-1} from both sides of (2.47) yields

$$Z_t = \upsilon + \omega t + (\rho - 1)X_{t-1} + \varepsilon_t, \qquad (2.48)$$

where

$Z_t = X_t - X_{t-1}$, $\rho = \phi_1$, $\upsilon = (\beta_o - \phi_1\beta_o + \phi_1\beta_1)$, and $\omega = (1-\phi_1)\beta_1 = (1-\rho)\beta_1$.

The expression in (2.41) suggests the OLS regression of Z_t on (1), *t*, and X_{t-1}. Here again, the Dickey-Fuller $\hat{\tau}$ statistic applies the same way as in the AR(1) process when testing the null hypothesis that

$$H_o : \rho = 1.$$

The only difference is that the percentiles for the limiting distribution takes into account the presence of time trend. The percentile values are given in Appendix F(b). Practical illustrations of these test procedures are given in Section 6.2 of Chapter 6.

2.4 Unit Root in the ARIMA (*p*, 1, *q*) Process

Let $Y_t = (1-B)^d X_t$, then an extension of the unit root approach to the ARIMA $(p,1,q)$ process due to Said (1982) employs the following relations

$$X_t = \rho X_{t-1} + Y_t \qquad (2.49a)$$

with

$$Y_t = \sum_{i=1}^{p} \phi_i Y_{t-i} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}, \qquad \{\varepsilon_t\} \sim i.i.d\, N(0,\sigma^2) \qquad (2.49b)$$

where $X_o = 0$. A test for a unit root in the ARIMA(1, 1, 1) is discussed in sub-section 2.4.1 below.

2.4.1 Testing for a Unit Root in the ARIMA(1, 1, 1) Process

The ARIMA (1, 1, 1) process is defined as

$$X_t = \rho X_{t-1} + Y_t \qquad (2.50a)$$

$$Y_t = \phi Y_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1}. \qquad (2.50b)$$

Writing ε_t as

$$\varepsilon_t = \sum_{j=0}^{\infty} \theta^j (Y_{t-j} - \phi Y_{t-j-1}),$$

$$\Rightarrow \qquad \theta\varepsilon_{t-1} = \sum_{j=0}^{\infty} \theta^{j+1}(Y_{t-j-1} - \phi Y_{t-j-2})$$

or $$\theta\varepsilon_{t-1} = \theta(Y_{t-1} - \phi Y_{t-2}) + \theta^2(Y_{t-2} - \phi Y_{t-3}) + \theta^3(Y_{t-3} - \phi Y_{t-4}) + \ldots \qquad (2.51)$$

Substituting (2.51) into (2.50b) yields

$$Y_t = \phi Y_{t-1} + \varepsilon_t - \theta(Y_{t-1} - \phi Y_{t-2}) - \theta^2(Y_{t-2} - \phi Y_{t-3}) - \theta^3(Y_{t-3} - \phi Y_{t-4}) - \ldots.$$

or

$$Y_t = (\phi - \theta)(Y_{t-1} + \theta Y_{t-2} + \theta^2 Y_{t-3} + \ldots) + \varepsilon_t. \qquad (2.52)$$

Again, substituting (2.52) into (2.50a) gives

$$X_t = \rho X_{t-1} + (\phi - \theta)(Y_{t-1} + \theta Y_{t-2} + \theta^2 Y_{t-3} + \ldots\ldots) + \varepsilon_t,$$

or $$X_t = \rho X_{t-1} + \sum_{j=1}^{\infty} \omega_j Y_{t-j} + \varepsilon_t, \qquad (2.53)$$

where ω_j is a function of ϕ and θ. Subtracting X_{t-1} from both sides of (2.53), we obtain

$$X_t - X_{t-1} = (\rho - 1)X_{t-1} + \sum_{j=1}^{\infty} \omega_j Y_{t-j} + \varepsilon_t. \qquad (2.54)$$

Testing the hypothesis that $\rho = 1$, we see from (2.50a) that

$$Y_t = X_t - X_{t-1} = Z_t. \qquad (2.55)$$

Hence, (2.54) can be re-expressed as

$$Z_t = (\rho - 1)X_{t-1} + \sum_{j=1}^{\infty} \omega_j Z_{t-j} + \varepsilon_t, \qquad (2.56)$$

suggesting the regression of Z_t on $X_{t-1}, Z_{t-1}, Z_{t-2}, \ldots\ldots\ldots, Z_{t-b}$, where b is an integer chosen as a function of n with the assumption that $b / T^3 \to 0$ and that there exists $c > 0,\ r > 0$ such that $bc > \sqrt[r]{T}$. Including a constant in the model, (2.56) becomes

$$Z_t = C + (\rho - 1)X_{t-1} + \sum_{j=1}^{b} \omega_j Z_{t-j} + \varepsilon_t . \qquad (2.57)$$

In the case of (2.57) the motivation is to regress Z_t on (1), $X_{t-1}, Z_{t-1}, Z_{t-2}, \ldots\ldots, Z_{t-b}$, where b is an integer chosen the same way as described above. In each case, the usual Dickey-Fuller $\hat{\tau}$ statistic applies the same way as in the AR(1) process when testing the null hypothesis that

$$H_o : \rho = 1 .$$

2.5 Summary

At the informal level, stationarity of a time series is tested by its correlogram, which is a graph of autocorrelation at various lags. For stationary time series, the correlogram tapers off quickly, whereas for non-stationary time series it dies off gradually. Non-stationarity established this way may be misleading in that it might not be able to establish whether it is due to some deterministic trend or a unit root. In this chapter, we have considered a formal check for stationarity using the unit root tests which is basically a concept of regression. This concept is found to be an important theoretical underpinning of stationarity or otherwise of a time series.

CHAPTER APPENDIX 2.1

For the AR(p)

$$\Phi(B)X_t = C + \varepsilon_t , \ \{\varepsilon_t\} \sim \text{WN}(0, \sigma^2) \qquad \text{A2.1}$$

where $\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$, $C = \mu(1 - \phi_1 - \phi_2 - \ldots - \phi_p)$, and $\mu = \frac{1}{T}\sum_{t=1}^{T} X_t$,

let $$\rho = \sum_{i=1}^{p} \phi_i \quad \text{and} \quad \alpha_j = -\sum_{i=j+1}^{p} \phi_i , \quad j = 1,2,\ldots,p\text{-}1.$$

Then

$$(1 - \rho B) - (\alpha_1 B + \alpha_2 B^2 + ..+ \alpha_{p-1} B^{p-1})(1 - B) = 1 - \rho B - (\alpha_1 B + \alpha_2 B^2 + ..+ \alpha_{p-1} B^{p-1} - \alpha_1 B^2 - \alpha_2 B^3 - ..- \alpha_{p-1} B^p)$$

$$(1-\rho B)-\sum_{j=1}^{p-1}\alpha_j B^j(1-B) = 1-(\rho+\alpha_1)B-(\alpha_2-\alpha_1)B^2-(\alpha_3-\alpha_2)B^3-..-(\alpha_{p-1}-\alpha_{p-2})B^{p-1}-(-\alpha_{p-1})B^p.$$

Using the following expressions

$$\rho+\alpha_1 = \sum_{i=1}^{p}\phi_i - \sum_{i=2}^{p}\phi_i = \phi_1 + \sum_{i=2}^{p}\phi_i - \sum_{i=2}^{p}\phi_i = \phi_1$$

$$\alpha_2-\alpha_1 = -\sum_{i=3}^{p}\phi_i + \sum_{i=2}^{p}\phi_i = -\sum_{i=3}^{p}\phi_i + \phi_2 + \sum_{i=3}^{p}\phi_2 = \phi_2$$

$$\alpha_3-\alpha_2 = -\sum_{i=4}^{p}\phi_i + \sum_{i=3}^{p}\phi_i = -\sum_{i=4}^{p}\phi_i + \phi_3 + \sum_{i=4}^{p}\phi_i = \phi_3$$

$$\vdots$$

$$\alpha_{p-1}-\alpha_{p-2} = -\sum_{i=p}^{p}\phi_i + \sum_{i=p-1}^{p}\phi_i = -\sum_{i=p}^{p}\phi_i + \phi_{p-1} + \sum_{p}^{p}\phi_p = -\phi_p + \phi_{p-1} + \phi_p = \phi_{p-1}.$$

the last equation becomes

$$(1-\rho B)-\sum_{j=1}^{p-1}\alpha_j B^j(1-B) = 1-\phi_1 B-\phi_2 B^2-....-\phi_{p-1}B^{p-1}-\phi_p B^p, \qquad \text{A2.2}$$

where we have used the fact that $-\alpha_{p-1} = \phi_p$. Equation A2.2 thus becomes

$$(1-\rho B)-\sum_{j=1}^{p-1}\alpha_j B^j(1-B) = \Phi(B). \qquad \text{A2.3}$$

Substituting A2.3 into A2.1 yields

$$\left[(1-\rho B)-\sum_{j=1}^{p-1}\alpha_j B^j(1-B)\right]X_t = C+\varepsilon_t$$

$$\Rightarrow \quad X_t - \rho BX_t - \sum_{j=1}^{p} \alpha_j \left(B^j - B^{j+1}\right) X_t = C + \varepsilon_t$$

$$\Rightarrow \quad X_t - \rho BX_t - \sum_{j=1}^{p-1} \alpha_j \left(B^j X_t - B^{j+1} X_t\right) = C + \varepsilon_t$$

$$\Rightarrow \quad X_t - \rho X_{t-1} = C + \sum_{j=1}^{p-1} \alpha_j \left(X_{t-j} - X_{t-j-1}\right) + \varepsilon_t$$

or $$X_t = C + \rho X_{t-1} + \sum_{j=1}^{p-1} \alpha_j \left(X_{t-j} - X_{t-j-1}\right) + \varepsilon_t \, . \qquad \text{A2.4}$$

Subtracting X_{t-1} from both sides of A2.4 simplifies to give

$$X_t - X_{t-1} = C + (\rho - 1) X_{t-1} + \sum_{j=1}^{p-1} \alpha_j \left(X_{t-j} - X_{t-j-1}\right) + \varepsilon_t \qquad \text{A2.5}$$

or $$Z_t = C + (\rho - 1) X_{t-1} + \sum_{j=1}^{p-1} \alpha_j Z_{t-j} + \varepsilon_t \quad , \qquad \text{A2.6}$$

where $Z_{t-j} = X_{t-j} - X_{t-j-1}$, j = 0,1,2, ..., p-1.

CHAPTER 3

REVIEW OF SOME OTHER UNIT ROOT TEST PROCEDURES

3.1 Introduction

In Chapter 2, our discussions were basically testing for non-stationarity in a given non-seasonal time series by testing for the presence of unit roots using the Dickey-Fuller and the Augmented Dickey-Fuller test procedure. In this chapter, we present a review of other unit root test procedures given a non-seasonal time series.

The layout of this chapter is as follows. The Phillips-Perron unit root test is discussed in Section 3.2. In Section 3.3, we discuss the frequency-domain test for stationarity, a test procedure based on periodogram ordinates. Section 3.4 considers the Reverse Dickey-Fuller Unit Root (RDFUR) test due to Leybourne (1995) while in Section 3.5, we consider the Lagrange Multiplier (LM) test for stationarity due to Schmidt and Phillips (1992). Section 3.6 presents a summary of results.

3.2 The Phillips-Perron Unit Root Test

In this section we shall give a summary of an alternative unit root test due to Phillips and Perron (1988). The test procedure takes into account the possibility of autocorrelation that might be present in the data when the series does not satisfy the AR(1) process given by

$$X_t = C + \rho X_{t-1} + \varepsilon_t . \qquad (3.1)$$

The strategy is basically the same as that of the Dickey-Fuller test except that the t-statistic is amended to incorporate any bias due to the autocorrelation in the error term of the Dickey-Fuller regression model

$$Z_t = C + (\rho - 1)X_{t-1} + \varepsilon_t , \qquad (3.2)$$

where $Z_t = X_t - X_{t-1}$. The bias results when the variance of the true population, σ_*^2, differs from the variance of the residuals, σ^2, in the regression model given in (3.2).

Consistent estimators of σ_*^2 and σ^2 are respectively given by

$$\hat{\sigma}^2 = \tfrac{1}{T}\sum_{t=1}^{T} \varepsilon_t^2 \quad , \tag{3.3a}$$

and $$\hat{\sigma}_*^2 = \tfrac{1}{T}\sum_{t=1}^{T} \varepsilon_t^2 + \tfrac{2}{T}\sum_{t=1}^{l}\sum_{t=k+1}^{T} \varepsilon_t\varepsilon_{t-k} = \hat{\sigma}^2 + \tfrac{2}{T}\sum_{t=1}^{l}\sum_{t=k+1}^{T} \varepsilon_t\varepsilon_{t-k}, \tag{3.3b}$$

where $k = 1,2,..,N$, and l is the lag truncation parameter that ensures that the autocorrelation of the residuals are completely captured using the first N autocovariances that are deemed relevant. The Newey-West estimators of σ^2 and $\hat{\sigma}_*^2$ are respectively given by

$$\hat{\gamma}_o = \hat{\sigma}^2 = \tfrac{1}{T}\sum_{t=1}^{T} \varepsilon_t^2 \qquad \text{and} \qquad \hat{\gamma}_{o(*)} = \hat{\gamma}_o + 2\sum_{k=1}^{N}\left[1 - \tfrac{k}{(N+1)}\right]\hat{\gamma}_k , \tag{3.4}$$

where $\hat{\gamma}_k = \tfrac{1}{T}\sum_{t=k+1}^{T} \varepsilon_t\varepsilon_{t-k}$. Under the null hypothesis

$$H_o : \rho = 1,$$

where $\{X_t\}$ is not necessarily an AR(1) process, the Phillip-Perron test statistic τ_{pp} is given by

$$\tau_{pp} = \left(\frac{\hat{\sigma}_*^2}{\hat{\sigma}^2}\right)\tau - \tfrac{T}{2}\left\{\frac{(\hat{\sigma}_*^2 - \hat{\sigma}^2)}{\sqrt{\hat{\sigma}_*^2 . \sum_{t=2}^{T} (X_{t-1} - \overline{X}_{-1})^2}}\right\}, \tag{3.5}$$

where $\overline{X}_{-1} = \tfrac{1}{T}\sum_{t=2}^{T} X_t$, and τ is the Dickey-Fuller test statistic under the null hypothesis. Critical values for the test statistic are the same as those used in the Dickey-Fuller test, (see Appendix F(b)).

When there is no autocorrelation,

$$\hat{\sigma}_*^2 = \hat{\sigma}^2, \tag{3.6}$$

and hence the Phillips-Perron test statistic given in (3.5) reduces to the Dickey-Fuller test since

$$\tau_{pp} = \tau. \tag{3.7}$$

Practical illustrations using Phillips-Perron test procedures are found in Chapter 6, Section 6.3.

3.3 The Periodogram

In this section, we quickly give a background to the term *periodogram* which is regarded as a foundation for the frequency-domain test to be discussed in Section 3.3.1. Given the univariate time series $\{X_t : t = 1,2,\ldots,T\}$ satisfying the AR(1) process

$$(X_t - \mu) = \rho(X_{t-1} - \mu) + \varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0,\sigma^2). \tag{3.8}$$

The periodogram ordinates of X_t is defined as

$$I(w_k) = \tfrac{T}{2}\left(a_k^2 + b_k^2\right), \tag{3.9}$$

where

$$a_k = \tfrac{2}{T}\sum_{t=1}^{T} X_t.\cos(w_k t) \tag{3.10a}$$

$$b_k = \tfrac{2}{T}\sum_{t=1}^{T} X_t.\sin(w_k t), \tag{3.10b}$$

and

$$w_k = \frac{2\pi k}{T}, \quad k = 0,1,2,\ldots,[T/2]. \tag{3.10c}$$

For $k > 0$,

$$\sum_{t=1}^{T}\cos(w_k t) = \sum_{t=1}^{T}\sin(w_k t) = 0, \tag{3.11}$$

and so a_k and b_k have mean 0 once X_t has a constant expected value. Furthermore, a_k and b_k are normally distributed if X_t is normally distributed. Normalizing a_k and b_k yield

$$A_k = \sqrt{\frac{2[1-\cos(w_k)]}{\sigma^2}}\left(\frac{2}{T}\right).\sum_{t=1}^{T} X_t.\cos(w_k t) \ , \tag{3.12a}$$

$$B_k = \sqrt{\frac{2[1-\cos(w_k)]}{\sigma^2}}\left(\frac{2}{T}\right).\sum_{t=1}^{T} X_t.\sin(w_k t) \, . \tag{3.12b}$$

respectively.

3.3.1 Frequency-Domain Test for Stationarity in Series with No Trend

If $\rho = 1$, the AR(1) process with no trend given in (3.8) reduces to a random walk process

$$X_t = X_{t-1} + \varepsilon_t \, . \tag{3.13}$$

The random walk process given in (3.13) has no autocovariance function and hence the spectrum of the series does not exist. Estimation of the spectrum of a series is the usual purpose of computing the periodogram for any sequence of numbers. A unit root process may have certain features in its periodogram making it a useful diagnostic tool.

In order to obtain estimates of the spectrum $f(w_k)$ of a time series at the frequencies w_k, the periodogram ordinates (3.9) is usually divided by 4π. If $\rho < 1$, (3.8) is stationary and its spectrum is given by

$$f(w_k) = \frac{\sigma^2}{2\pi[1+\rho^2 - 2\rho\cos(w_k)]} \ ,$$

or $$2\pi f(w_k) = \frac{\sigma^2}{1+\rho^2 - 2\rho\cos(w_k)}. \tag{3.14}$$

If $\rho = 1$, (3.14) becomes

$$2\pi f(w_k) = \frac{\sigma^2}{2[1-\cos(w_k)]} \quad \text{or} \quad \frac{2[1-\cos(w_k)]}{\sigma^2} = \frac{1}{2\pi f(w_k)}. \tag{3.15}$$

Akdi and Dickey (1998) showed that when $\rho = 1$, the application of Slutsky's theorem (Akdi and Dickey, 1998) to the normalised periodogram yields

$$A_k^2 + B_k^2 = \frac{2[1-\cos(w_k)]}{\sigma^2}\left(\frac{T}{2}\right)(a_k^2 + b_k^2) \longrightarrow z_1^2 + 3z_2^2, \tag{3.16}$$

where z_1 and z_2 are independent standard normal random variables. Substituting (3.9) and 3.15) in (3.16) yields the distributional result

$$\frac{I(w_k)}{2\pi f(w_k)} \longrightarrow z_1^2 + 3z_2^2. \tag{3.17}$$

For the AR(1) process (3.8), we state the proposed test statistic (from (3.17)) as

$$\tau_k^* = \frac{I_X(w_k)}{2\pi f(w_k)}. \tag{3.18}$$

For desirable power results, Akdi and Dickey (1998) set $k = 1$. Hence, we can rewrite the test statistic (3.18) as

$$\tau_1^* = \frac{I_X(w_1)}{2\pi f(w_1)}, \tag{3.19}$$

where $I_X(w_1)$ refers to the first periodogram ordinate of the undifferenced series. Thus, τ_1^* is distributed as $z_1^2 + 3z_2^2$ under the null hypothesis

$$H_0: \rho = 1. \tag{3.20}$$

The percentiles of $z_1^2 + 3z_2^2$ calculated by the mixture of chi-squares result from an approach due to Johnson, Kotz, and Balakrishnan (1994) are reported in Appendix F(c).

For higher order models, we define quantities in (3.19) based on the following representation

$$X_t - \mu = \rho(X_{t-1} - \mu) + W_t, \tag{3.21}$$

where W_t is an ARMA(p,q) process given by

$$W_t = \alpha_1 W_{t-1} + \alpha_2 W_{t-2} + \ldots + \alpha_p W_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q}, \tag{3.22}$$

and $\{\varepsilon_t\}$~WN(0, σ^2). To test the null hypothesis $H_0: \rho = 1$, Akdi and Dickey (1998) proposed the test statistic

$$\tau_1^{**} = \frac{I_X(w_1)}{2\pi f(w_1).\phi^2}, \tag{3.23}$$

where

$$\phi = \frac{1 - \theta_1 - \theta_2 - \ldots - \theta_q}{1 - \alpha_1 - \alpha_2 - \ldots - \alpha_p}. \tag{3.24}$$

The test statistic in (3.23) has the same distributional result reported in (3.17) and hence the same percentile values in Appendix F(c) apply in testing for a unit root. Evans and Dickey (1998) showed that

$$2\pi f(w_1) = \left(\frac{\sigma T}{2\pi}\right)^2, \tag{3.25}$$

and hence (3.23) becomes

$$\tau_1^{**} = \frac{4\pi^2}{\sigma^2\phi^2 T^2} I_X(w_1). \tag{3.26}$$

For $\rho = 1$, Evans and Dickey (1998) defined an estimate of $\sigma^2\phi^2$ as

$$\widehat{\sigma^2\phi^2} = \frac{1}{[\sqrt{T}]}\sum_{k=1}^{[\sqrt{T}]} \tfrac{1}{2} I_z(w_k), \tag{3.27}$$

where $I_z(w_k)$ is the k-th periodogram ordinate of the differenced series $Z_t = X_t - X_{t-1}$, and $w_k = \frac{2\pi k}{T-1}$. Applications of this approach are presented in Section 6.4 of Chapter 6 using real data sets.

3.3.2 Frequency-Domain Test for Stationarity in Series with A Trend

For models with time trends, the underlying model is an adjustment to the model given in (3.21):

$$X_t - (\beta_0 + \beta_1 t) = \rho\{X_{t-1} - [\beta_0 + \beta_1(t-1)]\} + W_t, \tag{3.28}$$

where W_t is as defined in (3.22). Since a time trend is neutralized in the first differenced series so that only a non-zero mean remains and periodogram ordinates are invariant to non-zero means, the addition of a time trend does not affect the estimation method based on the periodogram of the differenced series. Thus, an estimate of the quantity $\sigma^2\phi^2$ is obtained in the same manner as described in Section 3.3.1. Consequently, the critical values reported in Appendix F(c) also apply.

3.4 The RDFUR Test for Non-Stationarity

The Reverse Dickey-Fuller Unit Root (RDFUR) test, as the name goes is a unit root test procedure similar to the usual Dickey-Fuller (also termed the Forward Dickey-Fuller Unit Root - FDFUR) test. In this section, we review this test procedure. Our motivation for this method is that it can serve as a test for confirming a conclusion drawn as to whether a series is stationary or not. As it often the case, the autoregressive order p may not be the same when applying these the usual ADF test and the RDFUR test, but we become confidently sure when both approaches give the same conclusion.

For the AR(1) process

$$Z_t = C + (\rho - 1)X_{t-1} + \varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0, \sigma^2), \qquad (3.29)$$

where $Z_t = X_t - X_{t-1}$. The Dickey-Fuller unit root test statistic is given by

$$\hat{\tau} = \frac{\hat{\rho} - 1}{\hat{Se}(\hat{\rho} - 1)}, \qquad (3.30)$$

where $\hat{\rho}$ is the OLS regression estimate of ρ obtained by regressing Z_t on a constant and X_{t-1}. In the case of the RDFUR test, the same Dickey-Fuller test is applied to the *reverse* of Z_t. That is, if we let

$$X_0^* = X_{T+1}, X_1^* = X_T, X_2^* = X_{T-1}, X_3^* = X_{T-2}, \ldots, X_T^* = X_1, X_{T+1}^* = X_0, \qquad (3.31)$$

then the Dickey-Fuller test for the reverse series, $\hat{\tau}_*$, can be viewed as a t-test of the null hypothesis

$$H_o: \rho^* = 1, \qquad (3.32)$$

in the model

$$Z_t^* = C^* + (\rho^* - 1)X_{t-1}^* + \varepsilon_t^*, \qquad \{\varepsilon_t^*\} \sim WN(0, \sigma_*^2), \qquad (3.33)$$

where $Z_t^* = X_t^* - X_{t-1}^*$.

The corresponding test statistic for the reverse series is

$$\hat{\tau}_* = \frac{\hat{\rho}^* - 1}{\hat{S}e(\hat{\rho}^* - 1)}, \tag{3.34}$$

where $\hat{\rho}^*$ is the OLS estimate of ρ^* in (3.33). Equation (3.33) suggests the regression of Z_t^* on X_{t-1}^*. Under the null hypothesis $H_0\colon \rho^* = 1$, Leybourne (1995) showed that the reverse test statistic $\hat{\tau}_*$ has the same limiting distribution as the Dickey-Fuller test statistic, $\hat{\tau}$ (see Appendix F(a)). If a time trend is included in the model, (3.33) becomes

$$Z_t^* = (\alpha_o^* + \alpha_1^* t) + \rho^* X_{t-1}^* + \varepsilon_t^*, \qquad \{\varepsilon_t^*\} \sim WN(0, \sigma_*^2). \tag{3.35}$$

Again, (3.35) has the same structure as in the case of the usual process

$$Z_t = (\alpha_o + \alpha_1 t) + \rho X_{t-1} + \varepsilon_t. \tag{3.36}$$

Under the stationary alterative hypothesis, the limiting distributions of the test statistics for (3.35) and (3.36) are the same and hence critical values reported in Appendix F(b) apply. For the AR(p) process, the RDFUR test is based on the regression model

$$Z_t^* = \delta^* + (\rho^* - 1)X_{t-1}^* + \sum_{j=1}^{p-1} \alpha_j^* Z_{t-j}^* + \varepsilon_t^*, \tag{3.37}$$

where $Z_{t-j}^* = X_{t-j}^* - X_{t-j-1}^*$ for $j = 0,1,2,3,\ldots,p$. Equation (3.36) suggests regressing Z_t^* on $X_{t-1}^*, Z_{t-1}^*, Z_{t-2}^*, \ldots, Z_{t-p+1}^*$. Testing the null hypothesis of a unit root is done in a manner similar to the case of the AR(1) process. Within the limitation of this dissertation, we show that the application of the RDFUR test although employs an order p^* that may or may not be the same as the order p used in the usual Dickey-Fuller case, the same conclusion could be drawn regarding the presence of unit root when the RDFUR test is used. Practical illustrations of this methodology are reported in Chapter 6, Section 6.5.

3.5 Lagrange Multiplier (LM) Principle Test for Stationarity

This section considers testing for stationarity given the time series $\{X_t : t = 1,2,3,\ldots,T\}$ based on the LM (score) statistic suggested by Schmidt and Phillips (1992). The test is used to test the difference stationary model

$$Z_t = C + (\rho - 1)X_{t-1} + \varepsilon_t , \quad \text{with } |\rho| = 1, \tag{3.38}$$

against the trend stationary model

$$Z_t = (\beta_0 + \beta_1 t) + (\rho - 1)X_{t-1} + \varepsilon_t , \quad \text{with } |\rho| < 1. \tag{3.39}$$

In (3.38) and (3.39), we assume again that $\{\varepsilon_t\} \sim WN(0,\sigma^2)$. As shown by Schmidt and Phillips (1992), the score (or Lagrange Multiplier) principle gives rise to the following score test statistic

$$\tau_{sp} = \frac{\sum_{t=2}^{T} V_t \widetilde{S}_{t-1}}{\sum_{t=2}^{T} \widetilde{S}_{t-1}^2}, \tag{3.40}$$

where $Z_t = X_t - X_{t-1}$, $\qquad \widetilde{\beta} = \text{mean of } Z_t = \overline{Z} = \frac{1}{T-1}\sum_{t=2}^{T} Z_t = \frac{X_T - X_1}{T-1}$,

$$V_t = Z_t - \overline{Z} = Z_t - \widetilde{\beta} ,$$

$$\widetilde{S}_t = \sum_{i=2}^{t} V_i = \sum_{i=2}^{t} \left(Z_i - \overline{Z}\right) = X_t - X_1 - (t-1)\widetilde{\beta} ,$$

$$= X_t - X_1 - (t-1)\overline{Z} .$$

Equation (3.39) is a regression of Z_t on intercept, time trend t, and X_{t-1}, or equivalently a regression of $Z_t = X_t - X_{t-1}$ on the same variables. Schmidt and Phillips (1992) showed that the term $\sum V_t \widetilde{S}_{t-1}$ in (3.39) is the estimated regression coefficient of $\widetilde{S}_{t-1}$ in the regression

$$Z_t = \text{intercept} + (\rho - 1)\widetilde{S}_{t-1} + \text{error}, \qquad \text{for } t = 1,2,\ldots,T, \tag{3.41}$$

where $\widetilde{S}_{t-1}$ is the residual from an OLS regression of Z_t on intercept and time trend t. Under the null hypothesis

$$H_0 : \rho = 1, \tag{3.42}$$

the score test statistic (3.40) have non-standard distributions. The finite sample distributions of the score statistic is complicated. Under the null hypothesis $H_0 : \rho = 1$, Schmidt and Phillips (1992) obtained the following asymptotic result:

$$\tau_{sp} \longrightarrow -\left(2T\int_0^1 \overline{U}(r)^2 dr\right)^{-1}, \tag{3.43}$$

where $\overline{U}(r)$ is the standard Brownian motion. A table of critical values for the score statistic using a Monte Carlo simulation are reported in Appendix F(d), (Schmidt and Phillips, 1992).

3.6 Summary

In this chapter, we have reviewed some other tests for stationarity. With the advent of personal computers, we see that little effort is indeed required to write programs to handle them. Applications of these methodologies are given in Chapter 7.

CHAPTER 4

ESTIMATING THE DEGREE OF DIFFERENCING

4.1 Introduction

Differencing plays a major role in modelling and forecasting time series as proposed by Box and Jenkins (1970). The strategy is based on the mathematical application of the back-shift operator B defined by $B^k X_t = X_{t-k}$, where $\{X_t\}$ is the series to be modelled. The series is differenced d times until it is found to be stationary. The differenced series is then modelled as an $ARIMA(p,d,q)$ process. For non-seasonal series, the $ARIMA(p,d,q)$ process is represented as

$$\Phi(B)(1-B)^d X_t = \theta(B)\varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0,\sigma^2), \qquad (4.1)$$

where

$$\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots\ldots - \phi_p B^p,$$

and

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots\ldots - \theta_q B^q.$$

However, a major problem may arise regarding the estimation of the degree of differencing, d, in the estimation of the $ARIMA(p,d,q)$ process. In Chapter 2, it was emphasized that a series with a unit root in the AR operator with $d = 1$ is non-stationary. The Dickey-Fuller test and other tests were named as applicable tools for deciding, on the basis of the series, whether to use $d = 0$ or $d = 1$.

Nevertheless, some time series models may be very difficult to build. The difficulty may be due to the existence of slowly diminishing correlations in the series, in which case restricting d to non-negative integers may prove futile. To tackle this issue, Granger and Joyeux (1980) and Hosking (1981), among others, independently proposed fractional values of d.

The rest of the chapter are organised as follows. In Section 4.2, we discuss three used lag and spectral windows commonly employed in spectral analysis of time series. Section 4.3 presents two important methods for estimating the degree of differencing a given time assuming the series is non-stationary. A summary of this chapter is then given in Section 4.4. Chapter Appendix 4 contains some computer programs used to obtain the graphs in the chapter.

4.2 Smoothing the Spectrum - The Lag Window

Spectral analysis of a series gives quite substantial information about the nature of series. A high variance in the course of estimating the spectrum of the series may lead to invalid results. One way to reduce the variance of the sample spectrum of the series is by smoothing which involves choosing a suitable lag or spectral window. Here, one applies weights to the autocovariance function and then transform the smoothed autocovariance function.

For $W(x)$ satisfying

$$\left.\begin{array}{ll} |W(x)| \leq 1 & \\ W(x) = 1 & \\ W(x) = W(-x) & \\ W(x) = 0 \qquad for & |x| > 1 \end{array}\right\}, \qquad (4.2)$$

the weighted estimator of the spectral density function or the *smoothed spectrum* of a series of size T is given by

$$\hat{f}_x^*(\omega) = \frac{1}{2\pi}\sum_{k=-r}^{r} W\left(\frac{k}{r}\right)\hat{\gamma}_k e^{-i\omega k}, \qquad (4.3)$$

where $r < T$, and

$$\hat{\gamma}_k = \hat{\gamma}_{-k} = \frac{1}{T}\sum_{t=1}^{T-k}(X_t - \overline{X})(X_{t+k} - \overline{X}), \qquad k \geq 0. \qquad (4.4)$$

Next, define the Fourier transform

$$\hat{f}(\lambda) = \frac{1}{2\pi}\sum_{k=-(T-1)}^{T-1}\gamma_k e^{-i\lambda k}, \qquad (4.5)$$

then, the inverse transform of $\hat{f}(\lambda)$ is

$$\hat{\gamma}_k = \int_{-\pi}^{\pi} \hat{f}(\lambda) e^{i\lambda k} d\lambda, \qquad k = 0, \pm 1, \pm 2, \ldots, \pm (T-1). \tag{4.6}$$

Thus (4.3) becomes

$$\hat{f}^*(\omega) = \frac{1}{2\pi} \sum_{k=-(T-1)}^{T-1} W\left(\frac{k}{r}\right) \int_{-\pi}^{\pi} \hat{f}(\lambda) e^{i\lambda k} . e^{-i\omega k} d\lambda$$

$$= \int_{-\pi}^{\pi} \frac{1}{2\pi} \sum_{k=-(T-1)}^{T-1} W\left(\frac{k}{r}\right) e^{-i(\omega-\lambda)k} \hat{f}(\lambda) d\lambda$$

$$= \int_{-\pi}^{\pi} W^*(\omega - \lambda) \hat{f}(\lambda) d\lambda$$

or

$$\hat{f}^*(\omega) = \int_{-\pi}^{\pi} W^*(\lambda) \hat{f}(\omega - \lambda) d\lambda, \tag{4.7}$$

where

$$W^*(\lambda) = \frac{1}{2\pi} \sum_{k=-(T-1)}^{T-1} W\left(\frac{k}{r}\right) . e^{-i\omega k}, \tag{4.8}$$

is the *spectral window*. The Fourier transform of $W^*(\lambda)$ is $W(x)$, the weight function. The weight is referred to as *lag window*. In the next section, we present the three lag and spectral windows commonly used in spectral analysis.

4.2.1 Rectangular Window

With a rectangular window, the lag window based on the lag *k*, $W(k)$, is given by

$$W(k) = \begin{cases} 1, & |k| \leq (T-1) \\ 0, & |k| > (T-1) \end{cases}. \tag{4.9}$$

The spectral window, thus becomes

$$W^*(\omega) = \frac{1}{2\pi}\sum_{k=-(T-1)}^{T-1} W(k).e^{-i\omega k} = \frac{1}{2\pi}\sum_{k=-(T-1)}^{T-1} 1.e^{-i\omega k}$$

$$\Rightarrow \qquad W^*(k) = \frac{1}{2\pi}\left[1+2\sum_{k=1}^{T-1}\cos\omega k\right]. \qquad (4.11)$$

Employing the relations

$$\sum_{k=1}^{T}\cos\omega k = \cos\left[\frac{\omega(T+1)}{2}\right].\frac{\sin[\omega T/2]}{\sin[\omega/2]}, \qquad (4.12a)$$

$$\sum_{k=1}^{T}\sin\omega k = \sin\left[\frac{\omega(T+1)}{2}\right].\frac{\sin[\omega T/2]}{\sin[\omega/2]}, \qquad (4.12b)$$

the spectral window (4.11) becomes

$$W^*(\omega) = \frac{1}{2\pi}\left\{1+\frac{2\cos[\omega T/2].\sin[\omega(T-1)/2]}{\sin[\omega/2]}\right\}$$

which simplifies to yield

$$W^*(\omega) = \frac{1}{2\pi}\frac{\sin[\omega T/2]}{\sin[\omega/2]}. \qquad (4.13)$$

For time series of size $T = 10$, the rectangular spectral and lag windows are shown in Fig. 4.1a and Fig. 4.1b using the following *Matlab* and *SAS* programs in the Chapter Appendix (Program 4.1a and Program 4.1b).

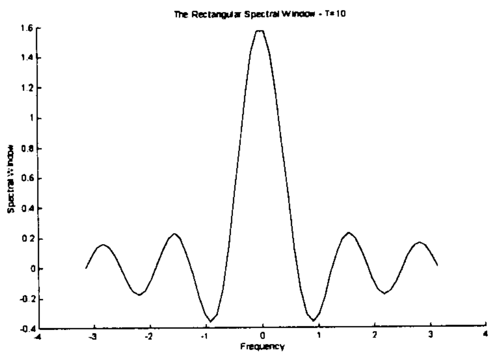


Fig. 4.1a: Rectangular Spectral Window (T=10)

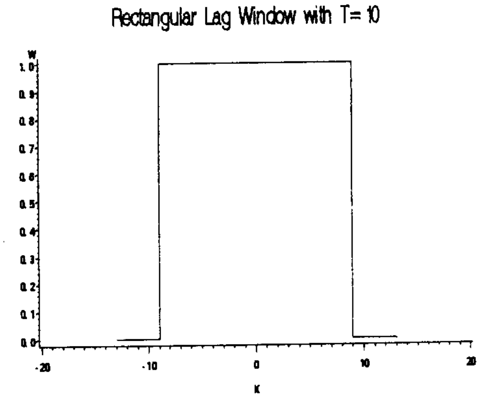


Fig. 4.1b: Rectangular Lag Window (T=10)

4.2.2 Bartlett Window

The lag window $W(k)$ formulated by Bartlett (1950) is as follows:

$$W(k) = \begin{cases} 1 - |k|/T, & |k| \le T, \\ 0, & |k| > T. \end{cases} \tag{4.14}$$

By definition, the spectral window is

$$W^*(\omega) = \frac{1}{2\pi}\sum_{k=-T}^{T}\left(1 - \frac{|k|}{T}\right)e^{-i\omega k} \quad = \quad = \frac{1}{2\pi T}\sum_{k=-T}^{T}(T - |k|)e^{-i\omega k} \tag{4.15}$$

$$W^*(\omega) \quad = \frac{1}{2\pi T}\sum_{j=0}^{T-1}\ \sum_{k=-j}^{j} e^{-i\omega k}$$

$$= \frac{1}{2\pi T}\sum_{j=0}^{T-1}\frac{\sin[\omega(j + \frac{1}{2})]}{\sin[\omega/2]}$$

$$= \frac{1}{2\pi T\sin[\omega/2]}\left\{\sin[\omega/2] + \sum_{j=1}^{T-1}\sin[\omega j + \frac{\omega}{2})]\right\}$$

$$W^*(\omega) = \frac{1}{2\pi T \sin(\omega/2)}\left\{\sin(\omega/2) + \cos(\omega/2)\sum_{j=1}^{T-1}\sin(\omega j)\right.$$

$$\left. + \cos(\omega/2)\sum_{j=1}^{T-1}\sin(\omega j)\right\}. \qquad (4.16)$$

Employing once again the relations in (4.12), equation (4.16) becomes

$$W^*(\omega) = \frac{1}{2\pi T[\sin(\omega/2)]^2}\{[\sin(\omega/2)]^2 + \sin[\omega(T-1)/2]\sin[\omega(T+1)/2]\},$$

which simplifies to give

$$W^*(\omega) = \frac{1}{2\pi T}\left\langle\frac{\sin(\omega T/2)}{\sin(\omega/2)}\right\rangle^2. \qquad (4.17)$$

Using the *Matlab* statements in Program 4.2 (in Chapter Appendix 4), the Bartlett spectral windows is shown in Fig. 4.2a. Fig. 4.2b is the corresponding lag window.

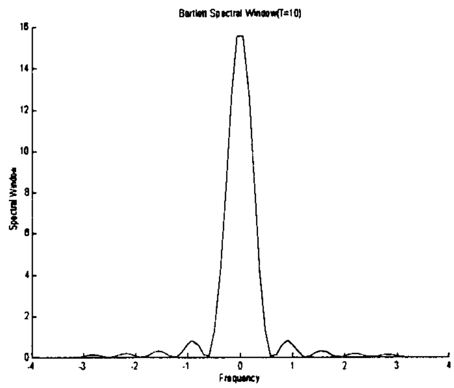


Fig. 4.2a: Bartlett Spectral Window (T=10)

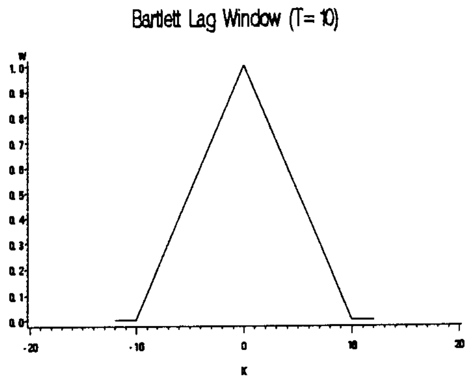


Fig. 4.2b: Bartlett Lag Window (T=10)

4.2.3 The Parzen Window

The Parzen window proposed by Parzen (1961a) is given by

$$W(k) = \begin{cases} 1 - 6(k/T)^2 + 6(|k|T)^3, & |k| \le T/2, \\ 2(1-|k|/T)^3, & (T/2) < |k| < T \\ 0 & |k| > T. \end{cases} \tag{4.18}$$

Thus, the corresponding spectral window $W^*(\omega)$ is

$$W^*(\omega) = \frac{1}{2\pi}\sum_{k=-T}^{T} W(\omega)\cos(\omega k)$$

$$W^*(\omega) = \frac{1}{2\pi}\left\{ \sum_{k=-T/2}^{T/2}[1 - 6(k/T)^2 + 6(|k|/T)^3]\cos(\omega k) + 2\sum_{|k|=T/2}^{T}(1-|k|/T)^3\cos(\omega k) \right\}, \tag{4.19}$$

which simplifies to give

$$W^*(\omega) = \frac{3}{8\pi T^3}\left\langle \frac{\sin(\omega T/4)}{(1/2)\sin(\omega/2)} \right\rangle^4 \left\langle 1 - (2/3)[\sin(\omega/2)]^2 \right\rangle. \tag{4.20}$$

Program 4.3 in Chapter Appendix 4 produces the Parzen spectral window shown in Fig. 4.3a. Fig. 4.3b is the corresponding Parzen lag windw.

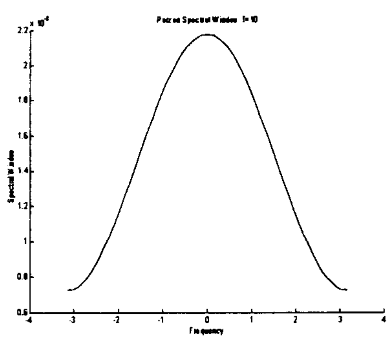


Fig. 4.3a: Parzen Spectral Window (T=10)

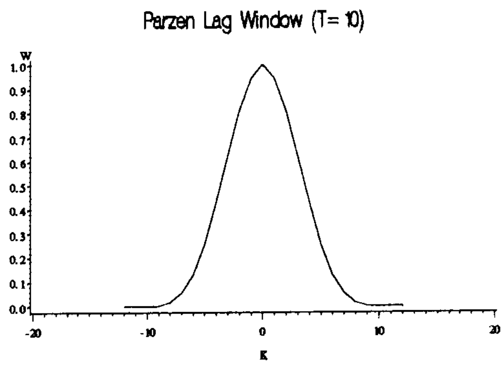


Fig. 4.3b: Parzen Lag Window (T=10)

4.3 Estimation of d

Several techniques have been proposed by different authors regarding the estimation of the degree of differencing, d. Notable among these are those due to Granger and Joyeux (1980) and Janecek (1982). In this section we focus on the estimation of d by three different methods, viz, the lag-window method, the periodogram method, and the smoothed-periodogram method.

4.3.1 The Lag-Window Method

If $Y_t = (1-B)^d X_t$, the ARIMA(p,d,q) process $\Phi(B)(1-B)^d X_t = \theta(B)\varepsilon_t$ becomes

$$\Phi(B)Y_t = \theta(B)\varepsilon_t$$

or

$$Y_t = \frac{\theta(B)}{\Phi(B)}\varepsilon_t . \tag{4.21}$$

Then for some scalar z, the polynomial $\theta(B)$ converges for all z whilst $1/\Phi(B)$ converges for $|z| \le 1$.

Hence, Y_t is a stationary series with a spectrum

$$f_y(\omega) = \frac{\sigma_\varepsilon^2}{2\pi}\frac{|\theta(e^{-i\omega})|^2}{|\Phi(e^{-i\omega})|^2} . \tag{4.22}$$

Now, $Y_t = (1-B)^d X_t$ implies

$$X_t = (1-B)^{-d} Y_t . \tag{4.23}$$

Let

$$f(z) = (1-z)^{-d}, \tag{4.24}$$

then

$$\frac{\partial^k f}{\partial z^k} = (d+k-1)(d+k-2)....(d+1)d(1-z)^{-d-k} . \tag{4.25}$$

Furthermore, the Taylor expansion for $f(z)$ about $z = 0$ is given by

$$f(z) = f(0) + \tfrac{z}{1!}\frac{\partial f}{\partial z} + \tfrac{z^2}{2!}\frac{\partial^2 f}{\partial z^2} + \tfrac{z^3}{3!}\frac{\partial^3 f}{\partial z^3} + \ldots \tag{4.26}$$

Employing (4.24) and (4.25) in (4.26) yields

$$(1-z)^{-d} = 1 + zd + \tfrac{z^2}{2!}d(d+1) + \tfrac{z^3}{3!}d(d+1)(d+2) + \ldots . \tag{4.27}$$

Equation (4.27) converges for all $|z| < 1$. The spectrum of $(1-z)$ is $(1-e^{-i\omega})(1-e^{i\omega}) = |1-e^{i\omega}|^2$. Therefore, $X_t = (1-B)^{-d}Y_t$ in (4.23) converges for all $|z| < 1$ and so $\{X_t\}$ stationary with spectrum

$$f_x(\omega) = \left(\left|1-e^{-i\omega}\right|^2\right)^{-d} \times f_y(\omega)$$

$$\Rightarrow \qquad f_x(\omega) = \left|1-e^{-i\omega}\right|^{-2d} f_y(\omega). \tag{4.28}$$

Multiplying both sides of (4.28) by $\dfrac{\hat{f}_x(\omega)}{f_y(0)}$ yields

$$\frac{f_x(\omega)\hat{f}_x(\omega)}{f_y(0)} = |1-e^{-i\omega}|^{-2d}\frac{f_y(\omega)\hat{f}_x(\omega)}{f_y(0)}, \tag{4.29}$$

or

$$\hat{f}_x(\omega) = |1-e^{-i\omega}|^{-2d} . f_y(0).\left(\frac{\hat{f}_x(\omega)}{f_x(\omega)}\right)\left(\frac{f_y(\omega)}{f_y(0)}\right). \tag{4.30}$$

Taking the logarithm of (4.30) gives

$$\ln \hat{f}_x(\omega) = \ln f_y(0) - d\ln\left(\left|1-e^{-i\omega}\right|^2\right) + \ln\left[\frac{\hat{f}_x(\omega)}{f_x(\omega)}\right] + \ln\left[\frac{f_y(\omega)}{f_y(0)}\right]. \tag{4.31}$$

Now,

$$\left|1-e^{-i\omega}\right|^2 = \left(1-e^{i\omega}\right)\left(1-e^{-i\omega}\right) = 2-(e^{i\omega}+e^{-i\omega}) = 2(1-\cos\omega), \qquad (4.32)$$

where we have employed the relation $(e^{i\omega}+e^{-i\omega}) = 2\cos\omega$. Using the trigonometric relation $\cos 2A = 1-2\sin^2 A$ implies $\cos\omega = 1-2\sin^2(\omega/2)$, and hence (4.32) becomes

$$\left|1-e^{-i\omega}\right|^2 = 2\left[1-1+2\sin^2(\omega/2)\right] = 4\sin^2(\omega/2). \qquad (4.33)$$

Substituting (4.33) into (4.31) and introducing the subscript $j = 1,2,3,\ldots,T$ yields

$$\ln\hat{f}_x(\omega_j) = \ln f_y(0) - d\ln\left(4\sin^2 {\omega_j}/{2}\right) + \ln\left[\frac{\hat{f}_x(\omega_j)}{f_x(\omega_j)}\right] + \ln\left[\frac{f_y(\omega_j)}{f_y(0)}\right], \qquad (4.34)$$

where $\omega_j = \dfrac{2\pi j}{T}$. Employing a result due to Geweke and Porter-Hudak (1983) that for frequencies near zero,

$$\ln\left(\frac{f_y(\omega_j)}{f_y(0)}\right) \to 0,$$

equation (4.34) reduces to the simple linear regression

$$M_j = \beta_o + \beta_1 N_j + e_j, \qquad (4.35)$$

where $M_j = \ln\hat{f}_x(\omega_j)$, $N_j = \ln\left[4\sin^2 {\omega_j}/{2}\right]$, $e_j = \ln\left[\dfrac{\hat{f}_x(\omega_j)}{f_x(\omega_j)}\right]$, $\beta_o = \ln f_y(0)$ and $\beta_1 = -d$. For good results, Brockwell and Davis (1987) recommended using the first $\Lambda = T^{0.5}$ periodogram ordinates. Then, by a simple linear regression approach involving, the degree of differencing d is given by

$$\hat{d} = -\frac{\sum_{j=1}^{\Lambda}\left[\left(N_j-\overline{N}\right)\left(M_j-\overline{M}\right)\right]}{\sum_{j=1}^{\Lambda}\left(N_j-\overline{N}\right)^2}, \qquad (4.36)$$

where

$$\overline{M} = \sum_{j=1}^{\Lambda} M_j / \Lambda \qquad \text{and} \qquad \overline{N} = \sum_{j=1}^{\Lambda} N_j / \Lambda . \tag{4.37}$$

The estimator of $f_x(\omega)$, $\hat{f}_x(\omega)$, is obtained by using the lag window method (4.3), where

$$\hat{f}_x(\omega) = \frac{1}{2\pi} \sum_{k=-(T-1)}^{T-1} W\left(\frac{k}{r}\right) \hat{\gamma}_k . e^{-i\omega k}.$$

The window parameter $r = r(T)$ is chosen such that $r \in (0, T)$. $W(\cdot)$ represents any of the lag windows discussed in Section 4.2, or any other known lag window.

4.3.2 The Peridogram Method

For a stochastic process $\{X_t\}$ with absolute summable autocovariances γ_k, the spectrum is expressed as

$$f_x(\omega) = \frac{1}{2\pi}\left\{\gamma_o + 2\sum_{k=1}^{\infty} \gamma_k \cos(\omega k)\right\}. \tag{4.38}$$

Given a sample of size T, an obvious estimator becomes

$$\hat{f}_x(\omega) = \frac{1}{2\pi}\left\{\hat{\gamma}_o + 2\sum_{k=1}^{T-1} \hat{\gamma}_k \cos(\omega k)\right\}, \tag{4.39}$$

where

$$\hat{\gamma}_k = \frac{1}{T}\sum_{t=1}^{T-k} (X_t - \overline{X})(X_{t-k} - \overline{X}), \qquad k = 0, \pm 1, \pm 2, \ldots, \pm (T-1)$$

and

$$\overline{X} = \frac{\sum_{t=1}^{T} X_t}{T}.$$

The periodogram ordinates $I_x(\omega_j)$ for the jth frequency in complex terms is defined as

$$I_x(\omega_j) = \frac{2}{T}\left|\sum_{t=1}^{T} X_t e^{-i\omega_j k}\right|^2, \qquad j = 1,2,\dots,(T-1). \qquad (4.40)$$

Replacing X_t by $(X_t - \overline{X})$, (4.40) becomes

$$I_x(\omega_j) = \frac{2}{T}\left|\sum_{t=1}^{T}(X_t - \overline{X})e^{-i\omega_j t}\right|^2 = \frac{2}{T}\sum_{t=1}^{T}\sum_{\tau=1}^{T}(X_t - \overline{X})(X_\tau - \overline{X})e^{-i\omega_j(t-\tau)}. \qquad (4.41)$$

where $j = 1,2,\dots,(T-1)$. Setting $(t\text{-}\tau) = k$ and using the definition of $\hat{\gamma}_k$ in (4.39), equation (4.41) becomes

$$I_x(\omega_j) = 2\sum_{k=-(T-1)}^{(T-1)} \gamma_k e^{-i\omega_j k} \quad \Rightarrow \quad I_x(\omega_j) = 2\left[\gamma_o + 2\sum_{k=1}^{T-1} \gamma_k \cos(\omega_j k)\right]. \qquad (4.42)$$

Hassler (1993) showed that the periodogram, $I_x(\omega_j)$, of the $ARIMA(p,d,q)$ process given by

$$\Phi(B)(1-B)^d X_t = \theta(B)\varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0,\sigma_\varepsilon^2)$$

with $d < 0$ and $E(\varepsilon_t^4) < \infty$, is asymptotically given by

$$I_x(\omega_j) \cong \frac{2\pi}{\sigma_\varepsilon^2} f_x(\omega_j) I_\varepsilon(\omega_j), \qquad (4.43)$$

where $I_\varepsilon(\omega_j)$ is the periodogram of $\{\varepsilon_t : t = 1,2,\dots,T\}$.

Taking the logarithm of (4.29) and using the fact that $\ln\left(\left|1 - e^{-i\omega}\right|^{-2}\right) = \ln\left[4\sin^2\left(\frac{\omega}{2}\right)\right]$, equation (4.28) becomes

$$\ln f_x(\omega_j) = \ln f_y(\omega_j) - d\ln\left(4\sin^2\left(\omega_j/2\right)\right), \qquad j = 0,1,2,..,[n/2]. \qquad (4.44)$$

Adding $\ln I_x(\omega_j)$ to both sides of (4.44) and applying the conditions in (4.33) and (4.34) we obtain

$$\ln I_x(\omega_j) = \ln f_y(\omega_j) - d\ln\left(4\sin^2(\omega_j/2)\right) + \ln\left(\frac{I_x(\omega_j)}{f_x(\omega_j)}\right) \qquad (4.45)$$

which has the form of a simple regression equation

$$M_j = \beta_o + \beta_1 N_j + e_j, \qquad j = 1,2,\ldots,T^{0.5} \qquad (4.46)$$

where $M_j = \ln I_x(\omega_j)$, $N_j = \ln\left(4\sin^2(\omega_j/2)\right)$, $\beta_1 = -d$, $e_j = \ln\left(\frac{I_x(\omega_j)}{f_x(\omega_j)}\right) + a$

and $\beta_o = \ln f_y(\omega_j) - a$, with $a = E\left(-\ln\frac{I_x(\omega_j)}{f_x(\omega_j)}\right)$.

The ordinary least square regression of $\{M_j: j = 1,2,\ldots,T^{0.5}\}$ on $\{N_j: j = 1,2,\ldots,T^{0.5}\}$ leads to the estimator

$$\beta_1 = \frac{\sum_{h=1}^{\Lambda}(N_h - \overline{N})M_h}{\sum_{h=1}^{\Lambda}(N_h - \overline{N})^2}, \qquad (4.47)$$

where $\Lambda = T^{0.5}$. The estimator of d, $\hat{d}$ is then given by $\hat{d} = -\beta_1$.

4.3.3 The Smooth-Periodogram Method

In this section, we consider the estimation of d by smoothed periodogram using the Parzen lag window. An estimator of the smoothed periodogram $\hat{f}_x^*(\omega)$ is the real part in (4.3) given by

$$\hat{f}_x^*(\omega) = \frac{1}{2\pi}\sum_{k=-r}^{r} W\left(\frac{k}{r}\right)\hat{\gamma}_k.\cos(\omega_j k). \qquad (4.48)$$

Let $W\left(k/r\right)$ assume the Parzen lag window based on the following weighting function:

$$W(v) = W(-v) = \begin{cases} 1 - 6v^2 + 6|v|^3, & |v| \le \frac{1}{2} \\ 2(1 - |v|)^3, & \frac{1}{2} < |v| \le 1. \\ 0, & |v| > 1 \end{cases} \tag{4.49}$$

We employ the following two lemmas, lemma 4.2.1 and lemma 4.2.2, due to Priestley (1981) and Anderson (1971), respectively, to formulate a regression model using the smoothed periodogram and the Parzen lag window:

Lemma 4.2.1

$\hat{f}_x^*(\omega_j)$ is asymptotically unbiased with variance given by

$$Var\{\hat{f}_x^*(\omega_j)\} = \begin{cases} 0.539285(r/T).f^2(\omega_j), & \omega_j \ne 0,\pi \\ 1.07856(r/T).f^2(\omega_j), & \omega_j = 0,\pi \end{cases} \tag{4.50}$$

and $$\lim(T/r).\text{cov}[\hat{f}_x^*(\omega_1),\hat{f}_x^*(\omega_2)] = 0. \tag{4.51}$$

Lemma 4.2.2

If $d \in (-0.5,0)$ and $W(k/r)$ assumes the Parzen lag window then

$$\ln\left(\frac{\hat{f}_x^*(\omega)}{f_x(\omega)}\right) \sim N(0,\sigma_*^2),$$

where

$$\sigma_*^2 = Var\left[\ln\left(\frac{\hat{f}_x^*(\omega)}{f_x(\omega)}\right)\right] = \begin{cases} 0.539285(r/T), & \omega \ne 0,\pi \\ 1.07856(r/T), & \omega = 0,\pi. \end{cases} \tag{4.52}$$

Adding $\ln\{\hat{f}_x^*(\omega_j)\}$ to both sides of (4.44) and using the fact that $\ln\left\{\frac{f_y(\omega_j)}{f_y(0)}\right\} = 0$, we obtain

$$\ln\{\hat{f}_x^*(\omega_j)\} = \ln\{f_y(0)\} - d\ln\left\{4\sin^2(\omega_j/2)\right\} + \ln\left\{\frac{\hat{f}_x^*(\omega_j)}{f_x(\omega_j)}\right\}. \qquad (4.53)$$

Equation (4.53) is seen to be a simple regression of the form

$$M_j = \beta_o + \beta_1 N_j + e_j, \qquad j = 1,2,\ldots,\Lambda \qquad (4.54)$$

where $\beta_o = \ln\{f_y(0)\}$, $\beta_1 = -d$, $M_j = \ln\{\hat{f}_x^*(\omega_j)\}$, and $e_j = \ln\left\{\frac{\hat{f}_x^*(\omega_j)}{f_x(\omega_j)}\right\}$. The value of Λ is chosen as before. The estimator of d is given by

$$d = -\beta_1 = -\frac{\sum_{h=1}^{\Lambda}(N_h - \overline{N})M_h}{\sum_{h=1}^{\Lambda}(N_h - \overline{N})^2}. \qquad (4.55)$$

From (4.48) it is seen that the periodogram is just a weighted average of the Fourier transform of the sample autocovariance. To reduce the computational cost of computing autocovariances for series with large observations, Fuller (1979) proposed the an alternative method of obtaining $f_x^*(\omega_j)$. The procedure involves the application of weights to the estimated autocovariance function $\hat{\gamma}_k$ and transforming the *smoothed* autocovariance function. The smoothing process adopts the following lag window:

$$W(v) = \begin{cases} 1, & v = 0 \\ 0, & |v| > 1 \end{cases} \qquad \text{with} \qquad |W(v)| < 1 \text{ for all } v. \qquad (4.56)$$

4.4 Summary

In this chapter, we have discussed some three methods, basically regression methods, for computing the degree of differencing, *d*. We have shown that the regression of the sample spectrum can be used to determine the differencing degree. We have also shown how practicable the regression of the periodogram and the smoothed periodogram could be used to estimate this degree.

CHAPTER APPENDIX 4

Program 4.1a:

```
EDU» f=linspace(-pi,pi,60);
EDU» W=sin(5*f)./((2*pi)*sin(f/2));
EDU» plot(f,W);
EDU» xlabel('Frequency')
EDU» ylabel('Spectral Window')
EDU» box off
EDU»
```

Program 4.1b:

```
title 'Rectangular Lag Window with T=5';
data rectwin;
  input w k;
cards;
 0    -13
 0    -12
 0    -11
 0    -10
 0     -9
 1     -9
 1     -7
 1     -6
 1     -5
 1     -4
 1     -4
 1     -2
 1     -1
 1      0
 1      1
 1      2
 1      4
 1      4
 1      5
 1      6
 1      7
 1      9
 0      9
 0     10
 0     11
 0     12
 0     13
;
symbol1 i=join v=none;
proc gplot data=rectwin;
   plot w*k;
run;
quit;
```

Program 4.2:

```
EDU» f=linspace(-pi,pi,60);
EDU» W=sin(5*f).^2./((2*pi)*(sin(f/2).^2));
EDU» plot(f,W);
EDU» box off
EDU» xlabel('Frequency')
EDU» ylabel('Spectral Window')
EDU»
```

Program 4.3:

```
EDU» f=linspace(-pi,pi,60);
EDU» A=sin(2.5*f);B=sin(f/2);C=A/B;D=C.^4;E=1-((2/3)*(B.^2));F=0.006*pi;
EDU» W=F*D*E;
EDU» plot(f,W)
EDU» xlabel('Frequency')
EDU» ylabel('Spectral Window')
EDU» title('Parzen Spectral Window')
EDU» title('Parzen Spectral Window T=10')
EDU» box off
EDU»
```

CHAPTER 5

TIME-DEPENDENT SPECTRAL ANALYSIS OF NON-STATIONARY TIME SERIES

5.1 Introduction

Spectral analysis is a fundamental tool used to study the cyclical behaviour of a series. In spectral analysis of time series, the primary objective is to obtain the spectrum of the series by decomposing the series into sums or integrals of sine and cosine functions. In the classical approach to defining spectrum, the time series is assumed to be stationary. With this assumption all stationary processes { X_t } can be represented in the form

$$X_t = \int_{-\pi}^{\pi} e^{itw} dz(w), \qquad (5.1)$$

where $z(w)$ is a complex, random function with

$$\operatorname{cov}\left[dz(w_1), \overline{dz(w_2)}\right] = E\left[dz(w_1)\overline{dz(w_2)}\right] = \begin{cases} 0, & w_1 \neq w_2 \\ dw, & w_1 = w_2. \end{cases}$$

Equation (5.1) is usually referred to as the Cramer representation of { X_t }. The spectrum, $f(w)$, of { X_t } is given by

$$f(w) = \int_{-\infty}^{\infty} \gamma_x . e^{-iwt} dw \qquad (5.2)$$

where γ_x is the autocovariance function of { X_t }.

However, in practice, most series are non-stationary. This means that the two statements in (5.1) and (5.2) do not hold in the case of non-stationary series and this calls for real modifications. One of such ways of modifying them is through evolutionary spectral analysis, where it is assumed that the process changes slowly in its spectral characteristics, ie, at each time point, a stationary interval can be defined within which the process becomes approximately stationary. Analysing such time-dependent time series spectrally is referred to as evolutionary spectral analysis.

An example of a non-stationary process { X_t } is

$$X_t = \begin{cases} X_{1t}, & t \le c \\ X_{2t}, & t > c \end{cases}, \tag{5.3}$$

where both { X_{1t} } and { X_{2t} } are stationary processes but with different autocovariance functions.

In this chapter, our discussions will be based on time-dependent spectral analysis of non-stationary time series - the concept of evolutionary spectral analysis. Evolutionary spectra have essentially the same type of physical interpretations as the stationary case, the difference being the fact that whereas the spectrum of a stationary process describes the frequencies over all time, the evolutionary spectrum describes the spectrum at each instant time t, and hence the word *evolution*. It should, however, be emphasized that the decomposition into sine and cosine functions of a time-varying quantity is based on the theory of Fourier transforms, a transformation that employs the complex form of the Fourier integral. In Section 5.2, we shall review the theory of the Complex Fourier Integral. Section 5.3 presents an overview of the Fourier transforms.

5.2 Complex Fourier Integral

Let X_t be defined for all t, and assume that

$$\int_{-\infty}^{\infty} |X_t| dt < \infty \quad . \tag{5.4}$$

Then

$$X_t = \frac{1}{\pi}\int_0^{\infty} \{M(w)\cos(wt) + N(w)\sin(wt)\} dw \, , \tag{5.5}$$

where $-\pi < w < \pi$, is the Fourier integral representation of X_t. $M(w)$ and $N(w)$ are the Fourier integral coefficients and are defined by

$$M(w) = \tfrac{1}{T}\int_{-\infty}^{\infty} X_t \cos(wt) dt \quad \text{and} \quad N(w) = \tfrac{1}{T}\int_{-\infty}^{\infty} X_t \sin(wt) dt \, . \tag{5.6}$$

Employing (5.6) and the relations

$$\cos\theta = \tfrac{1}{2}(e^{i\theta} + e^{-i\theta}) \qquad \text{and} \qquad \sin\theta = \tfrac{1}{2i}(e^{i\theta} - e^{-i\theta}) = -\tfrac{i}{2}(e^{i\theta} - e^{-i\theta}),$$

where $i = \sqrt{-1}$, equation (5.5) becomes

$$\tfrac{1}{\pi}\int_0^\infty \{M(w)\cos(wt) + N(w)\sin(wt)\}dw$$

$$= \tfrac{1}{\pi}\int_0^\infty \{M(w).\tfrac{1}{2}(e^{iwt} + e^{-iwt}) - N(w).\tfrac{i}{2}(e^{iwt} - e^{-iwt})\}dw$$

or

$$\tfrac{1}{\pi}\int_0^\infty \{M(w)\cos(wt) + N(w)\sin(wt)\}dw$$

$$= \tfrac{1}{\pi}\int_0^\infty \{\tfrac{1}{2}[M(w) - i.N(w)]e^{iwt} + \tfrac{1}{2}[M(w) + i.N(w)]e^{-iwt}\}dw \tag{5.7}$$

Setting

$$R(w) = \tfrac{1}{2}[M(w) - i.N(w)], \qquad \text{then} \qquad \overline{R(w)} = \tfrac{1}{2}[M(w) + i.N(w)]. \tag{5.8}$$

equation (5.7) becomes

$$\frac{1}{\pi}\int_0^\infty \{M(w)\cos(wt) + N(w)\sin(wt)\}dw = \tfrac{1}{\pi}\int_0^\infty R(w).e^{iwt}dw + \tfrac{1}{\pi}\int_0^\infty \overline{R(w)}.e^{-iwt}dw. \tag{5.9}$$

Inserting integral formulae for the Fourier integral coefficients (5.6), we have

$$R(w) = \tfrac{1}{2}[M(w) - i.N(w)] = \tfrac{1}{2}\left\{\int_{-\infty}^\infty X_t.\cos(wt)dt - i.\int_{-\infty}^\infty X_t.\sin(wt)dt\right\}$$

$$\Rightarrow \qquad R(w) = \tfrac{1}{2}\int_{-\infty}^\infty X_t\{\cos(wt) - i\sin(wt)\}dt = \tfrac{1}{2}\int_{-\infty}^\infty X_t.e^{-iwt}dt. \tag{5.10}$$

Similarly,

$$\overline{R(w)} = \tfrac{1}{2}\int_{-\infty}^{\infty} X_t.e^{iwt}dt = R(-w) \ . \qquad (5.11)$$

Substituting (5.10) into (5.9) yields

$$\frac{1}{\pi}\int_0^{\infty}\{M(w)\cos(wt)+N(w)\sin(wt)\}dw = \tfrac{1}{\pi}\int_0^{\infty} R(w).e^{iwt}dw + \tfrac{1}{\pi}\int_0^{\infty} R(-w).e^{-iwt}dw \ . \quad (5.12)$$

Setting $v = -w$ implies $\partial v = -\partial w$ and hence the Complex Fourier integral (5.12) becomes

$$\frac{1}{\pi}\int_0^{\infty}\{M(w)\cos(wt)+N(w)\sin(wt)\}dw = \tfrac{1}{\pi}\int_0^{\infty} R(w).e^{iwt}dw - \tfrac{1}{\pi}\int_0^{-\infty} R(v).e^{ivt}dv$$

$$= \tfrac{1}{\pi}\int_0^{\infty} R(w).e^{iwt}dw + \tfrac{1}{\pi}\int_{-\infty}^{0} R(w).e^{iwt}dw$$

$$\Rightarrow \qquad \frac{1}{\pi}\int_0^{\infty}\{M(w)\cos(wt)+N(w)\sin(wt)\}dw = \tfrac{1}{\pi}\int_{-\infty}^{\infty} R(w).e^{iwt}dw \ , \qquad (5.13)$$

is the Complex Fourier integral representation of X_t. In (5.13), $R(w) = \tfrac{1}{2}\int_{-\infty}^{\infty} X_t.e^{-iwt}dt$.

5.3 Fourier Transforms (FT) and Inverse Fourier Transform

The *Fourier transform* $\Omega(w)$ of the function X_t for which $\int_{-\infty}^{\infty} |X_t| dt < \infty$ is defined as

$$\Omega(w) = \int_{-\infty}^{\infty} X_t.e^{-iwt}dt \ . \qquad (5.14)$$

where $-\pi < w < \pi$.

Retrieving X_t given its Fourier transform $\Omega(w)$ assumes the equality between the Complex Fourier Integral and X_t, that is

$$X_t = \tfrac{1}{\pi}\int_{-\infty}^{\infty} R(w).e^{iwt}dw \,. \tag{5.15}$$

Substituting the formula for $R(w)$ in (5.15), we obtain

$$X_t = \tfrac{1}{\pi}\int_{-\infty}^{\infty}\left[\tfrac{1}{2}\int_{-\infty}^{\infty} X_\xi.e^{-iw\xi}d\xi\right].e^{iwt}dw = \tfrac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} X_\xi.e^{-iw\xi}d\xi\right].e^{iwt}dw$$

$$\Rightarrow \qquad X_t = \tfrac{1}{2\pi}\int_{-\infty}^{\infty}\Omega(w).e^{iwt}dw \,, \tag{5.16}$$

as the *inverse Fourier transform* of $\Omega(w)$.

5.3.1 Fast Fourier Transforms (FFT)

FFT is an algorithm for efficiently computing the values for a discrete Fourier transform. More often than not, analysts are interested in such properties of X_t as the amplitude and the periods, knowing only measured values at equally spaced time intervals. Obtaining such information employs the Fourier integral coefficients $M(w)$ and $N(w)$ to obtain approximate values for the discrete Fourier transform. From (5.6a) and (5.6b), we have

$$M(w) = \tfrac{1}{T}\int_{-\infty}^{\infty} X_t \cos(wt)dt \,, \qquad \text{and} \qquad N(w) = \tfrac{1}{T}\int_{-\infty}^{\infty} X_t \sin(wt)dt \,,$$

$$\Rightarrow \qquad R(w) = M(w) - i.N(w) = \tfrac{1}{T}\int_{-\infty}^{\infty} X_t[\cos(wt) - i\sin(wt)]dt$$

or

$$R(w) = \tfrac{1}{T}\int_{-\infty}^{\infty} X_t.e^{-iwt}dt \,. \tag{5.17}$$

Given the univariate time series $\{X_t : t = 1,2,3,\ldots,T\}$, the discrete form of (5.17) which is

$$H(w) = \tfrac{1}{T}\sum_{t=1}^{T} X_t . e^{-iwt}, \tag{5.18}$$

serves as an approximation to the discrete Fourier transform. To illustrate the algorithm, let w be defined in terms of the variable f as

$$w = \tfrac{2\pi f}{T}, \qquad -\pi < w < \pi . \tag{5.19}$$

Next, suppose T is even and that we are required to find the discrete Fourier transform of X_t. One approach is to partition X_t as follows

$$X_{1,t} = X_{2t-1} \tag{5.20a}$$

$$X_{2,t} = X_{2t}, \qquad t = 1,2,\ldots,T/2 . \tag{5.20b}$$

Then each of $X_{1,t}$ and $X_{2,t}$ consists of $T/2$ observations and hence

$$H_f = \tfrac{1}{T}\sum_{t=1}^{T} X_t . e^{-iwt} = \tfrac{1}{T}\sum_{t=1}^{T} X_t . e^{-i2\pi ft/T}, \qquad 0 \le f \le (\tfrac{T}{2} - 1)$$

$$H_f = \tfrac{1}{T}\left\{\sum_{t=1}^{T/2} X_{1,t} . e^{-i2\pi f(2t)/T} + \sum_{t=1}^{T/2} X_{2,t} . e^{-i2\pi f(2t-1)/T}\right\}$$

$$= \tfrac{1}{T}\sum_{t=1}^{T/2} X_{1,t} . e^{-i2\pi ft/(T/2)} + \tfrac{1}{T} . e^{i2\pi f/T}\sum_{t=1}^{T/2} X_{2,t} . e^{-i2\pi ft/(T/2)}$$

$$= \tfrac{1}{2}\left\{\tfrac{2}{T}\sum_{t=1}^{T/2} X_{1,t} . e^{-i2\pi ft/(T/2)} + \tfrac{2}{T} . e^{i2\pi f/T}\sum_{t=1}^{T/2} X_{2,t} . e^{-i2\pi ft/(T/2)}\right\}$$

$$\Rightarrow \qquad H_f = \tfrac{1}{2}\left\{H_{1,f} + e^{i2\pi f/T} . H_{2,f}\right\}, \qquad 0 \le f \le (\tfrac{T}{2} - 1) \tag{5.21}$$

where

$$H_{1,f} = \tfrac{2}{T}\sum_{t=1}^{T/2} X_{1,t}.e^{-i2\pi ft/(T/2)} \quad \text{and} \qquad H_{2,f} = \tfrac{2}{T}\sum_{t=1}^{T/2} X_{2,t}.e^{-i2\pi ft/(T/2)} \quad , \tag{5.22}$$

are respectively the $(T/2)$ -point discrete Fourier transforms of $X_{1,t}$ and $X_{2,t}$.

Now, since

$$e^{-i2\pi f/(T/2)} = e^{-i2\pi f/(T/2)}.e^{i2\pi} = e^{-i2\pi[f-(T/2)]/(T/2)} , \tag{5.23}$$

we have

$$H_{f+\frac{T}{2}} = \tfrac{1}{2}\left\{\tfrac{2}{T}\sum_{t=1}^{T/2} X_{1,t}.e^{-i2\pi ft/(T/2)} + \tfrac{2}{T}.e^{i(2\pi/T)[f+(T/2)]}\sum_{t=1}^{T/2} X_{2,t}.e^{-i2\pi ft/(T/2)}\right\}$$

or

$$H_{f+\frac{T}{2}} = \tfrac{1}{2}\left\{H_{1,f} + e^{i(2\pi/T)[f+\frac{T}{2}]}.H_{2,f}\right\} \qquad , \; 0 \le f \le (\tfrac{T}{2}-1) \tag{5.24}$$

where $H_{1,f}$ and $H_{2,f}$ are as defined in (5.22). Here, we have shown that the Fourier transform of X_t can be obtained from the Fourier series of the half series $X_{1,t}$ and $X_{2,t}$. In a similar sense, when $(T/2)$ is even, $X_{1,t}$ and $X_{2,t}$ may be partitioned into two series, $X^*_{1,t}$, $X^*_{2,t}$ and $X^{**}_{1,t}$, $X^{**}_{2,t}$ respectively and used to construct the transforms $H_{1,f}$ and $H_{2,f}$ from the transforms of the series of length $(T/4)$. The procedure is followed for a series of length 2^l (l is a prime number) until partitions of only one term has been achieved, for which the Fourier transform is equal to the term itself.

5.4 Evolutionary Spectra: The Univariate Case

In Section 5.1, we established the notion that if a process is non-stationary it cannot be represented in form of (5.1) and (5.2), and hence cannot talk about the spectrum of the series. Instead, Priestly (1965), considered the *evolutionary spectrum* for the non-stationary series X_t by generalizing the spectral decomposition of a stationary time series to

$$X_t = \int_{-\pi}^{\pi} \Omega_t(w)e^{iwt}dz(w) \; , \tag{5.25}$$

with $$E\left(|dz(w)|^2\right) = f(w)dw, \tag{5.26}$$

where $f(w)$ represents the spectrum of the stationary series $\int_{-\pi}^{\pi} e^{iwt} dz(w)$ and $\Omega_t(w)$, the Fourier transform of X_t. The evolutionary spectrum at time t will is defined as

$$dF_t(w) = E\left(|\Omega_t(w)dz(w)|^2\right) = |\Omega_t(\omega)|^2 E\left(|dz(\omega)|^2\right)$$

$$\Rightarrow \qquad dF_t(w) = |\Omega_t(w)|^2 f(w)dw. \tag{5.27}$$

The evolutionary spectral density $f_t(w)$, at time t is obtained by differentiating through (5.27) with respect to w. That is,

$$\frac{dF_t(w)}{dw} = \frac{|\Omega_t(w)|^2 f(w)dw}{dw}$$

$$\Rightarrow \qquad f_t(w) = |\Omega_t(w)|^2 f(w) \ . \tag{5.28}$$

5.5 The Uniformly Modulated Process

In this section, we discuss an interesting example of a non-stationary series that satisfies (5.25), that is

$$X_t = \int_{-\pi}^{\pi} \Omega_t(w) e^{iwt} dz(w).$$

Let { X_t } be a continuous process defined by

$$X_t = \Xi_t . Y_t \tag{5.29}$$

where Y_t is a stationary process with spectral density function $f_y(w)$ and Ξ_t, some function of time t. Since Y_t is stationary, it can be represented as

$$Y_t = \int_{-\pi}^{\pi} e^{i\omega t} dz(w), \tag{5.30}$$

where $z(w)$ is as defined in (5.1). Substituting (5.30) into (5.29) gives

$$X_t = \int_{-\pi}^{\pi} \Xi_t . e^{iwt} dz(w) \tag{5.31}$$

Then by (5.27)

$$dF_{x,t}(w) = |\Xi_t|^2 \, dF_y(w) \ , \tag{5.32}$$

where $dF_{x,t}(w)$ and $dF_y(w)$ are respectively the evolutionary spectra of X_t and Y_t . Thus, if an estimate of the spectrum is formed by using Ξ_t as though X_t were stationary, then by (5.32) the function that we should be actually estimating is

$$\hat{f}_{x,t}(w) = |\Xi_t|^2 \, \hat{f}_y(w) \tag{5.33}$$

where $\hat{f}_{x,t}(w)$ and $\hat{f}_y(w)$ are the evolutionary spectrum of X_t and the spectrum of Y_t respectively. In this case we expect that for each value of t, the shape of the evolutionary spectra shouldn't differ from that of the spectrum of the stationary series.

Example 5.1 - Evolutionary Spectra with Artificial Data

In this example, we illustrate the validity of the evolutionary spectra with realizations of artificial time-dependent non-stationary process generated from the stationary AR(2) process

$$Y_t = 0.8Y_{t-1} - 0.4Y_{t-2} + \varepsilon_t , \qquad \{\varepsilon_t\} \sim WN(0,100^2) . \tag{5.34}$$

Assuming the modulated process X_t is representable by

$$X_t = e^{-\frac{t^2}{2\alpha^2}} . Y_t , \qquad \text{where } \alpha = 200 , \tag{5.35}$$

then the evolutionary spectrum of X_t is given by

$$f_{x,t}(w) = \left[e^{-\frac{t^2}{2a^2}} \right]^2 \cdot f_y(w) \qquad (5.36)$$

where $f_y(w)$ is the spectrum of Y_t and $\Xi_t = e^{-\frac{t^2}{2a^2}}$. Then by the concept of evolutionary spectra, we will expect the estimated evolutionary spectrum of the modulated process X_t to have the same shape as that of the spectrum of Y_t at each time point $t = 1, 2, 3,.., 100$. Using the SAS program in Chapter Appendix 5.1 produces the estimated spectrum of Y_t in Fig. 5.1, the estimated evolutionary for t = 20, 40, 60, and 80 in Fig 5.2a, Fig. 5.2b, Fig. 5.2c, and Fig. 5.2d, just to use a few time points to illustrate the concept.

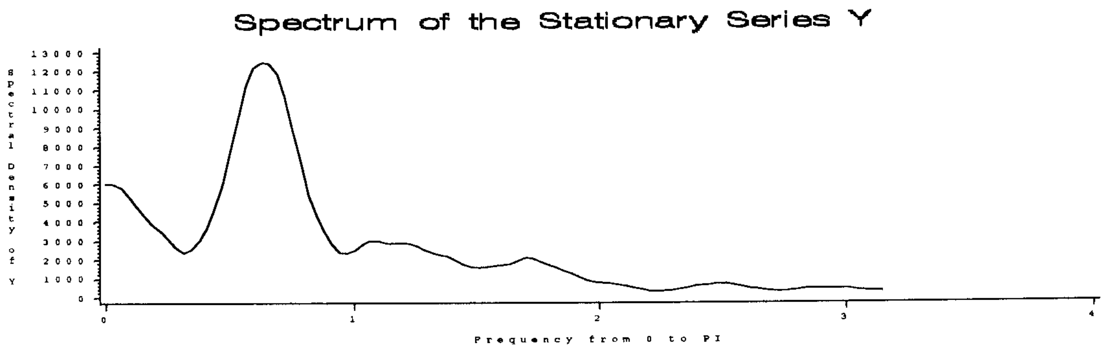


Fig. 5.1: Spectrum of Y_t

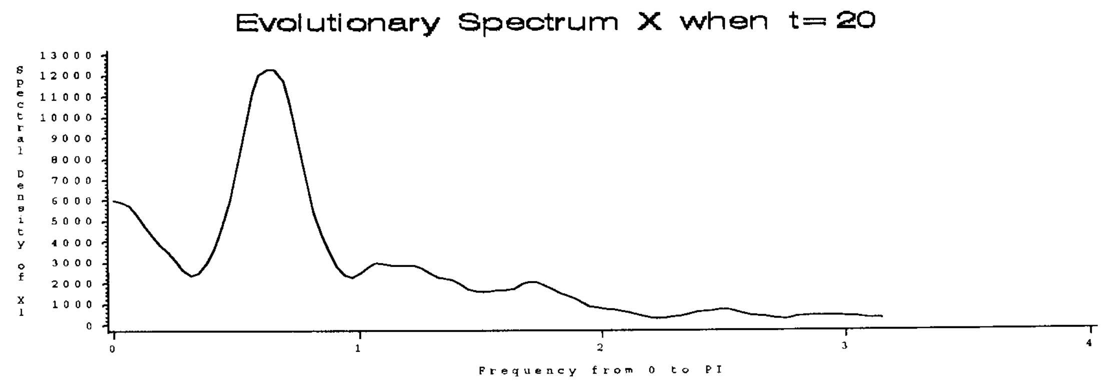


Fig. 5.2a: Evolutionary Spectrum of X_t for $t = 20$.

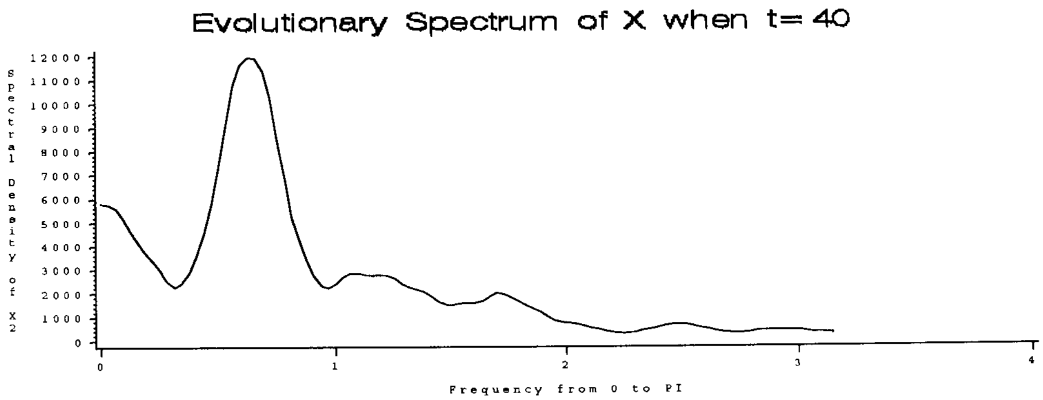


Fig. 5.2b: Evolutionary Spectrum of X_t for $t = 40$.

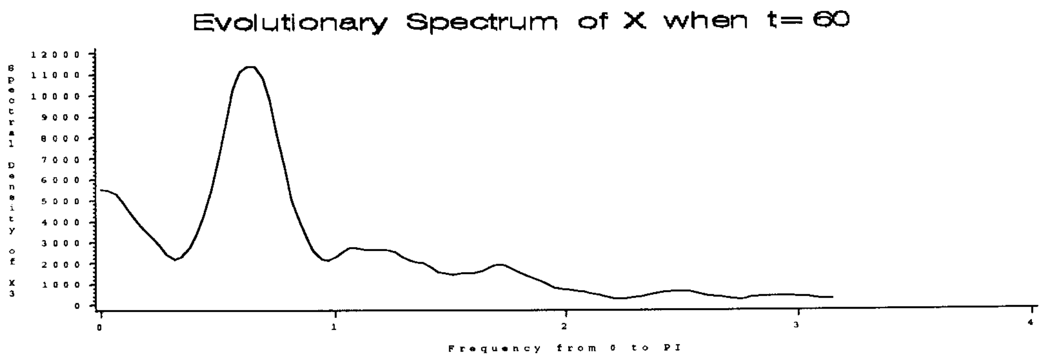


Fig. 5.2c: Evolutionary Spectrum of X_t for $t = 60$.

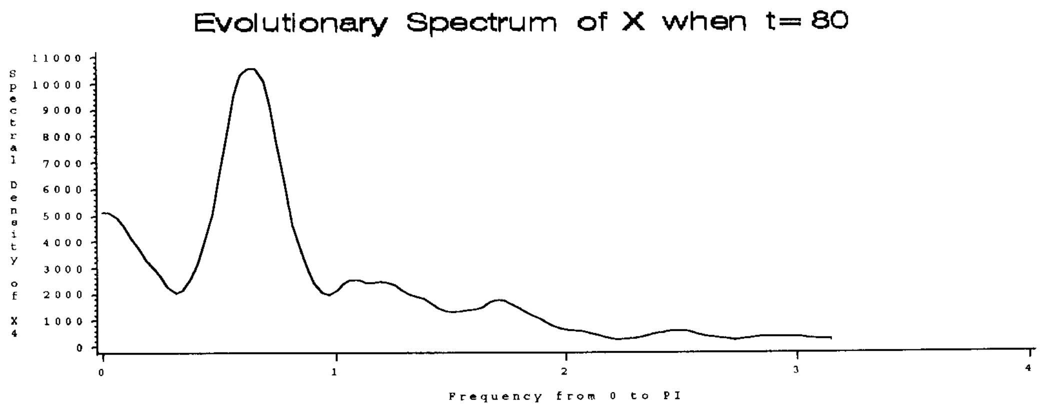


Fig. 5.2d: Evolutionary Spectrum of X_t for $t = 80$.

Comparing Fig. 5.1 and Fig. 5.2a - Fig. 5.2d, it is seen that the shapes of the evolutionary spectra of the time-dependent non-stationary series X_t ($t = 20$, 40, 60, and 80) are exactly the same as the spectrum for the stationary series Y_t.

5.6 Evolutionary Cross-Spectra: The Bivariate Case

An extension of the concepts outlined above can easily be made to handle the bivariate case. Let $X_{1,t}$ and $X_{2,t}$ be two non-stationary processes with different stationary spectral functions $f_{1,t}(w)$ and $f_{2,t}(w)$. Then we have

$$X_{1,t} = \int_{-\pi}^{\pi} \Omega_t(w).e^{iwt} f_{1,t}(w)dw$$
$$X_{2,t} = \int_{-\pi}^{\pi} \Omega_t(w)e^{iwt} f_{2,t}(w)dw \qquad (5.37)$$

with $E(|dz_1(w)|^2) = f_{1,t}(w)dw$ and $E(|dz_2(w)|^2) = f_{2,t}(w)dw$.

Let

$$X_t = X_{1,t} + X_{2,t}. \qquad (5.38)$$

For all t and τ, denote the autocovariance functions of $\{X_t\}$, $\{X_{1,t}\}$ and $\{X_{2,t}\}$ by $\gamma_{12,(t,\tau)}$, $\gamma_{1(t,\tau)}$, and $\gamma_{2(t,\tau)}$, then

$$\gamma_{12(t,\tau)} = \gamma_{1(t,\tau)} + \gamma_{2(t,\tau)} \qquad (5.39)$$

Now,

$$\gamma_{12(t,\tau)} = E(X_t\overline{X_\tau}) = \int_{-\pi}^{\pi} \Omega_t(w).e^{iwt}dz(w)\overline{\int_{-\pi}^{\pi} \Omega_\tau(w).e^{iw\tau}dz(w)}$$

$$= \int_{-\pi}^{\pi} \Omega_t(w).e^{iwt}\overline{\Omega_\tau(w)}.\overline{e^{iw\tau}}dz(w) = \int_{-\pi}^{\pi} \Omega_t(w)\overline{\Omega_\tau(w)}.e^{iwt}.e^{-iw\tau}dz(w)$$

$$\Rightarrow \quad \gamma_{12(t,\tau)} = \int_{-\pi}^{\pi} \Omega_t(w)\overline{\Omega_\tau(w)}.e^{iw(t-\tau)}dz(w). \qquad (5.40)$$

Similarly, we have

$$\gamma_{1(t,\tau)} = \int_{-\pi}^{\pi} \Omega_t(w)\overline{\Omega_\tau(w)}.e^{iw(t-\tau)}dz_1(w) \quad \text{and} \quad \gamma_{2(t,\tau)} = \int_{-\pi}^{\pi} \Omega_t(w)\overline{\Omega_\tau(w)}.e^{iw(t-\tau)}dz_2(w). \tag{5.41}$$

Substituting (5.39) and (5.40) in (5.38) yields

$$\int_{-\pi}^{\pi} \Omega_t(w)\overline{\Omega_\tau(w)}.e^{iw(t-\tau)}dz(w) = \int_{-\pi}^{\pi} \Omega_t(w)\overline{\Omega_\tau(w)}.e^{iw(t-\tau)}\{dz_1(w) + dz_2(w)\}$$

$$\Rightarrow \qquad dz(w) = dz_1(w) + dz_2(w)$$

and hence $$f_t(w)dw = f_{1,t}(w)dw + f_{2,t}(w)dw. \tag{5.42}$$

Thus,

$$f_t(w) = f_{1,t}(w) + f_{2,t}(w). \tag{5.43}$$

It follows from (5.43) that the evolutionary cross-spectrum of the bivariate non-stationary process $\{X_t\} = [\{X_{1,t}\} \cup \{X_{2,t}\}]$ has a physical interpretation similar to the cross-spectrum of a bivariate stationary process. From (5.39), the variance of the series is obtained by setting $t = \tau$, and hence

$$\gamma_{12(t,t)} = \int_{-\pi}^{\pi} |\Omega_t(w)|^2 dz(w) \qquad \text{or} \qquad \text{var}(X_t) = \int_{-\pi}^{\pi} dF_t(w) = \int_{-\pi}^{\pi} f_t(w)dw. \tag{5.44}$$

If the functions $\Omega_t(w)$ are standardized so that for all ω, $\Omega_0(w) = 1$, then $f_t(w)d(w)$ is the spectrum of X_t at $t = 0$ and $|\Omega_t(w)|^2$ is the change in the spectrum with respect to $t = 0$.

5.6.1 Evolutionary Co-Spectrum and Evolutionary Quadrature Spectrum

Our discussions on the evolutionary cross-spectral analysis of the class of non-stationary bivariate processes cannot be concluded without talking about some other important functions as the evolutionary co-spectrum and the evolutionary quadrature spectrum.

Setting $\tau = t - k$, where the two series are separated by lag k, (5.39) becomes

$$\gamma_{12(t,t-k)} = \int_{-\pi}^{\pi} \Omega_t(w)\overline{\Omega_{t-k}(w)}.e^{iwk}dw \quad . \tag{5.40}$$

Then, by Fourier transform we can define from (5.42), the evolutionary cross-spectrum $f_t(w)$ as

$$f_t(w) = \tfrac{1}{2\pi} \int_{-\pi}^{\pi} \gamma_{12(t,t-k)}.e^{-iwk}dw \quad , \tag{5.46}$$

where $\gamma_{12(t,t-k)}$ is as defined in (5.45). In terms of sine and cosine, (5.46) becomes

$$f_t(w) = \tfrac{1}{2\pi} \int_{-\pi}^{\pi} \gamma_{12(t,t-k)}[\cos(wk) - i\sin(wk)]dw$$

$$\Rightarrow \quad f_t(w) = \tfrac{1}{2\pi} \int_{-\pi}^{\pi} \gamma_{12(t,t-k)}.\cos(wk)dw - i.\tfrac{1}{2\pi} \int_{-\pi}^{\pi} \gamma_{12(t,t-k)}.\sin(wk)dw \, . \tag{5.47}$$

Hence, we can write (5.46) as

$$f_t(w) = C_{12,t}(w) - i.Q_{12,t}(w), \tag{5.48}$$

where

$$C_{12,t}(w) = \tfrac{1}{2\pi} \int_{-\pi}^{\pi} \gamma_{2(t,t-k)}.\cos(wk)dw \quad , \tag{5.49}$$

and

$$Q_{12,t}(w) = \tfrac{1}{2\pi} \int_{-\pi}^{\pi} \gamma_{12(t,t-k)}.\sin(wk)dw, \tag{5.50}$$

are respectively the evolutionary co-spectrum and evolutionary quadrature spectrum of $X_{1,t}$ and $X_{2,t-k}$. Thus the evolutionary cross-spectrum $f_t(w)$ may be defined as the Fourier transform of the evolutionary cross-covariance function. The evolutionary co-spectrum between $X_{1,t}$ and $X_{2,t}$ is the evolutionary cross-covariance between *in-phase* components for a particular frequency, w.

Similarly, the evolutionary quadrature spectrum is the evolutionary cross-covariance between *out of phase* components for a particular frequency, w.

5.6.2 Evolutionary Cross-Amplitude and Evolutionary Phase Spectrum

The evolutionary cross-spectrum now has imaginary and real components. The sum of the squares of these components

$$A_t(w) = C_{12,t}^2(w) + Q_{12,t}^2(w), \tag{5.51}$$

becomes the evolutionary cross-amplitude. The evolutionary phase-spectrum is defined as

$$\varphi_t(w) = \tan^{-1}\left(\frac{-Q_{12,t}(w)}{C_{12,t}(w)}\right). \tag{5.52}$$

The evolutionary cross-amplitude represents measures the evolutionary cross-covariance between the components of the two series at a particular frequency, w, whilst the evolutionary phase spectrum gives lead-lag relationships for a particular frequency component.

5.6.3 Evolutionary Gain-Spectrum and Coherence

The evolutionary gain-spectrum $G_t(w)$ and the coherence $K_t^2(w)$ are respectively defined by

$$G_t(w) = \frac{|f_t(w)|}{f_{1,t}(w)} \quad \text{and} \quad K_t^2(w) = \frac{|f_t(w)|^2}{f_{1,t}(w).f_{2,t}(w)}. \tag{5.53}$$

From (5.53), we have

$$G_t(w) = \frac{|f_t(w)|}{f_{1,t}(w)} = \frac{|\text{cov}[dz_1(w), dz_2(w)]|}{\text{var}[dz_1(w)]}, \qquad (5.54)$$

which implies that the evolutionary gain function $G_t(w)$ is simply the absolute OLS regression coefficient of $X_{1,t}$ at a particular frequency, w.

Similarly, we can write (5.54) as

$$K_t^2(w) = \frac{|f_t(w)|^2}{f_{1,t}(w).f_{2,t}(w)} = \frac{\{\text{cov}[dz_1(w), dz_2(w)]\}^2}{\text{var}[dz_1(w)].\text{var}[dz_2(w)]}. \qquad (5.55)$$

From (5.55), we can describe $K_t^2(w)$ is simply the square of the evolutionary cross-covariance coefficient between the two series at a particular frequency, w. Thus, a value of $K_t^2(w)$ close to 1 implies that the w-frequency components of the two series are highly linearly related, whereas a value near zero implies they are slightly linearly related.

5.7 Summary

In this chapter, we were able to estimate the evolutionary spectrum or non-stationary time series with time changing-spectra, and also find relationships between pairs of such series, highly satisfactorily, employing the methods devised for stationary time series.

CHAPTER APPENDIX 5.1

```
data simdat;
   phi1 = 0.8;
   phi2 =-0.4;
     e2 = sqrt(10000)*rannor(0);
     e1 = phi1*e2 + sqrt(10000)*rannor(0);
    do t=1 to 120;
      y = phi1*e1 + phi2*e2 + sqrt(10000)*rannor(0);
     c1 = exp(-0.5*((20/200)**2));
     c2 = exp(-0.5*((40/200)**2));
     c3 = exp(-0.5*((60/200)**2));
     c4 = exp(-0.5*((80/200)**2));
     x1 = c1*y;
     x2 = c2*y;
     x3 = c3*y;
     x4 = c4*y;
    if t>20 then output;
     e2 = e1;
     e1 = y;
end;

title'Spectrum of the Stationary Series Y';
proc spectra data=simdat out=b p s adjmean whitetest;
   var y;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b;
run;
symbol1 i=splines v=none;
proc gplot data=b;
   plot p_01 * freq;
   plot s_01 * freq;
run;

title'Evolutionary Spectrum X when t=20';
proc spectra data=simdat out=b1 p s adjmean whitetest;
   var x1;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b1;
run;
symbol1 i=splines v=none;
proc gplot data=b1;
   plot p_01 * freq;
   plot s_01 * freq;
run;

title'Evolutionary Spectrum of X when t=40';
proc spectra data=simdat out=b2 p s adjmean whitetest;
   var x2;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b2;
run;
symbol1 i=splines v=none;
proc gplot data=b2;
   plot p_01 * freq;
   plot s_01 * freq;
run;
```

```
title'Evolutionary Spectrum of X when t=60';
proc spectra data=simdat out=b3 p s adjmean whitetest;
   var x3;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b3;
run;
symbol1 i=splines v=none;
proc gplot data=b3;
   plot p_01 * freq;
   plot s_01 * freq;
run;

title'Evolutionary Spectrum of X when t=80';
proc spectra data=simdat out=b4 p s adjmean whitetest;
   var x4;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b4;
run;
symbol1 i=splines v=none;
proc gplot data=b4;
   plot p_01 * freq;
   plot s_01 * freq;
run;
quit;
```

CHAPTER 6

MONTE CARLO STUDY OF THREE COMMONLY USED UNIT ROOT TESTS

6.1 Introduction

Statistical properties of many estimation and hypothesis test procedures considered are known only asymptotically. This holds true in all types of models. The accuracy of asymptotic theory plays a vital role when it comes to interpreting such results. Basically, the accuracy of asymptotic theory is determined by the sample size - the larger the sample, the more the accuracy to allow us to interpret results with confidence. Due to the accuracy of asymptotic theory, interpretations of such estimates as parameter and test statistics based on exact finite-sample are rarely used. One way to deal with the accuracy of the asymptotic theory is to investigate the finite sample properties of estimators and test statistics by using Monte Carlo simulations. In this approach, quantities of interest are approximated by generating many random realizations of some statistical process and averaging them in some way.

In this chapter, we apply Monte Carlo simulations to study the performance of three unit root tests procedures that are frequently used in practice. This include the Augmented Dickey-Fuller (ADF), the Phillips-Perron (PP), and the Reverse Dickey-Fuller Unit Root (RDFUR) unit root test procedures.

6.2 Designing Monte Carlo Simulations

The number of replications performed in Monte Carlo studies differ in different situations. If the researcher is interested in calculating the size of a test statistic (that is the probability p of rejecting the null hypothesis when it is true) at some nominal level, the situation can be viewed as independent Bernoulli trials. Let this nominal level be 0.05. In this case if for R replications r rejections are obtained, then the estimate of p is $\hat{p} = r/R$. Let's allow the 95% confidence interval on p to have a length of 0.015. Then using the normal approximation to the binomial, the confidence interval covers $2 \times 1.96 = 3.92$ standard errors. We therefore seek the relation

$$3.92\left[\frac{p(1-p)}{R}\right]^{1/2} = 0.015,$$

or $$R = p(1-p)\left(\frac{3.92}{0.02}\right)^2 \text{ replications.} \qquad (6.1)$$

Throughout our Monte Carlo study, we shall use a nominal level of 0.05. This means that the 95% confidence interval on p requires roughly 2000 replications. If, however, the aim of the researcher is to compare two or more test statistics or estimators, a smaller number of replications gives the same level of accuracy as a larger one. All simulations will be carried out in SAS using the macro processor *%macro* with the PROC ARIMA statement to obtain pseudo-random variates.

6.3 Test Criteria

Given the series $\{X_t : t = 1,2,\ldots,T\}$, the standard univariate AR(1) process is given by

$$X_t - \mu = \rho(X_{t-1} - \mu) + \varepsilon_t \qquad \{\varepsilon_t\} \sim WN(0,\sigma^2), \qquad (6.2)$$

where μ is the mean of the series. If $\mu = 0$, we obtain the AR(1) process

$$X_t = \rho X_{t-1} + \varepsilon_t. \qquad (6.3a)$$

Subtracting X_{t-1} from both sides of (6.3a) equation yields

$$Z_t = (\rho - 1)X_{t-1} + \varepsilon_t, \qquad (6.3b)$$

where $Z_t = X_t - X_{t-1}$. If $\mu \neq 0$, we have an AR(1) process with drift

$$X_t = C + \rho X_{t-1} + \varepsilon_t, \qquad (6.4a)$$

where $C = (1-\rho)\mu$. Again, subtracting X_{t-1} from both sides of (6.4a) yields

$$Z_t = C + (\rho - 1)X_{t-1} + \varepsilon_t, \qquad (6.4b)$$

where $Z_t = X_t - X_{t-1}$. Lastly, if we replace μ by a linear trend, we have

$$X_t - [\delta + \gamma t] = \rho\{X_{t-1} - [\delta + \gamma(t-1)]\} + \varepsilon_t$$

$$\Rightarrow \qquad X_t = \beta_0 + \beta_1 t + \rho X_{t-1} + \varepsilon_t, \qquad (6.5a)$$

where $\beta_0 = \delta - \delta\rho + \rho\gamma$ and $\beta_1 = \gamma - \gamma\rho$. Subtracting X_{t-1} from both sides of (6.5a) gives

$$Z_t = \beta_0 + \beta_1 t + (\rho - 1)X_{t-1} + \varepsilon_t, \qquad (6.5b)$$

where $Z_t = X_t - X_{t-1}$. In our Monte Carlo study, we simulate the finite sample null distribution of the three test statistics using the data generating process (DGP)

$$X_t = \rho X_{t-1} + \varepsilon_t, \quad X_0 = 0 \quad \{\varepsilon_t\} \sim WN(0,1), \qquad (6.6)$$

for $t = 1,2,3\ldots,T$. The AR parameter ρ assumes the values 1, 0.9, 0.8, 0.7, and 0.6. The null hypothesis that $H_o\!:\rho = 1$ is of utmost importance in applications in that it tells whether it is appropriate to transform the series by differencing or not. The test statistics will be calculated from fitted regressions which includes a constant to ensure invariance to their respective starting values X_0 (Leybourne, 1995). We also obtain test statistics based on fitted regressions that include a trend. In a same manner, the inclusion of a trend makes the test statistics invariant to a non-zero drift in the simulated DGP, (6.6). We study the sampling distribution based on 2000 replications of each AR process. Each simulation is performed for $T = 100$ observations. Our Monte Carlo simulations will be used to study the sampling distributions, the size, and the powers of three unit root test statistics.

Although we consider studying all three processes, we shall pay particular attention to only two models, (6.4a) and (6.5a), because in practice they are regarded as realistic data descriptions rather than a zero-drift random walk process. All SAS programs used to obtain these simulation results are found in Chapter Appendices 6.1-6.4.

6.3.1 Augmented Dickey-Fuller (ADF) Unit Root Test Criterion

For the processes (6.3b), (6.4b), and (6.5b), the Augmented Dickey-Fuller (ADF) unit root test statistic is given by

$$\hat{\tau}_{df} = \frac{\hat{\rho} - 1}{Se(\hat{\rho} - 1)}, \qquad (6.7)$$

where $Se(\hat{\rho} - 1)$ is the standard error of the coefficient of X_{t-1}.

For $\rho = 1$, the percentiles of the asymptotic distribution of (6.7) based on each process have been reported by Fuller (1976).

6.3.2 Phillips-Perron (PP) Unit Root Test Criterion

The test regressions for the Phillips-Perron (PP) test are the same as those for the ADF test listed in (6.2). Correcting for higher order serial correlation, the PP test adjusts the t-statistic of the $(\rho - 1)$ coefficient from the AR(1) regression to account for the serial correlation that may be in the error term, ε. The test procedure employs the Newey - West heteroskedasticity autocorrelation consistent estimates of the variances of the residuals in a regression model, σ^2, and in the true population, σ_*^2

$$\hat{\sigma}^2 = \tfrac{1}{T}\sum_{t=1}^{T} \varepsilon_t^2 , \tag{6.8}$$

and

$$\hat{\sigma}_*^2 = \hat{\sigma}^2 + 2\sum_{k=1}^{N}\left[1 - \tfrac{1}{(N+1)}\right]\hat{\gamma}_k , \tag{6.9}$$

where N is the relevant number of autocovariances, and $\gamma_k = \sum \varepsilon_t \varepsilon_{t-k}$ for $t = k+1,\ldots,T$. The PP test statistic is computed as

$$\hat{\tau}_{pp} = \left(\frac{\hat{\sigma}_*^2}{\hat{\sigma}^2}\right)\hat{\tau}_{df} - \frac{T}{2}\left(\frac{(\hat{\sigma}_*^2 - \hat{\sigma}^2)}{\sqrt{\sigma_*^2 \sum_{t=2}^{T}(X_{t-1} - \mu)^2}}\right), \tag{6.10}$$

where μ is the mean of $X_2, X_3, \ldots, X_T$, and $\hat{\tau}_{df}$ is the ADF test statistic given in (6.7). The asymptotic distribution of the PP test statistic is the same as the ADF test statistic. Based on number of observations used in the test regression, the Newey-West automatic truncation lag selection N is given by

$$N = \left[4\left(\frac{T}{100}\right)^{2/9}\right], \tag{6.11}$$

where [ ] refers to the largest integer not exceeding the argument.

6.3.3 The RDFUR Unit Root Test Criterion

The Reverse Dickey-Fuller Unit Root (RDFUR) test criterion is performed the same way as the ADF test criterion. The only exception here is that the series is reversed before it is analysed.

6.4 Sampling Distributions

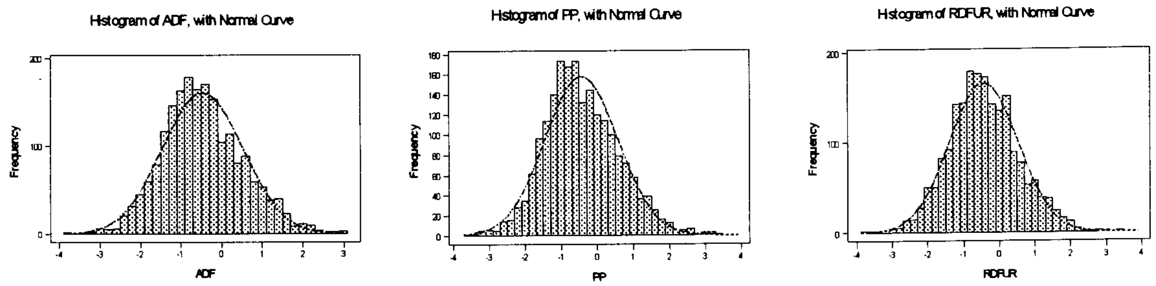
In this sub-section, we compare the sampling distributions for the test statistics. We simulated samples of size $T = 100$ using the data generating process (DGP), the random walk with no drift

$$X_t = \rho X_{t-1} + \varepsilon_t, \qquad X_0 = 0 \qquad \{\varepsilon_t\} \sim WN(0,1).$$

We then include a constant term and a linear time trend to obtain the processes (6.3a), (6.4a), and (6.5a). For convenience, we shall refer to these processes as Case I, Case II, and Case III, respectively. The sampling distributions are summarized in Table 6.1. The table was generated with 2000 replications. Figures 6.1, 6.2, and 6.3 display the histogram-normal distributions of the three test statistics based on the three cases are reported. A visual examination of the plots in figures 6.1 - 6.3 seem to be approximately normally.

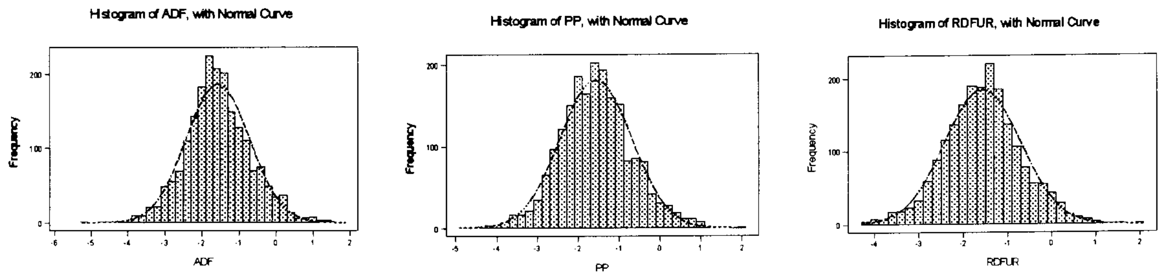
Table 6.1: Sampling Distributions for ADF, PP, and RDFUR Test Statistics, with $\rho = 1$

Case	Test Statistic	Mean	Median	Standard Deviation	Skewness	Kurtosis	Jarque-Bera Statistic	Jarque-Bera p Value	Fraction of False Rejection
I	ADF PP RDFUR	-0.4572 -0.4303 -0.4240	-0.5434 -0.5374 -0.4822	0.9980 1.0140 0.965	0.2922 0.3504 0.2212	3.2340 3.242 3.1482	33.0320 45.8236 18.1331	0.0000 0.0000 0.0001	0.0540 0.0475 0.0515
II	ADF PP RDFUR	-1.5574 -1.5665 -1.5529	-1.5994 -1.5828 -1.5508	0.8577 0.8804 0.8641	0.1864 0.1746 0.0810	3.4401 3.3980 3.3504	27.7157 23.3604 12.4193	0.0000 0.0000 0.0020	0.0570 0.0530 0.0545
III	ADF PP RDFUR	-2.1980 -2.2706 -2.2201	-2.1673 -2.2586 -2.1944	0.7835 0.7992 0.7981	-0.0434 0.0920 0.0380	3.3960 3.6026 3.3002	13.6957 33..0888 4.1547	0.0011 0.0000 0.1253	0.0530 0.0730 0.0640



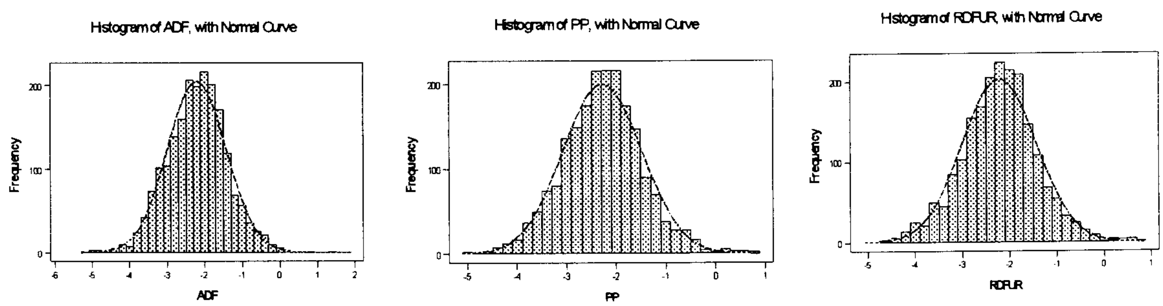
(6.1a) (6.1b) (6.1c)

Fig. 6.1: Histogram-Normal Distributions of test statistics ADF-(6.1a), PP-(6.1b), and RDFUR-(6.1c) Based on Case I



(6.2a) (6.2b) (6.2c)

Fig. 6.2: Histogram-Normal Distributions of test statistics ADF-(6.2a), PP-(6.2b), and RDFUR-(6.2c) Based on Case II



(6.3a) (6.3b) (6.3c)

Fig. 6.3: Histogram-Normal Distributions of test statistics ADF-(6.3a), PP-(6.3b), and RDFUR-(6.3c) Based on Case III

However, since for a normal distribution the value of skewness is 0, and the value of kurtosis is 3, we conclude that none of the test statistics is normally distributed. This is confirmed by the fact that p- values of the Jarque-Bera are smaller than the nominal level of 0.05. The results from Table 6.1 indicate that with the exception of the ADF test statistic which is slightly negatively skewed in Case III, the rest are right-skewed based on the three cases.

6.5 Empirical Size Comparisons

In this section, we investigate the sizes of our test statistics at the nominal 0.05 level - that is the probability of rejecting the null hypothesis of a unit root when it is in fact true. Using R = 2000 replications, the proportions of wrongful rejections (the probability of committing a Type I error) are reported in the last column of Table 6.1. Table values supplied by Fuller (1976) were used for the tests. The estimated sizes are all close to 0.05. The largest deviations were obtained in Case III for PP and RDFUR. The observed sizes tend to be slightly larger than the target size for the test. In Case I and Case II, we find that the probability of our committing a Type I error is least in Phillip-Perron (PP) unit root test procedure (ie 4.75%) compared with the other two test procedures. Even here, the RDFUR test procedure performs better than the celebrated ADF test procedure. However, the PP test has its worst performance in Case III, with the ADF test procedure performing best in this case.

The last column of Table 6.1 shows in all the three case, Case I, Case II, and Case III, that the probability of committing a Type I error (fraction of false rejection) was greater than our nominal level of 0.05 for all the three test statistics, except in one case. Using this conventionally acceptable nominal level of 0.05, we conclude that the null hypothesis of a unit root will be rightly accepted using these three test statistics.

6.6 Empirical Power Comparisons

Our study of empirical power comparisons will be based entirely on the probability of committing a Type II error - that is the probability of accepting the null hypothesis of a unit root while it is in fact false.In other words, our power comparisons will be based on the ability of a test statistic to correctly reject a false null hypothesis. Here, we simulate $T = 50, 100, 250$ observations Case I, Case II, and Case III, but $\rho < 1$. Specifically, we consider the cases where $\rho = 0.9, 0.8, 0.7$, and 0.6. The null hypothesis we consider is $H_0 : \rho = 1$, the false hypothesis.

Since comparing two or more test statistics does not necessarily require larger replications, simulation results will be generated using 1000 replications. Table 6.2 reports the proportion of accepting the false null hypothesis of a unit root for the three test statistics. There are several conclusions to be drawn from the results presented in Table 6.2. First, we recognise that for each case and each sample size T, power decreases as ρ increases monotonically. This is not surprising because we know from theory that the closer the true value of ρ is to the hypothesized value, the greater the probability of a Type II error, and hence the lesser the power (power is 1 minus the probability of committing a Type II error).

Table 6.2: Empirical Power of ADF, PP, and RDFUR Tests

			ρ			
Sample Size T	Case	Test	0.9	0.8	0.7	0.6
50	I	ADF PP RDFUR	0.306 0.360 0.334	0.707 0.801 0.686	0.920 0.967 0.920	0.985 0.998 0.973
50	II	ADF PP RDFUR	0.106 0.129 0.113	0.270 0.361 0.257	0.505 0.678 0.490	0.717 0.907 0.701
50	III	ADF PP RDFUR	0.089 0.097 0.086	0.154 0.205 0.171	0.316 0.434 0.311	0.467 0.715 0.473
100	I	ADF PP RDFUR	0.714 0.766 0.732	0.993 0.998 0.993	1.000 1.000 1.000	1.000 1.000 1.000
100	II	ADF PP RDFUR	0.293 0.351 0.278	0.818 0.879 0.787	0.979 0.996 0.971	0.997 1.000 0.999
100	III	ADF PP RDFUR	0.100 0.125 0.167	0.416 0.573 0.556	0.756 0.946 0.879	0.930 1.000 0.984
250	I	ADF PP RDFUR	1.000 0.999 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 0.999 1.000
250	II	ADF PP RDFUR	0.935 0.957 0.950	1.000 1.000 1.000	1.000 1.000 1.000	1.000 0.999 1.000
250	III	ADF PP RDFUR	0.775 0.856 0.796	1.000 1.000 0.999	1.000 1.000 1.000	1.000 0.999 1.000

Second, the PP test statistics appear to be more powerful compared with the ADF and RDFUR test statistics. Third, for $T = 50$, the performance of ADF and RDFUR were similar, no matter the case. Detailed graphical representations of the empirical powers for the three cases are given for figures 6.4a, 6.4b, and 6.4c. (see Chapter Appendix 6.5A for the programs used to obtain these graphs).

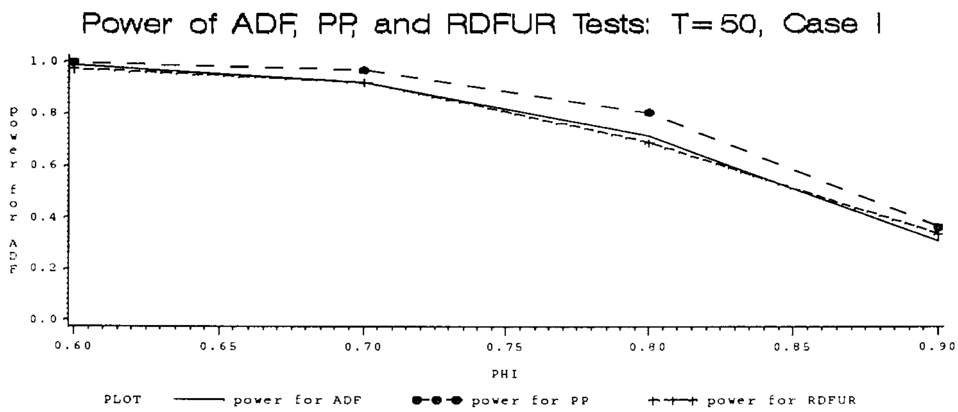


Fig. 6.4a: Empirical Power of ADF, PP, and RDFUR Tests - $T = 50$, Case I

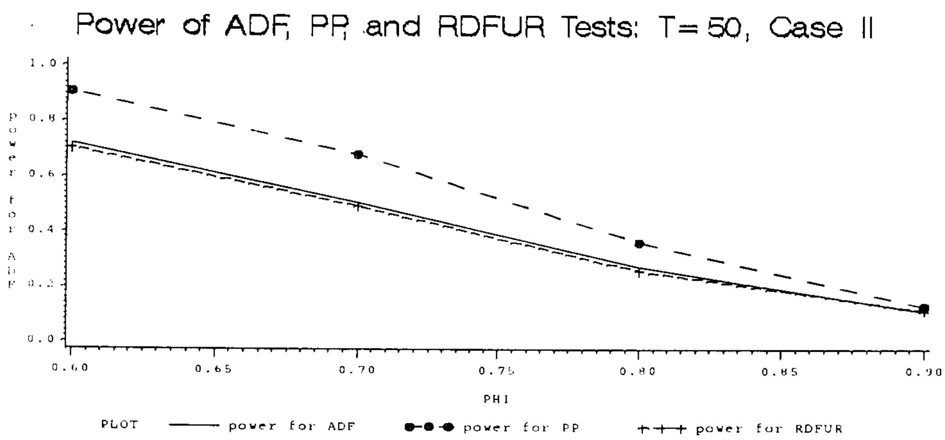


Fig. 6.4b: Empirical Power of ADF, PP, and RDFUR Tests - $T = 50$, Case II

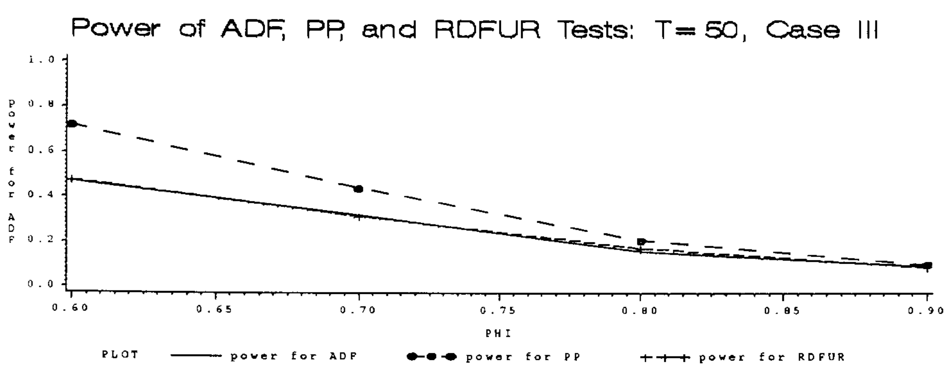


Fig. 6.4c: Empirical Power of ADF, PP, and RDFUR Tests - $T = 50$, Case III

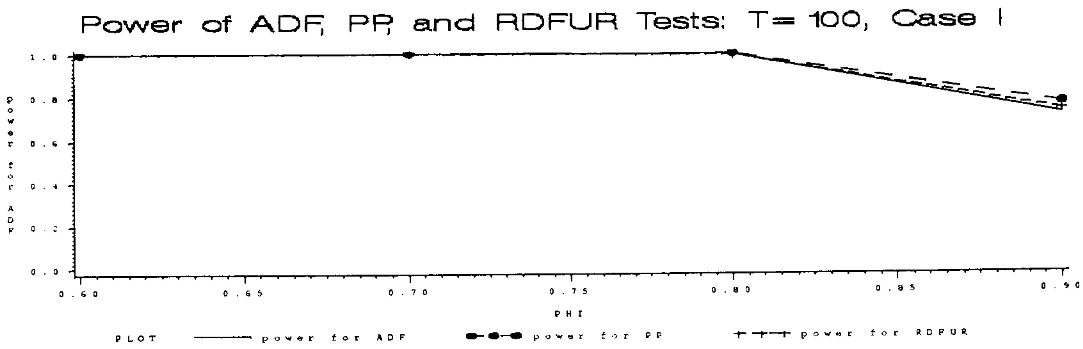


Fig. 6.5a: Empirical Power of ADF, PP, and RDFUR Tests - $T = 100$, Case I

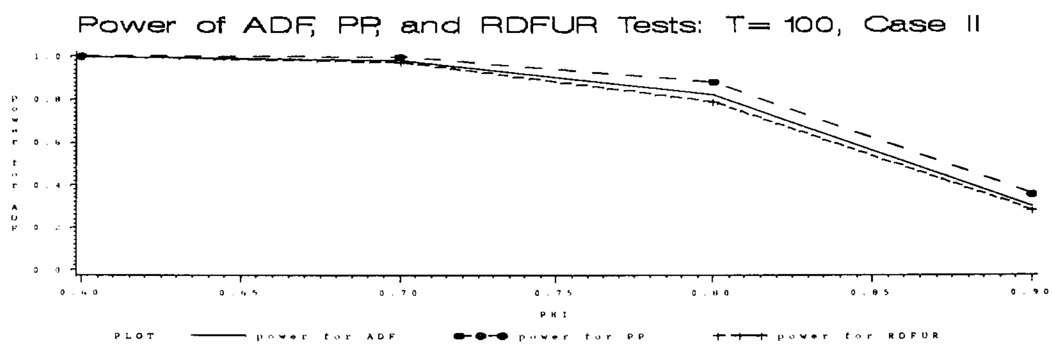


Fig. 6.5b: Empirical Power of ADF, PP, and RDFUR Tests - $T = 100$, Case II

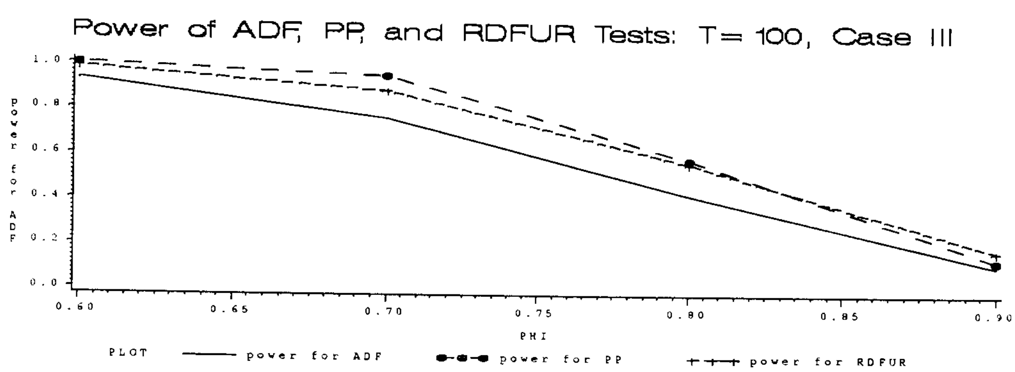


Fig. 6.5c: Empirical Power of ADF, PP, and RDFUR Tests - $T = 100$, Case III

Similar conclusions can be drawn from the case where $T = 100$. A look at Fig. 6.5a and Fig. 6.5b (Case I and Case II) suggest that the performances of the three test statistics are virtually similar, even though PP appears to be more powerful amongst the three. For $T = 100$, ADF has its worst performance whilst PP continues to prove its dominance.

Lastly, it is evident from Fig. 6.6 ($T = 250$) that while all the threes tests seem to perform equally powerful for $\rho \leq 0.8$ in all three cases, PP continues to be more powerful for $0.8 < \rho < 1$ in Case II and Case III. Furthermore, we find that in Case II and Case III, RDFUR performs better than the celebrated ADF for $0.8 < \rho < 1$, on the average.

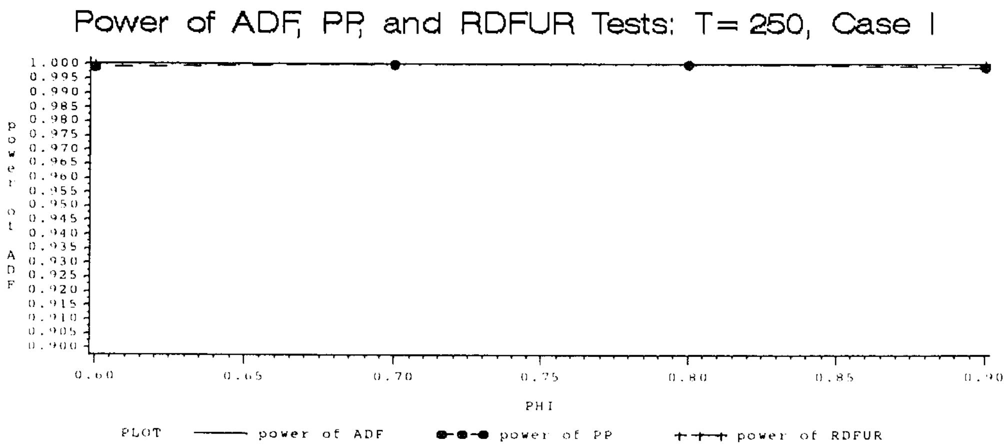


Fig. 6.6a: Empirical Power of ADF, PP, and RDFUR Tests - $T = 250$, Case I

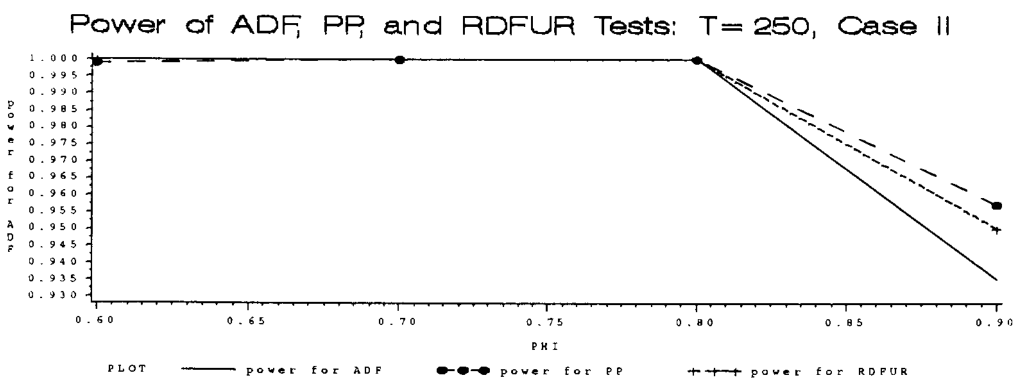


Fig. 6.6b: Empirical Power of ADF, PP, and RDFUR Tests - $T = 250$, Case II

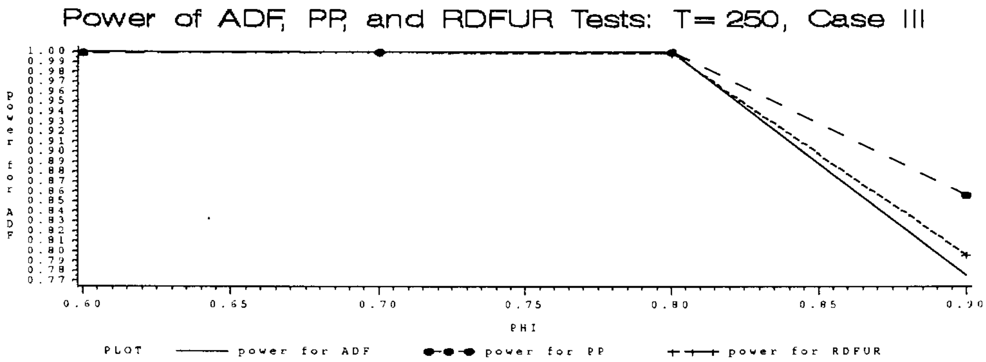


Fig. 6.6c: Empirical Power of ADF, PP, and RDFUR Tests - $T = 250$, Case III

Table 6.3a: Empirical Power of ADF, PP, and RDFUR Tests Based on Case I

		ρ			
Test	Sample Size T	0.9	0.8	0.7	0.6
ADF	50	0.306	0.707	0.920	0.985
	100	0.714	0.993	1.000	1.000
	250	1.000	1.000	1.000	1.000
PP	50	0.360	0.801	0.967	0.998
	100	0.766	0.998	1.000	1.000
	250	0.999	1.000	1.000	0.999
RDFUR	50	0.334	0.686	0.920	0.973
	100	0.732	0.993	1.000	1.000
	250	1.000	1.000	1.000	1.000

Table 6.3b: Empirical Power of ADF, PP, and RDFUR Tests Based on Case II

		ρ			
Test	Sample Size T	0.9	0.8	0.7	0.6
ADF	50	0.106	0.270	0.505	0.717
	100	0.293	0.818	0.979	0.997
	250	0.935	1.000	1.000	1.000
PP	50	0.129	0.361	0.678	0.907
	100	0.351	0.879	0.996	1.000
	250	0.957	1.000	1.000	0.999
RDFUR	50	0.113	0.257	0.490	0.701
	100	0.278	0.787	0.971	0.999
	250	0.950	1.000	1.000	1.000

Table 6.3c: Empirical Power of ADF, PP, and RDFUR Tests Based on Case III

		ρ			
Test	Sample Size T	0.9	0.8	0.7	0.6
ADF	50	0.089	0.154	0.316	0.467
	100	0.100	0.416	0.756	0.930
	250	0.775	1.000	1.000	1.000
PP	50	0.097	0.205	0.311	0.715
	100	0.125	0.573	0.946	1.000
	250	0.856	1.000	1.000	0.999
RDFUR	50	0.086	0.171	0.311	0.473
	100	0.167	0.556	0.879	0.984
	250	0.796	0.999	1.000	1.000

In Table 6.3, we summarize the empirical powers for the three test for easy assimilation. We also include plots of these powers in Fig. 6.7, Fig. 6.8, and Fig. 6.9. We conclude from Fig. 6.7-Fig.6.9 that based on sample size, the powers increase with sample size T (see Chapter Appendix 6.5B for the programs used to obtain the graphs).

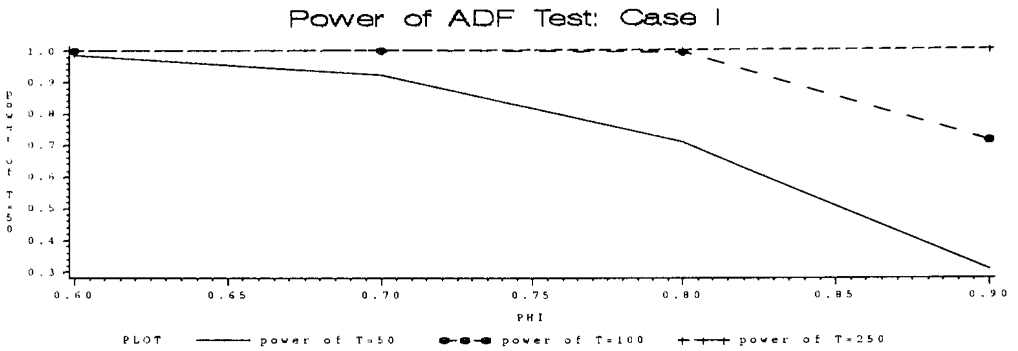


Fig. 6.7a: Size-Power Study of ADF Test - Case I

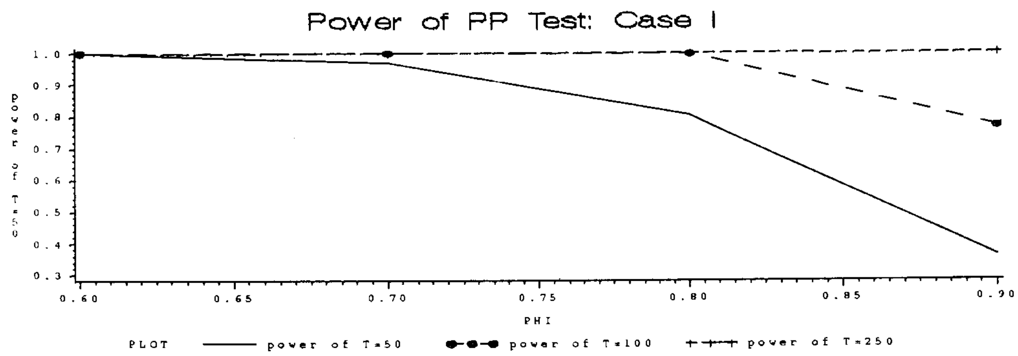


Fig. 6.7b: Size-Power Study of PP Test - Case I

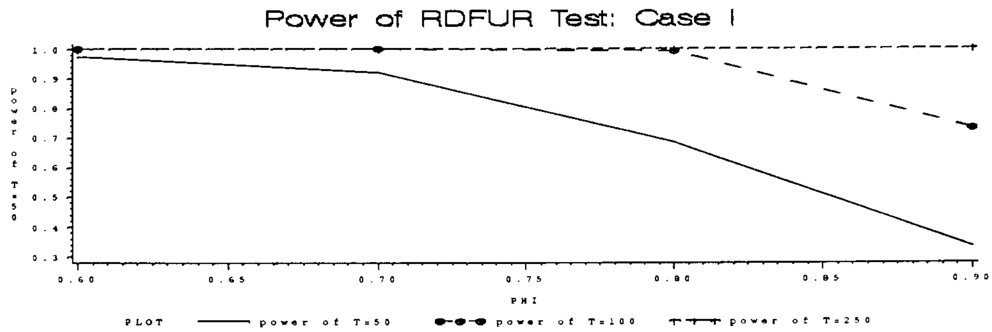


Fig. 6.7c: Size-Power Study of RDFUR Test - Case I

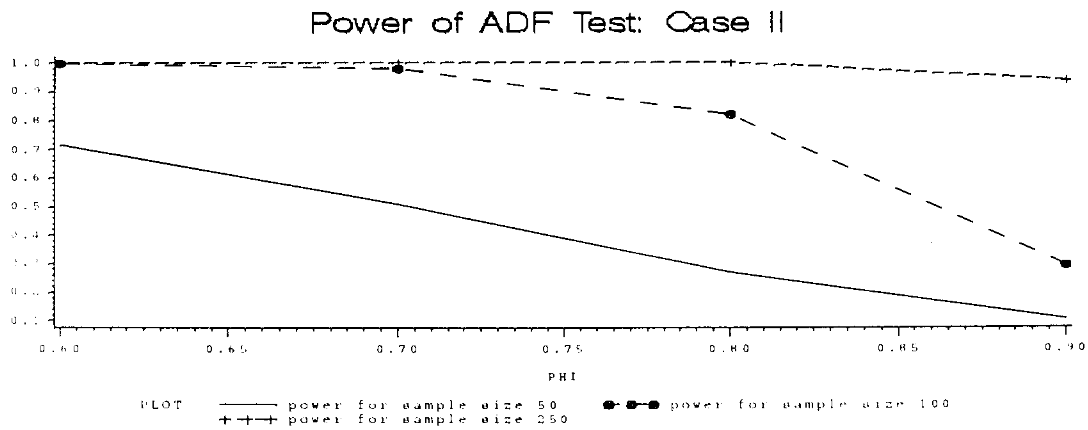


Fig. 6.8a: Size-Power Study of ADF Test - Case II

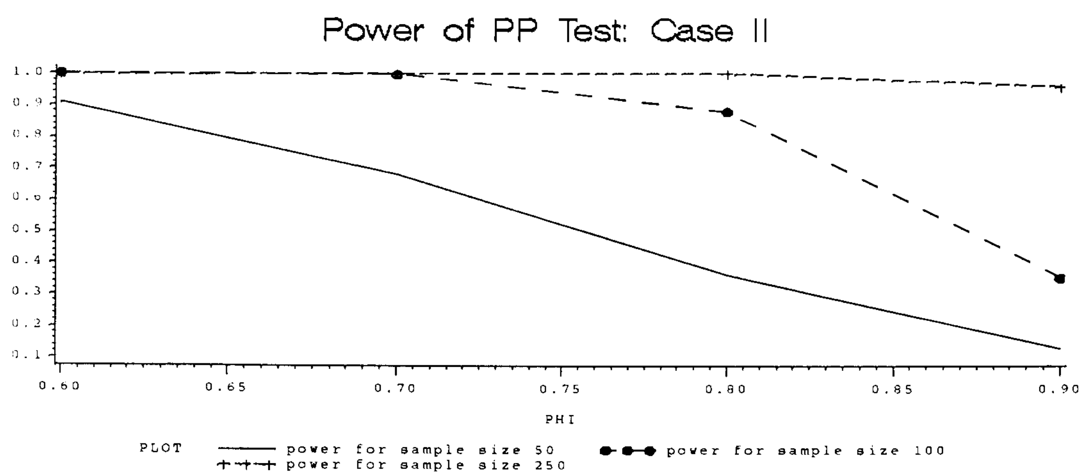


Fig. 6.8b: Size-Power Study of PP Test - Case II

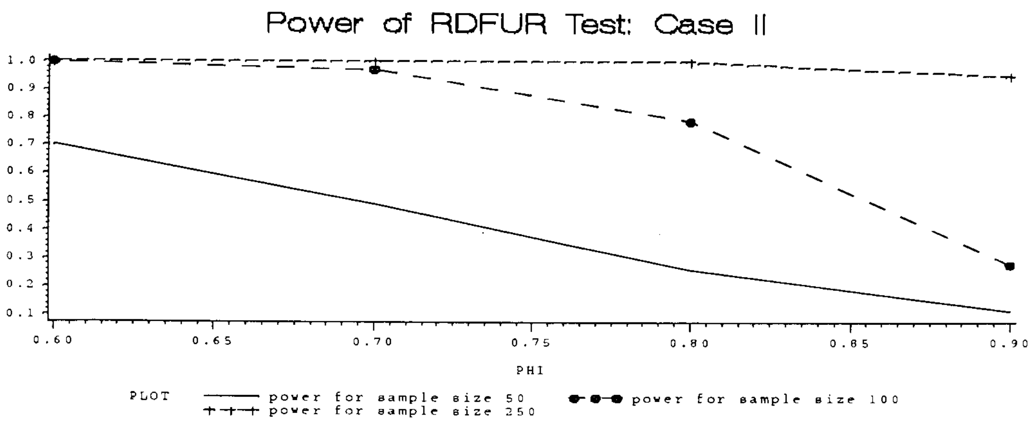


Fig. 6.8c: Size-Power Study of RDFUR Test - Case II

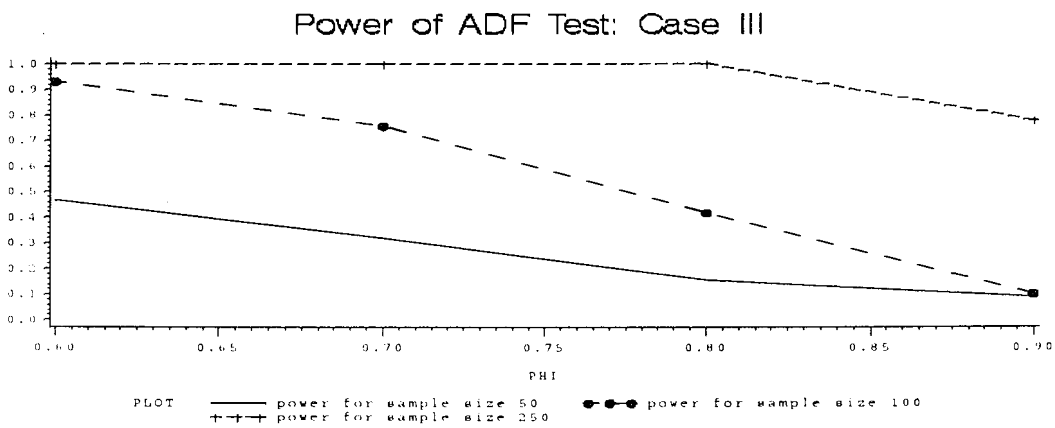


Fig. 6.9a: Size-Power Study of ADF Test - Case III

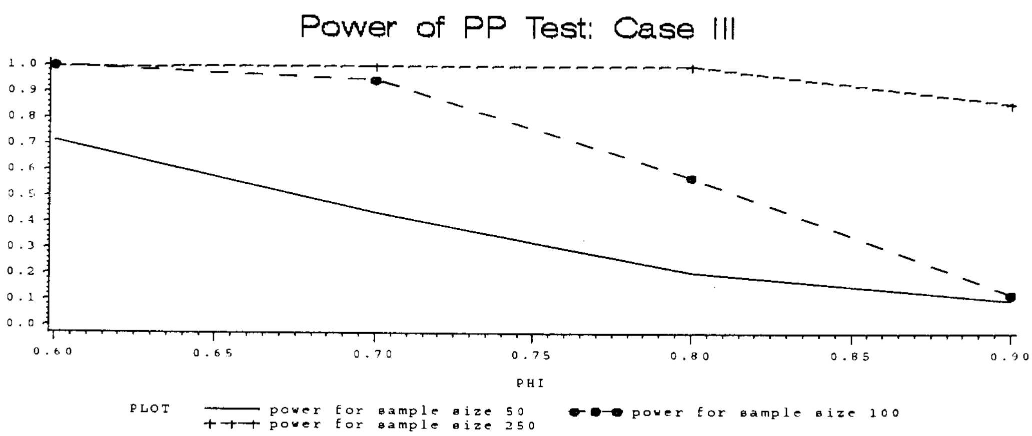


Fig. 6.9b: Size-Power Study of PP Test - Case III

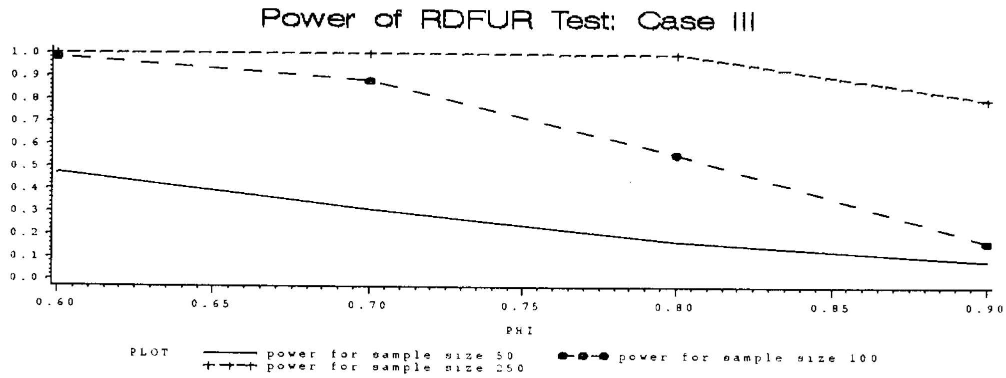


Fig. 6.9c: Size-Power Study of RDFUR Test - Case III

6.7 Conclusion

In this chapter, we considered Monte Carlo studies of three most commonly used unit root tests, namely the Augmented Dickey-Fuller (ADF), the Phillips-Perron (PP), and the Reversed Dickey-Fuller Unit Root (RDFUR) tests. The study revealed that among these three test procedures, the most powerful test is the PP test. We also established that these three test statistics are powerful enough to accept(reject) the true(false) null hypothesis of a unit root in most time series. Furthermore, our simulation evidence suggests that the power increases with sample size T.

CHAPTER APPENDIX 6.1

Program 6.1A: Simulating Pseudo-Random Variables for ADF Test

```
%macro sim(num);
%do i=1 %to 2000;                /*2000 simulations for ρ=1 and 1000 simulations for ρ=0.9,0.8,0.7,0.6*/
data sim&num;
   phi = 1.0;                    /*phi=1.0 for 2000 simulations and phi=0.9,0.8,0.7,0.6 for 1000 simulations*/
     a = sqrt(1)*rannor(0);
  do t = 1 to 100;               /*100 observations. Also used 50 and 250 observations where appropriate*/
     x = phi*a + sqrt(1)*rannor(0);
     a = x;
   if t>0 then output;
end;

proc arima data=sim&num;
   identify var=x stationarity=(ADF=1);   /*Computes test statistics by adding no constant to the AR(1) process,
run;                                        by adding a constant to the AR(1), and by adding a linear trend to
                                            the AR(1) process */

proc transpose out=a&num ;
run;

data b&num;
 set a&num;
 proc append base=combo data=b&num;
%end;
run;
%mend;
%sim;
run;
```

Program 6.1B: Simulating Pseudo-Random Variables for PP Test

```
%macro sim(num);
%do i=1 %to 2000;                /*2000 simulations for ρ=1 and 1000 simulations for ρ=0.9,0.8,0.7,0.6*/

data sim&num;
   phi = 1.0;                    /*phi=1.0 for 2000 simulations and phi=0.9,0.8,0.7,0.6 for 1000 simulations*/
     a = sqrt(1)*rannor(0);
  do t = 1 to 100;               /*100 observations. Also used 50 and 250 observations where appropriate*/
     x = phi*a + sqrt(1)*rannor(0);
     a = x;
   if t>0 then output;
end;

proc arima data=sim&num;
   identify var=x stationarity=(PP=4);  /*Computes test statistics by adding no constant to the AR(1) process,
run;                                    by adding a constant to the AR(1) process, and by adding a linear trend
                                         to the AR(1) process. Newey-West suggests PP=3 for 50 observations and
                                         PP=4 for 100 and 250 observations */

proc transpose out=a&num ;
run;

data b&num;
 set a&num;
 proc append base=combo data=b&num;
%end;
run;
%mend;
%sim;
run;
```

Program 6.1C: Simulating Pseudo-Random Variables for RDFUR Test

```
%macro sim(num);
%do i=1 %to 2000;                /*2000 simulations for ρ=1 and 1000 simulations for ρ=0.9,0.8,0.7,0.6*/
data sim&num;
   phi = 1.0;                    /*phi=1.0 for 2000 simulations and phi=0.9,0.8,0.7,0.6 for 1000 simulations*/
     a = sqrt(1)*rannor(0);
  do t = 1 to 100;                   /*100 observations. Also used 50 and 250 observations where appropriate*/
     x = phi*a + sqrt(1)*rannor(0);
     a = x;
   if t>0 then output;
end;

proc sort data=sim&num out=rev&num;   /*Reverses the Simulated Series, ie it turns the series upside-down*/
   by descending t;

proc arima data=sim&num;
   identify var=x stationarity=(ADF=1);   /*Computes test statistics by adding no constant to the AR(1) process,
run;                                          by adding a constant to the AR(1) process, and by adding a linear
                                              trend to the AR(1) process */

proc transpose out=a&num ;
run;

data b&num;
 set a&num;
 proc append base=combo data=b&num;
%end;
run;
%mend;
%sim;
run;
```

CHAPTER APPENDIX 6.2

Programs for Selecting Only the Test Statistics from the *SAS* Output (ADF)

Program 6.2A: Based on T = 50 Observations

```
data a;
 infile 'c:\mcadf50\0.9.dat';   /* 0.9 means when ρ=0.9. It's replaced by 1.0,0.8,0.7,0.6 where appropriate*/
 input #79 @48 tadf1 / @48 tadf2 / / / / @48 tadf3 #88;
   ADFI=tadf1;      /* With No Constant */
  ADFII=tadf2;      /* With Constant */
 ADFIII=tadf3;      /* With Linear Trend */
   keep ADFI ADFII ADFIII;
 proc print;
 run;
```

Program 6.2B: Based on T = 100 Observations

```
data b;
 infile 'c:\mcadf100\0.9.dat';
 input #121 @48 tadf1 / / / / @48 tadf2 / / / @48 tadf3 #130;
   ADFI=tadf1;        /* With No Constant */
  ADFII=tadf2;        /* With Constant */
 ADFIII=tadf3;        /* With Linear Trend */
   keep ADFI ADFII ADFIII;
 proc print;
 run;
```

Program 6.2C: Based on T = 250 Observations

```
data c;
 infile 'c:\mcadf250\0.9.dat';    /* 0.9 means when ρ=0.9. It's replaced by 1.0,0.8,0.7,0.6 where appropriate*/
 input #121 @48 tadf1 / / / / @48 tadf2 / / / @48 tadf3 #130;
   ADFI=tadf1;            /* With No Constant */
  ADFII=tadf2;            /* With Constant */
 ADFIII=tadf3;            /* With Linear Trend */
   keep ADFI ADFII ADFIII;
 proc print;
 run;
```

CHAPTER APPENDIX 6.3

Programs for Selecting Only the Test Statistics from the *SAS* Output (PP)

Program 6.3A: Based on T = 50 Observations

```
data a;                          /* 0.9 means when ρ=0.9. It's replaced by 1.0,0.8,0.7,0.6 where appropriate*/
 infile 'c:\mcpp50\0.9.dat';
 input #82 @59 tpp1 / / / / / / @59 tpp2 / / / / / @59 tpp3 #94;
   PPI=tpp1;                        /* With No Constant */
  PPII=tpp2;                        /* With Constant */
 PPIII=tpp3;                        /* With Linear Trend */
   keep PPI PPII PPIII;
 proc print;
 run;
```

Program 6.3B: Based on T = 100 Observations

```
data b;                             /* 0.9 means when ρ=0.9. It's replaced by 1.0,0.8,0.7,0.6 where appropriate*/
 infile 'c:\mcpp100\0.9.dat';
 input #125 @59 tpp1 / / / / / / / / / / / / @59 tpp2 / / / / / / @59 tpp3 #144;
   PPI=tpp1;               /* With No Constant */
  PPII=tpp2;               /* With Constant */
 PPIII=tpp3;               /* With Linear Trend */
   keep PPI PPII PPIII;
 proc print;
 run;
```

Program 6.3C: Based on T = 250 Observations

```
data c;
 infile 'c:\mcpp250\0.9.dat';
 input #125 @59 tpp1 / / / / / / / / / / / / @59 tpp2 / / / / / / @59 tpp3 #144;
   PPI=tpp1;                 /* With No Constant */
  PPII=tpp2;                 /* With Constant */
 PPIII=tpp3;                 /* With Linear Trend */
   keep PPI PPII PPIII;
proc print;
run;
```

CHAPTER APPENDIX 6.4

Programs for Selecting Only the Test Statistics from the *SAS* Output (RDFUR)

Same programs as those used for the ADF case in Chapter Appendix 6.2. The only difference is that the series is reversed
by the statements

```
proc sort data=sim&num out=rev&num;
   by descending t;
```

in Program 6.1C before analysis.

CHAPTER APPENDIX 6.5

Program for Plotting the Power Graphs

Program 6.5A: Power Graphs Based on Various Test Statistics

```
data rino;
   input phi pw1 pw2 pw3 @@;
   keep phi pw1-pw3;
 label pw  = 'power'
       pw1 = 'power for ADF'
       pw2 = 'power for PP'
       pw3 = 'power for RDFUR';
cards;
 0.9     -       -        -
 0.8     -       -        -        /* The dashes are the appropriate powers at various ρ */
 0.7     -       -        -
 0.6     -       -        -
;

proc gplot;
   plot (pw1-pw3)*phi / overlay vaxis=r to 1 by s legend haxis=0.6 to 0.9 by 0.05;
   symbol1 l=1 i=join v=none;
   symbol2 l=2 i=join v=dot;          /* r and s are respectively the minimum and maximum powers
   symbol3 l=3 i=join v=plus;            appropriately chosen in a given case */
   symbol1 l=1 i=join c=none;
   symbol2 l=2 i=join c=dot;
   symbol3 l=3 i=join c=plus;
run;
quit;
```

Program 6.5B: Power Graphs Based on the Sample Size *T*

```
data rino;
   input phi p50 p100 p250 @@;
   keep phi p50 p100 p250;
 label p50  = 'power of sample size 50'
       p100 = 'power of sample size 100'
       p250 = 'power of sample size 250';
cards;
 0.9     -      -      -
 0.8     -      -      -    /* The dashes are the appropriate powers at various sample sizes */
 0.7     -      -      -
 0.6     -      -      -
;

proc gplot;
   plot (p50 p100 p250)*phi / overlay vaxis=r to 1 by s legend haxis=0.6 to 0.9 by 0.05;
   symbol1 l=1 i=join v=none;
   symbol2 l=2 i=join v=dot;      /* r and s are respectively the minimum and maximum powers
   symbol3 l=3 i=join v=plus;        appropriately chosen in a given case */
   symbol1 l=1 i=join c=none;
   symbol2 l=2 i=join c=dot;
   symbol3 l=3 i=join c=plus;
run;
quit;
```

CHAPTER 7

PRACTICAL ILLUSTRATIONS - 1

7.1 Introduction

In this chapter, we shall illustrate the methodologies discussed in Chapter 2 and Chapter 3 with three non-seasonal time series. The series to be used in the illustrations are data on:

Series 2: *Eskom* stock yields traded on the Johannesburg Stock Exchange

Series 3: Gold shares traded on the Johannesburg Stock Exchange

Series 4: Consumer Price Index (CPI) for South Africa at 1995 prices.

Series 2, Series 3, and Series 4 are monthly observations. Series 2 covers the period January 1990 - June 1999. Series 3 covers the period January 1990 to April 1999 whilst Series 4 are observations from January 1994 to October 1999.

Section 7.2 and Section 7.3 illustrate the Dickey-Fuller and Phillips-Perron unit root tests. In Section 7.4, we shall test for unit roots in all the three data sets using the frequency domain approach. In Section 7.5, we illustrate testing for unit roots using the RDFUR test for stationary. Section 7.6 gives a summary of results. All computer programs are contained in Appendix G. Statistical softwares used in the analysis are SAS/ETS software, *Minitab*, *Matlab*, and *E-Views*.

7.2 The Dickey-Fuller Unit Root Tests for Stationarity

In this section, we consider testing for unit roots in Series 2, Series 3, and Series 4. Each series is discussed separately and comprehensively.

7.2.1 Testing for a Unit Root in Series 2

Fig. 7.1 is a plot of Series 2. Accompanying this plot in Fig. 7.2 and Fig. 7.3 are the plots of the autocorrelation functions and partial autocorrelation functions using Program 7.1 (Appendix G).

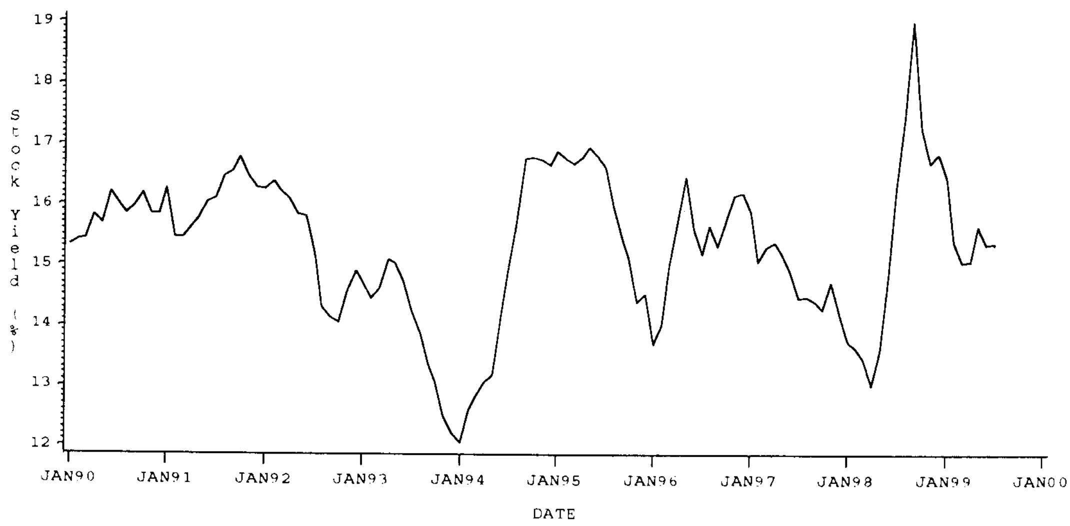


Fig. 7.1: Series 2

```
                Name of variable = X.
                Mean of working series =    15.278
                Standard deviation      = 1.223528
                Number of observations =       115

                              Autocorrelations
Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1         Std
  1   1.367296    0.91334  |                .   |******************   |   0.093250
  2   1.152035    0.76955  |              .     |***************      |   0.152327
  3   0.909779    0.60773  |             .      |************         |   0.183038
  4   0.659303    0.44041  |            .       |*********            |   0.199815
  5   0.427562    0.28561  |            .       |****** .             |   0.208084
  6   0.227023    0.15165  |            .       |***    .             |   0.211466
  7   0.064607    0.04316  |            .       |*      .             |   0.212409
  8  -0.062025   -0.04143  |            .      *|       .             |   0.212486
  9  -0.162242   -0.10838  |           .      **|        .            |   0.212556
 10  -0.226325   -0.15118  |           .     ***|        .            |   0.213036
 11  -0.294795   -0.19692  |           .    ****|        .            |   0.213967
 12  -0.338786   -0.22631  |           .   *****|        .            |   0.215537
 13  -0.350364   -0.23404  |           .   *****|        .            |   0.217593
 14  -0.344418   -0.23007  |           .   *****|        .            |   0.219771
 15  -0.308045   -0.20577  |           .    ****|        .            |   0.221856
 16  -0.272273   -0.18188  |           .    ****|        .            |   0.223509
 17  -0.222549   -0.14866  |           .     ***|        .            |   0.224793
 18  -0.160883   -0.10747  |           .      **|        .            |   0.225646
 19  -0.118669   -0.07927  |           .      **|        .            |   0.226090
 20  -0.079356   -0.05301  |           .       *|        .            |   0.226332
 21  -0.084887   -0.05670  |           .       *|        .            |   0.226440
 22  -0.119239   -0.07965  |           .      **|        .            |   0.226563
 23  -0.157379   -0.10513  |           .      **|        .            |   0.226807
 24  -0.186327   -0.12447  |           .      **|        .            |   0.227230
                             "." marks two standard errors
```

Fig. 7.2: Sample acf's for Series 2

```
                          Partial Autocorrelations

 Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
   1     0.91334   |                .   |******************  |
   2    -0.38990   |          ********|   .                  |
   3    -0.09298   |                . **|   .                 |
   4    -0.10692   |                . **|   .                 |
   5    -0.01817   |                .   |   .                 |
   6    -0.02523   |                .  *|   .                 |
   7    -0.00961   |                .   |   .                 |
   8    -0.02953   |                .  *|   .                 |
   9    -0.05299   |                .  *|   .                 |
  10     0.02765   |                .   |*  .                 |
  11    -0.17991   |                ****|   .                 |
  12     0.07661   |                .   |** .                 |
  13     0.01813   |                .   |   .                 |
  14    -0.02472   |                .   |   .                 |
  15     0.06540   |                .   |*  .                 |
  16    -0.10958   |                . **|   .                 |
  17     0.07776   |                .   |** .                 |
  18     0.00267   |                .   |   .                 |
  19    -0.08600   |                . **|   .                 |
  20     0.03209   |                .   |*  .                 |
  21    -0.19685   |                ****|   .                 |
  22    -0.01428   |                .   |   .                 |
  23    -0.01280   |                .   |   .                 |
  24     0.05415   |                .   |*  .                 |
```

Fig. 7.3: Sample pacf's for Series 2

The sample acf's pacf's in Fig. 7.2 and Fig. 7.3 suggest an AR(2) process to Series 2. Then testing for a unit root in the AR(2) process, with $p = 2$, the regression model in equation (2.35) assumes the following representation

$$Z_t = \delta + (\rho - 1)X_{t-1} + \sum_{j=1}^{1} \alpha_j Z_{t-j} + \varepsilon_t,$$

$$Z_t = \delta + (\rho - 1)X_{t-1} + \alpha_1 Z_{t-1} + \varepsilon_t \qquad (7.1)$$

where $\delta = \mu\left(1 - \sum_{i=1}^{2} \phi_i\right)$, and $Z_{t-j} = X_{t-j} - X_{t-j-1}$, $j = 0,1$.

Equation (7.1) suggests regressing Z_t on (1), X_{t-1}, and Z_{t-1} for $t = 3, 4, 5, \ldots, 115$, a total of 113 observations. Summary of the OLS results given in Table 7.1 were obtained using the following SAS statements in Program 7.2 (Appendix G).

Table 7.1 Regression Results - Series 2

Dependent Variable: Z

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	5.63102	2.81551	12.802	0.0001
Error	110	24.19120	0.21992		
C Total	112	29.82223			

Root MSE = 0.46896	R-square = 0.1888	C.V. = -58880.07256
Dep Mean =-0.00080	Adj R-sq = 0.1741	

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	1.839327	0.56000055	3.285	0.0014
X1	1	-0.120445	0.03654191	-3.296	0.0013
Z1	1	0.389872	0.08778325	4.441	0.0001

The fitted regression model is

$$\hat{Z}_t = \underset{(0.5600)}{1.8393} - \underset{(0.0365)}{0.1204}X_{t-1} + \underset{(0.0878)}{0.3899}Z_{t-1} \ , \qquad (7.2)$$

Figures in parentheses below the coefficients are standard errors. The estimate of $(\rho - 1)$, which is the coefficient of X_{t-1} is -0.1204 with standard error 0.0365. The test statistic for testing the presence of a unit root in Series 2 is

$$\hat{\tau} = \frac{\hat{\rho} - 1}{\hat{S}e(\hat{\rho} - 1)} = \frac{-0.1204}{0.0365} = -3.2986. \qquad (7.3)$$

At the 0.05 level of significance the critical value is -2.8891 [see Appendix F(a)]. Since $-3.2986 < -2.8891$, the unit root hypothesis that Series 1 contains a unit root is rejected. We therefore conclude that *Eskom* stock yields (Series 2) traded on the JSE from January 1990 to July 1999 is stationary, and hence the series does not require any differencing.

7.2.2 Testing for a Unit Root in Series 3

Series 3 is presented pictorially in Fig. 7.4. Fig. 7.5 and Fig. 7.6 are respectively the plots of the autocorrelation functions and partial autocorrelation functions using Program 7.3 (Appendix G).

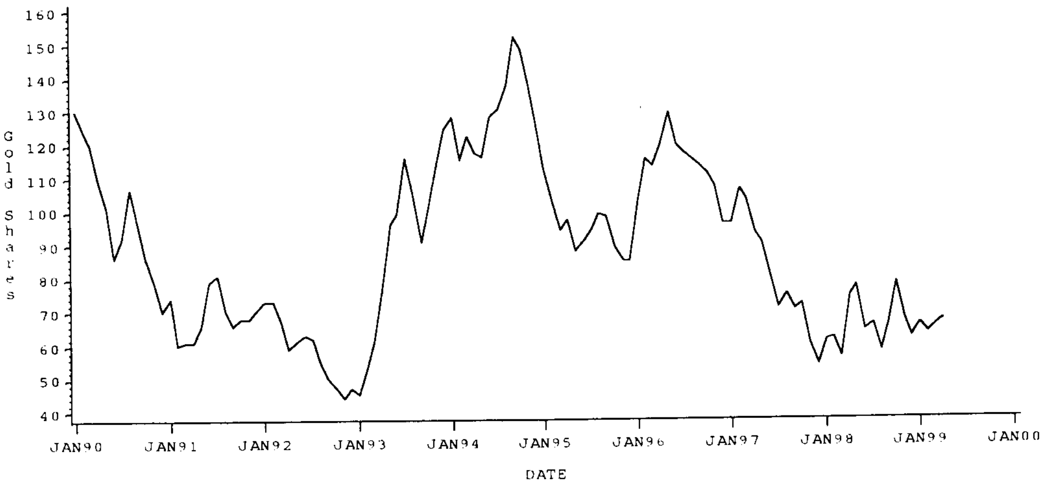


Fig. 7.4: Series 3

```
Name of variable   = X.                Mean of working series = 88.14286
Standard deviation = 25.55067          Number of observations = 112

                                  Autocorrelations

Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       Std
  1    608.935    0.93275  |                .   |******************* |  0.094491
  2    551.826    0.84527  |              .     |*****************   |  0.156412
  3    498.112    0.76300  |             .      |***************     |  0.192934
  4    445.092    0.68178  |            .       |**************      |  0.218218
  5    405.433    0.62103  |            .       |************        |  0.236474
  6    367.975    0.56365  |           .        |***********         |  0.250613
  7    324.770    0.49748  |           .        |**********          |  0.261687
  8    276.847    0.42407  |          .         |********  .         |  0.269999
  9    224.595    0.34403  |          .         |*******   .         |  0.275882
 10    176.332    0.27010  |          .         |*****     .         |  0.279686
 11    131.567    0.20153  |          .         |****      .         |  0.282006
 12  92.029701    0.14097  |          .         |***       .         |  0.283289
 13  57.217019    0.08764  |          .         |**        .         |  0.283914
 14  31.042092    0.04755  |          .         |*         .         |  0.284156
 15  11.483236    0.01759  |          .         |          .         |  0.284227
 16  -3.550109   -0.00544  |          .         |          .         |  0.284236
 17 -19.702077   -0.03018  |          .        *|          .         |  0.284237
 18 -45.044096   -0.06900  |          .        *|          .         |  0.284266
 19 -71.082544   -0.10888  |          .       **|          .         |  0.284415
 20 -91.655430   -0.14040  |          .      ***|          .         |  0.284787
 21   -108.566   -0.16630  |          .      ***|          .         |  0.285405
 22   -120.792   -0.18503  |          .     ****|          .         |  0.286269
 23   -123.292   -0.18886  |          .     ****|          .         |  0.287334
 24   -127.347   -0.19507  |          .     ****|          .         |  0.288440
                                 "." marks two standard errors
```

Fig. 7.5: Sample acf's for Series 3

```
                          Partial Autocorrelations

Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    0.93275   |                .   |*******************  |
  2   -0.19044   |                ****|   .                 |
  3    0.01635   |                .   |   .                 |
  4   -0.05386   |                .  *|   .                 |
  5    0.11789   |                .   |** .                 |
  6   -0.06287   |                .  *|   .                 |
  7   -0.09302   |                . **|   .                 |
  8   -0.08854   |                . **|   .                 |
  9   -0.07227   |                .  *|   .                 |
 10    0.00284   |                .   |   .                 |
 11   -0.05042   |                .  *|   .                 |
 12   -0.00611   |                .   |   .                 |
 13   -0.02225   |                .   |   .                 |
 14    0.06876   |                .   |*  .                 |
 15    0.01981   |                .   |   .                 |
 16    0.01847   |                .   |   .                 |
 17   -0.05856   |                .  *|   .                 |
 18   -0.12851   |                .***|   .                 |
 19   -0.01657   |                .   |   .                 |
 20    0.00527   |                .   |   .                 |
 21   -0.02885   |                .  *|   .                 |
 22   -0.03715   |                .  *|   .                 |
 23    0.07903   |                .   |** .                 |
 24   -0.04666   |                .  *|   .                 |
```

Fig. 7.6: Sample pacf's for Series 3

Fig. 7.2 and Fig. 7.3 suggest an AR(1) process to Series 3. Testing for a unit root in Series 3 is based on the following regression model

$$Z_t = \delta + (\rho - 1)X_{t-1} + \varepsilon_t, \qquad (7.4)$$

where $\delta = \mu(1 - \phi_1)$ and $Z_t = X_t - X_{t-1}$. The process in (7.4) suggests regressing Z_t on (1) and X_{t-1} for $t = 2,3,4, ...,112$, a total of 111 observations. The regression results in Table 7.2 were obtained using Program 7.4 (Appendix G). The fitted model is

$$\hat{Z}_t = \underset{(2.7968)}{4.8516} - \underset{(0.0304)}{0.0613}X_{t-1}, \qquad (7.5)$$

Figures in parentheses are standard errors of the coefficients.

Table 7.2: Regression Results - Series 3

Dependent Variable: Z

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	273.49217	273.49217	4.069	0.0461
Error	109	7325.75108	67.20873		
C Total	110	7599.24324			

Root MSE	8.19809	R-square	0.0360
Dep Mean	-0.56757	Adj R-sq	0.0271
C.V.	-1444.42587		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	4.851561	2.79681976	1.735	0.0856
X1	1	-0.061349	0.03041202	-2.017	0.0461

The estimate of $(\rho - 1)$ of X_{t-1} is -0.0613 with standard error 0.0304. The test statistic for testing the presence of a unit root in Series 3 is

$$\hat{\tau} = \frac{\hat{\rho} - 1}{\hat{S}e(\hat{\rho} - 1)} = \frac{-0.0613}{0.0304} = -2.0164. \qquad (7.6)$$

At the 0.05 level of significance the critical value is -2.8893. Since $-2.0164 > -2.8893$, the unit root hypothesis that Series 3 contains a unit root is accepted. We conclude that Gold Shares (Series 3) traded on the JSE from January 1990 to April 1999 is non-stationary, and hence differencing is required to induce stationarity.

7.2.3 Testing for a Unit Root in Series 4

A plot of Series 4 is shown in Fig. 7.7. Plots of the autocorrelation functions and partial autocorrelation functions of the series using Program 7.5 (Appendix G) are given in Fig. 7.7 and Fig. 7.8.The sample acf's pacf's in Fig. 7.2 and Fig. 7.3 suggest an AR(1) process to Series 4. The Box-Jenkins approach suggests that d could be equal to 2. However, the persistent upward trend in Fig. 7.7 also tells us that a deterministic time trend should be added to the AR(1) process. The AR(1) process with a deterministic time trend is

$$X_t = (\beta_o + \beta_1 t) + \rho X_{t-1} + \varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0, \sigma^2). \qquad (7.7)$$

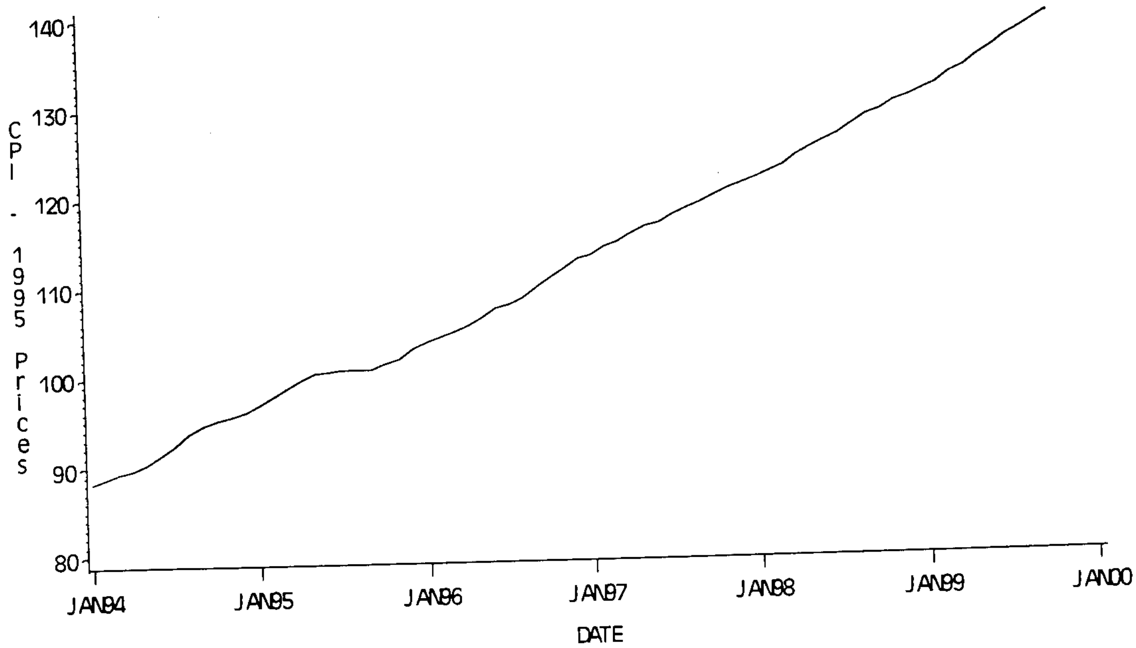


Fig. 7.7: Series 4

```
                     Name of variable = X.
                     Mean of working series = 111.9843
                     Standard deviation     = 14.40506
                     Number of observations =       70

                                  Autocorrelations

Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      Std
  1    198.185     0.95508  |               .    |******************* |   0.119523
  2    188.792     0.90981  |            .       |******************  |   0.200868
  3    179.456     0.86482  |          .         |*****************   |   0.252979
  4    170.058     0.81953  |        .           |****************    |   0.292177
  5    160.832     0.77507  |       .            |****************    |   0.323353
  6    151.779     0.73145  |      .             |***************     |   0.348885
  7    143.051     0.68938  |     .              |**************.     |   0.370144
  8    134.537     0.64835  |    .               |*************  .    |   0.388053
  9    126.334     0.60882  |    .               |************   .    |   0.403232
 10    118.088     0.56908  |   .                |***********     .   |   0.416157
 11    109.770     0.52900  |   .                |***********     .   |   0.427129
 12    101.462     0.48896  |   .                |**********      .   |   0.436388
 13  93.328464     0.44976  |  .                 |*********        .  |   0.444146
 14  85.373906     0.41143  |  .                 |********         .  |   0.450605
 15  77.646512     0.37419  |  .                 |*******          .  |   0.455940
 16  70.302713     0.33880  |  .                 |*******          .  |   0.460307
 17  63.044650     0.30382  | .                  |******            . |   0.463855
                              "." marks two standard errors
```

Fig. 7.8: Sample acf's for Series 4

```
                              Partial Autocorrelations
Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.95508  |                 .    |******************* |
  2    -0.02696  |                 .   *|    .                |
  3    -0.02050  |                 .    |    .                |
  4    -0.02760  |                 .   *|    .                |
  5    -0.01539  |                 .    |    .                |
  6    -0.01561  |                 .    |    .                |
  7    -0.00735  |                 .    |    .                |
  8    -0.01331  |                 .    |    .                |
  9    -0.00785  |                 .    |    .                |
 10    -0.02716  |                 .   *|    .                |
 11    -0.02896  |                 .   *|    .                |
 12    -0.02543  |                 .   *|    .                |
 13    -0.01657  |                 .    |    .                |
 14    -0.01671  |                 .    |    .                |
```

Fig. 7.9: Sample pacf's for Series 4

Subtracting X_{t-1} from both sides of (7.8) yields

$$X_t - X_{t-1} = \beta_o + \beta_1 t + (\rho - 1)X_{t-1} + \varepsilon_t$$

or $$Z_t = \beta_o + \beta_1 t + (\rho - 1)X_{t-1} + \varepsilon_t, \quad \text{where } Z_t = X_t - X_{t-1}. \qquad (7.8)$$

Here, equation (7.8) suggests regressing Z_t on (1), t, and X_{t-1} for $t = 2,3,4, \ldots,70$, a total of 69 observations. The regression results in Table 7.3 were obtained using Program 7.6 .

Table 7.3: Regression Results - Series 4

Dependent Variable: Z

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	0.32189	0.16095	2.311	0.1071
Error	66	4.59579	0.06963		
C Total	68	4.91768			

Root MSE = 0.26388 R-square = 0.0655 C.V.= 36.27050
Dep Mean = 0.72754 Adj R-sq = 0.0371

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	1.924610	3.18763085	0.604	0.5481
T	1	0.014346	0.02652752	0.541	0.5905
X1	1	-0.015482	0.03733525	-0.415	0.6797

The fitted regression model is

$$\hat{Z}_t = \underset{(3.2141)}{1.9246} + \underset{(0.0265)}{0.0143t} - \underset{(0.0373)}{0.0155X_{t-1}} \quad , \tag{7.9}$$

Standard errors of the coefficients are in parentheses. The estimate of $(\rho - 1)$ is -0.0155 with standard error 0.0373. The Dickey-Fuller test statistic

$$\hat{\tau} = \frac{\hat{\rho} - 1}{\hat{S}e(\hat{\rho} - 1)} = \frac{-0.0155}{0.0373} = -0.4155. \tag{7.10}$$

exceeds the 5% critical value of -2.9148. The null hypothesis of a unit root in Series 4 is thus accepted. We conclude that the Consumer Price Index (Series 4) from January 1994 to October 1999 is non-stationary and that it requires detrending and differencing to attain stationarity.

7.3 Phillips-Perron Unit Root Test for Stationarity

In the section, we illustrate the Phillip-Perron unit roots test procedure using the three series, namely Series 2, Series 3, and Series 4. We will then compare the results with the results in Section 7.2 using the Dickey-Fuller test. Results that will be reported here are all obtained using *EViews*.

7.3.1 Phillips-Perron Unit Root Test on Series 2

Results from the Phillip-Perron unit root test on Series 2 are given in Table 4.4. From the results in Table 7.4, the estimated Phillips-Perron test equation is given by

$$\hat{Z}_t = \underset{(0.5897)}{1.3238} - \underset{(0.03848)}{0.0866X_{t-1}}. \tag{7.11}$$

The Newey-West estimator suggests using the first $N = 4$ autocovariances. The Phillips-Perrron test statistic $\hat{\tau}_{pp}$ is given by

$$\hat{\tau}_{pp} = -2.940378. \tag{7.12}$$

Table 7.4: Phillips-Perron Unit Root Test on Series 2

PP Test Statistic	-2.940378	1% Critical Value*	-3.4885
		5% Critical Value	-2.8868
		10% Critical Value	-2.5801

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 4	(Newey-West suggests: 4)
Residual variance with no correction	0.250351
Residual variance with correction	0.439887

Phillips-Perron Test Equation
Dependent Variable: D(ESKOM)
Method: Least Squares
Sample(adjusted): 1990:02 1999:07
Included observations: 114 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ESKOM(-1)	-0.086641	0.038473	-2.251985	0.0263
C	1.323751	0.589675	2.244884	0.0267
R-squared	0.043319	Mean dependent var		8.77E-05
Adjusted R-squared	0.034777	S.D. dependent var		0.513812
S.E. of regression	0.504798	Akaike info criterion		1.488072
Sum squared resid	28.53999	Schwarz criterion		1.536076
Log likelihood	-82.82013	F-statistic		5.071439
Durbin-Watson stat	1.287472	Prob(F-statistic)		0.026273

Comparing the Phillips-Perron statistic with the 5% critical value of –2.8868, we see that –2.940378 < -2.8868. We again conclude that *Eskom* stock yields (Series 2) traded on the JSE from January 1990 to July 1999 is stationary, and hence the series does not require any differencing.

7.3.2 Phillips-Perron Unit Root Test on Series 3

Table 7.5 contains results from the Phillip-Perron unit root test on Series 3. The estimated Phillips-Perron test equation is

$$\hat{Z}_t = \underset{(2.7968)}{4.8516} - \underset{(0.0304)}{0.0613}X_{t-1}. \qquad (7.13)$$

The Phillips-Perron test statistic is $\hat{\tau}_{pp} = -2.254683$. The 5% critical value is -2.8874. Since -2.254683 > -2.8874, we accept the unit root null hypothesis and conclude that Gold Shares (Series 3) traded on the JSE from January 1990 to April 1999 is non-stationary, and hence differencing is required to induce stationarity.

Table 7.5: Phillips-Perron Unit Root Test on Series 3

PP Test Statistic	-2.254683	1% Critical Value*	-3.49
		5% Critical Value	-2.8874
		10% Critical Value	-2.5804

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 4	(Newey-West suggests: 4)	
Residual variance with no correction		65.99776
Residual variance with correction		90.68927

Phillips-Perron Test Equation
Dependent Variable: D(GOLDSHARE)
Method: Least Squares
Sample(adjusted): 1990:02 1999:04
Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GOLDSHARE(-1)	-0.061349	0.030412	-2.017249	0.0461
C	4.851561	2.79682	1.734671	0.0856

R-squared	0.035989	Mean dependent var	-0.567568
Adjusted R-squared	0.027145	S.D. dependent var	8.31168
S.E. of regression	8.198093	Akaike info criterion	7.063534
Sum squared resid	7325.751	Schwarz criterion	7.112354
Log likelihood	-390.0261	F-Statistic	4.069296
Durbin-Watson stat	1.52251	Prob(F-statistic)	0.046127

7.3.3 Phillips-Perron Unit Root Test on Series 4

The Phillips-Perron unit root test results on Series 2 are found in Table 7.6. The estimated Phillips-Perron test equation is

$$\hat{Z}_t = -0.0155 + 1.9533t + 0.0143X_{t-1} . \qquad (7.14)$$
$$\qquad (0.0373) \qquad (3.2405) \qquad (0.0265)$$

The Phillips-Perron test statistic is $\hat{\tau}_{pp} = -1.0269$ and the 5% critical value is -3.4749. Since -1.0269 > -3.4749, we accept the null hypothesis that data on Consumer Price Index (Series 4) from January 1994 to October 1999 is non-stationary and that it requires detrending and differencing to attain stationarity.

Table 7.6: Phillips-Perron Unit Root Test on Series 4

PP Test Statistic	-1.026887	1%	Critical Value*	-4.0948
		5%	Critical Value	-3.4749
		10%	Critical Value	-3.1645

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 3 (Newey-West suggests: 3)
Residual variance with no correction 0.066606
Residual variance with correction 0.116479

Phillips-Perron Test Equation
Dependent Variable: D(CPI)
Method: Least Squares
Sample(adjusted): 1994:02 1999:10
Included observations: 69 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CPI(-1)	-0.015482	0.037335	-0.414688	0.6797
C	1.953302	3.240522	0.602774	0.5487
t	0.014346	0.026528	0.540787	0.5905

R-squared	0.065456	Mean dependent var	0.727536
Adjusted R-squared	0.037136	S.D. dependent var	0.268922
S.E. of regression	0.263881	Akaike info criterion	0.215868
Sum squared resid	4.595790	Schwarz criterion	0.313003
Log likelihood	-4.447437	F-statistic	2.311332
Durbin-Watson stat	1.235898	Prob(F-statistic)	0.107101

7.4 Frequency Domain Test for Stationarity

In this section, we shall illustrate the methodology using Series 2, Series 3, and Series 4. Series 2 is made up of 115 observations and hence the number of periodogram ordinates required is $\left[\sqrt{T}\right] = \left[\sqrt{115}\right] = 10$. Similarly, Series 3 and Series 4 require 10 and 8 periodogram ordinates respectively.

7.4.1 Frequency Domain Test for Stationarity on Series 2

Using the SAS program in Program 7.7, the periodogram ordinates of Series 2 are given in Appendix B. The first perodogram ordinate of the undifferenced series is $I_X(w_1) = 11.1613$.

Next, we estimate the quantity $\sigma^2\phi^2$ using the periodogram of the differenced series $Z_t = X_t - X_{t-1}$.

Table 7.7: Periodogram Analysis Results - Series 2

OBS	FREQ	PERIOD	P_01	S_01
2	0.05512	114.000	0.03430	0.01957
3	0.11023	57.000	0.49236	0.03788
4	0.16535	38.000	0.56144	0.05876
5	0.22046	28.500	0.53428	0.08494
6	0.27558	22.800	2.68777	0.10770
7	0.33069	19.000	1.01021	0.11350
8	0.38581	16.286	2.15378	0.11244
9	0.44093	14.250	0.34640	0.09584
10	0.49604	12.667	1.93284	0.09240
11	0.55116	11.400	0.14653	0.08474

Table 7.7 contains the first 10 periodogram ordinates of the differenced series. The estimate of $\sigma^2\phi^2$ is

$$\sigma^2\phi^2 = \tfrac{1}{\sqrt{T}}\sum_{k=1}^{\sqrt{T}} \tfrac{1}{2}.I_z(w_k) = \tfrac{1}{10}\left(\tfrac{1}{2}\right)(0.03430 + 0.49236 + \ldots + 0.14653) = 0.4950.$$

The test statistic

$$\tau_1^{**} = \left(\frac{4\pi^2}{\sigma^2\phi^2 T^2}\right) I_X(w_1) \qquad = \frac{4\pi^2}{0.4950(115^2)} \times 11.1613 = 0.0673 \qquad (7.15)$$

is less than the 5% critical value of 0.1780 (see Appendix G(c)). Hence, we reject the null hypothesis that

$$H_o: \rho = 1,$$

and conclude that the series does not contain a unit root. We conclude that *Eskom* stock yields (Series 2) traded on the JSE from January 1990 to July 1999 is stationary.

7.4.2 Frequency Domain Test for Stationarity on Series 3

Here, we use the SAS program in Program 7.7 with the observations replaced by those of Series 3. The first periodogram ordinates of undifferenced Series 3 is $I_X(w_1)$ = 24124.25. Using the first differenced Series 3, the first 10 periodogram ordinates are reported in Table 7.8.

Table 7.8: Periodogram Analysis Results - Series 3

OBS	FREQ	PERIOD	P_01	S_01
2	0.05661	111.000	229.652	17.0044
3	0.11321	55.500	130.383	17.1609
4	0.16982	37.000	112.259	17.3828
5	0.22642	27.750	606.027	18.5998
6	0.28303	22.200	2.127	15.4000
7	0.33963	18.500	143.548	13.5832
8	0.39624	15.857	194.938	11.7487
9	0.45284	13.875	94.352	9.3962
10	0.50945	12.333	52.652	10.3472
11	0.56605	11.100	153.189	12.4967

Our estimate of $\sigma^2\phi^2$ is given by

$$\sigma^2\phi^2 = \frac{1}{\sqrt{T}}\sum_{k=1}^{\sqrt{T}} \tfrac{1}{2}.I_z(w_k^*) = \tfrac{1}{10}\left(\tfrac{1}{2}\right)(229.625 + 130.383 + \ldots + 153.189) = 83.3238.$$

Thus, the test statistic is given by

$$\tau_1^{**} = \left(\frac{4\pi^2}{\sigma^2\phi^2 T^2}\right) I_X(w_1) \quad = \frac{4\pi^2}{83.3238(112^2)} \times 24124.35 = 0.9112. \qquad (7.16)$$

The 5% critical value is 0.1780. Since $0.9112 > 0.1780$, we cannot reject the null hypothesis that $H_o\!:\rho = 1$, and conclude that Gold Shares (Series 3) traded on the JSE from January 1990 to April 1999 is non-stationary.

7.4.3 Frequency Domain Test for Stationarity on Series 4

Using the undifferenced Series 4 and Program 7.7 with the observations replaced by those of Series 4, we obtain $I_X(w_1)$ = 8728.44 as the first periodogram ordinate. The first 8 periodogram ordinates of first-differenced Series 4 are contained in Table 7.9. The estimate of $\sigma^2\phi^2$ is given by

$$\sigma^2\phi^2 = \frac{1}{\sqrt{T}}\sum_{k=1}^{\sqrt{T}} \tfrac{1}{2}.I_z(w_k^*) = \tfrac{1}{10}\left(\tfrac{1}{2}\right)(0.29484 + 0.40007 + \ldots + 0.61998) = 2.65265.$$

Table 7.9: Periodogram Analysis Results - Series 4

OBS	FREQ	PERIOD	P_01	S_01
2	0.09106	69.0000	0.29484	0.025593
3	0.18212	34.5000	0.40007	0.024655
4	0.27318	23.0000	0.38613	0.021838
5	0.36424	17.2500	0.11975	0.019728
6	0.45530	13.8000	0.19005	0.020977
7	0.54636	11.5000	0.02224	0.023285
8	0.63742	9.8571	0.61959	0.028236
9	0.72849	8.6250	0.61998	0.029081

For the test statistic, we have

$$\tau_1^{**} = \left(\frac{4\pi^2}{\sigma^2\phi^2T^2}\right)I_X(w_1) \qquad = \frac{4\pi^2}{2.65265(70^2)} \times 8728.44 = 26.5106. \qquad (7.17)$$

Since the test statistic of 26.5106 is greater than the 5% critical value of 0.178, we reject the null hypothesis that $H_o : \rho = 1$, and conclude that the Series 4 contains a unit root. This means that data on Consumer Price Index (Series 4) from January 1994 to October 1999 is non-stationary.

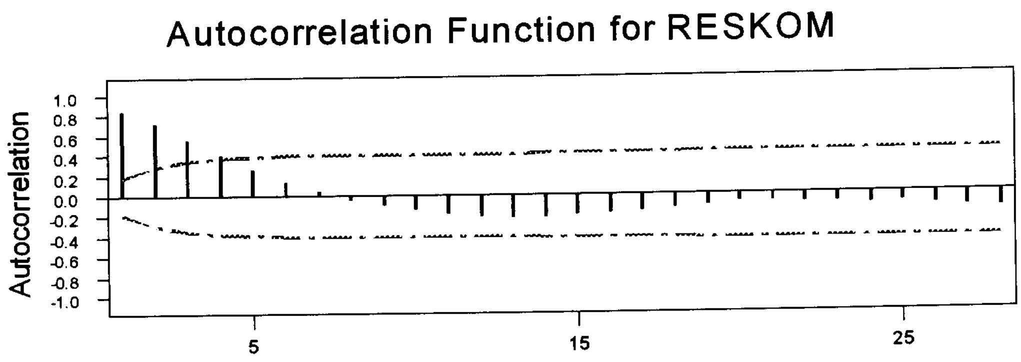
7.5 The RDFUR Stationarity Test

In this section, we apply the Reverse Dickey-Fuller Unit Root (RDFUR) test to Series 2, Series 3, and Series 4. The reverse data sets are found in Appendix E.

7.5.1 RDFUR Stationary Test on Reversed Series 2

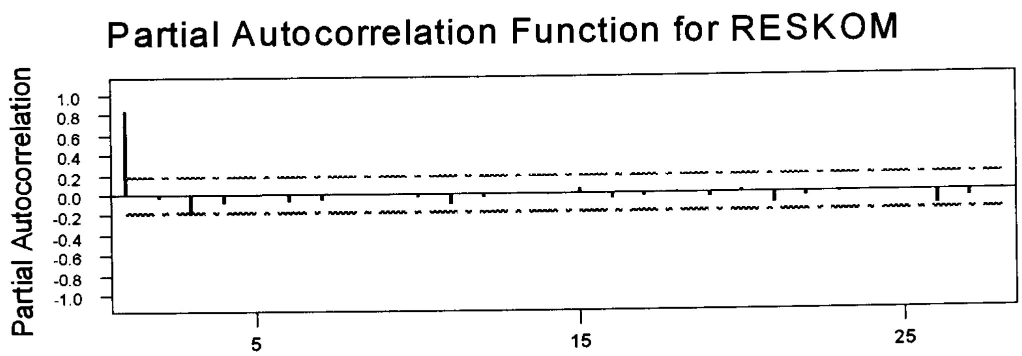
Appendix E-1 contains reverse Series 2. Fig. 7.10 and Fig. 7.11 are the sample acf's and pacf's of reversed Series 2. The sample acf's and pacf's suggest fitting an AR(1) process to reversed Series 2. The output from *E-Views* is given in Table 7.10. The fitted AR(1) process is

$$\hat{Z}_t^* = \underset{(0.75186)}{2.21597} - \underset{(0.04898)}{0.14480}X_{t-1}^*, \qquad (7.18)$$



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.86	9.17	86.32	10	-0.14	-0.69	221.10	19	-0.12	-0.55	261.56	28	-0.16	-0.74	278.98
2	0.72	4.93	148.40	11	-0.18	-0.89	225.33	20	-0.08	-0.39	262.58				
3	0.56	3.20	185.86	12	-0.21	-1.02	230.95	21	-0.09	-0.41	263.68				
4	0.40	2.13	205.60	13	-0.22	-1.07	237.37	22	-0.09	-0.44	264.96				
5	0.27	1.37	214.46	14	-0.22	-1.07	243.95	23	-0.10	-0.48	266.50				
6	0.14	0.71	216.96	15	-0.20	-0.94	249.12	24	-0.10	-0.48	268.12				
7	0.03	0.17	217.11	16	-0.18	-0.86	253.56	25	-0.10	-0.44	269.49				
8	-0.04	-0.22	217.35	17	-0.16	-0.76	257.13	26	-0.12	-0.55	271.64				
9	-0.10	-0.50	218.65	18	-0.13	-0.63	259.65	27	-0.15	-0.67	274.94				

Fig. 7.10: Sample acf's of Reversed Series 2



Lag	PAC	T	Lag	PAC	T	Lag	PAC	T	Lag	PAC	T
1	0.86	9.17	10	-0.02	-0.22	19	-0.04	-0.39	28	0.02	0.20
2	-0.03	-0.37	11	-0.10	-1.04	20	0.03	0.34			
3	-0.19	-2.03	12	-0.02	-0.25	21	-0.12	-1.24			
4	-0.08	-0.87	13	-0.00	-0.00	22	-0.04	-0.47			
5	-0.02	-0.20	14	-0.01	-0.13	23	-0.01	-0.12			
6	-0.07	-0.80	15	0.05	0.58	24	0.01	0.08			
7	-0.05	-0.55	16	-0.06	-0.60	25	0.01	0.07			
8	0.01	0.07	17	-0.03	-0.33	26	-0.14	-1.51			
9	-0.01	-0.11	18	0.01	0.16	27	-0.07	-0.78			

Fig. 7.11: Sample pacf's of Reversed Series 2

Table 7.10: Dickey-Fuller Test on Reversed Series 2

ADF Test Statistic	-2.956684	1% Critical Value*	-3.4885
		5% Critical Value	-2.8868
		10% Critical Value	-2.5801

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RESKOM)
Method: Least Squares
Sample(adjusted): 1990:02 1999:07
Included observations: 114 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RESKOM(-1)	-0.144803	0.048975	-2.956684	0.0038
C	2.21597	0.751857	2.947328	0.0039

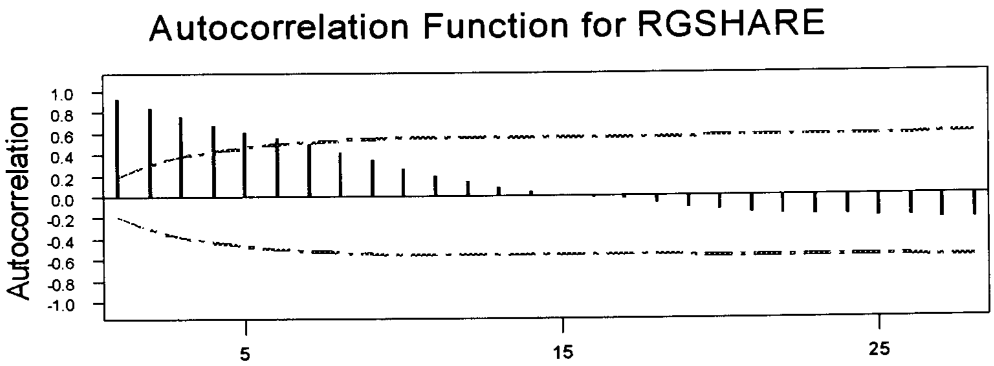
R-squared	0.072402	Mean dependent var	-8.77E-05
Adjusted R-squared	0.06412	S.D. dependent var	0.655489
S.E. of regression	0.634126	Akaike info criterion	1.944251
Sum squared resid	45.03701	Schwarz criterion	1.992254
Log likelihood	-108.8223	F-statistic	8.741978
Durbin-Watson stat	1.940602	Prob(F-statistic)	0.003793

The Dickey-Fuller test statistic for reversed Series 2 is $\hat{\tau}_* = -2.9567$. The 5% critical value is -2.8868. Since -2.9567 < -2.8868, we reject the null hypothesis that $\rho^* = 1$ and conclude that Series 2 (*Eskom* stock yields traded on the JSE from January 1990 to July 1999) is stationary.

7.5.2 RDFUR Stationary Test on Reversed Series 3

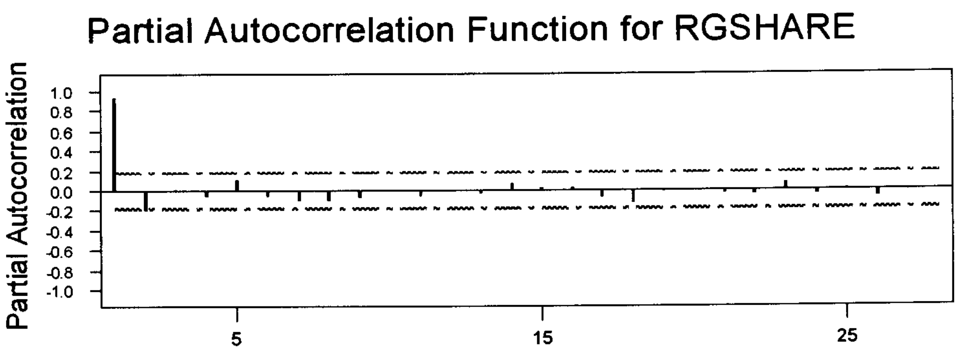
Reversed Series 3 is found in Appendix E-2. The sample acf's and pacf's of reversed Series 3 are shown in Fig. 7.12 and Fig. 7.13.Here again, the sample acf's and pacf's clearly suggest fitting an AR(1) process to reversed Series 3. From the regression results in Table 6.11, the fitted AR(1) process is

$$\hat{Z}_t^* = \underset{(2.8500)}{4.5006} - \underset{(0.0312)}{0.0448} X_{t-1}^* \,, \qquad (7.19)$$



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.93	9.86	99.90	10	0.27	0.96	464.24	19	-0.11	-0.38	475.43	28	-0.24	-0.82	527.11
2	0.84	5.40	182.55	11	0.20	0.71	469.28	20	-0.14	-0.50	478.18				
3	0.76	3.95	250.43	12	0.14	0.49	471.75	21	-0.17	-0.59	482.08				
4	0.68	3.12	305.07	13	0.09	0.30	472.69	22	-0.19	-0.65	486.94				
5	0.62	2.62	350.77	14	0.04	0.16	472.95	23	-0.19	-0.66	492.07				
6	0.56	2.25	388.82	15	0.01	0.05	472.98	24	-0.19	-0.68	497.58				
7	0.50	1.90	418.75	16	-0.01	-0.03	472.99	25	-0.20	-0.70	503.58				
8	0.42	1.57	440.66	17	-0.03	-0.11	473.12	26	-0.22	-0.74	510.45				
9	0.34	1.24	455.21	18	-0.07	-0.25	473.79	27	-0.23	-0.79	518.41				

Fig. 7.12: Sample acf's of Reversed Series 3



Lag	PAC	T	Lag	PAC	T	Lag	PAC	T	Lag	PAC	T
1	0.93	9.86	10	-0.00	-0.02	19	-0.01	-0.14	28	0.01	0.07
2	-0.19	-1.99	11	-0.05	-0.52	20	-0.00	-0.00			
3	0.02	0.19	12	-0.01	-0.08	21	-0.03	-0.28			
4	-0.05	-0.58	13	-0.03	-0.28	22	-0.04	-0.41			
5	0.12	1.23	14	0.07	0.69	23	0.07	0.79			
6	-0.06	-0.60	15	0.03	0.28	24	-0.04	-0.45			
7	-0.10	-1.02	16	0.03	0.27	25	0.01	0.08			
8	-0.09	-0.96	17	-0.06	-0.69	26	-0.07	-0.77			
9	-0.07	-0.72	18	-0.12	-1.32	27	-0.01	-0.13			

Fig. 7.13: Sample pacf's of Reversed Series 3

Table 7.11: Dickey-Fuller Unit Root Test on Reversed Series 3

ADF Test Statistic	-1.43652	1% Critical Value*	-3.49
		5% Critical Value	-2.8874
		10% Critical Value	-2.5804

*MacKinnon critical values for rejection of hypothesis of a unit root.

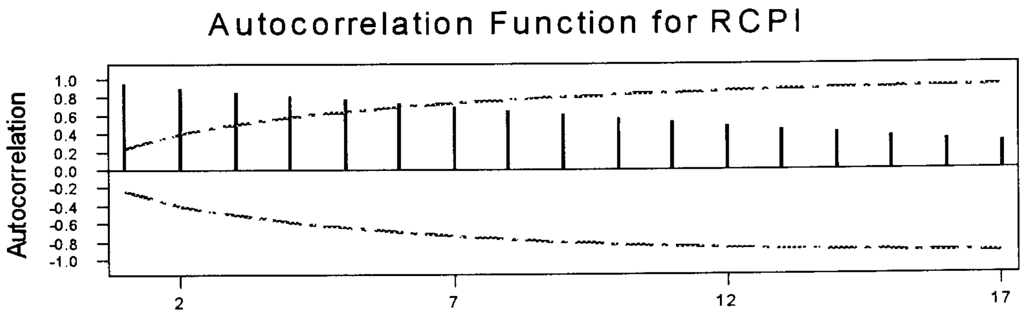
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RGSHARE)
Method: Least Squares
Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RGSHARE(-1)	-0.044827	0.031205	-1.43652	0.1537
C	4.500625	2.849962	1.579188	0.1172

Since the Dickey-Fuller test statistic for reversed Series 3, $\hat{\tau}_* = -1.43652$ is greater than the 5% critical value -2.8874, we cannot reject the null hypothesis that $\rho^* = 1$. We conclude that Series 3 (gold shares traded on the JSE from January 1990 to April 1999) is non-stationary.

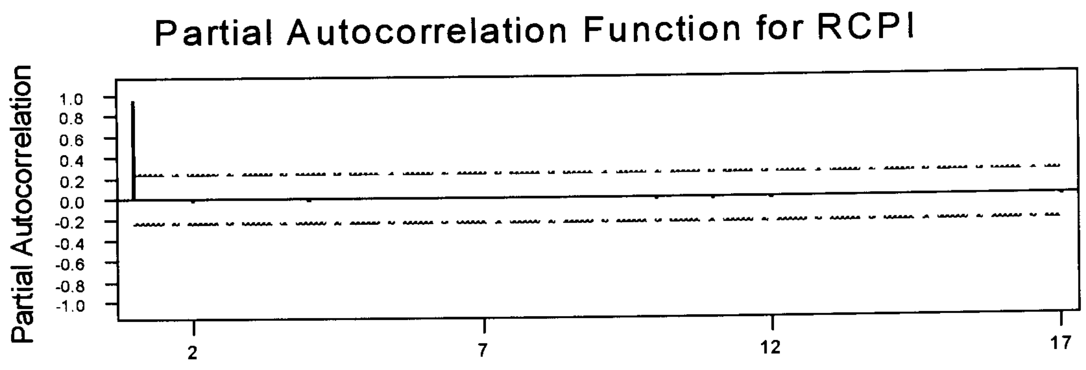
7.5.3 RDFUR Stationary Test on Reversed Series 4

The sample acf's and pacf's of reversed Series 4 in Appendix E-3 are as shown in Fig. 7.14 and Fig. 7.15.



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.96	7.99	66.63	8	0.65	1.67	396.43	15	0.37	0.82	544.90
2	0.91	4.53	127.98	9	0.61	1.51	427.06	16	0.34	0.74	555.61
3	0.86	3.42	184.24	10	0.57	1.37	454.26	17	0.30	0.65	564.39
4	0.82	2.80	235.53	11	0.53	1.24	478.17				
5	0.78	2.40	282.11	12	0.49	1.12	498.94				
6	0.73	2.10	324.24	13	0.45	1.01	516.83				
7	0.69	1.86	362.26	14	0.41	0.91	532.06				

Fig. 7.14: Sample acf's of Reversed Series 4



Lag	PAC	T	Lag	PAC	T	Lag	PAC	T
1	0.96	7.99	8	-0.01	-0.11	15	-0.01	-0.12
2	-0.03	-0.23	9	-0.01	-0.07	16	-0.01	-0.05
3	-0.02	-0.17	10	-0.03	-0.23	17	-0.02	-0.18
4	-0.03	-0.23	11	-0.03	-0.24			
5	-0.02	-0.13	12	-0.03	-0.21			
6	-0.02	-0.13	13	-0.02	-0.14			
7	-0.01	-0.06	14	-0.02	-0.14			

Fig. 7.15: Sample pacf's of Reversed Series 4

In addition to the AR(1) process being suggested by the sample acf's and pacf's, the downward trend of the reversed Series 4 (not shown here) suggests that we include a time trend in the model. Here again, the Box-Jenkins approach suggests *d* to be 2. Regression results for the time-trend AR(1) reversed Series 4 are given in Table 7.12.

Table 7.12: Dickey-Fuller Unit Root Test on Reversed Series 4

ADF Test Statistic	-2.055859	1% Critical Value*	-4.0948
		5% Critical Value	-3.4749
		10% Critical Value	-3.1645

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RCPI)
Method: Least Squares
Included observations: 69 after adjusting endpoints

Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
RCPI(-1)	-0.072324	0.03518	-2.055859	0.0438
C	9.082669	4.829517	1.880658	0.0644
t	-0.048174	0.025117	-1.917976	0.0594

From the reults in Table 7.12, the fitted AR(1) process is

$$\hat{Z}_t^* = \underset{(4.8295)}{9.0827} - \underset{(0.0251)}{0.0482t} - \underset{(0.0352)}{0.0723X_{t-1}^*}, \tag{7.20}$$

The Dickey-Fuller test statistic for reversed Series 4 is $\hat{\tau}_* = -2.055859$. The 5% critical value is -3.4749. We reject the null hypothesis that $\rho^* = 1$ since -2.055859 > -3.4749 and conclude that Series 4 is non-stationary.

7.6 Summary

In this chapter, we have used real data sets to illustrate the methodologies in Chapter 2 and Chapter 3. Results from using both the Dickey-Fuller, and Phillips-Perron, and the Frequency-Domain tests for stationarity gave the same conclusions. These test procedures established that Series 2 is stationary whilst Series 3 and Series 4 are non-stationary. These conclusions apply solely on data used and not in general.

CHAPTER 8

PRACTICAL ILLUSTRATIONS - 2

8.1 Introduction

This chapter illustrates the methodologies discussed in the chapters 4 and 5 with 6 data sets - Series 1, Series 2, Series 3, Series 4, and two simulated series. In Section 8.2 and Section 8.3, we determine the differencing degree in all the series using the periodogram method, and the lag-window method. In Section 8.4, we illustrate the Evolutionary Spectral Analysis using data on the effective nominal exchange rate of the South African rand indexed 1990=100. The effective nominal exchange rate is the weighted average exchange rate of the rand against six most important currencies. We also illustrate the Bivariate Evolutionary Cross-Spectral Analysis using the two simulated time series data. Section 8.5 summarizes the results.

8.2 Degree of Differencing, *d* - The Periodogram Method

In this section, we employ the periodogram method to obtain the degree of differencing d needed to transform Series 2, Series 3, and Series 4, assuming they are non-stationary series.

8.2.1 Degree of Differencing in Series 2

In this sub-section we illustrate the periodogram method of estimating the differencing degree in Series 2. Series 2 contains 115 observations and hence the regression analysis will involve approximately the first

$$\Lambda = 115^{0.5} \cong 11 \text{ periodogram ordinates.}$$

The SAS statements in Program 8.7 generate the sample smoothed periodogram of Series 2, using the triangular weighting in (4.57). The corresponding SAS output is given Appendix B. Spectral densities for the first 11 frequencies are reproduced in Table 8.1 below. Program 8.1 is a SAS program for regressing $M = \ln I_x(\omega)$ on $N = \ln[4\sin^2(\omega/2)]$ with its corresponding SAS output given in Appendix C. The regression results are given in Table 8.2.

Table 8.1: Periodogram Analysis Results - Series 2

OBS	FREQ	PERIOD	P_01	S_01
2	0.05464	115.000	11.1613	1.54352
3	0.10927	57.500	38.9346	1.71100
4	0.16391	38.333	22.2028	1.74219
5	0.21855	28.750	9.7468	1.67976
6	0.27318	23.000	35.2096	1.58162
7	0.32782	19.167	11.5292	1.22804
8	0.38245	16.429	14.4221	0.95710
9	0.43709	14.375	1.1525	0.66136
10	0.49173	12.778	7.8980	0.43508
11	0.54636	11.500	0.7407	0.31059

Table 8.2: Regression Results for the Estimation of d in Series 2 - Periodogram Method

Dependent Variable: M

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	4.59922	4.59922	3.334	0.1053
Error	8	11.03634	1.37954		
C Total	9	15.63555			

Root MSE	1.17454	R-square	0.2942
Dep Mean	2.20348	Adj R-sq	0.2059
C.V.	53.30373		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.829891	0.83898143	0.989	0.3516
N	1	-0.490077	0.26840441	-1.826	0.1053

From the results in Table 8.2,

$$\beta_1 = -\frac{\sum_{h=1}^{\Lambda}(N_h - \overline{N})M_h}{\sum_{h=1}^{\Lambda}(N_h - \overline{N})^2} = -0.490077 \quad \Rightarrow \quad \hat{d} = -\beta_1 = 0.49. \qquad (8.1)$$

Thus, the smoothed periodogram method suggests that the degree of differencing d to transform Series 2 to a stationary series is 0.49. Since $0.49 \in [-0.5, 0.5]$, we conclude that Series 2 is stationary, confirming the conclusion drawn in sub-section 8.2.1 that Series 2 is stationary.

8.2.2 Differencing Degree for Series 3

As a second example, we consider determining the of differencing degree for Series 3. Series 3 comprises 112 observations and hence the regression analysis will involve approximately the first $\sqrt{112} = 10$ periodogram ordinates. Using Program 7.7 with the data replaced by Series 3 yields the results in Table 8.3. Program 8.2 was used to obtain the regression results in Table 8.4.

From the results in Table 8.4,

$$\beta_1 = -\frac{\sum_{h=1}^{\Lambda} (N_h - \overline{N}) M_h}{\sum_{h=1}^{\Lambda} (N_h - \overline{N})^2} = -0.9392 \quad \Rightarrow \quad \hat{d} = -\beta_1 = 0.94. \qquad (8.2)$$

Table 8.3: Periodogram Analysis Results - Series 3

OBS	FREQ	PERIOD	P_01	S_01
2	0.05610	112.000	24124.35	1516.67
3	0.11220	56.000	13816.62	1260.37
4	0.16830	37.333	12641.32	953.67
5	0.22440	28.000	7275.42	644.25
6	0.28050	22.400	925.15	390.36
7	0.33660	18.667	3682.00	250.51
8	0.39270	16.000	611.09	144.32
9	0.44880	14.000	1591.32	95.00
10	0.50490	12.444	656.40	72.98
11	0.56100	11.200	314.36	49.70

The differencing degree of $0.94 \notin [-0.5, 0.5]$ suggests that Series 3 is non-stationary, confirming the conclusion drawn in sub-section 7.2.2 that Series 3 is non-stationary.

Table 8.4: Regression Results for the Estimation of d in Series 3 - Periodogram Method

Dependent Variable: M

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	16.88228	16.88228	31.415	0.0005
Error	8	4.29922	0.53740		
C Total	9	21.18150			

Root MSE	0.73308	R-square	0.7970
Dep Mean	7.90277	Adj R-sq	0.7717
C.V.	9.27621		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	5.319584	0.51589960	10.311	0.0001
N	1	-0.939179	0.16756473	-5.605	0.0005

8.2.3 Differencing Degree for Series 4

Series 4 contains 70 observations and hence requires the first $\sqrt{70} = 8$ periodogram ordinates. Again, using Program 7.7 with the data replaced with that of Series 4, the first 8 periodogram ordinates are reported in Table 8.5. Results from the regression of $M = \ln I_x(\omega) = \ln P\_01$ on

$N = \ln\left[4\sin^2(\omega/2)\right] = \ln\left[4\sin^2(FREQ/2)\right]$ are given in Table 8.6.

Table 8.5: Periodogram Analysis Results - Series 4

OBS	FREQ	PERIOD	P_01	S_01
2	0.08976	70.0000	8728.44	444.955
3	0.17952	35.0000	2005.68	325.286
4	0.26928	23.3333	1141.79	196.983
5	0.35904	17.5000	601.18	101.862
6	0.44880	14.0000	381.50	44.986
7	0.53856	11.6667	269.67	28.278
8	0.62832	10.0000	226.77	19.300
9	0.71808	8.7500	163.66	14.143

Table 8.6: Regression Results for the Estimation of d in Series 4 - Periodogram Method

Dependent Variable: M

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	12.36059	12.36059	2377.841	0.0001
Error	6	0.03119	0.00520		
C Total	7	12.39178			

Root MSE	0.07210	R-square	0.9975
Dep Mean	6.52255	Adj R-sq	0.9971
C.V.	1.10538		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	4.436722	0.04979405	89.101	0.0001
N	1	-0.953628	0.01955633	-48.763	0.0001

From the results in Table 8.6,

$$\beta_1 = -\frac{\sum_{h=1}^{\Lambda}(N_h - \overline{N})M_h}{\sum_{h=1}^{\Lambda}(N_h - \overline{N})^2} = -0.9536 \quad \Rightarrow \quad \hat{d} = -\beta_1 = 0.95. \qquad (8.3)$$

Since the differencing degree of $0.95 \notin [-0.5, 0.5]$ suggests that Series 4 is non-stationary, confirming the conclusion drawn in sub-section 7.2.3 that Series 4 is non-stationary.

8.3 The Lag-Window Method of Estimating d

This section illustrates the lag-window method of estimating the degree of differencing d using Series 2, Series 3, and Series 4.

8.3.1 Differencing Degree for Series 2

Series 3 contains 112 observations and hence requires its first $\sqrt{112}$ = 10.583=10 spectral densities. The estimated spectral densities S_01 $= \hat{f}_x(\omega)$ for Series 2 in Table 8.1. Using Program 8.3 which regresses $M = \ln \hat{f}_x(\omega)$ on $N = \ln\left[4\sin^2(\omega/2)\right]$, the regression results are given in Table 8.7.

Table 8.7: Regression Results for the Estimation of d in Series 2 - Lag-Window Method

Dependent Variable: M

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	3.18762	3.18762	12.479	0.0064
Error	9	2.29891	0.25543		
C Total	10	5.48653			

Root MSE	0.50541	R-square	0.5810
Dep Mean	-0.10827	Adj R-sq	0.5344
C.V.	-466.80226		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	-1.115654	0.32333016	-3.451	0.0073
N	1	-0.381104	0.10788230	-3.533	0.0064

From the results in Table 8.7, we have

$$\beta_1 = -\frac{\sum_{h=1}^{\Lambda}(N_h - \overline{N})M_h}{\sum_{h=1}^{\Lambda}(N_h - \overline{N})^2} = -0.3811 \quad \Rightarrow \quad \hat{d} = 0.38 . \qquad (8.4)$$

Since $0.38 \in [-0.5, 0.5]$, we conclude that Series 2 stationary. This supports our conclusion that Series 2 does not contain a unit root.

8.3.2 Differencing Degree for Series 3

In the case of Series 3, estimating d involves the first 10 spectral densities S_01 in Table 8.3. Using Program 8.4, the regression of $M = \ln \hat{f}_x(\omega)$ on $N = \ln[4\sin^2(\omega/2)]$ yields the following results in Table 8.8.

Table 8.8: Regression Results for the Estimation of d in Series 2 - Lag-Window Method

Dependent Variable: M

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	11.86742	11.86742	49.183	0.0001
Error	8	1.93033	0.24129		
C Total	9	13.79774			

Root MSE = 0.49121	R-square = 0.8601	C.V.= 8.61710
Dep Mean = 5.70045	Adj R-sq = 0.8426	

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	3.534652	0.34568938	10.225	0.0001
N	1	-0.787428	0.11228027	-7.013	0.0001

From the results in Table 8.8, we have

$$\beta_1 = -\frac{\sum_{h=1}^{\Lambda}(N_h - \overline{N})M_h}{\sum_{h=1}^{\Lambda}(N_h - \overline{N})^2} = -0.7874 \quad \Rightarrow \quad \hat{d} = 0.79\,. \qquad (8.5)$$

Since $0.79 \notin [-0.5, 0.5]$ implies Series 3 is non-stationary. This also confirms the conclusion drawn in sub-section 7.2.2 that Series 3 is non-stationary.

8.3.3 Differencing Degree for Series 4

Using the first 8 spectral densities $\hat{f}_x(\omega) =$ S_01 in Table 8.5 and performing the regression of $M = \ln \hat{f}_x(\omega)$ on $N = \ln[4\sin^2(\omega/2)]$ yields the following results in Table 8.9. The differencing degree d is

$$\beta_1 = -\frac{\sum_{h=1}^{\Lambda}(N_h - \overline{N})M_h}{\sum_{h=1}^{\Lambda}(N_h - \overline{N})^2} = -0.900637 \quad \Rightarrow \quad \hat{d} = 0.90\,. \qquad (8.6)$$

Table 8.9: Regression Results for the Estimation of d in Series 4 - Lag-Window Method

Dependent Variable: M

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	11.02506	11.02506	56.917	0.0003
Error	6	1.16223	0.19370		
C Total	7	12.18728			

Root MSE	0.44012	R-square	0.9046
Dep Mean	4.31840	Adj R-sq	0.8887
C.V.	10.19171		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	2.348478	0.30396171	7.726	0.0002
N	1	-0.900637	0.11937923	-7.544	0.0003

The fact that $d = 0.90 \notin [-0.5, 0.5]$ implies Series 4 is non-stationary. This conclusion also confirms the conclusion drawn in sub-section 7.2.3 that Series 4 is non-stationary.

8.4 Evolutionary Spectral Analysis

In Section 8.4, we apply the evolutionary spectral analysis to the effective nominal rate of the South African Rand indexed 1990=100. In Section 8.5, we illustrate the evolutionary cross-spectral analysis using two simulated time series, and Section 8.6 summarizes the results. We shall refer to this series as Series 1. Accompanying the plot of Series 1 in Fig. 8.1 is the sample autocorrelation functions (acf's) in Fig. 8.2. The sample acf's and pacf's indicate that Series 1 is non-stationary. However, Fig. 8.1 clearly shows that Series 1 could contain a linear trend and therefore suggest decomposing the series as

$$X_t = X_t^{*}.X_t^{**}, \tag{8.7}$$

where X_t = Series 1, X_t^{*} = trend, and X_t^{**} = detrended series. A unit root test on the detrended series (not shown here) reveals that it is indeed stationary.

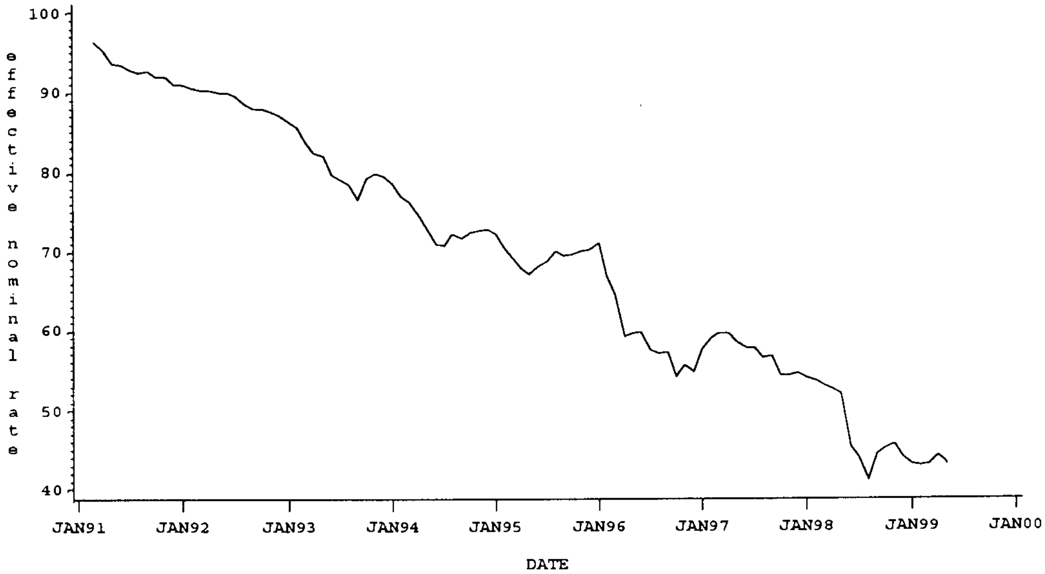


Fig. 8.1: Series 1 (Effective Nominal Rate of the South African Rand = 1993:3 - 1999:5)

```
ACF of C1
           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
   1   0.967                          XXXXXXXXXXXXXXXXXXXXXXXXX
   2   0.935                          XXXXXXXXXXXXXXXXXXXXXXXX
   3   0.900                          XXXXXXXXXXXXXXXXXXXXXXXX
   4   0.866                          XXXXXXXXXXXXXXXXXXXXXXX
   5   0.833                          XXXXXXXXXXXXXXXXXXXXXX
   6   0.800                          XXXXXXXXXXXXXXXXXXXXX
   7   0.769                          XXXXXXXXXXXXXXXXXXXX
   8   0.737                          XXXXXXXXXXXXXXXXXXX
   9   0.704                          XXXXXXXXXXXXXXXXXXX
  10   0.669                          XXXXXXXXXXXXXXXXXX
  11   0.635                          XXXXXXXXXXXXXXXXX
  12   0.603                          XXXXXXXXXXXXXXXX
  13   0.576                          XXXXXXXXXXXXXXX
  14   0.549                          XXXXXXXXXXXXXXX
  15   0.521                          XXXXXXXXXXXXXX
  16   0.493                          XXXXXXXXXXXXX
  17   0.467                          XXXXXXXXXXXXX
  18   0.442                          XXXXXXXXXXXX
  19   0.417                          XXXXXXXXXXX
  20   0.392                          XXXXXXXXXXX
  21   0.367                          XXXXXXXXXX
  22   0.342                          XXXXXXXXXX
  23   0.317                          XXXXXXXXX
  24   0.292                          XXXXXXXX
  25   0.270                          XXXXXXXX
```

Fig. 8.2: Sample acf's for Series 1

Using the *Minitab* program in Chapter Appendix 8.1, the decomposed series X_t^* and X_t^{**} are given in Table 8.11. We have included the product of the two series in the table to confirm that the product of the two series yields the original series X_t. Table 8.10 reports the fitted line using the same program.

Table 8.10: Fitted Trend Line, X_t

The regression equation: X_t = 96.9 - 0.547*t

Predictor	Coef	StDev	T	P
Constant	96.8916	0.4871	198.92	0.000
t	-0.547169	0.008458	-64.69	0.000

S = 2.405 R-Sq = 97.7% R-Sq(adj) = 97.7%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	24206	24206	4185.20	0.000
Error	97	561	6		
Total	98	24767			

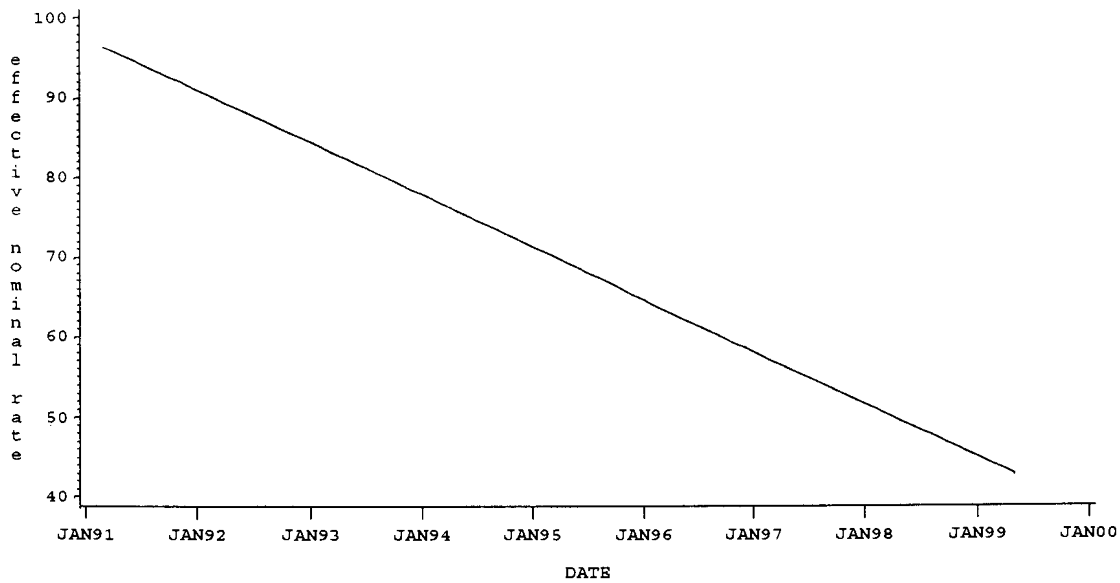


Fig. 8.3: Trend Line (A Decomposition of Series 1), X_t^*

Table 8.11: Series 1, Trend and De-Trended Components

YEAR	MON	NOM RATE	TREN	DETR	TR*DET	YEAR	MONTH	NOM RATE	TREND	DETREND	TR*DET
1991	Mar	96.33	96.3444	0.99985	96.33	1995	May	67.15	68.9860	0.97339	67.15
1991	Apr	95.15	95.7972	0.99324	95.15	1995	Jun	68.20	68.4388	0.99651	68.20
1991	May	93.51	95.2501	0.98173	93.51	1995	Jul	68.67	67.8916	1.01146	68.67
1991	Jun	93.34	94.7029	0.98561	93.34	1995	Aug	69.92	67.3445	1.03824	69.92
1991	Jul	92.83	94.1557	0.98592	92.83	1995	Sep	69.41	66.7973	1.03911	69.41
1991	Aug	92.49	93.6086	0.98805	92.49	1995	Oct	69.67	66.2501	1.05162	69.67
1991	Sep	92.67	93.0614	0.99579	92.67	1995	Nov	69.89	65.7029	1.06373	69.89
1991	Oct	91.88	92.5142	0.99314	91.88	1995	Dec	70.12	65.1558	1.07619	70.12
1991	Nov	92.01	91.9671	1.00047	92.01	1996	Jan	71.11	64.6086	1.10063	71.11
1991	Dec	91.00	91.4199	0.99541	91.00	1996	Feb	66.95	64.0614	1.04509	66.95
1992	Jan	91.02	90.8727	1.00162	91.02	1996	Mar	64.50	63.5143	1.01552	64.50
1992	Feb	90.47	90.3255	1.00160	90.47	1996	Apr	59.21	62.9671	0.94033	59.21
1992	Mar	90.35	89.7784	1.00637	90.35	1996	May	59.55	62.4199	0.95402	59.55
1992	Apr	90.26	89.2312	1.01153	90.26	1996	Jun	59.88	61.8728	0.96779	59.88
1992	May	90.00	88.6840	1.01484	90.00	1996	Jul	57.41	61.3256	0.93615	57.41
1992	Jun	89.99	88.1369	1.02103	89.99	1996	Aug	57.16	60.7784	0.94047	57.16
1992	Jul	89.54	87.5897	1.02227	89.54	1996	Sep	57.32	60.2313	0.95167	57.32
1992	Aug	88.56	87.0425	1.01743	88.56	1996	Oct	54.19	59.6841	0.90795	54.19
1992	Sep	88.00	86.4954	1.01740	88.00	1996	Nov	55.66	59.1369	0.94121	55.66
1992	Oct	87.95	85.9482	1.02329	87.95	1996	Dec	54.74	58.5898	0.93429	54.74
1992	Nov	87.49	85.4010	1.02446	87.49	1997	Jan	57.64	58.0426	0.99306	57.64
1992	Dec	87.04	84.8539	1.02576	87.04	1997	Feb	58.91	57.4954	1.02460	58.91
1993	Jan	86.28	84.3067	1.02341	86.28	1997	Mar	59.60	56.9482	1.04656	59.60
1993	Feb	85.52	83.7595	1.02102	85.52	1997	Apr	59.62	56.4011	1.05707	59.62
1993	Mar	83.59	83.2124	1.00454	83.59	1997	May	58.56	55.8539	1.04845	58.56
1993	Apr	82.39	82.6652	0.99667	82.39	1997	Jun	57.74	55.3067	1.04400	57.74
1993	May	81.86	82.1180	0.99686	81.86	1997	Jul	57.74	54.7596	1.05443	57.74
1993	Jun	79.58	81.5708	0.97559	79.58	1997	Aug	56.56	54.2124	1.04330	56.56
1993	Jul	79.05	81.0237	0.97564	79.05	1997	Sep	56.79	53.6652	1.05823	56.79
1993	Aug	78.38	80.4765	0.97395	78.38	1997	Oct	54.33	53.1181	1.02282	54.33
1993	Sep	76.47	79.9293	0.95672	76.47	1997	Nov	54.39	52.5709	1.03460	54.39
1993	Oct	79.17	79.3822	0.99733	79.17	1997	Dec	54.60	52.0237	1.04952	54.60
1993	Nov	79.69	78.8350	1.01085	79.69	1998	Jan	54.04	51.4766	1.04980	54.04
1993	Dec	79.46	78.2878	1.01497	79.46	1998	Feb	53.70	50.9294	1.05440	53.70
1994	Jan	78.57	77.7407	1.01067	78.57	1998	Mar	53.05	50.3822	1.05295	53.05
1994	Feb	76.91	77.1935	0.99633	76.91	1998	Apr	52.54	49.8351	1.05428	52.54
1994	Mar	76.19	76.6463	0.99405	76.19	1998	May	51.99	49.2879	1.05482	51.99
1994	Apr	74.52	76.0992	0.97925	74.52	1998	Jun	45.35	48.7407	0.93043	45.35
1994	May	72.80	75.5520	0.96357	72.80	1998	Jul	43.98	48.1935	0.91257	43.98
1994	Jun	70.96	75.0048	0.94607	70.96	1998	Aug	41.19	47.6464	0.86449	41.19
1994	Jul	70.75	74.4577	0.95020	70.75	1998	Sep	44.42	47.0992	0.94312	44.42
1994	Aug	72.23	73.9105	0.97726	72.23	1998	Oct	45.10	46.5520	0.96881	45.10
1994	Sep	71.66	73.3633	0.97678	71.66	1998	Nov	45.68	46.0049	0.99294	45.68
1994	Oct	72.24	72.8161	0.99209	72.24	1998	Dec	43.94	45.4577	0.96661	43.94
1994	Nov	72.59	72.2690	1.00444	72.59	1999	Jan	43.20	44.9105	0.96191	43.20
1994	Dec	72.70	71.7218	1.01364	72.70	1999	Feb	43.00	44.3634	0.96927	43.00
1995	Jan	72.17	71.1746	1.01398	72.17	1999	Mar	43.12	43.8162	0.98411	43.12
1995	Feb	70.31	70.6275	0.99551	70.31	1999	Apr	44.18	43.2690	1.02105	44.18
1995	Mar	68.98	70.0803	0.98430	68.98	1999	May	43.37	42.7219	1.01517	43.37
1995	Apr	67.84	69.5331	0.97565	67.84						

TR*DET = TREND*DE-TRENDED SERIES

From the results in Table 8.10, we obtain the trend, $X_t^* = 96.8916 - 0.5472t$ is shown pictorially in Fig. 8.3. Next, we look at the nature of the de-trended series, whether stationary or non-stationary. A plot of the de-trended series is shown in Fig. 8.4. Fig. 8.5 and Fig. 8.6 are the corresponding sample acf's and pacf's of the de-trended series.

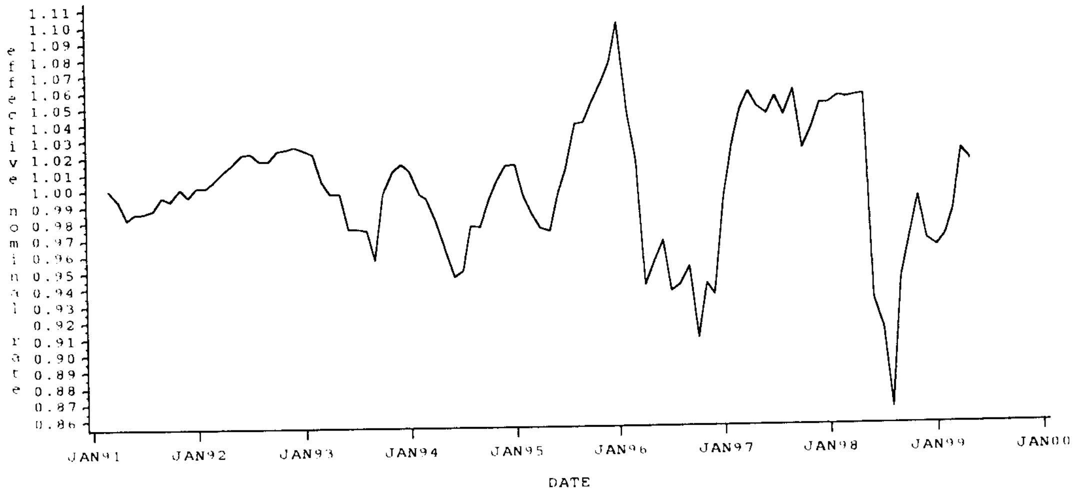


Fig. 8.4: De-trended Series (A Decomposition of Series 1), X_t^{**}

```
ACF of C4

          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
  1    0.815                         XXXXXXXXXXXXXXXXXXXXX
  2    0.600                         XXXXXXXXXXXXXXXX
  3    0.322                         XXXXXXXXX
  4    0.139                         XXXX
  5    0.020                         XX
  6   -0.099                       XXX
  7   -0.211                    XXXXXX
  8   -0.292                  XXXXXXXX
  9   -0.366                XXXXXXXXXX
 10   -0.353                XXXXXXXXXX
 11   -0.353                XXXXXXXXXX
 12   -0.301                 XXXXXXXXX
 13   -0.295                  XXXXXXXX
 14   -0.301                 XXXXXXXXX
 15   -0.312                 XXXXXXXXX
 16   -0.320                 XXXXXXXXX
 17   -0.272                  XXXXXXXX
 18   -0.159                     XXXXX
 19   -0.048                        XX
 20    0.085                         XXX
 21    0.135                         XXXX
 22    0.200                         XXXXXX
```

Fig. 8.5: Sample acf's for the De-trended Series, X_t^{**}

```
PACF of C4

         -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
           +----+----+----+----+----+----+----+----+----+----+
  1   0.815                         XXXXXXXXXXXXXXXXXXXXX
  2  -0.192                    XXXXXX
  3  -0.323                 XXXXXXXXX
  4   0.127                         XXXX
  5   0.045                         XX
  6  -0.284                  XXXXXXXX
  7  -0.133                      XXXX
  8   0.082                         XXX
  9  -0.191                    XXXXXX
 10   0.040                         XX
 11  -0.103                      XXXX
 12  -0.019                         X
 13  -0.177                     XXXXX
 14  -0.166                     XXXXX
 15  -0.076                       XXX
 16  -0.139                      XXXX
 17  -0.017                         X
 18   0.116                         XXXX
 19  -0.080                       XXX
 20  -0.015                         X
 21  -0.085                       XXX
 22   0.036                         XX
 23  -0.054                        XX
 24  -0.002                         X
 25   0.093                         XXX
```

Fig. 8.6: Sample pacf's for the De-trended Series, X_t^{**}

The sample acf's and pacf's show that the de-trended series is stationary and that an AR(1) process or an AR(3) process could be appropriate for the de-trended series.

Table 8.12a: Unit Root Tests and Model Selection

ADF Test Statistic	-4.002409	1% Critical Value*	-3.5226
		5% Critical Value	-2.9017
		10% Critical Value	-2.5879

*MacKinnon critical values for rejection of hypothesis of a unit root.

Method: Least Squares Augmented Dickey-Fuller Test Equation
Dependent Variable: D(NOMRATE)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
NOMRATE(-1)	-0.272037	0.067968	-4.002409	0.0002
D(NOMRATE(-1))	0.236946	0.111594	2.123281	0.0374
D(NOMRATE(-2))	0.370612	0.114913	3.225145	0.0019
C	0.271915	0.067795	4.010842	0.0002

Akaike info criterion = -5.180084 Schwarz criterion = -5.053603

Table 8.12b: Unit Root Tests and Model Selection

ADF Test Statistic	-2.270053	1% Critical Value*	-3.5200
		5% Critical Value	-2.9006
		10% Critical Value	-2.5874

*MacKinnon critical values for rejection of hypothesis of a unit root.

Method: Least Squares Augmented Dickey-Fuller Test Equation
Dependent Variable: D(NOMRATE)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
NOMRATE(-1)	-0.146828	0.064680	-2.270053	0.0262
C	0.147157	0.064574	2.278885	0.0256

Akaike info criterion = -5.054236 Schwarz criterion = -4.991964

Using *EViews*, we simultaneously check whether the de-trended series is stationary or non-stationary, and also find out which of the two series best describes the de-trended series. Results from the two fitted processes for X_t^{**} are given in Table 8.12. Both the AIC and the Schwartz SBC criteria select the AR(3) process for X_t^{**}.

Furthermore, since the ADF test statistic, - 4.002409, is less than all the critical values, we reject the null hypothesis that

$$H_o: X_t^{**} \text{ contains a unit root,} \tag{8.7}$$

and conclude that the de-trended series is stationary. Using the SAS program in Chapter Appendix 8.2, the spectrum for X_t^{**} is shown in Fig. 8.7.

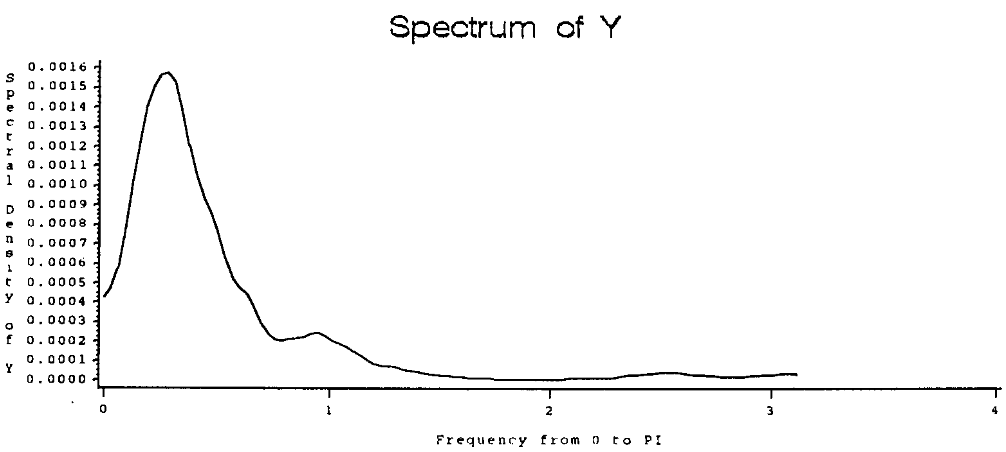


Fig. 8.7: Spectrum for the De-trended Series, X_t^{**}

From the concept of evolutionary spectral analysis described above, setting

$$X_t^* = \Xi_t = 96.8916 - 0.5472t \quad \text{and} \quad X_t^{**} = Y_t,$$

and hence

$$X_t = (96.8916 - 0.5472t)Y_t. \tag{8.8}$$

We shall expect that for each $t = 1,2,\ldots,99$, the evolutionary spectrum should take the same shape as the spectrum for the stationary series, $X_t^* = \Xi_t$. Evolutionary spectra for $t = 20$, 40, 60, and 80 using the SAS program in Chapter Appendix 8.2 are shown in Fig. 8.8.

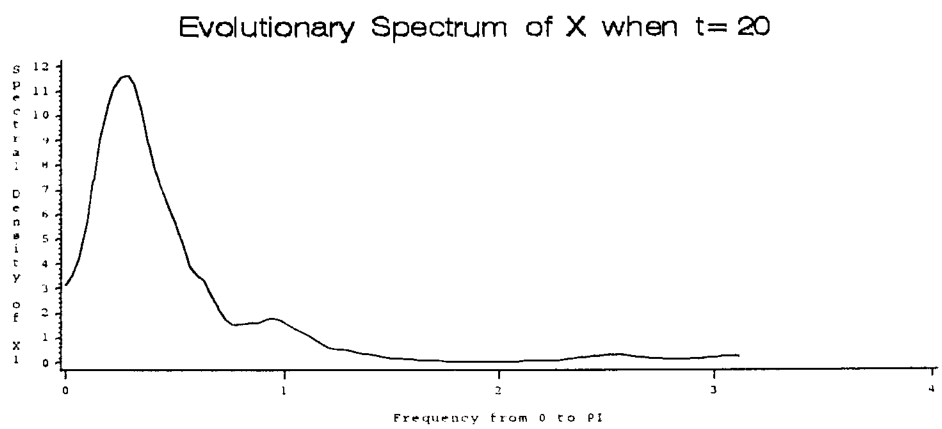


Fig. 8.8a: Evolutionary Spectrum for Series 1 when $t = 20$

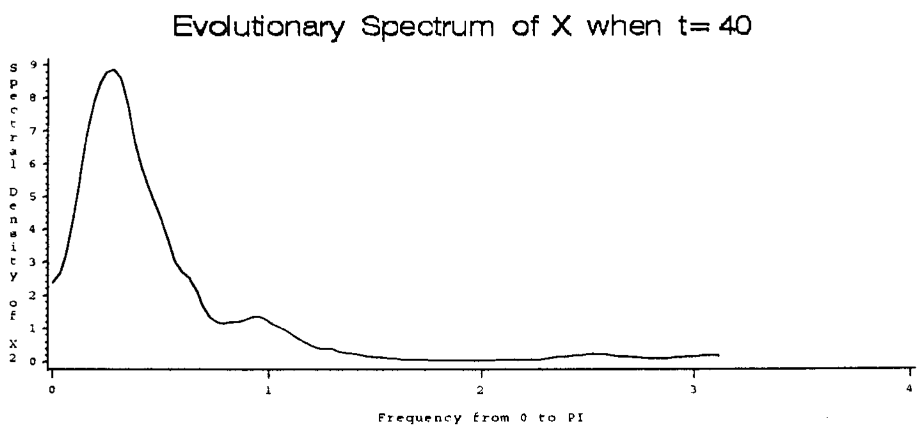


Fig. 8.8b: Evolutionary Spectrum for Series 1 when $t = 40$

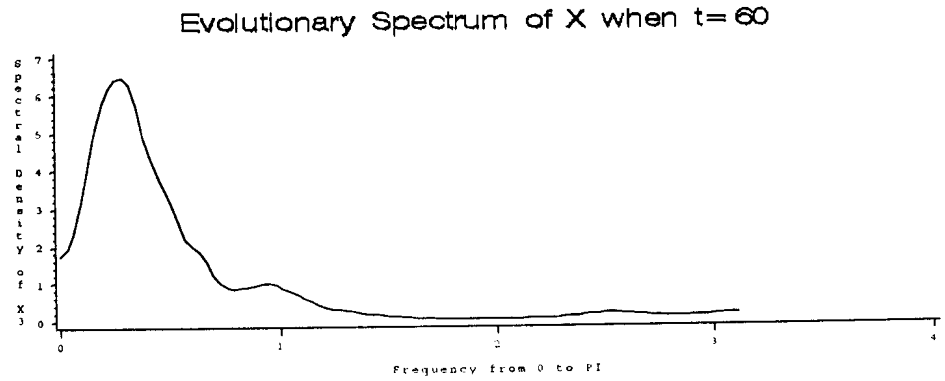


Fig. 8.8c: Evolutionary Spectrum for Series 1 when $t = 60$

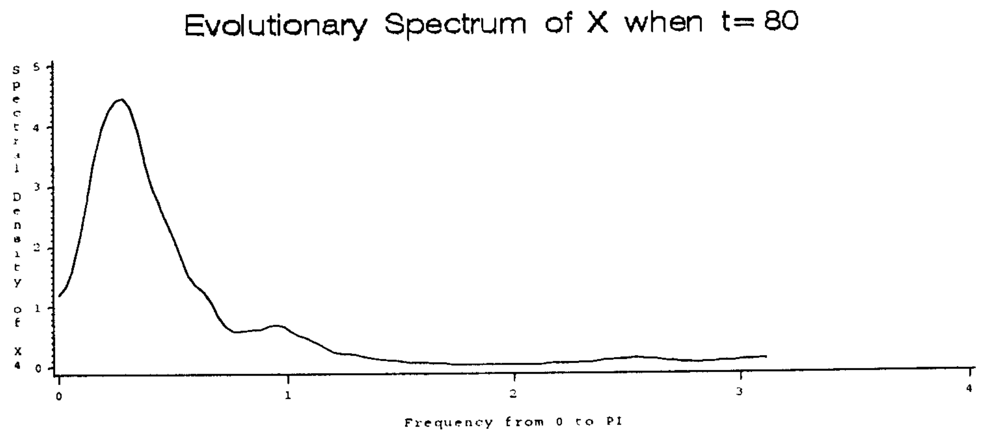


Fig. 8.8d: Evolutionary Spectrum for Series 1 when $t = 80$

We have shown that with an artificial time series and a real time series that for a series that changes slowly with time, the shapes of the evolutionary spectra at each time point $t = 1, 2, .., T$ are exactly the same as the spectrum of the stationary series. We therefore conclude that for non-stationary time series with time-changing spectra, the evolutionary spectral analysis of the series can be done exactly the same way as though the series were stationary.

8.5 Evolutionary Cross-Spectral Analysis

In Chapter 5, we considered an example on evolutionary spectral analysis using a simulated time dependent non-stationary. Here, we illustrate the concept of bivariate evolutionary spectral analysis using two simulated time dependent non-stationary series $X_{1,t}$ and $X_{2,t}$. Let

$$\{W_t\}=\begin{pmatrix} X_{1,t} \\ X_{2,t} \end{pmatrix}. \tag{8.9}$$

Consider the stationary time series $\{Y_t\}$ satisfying the AR(2) process

$$Y_t = 0.8Y_{t-1} - 0.4Y_{t-2} + \varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0,100^2). \tag{8.10}$$

We examine the validity of the estimation procedure by constructing two artificial non-stationary processees $\{X_{j,t}: j = 1,2; t = 21,22,\ldots,120\}$ from the model

$$X_{j,t} = \Xi_{j,t} Y_t, \tag{8.11}$$

where we define $\Xi_{1,t}$ and $\Xi_{2,t}$ as

$$\Xi_{1,t} = \exp\left\{ -\frac{1}{2}\left(\frac{t}{200}\right)^2 \right\}, \tag{8.12}$$

$$\Xi_{2,t} = \exp\left\{ -\frac{1}{2}\left(\frac{t}{80}\right)^2 \right\}. \tag{8.13}$$

Using Program 8.5, results from the evolutionary spectral analysis of { $X_{1,t}$ } and { $X_{2,t}$ } are given in Appendix D. Plots of the evolutionary co-spectrum, the evolutionary quadrature spectrum, the evolutionary coherency, and the evolutionary phase spectrum of { $X_{1,t}$ } by { $X_{2,t}$ } are respectively given in Fig. 8.12, Fig. 8.13, Fig. 8.14, and Fig. 8.15.

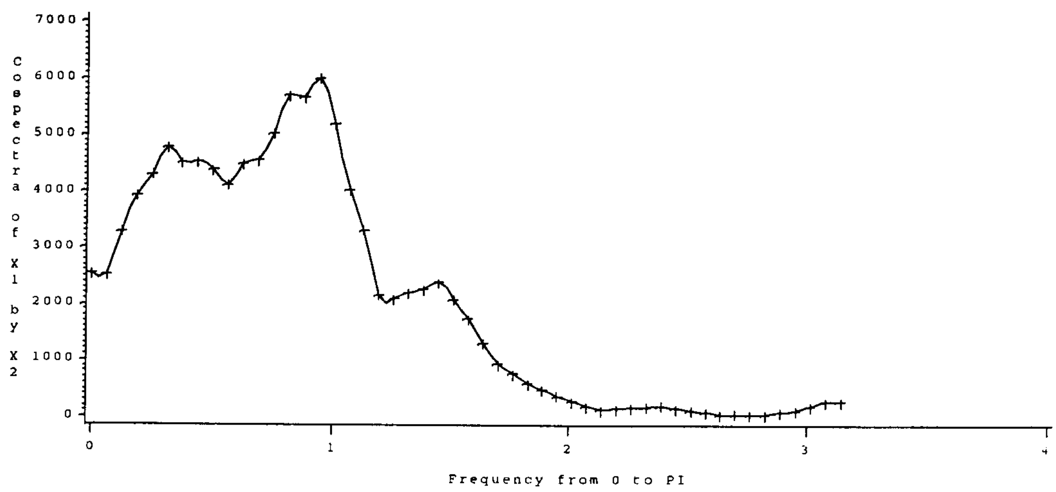


Fig. 8.12: Evolutionary Co-spectrum of $X_{1,t}$ by $X_{2,t}$

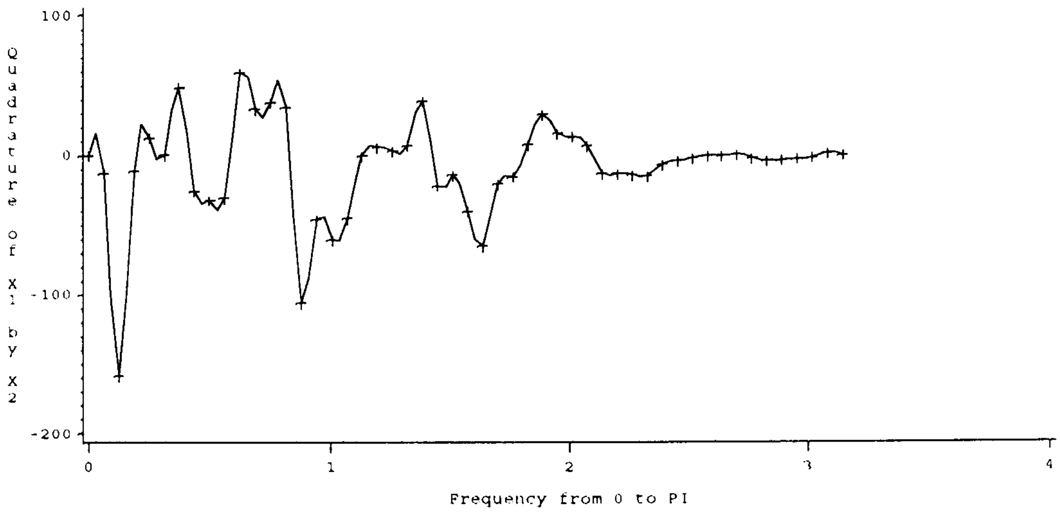


Fig. 8.13: Evolutionary Quadrature Spectrum of $X_{1,t}$ by $X_{2,t}$

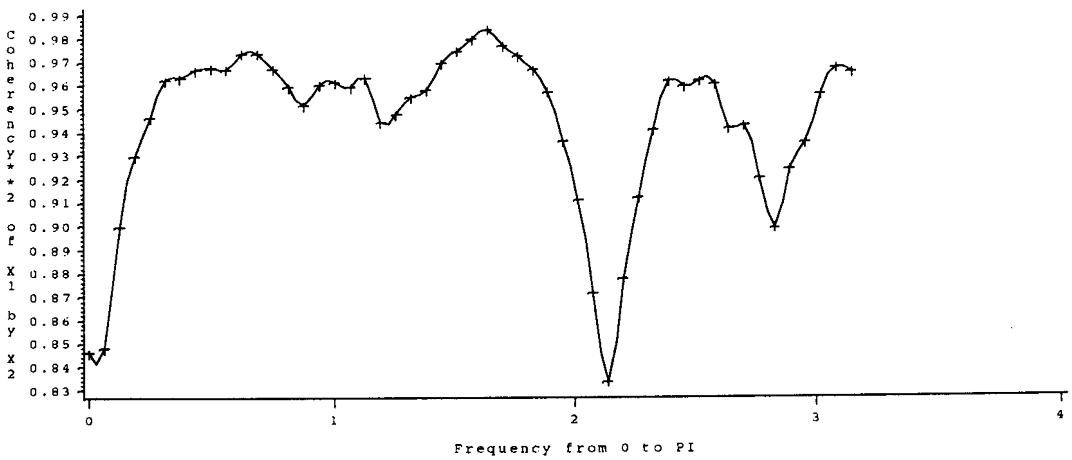


Fig. 8.14: Evolutionary Coherency of $X_{1,t}$ by $X_{2,t}$

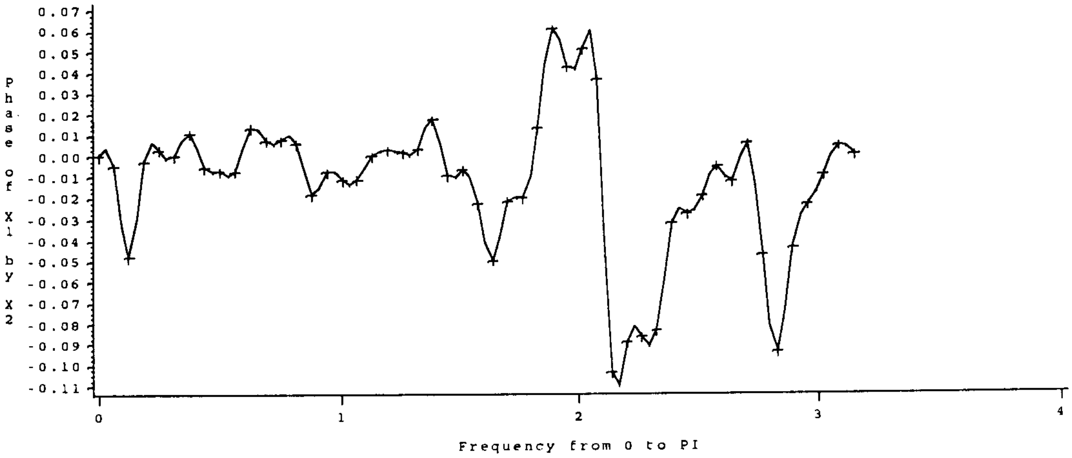


Fig. 8.15: Evolutionary Phase Spectrum of $X_{1,t}$ by $X_{2,t}$

From Fig. 8.15 (and the results in Appendix D), it is seen that the evolutionary coherence values K_01_02 are extremely high (almost 1) for the frequencies, $0 \leq w \leq 2\pi$. The lowest coherency of 0.83370 at frequency of 2.19911 is even high enough. The nearness of these coherences to 1 is an indication that the two series are highly related between these frequencies. The phase of $X_{1,t}$ by $X_{2,t}$ over the frequency range [2.07345, 2.13628] is a straight line with slope

$$\text{slope} = \frac{0.03612 - [-0.10361]}{2.07345 - 2.13628} \cong -2.2. \qquad (8.14)$$

Assuming the data were monthly data, the direction of the slope (the negative sign) is an indication that $X_{2,t}$ leads $X_{1,t}$ and by a time lag of approximately 2.2 months.

8.6 Summary

In this chapter, an estimate of the degree of differencing were able confirms the conclusion drawn in Chapter 6 that Series 2 is stationary. The approach also confirms that Series 3 and Series 4 are non-stationary. We are also able to estimate the average spectrum and also find relationships between pairs of such series satisfactorily for non-stationary series with time-changing spectra. This is always possible using the methods devised for stationary time series.

CHAPTER APPENDIX 8.1

```
MTB > READ C1 → (CREATES COLUMN C1 FOR NOMRATE DATA)
DATA> 96.33
DATA> 95.15
DATA> 93.51
DATA> 93.34
DATA> 92.83
  .       .
  .       .
  .       .
DATA> 43.20
DATA> 43.00
DATA> 43.12
DATA> 44.18
DATA> 43.37
DATA> ENDOFDATA → (END OF DATA)
      99 rows read.
MTB > TSPLOT C1 → (PLOTS NOMRATE)
MTB > ACF C1 → (COMPUTES THE ACF'S FOR NOMRATE)
```

Autocorrelation Function

```
ACF of C1

           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
  1   0.967                          XXXXXXXXXXXXXXXXXXXXXXXXX
  2   0.935                          XXXXXXXXXXXXXXXXXXXXXXXX
  3   0.900                          XXXXXXXXXXXXXXXXXXXXXXXX
  4   0.866                          XXXXXXXXXXXXXXXXXXXXXXX
  5   0.833                          XXXXXXXXXXXXXXXXXXXXXX
  6   0.800                          XXXXXXXXXXXXXXXXXXXXX
  7   0.769                          XXXXXXXXXXXXXXXXXXXX
  8   0.737                          XXXXXXXXXXXXXXXXXXX
  9   0.704                          XXXXXXXXXXXXXXXXXXX
 10   0.669                          XXXXXXXXXXXXXXXXXX
 11   0.635                          XXXXXXXXXXXXXXXXX
 12   0.603                          XXXXXXXXXXXXXXXX
 13   0.576                          XXXXXXXXXXXXXXX
 14   0.549                          XXXXXXXXXXXXXXX
 15   0.521                          XXXXXXXXXXXXXX
 16   0.493                          XXXXXXXXXXXXX
 17   0.467                          XXXXXXXXXXXXX
 18   0.442                          XXXXXXXXXXXX
 19   0.417                          XXXXXXXXXXX
 20   0.392                          XXXXXXXXXXX
 21   0.367                          XXXXXXXXXX
 22   0.342                          XXXXXXXXXX
 23   0.317                          XXXXXXXXX
 24   0.292                          XXXXXXXX
 25   0.270                          XXXXXXXX
```

```
MTB > SET C2 → (CREATES A NEW COLUMN FOR TIME t)
DATA> 1:99 → (CREATES TIME t FOR 99 OBSERVATIONS)
DATA> REGR C1 1 C2 → (REGRESSES TREND DATA ON TIME)
```

Regression Analysis

```
The regression equation is
C1 = 96.9 - 0.547 C2

Predictor         Coef       StDev          T        P
Constant       96.8916      0.4871     198.92    0.000
C2           -0.547169    0.008458     -64.69    0.000

S = 2.405       R-Sq = 97.7%     R-Sq(adj) = 97.7%

Analysis of Variance

Source       DF          SS          MS         F         P
Regression    1       24206       24206   4185.20     0.000
Error        97         561           6
Total        98       24767

Unusual Observations
Obs        C2         C1        Fit   StDev Fit   Residual    St Resid
 58      58.0     70.120     65.156       0.251      4.964      2.08R
 59      59.0     71.110     64.609       0.253      6.501      2.72R
 68      68.0     54.190     59.684       0.286     -5.494     -2.30R
 90      90.0     41.190     47.646       0.416     -6.456     -2.73R

R denotes an observation with a large standardized residual

MTB > LET C3 = 96.8916 - 0.5472*C2 → (PRODUCES THE TREND DATA & STORES THEM ON A WORKSHEET)
MTB > LET C4 = C1/C3 → (PRODUCES THE DE-TRENDED DATA & STORES THEM ON A WORKSHEET)
MTB > TSPLOT C3 → (PLOTS THE TREND DATA)
MTB > TSPLOT C4 → (PLOTS THE DE-TRENDED DATA)
MTB > ACF C4 → (COMPUTES THE ACF'S FOR DE-TRENDED DATA)
```

Autocorrelation Function

```
ACF of C4

           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
  1   0.815                          XXXXXXXXXXXXXXXXXXXXX
  2   0.600                          XXXXXXXXXXXXXXXX
  3   0.322                          XXXXXXXXX
  4   0.139                          XXXX
  5   0.020                          XX
  6  -0.099                        XXX
  7  -0.211                     XXXXXX
  8  -0.292                   XXXXXXXX
  9  -0.366                 XXXXXXXXXX
 10  -0.353                 XXXXXXXXXX
 11  -0.353                 XXXXXXXXXX
 12  -0.301                  XXXXXXXXX
 13  -0.295                   XXXXXXXX
 14  -0.301                  XXXXXXXXX
 15  -0.312                  XXXXXXXXX
 16  -0.320                  XXXXXXXXX
 17  -0.272                   XXXXXXXX
 18  -0.159                      XXXXX
 19  -0.048                         XX
 20   0.085                          XXX
 21   0.135                          XXXX
 22   0.200                          XXXXXX
 23   0.229                          XXXXXXX
 24   0.271                          XXXXXXXX
 25   0.314                          XXXXXXXXX

MTB > PACF C4 → (COMPUTES THE PACF'S FOR DE-TRENDED DATA)
```

Partial Autocorrelation Function

```
PACF of C4

          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
           +----+----+----+----+----+----+----+----+----+----+
  1   0.815                         XXXXXXXXXXXXXXXXXXXXX
  2  -0.192                    XXXXXX
  3  -0.323                 XXXXXXXXX
  4   0.127                         XXXX
  5   0.045                         XX
  6  -0.284                  XXXXXXXX
  7  -0.133                      XXXX
  8   0.082                         XXX
  9  -0.191                    XXXXXX
 10   0.040                         XX
 11  -0.103                      XXXX
 12  -0.019                         X
 13  -0.177                     XXXXX
 14  -0.166                     XXXXX
 15  -0.076                       XXX
 16  -0.139                      XXXX
 17  -0.017                         X
 18   0.116                         XXXX
 19  -0.080                       XXX
 20  -0.015                         X
 21  -0.085                       XXX
 22   0.036                         XX
 23  -0.054                        XX
 24  -0.002                         X
 25   0.093                         XXX

MTB > STOP
```

CHAPTER APPENDIX 8.2

```
data detnom;
  input y @@;
  x1 = (96.8916 - 0.5472*(20))*y;
  x2 = (96.8916 - 0.5472*(40))*y;
  x3 = (96.8916 - 0.5472*(60))*y;
  x4 = (96.8916 - 0.5472*(80))*y;
  label y = 'detrended nomrate';
cards;
0.99985 0.99324 0.98173 0.98561 0.98592 0.98805 0.99579 0.99314 1.00047 0.99541 1.00162 1.00160
1.00637 1.01153 1.01484 1.02103 1.02227 1.01743 1.01740 1.02329 1.02446 1.02576 1.02341 1.02102
1.00454 0.99667 0.99686 0.97559 0.97564 0.97395 0.95672 0.99733 1.01085 1.01497 1.01067 0.99633
0.99405 0.97925 0.96357 0.94607 0.95020 0.97726 0.97678 0.99209 1.00444 1.01364 1.01398 0.99551
0.98430 0.97565 0.97339 0.99651 1.01146 1.03824 1.03911 1.05162 1.06373 1.07619 1.10063 1.04509
1.01552 0.94033 0.95402 0.96779 0.93615 0.94047 0.95167 0.90795 0.94121 0.93429 0.99306 1.02460
1.04656 1.05707 1.04845 1.04400 1.05443 1.04330 1.05823 1.02282 1.03460 1.04952 1.04980 1.05440
1.05295 1.05428 1.05482 0.93043 0.91257 0.86449 0.94312 0.96881 0.99294 0.96661 0.96191 0.96927
0.98411 1.02105 1.01517

title'Spectrum of Y';
proc spectra data=detnom out=b p s adjmean whitetest;
   var y;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b;
run;
symbol1 i=splines v=none;
```

```
proc gplot data=b;
   plot p_01 * freq;
   plot s_01 * freq;
run;

title'Evolutionary Spectrum of X when t=20';
proc spectra data=detnom out=b1 p s adjmean whitetest;
   var x1;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b1;
run;
symbol1 i=splines v=none;
proc gplot data=b1;
   plot p_01 * freq;
   plot s_01 * freq;
run;

title'Evolutionary Spectrum of X when t=40';
proc spectra data=detnom out=b2 p s adjmean whitetest;
   var x2;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b2;
run;
symbol1 i=splines v=none;
proc gplot data=b2;
   plot p_01 * freq;
   plot s_01 * freq;
run;

title'Evolutionary Spectrum of X when t=60';
proc spectra data=detnom out=b3 p s adjmean whitetest;
   var x3;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b3;
run;
symbol1 i=splines v=none;
proc gplot data=b3;
   plot p_01 * freq;
   plot s_01 * freq;
run;

title'Evolutionary Spectrum of X when t=80';
proc spectra data=detnom out=b4 p s adjmean whitetest;
   var x4;
   weights 1 2 3 4 3 2 1;
run;
proc print data=b4;
run;
symbol1 i=splines v=none;
proc gplot data=b4;
   plot p_01 * freq;
   plot s_01 * freq;
run;
quit;
```

CHAPTER 9

SUMMARY, CONCLUSIONS AND RECOMMENDED RESEARCH

9.1 Summary and Conclusions

In this study, we have considered some aspects of non-stationary time series usually encountered in practice. We have seen that the usual asymptotic results do not apply if any of the variables in a test regression model is generated by a non-stationary process. When an underlying assumption in a regression model is violated, some strange things happen, for instance, totally unrelated variables may lead to spurious regression. This problem may be explained by the fact that a great many economic time series trend upward over time. Two obvious ways to avoid violating the standard assumptions of regression when using such series are to difference or de-trend the series prior to its use. But differencing and de-trending are two different operations. When differencing is deemed appropriate, de-trending becomes inappropriate, and vice versa.

The choice between differencing and de-trending boils down to a choice between the models

$$X_t = C + \rho X_{t-1} + \varepsilon_t \tag{9.1}$$

and

$$X_t = (\beta_0 + \beta_1 t) + \rho X_{t-1} + \varepsilon_t , \tag{9.2}$$

where $\{\varepsilon_t\} \sim WN(0,\sigma^2)$. Techniques usually used for choosing between (9.1) and (9.2) are the unit root tests. For an AR(p) process,

$$X_t = C + \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t , \qquad \{\varepsilon_t\} \sim WN(0,\sigma^2), \tag{9.3}$$

the stationarity of the process depends on the roots of the polynomial equation

$$\Phi(B) = 0, \tag{9.4}$$

where $\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$ with $B^k X_t = X_{t-k}$. If all roots are outside the unit circle, the process is stationary. If a root is equal to or less than 1 in absolute value, the process is non-stationary. When a root is equal to 1 in absolute value, it is referred to as a unit root. When a process has a unit root it must be differenced to induce stationarity.

Next, if X_{t-1} is subtracted from both sides of (9.1) and (9.2), the reformulated versions are respectively given by

$$Z_t = C + (\rho - 1)X_{t-1} + \varepsilon_t, \tag{9.5a}$$

$$Z_t = (\beta_0 + \beta_1 t) + (\rho - 1)X_{t-1} + \varepsilon_t, \tag{9.5b}$$

where $Z_t = X_t - X_{t-1}$ and $\{\varepsilon_t\} \sim WN(0,\sigma^2)$. A test of the null hypotheses

$$\left.\begin{aligned} H_0&:\rho = 1 \\ H_1&:\rho < 1 \end{aligned}\right\} \tag{9.6}$$

is commonly referred to as a unit root test. A glance at (9.5) might appear that a unit root test could be done simply by using the usual *t*-statistic

$$\rho - 1 = 0 \tag{9.7}$$

in (9.5), but this is not so. Fact is, when $\rho = 1$, the process generating X_t is integrated of order 1, and hence the X_{t-1} will not satisfy the standard assumptions needed for asymptotic analysis. Consequently, the *t*-statistic does not have the N(0,1) distribution asymptotically. From our discussions, we have seen that the simplest tests for unit roots are the Dickey-Fuller (DF) and Phillips-Perron (PP) test procedures due to Dickey and Fuller (1979) and Phillips and Perron (1988), respectively. The DF test assumes that the error terms in the test regressions, (9.5a) and (9.5b), are serially uncorrelated. More often than not, this assumption becomes untenable because the regression functions for the test regressions do not depend on any economic/econometric variables. In such a case, the error terms will display serial correlation.

The simplest and a modified test for a unit root in the presence of serial correlation of unknown form is the Augmented Dickey-Fuller (ADF) test also due to Dickey and Fuller (1979). The ADF test procedure assumes that the error terms follow an AR process of known order. Empirical work by Said and Dickey (1984) and Phillips and Perron (1988) reveal that the ADF tests are asymptotically valid under much less restrictive assumptions.

Another way to obtain unit root test statistics that are valid despite the presence of serial correlation of unknown form is the use of the non-parametric unit root test due to Phillips and Perron (1988). This test procedure is non-parametric in that no parametric specification of the error process is required. The test statistic is based on the usual test regressions (9.5a) and (9.5b), but is modified so that serial correlation does not affect the asymptotic distribution. To confirm the nature of a given time series, whether stationary or non-stationary, we established that even though the order specified in the Reversed Dickey-Fuller unit root test may differ from the order in the Dickey-Fuller test, both test procedures give the same conclusion.

Also in our study, we explored a unit root test based on periodogram ordinates. The proposed test is distributed as a linear combination of two independent standard normal variables. One appealing property of this approach is its invariance to deterministic seasonal components and time trends.

A spectral estimator of the differencing degree parameter d is of paramount importance in the analysis of economic/financial time series, more especially in model specification/misspecification. Persistent fractional models have characteristics similar to those of non-stationary models. First, the autocorrelation of persistent fractional models decay very slowly, a characteristic found in non-stationary time series. Second, realizations of these two models have periodograms diverging at zero frequency. Using the power transfer function, we established that the spectral density of the ARIMA(p,d,q) process

$$f_x(w) = |1 - e^{-iw}|^{-2d} f_y(w), \tag{9.9}$$

where $f_x(w)$ and $f_y(w)$ are respectively the spectra of the undifferenced series $\{X_t : t = 1,2,\ldots,T\}$ and the stationary series $Y_t = (1-B)^d X_t$. The estimator of d is based on the fact that for non-zero frequencies $w_j\,(j = 1,2,\ldots,T^{0.5})$, the spectrum and the periodogram of an ARIMA process

$$\Phi(B)(1-B)^d X_t = \theta(B)\varepsilon_t, \qquad \{\varepsilon_t\} \sim WN(0,\sigma^2) \tag{9.10}$$

is dominated by $|1 - e^{-iw}|^2$ as in (9.9). If we take logarithms of both sides of (9.9), include $\hat{f}_x(w)$, an estimate of $f_x(w)$ with $w = w_j (j = 1,2,\ldots,T^{0.05})$, and introduce the disturbance term e_j, equation (9.9) becomes

$$\ln \hat{f}_x(w_j) = \beta_0 + \beta_1 \ln[4\sin^2(w_j/2)] + e_j \quad , \qquad (9.11)$$

where $\beta_0 = \ln f_y(0)$, $\beta_1 = -\hat{d}$, and $e_j = \ln\left[\frac{\hat{f}_x(w_j)}{f_x(w_j)}\right]$, for $j = 1,2,\ldots,T^{0.05}$.

In a similar manner, if we take logarithms on both sides of (9.9), replace $f_x(w)$ with the periodogram $I_x(w)$, equation (9.9) becomes

$$\ln I_x(w_j) = \beta_0 + \beta_1 \ln[4\sin^2(w_j/2)] + e_j \quad , \qquad (9.12)$$

where β_0 and $\beta_1 = -\hat{d}$ are some constants. The regression models (9.10) and (9.11) provide an estimator for the differencing degree to stationarize the ARIMA process. A series is regarded as stationary $d \in (-0.5, 0.5)$, and non-stationary if otherwise.

Thus, while traditional autocorrelation methods have difficulty distinguishing between stationary and non-stationary processes, we have shown that a spectral regression on the low-order frequencies is able to estimate the order required to stationarize a non-stationary time series. Additionally, we have shown that this order reveals the stationarity or the non-stationarity of the process, a further step to confirm the stationarity or otherwise of a time series. In all our practical illustrations, we were able to confirm the unit root test conclusions by determining the differencing degree.

In Chapter 5, we concentrated our discussions on developing and examining the concept of evolutionary spectral analysis. The concept of evolutionary spectrum analysis provides a great deal of insight when it comes to speech processing and seismology. The existence of evolutionary spectrum was established by assuming the following representation for the non-stationary series $\{X_t\}$:

$$X_t = \int_{\Lambda} \Omega_t(\omega).e^{i\omega t} dz(\omega), \qquad (9.13)$$

where $z(\omega)$ is an orthogonal process with $E|dz(\omega)|^2 = f(\omega)d(\omega)$. We were able to establish that for the non-stationary process the (evolutionary) spectrum at time *t*, $dF(\omega)$, is

$$dF(\omega) = |\Omega_t(\omega)|^2 f(\omega)d\omega. \qquad (9.14)$$

A time-dependent univariate non-stationary series was simulated from a stationary series (data on the effective nominal exchange rate of the South African Rand, indexed 1990=100) and evolutionary spectrum was obtained. The existence of evolutionary spectrum for the simulated non-stationary data gives us the clue that all other elements of interest in spectral analysis of stationary series are possible to estimate in the case of time-dependent non-stationary series. We were also able to apply the concept of evolutionary cross-spectral analysis of time-dependent non-stationary bivariate processes, using two simulated time-dependent series. Plots of evolutionary co-spectrum, evolutionary quadrature spectrum, evolutionary coherency and evolutionary gain spectrum were obtained.

We have also performed Monte Carlo study on the size and the power of three most commonly used unit root test statistics viz, ADF, PP, and RDFUR tests. Our Monte Carlo indicates that the PP test criterion is generally most powerful compared to the two other test criteria.

8.2 Recommended Research

The following recommended research appears to be equally important as a result of this study.

i. A Monte Carlo study of the ADF, the PP, and the RDFUR test statistics based on tiny interval values for ρ, say ρ = 1.000, 0.995, 0.990, 0.985, 0.980,...,0.600.

ii. Applying bootstrap methods to these three test statistics to find out whether we will arrive at the same conclusions drawn using Monte Carlo simulation methods.

iii. A Monte Carlo study of less familiar unit root tests.

APPENDICES, PROGRAMS AND BIBLIOGRAPHY

APPENDIX A

SERIES 1

EFFECTIVE NOMINAL EXCHANGE RATE OF THE SOUTH AFRICAN RAND (1990=100)

Month	1991	1992	1993	1994	1995	1996	1997	1998	1999
Jan	*	91.02	86.28	78.57	72.17	71.11	57.64	54.04	43.20
Feb	*	90.47	85.52	76.91	70.31	66.95	58.91	53.70	43.00
Mar	96.33	90.35	83.59	76.19	68.98	64.50	59.60	53.05	43.12
Apr	95.15	90.26	82.39	74.52	67.84	59.21	59.62	52.54	44.18
May	93.51	90.00	81.86	72.80	67.15	59.55	58.56	51.99	43.37
Jun	93.34	89.99	79.58	70.96	68.20	59.88	57.74	45.35	
Jul	92.83	89.54	79.05	70.75	68.67	57.41	57.74	43.98	
Aug	92.49	88.56	78.38	72.23	69.92	57.16	56.56	41.19	
Sep	92.67	88.00	76.47	71.66	69.41	57.32	56.79	44.42	
Oct	91.88	87.95	79.17	72.24	69.67	54.19	54.33	45.10	
Nov	92.01	87.49	79.69	72.59	69.89	55.66	54.39	45.68	
Dec	91.00	87.04	79.46	72.70	70.12	54.74	54.60	43.94	

SERIES 2

YIELDS ON ESKOM LOAN STOCK TRADED ON THE JSE (%)

MON	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Jan	15.32	16.25	16.23	14.70	12.04	16.86	13.68	15.83	13.72	16.39
Feb	15.42	15.46	16.37	14.43	12.60	16.72	14.00	15.04	13.59	15.34
Mar	15.43	15.46	16.20	14.59	12.80	16.64	14.94	15.26	13.42	15.00
Apr	15.81	15.62	16.06	15.08	13.04	16.75	15.69	15.35	13.00	15.02
May	15.69	15.77	15.82	15.01	13.16	16.92	16.42	15.16	13.57	15.60
Jun	16.19	16.02	15.79	14.71	14.17	16.77	15.56	14.87	14.84	15.30
Jul	16.03	16.09	15.14	14.23	14.87	16.58	15.16	14.43	16.22	15.33
Aug	15.83	16.45	14.28	13.84	15.65	15.91	15.63	14.45	17.37	
Sep	15.96	16.53	14.11	13.31	16.72	15.43	15.29	14.38	18.97	
Oct	16.18	16.77	14.03	13.07	16.75	15.07	15.71	14.24	17.19	
Nov	15.83	16.44	14.55	12.50	16.71	14.38	16.12	14.70	16.64	
Dec	15.83	16.25	14.88	12.20	16.62	14.50	16.16	14.19	16.78	

SERIES 3

GOLD SHARES TRADED ON THE JOHANNESBURG STOCK EXCHANGE

MON	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Jan	130	74	73	45	128	112	102	96	61	66
Feb	124	60	73	54	115	102	115	106	62	63
Mar	120	61	67	61	122	94	113	103	56	65
Apr	109	61	59	77	117	97	119	93	74	67
May	101	66	61	96	116	88	129	90	77	
Jun	86	79	63	99	128	91	119	80	64	
Jul	92	81	62	116	130	94	117	71	66	
Aug	107	70	55	105	137	99	115	75	58	
Sep	96	66	50	91	152	98	113	70	66	
Oct	86	68	47	100	148	89	111	72	78	
Nov	79	68	44	113	137	85	107	60	67	
Dec	70	71	47	124	123	85	96	54	62	

SERIES 4

CONSUMER PRICE INDEX FOR SOUTH AFRICA AT 1995 PRICES

MON	1994	1995	1996	1997	1998	1999
Jan	88.2	96.8	103.5	112.7	120.8	130.2
Feb	88.7	97.7	104.0	113.6	121.5	130.9
Mar	89.3	98.5	104.6	114.2	122.1	132.0
Apr	89.7	99.5	105.1	115.1	123.2	132.7
May	90.4	100.1	105.9	115.7	124.1	133.8
Jun	91.3	100.4	107.0	116.2	124.8	134.7
Jul	92.4	100.5	107.5	117.1	125.5	135.8
Aug	93.6	100.5	108.1	117.7	126.6	136.6
Sep	94.6	100.6	109.3	118.4	127.5	137.6
Oct	95.0	101.1	110.3	119.0	128.2	138.4
Nov	95.4	101.7	111.2	119.7	129.0	
Dec	96.0	102.7	112.4	120.3	129.6	

APPENDIX B

OBS	FREQ	PERIOD	P_01	S_01
1	0.00000	.	0.0000	1.55055
2	0.05464	115.000	11.1613	1.54352
3	0.10927	57.500	38.9346	1.71100
4	0.16391	38.333	22.2028	1.74219
5	0.21855	28.750	9.7468	1.67976
6	0.27318	23.000	35.2096	1.58162
7	0.32782	19.167	11.5292	1.22804
8	0.38245	16.429	14.4221	0.95710
9	0.43709	14.375	1.1525	0.66136
10	0.49173	12.778	7.8980	0.43508
11	0.54636	11.500	0.7407	0.31059
12	0.60100	10.455	1.6550	0.23670
13	0.65564	9.583	6.5224	0.22575
14	0.71027	8.846	1.0545	0.15772
15	0.76491	8.214	0.1251	0.12522
16	0.81955	7.667	1.5425	0.09677
17	0.87418	7.188	0.4151	0.06222
18	0.92882	6.765	1.3567	0.05758
19	0.98346	6.389	0.3217	0.04589
20	1.03809	6.053	0.2044	0.03635
21	1.09273	5.750	0.3205	0.03282
22	1.14736	5.476	0.2424	0.03056
23	1.20200	5.227	0.9501	0.03464
24	1.25664	5.000	0.0758	0.03090
25	1.31127	4.792	0.4812	0.02780
26	1.36591	4.600	0.4025	0.02322
27	1.42055	4.423	0.0061	0.01722
28	1.47518	4.259	0.0755	0.01591
29	1.52982	4.107	0.3447	0.01555
30	1.58446	3.966	0.2766	0.01522
31	1.63909	3.833	0.0044	0.01434
32	1.69373	3.710	0.2678	0.01481
33	1.74836	3.594	0.2132	0.01383
34	1.80300	3.485	0.0415	0.01356
35	1.85764	3.382	0.2725	0.01456
36	1.91227	3.286	0.1684	0.01294
37	1.96691	3.194	0.2270	0.01185
38	2.02155	3.108	0.0604	0.00979
39	2.07618	3.026	0.0129	0.00796
40	2.13082	2.949	0.1754	0.00759
41	2.18546	2.875	0.0468	0.00633
42	2.24009	2.805	0.1253	0.00614
43	2.29473	2.738	0.0476	0.00649
44	2.34936	2.674	0.0013	0.00750
45	2.40400	2.614	0.0837	0.00946
46	2.45864	2.556	0.2988	0.01083
47	2.51327	2.500	0.2129	0.00993
48	2.56791	2.447	0.0204	0.00750
49	2.62255	2.396	0.0014	0.00495
50	2.67718	2.347	0.0152	0.00305
51	2.73182	2.300	0.0727	0.00287
52	2.78646	2.255	0.0136	0.00308
53	2.84109	2.212	0.0490	0.00358
54	2.89573	2.170	0.0796	0.00392
55	2.95037	2.12963	0.00911	.0042245
56	3.00500	2.09091	0.06596	.0051176
57	3.05964	2.05357	0.06471	.0057502
58	3.11427	2.01754	0.13541	.0060279

APPENDIX C

Dependent Variable: M

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	4.59922	4.59922	3.334	0.1053
Error	8	11.03634	1.37954		
C Total	9	15.63555			

Root MSE	1.17454	R-square	0.2942
Dep Mean	2.20348	Adj R-sq	0.2059
C.V.	53.30373		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	0.829891	0.83898143	0.989	0.3516
N	1	-0.490077	0.26840441	-1.826	0.1053

Obs	Dep Var M	Predict Value	Residual
1	2.4125	3.6793	-1.2669
2	3.6619	3.0004	0.6615
3	3.1002	2.6035	0.4967
4	2.2769	2.3224	-0.0455
5	3.5613	2.1048	1.4565
6	2.4449	1.9274	0.5174
7	2.6688	1.7780	0.8908
8	0.1419	1.6489	-1.5070
9	2.0666	1.5355	0.5311
10	-0.3002	1.4346	-1.7348

Sum of Residuals	0
Sum of Squared Residuals	11.0363
Predicted Resid SS (Press)	22.7234

APPENDIX D

OBS	FREQ	PERIOD	S_01	S_02	CS_01_02	QS_01_02	K_01_02	A_01_02	PH_01_02
1	0.00000	.	4078.26	1873.41	2542.27	0.000	0.84593	2542.27	0.00000
2	0.06283	100.000	4028.57	1863.25	2522.82	-12.623	0.84793	2522.85	-0.00500
3	0.12566	50.000	4962.13	2437.19	3295.08	-158.012	0.89985	3298.87	-0.04792
4	0.18850	33.333	5607.18	2957.27	3926.38	-11.120	0.92972	3926.40	-0.00283
5	0.25133	25.000	5921.84	3312.18	4308.18	12.770	0.94628	4308.20	0.00296
6	0.31416	20.000	6414.29	3698.71	4777.49	0.409	0.96206	4777.49	0.00009
7	0.37699	16.667	6026.43	3513.83	4516.10	49.075	0.96325	4516.36	0.01087
8	0.43982	14.286	6119.81	3473.00	4532.47	-25.249	0.96659	4532.54	-0.00557
9	0.50265	12.500	5848.01	3383.80	4375.15	-31.793	0.96738	4375.27	-0.00727
10	0.56549	11.111	5324.19	3292.16	4116.19	-30.183	0.96667	4116.30	-0.00733
11	0.62832	10.000	5547.82	3729.56	4487.71	59.834	0.97353	4488.11	0.01333
12	0.69115	9.091	5386.88	3960.86	4557.30	33.215	0.97345	4557.42	0.00729
13	0.75398	8.333	6054.36	4319.25	5028.15	38.219	0.96686	5028.30	0.00760
14	0.81681	7.692	7139.17	4723.38	5687.42	34.119	0.95928	5687.52	0.00600
15	0.87965	7.143	7417.04	4522.42	5647.92	-105.826	0.95132	5648.91	-0.01873
16	0.94248	6.667	8184.24	4588.06	6004.43	-45.904	0.96020	6004.60	-0.00764
17	1.00531	6.250	7150.10	3904.62	5180.13	-60.238	0.96128	5180.49	-0.01163
18	1.06814	5.882	5547.90	3046.04	4025.80	-45.092	0.95917	4026.05	-0.01120
19	1.13097	5.556	4518.82	2526.38	3315.94	-0.191	0.96314	3315.94	-0.00006
20	1.19381	5.263	2924.99	1705.26	2170.22	5.687	0.94426	2170.22	0.00262
21	1.25664	5.000	2762.17	1670.08	2090.79	3.159	0.94762	2090.79	0.00151
22	1.31947	4.762	2867.90	1780.32	2208.06	7.444	0.95492	2208.07	0.00337
23	1.38230	4.545	2821.25	1892.03	2260.89	39.127	0.95790	2261.23	0.01730
24	1.44513	4.348	2882.89	2057.74	2398.03	-21.817	0.96946	2398.13	-0.00910
25	1.50796	4.167	2436.10	1837.34	2088.41	-13.781	0.97446	2088.45	-0.00660
26	1.57080	4.000	1988.82	1554.02	1739.49	-39.745	0.97953	1739.95	-0.02284
27	1.63363	3.846	1493.29	1154.20	1300.44	-64.878	0.98364	1302.06	-0.04985
28	1.69646	3.704	1111.23	809.86	937.27	-20.212	0.97660	937.49	-0.02156
29	1.75929	3.571	960.79	629.66	766.77	-15.411	0.97223	766.93	-0.02010
30	1.82212	3.448	783.85	462.04	591.57	7.746	0.96643	591.62	0.01309
31	1.88496	3.333	660.73	372.35	484.33	29.389	0.95699	485.23	0.06061
32	1.94779	3.226	520.69	286.46	373.31	15.755	0.93597	373.64	0.04218
33	2.01062	3.125	384.61	209.38	270.52	13.710	0.91110	270.87	0.05064
34	2.07345	3.030	280.34	149.55	190.98	6.901	0.87110	191.10	0.03612
35	2.13628	2.941	189.69	102.47	126.62	-13.166	0.83370	127.30	-0.10361
36	2.19911	2.857	208.59	113.31	143.43	-12.831	0.87745	144.01	-0.08922
37	2.26195	2.778	230.32	127.95	163.31	-14.143	0.91183	163.92	-0.08639
38	2.32478	2.703	253.03	136.59	179.70	-15.009	0.94078	180.32	-0.08333
39	2.38761	2.632	286.45	145.79	200.28	-6.439	0.96153	200.39	-0.03214
40	2.45044	2.564	222.05	110.98	153.70	-4.300	0.95938	153.76	-0.02797
41	2.51327	2.500	170.96	80.88	115.28	-2.216	0.96151	115.31	-0.01922
42	2.57611	2.439	119.85	55.51	79.92	-0.413	0.96009	79.92	-0.00516
43	2.63894	2.381	73.14	33.58	48.07	-0.602	0.94121	48.08	-0.01253
44	2.70177	2.326	80.13	35.25	51.60	0.283	0.94261	51.60	0.00549
45	2.76460	2.273	77.96	32.04	47.89	-2.267	0.92031	47.95	-0.04730
46	2.82743	2.222	73.86	27.32	42.41	-3.981	0.89899	42.59	-0.09360
47	2.89027	2.174	128.21	51.56	78.08	-3.440	0.92395	78.15	-0.04404
48	2.95310	2.128	185.07	76.64	115.13	-2.700	0.93507	115.16	-0.02344
49	3.01593	2.083	295.39	132.04	193.08	-1.822	0.95592	193.09	-0.00944
50	3.07876	2.041	407.97	188.26	272.51	1.236	0.96697	272.52	0.00453
51	3.14159	2.000	413.46	189.11	274.69	0.000	0.96498	274.69	0.00000

APPENDIX E

REVERSED SERIES 2: E - 1

MON	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990
Dec	*	16.78	14.19	16.16	14.50	16.62	12.20	14.88	16.25	15.83
Nov	*	16.64	14.70	16.12	14.38	16.71	12.50	14.55	16.44	15.83
Oct	*	17.19	14.24	15.71	15.07	16.75	13.07	14.03	16.77	16.18
Sep	*	18.97	14.38	15.29	15.43	16.72	13.31	14.11	16.53	15.96
Aug	*	17.37	14.45	15.63	15.91	15.65	13.84	14.28	16.45	15.83
Jul	15.33	16.22	14.43	15.16	16.58	14.87	14.23	15.14	16.09	16.03
Jun	15.30	14.84	14.87	15.56	16.77	14.17	14.71	15.79	16.02	16.19
May	15.60	13.57	15.16	16.42	16.92	13.16	15.01	15.82	15.77	15.69
Apr	15.02	13.00	15.35	15.69	16.75	16.04	15.08	16.06	15.62	15.81
Mar	15.00	13.42	15.26	14.94	16.64	12.80	14.59	16.20	15.46	15.43
Feb	15.34	13.59	15.04	14.00	16.72	12.60	14.43	16.37	15.46	15.42
Jan	16.39	13.72	15.83	13.68	16.86	12.04	14.70	16.23	16.25	15.32

REVERSED SERIES 3: E -2

MON	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990
Dec	*	62	54	96	82	123	124	47	71	70
Nov	*	67	60	107	85	137	113	44	68	79
Oct	*	78	72	111	89	148	100	47	68	86
Sep	*	66	70	113	98	152	91	50	66	96
Aug	*	58	75	115	99	137	105	55	70	107
Jul	*	66	71	117	94	130	116	62	81	92
Jun	*	64	80	119	91	128	99	63	79	86
May	*	77	90	129	88	116	96	61	66	101
Apr	67	74	93	119	97	117	77	59	61	109
Mar	65	56	103	113	94	122	61	67	61	120
Feb	63	62	106	115	102	115	54	73	60	124
Jan	66	61	96	102	112	128	45	73	74	130

REVERSED SERIES 4: E - 3

MON	1999	1998	1997	1996	1995	1994
Dec	*	129.6	120.3	112.4	102.7	96.0
Nov	*	129.0	119.7	111.2	101.7	95.4
Oct	138.4	128.2	119.0	110.3	101.1	95.0
Sep	137.6	127.5	118.4	109.3	100.6	94.6
Aug	136.6	126.6	117.7	108.1	100.5	93.6
Jul	135.8	125.5	117.1	107.5	100.5	92.4
Jun	134.7	124.8	116.2	107.0	100.4	91.3
May	133.8	124.1	115.7	105.9	100.1	90.4
Apr	132.7	123.2	115.1	105.1	99.5	89.7
Mar	132.0	122.1	114.2	104.6	98.5	89.3
Feb	130.9	121.5	113.6	104.0	97.7	88.7
Jan	130.2	120.8	112.7	103.5	96.8	88.2

APPENDIX F

Sample size T	Probability that $(\hat{\rho}-1)/\hat{S}e(\hat{\rho}-1)$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-3.75	-3.33	-3.00	-2.63	-0.37	0.00	0.34	0.72
50	-3.58	-3.22	-2.93	-2.60	-0.40	-0.03	0.29	0.66
100	-3.51	-3.17	-2.89	-2.58	-0.42	-0.05	0.26	0.63
250	-3.46	-3.14	-2.88	-2.57	-0.42	-0.06	0.24	0.62
500	-3.44	-3.13	-2.87	-2.57	-0.43	-0.07	0.24	0.61
∞	-3.44	-3.12	-2.86	-2.57	-0.44	-0.07	0.23	0.60

F(a): *Percentile Values for the Dickey-Fuller Test and Phillips-Perron Test Based on Estimated OLS with a constant, C.*
Source: Hamilton (1994)

Sample size T	Probability that $(\hat{\rho}-1)/\hat{S}e(\hat{\rho}-1)$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-4.38	-3.95	-3.60	-3.24	-1.14	-0.80	-0.50	-0.15
50	-4.15	-3.80	-3.50	-3.18	-1.19	-0.87	-0.58	-0.24
100	-4.04	-3.73	-3.45	-3.15	-1.22	-0.90	-0.62	-0.28
250	-3.99	-3.69	-3.43	-3.13	-1.23	-0.92	-0.64	-0.31
500	-3.98	-3.68	-3.42	-3.13	-1.24	-0.93	-0.65	-0.32
∞	-3.96	-3.66	-3.34	-3.12	-1.25	-0.94	-0.66	-0.33

F(b): *Percentile Values for the Dickey-Fuller Test and Phillips-Perron Test Based on Estimated OLS with a time trend* $(\beta_o + \beta_1 t)$. *Source: Hamilton (1994)*

α	0.001	0.01	0.025	0.05	0.10	0.20	0.50	0.90	0.95	0.975	0.99
τ^*	0.0035	0.0348	0.0880	0.1780	0.3680	0.7900	2.5400	9.4800	12.8500	16.3700	21.1700

F(c): *Percentile Values for* $z_1^2 + 3z_2^2$.

Sample size T	Critical Values for $\tilde{\tau}_{sp}$							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-3.90	-3.50	-3.18	-2.85	-1.28	-1.17	-1.08	-1.00
50	-3.73	-3.39	-3.11	-2.80	-1.29	-1.16	-1.08	-0.99
100	-3.63	-3.32	-3.06	-2.77	-1.29	-1.17	-1.07	-0.97
250	-3.61	-3.30	-3.04	-2.76	-1.29	-1.16	-1.07	-0.97
500	-3.59	-3.29	-3.04	-2.75	-1.29	-1.16	-1.07	-0.98
1000	-3.58	-3.28	-3.02	-2.75	-1.29	-1.16	-1.07	-0.98
2000	-3.56	-3.27	-3.02	-2.75	-1.29	-1.16	-1.07	-0.97

F(d): *Critical Values for the Score Test Based on Estimated OLS with a time trend* $(\alpha + \beta t)$. *Source: Schmidt and Phillips (1992)*

APPENDIX G

Program 7.1

```
data eskom;
   input x @@;
cards;
15.32 15.42 15.43 15.81 15.69 16.19 16.03 15.83 15.96 16.18 15.83 15.83
16.25 15.46 15.46 15.62 15.77 16.02 16.09 16.45 16.53 16.77 16.44 16.25
16.23 16.37 16.20 16.06 15.82 15.79 15.14 14.28 14.11 14.03 14.55 14.88
14.70 14.43 14.59 15.08 15.01 14.71 14.23 13.84 13.31 13.07 12.50 12.20
12.04 12.60 12.80 13.04 13.16 14.17 14.87 15.65 16.72 16.75 16.71 16.62
16.86 16.72 16.64 16.75 16.92 16.77 16.58 15.91 15.43 15.07 14.38 14.50
13.68 14.00 14.94 15.69 16.42 15.56 15.16 15.63 15.29 15.71 16.12 16.16
15.83 15.04 15.26 15.35 15.16 14.87 14.43 14.45 14.38 14.24 14.70 14.19
13.72 13.59 13.42 13.00 13.57 14.84 16.22 17.37 18.97 17.19 16.64 16.78
16.39 15.34 15.00 15.02 15.60 15.30 15.33
 ;
symbol1 i=join  v=none;

proc gplot data=eskom;
   plot x*t;
run;

proc arima data=eskom;
   identify var=x;
run;
```

Program 7.2

```
data eskom;
    input x @@;
    x1 = lag(x);
    x2 = lag(x1);
     z =  x - x1;
    z1 = x1 - x2;
cards;
15.32 15.42 15.43 15.81 15.69 16.19 16.03 15.83 15.96 16.18 15.83 15.83
16.25 15.46 15.46 15.62 15.77 16.02 16.09 16.45 16.53 16.77 16.44 16.25
16.23 16.37 16.20 16.06 15.82 15.79 15.14 14.28 14.11 14.03 14.55 14.88
14.70 14.43 14.59 15.08 15.01 14.71 14.23 13.84 13.31 13.07 12.50 12.20
12.04 12.60 12.80 13.04 13.16 14.17 14.87 15.65 16.72 16.75 16.71 16.62
16.86 16.72 16.64 16.75 16.92 16.77 16.58 15.91 15.43 15.07 14.38 14.50
13.68 14.00 14.94 15.69 16.42 15.56 15.16 15.63 15.29 15.71 16.12 16.16
15.83 15.04 15.26 15.35 15.16 14.87 14.43 14.45 14.38 14.24 14.70 14.19
13.72 13.59 13.42 13.00 13.57 14.84 16.22 17.37 18.97 17.19 16.64 16.78
16.39 15.34 15.00 15.02 15.60 15.30 15.33

;
proc reg data=eskom;
   model z = x1 z1;
run;
```

Program 7.3

```
data gold;
   input x @@;
   t = _n_;
cards;
130 124 120 109 101  86  92 107  96  86  79  70
 74  60  61  61  66  79  81  70  66  68  68  71
 73  73  67  59  61  63  62  55  50  47  44  47
 45  54  61  77  96  99 116 105  91 100 113 124
128 115 122 117 116 128 130 137 152 148 137 123
112 102  94  97  88  91  94  99  98  89  85  85
102 115 113 119 129 119 117 115 113 111 107  96
 96 106 103  93  90  80  71  75  70  72  60  54
 61  62  56  74  77  64  66  58  66  78  67  62
 66  63  65  67
;

symbol1 i=join  v=none;

proc gplot data=gold;
   plot x*t;
run;

proc arima data=gold;
   identify var=x;
run;
```

Program 7.4

```
data gold;
   input x @@;
   x1 = lag(x);
    z =  x - x1;
cards;
130 124 120 109 101  86  92 107  96  86  79  70
 74  60  61  61  66  79  81  70  66  68  68  71
 73  73  67  59  61  63  62  55  50  47  44  47
 45  54  61  77  96  99 116 105  91 100 113 124
128 115 122 117 116 128 130 137 152 148 137 123
112 102  94  97  88  91  94  99  98  89  85  85
102 115 113 119 129 119 117 115 113 111 107  96
 96 106 103  93  90  80  71  75  70  72  60  54
 61  62  56  74  77  64  66  58  66  78  67  62
 66  63  65  67
;

proc reg data=gold;
   model z = x1;
run;
```

Program 7.5

```
data cpi;
   input x @@;
   t = _n_;
 cards;
 88.2  88.7  89.3  89.7  90.4  91.3
 92.4  93.6  94.6  95.0  95.4  96.0
 96.8  97.7  98.5  99.5 100.1 100.4
100.5 100.5 100.6 101.1 101.7 102.7
103.5 104.0 104.6 105.1 105.9 107.0
107.5 108.1 109.3 110.3 111.2 112.4
112.7 113.6 114.2 115.1 115.7 116.2
117.1 117.7 118.4 119.0 119.7 120.3
120.8 121.5 122.1 123.2 124.1 124.8
125.5 126.6 127.5 128.2 129.0 129.6
130.2 130.9 132.0 132.7 133.8 134.7
135.8 136.6 137.6 138.4
 ;

 symbol1 i=join  v=none;

 proc gplot data=cpi;
    plot x*t;
 run;

 proc arima data=cpi;
    identify var=x;
 run;
```

Program 7.6

```
data cpi;
   input x @@;
   t  = _n_ + 1;
   x1 = lag(x);
   z  =  x - x1;
 cards;
  88.2  88.7  89.3  89.7  90.4  91.3
  92.4  93.6  94.6  95.0  95.4  96.0
  96.8  97.7  98.5  99.5 100.1 100.4
 100.5 100.5 100.6 101.1 101.7 102.7
 103.5 104.0 104.6 105.1 105.9 107.0
 107.5 108.1 109.3 110.3 111.2 112.4
 112.7 113.6 114.2 115.1 115.7 116.2
 117.1 117.7 118.4 119.0 119.7 120.3
 120.8 121.5 122.1 123.2 124.1 124.8
 125.5 126.6 127.5 128.2 129.0 129.6
 130.2 130.9 132.0 132.7 133.8 134.7
 135.8 136.6 137.6 138.4
 ;

 proc reg data=cpi;
    model z = t x1;
 run;
```

Program 7.7

```
data eskom;
   input x @@;
cards;
15.32 15.42 15.43 15.81 15.69 16.19 16.03 15.83 15.96 16.18 15.83 15.83
16.25 15.46 15.46 15.62 15.77 16.02 16.09 16.45 16.53 16.77 16.44 16.25
16.23 16.37 16.20 16.06 15.82 15.79 15.14 14.28 14.11 14.03 14.55 14.88
14.70 14.43 14.59 15.08 15.01 14.71 14.23 13.84 13.31 13.07 12.50 12.20
12.04 12.60 12.80 13.04 13.16 14.17 14.87 15.65 16.72 16.75 16.71 16.62
16.86 16.72 16.64 16.75 16.92 16.77 16.58 15.91 15.43 15.07 14.38 14.50
13.68 14.00 14.94 15.69 16.42 15.56 15.16 15.63 15.29 15.71 16.12 16.16
15.83 15.04 15.26 15.35 15.16 14.87 14.43 14.45 14.38 14.24 14.70 14.19
13.72 13.59 13.42 13.00 13.57 14.84 16.22 17.37 18.97 17.19 16.64 16.78
16.39 15.34 15.00 15.02 15.60 15.30 15.33
;

proc spectra data=eskom out=b p s adjmean whitetest;
   var x;
   weights 1 2 3 4 3 2 1;
run;

proc print data=b;

symbol1 i=splines v=none;
proc gplot data=b;
   plot p_01 * freq;
   plot s_01 * freq;
run;
```

Program 8.1

```
data eskom;
   input P_01 freq @@;
   M=log(P_01);
   A=2*sin(freq/2);
   N=log(A*A);
cards;
11.1613      0.05464
38.9346      0.10927
22.2028      0.16391
 9.7468      0.21855
35.2096      0.27318
11.5292      0.32782
14.4221      0.38245
 1.1525      0.43709
 7.8980      0.49173
 0.7407      0.54636
;
proc reg data=eskom;
   model M = N;
run;
```

Program 8.2

```
data gold;
   input P_01 freq @@;
   M=log(P_01);
   A=2*sin(freq/2);
   N=log(A*A);
 cards;
 24124.3       0.0561
 13816.6       0.1122
 12641.3       0.1683
  7275.4       0.2244
   925.1       0.2805
  3682.0       0.3366
   611.1       0.3927
  1591.3       0.4488
   656.4       0.5049
   314.4       0.5610
 ;

 proc reg data=gold;
   model M = N;
 run;
```

Program 8.3

```
data eskom;
   input S_01 freq @@;
   M=log(S_01);
   A=2*sin(freq/2);
   N=log(A*A);
cards;
1.54352        0.05464
1.71100        0.10927
1.74219        0.16391
1.67976        0.21855
1.58162        0.27318
1.22804        0.32782
0.95710        0.38245
0.66136        0.43709
0.43508        0.49173
0.31059        0.54636
;
proc reg data=eskom;
   model M = N;
run;
```

Program 8.4

```
data gold;
   input S_01 freq @@;
   M=log(S_01);
   A=2*sin(freq/2);
   N=log(A*A);
 cards;
 1516.67      0.0561
 1260.37      0.1122
  953.67      0.1683
  644.25      0.2244
  390.36      0.2805
  250.51      0.3366
  144.32      0.3927
   95.00      0.4488
   72.98      0.5049
   49.70      0.5610
 ;

 proc reg data=gold;
    model M = N;
 run;
```

Program 8.5

```
data change;
   input x @@;
   t = _n_;
cards;
1.23 0.85 0.48 0.24 0.48 0.72 0.47 0.00 0.12 0.47 0.47 1.05 1.04 0.57 0.68 1.13 0.89
1.32 1.31 0.32 0.11 0.43 0.96 1.69 0.94 0.82 1.02 0.61 0.40 0.00 0.10 0.50 0.90 0.49
0.98 0.19 0.10 0.48 0.38 1.06 0.66 0.57 0.94 0.74 1.38 0.73 0.99 0.63 0.27 0.62 0.09
0.35 0.00 0.18 0.09 0.70 0.17 0.17 0.26 0.09 0.17 0.17 0.52 0.00 0.43 1.02 0.68 0.59
0.25 0.08 0.08 0.16 0.50 0.91 0.16 0.90 0.89 0.80 0.40 0.24
;
symbol1 i=join  v=none;
proc gplot data=price;
   plot x*t;
run;
proc arima data=price;
   identify var=x;
run;
   estimate p=1 noconstant method=ml;
run;
```

Program 8.6

```
data change;
   input price @@;
    t = _n_;
   x1 = (0.005*t)*price;
   y1 = lag(x1);
   y2 = lag(y1);
   y3 = lag(y2);
   y4 = lag(y3);
   y5 = lag(y4);
   x2 = (1/12)*(x1+2*y1+3*y2+3*y3+2*y4+y5);
cards;
1.23 0.85 0.48 0.24 0.48 0.72 0.47 0.00 0.12 0.47 0.47 1.05 1.04 0.57 0.68 1.13 0.89
1.32 1.31 0.32 0.11 0.43 0.96 1.69 0.94 0.82 1.02 0.61 0.40 0.00 0.10 0.50 0.90 0.49
0.98 0.19 0.10 0.48 0.38 1.06 0.66 0.57 0.94 0.74 1.38 0.73 0.99 0.63 0.27 0.62 0.09
0.35 0.00 0.18 0.09 0.70 0.17 0.17 0.26 0.09 0.17 0.17 0.52 0.00 0.43 1.02 0.68 0.59
0.25 0.08 0.08 0.16 0.50 0.91 0.16 0.90 0.89 0.80 0.40 0.24
;
proc spectra data=change out=b cross s k ph;
   var x1 x2;
   weights 1 2 3 4 3 2 1;
proc print data=b;
symbol1 i=splines v=plus;
proc gplot data=b;
   plot s_01*freq;
   plot s_02*freq;
   plot cs_01_02*freq;
   plot qs_01_02*freq;
   plot k_01_02*freq;
   plot ph_01_02*freq;
run;
quit;
```

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