5 MATHEMATICAL MODEL VALIDATION

5.1 Preamble

In this chapter, the mathematical model proposed in Chapter 3 is correlated with the experimental results discussed in Chapter 4. The mathematical model is validated with quantitative, as well as qualitative results. The reason for performing quantitative, as well as qualitative comparisons is that for the basic strut characteristics, the absolute forces and displacements are important, while for the SDOF simulations over a random terrain, peak, mean, or RMS values are important. The purpose of comparing the measured and simulated values is not only to determine the accuracy of the simulation model, but also to determine the deficiencies of the simulation model.

The mathematical model validation is discussed under two headings, namely basic strut modelling and SDOF modelling. The reason for treating the validation as two separate parts is that for the basic strut model the relative displacement signal was provided by the Schenck actuator, while for the SDOF model the sprung mass dynamics are also taken into account.

Since the ultimate goal of developing the mathematical model is to integrate it into a full multi-body 3D vehicle simulation (not part of this study), the signals recorded during the experimental tests are used as inputs to the simulation models. These signals include the actuator displacement signal, as well as the damper valve and spring valve switching signals. The recorded signals are used for the following reasons:

- Only the mathematical model is validated and not the control strategies (programming thereof), sensors and data acquisitioning equipment.
- To ensure repeatability (input signals stay constant i.e. not a function of model behaviour)

All the correlation results are supplied in graphical form in Appendix E.

5.2 Basic strut model validation

The basic strut configuration consists of the suspension strut, actuated by the Schenck hydropuls in a rigid characterisation frame (see Figure 4-1). The Schenck displacement signals, as well as the valve switching signals recorded during the experimental characterisations are used in the simulations.

5.2.1 Passive characteristics

The passive spring characteristics are validated by comparing the P-V diagrams of the measured and simulation results. By passive, it is implied that none of the valves are switched during the characterisation and only one state of the spring/damper unit is tested. Two simulations, one for each spring setting, were performed. The passive characterisations verify the correct working of the hydro-pneumatic spring model, as well as the "steady state" operation of the hydraulic flow model. Figure 5-1 indicates the correlation for the passive spring states.

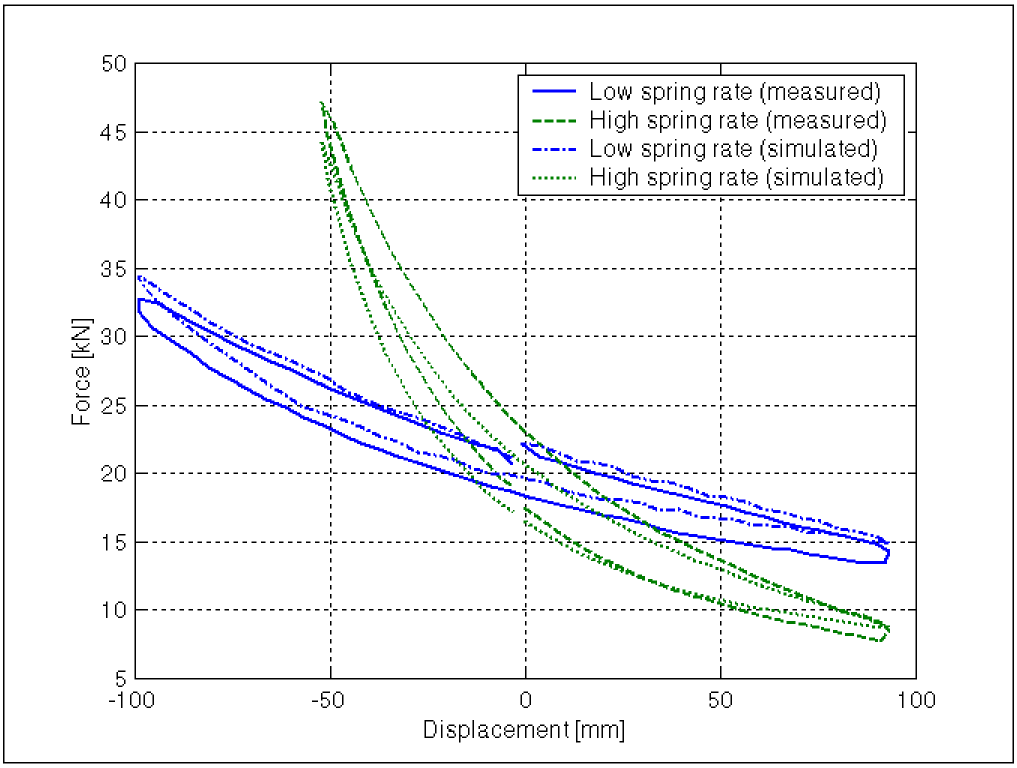

Figure 5-1: Passive spring characteristic validation (0.01*m/s*)

From this figure, it can be seen that good correlation is obtained for the passive spring states. Both the low spring (spring "off") and high spring (spring "on") characteristics are in good agreement with the measured data. This data is confirmed by the results obtained by Els (1993). It is evident that the passive spring behaviour can be predicted with high precision and that results similar to that predicted by Els (1993) are obtained.

5.2.2 Workspace tests

In order to determine the workspace of the strut and to validate the spring valve switching, the valve is opened and closed during a series of compression and extension strokes. This was done at excitation speeds of 0.001*m/s*, 0.01*m/s* and 0.1*m/s*. A relative strut displacement (peak-to-peak) of 200*mm* was used, while the spring valve was switched selectively in order to achieve both low and high spring characteristics. The displacement and switching signal used for the characterisation at 0.001*m/s* (nearly isothermal) is shown in Figure 5-2.

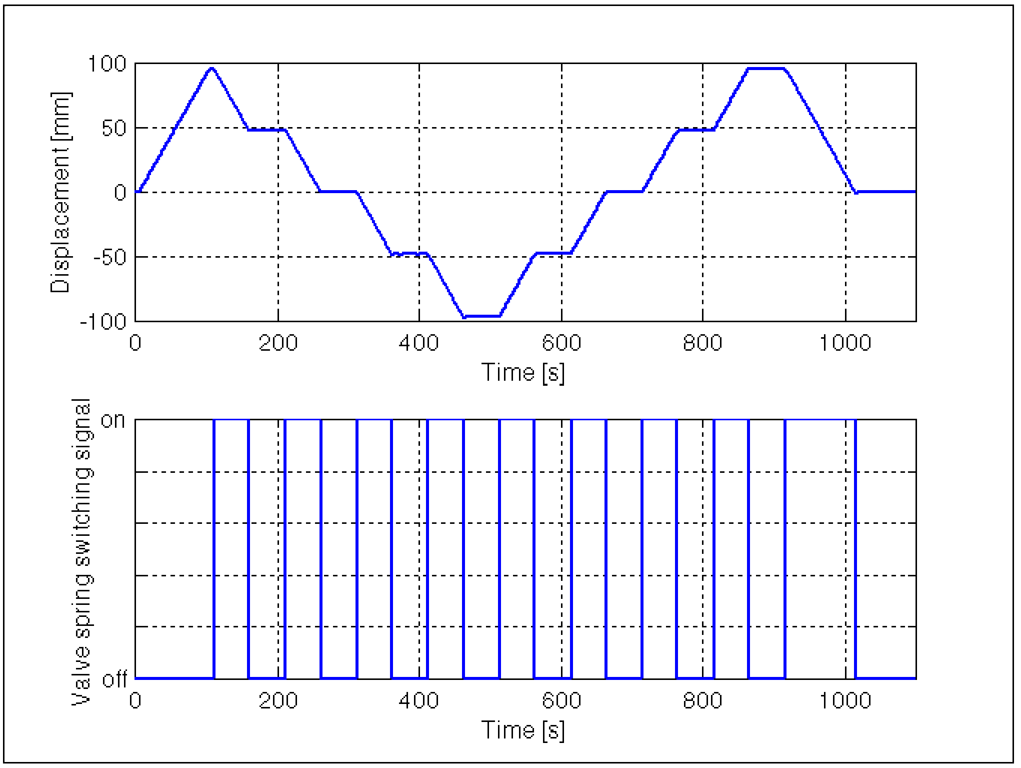

Figure 5-2: Workspace characterisation

From this figure, it can be seen that the displacement and switching signal is constructed so that the strut is first extended (positive displacement) and then compressed on the high characteristics for a quarter of the stroke. The spring valve is opened and the strut remains static for a few seconds. After the waiting period, the valve is again closed and another quarter compression stroke is performed. This process is continued, until a maximum stroke of approximately 100*mm* is reached. The strut is extended in the same way as it was compressed, until a rebound displacement of 100*mm* is reached. The strut is then returned to the static position.

The input signals shown in Figure 5-2 is used as input to the simulation model. The damper is not included in this model, since the characterisation configuration did not have a damper present. The result of this series of compression and rebound movements are displayed in Figure 5-3 and Figure 5-4.

Figure 5-3 indicates the force versus time results for the simulation, as described above, while Figure 5-4 indicates the force versus displacement results. From these two figures, it can be seen that good correlation was obtained between measured and simulated data for the low speed characterisations. The results of the simulations at 0.01*m/s* and 0.1*m/s* are supplied in Appendix E.

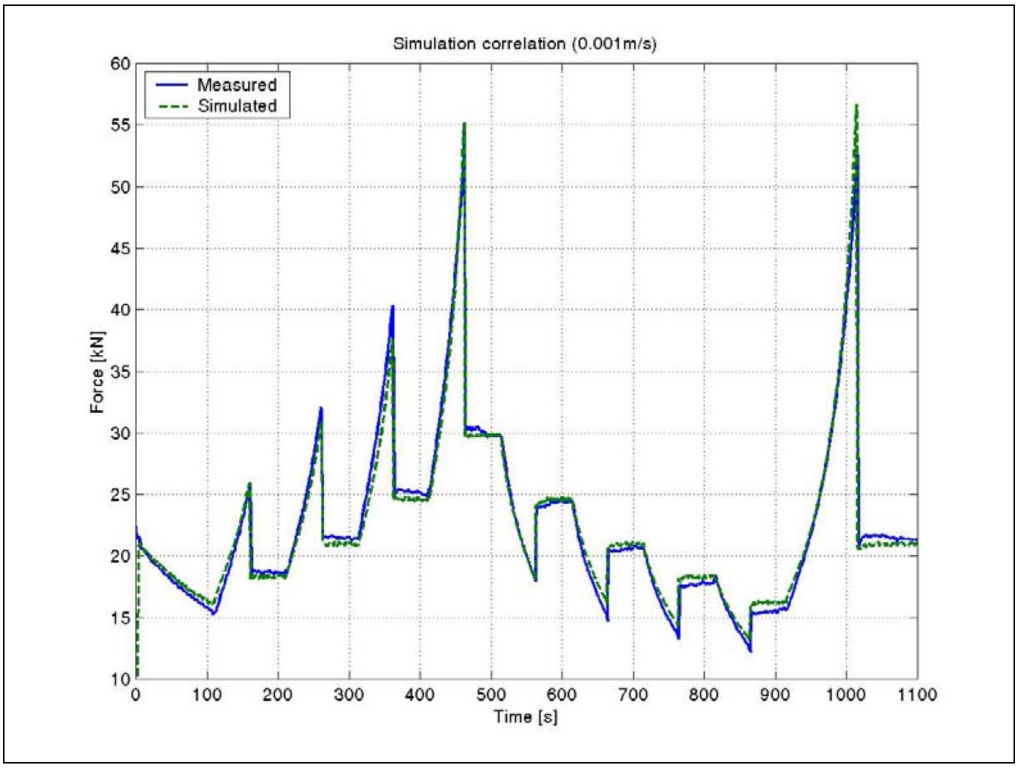

Figure 5-3: Force versus time correlation (0.001*m/s*)

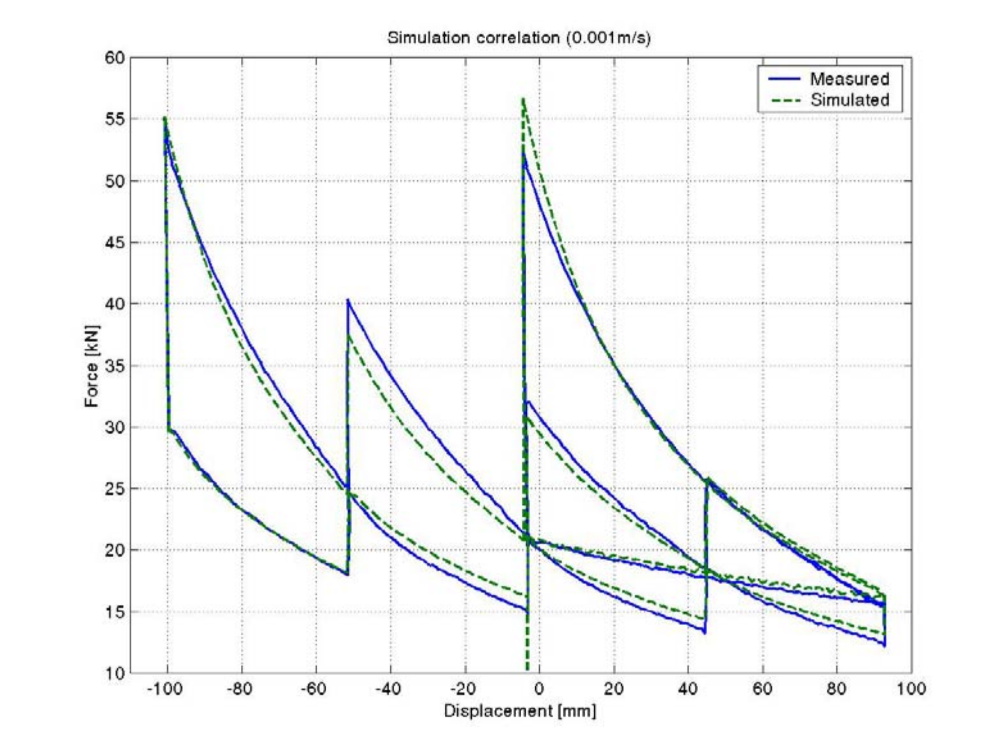

Figure 5-4: Force versus displacement correlation (0.001*m/s*)

From the figures presented in section E.1 of Appendix E, it can be seen that the correlation was not as good at the higher excitation speeds, as it was at the lower speeds. This may be attributed to the fact that flow losses and friction were nor modelled in detail.

5.3 SDOF model validation

For the single degree of freedom simulations, the complete spring-mass-damper system was modelled and compared with the measured results. The relative strut displacement was used as comparison criteria, since this parameter could easily be measured and does not present filtering and resonance problems, as do accelerometers. As explained previously, the measured actuator displacement and valve switching signals were used as input to the mathematical model.

Three types of tests were performed on the SDOF setup, namely step response, random input response and a sine sweep. The correlation results of these three simulations will be discussed in the following paragraphs.

5.3.1 Step response

The step response of a SDOF system gives and indication of the natural frequency and damping behaviour of the system, therefore the step response correlation indicates whether the dynamic properties (spring stiffness, SDOF mass, damping force etc.) of the model are accurate.

For the step response simulations, the basic SDOF model is used, with the measured Schenck displacement as input. Figure 5-5 indicates the measured actuator displacement that was used in the step response SDOF simulations. From this figure, it can be seen that the actuator supplied a step input of approximately 30*mm*. The actuator overshoot is also clearly visible in this figure.

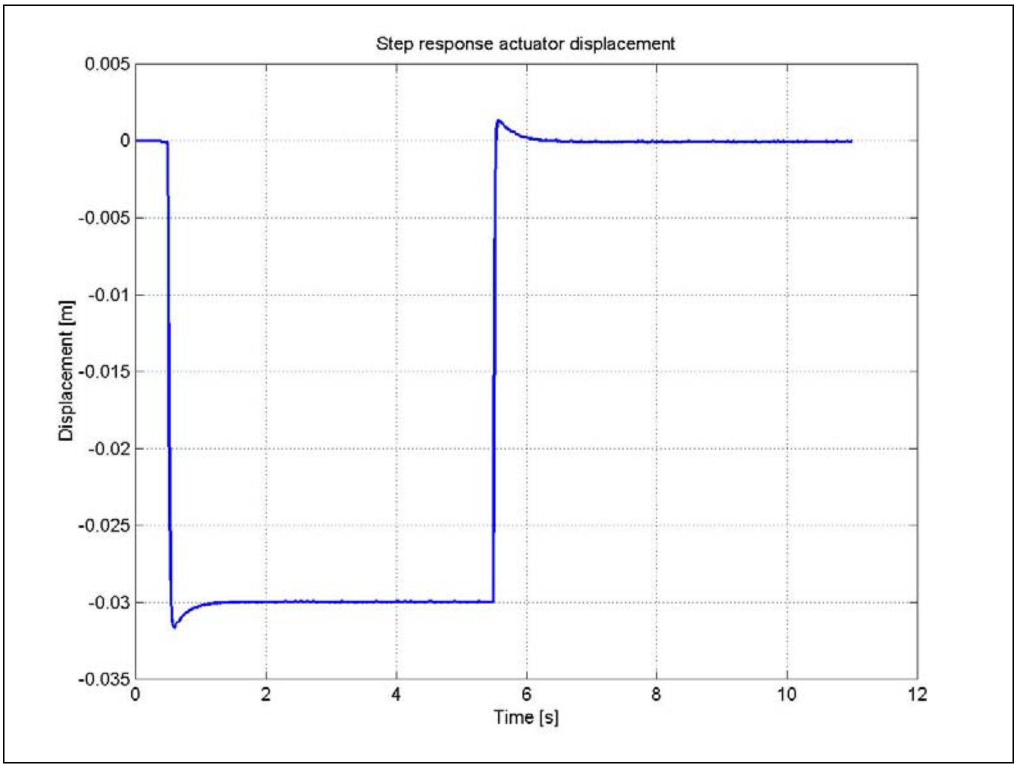

Figure 5-5: Measured actuator displacement used for SDOF simulations

Figure 5-6 indicates the recorded and simulated relative displacement step response of the strut unit in the passive state with both spring and damper off.

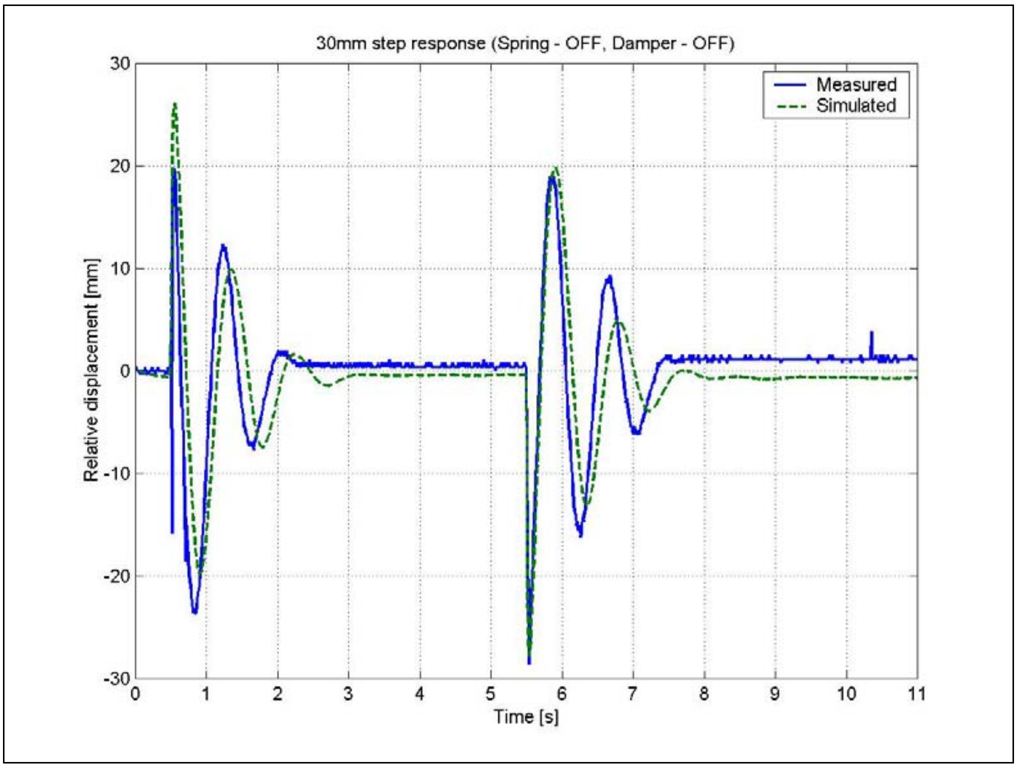

Figure 5-6: 30*mm* step response (Spring – OFF, Damper – OFF)

From Figure 5-6 it can be seen that the correlation between measured and simulated data is good and that the model is able to predict the amplitude, as well as phase response to the step input with reasonable accuracy. Both spring and damper was in the OFF state for this test, thereby indicating the passive response.

The correlation for different combinations of spring and damper settings and input amplitudes are supplied in section E.2 of Appendix E. From Figures E-8 to E-16 it can be see that good correlation was achieved for various combinations of spring and damper settings and that reasonable correlation was obtained with the damper controlled according to the strategy of Karnopp (Nell 1993) (Figure E-12).

5.3.2 Random input response

For the random input response simulations, the recorded Schenck displacement signal was used as input to the simulation model. The input signal represents the left hand lane of the Belgian paving track at the Gerotek Vehicle Testing Facility. The Belgian paving track is a 100*m* long cobble stone paving and can be used as a realistic random input for off-road vehicles. Figure 5-7 indicates the vertical displacement versus time at a vehicle speed of 25km/h over the Belgian paving track (left lane).

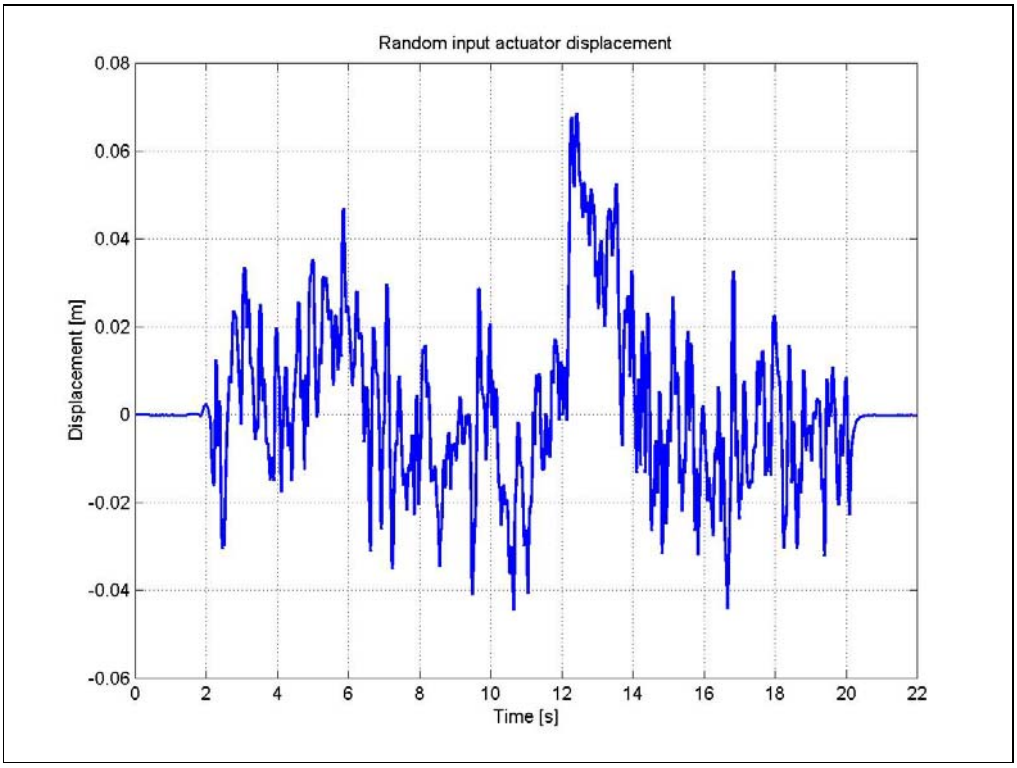

Figure 5-7: Random input actuator displacement (Belgian paving - left lane)

Figure 5-9 indicates the correlation between measured and simulated values for the random input (Belgian paving - Figure 5-8 shows the PSD of this track). From this figure, it can be seen that the correlation was good and that the simulated peak displacement is higher than what was measured. A possible reason for this is that not all the losses are accounted for in the simulation model.

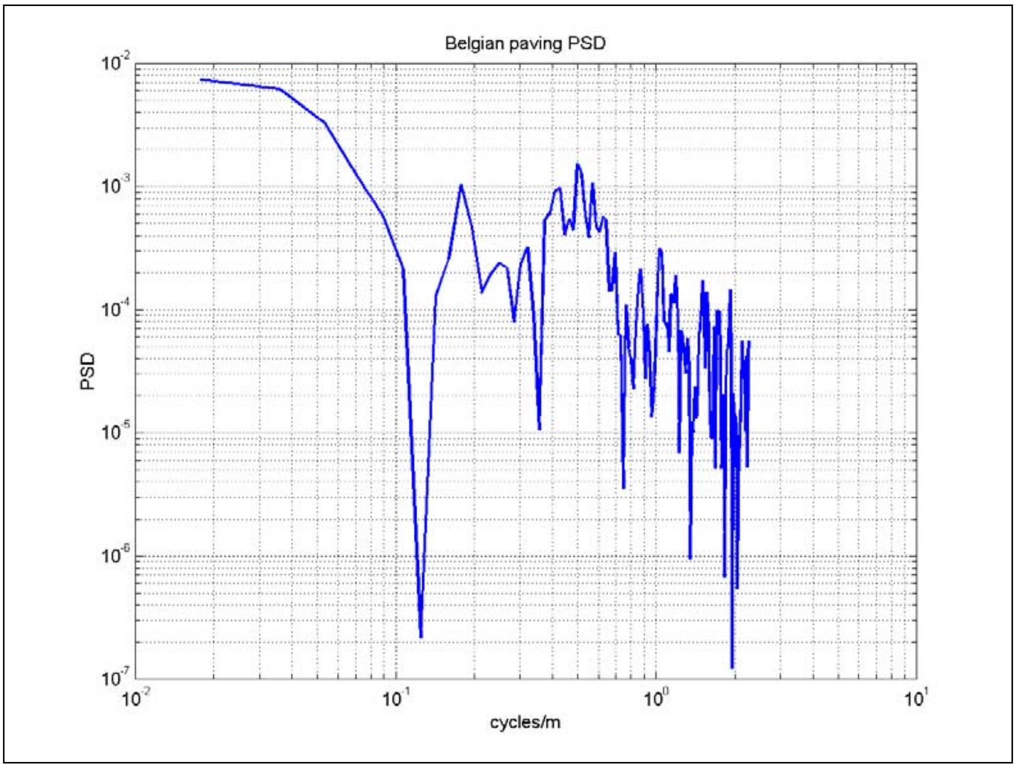

Figure 5-8: PSD of Belgian paving track (left lane)

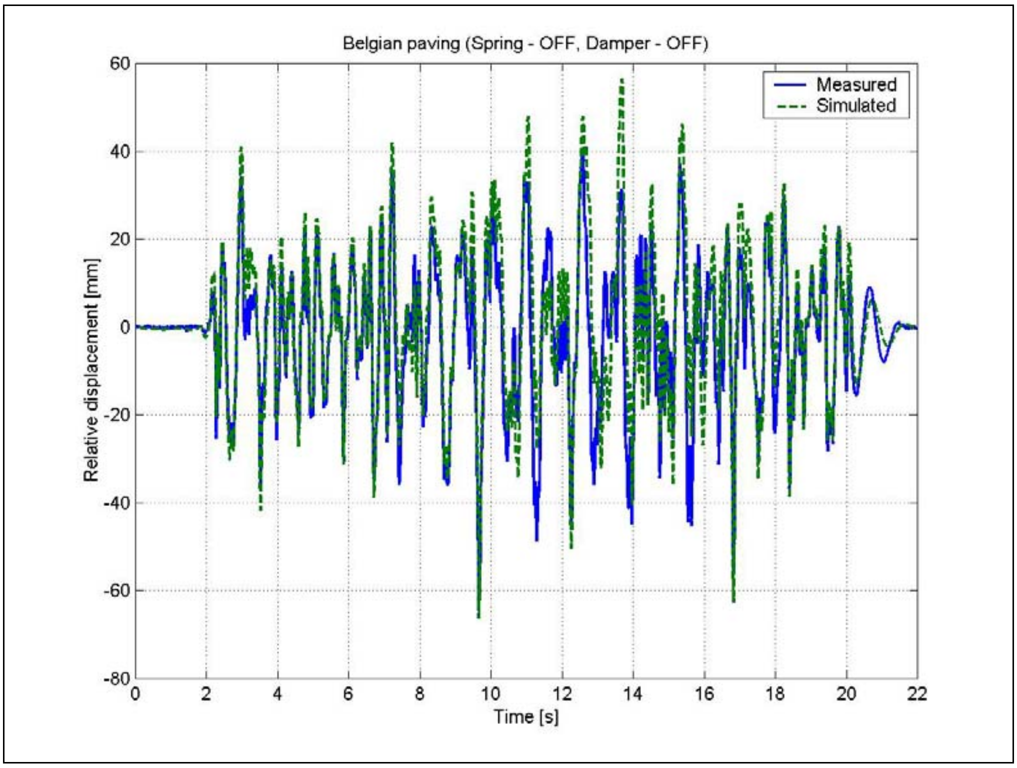

Figure 5-9: Correlation over Belgian paving (Spring – OFF, Damper – OFF)

Figures E-17 to E-26 indicate the random input correlation for some other spring and damper settings. Figure 5-10 shows the summary of statistical correlation between measured and simulated results. The configurations are defined in Table 5-1.

Table 5-1: Spring/damper configuration for random input tests

Configuration no.	Figure no.	Spring state	Damper state
1	Figure E-17	OFF	OFF
2	Figure E-18	ON	OFF
3	Figure E-19	OFF	ON
4	Figure E-20	ON	ON
5	Figure E-21	ON	Karnopp
6	Figure E-22	OFF	Karnopp
7	Figure E-23	ON	Hölscher & Huang
8	Figure E-24	OFF	Hölscher & Huang
9	Figure E-25	Karnopp	Karnopp
10	Figure E-26	Height adjustment	OFF

From Figure 5-10, it can be seen that good correlation was obtained, except for configuration 10 (Spring - height adjustment, Damper - OFF). Additional statistical information can be found in Figures E-27 and E-28.

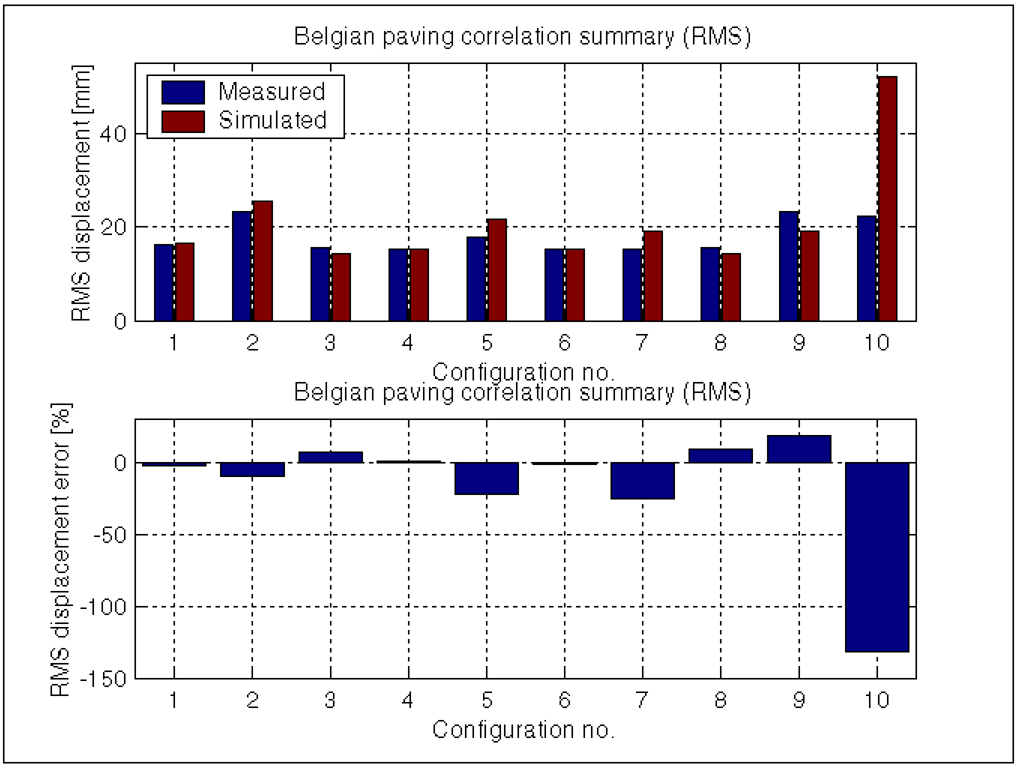

Figure 5-10: Belgian paving correlation summary (RMS)

5.3.3 Sine sweep

Although it is not valid to determine the frequency response of the strut with a sine sweep input since the system is highly non-linear, the sine sweep gives an indication of the system's natural frequency for a certain excitation amplitude. Sine sweep tests were therefore performed not to characterise the system in the frequency domain, but to determine whether the mathematical model is be able to reproduce the response to a sine sweep input.

A 78*s* sine sweep signal was used with a frequency content between 0,1*Hz* and 15*Hz*. The sine sweep input signal can be seen in Figure E-29 in Appendix E. Figure 5-11 indicates the sine sweep response for the spring in the "OFF" sate and the damper in the "ON" state. From this figure, it can be seen that the relative displacement decreases with an increase in frequency. This is because the damper characteristic in rebound is higher than in compression. This is a typical tendency of a wheeled vehicle damper. From Figure 5-11, it can be seen that the simulation model is able to reproduce this type of behaviour. More detailed results are supplied in Appendix E.

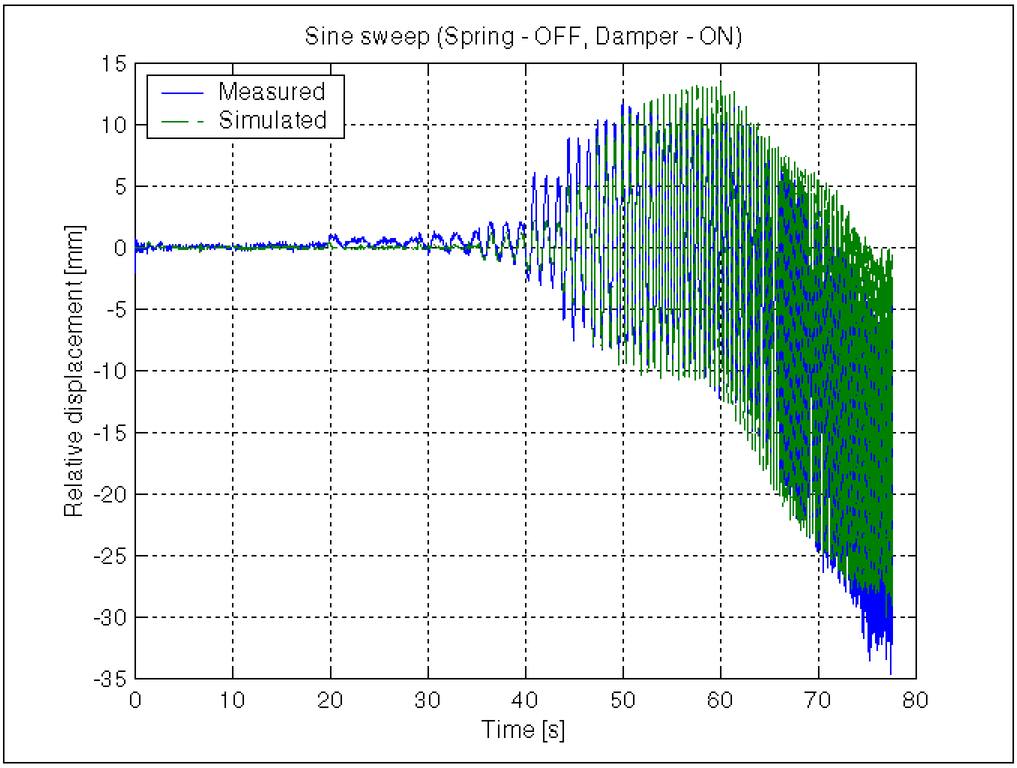

Figure 5-11: Sine sweep correlation (Spring - OFF, Damper - ON)

In order to determine whether the simulation model provides an accurate representation of the strut in the frequency domain, the transmissibility between the input displacement and the sprung mass response for the measured, as well as the simulated data was determined (see Figures E-38 to E-45). Figure 5-12 indicates the transmissibility for both the spring and damper in the "OFF" state. Also supplied in this figure is the coherence between the two displacement signals, giving an indication of the level of confidence in the transmissibility. From this figure, it can be seen that the coherence (for measured and simulated) is high below 10*Hz*. The transmissibility graph indicates that the simulated data has a higher transmissibility in the region of resonance, but that the region of resonance is the same for measured and simulated data.

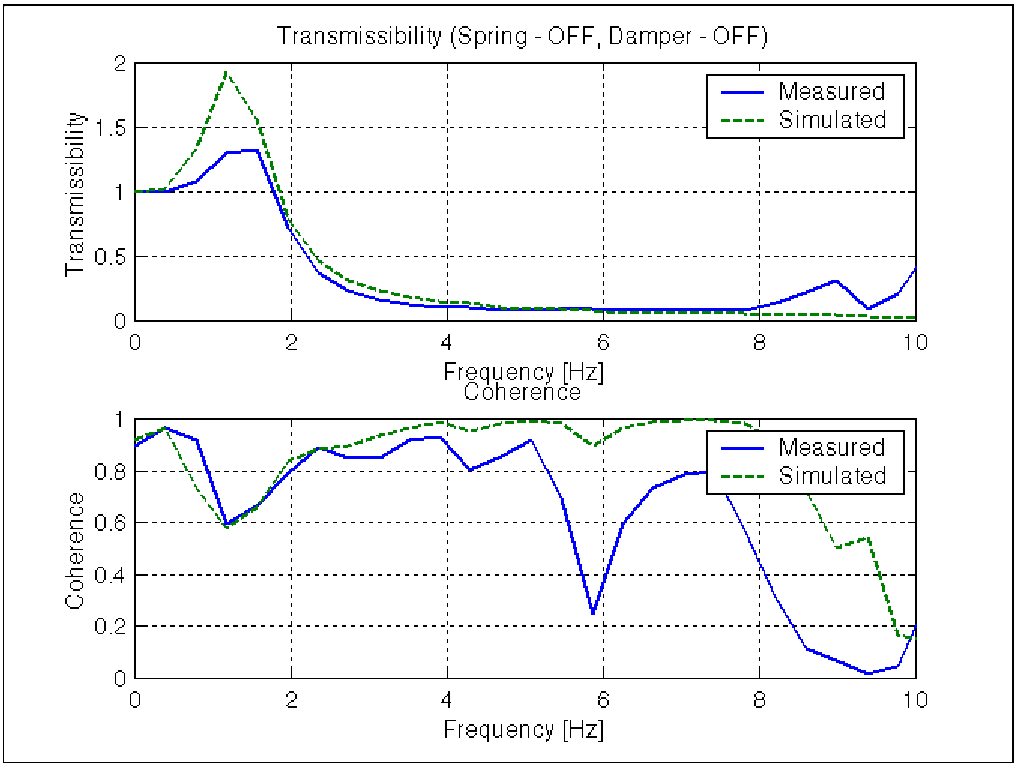

Figure 5-12: Transmissibility (Spring - OFF, Damper - OFF)

5.4 Closing

In conclusion, it can be said that the current mathematical model provides an adequate representation of the semi-active hydro-pneumatic spring/damper system. In the preceding paragraphs, it was illustrated that the mathematical model is able to predict the force versus displacement characteristics of the strut for different excitation frequencies.

For the SDOF modelling, good correlation was obtained for the step response, random input response, as well as the sine sweep simulations. Although the correlation for the time domain response of the random input simulations is not very good in all cases, the statistical correlation (RMS) was good. It is also illustrated that the simulation model is able to predicted phenomenon such as suspension squat at high frequencies, due to the unsymmetrical damper characteristics. Discrepancies between measured and simulated results can be attributed to amongst others gas leaks in the experimental units, unaccounted effects such as bearing friction, hydraulic damping effects in the pipes and valve response uncertainty.

The current model can be further refined by introducing a more sophisticated valve model to better simulate the strut at higher excitation frequencies. The solving time of this mathematical model is sufficiently fast (5-10 minutes of CPU time for 10*s* real time on a Pentium II 333*MHz* computer), in order to be incorporated into a full 3D, multi wheeled vehicle model.