CHAPTER 6: LINEAR MODEL DEVELOPMENT

6.1 INTRODUCTION

In this dissertation a model based controller design method is used which requires a linear plant model. The control law is then developed for the linear model and verified after which it is applied to the actual process, which might not be linear at all. Despite the difference between the process and the linear model, the control law should perform well for the same set of operating conditions. It is with the controller design in mind that a linear model is developed in this chapter.

The linear model is derived from the combined furnace and off-gas non-linear model. In itself it introduces no new assumptions other than the assumptions and approximations necessary to perform the linearization. In order to do the linearisation a nominal steady state set of conditions must be chosen. The linearisation then determines the gradients of all the differential equations for the specific set of conditions. A state-space set of matrices is then composed of all the gradients.

In this context the product of linearisation is a set of linear differential equations. With any linearisation the assumption is that the non-linear differential equations are sufficiently linear (in the region of the nominal steady state (NSS)) that the linearized differential equations will give state trajectories that approximate the state trajectories of the non-linear differential equations.

In Section 6.2 the method of linearisation is illustrated. In Section 6.3 the method for model adjustment is discussed. Section 6.4 shows that the linearized model approximates the non-linear model sufficiently close to be used for control design or as an internal model for a controller.

6.2 LINEARISATION

The required form for a linear state-space representation of the off-gas system model is:

$$\begin{aligned} \frac{d(\delta x(t))}{dt} &= A.\delta x(t) + B.\delta u(t) + E.\delta d(t) \\ y(t) &= C.\delta x(t) + D.\delta u(t) + F.\delta d(t) \end{aligned} \tag{6-1}$$

Where the following vector and matrix dimensions are used:

$$\mathbf{x}(t) \in \Re^{17x1}, \mathbf{u}(t) \in \Re^{2x1}, \mathbf{d}(t) \in \Re^{5x1}, \mathbf{y}(t) \in \Re^{6x1}$$

$$\mathbf{A} \in \Re^{17x17}, \mathbf{B} \in \Re^{17x2}, \mathbf{C} \in \Re^{6x17}$$

$$\mathbf{D} \in \Re^{6x2}, \mathbf{E} \in \Re^{17x5}, \mathbf{F} \in \Re^{6x5}$$

Linear system fundamentals are discussed by Reid [38].

The linearisation is done about a steady-state condition denoted by x^*, u^*, d^*. The numerical values for this particular steady-state condition can be determined from industrial furnace practice.

The next step in the linearisation is to determine the elements of the matrices A, B, C, D, E and F. This can be done using a Taylor series expansion and is illustrated by example. The state equation for silicon in liquid steel is:

$$\dot{x}_4 = f_4(x(t), d(t)) = -k_{dSi}\left(\frac{x_4/M_{Si}}{x_2/M_{Fe} + x_3/M_C + x_4/M_{Si}} - k_{XSi}\left(\frac{x_6 M_{FeO}}{x_7 M_{slag}} + \frac{x_8 M_{FeO}}{x_7 M_{SiO_2}} + 1\right)\right) \quad (6\text{-}2)$$

The relevant elements in **A** are obtained as:

$$A_{42} = \frac{\partial \dot{x}_4}{\partial x_2} = \frac{k_{dSi} x_4^* / (M_{Si} M_{Fe})}{\left(x_2^*/M_{Fe} + x_3^*/M_C + x_4^*/M_{Si}\right)^2} \quad (6\text{-}3)$$

$$A_{43} = \frac{\partial \dot{x}_4}{\partial x_3} = \frac{k_{dSi} x_4^* / (M_{Si} M_C)}{\left(x_2^*/M_{Fe} + x_3^*/M_C + x_4^*/M_{Si}\right)^2} \quad (6\text{-}4)$$

$$A_{44} = \frac{\partial \dot{x}_4}{\partial x_4} = \frac{-\frac{k_{dSi}}{M_{Si}}\left(\frac{x_2^*}{M_{Fe}} + \frac{x_3^*}{M_C}\right)}{\left(\frac{x_2^*}{M_{Fe}} + \frac{x_3^*}{M_C} + \frac{x_4^*}{M_{Si}}\right)^2} \quad (6\text{-}5)$$

$$A_{46} = \frac{\partial \dot{x}_4}{\partial x_6} = \frac{k_{dSi} k_{XSi} M_{FeO}}{x_7^* M_{slag}} \quad (6\text{-}6)$$

$$A_{47} = \frac{\partial \dot{x}_4}{\partial x_7} = -k_{dSi} k_{XSi}\left(\frac{x_6^* M_{FeO}}{(x_7^*)^2 M_{slag}} + \frac{x_8^* M_{FeO}}{(x_7^*)^2 M_{SiO_2}}\right) \quad (6\text{-}7)$$

$$A_{48} = \frac{\partial \dot{x}_4}{\partial x_8} = \frac{k_{dSi} k_{XSi} M_{FeO}}{x_7 M_{SiO_2}} \quad (6\text{-}8)$$

With $A_{4N} = 0$ if $N \neq \{2\text{-}4\}$ or $\{6\text{-}8\}$; $B_{4N} = 0$ for $N = \{1\text{-}2\}$; $E_{4N} = 0$ for $N = \{1\text{-}4\}$

The same calculations can be done for all elements of all matrices, using the same method as above. The required form as given in (6-1) is then obtained and the result is given in Appendix C.

6.3 METHOD OF MODEL ADJUSTMENT

The aim is to maximise the range over which the linear model is valid, so that the model based controller will perform good over a wide range. To begin with it is necessary to explain how the initial linear model version was obtained. A nominal steady state (NSS) was determined by taking

the time-average (by integration) of the various states, as given by the non-linear simulation. Then the NSS was used to determine the linear model matrix element values and adjusted such that the differential equation gradient conforms to the gradient obtained from the non-linear model (for deviation from the initial conditions). Since the NSS was determined by time-average over the full non-linear simulation, it is an approximation for an actual steady state over a wide range.

The NSS and the initial conditions for the linear model do not have to be the same. The NSS selection is manipulated to adjust the simulation response, while the initial conditions are merely the same as that for the non-linear model simulation. The NSS adjustment is done to achieve good correlation between the non-linear model simulation and the linear model simulation for the whole tap. After the NSS was adjusted to its final value a number of states compared exceptionally well with the non-linear simulation. These were in particular the liquid metal mass, solid scrap mass, fluid group temperature, and solid group temperature. The liquid and solid slag masses of the linear simulation did not agree exactly with the non-linear simulation, but were deemed acceptable.

Most of the other states also compared satisfactorily, although the linear simulation failed to follow the exact trends of the non-linear simulation. The linear simulation was inaccurate with respect to those states where the mass transfer is determined partly or completely by chemical reactions. These are the dissolved masses of carbon and silicon in the liquid metal, and the dissolved masses of FeO and SiO_2 in the liquid slag. The rates of change for these states are mainly determined by the reaction of FeO in the liquid slag with carbon and silicon in the liquid metal. The linear simulation at one stage allowed the FeO mass to go to a negative value. From a physical point of view, this is absurd. This is due to the fact that the linear model is a much-simplified model, and is unable to account for chemical reaction rates, while the non-linear model can. With the non-linear model, if there is no FeO to react, then there will be no reaction. Not so for the linear model, since it simply follows the trends dictated by the matrix-multiplication as in equation (6-1).

An attempt was made to correct the negative error in the FeO mass by means of adjustment to the NSS, as well as by means of adjustment of individual matrix elements. It was found that no NSS could remedy this problem, and that adjustment of individual matrix elements was just as futile. In addition, whenever the FeO mass followed a different trend, it influenced other states too, and almost invariably in an undesired manner, causing other states to behave completely erroneous. The best compromise was finally obtained, as shown in Section 6.4.

Finally the only states that could still not compare satisfactorily with those of the non-linear simulation were the gas-phase component masses and the relative pressure. These were then corrected by means of adjustments to individual matrix elements, without much effect on the other

states. Here the same problem of chemical reaction kinetics appears, due to the chemical reaction of the CO with oxygen from the leak-air. The relative pressure also causes trouble for the linear model: Where it is possible to place switches in the non-linear model, to determine whether the relative pressure is positive or negative, and act accordingly, it is not possible with the linear model. This explains the inaccuracy with respect to gas-phase components. Adjustment to individual matrix elements was limited to relations that were exaggerated by the linear model.

6.4 SIMULATION RESULTS

The linear model is now used in a pure simulation (not 1-step ahead simulation). With the same initial conditions given in Chapter 5 and the same disturbance model of Chapter 5 the following simulation results were obtained. Fig.6.1 shows the liquid metal and solid scrap masses. Fig.6.2 and Fig.6.3 show the carbon and silicon in solution in the liquid metal.

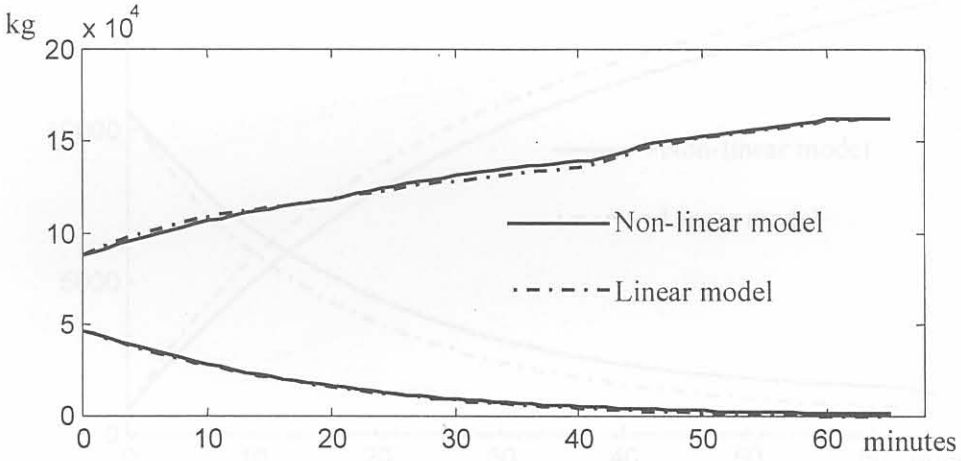

Figure 6.1 Liquid metal mass and solid scrap mass

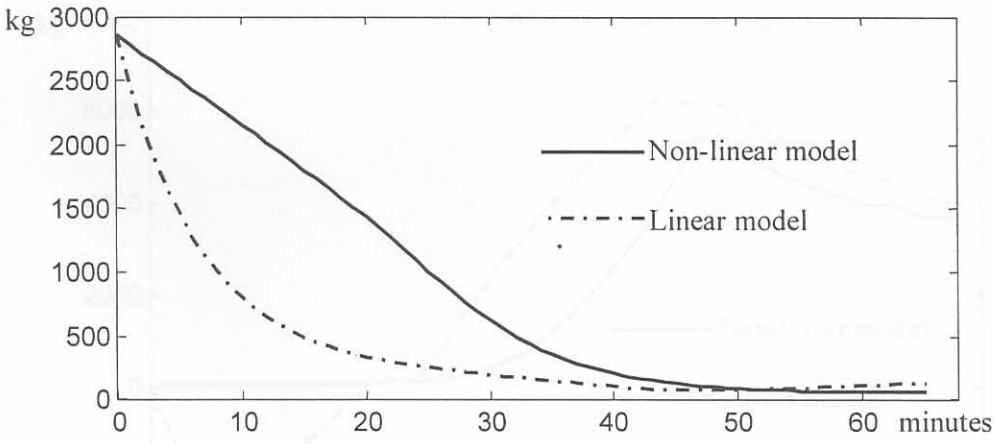

Figure 6.2 Mass of carbon in solution in liquid metal

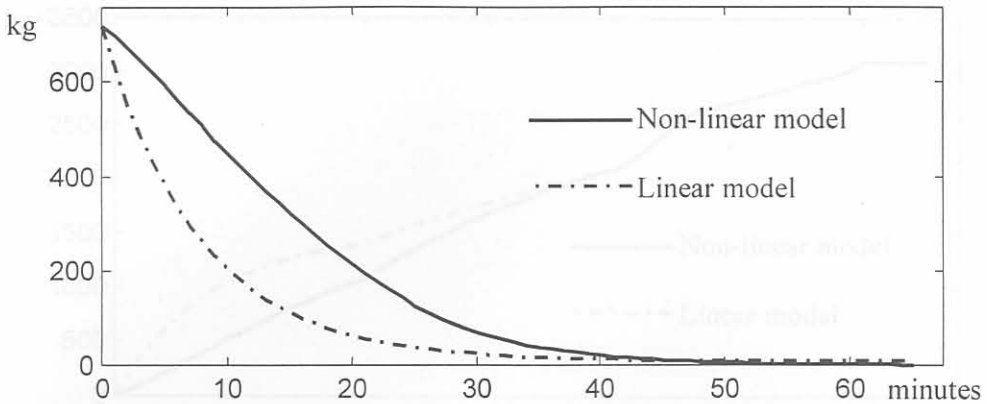

Figure 6.3 Mass of silicon in solution in liquid metal

In Fig.6.4 the comparison of the liquid and solid slag masses are shown. In Fig.6.5 the FeO in solution in the liquid slag is shown (the negative behaviour is discussed in Section 6.3). In Fig.6.6 the SiO_2 in solution in the liquid slag is shown.

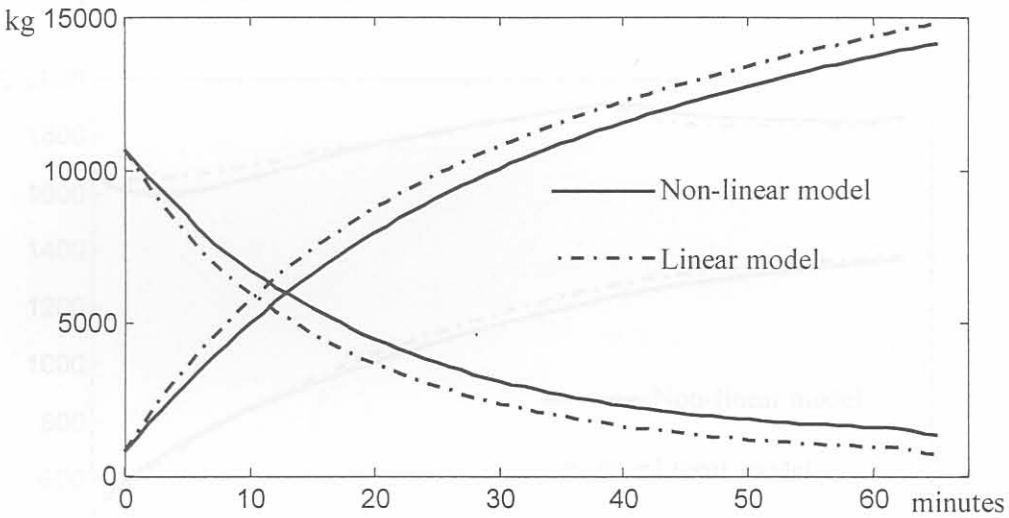

Figure 6.4 Liquid slag mass and solid slag mass

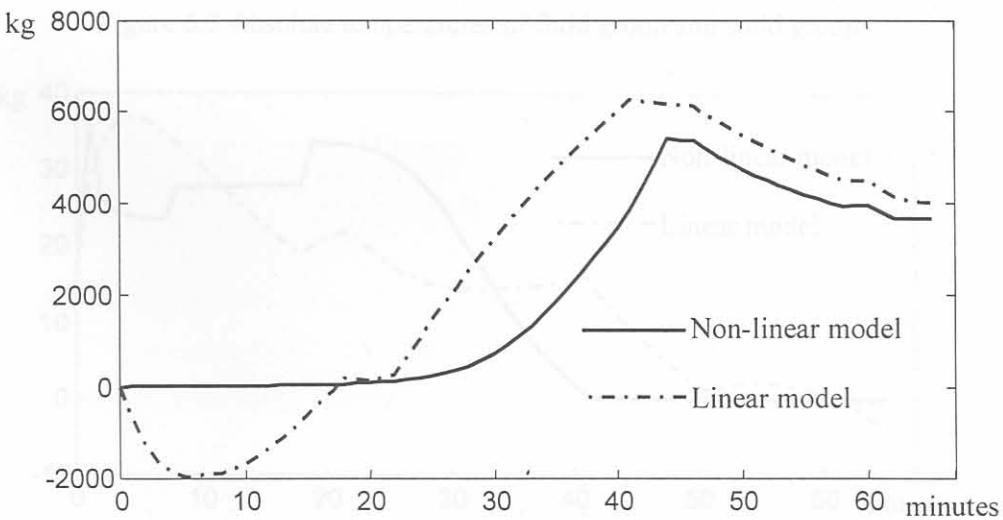

Figure 6.5 Mass of FeO in solution in liquid slag

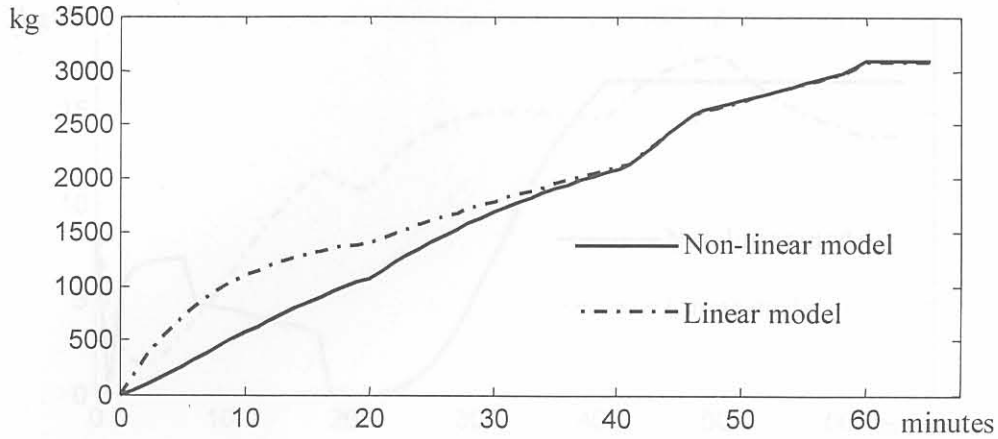

Figure 6.6 Mass of SiO_2 in solution in liquid slag

In Fig.6.7 the fluid group and solid group temperature are shown. In Fig.6.8 the mass of CO in the gas-phase is shown. Fig.6.9 shows the CO_2 mass in the gas-phase. The nitrogen in the gas-phase behaves quite similar to the CO_2.

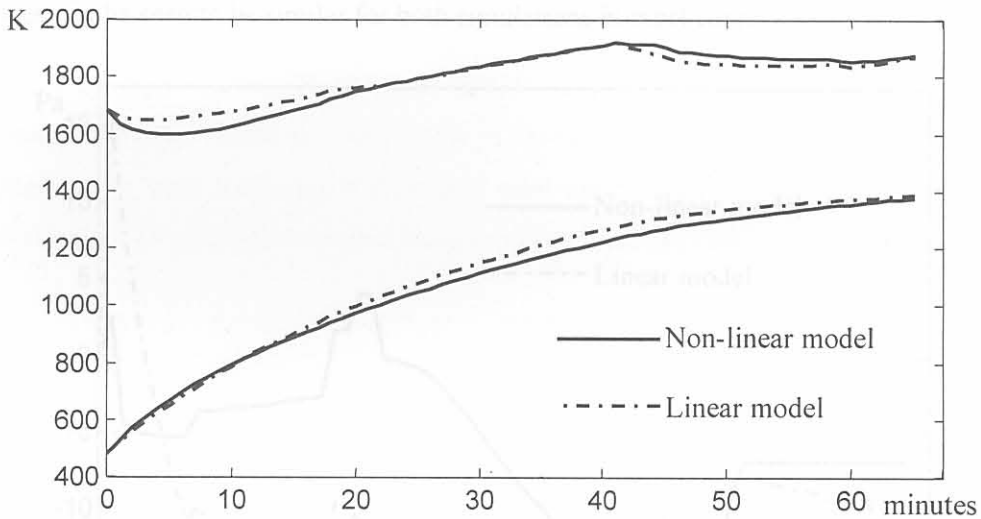

Figure 6.7 Absolute temperatures of fluid group and solid group

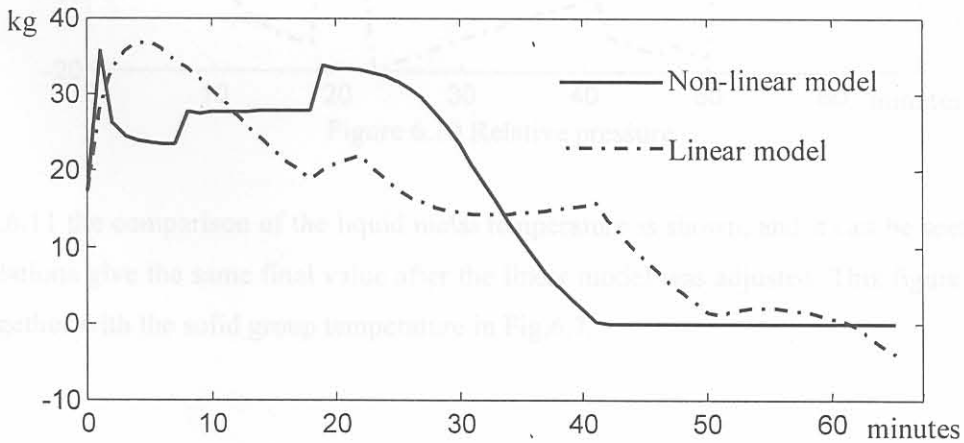

Figure 6.8 Mass of carbon-monoxide in gas-phase

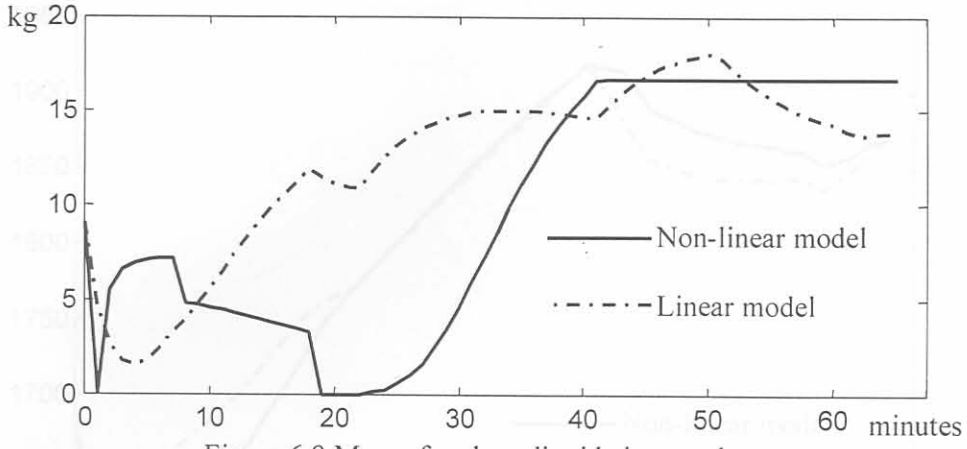

Figure 6.9 Mass of carbon-dioxide in gas-phase

There are four outputs that are considered here: The relative pressure, liquid metal temperature, off-gas CO mass-fraction and the off-gas temperature. The relative pressure is shown in Fig.6.10. Although the linear simulation diverges from the non-linear simulation, the effects of external disturbances can be seen to be similar for both simulations in most cases.

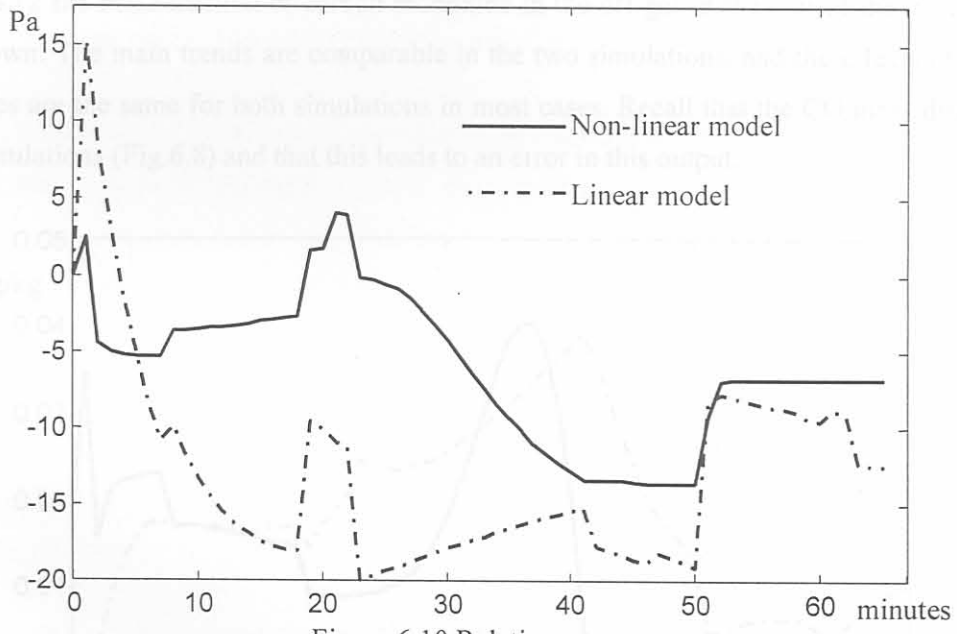

Figure 6.10 Relative pressure

In Fig.6.11 the comparison of the liquid metal temperature is shown, and it can be seen that the two simulations give the same final value after the linear model was adjusted. This figure was also shown together with the solid group temperature in Fig.6.7.

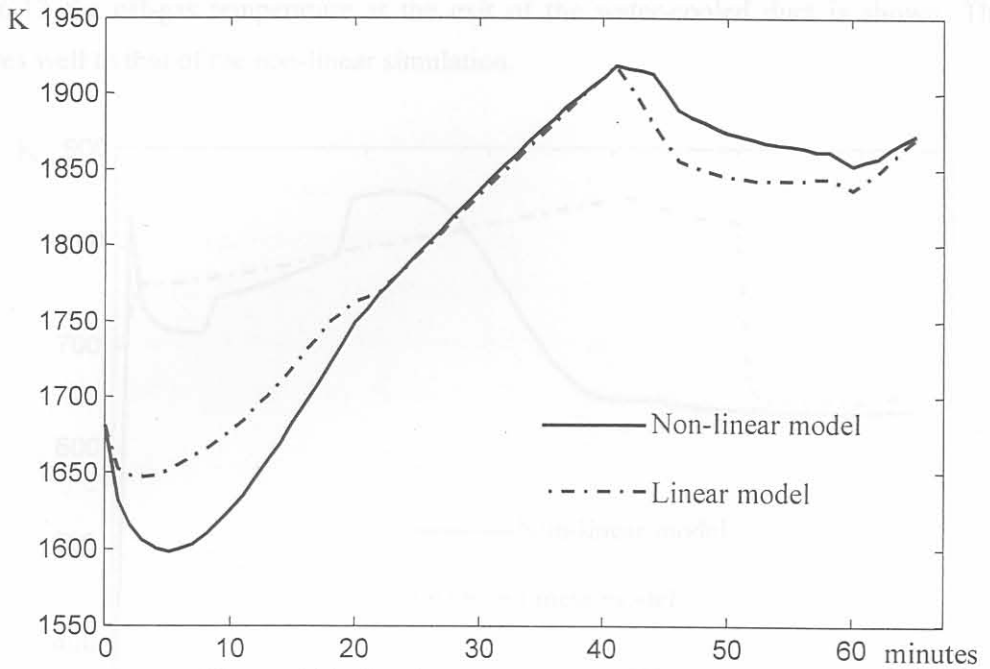

Figure 6.11 Absolute temperature of liquid metal

In Fig.6.12 the mass fraction of carbon monoxide in the off-gas at the exit of the water-cooled duct is shown. The main trends are comparable in the two simulations, and the effects of external disturbances are the same for both simulations in most cases. Recall that the CO mass differed for the two simulations (Fig.6.8) and that this leads to an error in this output.

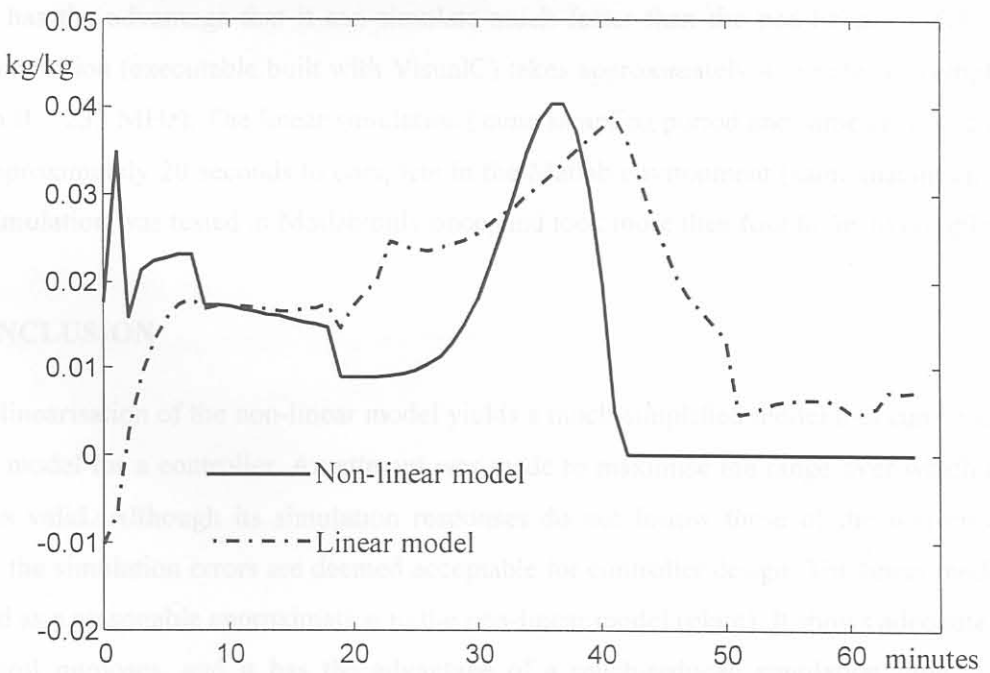

Figure 6.12 Mass fraction of CO in off-gas at water-cooled duct exit

In Fig.6.13 the off-gas temperature at the exit of the water-cooled duct is shown. This output compares well to that of the non-linear simulation.

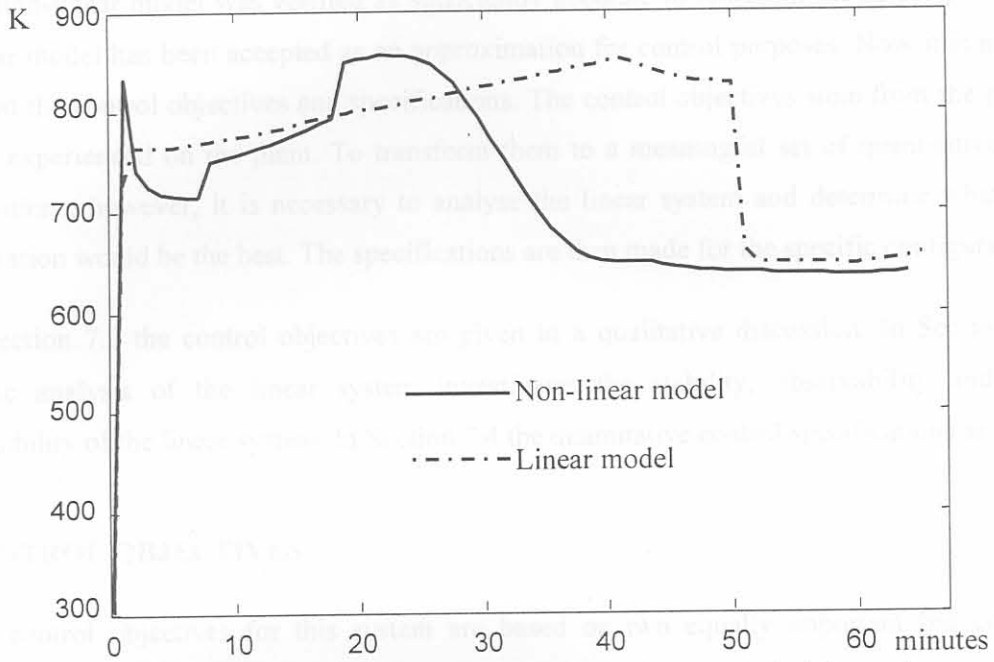

Figure 6.13 Absolute temperature of off-gas at water-cooled duct exit

Although a very good linear model approximation is always desired, it is not critical for control design purposes, since output-feedback will be employed. The linear model presented in this chapter has the advantage that it can simulate much faster than the non-linear model. The non-linear simulation (executable built with VisualC) takes approximately 4 minutes to complete (on a Pentium II – 233 MHz). The linear simulation (same sampling period and same process conditions) takes approximately 20 seconds to complete in the Matlab environment (same machine). The non-linear simulation was tested in Matlab only once, and took more than four hours to complete.

6.5 CONCLUSION

The linearisation of the non-linear model yields a much-simplified model that can be used as an internal model for a controller. An attempt was made to maximise the range over which the linear model is valid. Although its simulation responses do not follow those of the non-linear model exactly, the simulation errors are deemed acceptable for controller design. The linear model is then accepted as a reasonable approximation to the non-linear model (plant). It shows adequate accuracy for control purposes, and it has the advantage of a much-reduced simulation time. These two characteristics make it ideal for an internal model in Model Predictive Control.