CHAPTER 3: NON-LINEAR MODEL

3.1 INTRODUCTION

The emphasis of this chapter is on obtaining a model that is appropriate for control system simulation and verification. A model is a mathematical abstraction of an industrial process. There are many advantages of an appropriate model for a process [20,21]. The model can improve understanding of the process, and can be used to optimise operating conditions and design a control law for a new control strategy.

The non-linear state-space model discussed in this chapter was partly derived in [7]. There are static EAF mass- and energy-balance models cited in the literature e.g. [22]. However, this model needs to approximate the physical process in terms of its dynamic response due to control and disturbance input changes. The aim is to obtain the same type of response, with the same order of magnitude as is found in an industrial furnace.

There are other groups working on dynamic models, e.g. Morales *et al* [13] are working on a complete mathematical simulator for EAF operation. In the literature only fragments of this simulator are shown. Austin *et al* [23] developed a two-dimensional model for blast furnaces that represents the major chemical reactions and physical structures. This shows that dynamic models and simulators are becoming increasingly popular for all types of furnaces, including electric arc furnaces, as process engineers are working to optimise these processes.

Sridhar and Lahiri [24] developed models to predict EAF temperature distributions. References which focus on EAF gas flow include Gardin *et al* [25], who concentrated on gas flow above the liquid steel bath, and Zhulkovsky *et al* [26], who studied in-vessel off-gas post-combustion.

Fruehan [27] has done a study on the benefits of gas stirring in an EAF. In this study interaction between the FeO in slag and the carbon in the steel melt was investigated. Sarma *et al* [28] provided experimental results of the reduction of FeO in smelting slags by solid carbon.

Section 3.2 gives all the assumptions and simplifications. Section 3.3 discusses the non-linear model derivation. Subsections 3.3.1 to 3.3.15 give the separate derivations of the state equations and auxiliary equations. Section 3.4 shows the incorporation of the off-gas model.

3.2 ASSUMPTIONS TO FACILITATE MODELLING

The failure to choose an appropriate set of simplifying assumptions leads to either an overly complicated model or an overly simplistic model [20]. In this section a set of assumptions is given, which yields a seventeen-state non-linear state space model that is appropriate for this dissertation.

The assumptions are given here with motivation where necessary. This set of assumptions is appropriate for an unsteady state model that is intended for controller evaluation. It facilitates the modelling effort, and allows for simplification, without making the model trivial:

1. The liquid metal, molten slag and gas-phase are all at the same temperature. These elements form the fluid group. The mass of gas-phase elements is small in relation to the other fluids.

2. The solid scrap is at the same temperature as the solid slag additions that have not yet dissolved. These elements form the solid group. The solid group melt rates and the rate of increase of the solid group temperature (x_{13}) are proportional to the difference between the fluid group temperature (x_{12}) and the solid group temperature.

3. From Plöckinger & Etterich [29] it is seen that 48% of the arc heat flows to the steel melt by means of convection. A further 17% of the arc heat flows to the steel melt by means of radiation. There can be heat transfer from the arcs to the scrap, but since the greatest fraction of heat flows to the steel melt it is more convenient to make the assumption that all heat transfer from the arcs is to the liquid metal. This avoids the problem of establishing a division of the arc heat between the separate components in the furnace. To correct for the amount of energy transferred by the arc, the power input is multiplied by an efficiency factor in the disturbance model, before the disturbance model is applied as input to the plant model.

4. A power division coefficient describes heat transfer from the fluid group to the solid group. The heat transferred to the scrap can be utilised to increase the temperature of the solid group, or to melt part of the scrap. The coefficient $\sqrt{x_{13}/x_{12}}$ is used for the melting rates of the scrap and the solid slag, while $\left(1 - \sqrt{x_{13}/x_{12}}\right)$ is the coefficient used for the heating rate.

5. Fluxes added to the furnace do not melt immediately, but remain in an unmixed, solid condition until melted by the heat of the fluid group to form part of the liquid slag.

6. Because the concentration of Fe in the steel melt is very much greater than that of carbon or silicon, all oxygen blown into the steel reacts with Fe to form FeO in the slag.

7. All FeO and SiO_2 that form in the furnace are immediately dissolved in the molten slag.

8. A carrier gas blows graphite into the slag to reduce the FeO. The carrier gas is usually air and contains oxygen. All graphite is assumed to either react with FeO instantly or with the carrier gas during the ascent to the surface. This is based on the result that no carbon solids remain in the slag. The typical residence time of carbon particles in the slag is 8-10 seconds [13].

9. The amount of FeO that reacts with carbon and silicon is proportional to the FeO in the slag, from experimental evidence provided by Sarma *et al* [28].

10. The DRI contains 82.5% metallic iron, and when DRI is introduced into the furnace, 82.5% of the mass is added as iron to the steel melt, 4.5% as SiO_2 and 13% as FeO to the liquid slag.

11. The iron in the DRI melts instantaneously. This is based on the observation that the EAF temperature can be controlled by matching the DRI feed rate with the electrical power input: 33 kg/min per MW yields a constant temperature [16]. By implication the DRI effectively melts immediately upon entry, with the associated latent heat requirement effected immediately.

12. The scrap does not contain impurities that will be oxidised. In steel-making practice graded scrap is needed for low tramp element requirements in steel [30].

13. The average value (for temperatures from 600 K to 1800 K) for the specific heat of iron is used for the scrap. Cp(Fe(S)) = 0.039 kJ/(mol.K). Cp(Fe(L)) = 0.046 kJ/(mol.K) for the liquid metal.

14. Typical composition of slag additives is 2/3 burnt lime and 1/3 dolomite [30]. Burnt lime has the composition (97% CaO, 3% MgO) and doloma has the composition (57% CaO, 43% MgO) [31]. This is approximately 83% CaO and 17% MgO.

15. The CaO and MgO parts of the slag are lumped and treated as a single state variable, with an equivalent heat of fusion, heat of solution, molar mass and specific heat.

16. All elements entering the furnace are heated to the respective temperatures. Reagents are heated to 1680 K before they participate in reactions. All chemical reactions take place at an average temperature of 1680 K to prevent temperature from being a variable that determines chemical reaction power.

17. All energy is exchanged with the fluid group. The solid group receives energy from the fluid group. This means that the electric arc power is assumed to heat up the liquid metal first, which then heats up and melts the other components in the furnace.

18. The only gas-phase elements are CO, CO_2 and N_2. All O_2 reacts on entry, and H_2 is ignored.

19. The pressure in the furnace can be obtained from the ideal gas law.

3.3 NON-LINEAR MODEL DERIVATION

The EAF is described in a Multi-Input Multi-Output (MIMO) state space format with 17 states. There are four vectors that feature in the model: State vector **x**, manipulated variable vector **u**, measured disturbance vector **v** and unmeasured disturbance vector **w**.

The model is a vector function (**f**) of 17 ordinary differential equations (ODE) that describe the rate of change (with respect to time) of each state variable. The differential equations are mostly

non-linear, and are taken from the set that contains all functions of $\mathbf{x}$, $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, i.e. $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, \mathbf{v}, \mathbf{w})$ where the dimensions are: $\mathbf{x} \in \mathfrak{R}^{17 \times 1}, \mathbf{u} \in \mathfrak{R}^{2 \times 1}, \mathbf{v} \in \mathfrak{R}^{3 \times 1}, \mathbf{w} \in \mathfrak{R}^{2 \times 1}$. Appendix A gives a list of symbols. Standard symbols such as molar mass (M_x) and specific heat ($Cp(x)$) are not given.

3.3.1 Rate of Change of Solid Scrap

The rate of change of the mass of solid scrap ($\dot{x}_1$) in the furnace is proportional to the heat transfer rate from the liquid metal to the scrap. This in turn is proportional to the temperature difference between the liquid metal and the scrap. This is an approximation, since scrap melting involves both mass transfer (diffusion of carbon) and heat transfer. However, since the main aim of this model is to characterise the off-gas system, a formulation that yields a realistic scrap-melting rate is deemed acceptable. The liquid metal temperature is denoted as x_{12} and the solid scrap temperature as x_{13}. The heat transfer coefficient is $k_{ther1} = 0.24$ kW.K^{-1}.m^{-2}. This is an adjustable constant, depending on the observed melting time. The area of contact is assumed to be proportional to the mass of scrap in the furnace. A specific area of $k_{area1} = 0.005$ m^2.kg^{-1} is used, and this corresponds to sheets of thickness 0.05 m.

A partition coefficient is used to allocate power transferred to melt the scrap, as opposed to power transferred to heat the scrap, as discussed in assumption 3. The scrap that is melted must receive latent heat of fusion (λ_{Fe} [kJ/mol]) and specific heat multiplied by the temperature difference between the liquid group and the solid group.

The equation for the scrap melt rate in units of [kg.s^{-1}] is given by:

$$\dot{x}_1 = \frac{- M_{Fe} k_{ther1} k_{area1} (x_1)(x_{12} - x_{13}) \sqrt{x_{13}/x_{12}}}{\left[\lambda_{Fe} + Cp(Fe_{(S)})(x_{12} - x_{13}) \right]} \qquad (3\text{-}1)$$

3.3.2 Rate of Change of Liquid Metal Mass

The rate of change of liquid metal mass ($\dot{x}_2$) equation has six parts:

- Negative of equation (3-1), which gives the melt rate of steel scrap.
- Rate of oxidation of iron by oxygen to form FeO.
- Rate of iron reduced from FeO by graphite injection into the slag.
- Rate of iron reduced from FeO by dissolved carbon in the steel melt.
- Rate of iron reduced from FeO by dissolved silicon in the steel melt.
- Rate of metal added by DRI additions, which contain 82.5% metallic iron (see assumptions 10 and 11).

All oxygen blown into the steel reacts with Fe to form FeO (see assumption 6). The rate of Fe oxidation is then given by:

$$-\frac{2M_{Fe}v_1}{M_{O_2}} \tag{3-2}$$

The rate of reduction of FeO by the graphite is given by equation (3-3). It is proportional to the rate of graphite injection and the content of FeO in the slag (See assumption 8).

$$\frac{x_7 k_{gr} M_{Fe} w_2}{(x_6 + x_7 + x_8)M_C} \tag{3-3}$$

The rates of FeO reduction by dissolved carbon and dissolved silicon in the steel melt are discussed in Subsections 3.3.3 and 3.3.4. Please refer to these subsections for the development of the rate equations. Equations (3-8) and (3-9) give X_C and X_C^{eq} respectively and equations (3-15) and (3-16) give X_{Si} and X_{Si}^{eq} respectively. Thus the rate of change of liquid metal is given by:

$$\dot{x}_2 = \frac{M_{Fe} k_{ther1} k_{area1} (x_1)(x_{12} - x_{13})\sqrt{x_{13}/x_{12}}}{[\lambda_{Fe} + Cp(Fe_{(S)})(x_{12} - x_{13})]}$$
$$+ \frac{x_7 k_{gr} M_{Fe} w_2}{(x_6 + x_7 + x_8)M_C} + \left(\frac{M_{Fe}}{M_C}\right)k_{dC}\left(X_C - X_C^{eq}\right) \tag{3-4}$$
$$+ \left(\frac{2M_{Fe}}{M_{Si}}\right)k_{dSi}\left(X_{Si} - X_{Si}^{eq}\right) - \frac{2M_{Fe}v_1}{M_{O_2}} + 0.825v_3$$

3.3.3 Rate of Change in Dissolved Carbon

The rate of change of dissolved carbon ($\dot{x}_3$) in the steel melt is due to reaction with FeO in the slag. The reaction with FeO in the slag is driven by the deviation from equilibrium that is approximated as a hyperbolic relationship between the carbon in the steel melt and the FeO in the slag (mass percentages). For low-carbon melts ($\gamma_C = \gamma_C^{\infty}$), and for basic slags (in which $\gamma_{FeO} \approx 1$) the equilibrium product varies with temperature from 1.8 at 1500°C to 0.89 at 1700°C [31,32]. Since 1600°C is close to the final tap temperature, a constant equilibrium product was determined for 1600°C and used throughout the simulation.

The reaction between C and FeO is far from equilibrium throughout the steelmaking process, and therefore the exact value of the equilibrium constant is not critical. At 1600°C Turkdogan [31] (equation 6.6, p.183) gives equilibrium by:

$$(\%FeO)(\%C)_{equilibrium} = 1.25 = k_{C\%} \tag{3-5}$$

The relationship between $k_{C\%}$ and the molar fractions is approximated by equation (3-6). Equation (3-7) gives the relationship for the dimensionless equilibrium concentration constant k_{XC}.

$$k_{C\%} = X_{FeO} X_C \left(\frac{M_{FeO} M_C}{M_{slag} M_{Fe}} 100^2 \right) \tag{3-6}$$

$$k_{XC} = X_{FeO} X_C = k_{C\%} \left(\frac{M_{slag} M_{Fe}}{M_{FeO} M_C \cdot 100^2} \right) \tag{3-7}$$

The average molar mass of steelmaking slag is $M_{slag} = 0.0606$ kg/mol [31]. From equation (3-7) a value of $k_{XC} = 491*10^{-6}$ was obtained. X_C is the molar fraction of carbon in the steel given by equation (3-8). X_C^{eq} is the equilibrium molar fraction of carbon in the steel given by equation (3-9). X_{FeO} is the molar fraction of FeO in the slag given by equation (3-10).

$$X_C = \frac{x_3/M_C}{x_2/M_{Fe} + x_3/M_C + x_4/M_{Si}} \tag{3-8}$$

$$X_C^{eq} = k_{XC} \left(\frac{x_6 M_{FeO}}{x_7 M_{slag}} + \frac{x_8 M_{FeO}}{x_7 M_{SiO_2}} + 1 \right) \tag{3-9}$$

$$X_{FeO} = \frac{x_7/M_{FeO}}{x_6/M_{slag} + x_7/M_{FeO} + x_8/M_{SiO_2}} \tag{3-10}$$

The reaction rate is controlled by the difference of the actual carbon content from the equilibrium carbon content as in Fruehan [27] (equation 10, p.37). For this equation the rate constant was determined to give a reasonable decarburisation rate. The rate constant has units of kg/s since the differential equation for carbon is in units of kg/s.

The decarburisation rate constant was determined as $k_{dC} = 48$ kg/s. The rate of decarburisation due to reaction with FeO in the slag is given by equation (3-11) where X_C and X_C^{eq} are given by equations (3-8) and (3-9):

$$\dot{x}_3 = -k_{dC} \left(X_C - X_C^{eq} \right) \tag{3-11}$$

3.3.4 Rate of Change in Dissolved Silicon

The rate of decrease of the silicon ($\dot{x}_4$) in the steel melt is due to reaction with FeO in the slag. The reaction with FeO in the slag is driven by the deviation from equilibrium that is approximated as a hyperbolic relationship between the Si in the steel melt and the $(FeO)^2$ in the slag (mass percentages). This is similar to the treatment of the carbon in the steel melt:

$$(\%FeO)^2 (\%Si) = 0.00057 = k_{Si\%} \tag{3-12}$$

The relationship between $k_{Si\%}$ and the molar fractions is approximated by equation (3-13). Equation (3-14) gives the relationship for the dimensionless equilibrium concentration constant.

$$k_{Si\%} = X_{FeO}^2 X_{Si} \left(\frac{M_{FeO}^2 M_{Si}}{M_{slag}^2 M_{Fe}} 100^3 \right) \tag{3-13}$$

$$k_{XSi} = X_{FeO}^2 X_{Si} = k_{Si\%} \left(\frac{M_{slag}^2 M_{Fe}}{M_{FeO}^2 M_{Si} \cdot 100^3} \right) \tag{3-14}$$

From equation (3-14) a value of $k_{XSi} = 8.08 \times 10^{-8}$ was obtained. X_{Si} is the molar fraction of silicon in the steel given by equation (3-15). X_{Si}^{eq} is the equilibrium molar fraction of silicon in the steel given by equation (3-16):

$$X_{Si} = \frac{x_4/M_{Si}}{x_2/M_{Fe} + x_3/M_C + x_4/M_{Si}} \tag{3-15}$$

$$X_{Si}^{eq} = k_{XSi} \left(\frac{x_6 M_{FeO}}{x_7 M_{slag}} + \frac{x_8 M_{FeO}}{x_7 M_{SiO_2}} + 1 \right)^2 \tag{3-16}$$

The reaction rate is controlled by the difference of the actual silicon content from the equilibrium silicon content, similar to equation (3-11) for carbon. The rate constant was determined as $k_{dSi} = 144$ kg/s. The rate of change in silicon due to reaction with FeO in the slag is given by:

$$\dot{x}_4 = -k_{dSi} \left(X_{Si} - X_{Si}^{eq} \right) \tag{3-17}$$

3.3.5 Rate of Change of Solid Slag Mass

The rate of change of solid slag mass ($\dot{x}_5$) is dependent on the introduction of new fluxes to the furnace and the rate at which the slag is melting. Similar to the scrap melting approach the melting rate is assumed to be proportional to the heat transfer rate from the liquid metal to the solid slag, which is proportional to the temperature difference between the liquid metal and the solid slag.

The heat transfer coefficient is $k_{ther5} = 0.0125$ kW.K^{-1}.m^{-2} which can be adjusted depending on the observed slag melting rate. The area of contact is assumed to be proportional to the mass of solid slag in the furnace, and a specific area of $k_{area5} = 0.12$ m^2.kg^{-1} is used, corresponding to cubic blocks (density 2500 kg/m^3) with 0.02 m sides.

The slag that is being melted must receive latent heat of fusion (λ_{Slag}) and specific heat multiplied by the temperature difference between the liquid group and the solid group. λ_{Slag} and $C_p(Slag_{(X)})$ has units of kJ/g-atom (kJ/(g-atom.K)) instead of kJ/g-mol (kJ/(g-mol.K)). The equation for the solid slag melt rate in units of [kg.s^{-1}] is given by:

$$\dot{x}_5 = \frac{-M_{Slag} k_{ther5} k_{area5} (x_5)(x_{12} - x_{13})\sqrt{x_{13}/x_{12}}}{\left[\lambda_{Slag} + Cp(Slag_{(S)})(x_{12} - x_{13}) \right]} + w_1 \tag{3-18}$$

3.3.6 Rate of Change of Liquid Slag Mass

The rate of increase of molten slag ($\dot{x}_6$) is equal but of opposite sign to the solid slag melting rate.

$$\dot{x}_6 = \frac{M_{slag} k_{ther5} k_{area5} (x_5)(x_{12} - x_{13})\sqrt{x_{13}/x_{12}}}{[\lambda_{Slag} + Cp(Slag_{(S)})(x_{12} - x_{13})]} \tag{3-19}$$

3.3.7 Rate of Change of Iron-oxide Mass in Slag

The rate of change of FeO in the slag ($\dot{x}_7$) is dependent on five rate mechanisms.

- The first rate is the same as in equation (3-2), with M_{FeO} replacing M_{Fe};
- The second rate is the same as in equation (3-3), with M_{FeO} replacing M_{Fe};
- The third rate is the FeO that is introduced into the furnace with the DRI;
- The rate of FeO reduction by carbon out of the steel melt;
- The rate of FeO reduction by silicon out of the steel melt.

Equations (3-8), (3-9), (3-15) and (3-16) give X_C, X_C^{eq}, X_{Si} and X_{Si}^{eq} respectively. The rate of change of FeO in the slag is then given by:

$$\dot{x}_7 = \frac{2M_{FeO} v_1}{M_{O_2}} - \frac{x_7 k_{gr} M_{FeO} w_2}{(x_6 + x_7 + x_8)M_C} + 0.13 v_3$$
$$- \left(\frac{M_{FeO}}{M_C}\right) k_{dC}\left(X_C - X_C^{eq}\right) - \left(\frac{2M_{FeO}}{M_{Si}}\right) k_{dSi}\left(X_{Si} - X_{Si}^{eq}\right) \tag{3-20}$$

3.3.8 Rate of Change of Silicon-dioxide Mass in Slag

The rate of change of SiO_2 in the slag ($\dot{x}_8$) is equal but of opposite sign to equation (3-17) plus the SiO_2 introduced with DRI to the furnace. Since the molar masses are different this equation must be multiplied by the ratio of M_{SiO2} to M_{Si}. Equations (3-15) and (3-16) give X_{Si} and X_{Si}^{eq} respectively. The rate of change of SiO_2 in the slag is then given by:

$$\dot{x}_8 = \left(\frac{M_{SiO_2}}{M_{Si}}\right) k_{dSi}\left(X_{Si} - X_{Si}^{eq}\right) + 0.045 v_3 \tag{3-21}$$

3.3.9 Carbon-monoxide in Furnace Gas-phase

The CO equation has 4 components:

- Rate of CO produced by decarburisation.
- Rate of CO produced by graphite injection into slag with FeO.
- Rate of CO combustion in leak-air from outside.
- Rate of CO extracted from the furnace through the off-gas door.

The rate of CO produced by decarburisation is the negative of equation (3-11). Since the molar masses are different this equation must be multiplied by the ratio of M_{CO} to M_C. This is shown by:

$$\left(\frac{M_{CO}}{M_C}\right)k_{dC}\left(X_C - X_C^{eq}\right) \tag{3-22}$$

The rate of CO produced by injection of graphite into the slag is proportional to the rate of graphite injection (d_5). Following from assumption 8, all the carbon introduced by the graphite injection reacts instantly with FeO to form CO and iron. This rate of production is represented by the following equation:

$$d_5 M_{CO}/M_C \tag{3-23}$$

The rate of CO combustion is proportional to the rate of leak-air entry, which is proportional to the pressure difference between the outside atmospheric pressure and the furnace pressure. The pressure difference can be defined as the relative pressure of the furnace with respect to the atmospheric pressure. This is treated as state variable x_{14}, referred to here as the relative pressure. Note that the relative pressure will normally be negative because of the extraction force exercised by the off-gas extraction fan.

Air that leaks into the furnace is assumed to have the normal air composition of 79% nitrogen and 21% oxygen where other gases are ignored. This amounts to 7.3 mole O_2 and 27.4 mole N_2 per kilogram of air. Based on the assumption that all the oxygen in the leak-air combusts with CO in the furnace freeboard, the rate of CO combustion is:

$$2M_{CO}k_{AIR1}k_{PR}x_{14} \tag{3-24}$$

The rate of extraction of CO from the EAF as a fraction of the total gas extraction rate equals the CO fraction of the gas composition in the furnace. The total gas extraction rate from the EAF is given by equation (3-56). Please see Section 3.4 for an explanation. The total gas extraction rate from the EAF is then given in advance here by equation (3-25).

$$\frac{h_d\left[\left(2K_M/t_d\tau_1\tau_2\right)x_{15} - \left(K_M/\tau_1\tau_2\right)x_{16}\right]}{k_U u_2 + h_d} \tag{3-25}$$

Combining the three components as discussed, the equation for the rate of change of CO in the gas-phase within the furnace is obtained. When the relative pressure is negative, equation (3-26) is used, however when the relative pressure is positive, (3-27) is used. This takes into account that the furnace gas composition differs from the composition of air. Equations (3-8) and (3-9) give X_C and X_C^{eq} respectively.

$$\dot{x}_9 = \frac{M_{CO}d_5}{M_C} - \frac{\left[(2K_M/t_d\tau_1\tau_2)x_{15} - (K_M/\tau_1\tau_2)x_{16}\right]h_d x_9}{(k_U u_2 + h_d)(x_9 + x_{10} + x_{11})}$$
$$+ \left(\frac{M_{CO}}{M_C}\right)k_{dC}\left(X_C - X_C^{eq}\right) + 2M_{CO}k_{AIR1}k_{PR}x_{14} \qquad (3\text{-}26)$$

$$\dot{x}_9 = \frac{M_{CO}d_5}{M_C} - \frac{\left[(2K_M/t_d\tau_1\tau_2)x_{15} - (K_M/\tau_1\tau_2)x_{16}\right]h_d x_9}{(k_U u_2 + h_d)(x_9 + x_{10} + x_{11})}$$
$$+ \left(\frac{M_{CO}}{M_C}\right)k_{dC}\left(X_C - X_C^{eq}\right) - \frac{k_{PR}x_{14}x_9}{(x_9 + x_{10} + x_{11})} \qquad (3\text{-}27)$$

3.3.10 Carbon-dioxide in Furnace Gas-phase

The CO_2 equation has 2 components:

- The rate of CO combustion to CO_2.
- The rate of CO_2 extraction from the furnace through the off-gas door.

The rate of CO combustion to CO_2 is the negative of equation (3-24), with M_{CO2} replacing M_{CO}. The rate of CO_2 extraction from the furnace is the same fraction of the total gas extraction as the CO_2 fraction of the gas composition in the furnace. The total gas extraction was discussed in the previous section and is explained in more detail in Section 3.4. Combining the two components as discussed, the equation for the rate of change of CO_2 in the gas-phase is obtained. Equation (3-28) is used for negative relative pressure and equation (3-29) for positive relative pressure.

$$\dot{x}_{10} = -\frac{\left[(2K_M/t_d\tau_1\tau_2)x_{15} - (K_M/\tau_1\tau_2)x_{16}\right]h_d x_{10}}{(k_U u_2 + h_d)(x_9 + x_{10} + x_{11})} - 2M_{CO2}k_{AIR1}k_{PR}x_{14} \qquad (3\text{-}28)$$

$$\dot{x}_{10} = -\frac{\left[(2K_M/t_d\tau_1\tau_2)x_{15} - (K_M/\tau_1\tau_2)x_{16}\right]h_d x_{10}}{(k_U u_2 + h_d)(x_9 + x_{10} + x_{11})} - \frac{k_{PR}x_{14}x_{10}}{(x_9 + x_{10} + x_{11})} \qquad (3\text{-}29)$$

3.3.11 Nitrogen in Furnace Gas-phase

The N_2 equation has 3 components:

- The rate of N_2 leaking into the furnace.
- The rate of N_2 blown into the furnace by the graphite injection system.
- The rate of N_2 extraction from the furnace through the off-gas door.

The rate of N_2 leaking into the furnace is proportional to the relative pressure in the furnace.

$$M_{N2}k_{AIR2}k_{PR}x_{14} \qquad (3\text{-}30)$$

The rate of N_2 blown into the EAF by the graphite injection system is a fraction of the graphite injection rate. For every 150 kg graphite that is injected 1 kg of N_2 is injected as carrier gas.

The rate of N_2 extraction from the furnace is the same fraction of the total gas extraction as the N_2 fraction of the gas composition in the furnace. The total gas extraction was discussed in Subsection 3.3.9 and is explained in more detail in Section 3.4. Combining the two components, the equation for the rate of change of N_2 in the gas-phase is obtained. Equation (3-31) is for negative relative pressure and (3-32) for positive relative pressure:

$$\dot{x}_{11} = -\frac{\left[(2K_M/t_d\tau_1\tau_2)x_{15} - (K_M/\tau_1\tau_2)x_{16}\right]h_d x_{11}}{(k_U u_2 + h_d)(x_9 + x_{10} + x_{11})} - M_{N2}k_{AIR2}k_{PR}x_{14} + \frac{w_2}{150} \tag{3-31}$$

$$\dot{x}_{11} = -\frac{\left[(2K_M/t_d\tau_1\tau_2)x_{15} - (K_M/\tau_1\tau_2)x_{16}\right]h_d x_{11}}{(k_U u_2 + h_d)(x_9 + x_{10} + x_{11})} - \frac{k_{PR}x_{14}x_{11}}{(x_9 + x_{10} + x_{11})} + \frac{w_2}{150} \tag{3-32}$$

3.3.12 Temperature of Liquid Metal

The total amount of power generated in the furnace is equal to the sum of all power sources and losses. The power loss through the water-cooled furnace wall is assumed to be proportional to the temperature difference between the bath temperature and the atmospheric temperature. The total power is equal to $(p_t + v_2 - k_{VT}x_{12})$ where p_t is a function of the power released by chemical reactions, power consumed by heating up the elements in the furnace, and power loss to melt scrap. Variable v_2 is the electric arc power. The function p_t will be discussed in Subsection 4.15. The rate of temperature change of the liquid metal is the total power divided by the heat capacity of all the elements that are heated up to the liquid metal temperature. It is assumed that the heat transfer between the liquid metal and the slag is sufficiently fast that the slag and the liquid metal are at the same temperature. The equation for the rate of temperature change is then:

$$\dot{x}_{12} = \frac{p_t + v_2 - k_{VT}.(x_{12} - T_{air})}{\dfrac{x_2 C_p(Fe_{(L)})}{M_{Fe}} + \dfrac{x_3 C_p(C)}{M_C} + \dfrac{x_4 C_p(Si)}{M_{Si}} + \dfrac{(2x_6 + 2x_7 + 3x_8)}{M_{slag}}C_p(Slag_{(L)})} \tag{3-33}$$

$C_p(Slag_{(L)})$ is obtained from the enthalpy equation for liquid steelmaking slag [33].

$$H_T = -356.31 + 0.0405T \text{ kJ/g-atom} \tag{3-34}$$

3.3.13 Scrap and Solid Slag Temperature

The liquid metal heats the scrap and the solid slag. The heat transfer is proportional to the temperature difference between the liquid metal and the scrap. The heat transfer coefficients are $k_{ther1} = 0.24$ kW.K^{-1}.m^{-2} for scrap and $k_{ther5} = 0.0125$ kW.K^{-1}.m^{-2} for solid slag. The constants $k_{area1} = 0.005$ m^2.kg^{-1} for scrap and $k_{area5} = 0.12$ m^2.kg^{-1} for solid slag are used. The heat that is transferred divided by the heat capacity gives the rate of change for the scrap and solid slag temperature:

$$\dot{x}_{13} = \frac{\left[k_{Ther1}k_{Area1}x_1\left(x_{12} - x_{13}\right) + k_{Ther5}k_{Area5}x_5\left(x_{12} - x_{13}\right)\right]\left(1 - \sqrt{x_{13}/x_{12}}\right)}{x_1 Cp(Fe_{(S)})/M_{Fe} + 2x_5 Cp(Slag_{(S)})/M_{Slag}} \qquad (3\text{-}35)$$

$C_p(Slag_{(S)})$ is obtained from the enthalpy equation for solid steelmaking slag [33].

$$H_T = -348.35 + 0.03T \text{ kJ/g-atom} \qquad (3\text{-}36)$$

3.3.14 Relative Pressure

To get the rate of change for the relative pressure, it is assumed that the atmospheric pressure remains constant. The rate of change of the relative pressure is then only dependent on the rate of change of the furnace pressure and is thus independent of the atmospheric pressure. One approach to obtain an equation for the rate of change of pressure is to use the ideal gas law, to solve it for pressure, and then take the first derivative with respect to time. In doing this, for simplicity it is assumed that the freeboard volume remains constant. Then only the molar quantity and the temperature rates have to be considered. Since the freeboard volume (175 m^3) is large with respect to the volume of the solids and liquids in the furnace, this is acceptable. For temperature variation, multiply the function f_{12} by the molar quantity. When the molar quantity is considered, add the functions f_9, f_{10} and f_{11} and then multiply by the current temperature. Multiply both parts with the universal gas constant R, and divide both with the freeboard volume.

When all the components of the equation are combined, the rate of change for the relative pressure is obtained:

$$\dot{x}_{14} = \left(\frac{x_9}{M_{CO}} + \frac{x_{10}}{M_{CO_2}} + \frac{x_{11}}{M_{N_2}}\right)\frac{Rf_{12}}{Vol} + \frac{Rx_{12}}{Vol}\left(f_9 + f_{10} + f_{11}\right) \qquad (3\text{-}37)$$

f_9 to f_{12} are given in Subsections 3.3.9 to 3.3.12, they are the actual differential equation functions of the states x_9 to x_{12}.

3.3.15 Thermo-chemical Energy Exchange

Enthalpies of formation, molar heats of fusion, and molar heat capacities were obtained from Kubaschewski [32]. These are usually temperature dependent, and average values for the relevant temperature range are used. Square brackets indicate that the reagent/product is in solution, and for these the enthalpy of solution must be taken into account.

The average enthalpy of formation for CO is ΔH_{CO} = -117 kJ/mol, and for compositions 0% to 4% C in liquid Fe solution, ΔH_{C_S} = 27 kJ/mol [34]. The average enthalpy of formation from elements for CO_2 is ΔH_{CO2} = -396 kJ/mol.

Since the activity coefficient of FeO is not a strong function of temperature [31] it can be assumed that the enthalpy of solution for FeO in the slag is sufficiently small to be ignored. The average enthalpy of formation for FeO is ΔH_{FeO} = -243 kJ/mol.

The average enthalpy of formation for SiO_2 is ΔH_{SiO2} = -946 kJ/mol. For low Si in liquid Fe the enthalpy of solution is ΔH_{Si_S} = -132 kJ/mol [35]. The average enthalpy of solution for SiO_2 in slag is ΔH_{SiO2_S} = -45 kJ/mol.

Equations (3-38) to (3-49) give the thermo-chemical heat equations. In each case ΔH_{**} is the enthalpy of formation from elements except where it is indicated by an underscore (_s) used for enthalpy of solution. (Elements and products are at a temperature of 1680 K, see assumption 16.)

p_1 = reaction ([C] + [FeO] → Fe + CO)

$$p_1 = \left(\Delta H_{C\_S} + \Delta H_{FeO} - \Delta H_{CO}\right)\left(k_{dC}/M_C\right)\left(X_C - X_C^{eq}\right) \tag{3-38}$$

X_C and X_C^{eq} are given by equations (3-8) and (3-9) respectively.

p_2 = reaction (Fe + ½ O_2 → FeO)

$$p_2 = -\left(\Delta H_{FeO}\right)2v_1/M_{O_2} \tag{3-39}$$

p_3 = reaction (CO + ½ O_2 → CO_2)

$$p_3 = -\left(\Delta H_{CO2} - \Delta H_{CO}\right)\left(2k_{air1}\right)\left(k_{PR}x_{14}\right) \tag{3-40}$$

p_4 = reaction ([Si] + [FeO] → Fe + [SiO_2])

$$p_4 = \left(\Delta H_{Si\_S} - \Delta H_{SiO_2} - \Delta H_{SiO_2\_S}\right)\left(k_{dSi}/M_{Si}\right)\left(X_{Si} - X_{Si}^{eq}\right) \tag{3-41}$$

Equations (3-15) and (3-16) give X_{Si} and X_{Si}^{eq} respectively.

p_5 = heating loss to oxygen stream

$$p_5 = -\left(v_1/M_{O2}\right)\left(x_{12} - T_{O2}\right)C_P(O_2) \tag{3-42}$$

p_6 = heating loss to oxygen in leak-air stream

$$p_6 = -k_{air1}k_{PR}x_{14}\left(x_{12} - T_{AIR}\right)C_P(O_2) \tag{3-43}$$

p_7 = heating loss to nitrogen in leak-air stream

$$p_7 = -k_{air2}k_{PR}x_{14}(x_{12} - T_{AIR})C_P(N_2)$$ (3-44)

p_8 = heating loss to Slag stream

$$p_8 = -2w_1 Cp(Slag_{(S)})(x_{13} - T_{Slag})/M_{slag}$$ (3-45)

p_9 = heating loss to DRI stream

$$p_9 = -0.825v_3 \frac{\lambda_F(Fe) + Cp(Fe_{(S)})(x_{12} - T_{DRI})}{M_{Fe}}$$ (3-46)

p_{10} = heating and melting loss to solid steel and solid slag

$$p_{10} = -k_{Ther1}k_{Area1}x_1(x_{12} - x_{13})$$
$$- k_{Ther5}k_{Area5}x_5(x_{12} - x_{13})$$ (3-47)

p_{11} = heat of reaction (C + [FeO] $\rightarrow$ Fe + CO) from graphite injection

$$p_{11} = \frac{x_7 k_{gr} w_2 (\Delta H_{FeO} - \Delta H_{CO})}{(x_6 + x_7 + x_8)M_C}$$ (3-48)

$$p_t = p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + p_9 + p_{10} + p_{11}$$ (3-49)

3.4 OFF-GAS MODEL INCORPORATION

The off-gas model will be treated in Chapter 4, but it is necessary to give a short description of it here, to show how it is incorporated into this model. In transfer function form, the off-gas model consists of a transfer function with two poles and dead time. The transfer function form is shown in equation (3-50).

$$\dot{m}(s) = \frac{K_M e^{-t_d s}}{(\tau_1 s + 1)(\tau_2 s + 1)} P(s)$$ (3-50)

After applying a Padé approximation to the dead-time function and converting the system from transfer function format to state-space format three states are added to the state space, x_{15} to x_{17}. Their equations are given here:

$$\dot{x}_{15} = x_{16}$$ (3-51)

$$\dot{x}_{16} = x_{17}$$ (3-52)

$$\dot{x}_{17} = -2/(t_d \tau_1 \tau_2)x_{15} - (2\tau_1 + 2\tau_2 + t_d)/(t_d \tau_1 \tau_2)x_{16} - (1/\tau_1 + 1/\tau_2 + 2/t_d)x_{17} + u_1$$ (3-53)

The equivalent mass-flow rate in the water-cooled duct is then given by equation (3-54):

$$\dot{m} = [2K_M/(t_d \tau_1 \tau_2)]x_{15} - [K_M/(\tau_1 \tau_2)]x_{16}$$ (3-54)

Expressed as fraction of $\dot{m}$, the rate of gas extraction from the EAF is:

$$\frac{h_d}{k_U u_2 + h_d} \tag{3-55}$$

When equations (3-54) and (3-55) are combined they give the gas extraction rate from the EAF:

$$\frac{h_d \left[(2 K_M / t_d \tau_1 \tau_2) x_{15} - (K_M / \tau_1 \tau_2) x_{16} \right]}{k_U u_2 + h_d} \tag{3-56}$$

The complete derivation of this relationship is given in Chapter 4.

3.5 CONCLUSION

The set of simplifying assumptions was given to facilitate the modelling effort. The derivation of the EAF model was shown in detail. Each state variable was associated with an appropriate differential equation. The use of thermodynamic and chemical first principles was discussed. The use of empirical relations asserted by industry experts and adjustable coefficients were also discussed. The incorporation of the off-gas model into a combined model was shown.