Chapter 1

Introduction

Quarks are usually bound in hadronic states. However, lattice calculations of Quantum Chromodynamics (QCD) [3, 4, 5, 6, 7] predict that at high temperature and pressure, the hadrons essentially melt and the quarks and gluons become asymptotically free. Such a state is called a quark-gluon plasma (QGP). One of the primary objectives of colliding heavy ions at very high energies is to study this new phase of matter, the QGP. Collisions of nuclei with highly relativistic speeds are expected to produce small volumes of matter in which the quarks and gluons, ordinarily confined to protons and neutrons, interact freely with each other[1].

When the density of quarks and antiquarks in a system is low, the quarks are confined in individual hadrons, surrounded by *normal* vacuum. However, as the density is raised, by increasing temperature or baryon density, the hadrons begin to overlap and matter is expected eventually to undergo a transition to the QGP phase, in which the quarks and gluons are no longer locally confined, but are free to roam over the entire system. If one imagines hadrons as surrounded by little islands of *perturbative* vacuum, as in bag

[1] For a brief introduction of quarks and gluons see Appendix A.

models, then at sufficiently high density, the inbetween regions of *normal* vacuum are squeezed out, and the space becomes filled with *perturbative* vacuum [8].

In the absence of a complete solution of QCD, one can describe confinement and possible deconfinement of quarks to a first approximation in terms of a picture of vacuum having two possible phases. The first, the *normal* vacuum outside hadrons, is that in the absence of physical quarks and color fields. Quarks and gluons modify the vacuum in their neighborhood, transforming it into a second, high energy state, the *perturbative* vacuum, the form of the vacuum inside hadrons. The crucial difference between between these two states is that the *normal* vacuum excludes physical quark and gluon fields, while they can propagate freely throughout the *perturbative* vacuum. In terms of quark masses, one would say that in the *normal* vacuum the mass of an isolated quark is infinite (provided confinement is exact), while in the *perturbative* vacuum the quarks have the current mass values (see Appendix A) [8].

In addition to man-made production of the QGP in heavy ion collisions where heavy ions are accelerated to relativistic energies and made to collide, the QGP can be found in cosmic rays, supernovae, neutron stars and the early universe [8].

Substantial theoretical research has been carried out to study the phase transition between hadronic matter and the QGP [1, 2, 9, 10, 11, 12]. When calculating the QGP signatures in relativistic nuclear collisions, the distribution functions of quarks and gluons are traditionally described by Boltzmann-Gibbs (BG) statistics. Here we investigate the effect of the non-extensive form of statistical mechanics proposed by Tsallis [13] on the formation of a QGP [14]. The crucial difference between the hadronic and QGP phase is the relative importance of *short-range* and *long-range* interactions among the constituents on either side of the anticipated phase transition. The hadronic phase is characterized by a dominant *short-range* interaction among hadrons (which lends itself to BG statistics) while the QGP phase has a greatly reduced *short-range* interaction (due to "asymptotic freedom") and consequently a dominant *long-range* interaction. We suggest to account for the effects of the dominant part of this *long-range* interaction by a change in statistics for the constituents in the QGP phase.

Since hadron-hadron interactions are of *short-range*, the BG statistics is successful in describing particle production ratios seen in relativistic heavy ion collisions below the phase transition [15, 16, 17, 18, 19, 20]. Our motivation for the use of generalized statistics in the QGP phase lies in the necessity to include the *long-range* interactions on the QGP side. Recently Hagedorn's [21] statistical theory of the momentum spectra produced in heavy ion collisions has been generalized using Tsallis statistics to provide a good description of e^+e^- annihilation experiments [22, 23]. Furthermore, Walton and Rafelski [24] studied a Fokker-Planck equation describing charmed quarks in a thermal quark-gluon plasma and showed that Tsallis statistics were relevant. These results suggest that perhaps BG statistics may not be adequate in the quark-gluon phase.

It has been demonstrated [25, 26] that the non-extensive statistics can be considered as the natural generalization of the extensive BG statistics in the presence of *long-range* interactions, *long-range* microscopic memory, or *fractal space-time* constraints. It was suggested in [27, 28] that the extreme conditions of high density and temperature in ultra-relativistic heavy ion collisions can lead to memory effects and *long-range* color interactions. We deem the non-dominant part of this *long-range* interaction negligible for the purpose of the phase diagram which we study here in detail. This latter view is supported by the empirical insensitivity of the phase diagram to *details* of the interaction among the constituents on either side of the phase transition. Therefore, we use the generalized statistics of Tsallis to describe the QGP phase while maintaining the usual BG statistics in the hadron phase (as we shall see this may also be regarded as choosing Tsallis statistics in the hadron phase with the Tsallis parameter q=1). In chapter two, we will discuss the generalized statistics proposed by Tsallis and formulate the distribution functions of fermions and bosons from maximum entropy principle. In chapter three, we will apply the distribution functions formulated in chapter two to the constituents of the QGP and study the effect of the generalized statistics on the formation of a QGP.