

# Generalized statistics and the formation of a quark-gluon plasma

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by

Supervisor: Professor H.G. Miller

Department: Physics

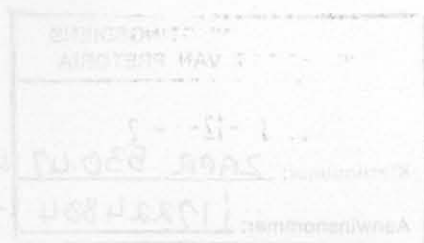
Degree: Magister Scientiae

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## Abstract

Substantial theoretical research has been carried out to study the phase transition between hadronic matter and a quark-gluon plasma (QGP). When calculating the QGP signatures in relativistic nuclear collisions, the distribution functions of quarks and gluons are traditionally described by Boltzmann-Gibbs (BG) statistics. Here we investigate the effect of both extensive and non-extensive forms of statistical mechanics on the formation of the QGP. We suggest to represent the dominant part of the *long-range* interactions among the constituents in the QGP by a change in the statistics of the system in this phase, and we study the relevance of this statistics for the phase transition. The results show that for small deviations ( $\approx 10\%$ ) from BG statistics in the QGP phase, the critical temperature for the formation of a QGP does not change substantially for a large variation of the chemical potential. This can be interpreted as the formation of a QGP occurs at a critical temperature

which is almost independent of the total number of baryons participating in heavy ion collision. The resulting insensitivity of the critical temperature to the total number of baryons presents a clear experimental signature for the existence of fractal statistics for the constituents of the QGP.

and constructive remarks.

I would also like to thank R. Q. Odendaal for the help he offered me in fixing computer related problems.

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Quarks are usually bound in hadronic states. However, lattice calculations of Quantum Chromodynamics (QCD) [3, 4, 5, 6, 7] predict that at high temperature and pressure, the hadrons essentially melt and the quarks and gluons become asymptotically free. Such a state is called a quark-gluon plasma (QGP). One of the primary objectives of colliding heavy ions at very high energies is to study this new phase of matter, the QGP. Collisions of nuclei with highly relativistic speeds are expected to produce small volumes of matter in which the quarks and gluons, ordinarily confined to protons and neutrons, interact freely with each other<sup>2</sup>.

When the density of quarks and antiquarks in a system is low, the quarks are confined in individual hadrons, surrounded by normal vacuum. However, as the density is raised, by increasing temperature or baryon density, the hadrons begin to overlap and matter is expected eventually to undergo a transition to the QGP phase, in which the quarks and gluons are no longer locally confined, but are free to roam over the entire system. If one imagines hadrons as surrounded by little islands of perturbative vacuum, as in bag

<sup>2</sup>For a brief introduction of quarks and gluons see Appendix A.