

# **Investigating the interaction of mathematics teachers with learners' mathematical errors**

by

Johanna Cornelia (Hanlie) Verwey

Submitted in fulfilment of the requirements for the degree

**Magister Educationis**

Faculty of Education

University of Pretoria

July 2010

Doctor Hannah Barnes

Doctor Gerrit Stols



UNIVERSITEIT VAN PRETORIA  
UNIVERSITY OF PRETORIA  
YUNIBESITHI YA PRETORIA

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## ABSTRACT

This study investigated the interaction of mathematics teachers with learners' mathematical errors. The teachers' verbal interaction with learners' errors during learning periods and their written interaction in assessment tasks were explored. The study was contextualized in grade 9 secondary school classrooms in the Gauteng province of South Africa. The investigation was epistemologically underpinned by constructivism/socio-constructivism. The investigation was qualitatively approached through four case studies. Structured and semi-structured interviews, classroom observations and learners' written assessment tasks were employed as sources of data. The participating teachers were described in terms of their beliefs about mathematics, their beliefs about learners' mathematical errors, their observed prevalent teaching approach and their professed and enacted interaction with learners' mathematical errors. Within-case and cross-case comparisons ensued. The findings proposed that when teachers believed that the value of learners' errors was vested in the corrections thereof, rather than employing these opportunities for discussion, valuable opportunities for learners to develop and improve their meta-cognitive abilities might potentially be lost. The findings further indicated that a focus on the mere correction of learners' errors probably denied learners opportunities to develop a mathematical discourse. The results of the investigation illuminated that an emphasis on achievement during assessment, together with a disapproving disposition towards errors among teachers and learners, were hindrances. They acted as barriers to engendering a socio-constructivist learning environment in which interactions with learners' errors could enhance learning and establish a negotiating mathematical community. A concurrence between the teachers' prevalent teaching approach and their mathematical beliefs was confirmed. However, in two of the four cases, a dissonance was revealed between their prevalent teaching approach and their interaction with learners' errors. Interaction with learners' mathematical errors was hence identified as a separate and discrete component of a teacher's practice. The findings suggest the explicit inclusion of error-handling in reform-oriented teacher-training and professional development courses to utilize learners' mathematical errors more constructively.

Keywords: mathematics teachers, learners' mathematical errors, teaching mathematics, learning mathematics, interaction, instructional scaffolding, formative assessment, socio-constructivism, secondary school

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*It's never too late to be what you might have been (George Eliot).*



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## LIST OF ACRONYMS

TIMSS	Trends in International Mathematics and Science Study
PGCE	Post Graduate Certificate in Education
DoE	Department of Education
GET	General Education and Training
FET	Further Education and Training
GDE	Gauteng Department of Education
MMAP	The Middle School Mathematics through Applications Project
PCK	Pedagogical content knowledge
ZPD	Zone of proximal development
B Sc	Bachelor of Science/Baccalaureus Scientiae
NCS	National Curriculum Statement

## **1.1 INTRODUCTION**

There is constant public reference to the poor performance of South African learners in the Trends in International Mathematics and Science Study (TIMSS) (Reddy, 2005), the lack of grade 12 learners who achieve in mathematics, the small number of students in tertiary mathematics and the shortage of mathematics teachers in South Africa (Bernstein, 2007; Louw-Carstens, 2007; Rademeyer, 2009; Vollgraaff, 2008). Teachers' interactions with learners' mathematical errors may contribute to maintaining the unfavourable situation. The conduct and the attitude of the teacher influence the learners' behaviour in the mathematics classroom (Op't Eynde & De Corte, 2003). The way a teacher handles learners' errors during classroom interactions can be detrimental to the attitudes learners have towards mathematics (Ames & Archer, 1988) and can inhibit learners from studying mathematics at tertiary level (Dweck, 1986). Valuable opportunities to clarify learners' misconceptions and to develop the skills to reflect (Leu & Wu, 2005) are lost when the errors learners make are not discussed during classroom interactions. Leu and Wu (2005) identify the way teachers interact with learners' errors as an aspect of the superficial changes in teaching practice when curriculum reform necessitates teachers alter their practices. Curriculum reform has been a public point of discussion in South Africa for the last decade and the success of these reforms is unconvincing (Graven, 2002; Jita & Vandeyar, 2006; Newstead, 1999). Error-handling may hence be a contributing issue in the apparent disparity between curriculum reform policy and practice. It was within this context that I embarked on investigating the interaction of mathematics teachers with learners' mathematical errors in four urban high<sup>1</sup> schools in the Gauteng<sup>2</sup> province.

This chapter serves to introduce the systematic enquiry. It begins with an account of my personal background in section 1.2, with the problem statement ensuing in section 1.3, followed by the research questions in section 1.4. The rationale for the research is delineated in section 1.5. Drawing on scholarly literature, both international and national, section 1.6 offers a critical review and synthesis of the relevant literature. Section 1.7 is

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<sup>1</sup> The terms *high school* and *secondary school* are used interchangeably. The grades in a secondary school range from grade 8 to grade 12.

<sup>2</sup> Gauteng is one of the nine provinces of the Republic of South Africa.

centred on the research methodology. Woven into the research methodology is a delineation of the philosophical assumptions underpinning the research. A reflection on the limitations of the study is put forward in section 1.8. The chapter closes with an outline of the research report in section 1.9.

## **1.2 PERSONAL BACKGROUND**

Learners' mathematical errors have intrigued me since I embarked on tutoring and teaching mathematics more than two decades ago. However, it was only in 2006, while completing the Postgraduate Certificate in Education (PGCE), that I understood and appreciated mathematical errors from a constructivist perspective, through my exposure to scholarly literature. I endorsed a behaviouristic point of view on learners' mathematical errors during the initial weeks of the PGCE course. To meet the requirements of the PGCE programme, I had to develop a constructivist, problem-based, learner-centred approach to teaching. I was introduced to the theory of Realistic Mathematics Education in which Nelissen (1999, p. 2) describes the approach to errors as follows:

*Mathematics instruction means more than acquainting children with mathematical content, but also teaching them how mathematicians work, which methods they use and how they think. For this reason, children are allowed to think for themselves and perform their own detective work, are allowed to make errors because they can learn by their mistakes, are allowed to develop their own approach and learn how to defend it but also to improve it whenever necessary. This means that students learn to think about their own mathematical thinking, their strategies, their mental operations and their solutions.*

Two of Olivier's (1992; 1999) articles played a pivotal role in accomplishing the paradigm shift I had to make in terms of mathematics education. Olivier's (1999, p. 26) description of how learners misinterpreted the fact that the earth is round: visualizing the earth as a flat disc instead of a sphere, made an indelible impression on me. I found his account of teaching through transmission as a uniform approach to social, physical and conceptual knowledge (1999, p. 27) enlightening and convincing. His article on learners' misconceptions (1992) was instrumental in conceptualizing and realizing this investigation.

## **1.3 RESEARCH PROBLEM AND STATEMENT OF PURPOSE**

### **1.3.1 Research Problem**

The aim of this multiple-case study analysis, executed in four South African secondary schools in the Gauteng province, was to explore and describe the verbal and written interactions of mathematics teachers with learners' mathematical misconceptions or errors. In doing so, the research also investigated teachers' beliefs about mathematics, about learners' errors and the role errors could play in the teaching and learning of mathematics. However, exploring teachers' mathematical beliefs was not a primary focus of the study. Teachers' mathematical beliefs formed the backdrop against which their verbal and written interactions with learners' mathematical errors were researched.

### **1.3.2 Statement of Purpose**

The purpose of the study was to investigate the verbal and written interaction of secondary school mathematics teachers with learners' mathematical errors. To reach the purpose, teachers' beliefs about mathematics, about learners' errors and the role errors could play in the teaching and learning of mathematics were investigated. Additionally, teachers' verbal interactions with learners' errors during learning periods and their written interactions with errors in assessment tasks informed the investigation.

## **1.4 RESEARCH QUESTIONS**

The study was an empirical exploration of the way secondary school teachers interacted verbally with learners' mathematical errors during learning periods and in writing in assessment tasks. The study was guided by the following primary research question:

- How do secondary school mathematics teachers interact with learners' mathematical errors?

The subsequent secondary research questions were formulated to closely link to the primary question. Each of these was researched and addressed on its own to fully explore the primary research question (Maree & Van der Westhuizen, 2007).

- What beliefs about mathematics, about learners' mathematical errors and about the role errors can play in the teaching and learning of mathematics do mathematics teachers have?

- How do mathematics teachers interact verbally with learners' errors during learning periods?
- How do mathematics teachers interact in writing with learners' errors in assessment tasks?

## **1.5 RATIONALE**

I completed the PGCE with specialization in mathematics in 2006 at the University of Pretoria. My personal interest in the way teachers interacted verbally with learners' errors during learning periods and my awareness of the classroom dynamics at play developed during the two compulsory school-based periods. These impressions had been confirmed and reinforced during the three semesters I acted as lecturer in Subject Didactics Mathematics. During learning periods, learners were found to be exceptionally dependent on the teacher and showed a lack of self-confidence in their mathematical ability. Learners were extremely cautious of giving incorrect answers. The learners showed signs of uncertainty and anxiousness, especially prior to standardised assessments. By observing the mentor teachers in their classrooms, I could recognize their approaches as teacher-centred, instrumentalist (Ernest, 1988) and mechanistic (Askew & Carnell, 1998). The mathematical authority appeared to reside with the teachers who transmitted mathematical knowledge to passive and receptive learners. It was my impression that a perception was conveyed to learners that errors were forbidden, errors had to be avoided and errors were something to be ashamed of.

Learners' mathematical errors are essentially part of the learning process (Hartnett & Gelman, 1998; Leu & Wu, 2005; Olivier, 1992; Santagata, 2005), based on the constructivist theory of how new information is interpreted by a learner in terms of his or her existing knowledge structures (Olivier, 1992). Learning will be discussed in more detail in the literature review in chapter two. Empirical studies by Santagata (2005) and Heinze (2005) show disturbing evidence of how mathematics teachers handle learners' mathematical errors during classroom interactions. According to their reports, the correcting of errors is usually not the main focus of the classroom events, with teachers ignoring a considerable number of learners' errors during classroom discourse. These studies indicate that teachers are primarily in charge of deciding what is mathematically right or wrong, that teachers directly solve up to a third of all errors and that almost half of the errors are merely corrected, without explanation. The studies further report that



negligible segments of learning periods are devoted to handling errors, with the purpose of correcting learners' errors being the continuation of the planned course of learning periods. Santagata (2005) states that in instances during which errors are discussed by teachers and corrected by learners, hints are given to learners to get the correct answer, with the focus on arriving at the correct answer and not on analysing the wrong answer. Even teachers who engage with learners in discourse, do not "scaffold" self-regulation (Magnusson, Krajcik & Borko, 1999, p. 29; Wood, Bruner & Ross, 1976, p. 90) due to low levels of questioning, cueing learners to correct answers (Santagata, 2005), showing solutions, teaching rules and expecting correct answers (Meyer & Turner, 2002).

The results, as reported by the above-mentioned studies, may probably be extrapolated to the South African situation. Brodie (2008) confirms Santagata's (2005) conclusions when she states that teachers' main concerns with errors are to produce correct answers. An appropriate engagement with learners' errors, with the goal of enhancing understanding and learning, will probably only be realized in reform-oriented or socio-constructivist (Ellis & Berry III, 2005; Wood & Sellers, 1996) classrooms (Borasi, 1996). A number of studies indicate that large numbers of South African teachers do not hold constructivist or problem-solving views of mathematics or do not fully understand what constructivism entails (Jita & Vandeyar, 2006; Stols, Olivier & Grayson, 2007).

The importance of this study is embedded in the relevance of learners' errors in the learning process and the seeming disregard of the pedagogical value of errors that mathematics teachers portray. Teachers' awareness of the pedagogical value learners' errors carry in the learning process and empowerment of teachers with knowledge regarding learners' errors may add value to addressing the problems described in the preceding paragraphs.

Clear gaps exist in the literature. Literature on teachers' and learners' mathematical beliefs and teachers' resultant orientations towards the teaching of mathematics and towards learners' errors is available in abundance. Literature on learners' errors is readily available too. Very few studies, however, have analyzed teacher-learner interactions in relation to learners' mathematical errors in actual classroom settings (Heinze, 2005; Santagata, 2005). The majority of the studies, focusing on learners' mathematical errors, are of a diagnostic nature. The studies identify underlying reasons for making the errors and attempt to develop didactic measures to prevent or correct the errors. The focus of these diagnostic

studies is on avoiding the errors and not actually on utilizing the errors didactically. Hardly any studies focus on the way teachers interact with mathematical errors in the classroom. Borasi (1988, 1989, 1994, 1996) has reported elaborately on the employment of learners' mathematical errors to enhance understanding and learning. However, she assigns prominence to the aim of facilitating learners to interact with their errors themselves, with exploration as purpose. Teachers are instrumental in creating an environment conducive to such an aim. Although the study by Leu and Wu (2005) centres on the relationship between the views and beliefs a teacher has of mathematics and the way the teacher handles learners' errors during classroom interactions, the teacher was an elementary school teacher of five- and six-graders. Their study had an intervention component of a values-cultivating programme to assist professional development of the teacher. The study by Santagata (2005) focuses on the influence cultural factors have on the way teachers interact with learners' errors and compare teachers in Italy and in the United States of America. The study by Heinze (2005) foregrounds the perspectives learners have of the way teachers interact with learners' errors. Olivier's (1992) work is not empirical and rather diagnostic by nature. Studies that discuss a possible relationship between teachers' beliefs about mathematics and about learners' errors and the way they interact with learners' errors are limited. This empirical, explorative study, focusing on how secondary school teachers interact with learners' mathematical errors both verbally and in writing, seeks to contribute to addressing this gap in the literature.

## **1.6 LITERATURE REVIEW**

*Mathematics teachers and learners' mathematical errors* are prominent in the primary research question, as stated in section 1.4. The constructs *teachers' beliefs about mathematics, teachers' beliefs about learners' mathematical errors, teaching mathematics and learning mathematics* are prominent in the abovementioned, secondary research questions. The preliminary literature review, as presented in this chapter, was conducted with a focus on these themes and constructs. Although Begg (2009) asserts that a number of theories about the teaching and learning of mathematics exist, I am only considering the most prominent learning theories. For the purposes of this study, the teaching and learning of mathematics and learners' mathematical errors are understood and described against the backdrop of behaviourism, constructivism and socio-constructivism. The learning of mathematics inevitably produces errors. This reality is confirmed by the literature and by own experience. *Learners' mathematical errors* are discussed from both the behaviouristic

and the constructivist perspective. In this chapter, *mathematics teachers* are delineated in terms of mathematical beliefs and prevalent teaching approaches, including interactions with learners' errors, consistent with the behaviouristic and constructivist learning theories respectively.

### **1.6.1 Clarification of Main Terms and Constructs**

#### ***Mathematics***

The Department of Education (DoE) (2002, p. 4) portrays the national ideal for the teaching and learning of mathematics in South Africa with the following official definition of mathematics:

*Mathematics is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. Through this process, new mathematical ideas and insights are developed. Mathematics uses its own specialised language that involves symbols and notations for describing numerical, geometric and graphical relationships. Mathematical ideas and concepts build on one another to create a coherent structure. Mathematics is a product of investigation by different cultures – a purposeful activity in the context of social, political and economic goals and constraints.*

#### ***Misconceptions, Mathematical Errors and Mistakes***

Errors have been metaphorically referred to as symptoms, implying that a learner's errors are symptomatic of a disease, the misconception, and as computer bugs, or unsuitable commands in a computer programme (Borasi, 1988). However, the referenced author prefers to compare learners' errors to the experience of going astray in a city, thus emphasising the learners' role in the process of error-handling (ibid.). Heinze (2005) defines errors from a pedagogical perspective as "counterexamples" (p. 106). The definition is based on that of Oser, Hascher and Spychiger (1999, as cited in Heinze, 2005) who refer to an error as "a process or a fact that does not comply with the norm", originating from their perspective of "negative expertise" (p. 106). Ya-amphan and Bishop (2004) portray a more superficial view of errors in describing errors as resulting from a lack of mastery of previously learned concepts, unfamiliarity with procedures and too little experience with particular approaches. From the conclusions they draw about the role of the textbook in this regard, one can classify their view as "an eclectic position in which researchers attempt to combine the notion of learning as active

construction with aspects of the representational view of mind” (Cobb, Yackel & Wood, 1992, p. 3). Olivier (1992) distinguishes between “slips, errors and misconceptions” (p. 3). Both slips and errors will result in wrong answers. Slips are not conceptually regarded as errors, but are careless, computational failures. Errors are described as symptomatic of misconceptions. The same errors will be made regularly in similar contexts due to learners’ misconceptions.

The terms *errors* and *mistakes* are used interchangeably in the literature (Borasi, 1996; Brodie, 2005; Heinze, 2005; Leu & Wu, 2005; Olivier, 1992; Santagata, 2005; Smith, DiSessa & Roschelle, 1993; Ya-amphan & Bishop, 2004). I prefer to use the term *errors*, albeit using the term *misconceptions* occasionally (see section 1.6.3). I take cognisance of the fact that the terms *misconceptions*, *alternative conceptions* or *implicit theories* instead of *errors* are perceived as terminology that is more positive, from a constructivist perspective. In addition to that, I acknowledge the use of the terms *student conceptions* or *alternative conceptions* instead of *misconceptions* as terminology that appreciates and regards learners’ contributions (Borasi, 1996). However, I accept Olivier’s (1992) delineation of mathematical misconceptions and errors for the purposes of this investigation. I appeal to readers not to infer any negative implications associated with the use of the word *errors*.

As the aim of this study was to investigate and describe teachers’ verbal and written interaction with learners’ mathematical errors, I am in resonance with Borasi’s (1996, pp. 169, 170) description of the scope of errors in her teaching experiment, albeit in a different context.

*... the term mathematical error will be interpreted in the most comprehensive way possible, so as to maximize the occasions when the proposed strategy could be used in the context of mathematics instruction. Thus, within this study, even borderline cases such as contradictions, tentative hypotheses, contrasting results, or results that do not make sense will be considered as legitimate starting points for the development of “error activities” designed to initiate and support student inquiry.*

I will hence consider all instances of learners’ mathematical errors and of learners’ mathematical contributions or learners’ questions, indicative of misconceptions, as relevant to this study.

Borasi uses concurrent conceptions for errors in a number of her publications. Errors are referred to as “meeting unexpected results” (1988, p. 23), “a contrast with our original expectations” (1988, p. 30), “a contrast with what is initially expected” (1989, p. 29), “does not meet one’s expectations”, “a prototypical example of an anomaly” (1994, p. 168) and “something puzzling” (1996, p. 164). Even and Tirosh (2002) concur with Borasi when they use the fragment “differently ... than what might be expected” (p. 234) to refer to learners’ conceptions. I hence apply the following definition to the term *errors*: A mathematical error is any response, contribution or question by a learner that puzzles or that contrasts with what is expected.

### **1.6.2 The Behaviouristic Perspective**

Behaviourist teachers believe that knowledge is transferable and that learners are passive recipients of information. Learners will understand what they see (Olivier, 1999). Behaviourists thus perceive the existing or prior knowledge learners have as irrelevant to learning new concepts (Gagné, 1983). Due to the fact that behaviourists assume that new information is isolated from previously obtained knowledge, learners’ errors are not important in the learning process (Olivier, 1992). Only correct knowledge or successful procedures have significance for behaviouristic learning (Heinze, 2005). Behaviourists regard learners’ errors as negative and without carrying any pedagogical value (Leu & Wu, 2005). Learners’ errors should preferably be avoided due to the possibility of reinforcing wrong procedures (Skinner, 1958, as cited in Santagata, 2005). Learners’ misconceptions and related errors can be overruled by teaching correct procedures (Gagné, 1983). Hence, from a behaviouristic perspective, learning takes place when correct procedures are enforced (Leu & Wu, 2005; Santagata, 2005).

### **1.6.3 The Constructivist/Socio-constructivist Perspective**

Learning, from a constructivist perspective, is considered not to take place by receiving transmitted information, but by an active process of interaction between learning experience and existing knowledge. Resnick, Nesher, Leonard, Magone, Omanson, and Peled (1989) describe the process of learning mathematics as “making conceptual sense of new mathematics instruction in terms of their (the learners’) already available knowledge” (p. 8). New information is organized and mentally structured into appropriate schemas through assimilation, accommodation or distortion (Olivier, 1992). General agreement exists among constructivists on the notions of construction of knowledge, activation of

existing cognitive structures and continual development of these cognitive structures. Although the acts of cognitive construction may vary from weak to strong, depending on different teaching styles, construction of knowledge takes place, nonetheless. All intellectual action is thus considered to be constructive, regardless of the educational paradigm that determines how the learning material is presented (Noddings, 1990; Olivier, 1999). Inappropriate cognitive structures result in learners' misconceptions (Olivier, 1992). This is in resonance with Brodie's (2005) definition of misconceptions as "underlying conceptual structures" (p. 179).

The mental processes of assimilating and accommodating new information and the resultant distortion of cognitive structures in particular, lead to learners' misconceptions (Olivier, 1992). Errors are regarded as examples of misconceptions or irregularities in learners' thinking and reflect the level of conceptual understanding learners have of mathematics (Leu & Wu, 2005). Resnick, et al. (1989) describe error-full rules as "intelligent constructions based on what is more often incomplete than incorrect knowledge" (p. 26). This is in resonance with Brodie (2005), who refers to cognitive structures as "usually a more limited version of a mature conceptual system" (p. 179). Hence, based on constructivism, errors cannot be avoided and are a normal part of the process of internalising new information in existing mental structures (Hartnett & Gelman, 1998). Olivier (1992) states unambiguously that "learners' errors are rational and meaningful efforts to cope with mathematics. Incorrect new learning is mostly the result of previous correct learning." (p. 10). Learners' errors therefore play a fundamental role in the process of learning within a constructivist framework and are of paramount importance, specifically in the teaching of mathematics (Borasi, 1996; Leu & Wu, 2005; Olivier, 1992; Santagata, 2005).

Constructivist learning is not merely cognitive, but entails a social aspect too (Cobb, et al., 1992). The previously mentioned authors state that mathematical learning is dependent on the social and cultural circumstances that pertain. This duality will be discussed in greater detail in the following chapter. A socio-constructivist classroom environment will be recognized by a focus on problem-solving, reflection on the process of problem-solving, acknowledgement of different strategies, learner participation and a community of learners working cooperatively and collectively towards consensus regarding the status of mathematical knowledge (Noddings, 1990; Szydlik, Szydlik & Benson, 2003). Reflection on the process of problem-solving

encompasses the acknowledgement of different strategies towards problem-solving while the community of learners are negotiating consensus (Szydlik, et al., 2003). Teachers should preferably encourage the public discussion of learners' errors and misconceptions in particular, simultaneously with reflection and negotiation (Clements & Battista, 1990). Teaching practices, compatible with socio-constructivist learning environments, involve facilitating learners to reflect on their errors and misconceptions. Such reflective discussions culminate in the negotiation of accepted mathematical knowledge (Clements & Battista, 1990) and adaptation of learners' perceptions of the errors they make (Szydlik, et al., 2003).

#### **1.6.4 Teachers' Mathematical Beliefs**

Teachers' mathematical beliefs determine their classroom practices and the way they interact with learners' errors (Bauersfeld, 1994; Leu & Wu, 2005). Ernest (1988) describes three sets of mathematical beliefs teachers have. Instrumentalist teachers believe that mathematics is a compilation of unconnected truths, rules and skills. Instrumentalist teachers believe that mathematics is learnt by receiving transmitted knowledge, complemented by practice and drill work (Ernest, 1988; 1991). Platonist teachers believe mathematics to be a constant, neutral, integrated body of knowledge. Learning is realized through the understanding of elucidative mathematical examples, explicated by the teacher. Teachers, who perceive mathematics as a compilation of rules or a neutral body of knowledge, usually regard learners' errors as negative and will normatively correct the errors (Bauersfeld, 1994; Brodie 2008; Ernest, 1988; 1991). Teachers holding a problem-solving view of mathematics believe that mathematics is a social construction or a fallible human construction. Learning is regarded as the active construction of knowledge, facilitated through exploration and negotiation, while the learner is developing autonomy (Borasi, 1996; Ernest, 1988; 1991). Teachers who believe that mathematics is "a practice of shared mathematizing" (Bauersfeld, 1994, p. 140), will negotiate meaning through discussion.

#### **1.6.5 Teachers' Approaches to Teaching and Interactions with Learners' Errors**

The instrumentalist view of mathematics (Ernest, 1988) corresponds to a perceived teacher's role of instructor and the strict following of a text. The mastery of skills is in resonance with this view and the learner is regarded as passive and accommodating.

Instrumentalist teachers, focusing on the mastering of skills, will probably avoid learners' errors and see errors as negative and contra-indicative to learning. The Platonist view (ibid.) corresponds to a perceived teacher's role of explainer and a modification of the textbook approach. Learning is the reception of knowledge. In these classrooms, teachers will probably correct errors themselves and do so immediately and privately.

One of the two categories of teachers, identified and described by Barkatsas and Malone (2005, p. 80), the category of "traditional-transmission-information processing" teachers, is comparable to the two abovementioned categories identified and described by Ernest (1988). The pedagogical approaches of the traditional-transmission-information processing, instrumentalist or Platonist teachers concur with the behaviouristic theory of learning. Teachers who are traditional-transmission-information processing inclined (Barkatsas & Malone, 2005), or behaviouristic, move quickly past the errors and learners are not given an opportunity to give feedback on their understanding of the error or an explanation of what was wrong. Brodie (2008) describes the frame of reference of these teachers as one of only two categories – "right or wrong" (p. 8). The ways in which these teachers manage errors, do not involve high-level reasoning processes or significant elaboration of the errors (Santagata, 2005). Such teachers regard errors as an indication of merely computational failure or carelessness with no pedagogical value (Leu & Wu, 2005). Olivier (1992) and Leu and Wu (2005) describe how learners' errors are misinterpreted by teachers who do not regard the pedagogical value of errors highly.

Constructivism, as a learning theory, is complemented by teaching practices such as challenging learners with problems in realistic context, engendering a classroom culture in which discussions are valued, analysing and solving problems from different perspectives and promoting meta-learning and problem-solving (Borasi, 1996; Maree, 2004). These pedagogical approaches correspond to a perceived teacher role of facilitator and the personal construction of a mathematics curriculum, which in turn, concur with the problem-solving view (Ernest, 1988) certain mathematics teachers may hold of mathematics. Brodie (2008) describes a tendency of increased learners' errors associated with such altered classroom practices. The creation of an environment in which errors will be accepted as a natural part of the learning process, with the concurrent utilization of the pedagogical value of the learners' errors, is thus paramount (Borasi, 1996; Olivier, 1992). Teachers are recommended to be attentive to learners' mathematical errors and adapt their teaching accordingly in order to facilitate the restructuring of mental structures



(Palincsar & Brown, 1984). Noddings (1990) uses the example of the learner, Benny, (Erlwanger, 1973, as cited in Noddings, 1990) to accentuate the responsibility of the teacher to address misconceptions through exploration in a mathematical environment conducive to such an approach. Only by engendering a tolerance of errors in the mathematics classroom (Olivier, 1992) and by changing the perception of learning to that of a social, public and collective process (Santagata, 2005), will learners' errors carry pedagogical value.

## **1.7 RESEARCH METHODOLOGY**

### **1.7.1 Research Paradigm**

The research paradigm, of which an investigator's theoretical perspectives are intrinsic components, is the foundation and the origin of ensuing resolutions regarding methodology and design (Mackenzie & Knipe, 2006). The research focus in this systematic enquiry was to understand an educational matter of practical interest (Ernest, 1997): the way secondary school mathematics teachers verbally interact with learners' mathematical errors during learning periods and in writing in assessment tasks. The research phenomenon was inextricably related to the context of the respective classroom communities (Lodico, Spaulding & Voegtle, 2006). I was present during classroom interactions and was, to a large degree, the research instrument (Nieuwenhuis, 2007 b). Certain bias and subjectivity, as described in section 1.7.4, existed on my part due to my own professional development and engagement with scholarly literature on the relevant topics (Maxwell, 2005). I was subjectively immersed (Maree & Van der Westhuizen, 2007) in the research process and inductively attributed meaning to the data (Henning, 2005). The research methodology was purely qualitative (Lodico, et al., 2006). Based on the preceding, prevailing characteristics, my theoretical perspectives are located within a constructivist-interpretive paradigm (Mackenzie & Knipe, 2006).

### **1.7.2 Philosophical Assumptions**

A researcher's methodological preference is a rational corollary of the researcher's worldview or ontological and epistemological assumptions (Hitchcock & Hughes, 1995). My ontological assumptions regarding the essential nature of social phenomena and what can be known (Punch, 2009) can be described as relativist (Denzin & Lincoln, 2005; Smith & Hodkinson, 2005). The study focused on the individual and attempted to

understand individual behaviour (Maree & Van der Westhuizen, 2007). Information related to teachers' constructions of their social worlds (Mertens, 2009; Nieuwenhuis, 2007 a): their mathematical beliefs, their perceptions of learners' errors and their classroom related actions, was obtained during interviews and through classroom observations. I conjectured and endeavoured to understand these multiple personal and subjective realities (Denzin & Lincoln, 2005; Merriam, 1991) through the interpretation of the participants' actions and dialogues. The participants' internal and subjective experiences were central to the study (Maree & Van der Westhuizen, 2007; Smith & Hodkinson, 2005). My epistemological assumptions concerning the nature, the acquisition and the transferral of knowledge along with the relationship between the knower and what can be known (Ernest, 1997; Nieuwenhuis, 2007 a; Punch, 2009) are constructivist/socio-constructivist. I constructed an understanding of teachers' mathematical beliefs and pedagogical orientations (Ernest, 1997) through the interpretation of their interactions with learners' errors (Leu & Wu, 2005). My understanding of the educational phenomenon was partial (Ernest, 1997), subjective (Nieuwenhuis, 2007 a), contextbound (Koro-Ljungberg, 2007) and dependent on my experience (Maxwell, 2005).

### **1.7.3 Methodology**

My methodological approach to the study was qualitative (Merriam, 1991) and essentially constructivist (Maree & Van der Westhuizen, 2007; Nieuwenhuis, 2007 a). The research focus was on understanding and interpreting the individual behaviour of mathematics teachers, pedagogically engaged in their classroom communities. My role in the processes of data collection and data analysis was dominant and instrumental. Meaning was, at least to some extent, inductively attributed to the data (Henning, 2005). The nature of the research account is descriptive (Merriam, 1991).

### **1.7.4 Role of the Researcher**

By attending learning periods as an observer, I immersed myself in the research situation created in the classroom (Maree & Van der Westhuizen, 2007). To attempt not to affect or alter the unique dynamics of the classroom situation (Nieuwenhuis, 2007 b), the role I assumed as observer, was neutral and traditional. Data were therefore not collected in an invasive manner. I recorded classroom events concurrent with fulfilling my role as observer. My role as interviewer was an interactive one (McMillan & Schumacher, 2001). These roles were created for the sole purpose of data collection (ibid.). My central

character and personal history (Maxwell, 2005; Rossman & Rallis, 2003), my mathematical training (Ernest, 1997) and my induction into the discipline of mathematics education (Merriam, 1991) were important elements in the research process. Albeit an apprehensiveness of personal bias is imperative, these subjective experiences are attributing to the uniqueness of this research account.

### **1.7.5 Case Study Design**

An interactive, qualitative case study design (Maree & Van der Westhuizen, 2007) was applied to obtain a clearer, indepth understanding (McMillan & Schumacher, 2001) of the way secondary school mathematics teachers verbally interacted with learners' mathematical errors during learning periods and in writing in assessment tasks. The research project qualified as a qualitative case study design on the basis of the comprehensive investigation of a limited group of participants, distinct in their description as grade 9 mathematics teachers (McMillan & Schumacher, 2001). This distinction is synonymous with the concept of bounded system that defines a case study enquiry (Henning, 2005). Although the teachers were at different schools, "the boundedness" (ibid., p. 40) was defined as the grade and the learning area they were teaching. To elicit a key strength of case study designs, I drew on multiple sources of and techniques in gathering the data (Nieuwenhuis, 2007 b). I obtained the data through mechanically recorded classroom observations, supplemented with researcher's field notes, by conducting and recording interviews and by the analysis of learners' written assessment tasks. Another advantage of a case study design, which makes the design appropriate for a Masters study, is that a researcher can undertake the research independently, without the need for a full research team (Cohen, Manion & Morrison, 2005). In addition to being categorized as interactive, this qualitative case study was designed as a multiple-case study, encompassing four cases (see ensuing section). The unit of analysis was indicated as a pedagogically engaged secondary school mathematics teacher. Additional features of this case study design contributed to the educational, descriptive and interpretive attributes thereof.

#### ***Selection of Participants***

The sample was conveniently selected. The aim of the study was not to statistically generalize the results (McMillan & Schumacher, 2001). A typical qualitative sample size can be as small as one participant (McMillan & Schumacher, 2001). Yin (2003) suggests a

selection of more than three cases when the selected cases portray inconsequential differences. I approached six of the partnership schools of the university where I acted as lecturer in Subject Didactics Mathematics. The staff at four of the six schools portrayed more enthusiasm for the study. I hence selected four schools on the basis of voluntary participation, approachability and for comparative purposes. The selected sample was a group of four grade 9 mathematics teachers. I decided to focus on grade 9 teachers, since grade 9 was the final year of the Senior Phase in the General Education and Training (GET) band of the South African school system (see section 3.6.2) and mathematics was compulsory for all learners. In contrast to that, once learners enter the Further Education and Training (FET) band, large numbers of learners may choose to change to mathematical literacy. The grade 9 groups might have ensured a larger variation in mathematical aptitude.

### ***Data Collection Strategies***

The case study design provided me with multiple sources of data to obtain indepth information over a sustained period of time. The interactive strategies for data collection were structured and semi-structured interviews, classroom observations and document analysis, supported by supplementary data collection techniques of video-recordings, audio tape-recordings and digital voice-recordings. Valid data are obtained by using a combination of various data collection techniques (McMillan & Schumacher, 2001). Although the sample of the study was small, I continually returned to every site and participant, over a two-week period for each (see section 3.6.3), to collect and confirm data (ibid.). Data collection ended according to the planned timelines set for the study.

The first secondary research question (see section 1.4) was answered by collecting data through interviews (see appendix A). Knowledge of the teachers' professed mathematical beliefs, including their beliefs about learners' errors and about the pedagogical value learners' errors carried, were obtained during the interviews. The second secondary research question was answered by analysing data collected through classroom observations. The observations focused on teachers' verbal interactions with learners' errors and were more open than standardised observations, structured in nature (Henning, 2005). The third secondary research question was answered by collecting data through the analysis of learners' written assessment tasks, as revised by the teacher.

### ***Data Analysis***

My paradigmatic assumptions are constructivist-interpretive and my epistemological assumptions constructivist, as previously indicated. For this paradigm, the data analysis most preferred is inductive. The exploratory nature of the study is in resonance with applying such data analysis in expectation of identifying the potentially multiple realities that are present in the data (Maree & Van der Westhuizen, 2007). The cognitive processing of ideas and facts during data collection can be regarded as initial and interim data analysis (McMillan & Schumacher, 2001).

The information obtained from the written, structured interviews was not quantitatively analysed, but qualitatively applied in order to get thick descriptions of the participants, their contexts and their beliefs. The mechanically recorded semi-structured interviews and sequences of teachers' verbal interactions with learners' errors during learning periods were transcribed prior to analysis (see appendix B). Document analysis of the written assessment tasks encompassed an analysis of the teachers' written interactions with learners' errors, including remarks and cues teachers wrote as feedback. Preliminary examinations of the data collected from the various sources preceded the coding thereof. The coding processes were executed with a combination of "deductively" and "inductively" (p. 155) derived categories through application of the Miles and Huberman model (1994) of "within-case data reduction and data display" (pp. 10, 11) (see appendix C).

### ***Quality Criteria***

As a qualitative researcher, I was the data collection instrument (Maxwell, 2005; Nieuwenhuis, 2007 b; Rossman & Rallis, 2003; Somekh & Lewin, 2005). Reference to validity and reliability of the human instrument in qualitative research is therefore usually made in terms of trustworthiness (Nieuwenhuis, 2007 b). Validity of qualitative research designs depends on the level of correspondence of collective meaning that the researcher and the participants attribute to interpretations and constructs. In addition to the conduct and actions characteristic of the participants, the natural occurrence of events, irrespective of the researcher's presence, will yield valid data (McMillan & Schumacher, 2001). It is impossible to replicate qualitative research results (Maree & Van der Westhuizen, 2007; Merriam, 1991). Reliability in qualitative studies is thus described as the consistency between the collected data and the results and depends on the logic of the particular

interpretation (Merriam, 1991). In order to enhance the trustworthiness of the research (McMillan & Schumacher, 2001, pp. 407–408), I conducted the fieldwork over a period of almost four months, visiting the participants daily during a two-week period for each. Triangulation was accomplished through multi-method strategies of data collection, namely interviews, classroom observations and document analysis. Multiple sources of data included written, structured interviews, transcripts of semi-structured interviews, transcripts of mechanically recorded classroom events and learners' written assessment tasks. The classroom events and classroom interactions were captured in triplicate as video-recordings, audio tape-recordings and digital voice-recordings. Digital voice-recordings and audio tape-recordings were made of the semi-structured interviews. The interviews were not captured on video-tape. Transcriptions of mechanically recorded interviews and classroom interactions were precise representations of the language used by the participants. This resulted in data of a more concrete nature than data collected by some instruments in other designs. The research participants were able to understand the precise and factual descriptions from field notes and from mechanical recordings.

### ***Ethical Considerations***

I obtained ethical clearance from the Ethics Committee in the Faculty of Education at the University of Pretoria prior to the commencement of the systematic enquiry. Written permission was granted to me on three managerial levels. I obtained permission from the Gauteng Department of Education (GDE), the relevant educational districts and the respective school principals. Participation was voluntary. The participating teachers signed letters of informed consent prior to the start of the data collection period (see appendix D). The participants were guaranteed my truthfulness and protection. The identities of the participants and the names and locations of the schools were not disclosed. Pseudonyms were used in the research account. Video-recordings were not made publicly accessible. Data were protected and securely stored.

### ***Scope and Delimitations***

The case study was delimited to four multiple cases, each representing a pedagogically involved grade 9 mathematics teacher as the unit of analysis. Of the four research sites, three were co-educational and one an all girls' school. Two of the research sites were

double medium<sup>1</sup> and two were English medium schools. Three of the schools were public, departmental schools and one a private school. All four schools had an urban location and were adequately resourced. Although the learners' mathematical errors and the teachers' interactions with these errors were fundamentally important to the study, a focus on the learners was beyond the scope of this study. The data analysis was delimited by a focus on a teacher's interactions with learners' mathematical errors. Error-coding was hence beyond the scope of this study. The study was not embedded in a particular learning outcome or topic.

## **1.8 LIMITATIONS**

My presence during learning periods might have had an influence on the dynamics of the classroom situation (Nieuwenhuis, 2007 b), although I made a conscious and deliberate attempt to avoid that. The exposure of the teachers to the research focus of error-handling might have had an influence on the way they interacted with learners' errors during the research period. Sections of the mechanically recorded classroom events were inaudible. However, the plethora of collected data might counteract the limitation. The postponement of the data analysis processes resulted in unresolved matters, as discussed in further chapters. Possible bias on my part could exist due to my prior experiences of mathematics education and my exposure to scholarly literature.

## **1.9 SYNOPSIS AND OUTLINE OF REPORT**

The research problem of investigating the verbal and written interactions of mathematics teachers with learners' mathematical errors is introduced in this chapter. The statement of purpose and the research questions pertaining to this study are delineated and the rationale for conducting the research is presented in this chapter. Overviews of the literature review and the research methodology are given. A succinct discussion of the limitations to the study concludes the chapter.

A literature review and a discussion of the conceptual framework guiding the study are presented in chapter two. The research paradigm, underlying philosophical assumptions and the research methodology are discussed in chapter three. The contextualization of the research is included in chapter three. The data are presented, per case, in chapter four to

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<sup>1</sup> Instruction was either given in Afrikaans or in English.

seven with classroom vignettes and contextual descriptions, structurally aligned with the three secondary research questions. Conclusions drawn from the study and resultant recommendations are presented in the final chapter.



## CHAPTER TWO: LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

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### 2.1 INTRODUCTION

The purpose of this chapter is to present the literature review appropriate for the investigation. Prior to synthesizing the review, I conducted a critical, investigative exploration of the research literature. The development of perceptive and critical research questions is one of the objectives of conducting a literature review (Yin, 2003). Reciprocally, the organizational structure of the literature review emerged from the research questions that evolved. Chapter two is concluded with an illuminative discussion of the generation and the application of the conceptual framework relevant to the study.

Preceding analysis of the primary research question (see section 1.4) enabled the identification and isolation of the themes *mathematics teachers* and *learners' mathematical errors*. Three secondary research questions were formulated from the primary research question. A dissection of the three secondary research questions proposed the following additional constructs:

- Teachers' beliefs about mathematics
- Teachers' beliefs about learners' mathematical errors
- Teachers' beliefs about the role errors can play in the teaching and learning of mathematics
- Teaching of mathematics
- Learning of mathematics
- Teachers' verbal interaction with learners' errors during learning periods
- Teachers' written interaction with learners' errors in assessment tasks

The constructs *teachers' beliefs about mathematics*, *teachers' beliefs about learners' mathematical errors* and *teachers' beliefs about the role errors can play in the teaching and learning of mathematics* were identified as pertaining to mathematics teachers and particularly to their mathematical beliefs. Research literature regarding teachers' views of and beliefs about mathematics, the teaching of mathematics and the learning of mathematics underpin this segment of the literature review.

The pair of constructs *teachers' verbal interactions with learners' errors during learning periods* and *teachers' written interactions with learners' errors in assessment tasks* pertains to mathematics teachers and is embedded in the general teaching approach. Literature on teacher-learner interactions, classroom discourse, teacher questioning, teacher listening and teachers' pedagogical content knowledge (PCK) was reviewed in order to synthesize a comprehensive foundation for describing and understanding teachers' actions in general and interactions with learners' errors in particular. Inevitably, a review of this literature directed the literature search to "instructional scaffolding" (Magnusson, et al., 1999, p. 29; Wood, et al., 1976, p. 90) and to assessment.

Irrespective of teachers' and researchers' ontological or epistemological assumptions, teachers experience and researchers report that learners make errors during the process of learning mathematics. However, these philosophical assumptions fundamentally inform theories on learning and hypotheses regarding learners' mathematical misconceptions and errors. The literature review hence encompasses sections concerning learning theories. These sections describe teaching approaches that complement the particular learning theories. Hypotheses regarding mathematical misconceptions and errors compatible to the respective learning theories are delineated in these sections. For the purpose of this study the constructs *learning mathematics* and *teaching mathematics* were assumed to be concurrent processes embedded in a learning theory.

For the purposes of investigating the interaction of mathematics teachers with learners' mathematical errors, the delineation of this literature review is hence presented according to the following structure:

#### *The Teaching and Learning of Mathematics*

- An Overview of Learning Theories for Mathematics
- Behaviourism: Learning, Learners' Mathematical Misconceptions and Errors, and Teaching
- Constructivism: Learning, Learners' Mathematical Misconceptions and Errors, and Teaching
- Socio-Constructivism: Learning and Teaching
- Constructivism and Socio-Constructivism

### *Mathematics Teachers*

- Teacher-Learner Interactions and Classroom Discourse
- Teacher Questioning and Listening
- Instructional Scaffolding (Magnusson, et al., 1999; Wood, et al., 1976) and the Zone of Proximal Development (ZPD) (Vygotsky, 1978)
- Assessment
- Teachers' Mathematical Beliefs
- Pedagogical Content Knowledge (PCK)

## **2.2 THE TEACHING AND LEARNING OF MATHEMATICS**

### **2.2.1 An Overview of Learning Theories**

Maree (2004) gives an overview of the prevalent and dominant perspectives on the teaching and learning of mathematics through the twentieth century, developing from expository teaching and rote learning, realistic contextualization, problem-solving in related learning areas and modern socio-constructivism, albeit in iterative and not chronological, systematic phases. Ellis and Berry III (2005) confirm Maree's (2004) description with their account of the "history of revisions in mathematics education" (p. 7) in the United States of America. They describe the development of models for mathematics education through the course of the twentieth century, predominantly situated in the "procedural-formalist" or traditional paradigm (p. 11). The overview includes Thorndike's Stimulus-Response Bond Theory, the Progressive Movement, New Math and Back-to-Basics (pp. 7–10). The overview concludes with reference to the emergence of the "cognitive-cultural" paradigm (pp. 12, 13) in which the constructivist and socio-constructivist theories are located. In the period during which the research was undertaken, the outcomes-based approach to teaching and learning, epistemologically underpinned by socio-constructivism, was the official and proclaimed pedagogy in the South African curriculum documents (DoE, 2002; Maree, 2004).

## 2.2.2 Behaviourism

### *Learning*

The behaviouristic approach to teaching and learning corresponds to a mechanistic worldview in which learners are epitomized as sedentary, uninvolved and blank. Learning is regarded as an overt and measurable change in behaviour (Askew & Carnell, 1998).

### *Learners' Mathematical Misconceptions and Errors*

From a behaviouristic perspective, knowledge is transmitted and conveyed without distortion or personal interpretation. Learners' existing knowledge or conceptions are regarded as insignificant and unrelated to the acquisition of new knowledge. In a behaviourist view, new information is superimposed on current knowledge. Behaviourists thus subscribe to the replacement of erroneous conceptions by teaching correct procedures (Olivier, 1992).

### *Teaching*

Behaviourism embraces the Functionalist model of education in which the teacher retains the authority, based on the supremacy of his or her knowledge, that is transmitted to receptive and novice learners (Askew & Carnell, 1998). Behaviouristic stimulus-response theories of learning underpin the pedagogical inclination towards reinforcing recently acquired information with contiguous (Gage & Berliner, 1998) and numerous examples, based on a stepwise approach and a gradual increase in the level of difficulty. A sequential teaching approach, underpinned by "Gagné's information processing model" will probably be observed in a behaviouristic classroom environment (Fraser, Loubser & Van Rooy, 1993, p. 43).

## 2.2.3 Constructivism

Although I take cognisance of the existence of different schools of thought relating to the psychological and sociological aspects of mathematics education (Cobb & Yackel, 1998; Ernest, 1994), it is beyond the scope of my study to execute a more profound review thereof. I concur with Brodie (2005), Cobb and Bowers (1999), Cobb and Yackel (1998) and Sfard (1998) in the proclamation of a dual and complementary perspective on mathematics education. Ernest (1994) describes this position as "complementarist" (p. 66) and expresses apprehension about a possible disregard of the sociological perspective and

the role of language (Lerman, 1994). I recognize the ambivalence in my study between learners' mathematical errors, residing in constructivism and mathematics teachers' interactions with learners' mathematical errors, located in socio-constructivism. However, I prefer to discuss the two theories separately, albeit acknowledging their significance as equivalent in the literature review.

For the purposes of my study, the "interactionist" perspective (Bauersfeld, 1994) will be pragmatically interpreted as socio-constructivist, on the premise of a number of equivalents. These equivalents are "the teacher's and students' interactive constitution of the classroom microculture" (Bauersfeld, 1994; Cobb & Yackel, 1998, p. 160), the fundamental role that classroom interaction (Bauersfeld, 1994; Cobb & Yackel, 1998) has to play in this study and the primary focus of the study on the pedagogical role of the teacher (Bauersfeld, 1994). My interpretation of the interactionist perspective as socio-constructivist is concurring with that of Ernest (1994) in his paper on the foundation and the essence of socio-constructivism. Pertaining to this study, the terms *socio-constructivism* and *social constructivism* are accepted as synonymous.

### ***Learning***

Learning, from a constructive perspective, is said to be an idiosyncratic process of interpretation and sense-making of experiences and information (Gatt & Vella, 2003). Although learning is influenced by the content taught to learners, the learning cannot be controlled (Olivier, 1992). New concepts that learners encounter during classroom instruction are interpreted in terms of existing knowledge structures (Olivier, 1992; Resnick, et al., 1989; Smith, et al., 1993).

Piaget and Kelly respectively, probably originated the concept of dynamic and evolving knowledge structures, albeit dissimilarities in their theories are recognized (Gatt & Vella, 2003). A compilation of consistent ideas is referred to as a mental schema, a cognitive device in the memory, which can be reinstated and intellectually employed (Olivier, 1992). These cognitive or mental knowledge structures or schemas are designed to process information through assimilation or accommodation with concurrent developmental maturation and complexity (Ernest, 1994; Gatt & Vella 2003; Olivier, 1992). Learning, from a constructivist perspective, is the combined and reciprocal action of a learner's mental schemas and newly acquired information. This happens through processes of integration or assimilation of information into mental schemas and

reconstruction of the mental schemas. This results in the accommodation of information (Olivier, 1992). From a constructivist perspective, learning results in adaptations to the mental structures (ibid.).

Learning, interpreted from a constructive perspective, would be impossible without the prior knowledge that learners have (Smith, et al., 1993). Learners dynamically participate in their own learning through interpretation, organisation and arrangement of the information into existing compilations of consistent ideas (Olivier, 1992). In addition to recognizing prior knowledge as a cognitive resource for the development of more sophisticated knowledge structures, there is existing evidence that “complex knowledge systems” (p. 148) encompass initial, less advanced structures (Smith, et al., 1993). Within a constructivist framework for learning, the prior knowledge that learners possess is a viable basis for the maturation of intricate systems of knowledge. The resultant, adequate knowledge structure can only emerge from some prior, embryonic structure. The learning process hence corresponds to the continuing refinement and sophistication of knowledge structures (ibid.).

### ***Learners’ Mathematical Misconceptions and Errors***

Constructivism, Piaget’s constructivism in particular, elucidates the occurrence of learners’ mathematical errors (Ernest, 1994; Olivier, 1992). According to Olivier (1992) errors are methodical, occur regularly and are symptomatic of misconceptions, caused by mental schemas that are absent, immature, inappropriate or irretrievable (Olivier, 1992). Hence, it is an oversimplification to merely categorize a conception as correct or erroneous. To understand why an error has been made, is of much more importance (Smith, et al., 1993).

The preliminary stages of learning, as previously discussed, are often characterised by errors due to the crudeness of existing knowledge structures and the inappropriate context in which these are applied (Hartnett & Gelman, 1998; Olivier 1992; Smith, et al., 1993). Learning is viewed as a process of modification or “refinement and reorganization” (Smith, et al., 1993, p. 116; Tobin & McRobbie, 1999) and not of substituting particular chunks of knowledge with more appropriate ones. Knowledge is much more complex than that implied by the replacement model (ibid.).

From a constructivist perspective, learners’ errors play a fundamental role in the learning process. Cognisance of this principle should preferably be taken in the teaching of

mathematics and expectantly prompt adjustments to teaching practice (Leu & Wu, 2005; Olivier, 1992; Santagata, 2005). Consequently, from a learner perspective, avoiding, or, from a teacher perspective, ignoring errors is thus incompatible with a constructivist framework for learning (Leu & Wu, 2005; Santagata, 2005; Smith, et al., 1993). Although learners' developmental knowledge structures may be immature, recognition of the potential productiveness of these structures is recommended (Brodie, 2005; Resnick, et al. 1989; Smith, et al., 1993). Flexibility in adapting teaching practice in order to facilitate the restructuring of learners' mental structures is desirable in a constructivist environment (Palincsar & Brown, 1984).

Although the terms *misconceptions*, *mistakes* or *errors* are used in the literature, (Borasi, 1996; Brodie, 2005; Heinze, 2005; Leu & Wu, 2005; Resnick, et al. 1989; Olivier, 1992; Santagata, 2005; Smith, et al., 1993; Ya-amphan & Bishop, 2004) emphasis should preferably be placed on the similarities between novice and expert knowledge, rather than on the inconsistencies (Smith, et al., 1993). It is hence proposed that learners' developmental knowledge structures be acknowledged as immature, but potentially productive (Brodie, 2005; Resnick, et al. 1989; Smith, et al., 1993).

### ***Teaching***

Constructivism is a theory of learning and not of teaching. Hence, to distinguish a distinct constructivist approach to teaching is illusory. However, expository teaching through the transmission of knowledge is in conflict with constructivist views on learning (Gatt & Vella, 2003; Olivier, 1992). Congruent to the preceding discussion on learning, a constructivist viewpoint on teaching would entail the creation of appropriate opportunities for learners to construct personal knowledge and to simultaneously confirm the correspondence of learners' personal knowledge to knowledge that is commonly acknowledged (Gatt & Vella, 2003).

For the reason that errors are a normal part of the process of internalising new information in existing mental structures (Hartnett & Gelman, 1998; Olivier, 1992; Smith, et al., 1993), the confrontation and replacement of learners' misconceptions through instructional methods are not concurrent with a constructivist approach to learning (Leu & Wu, 2005; Santagata, 2005; Smith, et al., 1993). The theory of advanced knowledge structures maturing from existing ones refutes the replacement model on the basis of an inability to provide an explanation for such an existing structure from which a more productive one

could evolve (Smith, et al., 1993). The theory of available and viable cognitive resources thus counteracts the claims of confronting and replacing misconceptions (ibid.). The confrontation and replacement of learners' misconceptions through instructional methods, during a social process that commences externally in the classroom, correspond to a behaviouristic approach to learning and are thus incompatible with a constructivist framework for learning (Leu & Wu, 2005; Santagata, 2005; Smith, et al., 1993). Probing to understand why an error has been made, is of much more importance and pedagogical value than merely categorizing a learner's contribution as correct or erroneous (Smith, et al., 1993). In addition to the previous argument, confrontation is inconsistent with socio-constructivism on the premise that negotiation is seriously challenged by portraying to learners that their contributions are faulty in principle (Noddings, 1990; Smith, et al., 1993; Szydlik, et al., 2003).

When teachers take cognisance of learners' errors and are sufficiently flexible to be able to adapt their teaching accordingly, teachers can facilitate the restructuring of learners' mental structures (Palincsar & Brown, 1984). Once classroom circumstances are favourable for learners to expose the cognitive frameworks that support their understanding, learners are enabled to reflect on these frameworks in an evaluative way and reconcile alternative conceptions (Tobin & McRobbie, 1999). Melis (2005), in resonance with Santagata (2005), discusses the importance of meta-cognitive skills in the restructuring of existing cognitive structures or prior knowledge. She refers to the connection between identifying and correcting errors and the stimulation of meta-cognitive activities. Meta-cognitive knowledge is the capability to assess and to monitor one's own learning (Hooper & Hokanson, 2000), the conscious and reflective management of one's own learning through planning, executing, monitoring and assessing own learning (Slabbert, De Kock & Hattingh, 2009).

Teachers who maintain practices to create constructivist classroom environments (Beswick, 2005) will give learners opportunities to give feedback on their understanding of errors or will provide explanations of what is wrong. Learner contributions will not merely be categorized as right or wrong, but allowance will be made for alternative solutions and different strategies will be acknowledged (Brodie, 2008; Szydlik, et al., 2003). Teachers maintaining these practices will manage errors in ways that involve high-level reasoning processes and will elaborate significantly on the errors (Santagata, 2005). Such teachers



will seriously attempt to interpret learners' errors correctly (Leu & Wu; 2005; Olivier, 1992).

Presenting learners with frequent and prevalent erroneous statements or procedures for their evaluation, without identifying the statement or procedure as fallacious per se, can develop learners' cognitive skills (Ruggiero, 1988). Interactions with learners' errors, conducive to enhancing their cognitive abilities, entail the following (ibid.):

- The learner is able to explain his or her approach or result.
- The learner is challenged to consider alternative approaches.
- Peers are consulted for their opinion.
- The learner is challenged with a question related to the origin of the error.
- The learner is obliged to clarify his or her thinking in greater detail.

#### **2.2.4 Socio-Constructivism**

The process of learning, from a socio-constructivist perspective, is fundamentally social (Cazden & Beck, 2003; Cobb, et al., 1992). Kaldrimidou, Sakonidis and Tzekaki (2004) indicate the social dimension of the classroom context in which constructed mathematical knowledge is discussed, compromised and agreed upon among the community of learners and teacher. This social dimension of the classroom context encompasses pedagogy, language, culture and interpersonal exchanges (Ernest, 1994). Language is instrumental in the social construction of knowledge and the process of sense-making (Tobin & McRobbie, 1999). Speech is interrelated with thinking in the human developmental process and is paramount in higher order and reflective thought, especially in adults. In the case of the more mature child, speech fulfils a preliminary function to thought and practice (Vygotsky, 1978). The importance of classroom discourse aligns with the views of Bauersfeld (1994), Ernest (1994) and Vygotsky (1978) that meaning is constructed through the use of words and symbols and speech is interrelated with thinking.

Discourse in a socio-constructivist classroom environment involves all members of the classroom community as collaborators in argumentation (Adler, Davis, Kazima, Parker & Webb, 2005; Cazden & Beck, 2003; Santagata, 2005; Tobin & McRobbie, 1999), facilitates reflection on errors and has negotiation as an outcome (Clements & Battista, 1990). The teacher does not play the role of the validating

authority who possesses all the knowledge, but as a member of the classroom community, the role of a participant in actual enquiry, through questioning (Mason, 2000). Acknowledging learners as members of the classroom community, their contributions are valued and accommodated in a socio-constructivist environment in which negotiation is engendered. An inclination towards the public discussion of errors exists (Heinze, 2005; Santagata, 2005). The way in which learners' contributions are approached and evaluated should preferably convey the potential productiveness (Brodie, 2005; Resnick, et al. 1989; Smith, et al., 1993) thereof to learners (Noddings, 1990; Smith, et al., 1993; Szydlik, et al., 2003; Tobin & McRobbie, 1999).

### ***Learning***

Learning, from an interactionist perspective, is a personal, interactive, participatory process of cultural adaptation (Bauersfeld, 1994). Internal representations, although these are individual or personal concepts, transpire through social interaction (ibid.). The focus of learning is altered from an individual concern to a process that is predominantly social (Cazden & Beck, 2003). Such an interactionist view portrays the duality or mutual inclusiveness of individual transformation through involvement in social interaction and the achievement of irreversible alterations in the social consistencies through personal involvement (Bauersfeld, 1994).

### ***Teaching***

In a socio-constructivist environment, the teacher's role during classroom interaction can be described as negotiating learners' mathematical conceptions and the outcomes learners are expected to reach, as envisioned by the teacher or described by the curriculum (Adler, et al., 2005). The teacher is ideally not providing information through transmission during classroom communication, but is participating as a "meta-cognitive coach" (Morine-Dershimer & Kent, 1999, p. 32). When learners actively participate as partners in a community of practice, the teacher plays the role of a supportive and specialist collaborator who elicits ideas, "scaffolds" (Magnusson, et al., 1999, p. 29; Wood, et al., 1976, p. 90) learners' thinking, supervises learners' interpretations, rephrases questions, encourages cooperative responsibility and orchestrates the course of the interaction (Kovalainen & Kumpulainen, 2007; Maree, 2004).

Rather than the transmission of knowledge, teaching in a socio-constructivist framework is perceived as a process of creating and maintaining a learning environment in which this interactive process is regulated (Bauersfeld, 1994; Tobin & McRobbie, 1999). As a member of the classroom community, the teacher participates in actual enquiry through questioning (Mason, 2000). A supportive and collective disposition will thus be engendered among all the members of the classroom community within an environment where mathematical thinking is encouraged (ibid.).

Socio-constructivist classrooms will be hallmarked by a lack of predictability, prominent learner participation and exploration through enquiry (Davis, 1997). Such a learning environment will be characteristic of a supportive and collective disposition among all the members of the classroom community while mathematical thinking is encouraged (Mason, 2000). A tolerance of errors, as a prerequisite for the pedagogical utilization of errors in the mathematics classroom, seems possible in such an atmosphere (Olivier, 1992).

### **2.2.5 Constructivism and Socio-constructivism**

The duality between constructivism and socio-constructivism is positively elaborated on in Brodie's (2005) discussion of "cognitive and situative perspectives" (p. 177) on learning. Sfard (1998), Cobb and Yackel (1998) as well as Cobb and Bowers (1999) are unanimous and unequivocal in their claim that learning is best understood from an integrative perspective of the two theories. Consequently, it is possible to apply both metaphors for learning, that of acquisition and of participation, as complementary to the understanding of the process of learning (Sfard, 1998). Knowledge is acquired through learning, irrespective of how learning is epistemologically perceived. The acquisition metaphor corresponds to a constructivist, a cognitive or a psychological perspective of personal knowledge construction (Brodie, 2005). However, learning through active participation within a social context is widely acknowledged (Bauersfeld, 1994; Ernest, 1994; Sfard, 1998). The participatory metaphor concurs with a socio-constructivist, a situative or a sociological perspective of learning through social interaction (Brodie, 2005). The statement: "A situative view is in fact an expanded cognitive view." (Greeno & MMAP, 1998, as cited in Brodie, 2005, p. 178) concisely depicts the connection. Learners' individual conceptual structures determine and steer their community-related contributions. A learner's cognitive structures are part-and-parcel of

the learner's participation in the community and will be "imposed" (p. 178) on the environment (Brodie, 2005).

Learning, from a cognitive view, predominantly takes place through a process of "self-regulation" (Brodie, 2005, p. 177). Concurrently, teaching will be aimed at creating the opportunities and environment for learners to individually acquire knowledge (Brodie, 2005). Noddings (1990) refers to the creation of a "mathematical environment" (p. 15) in which the teaching is focused on revealing learners' errors during teacher-learner interactions. These interactions entail explicit verbal descriptions of learners' thinking; it is a matter of *thinking-out-loud*. The process of writing in mathematics can have similar pedagogical value in developing meta-cognitive skills or self-regulation (Abel & Abel, 1988). Sound pedagogical principles thus involve knowledge of learners' thinking, identification of learners' errors and the facilitated process of correcting these errors (Noddings, 1990). Beswick (2005) is in agreement with Noddings (1990) when accentuating the importance of teachers' determined attempts to understand learners' [erroneous] mathematical constructions. Effective teaching will mean facilitating "cognitive restructuring" (p. 44) through the designing of appropriate learning experiences (Beswick, 2005). From a situative view, however, learning takes place through interacting with and participating in the classroom community. Teaching will entail the creation of such opportunities through posing appropriate problems to learners (Bauersfeld, 1994) and facilitating discussion effectively (Brodie, 2005).

Ernest (1994) prefers to promulgate and subscribe to a social constructivist theory of learning mathematics that implicitly encompasses both theories, mutually accounting for idiosyncratic mathematical knowledge construction and the social components of pedagogy, language, culture and interpersonal exchanges. This evolving, alternative theory is proposed to address the predicament of the constructivist metaphor that depicts the learner in isolation of the context and the community. The learning theory implies an indestructible cohesiveness among individual members of the classroom community, their social context and their development.

## **2.3 MATHEMATICS TEACHERS**

### **2.3.1 Teacher-Learner Interactions and Classroom Discourse**

Classroom discussion is paramount in the learning process, on condition that the discussion facilitates reflection on learners' existing knowledge with consequent refinement of current knowledge structures as the outcome (Clements & Battista, 1990; Smith, et al., 1993). Classroom discussions can play a fundamental role in developing learners' cognitive skills, on condition that teachers' questions compel learners to explicate, elucidate and endorse their personal understandings and that teachers listen to these appropriately (Ruggiero, 1988). In contrast to this ideal, the main focus of classroom communication, from the teacher's perspective, is often successful problem-solving, perceived as finding the correct solution to a problem (Martens, 1992).

In classrooms where learners are co-participating in conversation, a common discourse, which serves as a connection between the learners' language and professional language, can be recognized. If learners are not participating, however, two parallel discourses with little, if any, intersection, can be identified (Tobin & McRobbie, 1999). When learners are unable to appropriate the discourse, instrumental understanding (Skemp, 2006) will probably be inevitable and errors and misconceptions will be neglected (Tobin & McRobbie, 1999). Co-participation will only be realized when the power does not reside with the teacher alone and the teacher does not engage in a continuous "monologue" (ibid., p. 233). Tobin and McRobbie (1999) regard the creation of such a common discourse as the fundamental nature of PCK.

In classrooms where co-participation is not encouraged, avoidance of answering teachers' questions or answering questions incorrectly will probably prompt teachers to provide the correct answers without an opportunity for learners to resolve their lack of understanding or cognitive inconsistencies (Tobin & McRobbie, 1999). Reliance on algorithms and instrumental understanding may be plausible outcomes (Skemp, 2006; Tobin & McRobbie, 1999).

Socio-constructivist roles a teacher plays during teacher-learner interactions are that of negotiator (Adler, et al., 2005), interpreter and mediator (Ball, 2000) and supportive and specialist collaborator (Kovalainen & Kumpulainen, 2007). The role of an efficient teacher in the classroom community is to actively involve all learners and to orchestrate the

discourse to evolve from a common, accessible language to a more professional one (Tobin & McRobbie, 1999). Pedagogical knowledge encompasses an appreciation and acknowledgement of the importance of classroom discourse (Morine-Dershimer & Kent, 1999). Ideally, the nature of classroom discourse is described as explanatory and argumentative, while learners are actively involved as collaborators. Classroom discourse of this nature will encourage and sustain higher order thinking (Cazden & Beck, 2003; Santagata, 2005). Learners' meta-cognitive abilities are developed and refined when learners have opportunities to verbalize their own thoughts and understandings (Gatt & Vella, 2003; Ruggiero, 1988).

Opportunities for addressing contradictions in learners' thinking are created during discussions in which learners co-participate (Tobin & McRobbie, 1999). These discussions of learners' errors create favourable conditions to clarify learners' misconceptions and concurrently develop the skills to reflect (Leu & Wu, 2005). This implies a deviation from the confrontation and replacement model (Smith, et al., 1993). It is preferable for teachers to refrain from immediate evaluation (Mehan, 1979) of learner contributions, to elicit additional responses from learners and to involve peers in the discussion (Cazden & Beck, 2003; Szydlik, et al., 2003). Instead of providing correct answers when learners' errors occur, learners would benefit from being challenged to present proof for their solutions (Ruggiero, 1988; Tobin & McRobbie, 1999).

Classroom discourse in reform-oriented or socio-constructivist (Ellis & Berry III, 2005; Wood & Sellers, 1996) classrooms is expected to deviate from traditional patterns, in mathematics classrooms in particular (Cazden & Beck, 2003). An "asymmetry" (Cazden & Beck, 2003, p. 176) between the rights to speak, when and to whom, exists in the traditional classroom. The omnipresent pattern of classroom discourse in traditional classrooms is that of the teacher instigating the exchange, the learner providing a reply, which is assessed or warranted by the teacher (Mehan, 1979). In contrast to this, learners and teachers co-participating in discourse during the introduction of new topics create a climate conducive to teachers assisting learners in reorganizing extant cognitive structures to accommodate new knowledge (Tobin & McRobbie, 1999). Learners' co-participation in the classroom events provides teachers with opportunities to evaluate learners' understandings and is thus indispensable in exposing learner misconceptions (ibid.). However, more subdued, less responsive learners may result in an increase in teacher talk distinguished by a higher incidence of questions and, particularly, closed questions.

Hence, a potential strategy to consider during classroom interaction with more passive groups is the technique of nominating, which is when a learner is called upon by name to answer a question (Hargreaves, 1984).

### **2.3.2 Teacher Questioning and Listening**

Although non-traditional classroom discourse still encompasses questioning by teachers, the discourse pattern differs (Cazden & Beck, 2003). Asking a question may have one of three purposes (Mason, 2000, p. 103):

- To focus a learner's attention (Hargreaves, 1984).
- To assess a learner's knowledge or understanding.
- To enquire.

“... but rather, all questions could be seen as an attempt to provoke students into making sense, or put more prosaically, provoking students into constructing their own stories” (Mason, 2000, p. 107). Teacher questioning is preferably aimed at facilitating learners' reflective thinking and subsequent refinement of their knowledge structures (Clements & Battista, 1990; Smith, et al., 1993). The nature of appropriate teacher questions, in order to enhance learning, is open-ended and meta-cognitive and not focused on a fixed, predetermined response (Cazden & Beck, 2003; Mason, 2000). These questions are functional in learners' mathematical understanding (Mason, 2000).

Relevant and appropriate teacher questions and remarks are utilized (Adler, et al., 2005) to coordinate the course of the socio-constructivist classroom interaction (Kovalainen & Kumpulainen, 2007; Maree, 2004). A teacher's questioning skills involve the teacher's ability to pose a specific kind of question in an appropriate context and are paramount to the quality of learning outcomes that is reached during classroom interaction (Hargreaves, 1984).

Probing learners' thinking and the reasons for their thinking challenges teachers' questioning skills (Smith, 1999). In order for the teacher to prepare relevant analytical questions with which to interact with learners' errors, it is paramount for the teacher to understand these errors and misconceptions. In addition to the preceding condition, an ability to recognize potential learning opportunities during classroom interaction is an imperative pedagogical skill (ibid.).

Questions asked during classroom interaction can be categorized by distinguishing between factual questions and interpretive questions. Factual questions regularly only have one correct response and can thus be regarded as closed questions. Interpretive questions are typically open questions. Half-open questions are those questions that can usually be answered with a yes or a no response with the option to elaborate or justify further without an additional incentive by the teacher. In incidents where learners persistently respond to half-open questions with an abrupt reply, the option to deliberately follow up with open or interpretive questions is recommended (Hargreaves, 1984, pp. 46–49).

Listening in non-traditional classrooms, the essence of what is heard and thus understood from verbal contributions, is paramount (Cazden & Beck, 2003, Davis, 1997). Ball (2000) links PCK to what she calls “flexible hearing” (p. 243) of learner contributions while Davis (1997) links socio-constructivist teaching practices to what he refers to as “hermeneutic listening” (p. 369). Cazden and Beck (2003) are in resonance with Davis (1997) in accentuating the importance of listening in non-traditional classrooms. The ways in which teachers possibly listen to learner contributions during classroom interaction can be represented as (Davis, 1997, pp. 357, 361, 365):

- evaluative listening;
- interpretive listening; and
- hermeneutic listening.

The ways in which teachers listen to learner contributions in non-traditional classrooms will be distinguished by the following principles (Davis, 1997):

- Questions are open and not posed with the purpose of eliciting responses; teachers are probing learner’s conceptions and understanding.
- Preconceived answers are not anticipated; the listening is thus uninhibited.
- The teacher listens in a participatory way; authority is thus not maintained by the teacher.
- In listening, the teacher is actively making sense of learner contributions, interpreting and paraphrasing these.
- Learners determine the flow and structure of the learning period which can thus be described as dynamic and dependent on the context.



When a teacher is listening in anticipation of a preconceived answer, the listening can be described as “evaluative” (Davis, 1997, p. 359). This kind of listening is thus inhibited by the teacher’s expectation of a model answer. Not only is the act of listening inhibited, but so is the potential for learner contributions. By listening in an evaluative manner, a teacher maintains his or her own authority in the classroom. The corresponding teaching approach will be characterized by “unambiguous explanations and well-structured lessons” (pp. 360, 363) and will be expository and exemplary of transmission. Constructivist principles for learning will not be served by this approach. A measure of evaluative listening can be found in the degree to which a particular group of learners orchestrates the flow and the structure of the learning period. If the structure and the content of the lesson are independent of the context, the teacher probably listens in an evaluative way. The purpose of the questions, asked by the teacher who listens in an evaluative way, will usually be to elicit responses, as predetermined by the teacher.

Interpretive listening is characterized by an increase in learner participation. A teacher who listens interpretively will ask questions for the purpose of attaining information about learner’s conceptions and understanding. The responses to these questions will be more intricate and will reveal information about the way in which learners are making sense of the mathematical concepts. It will be difficult for the teacher who listens interpretively to anticipate the responses to questions posed by him or her and to pay attention to these responses will be demanding. During interpretive listening, the teacher is diverting from responding to learner contributions as either correct or incorrect and meaning is socially constructed to an extent. Interpretive listening is an active process of sense-making, of “reaching out” (ibid., p. 364) to learner contributions with the teacher continuously and subjectively interpreting and paraphrasing. Davis (1997, p. 364) concludes that ...

*it thus seems reasonable to suggest that the important distinguishing characteristic between conventional and constructivism-informed teaching is not to be found in the way the teacher speaks or structures her lessons (i.e., in the visible), but in the manner in which he or she listens (i.e., in the invisible).*

Although interpretive listening corresponds to constructivist principles for learning, the authority or expertise still resides with the teacher.

In a classroom community where the teacher listens hermeneutically, a predictable lesson structure is absent. Learners explore a mathematical topic through a process or inquiry.

The main purpose of a learning period is not the mastering of objective parts of knowledge. The knowledge is socially constructed through the process of inquiry and is dependent on the learners and their context. A teacher who listens hermeneutically plays a participatory role in the knowledge construction as a member of the classroom community. The teacher has the responsibility to create opportunities for learners to participate in and contribute to classroom discussion. A learning period will evolve from being a “complicated coordination” to a “complex dynamic” (ibid., p. 370).

### **2.3.3 Instructional Scaffolding and the Zone of Proximal Development**

Vygotsky (1978, p. 86) defines the ZPD as “the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers”. The “actual developmental level” (ibid., pp. 85, 86) is a product of mental maturation and prior learning.

The process of “scaffolding” (Wood, et al., 1976, p. 90) is used to assist learners in making correct sense of mathematical concepts. “Instructional scaffolds” (Magnusson, et al., 1999, p. 29) are not only provided by an expert, the teacher, in order for learners to reach outcomes, but to engender strategic thinking. The questions a teacher asks during classroom interaction can play a paramount role in the sense learners make of mathematics. Classroom interaction, as realized by a teacher’s questions, provides the social context referred to by Vygotsky in relation to the ZPD (Mason, 2000, p. 97). The nature of such learning is thus social (Vygotsky, 1978).

For the purposes of this study, “instructional scaffolding” (Magnusson, et al., 1999, p. 29; Wood, et al., 1976, p. 90) can be described as a learning process during which a more capable person, the teacher, through interaction by means of questions, enables someone less capable, a learner, to reach a goal: the goal being the modification or maturation of knowledge systems that may currently be a source of errors (Smith, et al., 1993). Interaction with those that are more capable is established through questioning or exposition (Vygotsky, 1978). The success of the scaffolding process is premised on the learner’s understanding of his or her misconception or the inappropriateness of the context (Smith, et al., 1993).

During the scaffolding process, teachers preferably need to make use of meta-cognitive questions in order to focus learners' attention on their own thinking (Mason, 2000). If the process of scaffolding is too explicit, learners can develop a dependence on the teacher. The final outcome of the process of scaffolding is fading: a phase that corresponds to independent and reflective thinking on the part of the learner. Scaffolding can therefore be described as the appropriate and temporary support of learners' thinking while teachers refrain from the mere transmission of knowledge (Mason, 2000). Instructional scaffolding is thus provided temporarily in order for learners to become autonomous (Magnusson, et al., 1999).

To maintain the constructivist ideals of higher order and reflective thinking (Cazden & Beck, 2003; Clements & Battista, 1990; Santagata, 2005) and autonomous thinking (Ernest, 1988; Kaldrimidou, et al., 2004), the outcomes of scaffolding (Magnusson, et al., 1999; Wood, et al., 1976) are expected to be self-regulation (Meyer & Turner, 2002) and independence (Mason, 2000). A learning atmosphere conducive to reaching these outcomes is identified by high-level teacher questions and teachers refraining from expository teaching and the teaching of rules (Meyer & Turner, 2002) that will result in instrumental understanding (Skemp, 2006). The primary focus of scaffolding learners' mental processes is the analyses of the learners' errors and not to arrive at the correct answers (Santagata, 2005). Analyses of learners' errors will ideally result in an understanding of the error or the inappropriateness of the context that resulted in the error (Smith, et al., 1993).

#### **2.3.4 Assessment**

Assessment potentially retains a dual purpose of establishing learners' results regarding academic performance or of ascertaining to what extent pedagogical approaches should preferably be adjusted in order to address learners' disparities in their mathematical understanding (Popham, 2007). The author is unambiguous in his claim that the pedagogical value of assessment *for* learning (p. 271) is exceeding that of assessment *of* learning (p. 271) by far.

Popham (2007) and Stiggins (2005) are unanimous in their portrayal of formative assessment as a process during which learning is enhanced through appropriate pedagogical decisions teachers make, based on information regarding learners' academic progress or levels of mathematical understanding. Popham (2007), in particular,

emphasises an appropriate and deliberate adjustment in teaching approach. Instructional decisions teachers make, should ideally be effected and refined by information the teacher obtains from assessments (Popham, 2007). The key elements of Popham's and Stiggins' definitions, impacting on this study, are the following:

- Information regarding learners' mathematical understanding, or for the purposes of this study, incomplete understanding or misunderstanding, is pedagogically utilized.
- The teachers' interactions with learners' mathematical errors are reactive and, during classroom interactions, instantaneous, albeit not when assessment is formal and written.
- The ways in which teachers deal with learners' incomplete understanding or misunderstanding result in the improvement of their learning and in the refinement of the learners' knowledge structures (Palincsar & Brown, 1984; Smith, et al., 1993; Tobin & McRobbie, 1999).

Shepard (2005, p. 66), however, defines formative assessment as follows:

*Occurring in the midst of instruction, formative assessment is a dynamic process in which supportive adults or classmates help learners move from what they already know to what they are able to do next, using their zone of proximal development.*

Although Shepard (2005) perceives the concepts formative assessment and instructional scaffolding as similar, these will be interpreted as interrelated, but separate, processes in this study. From my perspective, the focus during formative assessment is on obtaining relevant information, to be analytically utilized by the teacher. The focus during instructional scaffolding is on enabling learners, through appropriate questioning informed by teachers' insight, gained through formative assessment, to realize concrete adaptations in learners' mental structures.

Assessment does not only inform decisions regarding instructional content, instruction time and effective instructional approaches (Popham, 2007). Assessment provides evidence of learners' existing "levels of understanding" (ibid., p. 273) which, for the purposes of this study, will be accepted to include misconceptions or immature developmental knowledge structures (Brodie, 2005; Resnick, et al. 1989; Smith, et al., 1993) as portrayed by the errors learners make. Hence, in resonance with this study, formative assessment should

preferably result (Popham, 2007) in continuing refinement and sophistication of learners' knowledge structures and reconciliation of alternative conceptions (Palincsar & Brown, 1984; Smith, et al., 1993; Tobin & McRobbie, 1999).

It is recommendable for teachers to be conscious of assessing formatively, even during informal classroom interactions. In order to maximize the full educational potential of formative assessment, learners are to be co-responsible in managing and evaluating their own learning. In addition to that, formative assessment should ideally have no connotation with achievement (Popham, 2007).

### **2.3.5 Teachers' Mathematical Beliefs**

*However, regardless of whether one calls teacher thinking beliefs, knowledge, conceptions, cognitions, views, or orientations, with all the subtlety these terms imply, or how they are assessed, e.g. by questionnaires (or other written means), interviews, or observations, the evidence is clear that teacher thinking influences what happens in classrooms, what teachers communicate to students, and what students ultimately learn.*  
(Wilson & Cooney, 2003, p. 144)

Borasi (1996), Ernest (1991) and Cross (2009), among several other authors, are in resonance with Wilson and Cooney (2003) when they assert that teachers' mathematical beliefs determine their teaching approaches and their perceptions of teaching, learning and of learners' mathematical errors. A teacher's mathematical beliefs are fundamental in the perceived teacher's role in the mathematics classroom, the prevalent teaching approach, learning activities that are employed and types of assessments.

Borasi (1996) classifies teachers' mathematical beliefs as located either in a transmission paradigm or within an inquiry framework, exemplified by the varying prominence assigned to "product" versus "process" and "teacher's explanations" versus "learners' constructions" (p. 24). The assumptions that mathematics is a neutral body of knowledge, that learning entails the gradual and accumulative reception of information and that teaching involves the transmission of information, demonstrated with unambiguous examples are in accord with the transmission paradigm or the behaviouristic view of learning. On the other hand, the assumptions that mathematics is a fallible human construction, that learning entails the construction of knowledge through contextual investigations and that teaching comprises the creation of opportunities and milieus favourable for learners' investigations, are located within the inquiry framework or the

constructivist view of learning. Two of the three sets of mathematical beliefs that Ernest (1988) describes, those pertaining to instrumentalist teachers and to Platonist teachers, correspond to beliefs located in a transmission paradigm. The mathematical beliefs of teachers holding a problem-solving view of mathematics, are concurrent with beliefs situated in an inquiry framework.

Teachers holding an instrumentalist view of mathematics (Ernest, 1988) will maintain a classroom culture in which learners are expected to be passive and accommodating and not be allowed to develop autonomy (Ernest, 1988; Kaldrimidou, et al., 2004). Negotiation through discussion among the members of the classroom community will probably be completely absent in these classrooms (Bauersfeld, 1994; Tobin & McRobbie, 1999). Such teachers will usually be in charge of deciding what is right or wrong (Heinze, 2005; Santagata, 2005) and will probably not allow for alternative approaches or solutions (Brodie, 2008). The prevention or elimination of learners' errors is typical in classrooms supportive of the transmission paradigm (Borasi, 1996). However, teachers who subscribe to a constructivist view of learning, recognize learners' errors as inherent to the learning process and important in assessing learning (ibid.). A teacher subscribing to instrumentalist views of mathematics will correct learners' errors immediately, privately and normatively without discussion or analysis thereof (Bauersfeld, 1994; Beswick, 2005; Brodie 2008; Ernest, 1988; Mehan, 1979). Instrumentalist teachers will probably avoid open, interpretive questions during classroom interaction (Hargreaves, 1984) and will rely on questions with a predetermined response (Cazden & Beck, 2003). Their classroom discourse patterns will most likely be abrupt and evaluative and assertively maintained (Cazden & Beck, 2003; Mehan, 1979) while an "asymmetry" (Cazden & Beck, 2003, p. 176) between opportunities and right to speak will probably be witnessed.

The way teachers interact with learners' errors is a source of understanding teachers' beliefs and teaching practices (Leu & Wu, 2005). The view one has of mathematics will further be revealed in the kinds of questions being posed to learners during discussions (Mason, 2000). The use of open, interpretive questions will be less likely in a classroom where roles are distinctly defined and the authority of the teacher is maintained purposively (Hargreaves, 1984).

Inconsistencies between teachers' professed beliefs about mathematics and their classroom practices are well reported (Barkatsas & Malone, 2005; Cornelius-White, 2007). Ernest (1988) refers to these as espoused beliefs and enacted beliefs teachers have. The demands of the discipline influence teachers' practice (Beswick, 2005; Ernest, 1988). According to a report by Barkatsas and Malone (2005) the majority of teachers profess to have "contemporary-constructivist" (p. 80) views of mathematics. Their beliefs, however, are often articulated theoretically, in isolation of the practical context and do not correspond to their classroom practices or the way they handle learners' errors. The teaching of mathematics, which mainly entails classroom practices, depends on certain key elements: a teacher's mathematical beliefs, the constraints and opportunities of the teaching situation and the ability to reflect and the level thereof (Ernest, 1988). The situation in the schools regarding time constraints, curriculum demands, standardised examinations and expectations the school and the parents have of achievement result in the acceptance of the absolutist nature of mathematics, irrespective of the teachers' beliefs (Op't Eynde & De Corte, 2003). Curriculum reform necessitates teachers to alter their practices, but these changes are superficial (Leu & Wu, 2005). When teachers are under pressure, they will fall back to the transmission model and focus on the acquisition of mathematical knowledge and computational skills. Errors are then not usually analyzed or discussed, but rather corrected immediately and privately. According to Beswick (2005) very few teachers maintain practices to create constructivist classroom environments.

### **2.3.6 Pedagogical Content Knowledge (PCK)**

Specialist and proficient content knowledge alone does not account for the full spectrum of knowledge teachers are expected to have in order to fulfil all the roles as mathematics teachers (Bromme, 1994). Despite adequate understanding of mathematical content, teachers often do not know the content appropriately in order to fulfil their roles as interpreter and mediator of learner contributions (Ball, 2000). PCK encompasses knowledge of conceptions and preconceptions learners have or should have of particular (mathematical) content and an understanding of how these conceptions can easily be misunderstood by learners (Shulman, 1986). A teacher's PCK incorporates the orientation the teacher has towards teaching the particular learning area, which in turn will affect the teacher's perception of possible difficulties learners may have. The teacher's understanding and knowledge of common errors learners make are implicit (Magnusson, et al., 1999).

PCK is equivalent to knowledge appropriate to effectively address learners' mathematical errors (Ball, 2000; Shulman, 1986). Therefore, poor PCK, often associated with lower levels of conceptual understanding (Van Driel, Verloop & De Vos, 1998), can result in the reinforcement of learners' misconceptions through inappropriate forms of reasoning that teachers use to explain concepts to learners (Halim & Mohd.Meerah, 2002). Magnusson, et al. (1999) are in resonance with Bromme (1994) in claiming that teachers often do not possess the appropriate knowledge that enables them to address the difficulties learners have. A poor quality PCK can be an explanation for teachers' unawareness of learners' errors or their inability or negligence to address these (Magnusson, Borke, Krajcik & Layman, 1994, as cited in Magnusson, et al., 1999).

Pedagogical knowledge encompasses an appreciation and acknowledgement of the importance of classroom discourse (Morine-Dershimer & Kent, 1999). The ability to interpret, examine and evaluate learners' mathematical constructions is an integral component of "mathematics for teaching" (Adler, et al., 2005, p. 1). Ball (2000, p. 245) describes this aspect as follows:

*Being able to see and hear from someone else's perspective, to make sense of a student's apparent error or appreciate a student's unconventionally expressed insight requires this special capacity to unpack one's own highly compressed understandings that are the hallmark of expert knowledge.*

In order to reach the goal of negotiating learners' mathematical conceptions and the outcomes learners are expected to reach, relevant and appropriate teacher questions and remarks are utilized (Adler, et al., 2005). To "hear flexibly" (Ball, 2000, p. 243) entails processes of attentive listening, of interpretation or sense-making of learner contributions through questioning, or scaffolding (Wood, et al., 1976) or providing hints, and of observation (Ball, 2000).

Although no general, exact definition of PCK exists among scholars globally: all scholars are unanimous in acknowledging teachers' knowledge of learners' difficulties with learning and misconceptions of particular topics as a fundamental component of PCK (Van Driel, et al., 1998). PCK enables teachers to address learners' misconceptions effectively in order to result in learners' conceptual reorganizing of their knowledge (Shulman, 1986).



## 2.4 CONCEPTUAL FRAMEWORK

### 2.4.1 Development of Conceptual Framework

Subsequent to an analysis, analogous to the process previously elaborated on in this chapter, *mathematics teachers* and *learners' mathematical errors* were identified as instrumental and paramount in this research project. The two phenomena and their interrelations are embedded in the *teaching and learning of mathematics*.

Considering the prominence assigned to learners' mathematical errors in this study and taking cognisance of the irrefutable recognition, from a constructivist perspective, of the fundamental role learners' errors play in the process of learning mathematics (Leu & Wu, 2005; Olivier, 1992; Santagata, 2005), my assumptions, as a researcher, are dominantly constructivist. The theories of constructivism and socio-constructivism underpin this study and constitute the point of reference from which exploration and investigation have been executed and interpretation and understanding have been attained.

The development of a theoretical or conceptual framework for a case study design, primarily from the existing literature base, is paramount, irrespective of the exploratory nature of the design. Not only does the theory direct and regulate the collection of appropriate data and the selection of compatible data analysis strategies, but provides a theoretical platform for the analytic generalization of the research results (Yin, 2003). Based on the three secondary research questions and informed by the literature, *mathematics teachers* were portrayed in terms of the teachers' professed beliefs about mathematics, about learners' mathematical errors and about the role errors can play in the teaching and learning of mathematics, collectively referred to as their mathematical beliefs in the conceptual framework, their observed teaching approaches and their interactions with learners' errors. In an attempt to acquire an understanding of teachers' interaction with learners' errors, teachers were classified (Ernest, 1988) in terms of their professed mathematical beliefs<sup>1</sup> and aspects of their observed teaching approaches, including their interactions with learners' errors. The purple arrow in the conceptual framework depicts this encompassment of teachers' interactions with learners' errors by their teaching

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<sup>1</sup> The construct *teachers' mathematical beliefs* in the conceptual framework is collectively used for teachers' professed beliefs about mathematics, about learners' mathematical errors and about the role errors can play in the teaching and learning of mathematics.

approach and indicates an alignment between the teaching approach and interactions with errors.

Teachers' prevailing classroom practices and the way they interact with learners' errors are determined by the beliefs or personal philosophies teachers have about mathematics (Bauersfeld, 1994; Ernest, 1988; Leu & Wu, 2005). It was thus expected that teachers' mathematical beliefs would affect their interaction through discourse, the nature or typology of their listening, the typology of their questions and the instructional scaffolding they provided, their feedback during assessment, the predominant classroom culture that was sustained and their interactions with learners' errors. The implication was hence that, while the teachers were described in terms of their observed and recorded teaching actions, these actions simultaneously served as a source of information from which their mathematical beliefs could deductively be outlined. It would thus be possible to make inferences about teachers' beliefs about mathematics from observed and recorded classroom practices that served as a source of understanding teachers' beliefs and concurrent teaching practices (Leu & Wu, 2005). The blue and green arrows in the conceptual framework depict these reciprocal relations.

Several components of the classroom events and features of the classroom culture are reviewed in the literature as indicative of a teacher's views, beliefs or personal philosophies of the nature of mathematics, the learning of mathematics and the teaching of mathematics. Comparable components of events or features of classroom culture that were observed and recorded could be analysed in order to make conjectures of teachers' personal philosophies of mathematics. Cognisance of the following connections with teachers' views of and beliefs about mathematics was taken during the literature review:

- Teaching approaches (Bauersfeld, 1994; Brodie, 2005; Davis, 1997; Halim & Mohd.Meerah, 2002; Morine-Dershimer & Kent, 1999; Tobin & McRobbie, 1999) and the flexibility of teaching practice (Leu & Wu, 2005; Olivier, 1992; Palincsar & Brown, 1984; Santagata, 2005; Smith, 1999), including accommodation of alternative approaches (Brodie, 2008; Szydlik, et al., 2003).
- Prevailing teacher role (Adler, et al., 2005; Ball, 2000; Kovalainen & Kumpulainen, 2007; Maree, 2004; Tobin & McRobbie, 1999), encompassing decisive authority about mathematical appropriateness or correctness

(Heinze, 2005; Mason, 2000; Santagata, 2005) and the teacher's expectations of learners' dispositions and the degree of learners' autonomy (Ernest, 1988; Kaldrimidou, et al., 2004).

- Classroom discourse patterns (Cazden & Beck, 2003; Clements & Battista, 1990; Mehan, 1979; Smith, et al., 1993) encompassing dominance, authority and degree of collaboration (Adler, et al., 2005; Cazden & Beck, 2003; Mason, 2000; Santagata, 2005; Tobin & McRobbie, 1999), the prominence of negotiation through discussion (Bauersfeld, 1994; Clements & Battista, 1990; Noddings, 1990; Smith, et al., 1993; Szydlik, et al., 2003; Tobin & McRobbie, 1999), typology of listening (Ball, 2000; Davis, 1997) and the categories of teachers' questions (Cazden & Beck, 2003; Hargreaves, 1984).
- The teacher's inclination towards learners' mathematical errors (Brodie, 2005; Heinze, 2005; Noddings, 1990; Olivier, 1992; Resnick, et al. 1989; Santagata, 2005; Smith, et al., 1993; Tobin & McRobbie, 1999) and the teacher's interaction with learners' mathematical errors (Bauersfeld, 1994; Beswick, 2005; Brodie 2008; Ernest, 1988; Leu & Wu, 2005; Mehan, 1979; Olivier, 1992; Santagata, 2005; Smith, et al., 1993).

The mathematics teachers' espoused beliefs as professed to me and their enacted beliefs as conjectured from my observations of their pedagogical practices were linked to their actions related to learners' errors, whether in writing or verbally during classroom interaction and vice versa, as previously illuminated.

For the purpose of this study, learners' mathematical errors could be illuminated during learning periods, through classroom interaction and in written assessment tasks. The two orange arrows on the conceptual framework depict teachers' verbal and written interactions with learners' errors. Assessment could have a summative or a formative nature and be traditional or alternative. Based on the literature, the nature and the quality of written or instructional feedback to learners' errors as brought to the fore during assessment, could be formative or not. Teachers' feedback to learners' errors in written assessment tasks could be in a written form or, during a whole-class discussion, in an instructional form. Instructional feedback, resembling classroom interaction, would therefore adhere to similar possibilities of interaction through discourse.

## 2.4.2 Conceptual Framework

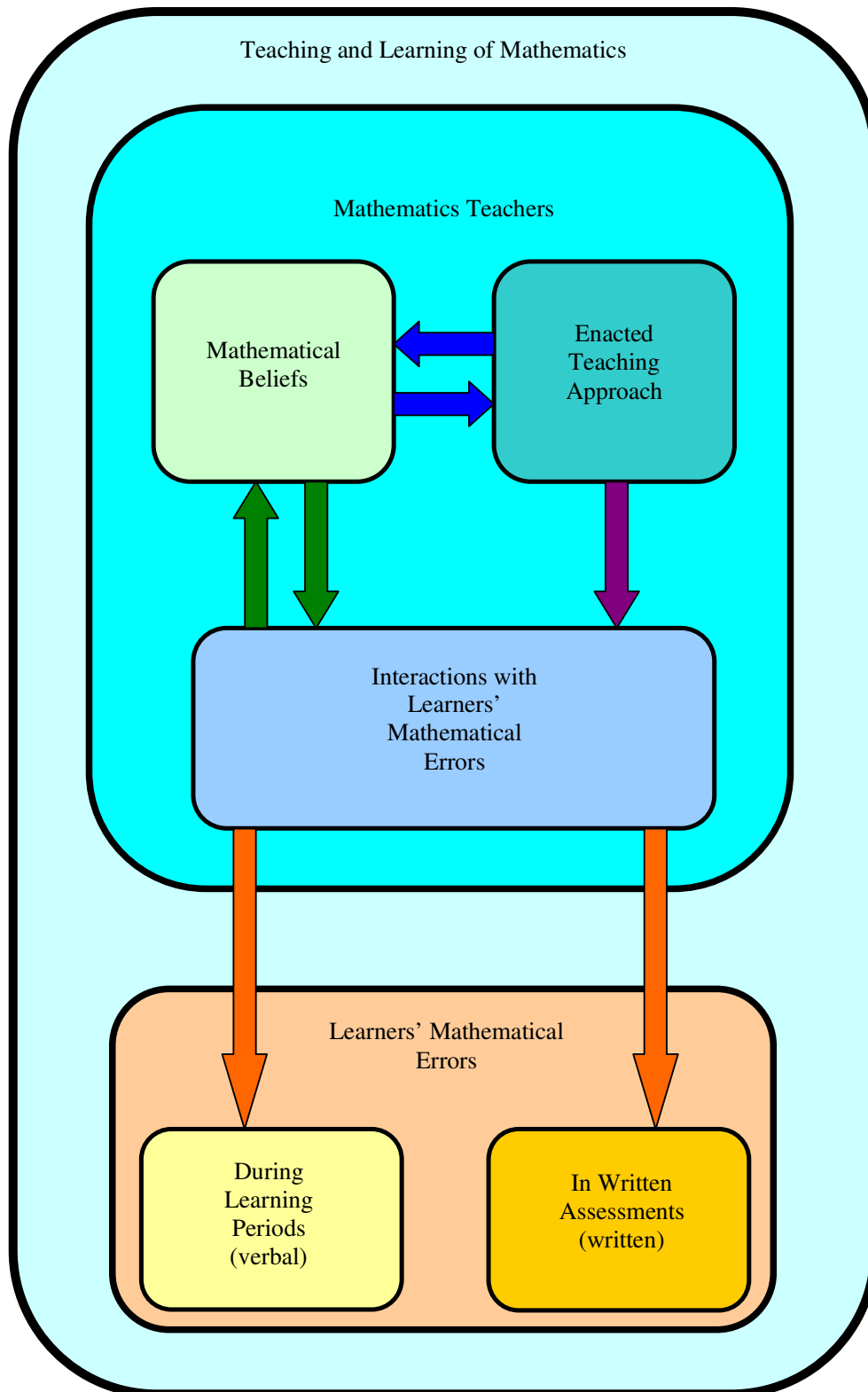


Figure 2-1 *Conceptual framework for the investigation*

## 2.5 SYNOPSIS

This chapter deals with the synthesis of a critical literature review according to the two salient themes, *mathematics teachers* and *learners' mathematical errors*, embedded in the *teaching and learning of mathematics*. The constructs, *learning mathematics* and *teaching mathematics*, as well as *learners' mathematical misconceptions and errors* are discussed with behaviouristic and constructivist underpinnings respectively. A clarification on the constructivist/socio-constructivist dualism ensues. Mathematics teachers are delineated according to teaching approaches and mathematical beliefs. Teaching approaches are described in terms of teacher-learner interactions and classroom discourse, teacher questioning and listening, instructional scaffolding and the ZPD and assessment. This is followed by an explication of the development of the conceptual framework from the existing literature base, emphasising the role of the conceptual framework in directing and regulating data collection and data analysis.

### **3.1 INTRODUCTION**

A considerable degree of inconsistency regarding research terminology exists in the literature (Mackenzie & Knipe, 2006). I take cognisance of the multiplicity of interpretations that prevail. One example is Maxwell's (2005) use of the term *research design* to refer to *research methodology*. Ernest (1997) represents *research methodology* as a fundamental theoretical structure with collective philosophical beliefs that determines the researcher's world view, thus analogous to a research paradigm. I concur with Kothari's (2006) definition of *research methodology* as the multi-dimensional science of research that encompasses the philosophical assumptions underpinning the research, the rationale for conducting the research, the statement of purpose, the research questions pertaining to the study, the sampling, the collection and the nature of the data, data analysis and the ultimate research report. Analogous to Maxwell's (2005) delineation, I prefer to integrate the concepts *conceptual framework*, *ethical considerations* and *quality criteria* in the research methodology. However, Mackenzie and Knipe (2006) are unequivocal in their claim that: "Without nominating a paradigm as the first step, there is no basis for subsequent choices regarding methodology, methods, literature or research design" (para. 4). The preceding claim resonates with Merriam's (1991) statement that the researcher's worldview is instrumental in the interpretation of observations and data analysis.

This chapter will hence delineate the research paradigm in which my theoretical perspectives are inherent, my philosophical assumptions that underpin the research and the methodology as demarcated in the preceding definitions. I prefer to confirm my paradigmatic, philosophical and methodological positions by mirroring these in the corresponding orientations, thus refraining from a discussion of dissimilar orientations. Following a dissection of the preceding definitions of *research methodology*, a number of aspects are identified. Not all of those aspects will be discussed in this chapter. The rationale for conducting the research, the statement of purpose and the research questions pertaining to the study are discussed in the first chapter of this research account (see sections 1.3, 1.4 and 1.5) while the conceptual framework is presented in the second chapter (see section 2.4). The *research design*, as described in this chapter (see section 3.6), encompasses the sampling criteria and process, the collection and the

nature of the data, the approach to data analysis, the quality criteria, the ethical considerations and the scope and delimitations of the study. My role as researcher will also be illuminated.

### **3.2 RESEARCH PARADIGM**

Nieuwenhuis (2007 a, pp. 47, 48) defines a paradigm as:

*A set of assumptions or beliefs about fundamental aspects of reality which gives rise to a particular worldview – it addresses fundamental assumptions taken on faith, such as beliefs about the nature of reality (ontology), the relationship between knower and known (epistemology) and assumptions about methodologies.*

Miscellaneous research paradigms will indicate a qualitative research approach as being appropriate to a particular systematic enquiry. Hence, to refer to a qualitative paradigm constitutes an oversimplification of a possible typology (Maxwell, 2005). An inclination towards purely qualitative research often corresponds to a theoretical perspective situated in a constructivist paradigm, also referred to as interpretive, naturalistic or social constructivist (Lodico, et al., 2006). The preceding descriptive labels concur with those used by Mackenzie and Knipe (2006, para. 6) when they recognize and apply the adjectives “interpretive” and “constructivist” as complementary in reference to the research paradigm characterized by practical interest, culminating in an understanding of social or educational phenomena (Ernest, 1997). Lodico, et al. (2006) confirm Ernest’s (1997) characterization of the paradigm with their description of systematic enquiry, compatible to this paradigm, as an endeavour to comprehend contextualized, social occurrences. The researcher is subjectively involved in the investigative process that is affected by the researcher’s narrative (Lodico, et al., 2006; Mackenzie & Knipe, 2006).

This investigation followed a qualitative approach, as illuminated in the remainder of this chapter. The research focus was an understanding of a practical, educational phenomenon: mathematics teachers’ interactions with learners’ mathematical errors. The enquiry was embedded in the research context of individual classroom communities. I was subjectively immersed in the research process, inextricably connected to my own narrative, as depicted in this chapter, acting as the research instrument (Maxwell, 2005; Nieuwenhuis, 2007 b; Rossman & Rallis, 2003; Somekh & Lewin, 2005). Analogous to the argument in the preceding paragraph, my theoretical perspective is located within a constructivist-interpretive paradigm (Mackenzie & Knipe, 2006).

### **3.3 PHILOSOPHICAL ASSUMPTIONS**

The eventual selection of research methods for data collection and data analysis is an outcome of the researcher's world view as culminated in the research methodology of choice (Hitchcock & Hughes, 1995). A researcher's world view encompasses four categories of assumptions. Ontological assumptions are concerned with the fundamental nature of social phenomena and what can be known (Punch, 2009). Epistemological assumptions are related to the nature, the acquisition and the transferral of knowledge and the relationship between the knower and what can be known (Ernest, 1997; Nieuwenhuis, 2007 a; Punch, 2009). A third category of assumptions, which is not addressed in this account, is the perception one has of human nature. Collectively, the former three perspectives on social phenomena determine the fourth category, research methodology (Burrell & Morgan, 1979). Congruent to the preceding view, Hitchcock and Hughes (1995) argue that the preferred research methodology corresponds to the researcher's epistemological perspective. The latter is inseparable from and shaped by the researcher's ontological assumptions.

#### **3.3.1 Ontology**

From a constructivist perspective, the social world, or what can be known, is socially and mentally constructed through individual encounters therewith (McMillan & Schumacher, 2001; Mertens, 2009; Nieuwenhuis, 2007 a). An assumption underpinning qualitative research is that the nature of being or existence is that of an integrated, whole system with several different aspects that are neither constant nor static, nor independent of individual perception. Multiple, subjective realities, based on interpretation, perception and beliefs, constructed through interaction, exist (Denzin & Lincoln, 2005; McMillan & Schumacher, 2001; Merriam, 1991). Koro-Ljungberg (2007) refers to "ontological flexibility" (p. 434). During the periods of fieldwork, I observed the participating teachers' constructions of reality, how they understood their worlds (Merriam, 1991). Their realities were independent of mine, as researcher or observer (Ernest, 1997). I constructed knowledge, which could only be partial, relevant to the participants' realities through the subjective interpretation of their discourses and actions. The internal and subjective experiences and realities of the participants (Cohen, et al., 2005; Ernest, 1997; Maree & Van der Westhuizen, 2007) were thus central to the study. Based on the preceding characteristics, my ontological



assumptions could be described as relativist (Denzin & Lincoln, 2005; Smith & Hodkinson, 2005).

### **3.3.2 Epistemology**

The involvement and the collaboration of the participating teachers were instrumental in this investigative process of constructing knowledge. The construction of partial and subjective knowledge, through discussion and compromise, was an attempt to come to an agreement on the teachers' mathematical beliefs and their interactions with learners' mathematical errors. The constructed knowledge was related to particular sets of learners and curriculum topics, existing in particular classrooms at a particular time in that academic year (Koro-Ljungberg, 2007). The knowledge was hence socially constructed through the interpretation of participants' dialogue and conduct (Ernest, 1997). My understanding of the educational phenomenon was subjective and was gained through the perspectives of the participating teachers who were involved in the phenomenon (Nieuwenhuis, 2007 a). Such knowledge is incomplete and tentative (Ernest, 1997). My epistemological assumptions are therefore constructivist or socio-constructivist.

### **3.4 METHODOLOGY**

The differentiation between quantitative and qualitative research is based upon diverse philosophical assumptions and not superficially on the nature of the collected data (Yin, 2003). Research methods, therefore, are not merely mechanical processes of data collection and data analysis (Cohen, et al., 2005; Ernest, 1997). Scientific-positivistic, naturalistic-interpretive and critical theory are recognized as the three principal research methodologies (Cohen, et al., 2005). For the purposes of this study, the adjectives *naturalistic-interpretive*, *qualitative* and *idiographic* will be accepted as synonymous. A qualitative or idiographic research methodology is essentially constructivist with a focus on the particular (Maree & Van der Westhuizen, 2007; Nieuwenhuis, 2007 a). To appreciate, which is to comprehend the significance or the importance of an event or a situation is the dominant purpose of a qualitative orientation to research (Merriam, 1991). The methodological approach to this study can be described as qualitative, based on the following characteristics of the investigation (ibid.):

- I was predominantly interested in the processes of how mathematics teachers interacted with learners' mathematical errors and why events were as perceived. I

attempted to understand and interpret individual behaviour in a specific context. The study focused on individual teachers in their classrooms, considered as their natural environment. The social contexts were not manipulated by me.

- I fulfilled a dominant and instrumental role in the research processes of data collection and data analysis. The subjective realities of the participants and the personal meanings they attributed to mathematics, the teaching and learning of mathematics and learners' errors were recorded during interviews, including a written, structured interview, in the format of a questionnaire. Knowledge concerning the realities of the participants was constructed by me through the interpretation of participants' discourse and actions. The information was personal and subjective and recorded in the participants' own words. I was subjectively immersed (Maree & Van der Westhuizen, 2007) in the research process and inductively attributing meaning to the data (Henning, 2005).
- I was, to a large degree, the research instrument (Maxwell, 2005; Nieuwenhuis, 2007 b; Rossman & Rallis, 2003; Somekh & Lewin, 2005). My physical presence as observer in the classrooms during the periods of fieldwork was hence inevitable.
- This research report has a distinct descriptive nature and my understanding of mathematics teachers' interactions with learners' mathematical errors is linguistically communicated in the account. Conjectures and conclusions were inductively derived from the collected and analysed data.

### **3.5 ROLE OF THE RESEARCHER**

*To my mind, constructionist research is not about evading the presumption to have validly described the world. It cannot be. Instead, constructionism is about the recognition that things could be otherwise and that we might make them so. It is about recognizing that our theories are answerable to our common lives before, during, and after their answerability to our common world. It is about recognizing that with claiming the power to have valuably and validly described the world inevitably comes the personal responsibility to defend our claims against all comers – that our legitimacy in doing so comes from nowhere else. And it is about recognizing that if it is anything, epistemology is an ethics of truth. It is about making normative claims as to how we might better, or more valuably, understand the worlds we inhabit. Researchers who present themselves as amoral, or as they might prefer, “value-free” and disembodied spectators on the workings of the social world are*

*mistaken. The truth is that we must live in the world if we would hope to understand it.*

(Weinberg 2007, p. 35)

The human researcher is the data collection instrument (Maxwell, 2005; Nieuwenhuis, 2007 b; Rossman & Rallis, 2003; Somekh & Lewin, 2005) in a qualitative investigation, engaging human capacities as susceptibility or receptiveness in data collection methods such as interviews and observations (Merriam, 1991). My roles of observer and interviewer were created for the sole purpose of data collection (McMillan & Schumacher, 2001). By attending learning periods as observer, I immersed myself in the research situation created in the classroom and became part of the observation (Koro-Ljungberg, 2007). My role as interviewer was an interactive one (McMillan & Schumacher, 2001), including the facet of me as a learner (Koro-Ljungberg, 2007; Marshall & Rossman, 2006; Rossman & Rallis, 2003) during the process of knowledge construction. From a constructivist epistemological perspective, knowledge is constructed by the researcher through the interpretation of participants' dialogue and conduct. Consequently, a researcher disposition of apprehensiveness and humbleness towards the knowledge generated through the research process is appropriate (Ernest, 1997; Rossman & Rallis, 2003).

The researcher's personality and essential character, in addition to the personal circumstances and experiences that have shaped his or her life, are recognized as important elements in the research process (Maxwell, 2005; Rossman & Rallis, 2003) and should preferably be incorporated in the research account. Merriam (1991) raises the issue of what impact the induction of a human being into a discipline, the induction into functioning successfully in a particular society as a member, has on that individual and his or her worldview. This is particularly relevant to my own situation and my induction into the field of mathematics education at a comparatively late stage of my life. With a Baccalaureus Scientiae (B Sc) degree, specializing in chemistry and mathematics majors, my academic background was scientific. My induction into the academia of mathematics education commenced more than twenty-five years later when I enrolled for the PGCE and had the privilege to act as a part-time lecturer in Subject Didactics Mathematics on completion of the PGCE. My exposure to academic literature, my professional development through reflection on my own mathematical beliefs and practice and my responsibility towards the students facilitated the transformation in my theoretical orientation that eventually led to this qualitative study. However, I give full consideration

to Ernest's (1997, p. 35) statement that: "Mathematical training often implants the assumptions of the scientific research paradigm. Thus, the first use of qualitative methods can be against the hidden backdrop of some or all of the assumptions of the scientific research paradigm."

### **3.6 RESEARCH DESIGN**

Yin (2003, p. 19) defines a research design as: "the logic that links the data to be collected (and the conclusion to be drawn) to the initial questions of study". He continues to state that: "In the most elementary sense, the design is the logical sequence that connects the empirical data to a study's initial research question and, ultimately, to its conclusions" (p. 20). A qualitative case study is an acknowledged non-experimental or descriptive research design (Merriam, 1991). Yin (2003) elaborates on the case study as an empirical investigation and accentuates the intentional and premeditated lodging of the research phenomenon in its actual context, the plethora of variables pertaining to the phenomenon and the collection of data from various sources. The constituents of a case study as a research design are the research questions pertaining to the study, the theoretical assumptions that direct the collection and the analysis of the data, if included, the units of analysis, the sensible rational thought and argument that connect the data to the theoretical assumptions and the principles applied during the analysis of the data (ibid.).

#### **3.6.1 Case Study Design**

A case study design was selected as an appropriate research strategy for this investigation, considering the mode of the primary research question guiding the investigation, the degree of control I had over the research phenomenon and the contemporariness or the realistic contextualization of the research phenomenon (Yin, 2003). The emphasis in the primary research question pertaining to this study is on *how* mathematics teachers interact with learners' mathematical errors in secondary school classrooms. As a "passive, direct" (ibid., p. 8) observant, I attempted to be as unobtrusive as possible while attending learning periods at each of the four schools respectively to follow the classroom events. The research phenomenon was a current issue, inseparable from the classroom context and the research participant as teacher within the specific context. The unique and significant individual components of each classroom setting were retained during the research process. In this holistically designed case study, a secondary school mathematics teacher,

pedagogically engaged in a grade 9 classroom, defines the unit of analysis (ibid.) or the bounded system that demarcates the case (Merriam, 1991).

Four defining features (Merriam, 1991, pp. 11–14) hallmark a qualitative case study. This study is *particularistic* in its focus on a teacher's interactions with learners' mathematical errors as phenomenon, contextually situated in a grade 9 mathematics classroom, with a grade 9 mathematics teacher as the unit of analysis. Based on narratives, contextual descriptions and classroom vignettes, the research report adheres to the condition of a *rich, thick description*. The *heuristic property* of this case study is embedded in the interconnectedness of a literature review, the systematic development of a conceptual framework and by generating a transparent chain of evidence to facilitate the reader in the process of argumentation and interpretation to eventually draw the final conclusions. Although the conceptual framework developed for this study provided direction during the data analysis process, the *inductive nature* of this study was confirmed by the prominence of inductive reasoning during the analysis of the data, as reported in section 3.6.4 of this chapter and in chapters four to seven.

The multiple-case design comprises four cases or units of analysis and is not located within a different methodological framework from the single or classic case study (Yin, 2003). Although the four selected schools demonstrated obvious differences, as delineated in section 3.6.2, it is my opinion that these dissimilarities were superficial and to a large degree inapplicable to the research questions. The classroom contexts displayed significant parallels in terms of the teachers' mathematical qualifications, the physical resources, the arrangement of the furniture in rows of learners facing the blackboard and the prevailing teaching approaches (see chapters four to seven). Four replications may hence confirm the conclusions in the final chapter, drawn from the research results (Yin, 2003). A plausible categorization of this multiple-case study is educational, descriptive and interpretive (ibid.). The research report is descriptively comprehensive, albeit not void of theory. Numerous variables relating to, inter alia, the teachers' mathematical beliefs including beliefs about learners' errors, the teachers' listening and questioning skills during interactions with errors, the prevailing teaching approaches, the classroom culture, all of which were contextually bound and not equally overt, were present in this study (Merriam, 1991).

### *Selection of Participants*

According to the criteria for qualitative sampling, non-probability sampling was used for the systematic investigation (Merriam, 1991). The selection of the sample was conveniently (Cohen, et al., 2005) done and I relied completely on the potential participants' voluntary compliance. Through my involvement as part-time lecturer in Subject Didactics Mathematics in the PGCE programme, I became familiar with a number of partnership schools during my visits to PGCE students at the respective schools. I approached the principals and/or the heads of the mathematics departments at those schools that are in relatively close proximity to where I live to ask for permission to conduct research at their schools. I contacted six schools and eventually selected the four schools that portrayed an enthusiastic and accommodating attitude towards the project. The prerogative for selecting three of the four research participants vested in the respective heads of the mathematics departments. I was acquainted with the research participant at the private school prior to approaching the school. The research participants were introduced to me by the heads of the mathematics departments at schools A, B and C. Informed consent was obtained from all four participants. The specific group of learners that I joined as observer at each school was selected by the individual research participant.

My decision to focus on grade 9 mathematics teachers was motivated by the logical assumption that a larger variation in learners' mathematical aptitude would be plausible. The Senior Phase in the GET band concludes with grade 9. Mathematics is compulsory for all learners in the GET band in South Africa. Once learners enter the FET band in grade 10, they have the option to choose mathematical literacy instead of mathematics. Large numbers of learners, particularly those achieving unsatisfactorily in mathematics, choose mathematical literacy in the FET band.

#### **3.6.2 Contextualization of the Research**

In this section of the chapter, the research is contextualized in terms of the research sites. The sites are individually presented, per case. Each presentation is demarcated according to a structure that includes the research site in terms of the school, the curriculum discussed during the fieldwork, the prescribed textbook, the classroom assessment in general and the standardised assessment task in particular

### *School A*

School A is a gender-specific, all-girls', public, secondary school, situated in close proximity to various other academic institutions in an affluent urban environment. The medium of instruction is English. The school is racially integrated. One thousand four hundred learners are accommodated on average.

During the course of the fieldwork at school A, the topics under discussion were solving variables from linear equations, solving variables from quadratic equations, solving problems in realistic context with prior translation of the information to an equation, solving problems that involved ratio, rate and proportion and the graphic representation of examples of direct and inverse relationships.

The prescribed mathematics textbook was a popular, recommendable one, as verified by the retailers at the local, academic bookstore. The authors attempted to present the mathematical content in a way that encouraged a problem-solving approach. The particular edition used in school A was the most recent one, adapted to the requirements of the National Curriculum Statement (NCS) for mathematics in South Africa.

One informal, non-standardised class test was administered to the learners during the fortnight of fieldwork. The first ten minutes of a learning period was utilized for that. The research participant, Alice, marked these class tests. The solutions were discussed with the learners during the following learning period. Alice emphasised the value of these class tests in preparation for formal assessments. These informal tests did not contribute to the data I collected for this study.

Learners at school A wrote a standardised term test<sup>1</sup> during the period in which I was conducting the fieldwork at the school. The learners had forty minutes in which to complete the test worth forty marks. Questions in realistic context<sup>2</sup> covered 57,5% of the total marks. The topics assessed in the paper were all discussed in the classroom during the period in which I collected data at the school. The participating teacher, Alice,

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<sup>1</sup> Based on the fact that the focus of my study did not encompass the measurement of learners' achievement, the individual performances of the learners were not recorded.

<sup>2</sup> The relevance of reporting the proportion of problems in realistic context resides in the prominence attributed to the context (Bauersfeld, 1994; Karagiorgi & Symeou, 2005) from a socio-constructivist perspective and is discussed in the final chapter.

followed her prevailing teaching approach during the subsequent memorandum discussion (see chapter four).

### ***School B***

School B is a co-educational, public, secondary, focus-school specialising in the fields of the Arts and Entrepreneurship. It is an urban school with modern buildings in a flourishing suburb. It is a dual-medium school and instruction is given both in English and in Afrikaans. Parallel-medium<sup>1</sup> education, however, takes place in the Senior Phase. The school is racially integrated. Seven hundred learners are accommodated on average.

During the course of the fieldwork at school B, the topics under discussion were the transition from number lines to a Cartesian plane, representing linear functions graphically on a Cartesian plane using a variety of approaches and finding the formulae of straight-line graphs with reference to intercepts and gradients. Solving variables from linear and quadratic equations were revised prior to writing a standardised test.

The prescribed textbook was not as widely recognized as the one used in school A. However, it appeared that the authors attempted to present the content according to a problem-solving approach. The textbook adhered to the requirements of the NCS for mathematics in South Africa. The specific edition was as recent as the textbook used in school A. The publishing company maintained that the content presented in the textbook was bona fide South African. A few strategic concepts were translated in all eleven, official, South African languages and were listed in each chapter. Although the research participant, Barry, discussed examples from the textbook and learners were given homework from the book, he supplied the learners with copies of personal notes. I did not have access to these supplementary notes.

Assessment in classroom B appeared to be formal and summative. No reference to regular, informal, formative assessment was made during the fortnight of classroom observations. The impression was given that alternative forms of assessment were applied under obligation. When a learner referred to a journal entry, Barry responded with:

*No, people, a journal entry is a type of portfolio assignment that you get. You do that once a year and then you never do it again.*

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<sup>1</sup> The medium of instruction in grade 8 and grade 9 is either English or Afrikaans.



Learners at school B wrote a standardised term test towards the end of my data collection period at the school. The learners had forty minutes in which to complete the test worth thirty-five marks. Eighty-eight percent (88%) of the topics assessed in the paper were discussed in the classroom during the time I had attended the classes as an observer while doing fieldwork. All questions were formulated theoretically in an isolated, naked context, although one question, worth one mark, was aimed at compelling learners to reflect. The subsequent memorandum discussion resembled an ordinary learning period (see chapter five).

### *School C*

School C is a co-educational, public, secondary school in a well-established suburban setting. Although it has been a parallel-medium school for more than a decade, it used to be a single-medium, Afrikaans school. The school accommodates eight hundred learners on average and is multi-cultural.

During the course of the fieldwork at school C, the topics under discussion were solving variables from linear equations, solving variables from quadratic equations, manipulating equations to change the subjects thereof and solving problems in realistic context with prior translation of the information to an equation.

The prescribed textbook was a compilation of numerous worksheets and strongly supported the instrumentalist view (Ernest, 1988) of teaching and learning mathematics. The approach in the textbook was that of recognizing patterns and applying subsequent algorithms. Although reference was made to the learning outcomes in the NCS for mathematics in South Africa, it was my opinion that the way in which the mathematics content was presented in the textbook, did not subscribe to the ideals underpinning the South African curriculum.

The learners at school C routinely wrote a ten-mark, ten-minute, informal, non-standardised class test on a Friday, but the research participant, Chloe, had the prerogative of altering the day on which such a test could be taken. During the two-week observation period, the learners wrote one class test on a Tuesday. Chloe prompted the test with:

*Friday's test move to tomorrow. I'm gonna give you three sums you must go and revise because I hear questions about things that you should know by now.*

The first ten minutes of a learning period was utilized for that. Peers marked one another's tests. Chloe subsequently discussed the solutions with the learners during the same learning period.

The learners wrote a standardised term test during the period in which I was collecting data at the school. The learners had sixty minutes in which to complete the test worth sixty marks, but were allowed to complete the test the following day during the last fifteen minutes of their mathematics period. A mere 23% of the topics assessed in the paper were discussed in the classroom during my data collection period at the school. Of those questions, none was formulated in realistic context. The subsequent memorandum discussion resembled an ordinary learning period and did not deviate from the prevailing teaching approach (see chapter six).

### ***School D***

School D is a private, Christian, co-educational school with six hundred and fifty learners ranging from grade 0 to grade 12. The school is situated on the church premises, a short distance from major shopping malls. The medium of instruction is English. The school is racially integrated.

During the course of the fieldwork at school D, the topics under discussion were solving problems that involved ratio, rate and proportion, solving problems in financial context and developing an understanding of simple and compound interest, solving problems that involved time, distance and speed and the graphic representation of examples of direct and inverse relationships. However, the research participant, Dawn, often made the learners write a quiz that covered Euclid geometrical concepts.

The prescribed mathematics textbook was a previous edition of the popular, recommendable one used in school A. This edition was adapted to the requirements of Curriculum 2005 and outcomes-based education in South Africa, but not to the requirements of the NCS for mathematics in South Africa per se. The authors attempted to present the mathematical content in a way that encouraged a problem-solving approach in both editions.

A reasonably high incidence of informal assessment was witnessed in Dawn's classroom. Learners wrote three pop quizzes during the two-week period of fieldwork. Dawn emphasized the benefit of these pop quizzes when learners had to do revision in

preparation for their examinations. Learners were requested to write a journal entry on their understanding of simple and compound interest during one learning period. One set of homework problems had to be handed in separately as a written assessment task. Learners were instructed to keep a three-day journal on their understanding of financial mathematics and to prepare a presentation thereof. Learners had been working on a summary of their mathematics syllabus during the observation period and were continuously reminded of that and of the advantage they would have during the examinations.

Learners at school D wrote a standardised term test towards the end of the time I spent at the school during fieldwork. The learners had forty minutes in which to complete the test worth forty-six marks. Questions in realistic context covered 76% of the total marks. All topics assessed in the paper were discussed in the classroom during the time I attended the classes as an observer during the period of fieldwork at the school. Although Dawn's teaching approach was evident of an inclination towards problem-based teaching (see chapter seven), the subsequent memorandum discussion corresponded to conventional teaching through transmission.

### **3.6.3 Data Collection Strategies**

Case studies are not merely contingent upon data collected from observations. The actuality of multiple sources of data is a distinct advantage in case study research. Consequently, three sources of data, recognized as sources frequently used in case study research, were used in this research project. In addition to observations, data were collected from interviews and documentation (Yin, 2003).

#### ***Interviews***

A written, structured interview, in the format of a questionnaire, with the questions formulated and organized in advance, was used to collect biographical information as well as information regarding the professed mathematical beliefs of the research participants (see appendix A). The information obtained from the structured interviews was used to get thick descriptions of the participants, their backgrounds and their contexts. The structured interview was piloted prior to the commencement of the fieldwork by administering the questionnaire to a group of PGCE students in Subject Didactics Mathematics. Ostensibly due to their parallel mathematical backgrounds and their then-recent experiences as novice

teachers in mathematics classrooms, no apparent ambiguities regarding the questions were reported (Merriam, 1991). My decision to request research participants to complete the structured interviews in their own time in writing was both strategic and pragmatic. Strategically I perceived this approach to be less cumbersome than posing the questions directly to them. Participants were allowed time to reflect on their own assumptions and practice and presumably to provide more elaborate responses. The pragmatic consideration was based on the availability of time.

Based on the exploratory nature of the research project, semi-structured interviews, in comparison to the initial structured interviews, were selected as most appropriate. “Interview guide approach” type (Cohen, et al., 2005, p. 271) semi-structured interviews, directed by a number of questions (see appendix A), were conducted in a one-on-one approach (Creswell, 2005) to elicit preconceived information from all the research participants. The mode of these semi-structured or open-ended interviews was conversational rather than investigative (Yin, 2003). I enjoyed the flexibility to react to each unique situation appropriately and to spontaneously explore related issues raised during the course of each interview (Merriam, 1991). I hence did not make decisions on the exact formulation of the questions or the sequence of the questions prior to the interviews. However, I did pursue the interview protocol, albeit in an unprejudiced style (Yin, 2003). All the semi-structured interviews were conducted in English. The four participants were fully bilingual and fluent in English (Merriam, 1991).

I conducted an initial, semi-structured interview, prior to the respective observation periods, with each participating teacher at the three departmental schools in order to obtain a subjective view from each participant's perspective. In order to ensure that I would eventually be able to triangulate the interview information with the data generated by the classroom observations and through document analysis, I had to focus on teachers’ beliefs about learners’ errors, in their capacities as teachers. An additional focus on how they professed to interact verbally with errors during learning periods was necessary. The interview questions had to be adequately concentrated on the generation of relevant information to answer the three respective secondary research questions (see section 1.4) (Maxwell, 2005).

Prior to conducting the second round of semi-structured interviews, subsequent to the respective periods of fieldwork at the three departmental schools, I had to interrogate my

own dispositions and my innate tendency to evaluate in order to portray an unbiased and neutral disposition (Merriam, 1991). Attributed to my own experience of occasionally teaching Subject Didactics Mathematics in the PGCE programme, I was oriented towards facilitating student teachers to reflect on their own practice and ultimately to transform their practice. As a preventative measure, I made a deliberate decision not to commence with preliminary data analysis through watching or listening to the mechanically recorded classroom events. This was to avoid posing basically judgemental questions to the participating teachers during the second round of interviewing. I retrospectively evaluated the respective second semi-structured interviews and concluded that these could not be described as *follow-up* interviews, as these three semi-structured interviews had not been sufficiently informed by preliminary data analysis, for reasons as discussed in this paragraph.

If the purpose of the first round of semi-structured interviews, prior to the respective observation periods, had been to obtain a subjective view from a participant's perspective, then reasonably, the main purpose of the second round of semi-structured interviews would be one of clarification and completion. These three semi-structured interviews thus served the purpose of narrowing my focus essentially on the research questions guiding the research project in order to obtain appropriate information I could transform into data (Maxwell, 2005). A combination of the two basic sets of questions was guiding the single semi-structured interview I conducted with Dawn, the participating teacher at the private school D, prior to the observation period.

The semi-structured interviews were audio-recorded with the use of a tape-recorder and a digital voice-recorder. Consent from the research participants was obtained prior to the mechanical recording of the respective interviews. The semi-structured interviews were not captured on video-tape. Mechanical recording of the semi-structured interviews and verbatim transcriptions of the digital voice-recordings of the semi-structured interviews ensured accurate and comprehensive data (Maxwell, 2005). Transcriptions of the semi-structured interviews, precise representations of the language used by the participants, were electronically sent to all four of the participating teachers. Not one of the participants expressed concern or doubt about any incongruities.

## ***Observations***

The main purpose of collecting data through classroom observations and mechanically recorded classroom activities was to answer the second secondary research question guiding the study. According to Junker's (Junker, 1960, as cited in Merriam, 1991, p. 93) typology of observations, my role as observer is compatible to the "observer as participant" category. I physically attended the learning periods, the participating teacher and the learners were aware of my presence in the classroom and the learners were informed about the purpose of the observations. However, my presence was as unobtrusive as possible and there was no degree of participation in the classroom activities, other than my attendance of the learning periods. I took the role of "complete observer" (ibid., p. 93) in terms of my abstinence from participation and involvement, but, based on the overt nature of the exploration, that of "observer as participant" (ibid., p. 93). Yin (2003, p. 92), however, refers to my role as observer in the classrooms during the periods of fieldwork, as that of "direct observer". Direct observations are conducted passively in a non-participating way. This is in resonance with Nieuwenhuis' (2007 b, p. 85) description of "observer as participant". According to him, the researcher does not engage or interfere in the research activities and does not influence the dynamics of the research situation, albeit his or her presence is evident. Creswell (2005, p. 212), however, refers to my role as observer, at the back of the classroom, as a "nonparticipant observer". I hence prefer to describe the role I fulfilled during learning periods as that of a *direct observer* or a *nonparticipant observer*.

I recorded components like the locations of the schools, school grounds, classroom layouts, classroom discipline, teaching practices and approaches and verbal interactions, from observations made during the periods of fieldwork (Merriam, 1991). These aspects are communicated in various chapters of this report (see section 3.6.2 and chapters four to seven). Classroom data were collected in threefold. A strategic position in each classroom for the audio tape-recorder was negotiated in consultation with the respective participating teachers. Each of the participants cordially agreed to wear the digital voice-recorder around their necks. I occupied a desk at the back of each of the four classrooms. The video-camera was secured on a tripod next to where I was seated. That enabled me to change the angle of the camera, thus capturing different sections of the classroom, different areas of the blackboard and to zoom in to what was written on the blackboard without being intrusive. In addition to the above-mentioned mechanical recordings, I took written field notes during the learning periods. The field notes could be

described as “running records” (Nieuwenhuis, 2007 b, p. 85). I included detail in the notes and the notes followed the sequence of the learning periods. I occasionally recorded personal interpretations, reflections or opinions in the field notes. The field notes I took during classroom observations correspond to what Creswell (2005, p. 223) refers to as “observation protocols”. My observations could not be described as structured. I made a rational decision not to employ checklists (Nieuwenhuis, 2007 b) or observation schedules (Cohen, et al., 2005) with predetermined typologies. I previously experienced the use of checklists as being exceptionally distracting. In addition to that, the focus of my research was not to collect numerical data (ibid.).

Clarity on the degree to which my presence in the classrooms altered the teaching approaches or the way in which the participating teachers interacted with learners’ mathematical errors, was not evident (Merriam, 1991). However, altering teaching practices inherent to the research participants (Frankenberg, 1982, as cited in Merriam, 1991) may be implausible against the backdrop of the multiplicity of the observations and the prolonged periods of field work.

### ***Documents***

The learners’ written assessment tasks were useful and important as a source of data. Although the assessments were not written or marked for research purposes, the written feedback from the research participants to the learners was directly feeding into one of the secondary research questions, consequently corresponding to the conceptual framework guiding the study. Except for the data from the written assessment tasks, limited other relevant data (Merriam, 1991) were available to answer the third secondary research question. In addition to the preceding motivation for using the written assessment tasks as a data source, data collected from documentation were used to substantiate the results obtained from data obtained from other sources (Yin, 2003). Besides the relevance of the written feedback, the substance of the assessments was implicitly relevant to the teachers’ mathematical beliefs (Bauersfeld, 1994; Karagiorgi & Symeou, 2005, Remillard, 2005; Sun, Kulm & Capraro, 2009). Cognisance was taken of the possible pitfalls of using documents as a source of data. I was capable of thorough comprehension of the content of these written assessments (Riley, 1963, as cited in Merriam, 1991) and the genuineness of the documents was beyond doubt. Numerous data analysis categories were inductively

(Miles & Huberman, 1994) developed from the examination of the teachers' written feedback on the learners' assessment scripts (Merriam, 1991).

The respective participants cordially gave me access to the written assessment tasks on completion of the marking thereof, the capturing of the learners' results and the subsequent discussions of the memorandums. Three forms of writing were recorded on the scripts; the learners' initial process of being assessed, the teachers' marking and the learners' ultimate copying of the correct mathematical procedures during the memorandum discussions. Those sections of the written assessment tasks on which learners' errors occurred, were scanned onto my desktop computer, thus enabling me to differentiate between the various phases of writing on each script, by colour. One of my aims was to avoid identification of the learners. Where possible, learners' names were not recorded. The learners' academic performance or measured achievement was beyond the scope of my study and was thus not recorded. Those instances where names and/or marks were recorded were incidental or inevitable.

#### **3.6.4 Data Analysis**

Three types of raw qualitative data were analyzed prior to the compilation of the research report (Merriam, 1991). Accounts of contexts, participants and incidents were obtained from the mechanically recorded data, from my field notes and from the structured interviews. The participants' verbatim language was obtained from the mechanical recordings of the classroom events and the semi-structured interviews. Excerpts from documents came from the written and marked assessment tasks.

##### ***Interviews***

On completion of the verbatim transcriptions (see appendix B) of the seven semi-structured interviews, I initiated the data analysis process by reading through hard copies of the transcriptions (Creswell, 2005), initially while I was listening to the digital voice-recordings and ultimately without the audible recordings. Only then did I approach the interview transcriptions with the purpose of continuing the data analysis process with the commencement of data reduction (Miles & Huberman, 1994).

During the data reduction phase of data analysis (Miles & Huberman, 1994), I compared the verbatim transcriptions of the seven semi-structured interviews I had conducted with the four participants and compiled a data matrix for the respective questions that were



posed to each of the four participating teachers. Although all the respective interviews were initiated similarly and the relevant questions were posed to each and every participant, the semi-structured interviews accommodated personal and individual deviations (Merriam, 1991). Due to this phenomenon, various important issues were raised by the different participants, but these issues were usually distinct and unique to each particular participant. Cross-case comparisons (Miles & Huberman, 1994) relating to those idiosyncratic issues were subsequently unattainable. I realized retrospectively that this could have been prevented, had I commenced with the data analysis while I was in the field. This limitation will be stated per se in the dissertation. The reality of conducting only one semi-structured interview with Dawn, at school D, did not contribute to my predicament to a greater extent than the phenomenon of individual deviations. With the prior experience of six preceding interviews, I was more competent in interviewing and was more focused on asking relevant questions.

Although I had formulated concepts and had designed the conceptual framework for the study prior to the data reduction process, I kept an open mind to recognize categories that could probably emerge inductively (Miles & Huberman, 1994) from the information in the transcriptions. Embedded in the conceptual framework were aspects regarding teachers and aspects regarding learners' errors. Those aspects relating to teachers were, in a straightforward way, what teachers *were* and what teachers *did*, both in relation to learners' mathematical errors, either illuminated during *classroom interaction* or through *written assessment*. From those aspects, I derived categories relating to teachers' beliefs and categories relating to teachers' responses (see appendix C).

I copied chunks of data from each semi-structured interview in the participant's own language on removable, self-adhesive notes, in a predetermined colour for each individual participant. Creswell (2005, p. 238) refers to this approach as the use of "in vivo" codes. Large sheets of cardboard were prepared by drawing matrices on the cardboard sheets. The notes were subsequently pasted to the cardboard matrices. Two processes of data analysis, namely "data reduction" and "data display" (Miles & Huberman, 1994, pp. 10, 11) were performed concurrently. I continuously verified the categories or codes I had deduced from the conceptual framework for the study to those emerging inductively (Miles & Huberman, 1994) from the data in semi-structured interview transcripts. A number of codes emerged inductively (see appendix C). When these processes were completed for each individual participant, I paraphrased the chunks of

data onto data matrices I created on Microsoft Word Documents (see appendix C). At this stage, these processes were limited to within-case data analysis. The preceding description captured the first phase of the analysis of semi-structured interview data, executed within-case, performed through data reduction and data display (ibid.).

### ***Observations***

The digital voice-recordings were downloaded onto my desk-top computer. Due to the convenience, the preciseness and the accessibility of these recordings, the digital voice-recordings ultimately formed the principal source of the classroom data. While listening to these recordings, I was able to transcribe teacher-learner interactions relating to learners' errors, so-called *error moments*<sup>1</sup> (see appendix B). Although I subscribe to a typology of errors (Olivier, 1992), I decided not to differentiate between teachers' interactions with slips, errors or misconceptions. The questions learners asked or the contributions learners made that were indicative of erroneous thinking or misconceptions were recognized as potential and relevant data (see section 1.6.1). I hence decided to utilize such instances in the data analysis process. While I was transcribing these *error moments*, I focused on general trends in each classroom, thus enabling me to construct a typical classroom vignette for each of the participants (see chapters four to seven). I decided not to physically extract or separately isolate the *error moments*. I could not justify a necessity for the particular process. I was able to isolate or extract these interactions by my approach to the transcription thereof. The practical implication was that sections of the recordings, those sections in which the teachers delivered monologues, without involving the learners, were not transcribed. However, sections that were relevant to the description of typical classroom vignettes were transcribed. The process finds resonance in Merriam's (1991, p. 82) description of the "interview log" that she developed for her postgraduate students. The transcription process of extracting the *error moments* from the mechanical recordings corresponded to an initial phase of data analysis through data reduction and resembled a within-case analysis strategy (Miles & Huberman, 1994).

A limitation of the digital voice-recordings was the inaudibility of some of the learner contributions. In order to compensate for this limitation, I listened to the duplicate audio tape-recordings of a few of the learning periods. In some instances the audibility of the

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<sup>1</sup> An *error moment* entails an excerpt of teacher-learner interactions in relation to errors.

learner contributions was in fact of a better quality. However, the slight improvement in the audibility of the audio tape-recordings in comparison to the immensity of the data and the resultant time it would take to listen to each recording in duplicate or in triplicate did not seem to be justifiable. I utilized my written field notes as a reference to clarify some of the learner contributions that were inaudible on the mechanical recordings.

I decided to use the video-recorded classroom events as an auxiliary source of data. The teachers' facial expressions and their body language were hence not employed as potential data. I randomly selected two to three video-recordings, of a possible ten video-recordings per research site, to watch. In addition to that, I watched the video-recordings of the four respective memorandum discussions. The audibility of the video-recordings did not prove to exceed that of the other mechanical recordings. The advantage that the video-recordings had over the other two types of mechanical recordings was the visual component that enabled me to provide rich, detailed descriptions of the research contexts. The visual material confirmed, supported and supplemented my written field notes.

Subsequent to the transcription of the teacher-learner interactions in relation to errors, the so-called *error moments*, I approached the process of data analysis deductively through data coding (Miles & Huberman, 1994) with eight potential data categories in mind. These potential, deductive categories were the following:

- Teacher's impassiveness or unresponsiveness towards learners' errors
- Interacting with learners' errors through verbal interjections or repetitive negative interjections
- Correction of learners' errors through transmission
- Instructional scaffolding through questioning
- Quality or value of teacher's questions
- Typology of teacher's listening
- Peer involvement in error discussion
- Teacher's classification of errors

When a teacher had not been involved in an interaction with a learner and did not respond to an erroneous contribution, I categorized that incident as *teacher's impassiveness* or *unresponsiveness*. The subsequent excerpt from Chloe's classroom is illustrative thereof.

T: *We have one over t equals c over s. How are we going to get t? Think!*

L1: *Times with one over t. Yes, you times it.*

T: *How can I get it to be just t? So, if I want t I have ...?*

L2: *Negative, negative.*

T: *s over c guys! I can just flip it around, but then I must flip both sides, because this is equations. What I do on one side, I have to do on the other side.*

In Chloe's case a phenomenon, that a teacher's impassiveness or unresponsiveness towards learners' errors was most obvious when the teacher was *steering learners to the teacher's preconceived approach* (see inductive codes), emerged from the data. A relatively strong contingency between the two categories is hence suggested.

The excerpt from Barry's classroom, presented below, serves to illustrate the interaction with a learner's error through *a repetitive, negative interjection*.

T: *I have to solve x, I have to work out what the x is, where do you start?*

L: *Sir, you make the denominators the same.*

T: *No, no, no, no, no, listen, listen, be very careful. You will make the denominators the same if it's an expression, there isn't an equal.*

I had to define a differentiation between instances categorized as the *correction of learners' errors through transmission*. Not one of the participating teachers portrayed consistency in their interaction with learners' errors. In some instances, teachers *corrected the errors promptly and without elucidation*. An example from Barry's classroom ensues.

T: *In this one x is ...?*

L: *Positive.*

T: *No, negative.*

In other instances, as in the subsequent quote from Alice's classroom, teachers *augmented the corrections through transmission by clarifying the errors* or the misconceptions.

L: *Can't you get all the fractions on the same side and the ...*

T: *Fractions on the same side? I thought in equations we try to get all the variables on one side and the numbers on the other. This fraction doesn't have a variable. This one does. This is actually perfect. We've got all the variables on the left and the constant on the right, even though it's a fraction. So I think let's rather go this route. Leave it the way it is.*

I hence subdivided the category *correction of learners' errors through transmission* according to the preceding motivation (see data matrix in appendix C).

The following passage from Chloe's classroom exemplifies an incidence of *instructional scaffolding through questioning*.

L: *So, in other words, will that be minus x plus minus x plus minus x?*

T: *Tell me, consecutive numbers, what do they do?*

L: *They're right after each other.*

T: *Okay, like?*

L: *One, two, three.*

T: *What did you do with one to get to two?*

L: *Plus.*

T: *Plus one.*

T: *So what do we do with x? Say now x is our first number. How do we get to the next consecutive number?*

L: *Also plus.*

T: *Plus one and then to the second number?*

L: *Plus one.*

I eventually decided to omit the two categories regarding teachers' questions and teachers' listening, namely *quality or value of teacher's questions* and *typology of teacher's listening* from the data matrix for the individual coding of the classroom data. I subsequently designed separate data matrices for the two phenomena (see appendix C).

No examples of the category *peer involvement in error discussion* were recorded during the period of fieldwork. A remark Alice made during her interaction with a learner's error

is quoted to serve as an example of the category *teacher's classification of learners' errors*.

*There's your mistake. So, multiplication error.*

The above-mentioned categories emerged deductively (Miles & Huberman, 1994) from the literature and from my personal experience. The following seven categories emerged inductively (ibid.) while I was transcribing the *error moments* from the digital voice-recordings:

- Enhancing the mathematical value of a learner's contribution by paraphrasing and rephrasing the contribution without emphasizing the learner's error(s), thus temporarily allowing the error(s) to elicit the mathematics from the learner.
- Entertaining learners with their mathematical errors.
- Learner's error is not detected and teacher accepts and confirms the statement or teacher evaluates the learner's contribution or approach incorrectly.
- Steering learners towards the teacher's preconceived approach.
- Involving learners in the negotiation regarding mathematical results.
- Not addressing the learner's precise error.
- Interaction with learners' errors through the employment of heuristics.

An excerpt from Dawn's classroom is presented to illustrate the category *enhancing the mathematical value of a learner's contribution by paraphrasing and rephrasing the contribution without emphasizing the learner's error(s), thus temporarily allowing the error(s) to elicit the mathematics from the learner*.

*L: They take it to the power of five and the power of three.*

*T: Well done! Well done! Do you see their mistake? They said five squared ( $5^2$ ) is 25, therefore three squared ( $3^2$ ) must be nine.*

In the extract that follows, Alice actually executed the learner's erroneous suggestion to convince the learner of the inappropriateness thereof. This serves as an example of the category *entertaining learners with their mathematical errors*.

*L: Don't you use nought as your  $x$ ?*

T: *Let's say  $x$  is nought. What is this bracket equal to?*

L: *Nought.*

The learner's peers responded in a chorus with: "minus one!"

T: *Oh, oh, minus one times three is minus three. Not gonna work.*

The subsequent example, from Alice's classroom, shows an instance in which a learner's erroneous statement was accepted and confirmed. The excerpt serves to illustrate the category *learner's error is not detected and teacher accepts and confirms the statement or teacher evaluates the learner's contribution or approach incorrectly*.

T: *Does the cost of hiring mountain bikes per hour (own emphasis) decrease the longer you hire the bikes?*

L: *No it gets more. (The rate per hour actually stayed constant.)*

T: *It gets more and it's in proportion.*

The category *steering learners towards the teacher's preconceived approach* can be illustrated by an excerpt from Chloe's classroom regarding the equation  $P = 2(l + b)$ .

T: *We got information. They gave us the perimeter; they gave us the breadth, but we don't know what the length of the particular room is. How are we going to get "l" alone?*

L: *Are we going to use the distributive law?*

T: *We are definitely going to divide. We could use the distributive law, but think about what we did yesterday where we specifically brought things into product form to make it easier to work with them. What will be easier than saying two l plus two b? Because we want l alone now, we're complicating things if we are going to remove the bracket. We'd rather want to keep the bracket, okay?*

The category *involving learners in the negotiation regarding mathematical results* is exemplified by an incident from Dawn's classroom.

T: *What is the answer to number seven LI?*

LI: *Ten thousand.*

T: *Ten thousand centimetres, which is? Which is? One kilometre, isn't that? Am I right?*

L2: *It's twenty times five thousand.*

T: *It's a hundred thousand, which is? Is it ten kilometres? It's not!*

T: *Come, show me.*

The example, from Chloe's classroom, cited below, illustrates an instance in which the teacher did not quite address the learner's question. The excerpt is relevant to the category *not addressing the learner's precise error*. The result of  $x = 3$  was obtained from  $-x = -3$ .

L: *Why did you put like three under minus three?*

T: *Why did I put three?*

L: *Under minus three equals to three and why didn't you leave it as minus three?*

T: *What you got here, when you move two  $x$  to the other side, you had two  $x$  (should have been  $x$ ) minus two  $x$ , okay? That was equal to three. You moved six to the other side. That gave you minus six and then if you say  $x$  minus two  $x$ . You're gonna get minus  $x$  and three minus six is minus three. If I have minus on both sides the minuses fall away and I'm left with  $x$  equals three. Is that it?*

The citation from Dawn's classroom serves as an illustration of employing heuristics in teachers' interactions with learners' errors. The incident is categorized as *interaction with learners' errors through the employment of heuristics*.

T: *What does an inverse relationship mean?*

L: *That ...*

T: *Draw and present it for me on the back.*

L: *y-axis ...*

T: *Right, y and ...?*

L: *x*

T: *Okay, cool! So, draw it quickly ... inverse proportion.*

L: *Inverse can be like that.*

T: *Which means that as ... as  $x$  increases,  $y$  ...?*

L: *Decreases.*



A data matrix with fourteen categories (see appendix C) was designed and utilized to classify the *error moments* transcribed from the mechanically recorded classroom activities. The classification was manually performed on my desk-top computer with the data matrix in a Microsoft Word Document. Two of the original eight deductive categories, those pertaining to teachers' questioning and listening skills, were employed in separate data matrices. One of the deductive categories, relating to the correction of learners' errors through transmission, was subdivided into two categories. In addition to the resultant seven deductive categories, seven categories evolved inductively.

While I was listening to the digital voice-recordings and watching the video-recordings of the classroom activities, I focused on teaching and management trends, both idiosyncratic and general, that constituted the respective classroom cultures. These aspects of the classroom dynamics enabled me to sketch a classroom vignette for each of the four research contexts (see chapters four to seven). Aspects that were considered related to classroom management, to teaching approach and to the nature of the classroom discourse (see appendix C). I copied examples of typical teacher questions to a Microsoft Word Document from which I classified the teacher questions. I inductively identified seven categories from which I designed a data matrix (see appendix C) to explore and describe the levels and types of teacher questions. I employed Davis' (1997) typology of listening to design a data matrix (see appendix C) with which to explore and describe the types of respective teachers' listening. As teachers' interactions with learners' errors constitute the primary focus of my study, little or no reference to that is made in the classroom vignette sections of the four contextual narratives (see sections 4.2, 5.2, 6.2 and 7.2). The particular concern is discussed in depth in the relevant sections of the four contextual narratives in chapters four to seven.

### ***Documents***

As a first phase of data analysis, through a process of data reduction (Miles & Huberman, 1994), I examined the scanned written assessment tasks with the aim of answering the third secondary research question of my study. The approach resembled a within-case analysis strategy (ibid.). I decided to focus on those questions in the written assessment tasks in which teachers' responses were signified by corrections or by markings and written remarks other than just conventional markings (ticks and crosses). Although I took the mere use of a conventional marking, indicating an incorrect mathematical

procedure or solution, into consideration during data analysis, I decided not to interrogate learners' reasoning in instances where the teacher refrained from doing so. This was a pragmatic decision I took during the process of data reduction. I had to analyze a plethora of data. As my focus was on teachers' interactions with learners' errors, inquiry into learners' errors, in addition to those that teachers responded to, was regarded as being beyond the scope of my study.

I initially approached the process of data analysis, through data reduction, deductively (Miles & Huberman, 1994) with potential data categories, embedded in the conceptual framework of my study, in mind. However, on completion of the first cycle of data reduction through the assignment of appropriate codes to chunks of data, I recognized that I had assigned a significant number of these chunks of data to more than one category. Through examining the written assessment tasks, I discovered certain teacher responses to learners' errors in written assessment tasks for which I initially did not define categories. I subsequently decided, based on these inductively emerging codes, to rearrange the data categories (see appendix C).

In some instances, teachers corrected learners' errors *without* a written, mathematical explanation of why the learner's effort or interpretation was erroneous. In other instances, the correction of the learner's error was augmented with a written, mathematical explanation. The correction of learners' errors could also be the provision of complete, correct, written solutions. Written comments could be a mathematical explanation of why a learner's effort or interpretation was erroneous, as previously mentioned. However, examples of an explanation *without* a correction of the learner's error were encountered too. Other written comments were an explanation of what the appropriate mathematical approach would be, instructional interjections, authoritative interjections, encouraging interjections or classifying learners' errors. The participating teachers used various markings (see appendix C) to indicate learners' errors. Teachers' written interaction with learners' errors included comments that could potentially compel the learner to reflection and critical assessment of his or her own thinking, facilitating the learner heuristically to understanding and scaffolding the learner's thinking instructionally. Examples of teachers' attempts to accommodate a learner's alternative approach were seen (see appendix C for a comprehensive and elucidative discussion of these data categories). A data matrix was utilized to arrange data categories resulting from the written assessment tasks (see appendix C).

### 3.6.5 Quality Criteria

*Trustworthiness ... is an umbrella term that ... offers a way to talk about the many steps that researchers take throughout the research process to ensure that their efforts are self-consciously deliberate, transparent and ethical – that they are, so to speak, enacting a classically “scientific attitude” of systematicity while simultaneously allowing the potential revisability of their results. As a tool of assessment, it facilitates discussion of criteria for judging the overall quality of a research study and the degree to which others – scholars, laypeople, policy actors – can build on its analysis.* (Schwartz-Shea, 2006, pp. 101, 102)

*Internal validity or credibility* (Lincoln & Guba, 1985) is a measure of the consistency between the research phenomenon and the interpretive research findings: “What is being observed are people’s constructions of reality, how they understand the world.” (Merriam, 1991, p. 167). Validity hence involves the authenticity of the researcher’s representations of these personal constructions by the provision of sufficient descriptions or demonstrations of the primary personal constructions, as validated by the original people (Lincoln & Guba, 1985). Huberman and Miles (2002) are in agreement with the emphasis on the consistency between the research phenomenon and the interpretive research findings when they assert that the concept of validity in qualitative research is intrinsically present in the relationship between the research report and the research phenomenon, rather than it being a technical matter and dependent on research procedures.

This investigation was characterized by repeated visits to the research sites and by *prolonged fieldwork*. Each participating teacher received at least eleven class visits during a two-week period of fieldwork at their respective schools. I attended the learning periods as a direct observer, while giving undivided attention to the classroom events. My mathematical training and my past experience of classroom observations (Merriam, 1991), enhanced the *persistent* quality of the *observations* (Lincoln & Guba, 1985). *Triangulation* was accomplished through the employment of multiple data collection strategies and the utilization of multiple sources of data. Data were collected through written, structured interviews and semi-structured interviews, through mechanical recordings of classroom events, augmented with field notes and through the analysis of written assessment tasks. Considerable sections of the research account portray the personal constructions of the research participants through the employment of *participant verbatim language*. Their personal definitions of mathematics and their professed beliefs about learners’ errors are quoted verbatim in the research account (Lincoln & Guba, 1985). The research account

was *peer reviewed* by at least two expert supervisors at regular intervals. *Member checking* as a validation process was included by submitting interview transcripts and contextual descriptions to the participating teachers for review. No inconsistencies were reported. The measures described in this paragraph were an attempt to ensure the *credibility* or *internal validity* of the research account (Lincoln & Guba, 1985; Merriam, 1991).

*External validity, applicability or transferability* (Lincoln & Guba, 1985) reflects the potential of generalization of the research results. Research findings can be generalized to research populations or to theory (Yin, 2003). However, the notion of generalizing the results of a research study, the degree to which the findings are relevant to other contexts, is viable on condition that the construct *generalization* is interpreted in terms of assumptions compatible to the qualitative research paradigm (Merriam, 1991). Yin (2003) subscribes to this differentiation in his distinction between analytical and statistical generalization. *External validity, applicability or transferability* in qualitative research therefore reflects the potential of analytic generalization of the research results, hence entailing the expansion of theory. *Transferability* is concerned with the relevance of the research findings to other contexts with the purpose of enhancing understanding of the other contexts (Huberman & Miles, 2002). The replication of the study through the execution of a multiple-case study can enhance the *external validity* (Merriam, 1991; Yin, 2003).

The results concluded from this investigation are embedded in detailed, *descriptive accounts* of the research participants and the research contexts as proposed in section 3.6.2 and presented in the narratives in chapters four to seven. I affirm the exactness of the reported physical and behavioural research actualities (Huberman & Miles, 2002). Each of the four cases is *categorized* in terms of its ordinariness of a grade 9 mathematics teacher in an urban, well-resourced, secondary school in the Gauteng province. A *cross-case analysis* is executed in the final chapter, focusing on the participants' professed mathematical beliefs, their observed teaching approaches, their observed classroom discourse and their interactions with learners' errors. An endeavour to enhance the *generalizability* or *transferability* of the research results is portrayed by the measures proposed in this paragraph (Lincoln & Guba, 1985; Merriam, 1991; Yin, 2003).

According to Yin (2003) *reliability or dependability* (Lincoln & Guba, 1985) does not imply a replication of the study in a comparable context, but an actual repetition thereof. Contrary to this, Merriam (1991) asserts that, since human behaviour is dynamic and inconsistent, the notion of repeating a study with the expectation of duplicating the results raises concern. However, parallel research results obtained from replicating an entire study will enhance the quality of a study (Merriam, 1991). Yin (2003) concurs that equivalent conclusions drawn from discrete cases will significantly escalate the potential to externally generalize the research results. Therefore, the rationale behind multiple-case designs is one of replication and reliability (ibid.).

Sincere attempts to convey *transparency* regarding my personal assumptions and theoretical orientation, the criteria for the sampling, the research participants and the contextual factors are presented in sections 3.2, 3.3, 3.5, 3.6 and in chapters four to seven. An appeal to the recognition of *triangulation* is made, based on the *parallel research results* and equivalent conclusions arrived at in the final chapter, albeit data were collected from a variety of sources (Cohen, et al., 2005). The research participants' professed mathematical beliefs concurred with their enacted classroom practices. In addition to that, in Alice's case and in Barry's case, their interactions with learners' errors were corresponding to their classroom practices too. I attempted to ensure the generation of an *audit trail* or a chain of evidence regarding the decision making concerning data collection and data analysis through extensive documentation, as presented in section 3.6 and in appendix C. I believe that the *dependability* of the investigation was advanced by the processes delineated in this paragraph (Lincoln & Guba, 1985; Merriam, 1991; Yin, 2003).

I confess and explicate my *bias*, personal *assumptions* and theoretical *orientation* in sections 3.3 and 3.5. My reflections on my role as researcher are proposed in various parts of this account. My effort to ensure the *conformability* of the research account is witnessed by an attempt to *openness and transparency* (Lincoln & Guba, 1985). I hence appeal to the recognition of the *trustworthiness* of this investigation, based on the adherence to the quality control criteria of *credibility*, *transferability*, *dependability* and *conformability*, as delineated in the preceding paragraphs.

### **3.6.6 Ethical Considerations**

#### ***Ethical Clearance***

I obtained ethical clearance from the Ethics Committee in the Faculty of Education at the University of Pretoria prior to the commencement of my systematic investigation. A designated form, explaining the purpose and the nature of the research, was completed and submitted to the Ethics Committee with the relevant documentation.

#### ***Permission***

The only purpose for granting permission to the researcher for creating the roles of observer and interviewer is to collect data (McMillan & Schumacher, 2001). As researcher, I had to obtain written permission from the GDE, relevant district officials and school principals. I approached the principals of the selected schools to make appointments. During informal discussions with the principals and the respective heads of the mathematics departments, the purpose and the nature of the research were explained in order to obtain oral permission in principle. A relevant form was completed and submitted to the GDE in order to obtain permission to conduct the research. Formal letters were written to the various educational districts and the principals of the schools subsequent to receiving departmental permission. The necessary forms were completed and permission was officially obtained on all three managerial levels prior to applying for ethical clearance.

#### ***Informed Consent and Voluntary Participation***

After verbal informed consent had been obtained from voluntary mathematics teachers, written informed consent was obtained from the participants. During individual meetings with each participant, the purpose and the nature of the research were explained to him or her. They were presented with letters of informed consent (see appendix D) explaining the research and emphasizing the fact that participation was voluntary. Participants were reminded of the voluntary nature of participation (McMillan & Schumacher, 2001). All participants had the opportunity to read the letters of informed consent and to ask clarifying questions prior to signing the consent forms. Although the learners contributed to the study implicitly, in that it was their errors and interactions with the teachers I was drawing on, data were not collected directly from the learners. They did not actively participate in the data collection process. Learners' privacy was not at stake, as the video-recordings

were not made publicly accessible. Even though it was not required of me to obtain informed assent from the learners or informed consent from the learners' parents, letters of information (see appendix D), explaining my presence in the classrooms, were handed out to the learners. In addition to that, the respective teachers explained my presence in the classrooms to the learners and reassured the learners of the protection of their privacy.

### ***Protection from Harm and Deception***

As researcher, I had the responsibility to protect the participants from physical, psychological, or harm of any nature. I remained apprehensive of possible means in which the participating teachers could be harmed. I guaranteed the participants of my truthfulness. I undertook to refrain from deceiving the participating teachers (McMillan & Schumacher, 2001).

### ***Privacy, Confidentiality and Anonymity***

I guaranteed the participants privacy, confidentiality and anonymity by not disclosing their names, the schools, the suburbs or the districts where the research was conducted (McMillan & Schumacher, 2001). Pseudonyms were used for the participants. Video-recordings were not made publicly accessible. The sole purpose of the recordings was for personal reference, to validate my field notes and to enrich descriptions of the contexts.

### **3.6.7 Scope and Delimitations**

The decision regarding single or multiple research sites is part-and-parcel of the scope of the case study (Yin, 2003). This study was conducted as a multiple-case study, comprising four cases. Of the four research sites, three were co-educational and one an all-girls' school. Two of the research sites were parallel-medium and two were English-medium schools. Three of the schools were public, departmental schools and one a private school. All four schools had an urban location and were adequately resourced.

The South African school system constitutes two bands, the GET band and the FET band. The GET band consists of three phases, the Foundation Phase, from grade R to grade 3, the Intermediate Phase, from grade 4 to grade 6, and the Senior Phase, from grade 7 to grade 9. The FET band comprises of grades 10, 11 and 12. Two of the ten grades in the GET band, grade 8 and grade 9, are acknowledged as secondary school education. In terms of the

school system, the research was hence contextualized in secondary schools, in the GET band, in grade 9.

All four of the participants were adequately qualified to teach mathematics at grade 9 level, albeit none of them had qualified in mathematics on a third year tertiary level. Three of the participants qualified in mathematics on second year and one participant on first year level. One participant obtained an Honours degree in Mathematics Education. Two of the four participants were Afrikaans-speaking and were teaching in their second language. However, all four of the participating teachers were fully bilingual and fluent in both Afrikaans and English. All four participants were white. Three of the four participants were female. Their teaching experience varied between two and nineteen years.

I investigated mathematics teachers' interactions with learners' errors in four grade 9 classrooms over a period of almost two school terms, with a time span of more than three months, from the middle of July to the middle of October. I spent 45 minutes or 90 minutes, depending on single or double learning periods, daily, for a two-week period in each participant's classroom. To spend two weeks in each of the four schools was a pragmatic decision. As the GDE does not allow research to be conducted during the fourth school term, the time allowed for fieldwork, subsequent to obtaining ethical clearance in July 2008, implied two-week visits to each of the three departmental schools. One of each participant's grade 9 groups of learners was selected by the particular research participant as the group I was joining as an observer.

The South African mathematics curriculum for schools is underpinned by modern socio-constructivism, described by Maree (2004) as: "the epistemological basis of the outcomes-based approach to teaching, learning and assessment in mathematics in South Africa" (p. 246). The sections of the grade 9 mathematics curriculum (DoE, 2002) that were discussed during the periods of fieldwork were the following:

- Learning Outcome 1: Numbers, Operations and Relationships: to solve problems in financial context and to solve problems that involve ratio rate and proportion.
- Learning Outcome 2: Patterns, Functions and Algebra: to represent and use relationships between variables, to draw graphs on the Cartesian plane, to solve equations, to find the product of binomials and to factorise algebraic expressions.



- Learning Outcome 4: Measurement: to solve ratio and rate problems involving time, distance and speed.

The study is, however, not embedded in a particular learning outcome or topic.

A focus on the learners and the coding of learners' mathematical errors were beyond the scope of this study. However, the learners' mathematical errors and the teachers' interactions with these errors were fundamentally important to the study. The contextualization of the research thus necessitated the presentation of information regarding class sizes and racial and gender compositions<sup>1</sup>. The class sizes varied between 22 and 37 learners. The composition of the classes portrayed mainly black and white learners with black learners as the majority in three of the four schools. The female learners formed a slight majority in two of the co-educational schools and a large majority in one of these.

### **3.7 LIMITATIONS**

Possible bias could exist on my part due to my experience of changing my own mathematical beliefs through my introduction to and involvement in the PGCE programme. It might have resulted in a critical and subjective way of looking at how mathematics teachers interacted with learners' errors. Through my exposure to scholarly literature, I became convinced about the pedagogical value that learners' mathematical errors carry. This might have influenced my perceptions of how teachers were supposed to interact with learners' mathematical errors.

My presence during learning periods might have had an influence on the dynamics of the classroom situation (Nieuwenhuis, 2007 b), although I attempted to avoid that as far as possible. The exposure of the teachers to the structured interview questions and the focus on error-handling during the semi-structured interviews might have had an influence on the way teachers interacted with errors during the research period. Their participation in the research project might have caused them to be more aware of their approach to learners' errors.

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<sup>1</sup> Although neither the race nor the gender of the participants holds relevance to the study, recounting race and gender is customary within the new, democratic South African dispensation.

The actuality of postponing the data analysis until after completion of the data collection process, might have limited the potential of cross-case comparisons (Miles & Huberman, 1994) relating to idiosyncratic issues raised by the individual participants during the semi-structured interviews. I found my role as interviewer exceptionally challenging. That might have impeded the value of the semi-structured interviews as a method of data collection. Sections of the mechanical recordings were inaudible. Potential data were lost due to the problem.

### **3.8 SYNOPSIS**

This chapter gives an account of the research methodology. The chapter opens with a confirmation of the research paradigm in which my theoretical perspective is located, delineates my philosophical assumptions and the research methodology in which the assumptions culminate. This is followed by a succinct personal narrative, explicating my role as researcher. The choice of a multiple-case study design is accounted for next. Flowing out of the case study design, data collection strategies and approaches to data analysis are particularized. Measures of quality control, ethical issues and the scope of the study are subsequently dealt with. A reflection on the limitations pertaining to the study concludes the chapter.

## CHAPTER FOUR:

## ALICE'S CONTEXTUAL NARRATIVE

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### 4.1 INTRODUCTION TO ALICE

Alice taught at school A (see section 3.6.2). She was a white English-speaking female with 19 years teaching experience. She had been teaching mathematics her entire teaching career, although she did not feel comfortable teaching mathematics beyond grade 10 level. She obtained a B Sc degree with mathematics at first year level and completed a Higher Educational Diploma. She defined mathematics as:

*A language people use to communicate ideas and understand how society functions and how the world works. We actually cannot teach people/children, but we help them find it within themselves.*

She described her strengths as a mathematics teacher as “patience, sympathy, kindness, thoroughness and questioning approach”.

She expressed the way she thought mathematics was best taught as:

*I like to hear first what learners think before explaining a new concept. I try to get it out of them, by asking questions, so that they actually come up with the solution. I think it's critical to link up with their prior knowledge, so as to start at a point they are comfortable with.*

### 4.2 CLASSROOM VIGNETTE

The learners were not observed gathering and lining up outside the classroom prior to the commencement of the learning period. However, the learners did not take their seats on entering the classroom. They remained standing and waited at their desks to be seated. Following the exchange of greetings, they were prompted by Alice to sit down. The atmosphere in the classroom was pleasant, relaxed and friendly. Alice was patient and not prone to raising her voice. In one or two exceptional instances, Alice had to address learners to establish order in the classroom.

Daily checking of individual homework and book-control were meticulously done by Alice. Each day, as the homework was allocated, the learners were usually supplied with the correct, final answers to the problems. During the following learning period, Alice would ask about possible problems learners had encountered with solutions not corresponding to those previously provided. Alice primarily demonstrated and explained

the solutions to these problems on the blackboard. In these instances, Alice completed the book-control during the final, tutorial-type segment of the learning period.

Occasionally, appointed learners would demonstrate their solutions on the blackboard while the learners were marking their homework and Alice was assisting individuals and doing book-control. Learners, called upon to demonstrate their solutions on the blackboard, wrote these down without providing explanations to or interacting with their peers. Alice used to interrupt her book-control to check the learners' solutions on the blackboard. Errors in these solutions were seldom observed. In instances where errors did occur on the blackboard, Alice corrected the errors immediately without involving other learners in a discussion. Learners per se were not requested to identify errors in their peers' work on the blackboard.

As and when the situation dictated, an overhead projector was used for learners to view the homework memorandum. Learners were allowed an opportunity to mark and to correct their mathematical attempts while Alice was interacting individually with the learners, executing book-control and encouraging and rewarding learners with stickers or stamps in their workbooks. Learners were allowed to discuss the mathematics with those in close proximity during this time.

Alice's commitment to this task of homework-control and her apparent focus during the execution thereof could probably be interpreted as driven by a concern for the learners' academic interests. She ensured that learners were not falling behind and that their workbooks were complete for proper assessment preparation. Alice explained her concern about the learners' workbooks:

*Your summary must be nice and neat 'cause this is what you refer to when you do your homework later.*

*It's good to draw the tables 'cause then you can see what's going on when you revise.*

She accepted an excuse regarding a lack of understanding from a learner who did not complete her homework. There were no apparent repercussions for the learner.

The following segment of a learning period usually entailed the typical introduction of a subsequent mathematical topic through expository teaching. Alice would announce a new mathematical topic to learners and advise learners to write a suitable heading in their scripts. She demonstrated and explained relevant mathematical examples on the

blackboard while the learners took these down from the blackboard, concurrently with the teaching. Alice seemingly accepted responsibility for the mathematical interpretation of problems. When learners encountered a problem regarding the graphical representation of cellular phone costs, Alice repeated the problem as stated in the textbook and confirmed the interpretation thereof. The crux of the matter was deciding upon the continuous or the discrete nature of the graph.

*Let me just tell you what they say, because ... they say ... here we go* (Alice reads from textbook)

*If it is per minute billing then you do not join.*

*If it's per second then you can.*

Alice attempted to involve the learners by asking stepwise contributions from them and by encouraging learner participation with occasional questions. The majority of questions that Alice asked learners during the discussion of new mathematical examples were related to reminding learners of procedural information, exemplified by the two excerpts that follow:

*What do you do normally when you want to get rid of the denominators?*

*Now how can I make  $x$  minus one zero?*

Alice did not exhibit an inclination towards asking reflective questions or questions that required mathematical interpretation from the learners.

Alice accommodated learners' mathematical contributions. She acknowledged learners' alternative approaches to solving mathematical problems during the discussion of the homework. She exhibited a deliberate attempt to recognize every learner's mathematical contribution and portrayed respect for each learner's opinion during the teaching. Alice attempted to accommodate learner contributions in her explanation and exposition of mathematical procedures. She allowed time to incorporate these learner contributions. She remained patient and made an effort to follow learners' thinking.

Alice often attempted to contextualize mathematical problems with personal anecdotes or by relating mathematical problems to learners' everyday lives. She referred to activities

like buying perfume, compact discs or cold drinks at the tuck shop or playing the Lotto<sup>1</sup>. Alice illustrated indirect or inverse proportion in the following way:

*Think of Survivor<sup>2</sup>, ladies. Let's say both tribes have to eat from it. On the other hand, only the one tribe. Fewer people eat the food, the longer the food will last.*

She referred to an example in realistic context to illustrate a continuous graph and how to decide on the discrete or continuous nature of a graph.

*What do you buy in the shops where they charge you per gram? All the stuff that they weigh, bananas, cheese, cold meat. They don't round up. They actually charge you per gram. If you buy cheese, it will say nought comma three two eight grams. Then you draw a solid line.*

The final phase of the learning period was typically utilized in a tutorial-like manner. Learners had the opportunity to start with their homework. The exercises learners had to try on their own for homework were similar to the mathematical examples Alice had previously demonstrated. Discussion among learners was allowed and encouraged during this segment of the learning period. These discussions were, however, seldom expanded to whole-class discussions. Alice suggested that learners work in pairs and compare their solutions. She motivated learners to persist with their mathematical attempts.

*You can talk to your partner next to you.*

*Just talk about it a bit more.*

While the learners continued with their homework, Alice attended to individual learners on demand. Consequently, much of the teacher-learner interaction happened on a one-to-one basis.

During the tutorial-type segment of the learning period, a tendency among learners to call Alice in order to look for their mathematical errors was observed. Alice was not observed asking learners to explain their thinking, but rather interpreted and verified learners' solutions with her own thinking. The authority to evaluate the quality and correctness of mathematical statements and solutions appeared to reside with Alice. Three exemplary excerpts are cited below.

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<sup>1</sup> The Lotto is a national gambling game of which a percentage of the proceeds is donated to charity.

<sup>2</sup> Survivor is a reality series broadcasted on national television.

*T: Huh, huh<sup>1</sup>, look. You see, there's your plus four. There's your mistake. There you had a minus, then that's wrong there. There's your error. You can go on from there.*

*L: Ma'am, I've got this one wrong.*

*T: I know. I'm looking, I'm looking.*

*L: I said four hundred and eighty divided by eight.*

*T: I wonder now where that eight's coming from?*

Alice would not hesitate to interrupt learners during the tutorial-type segment of the learning period when she was of the opinion that too many learners were struggling with a similar problem. She then discussed the problem publicly with the whole class and illustrated the approach on the blackboard. Alice invited learners to see her during break when learners did not have time to sort out their mathematical problems during learning periods.

### **4.3 ALICE'S BELIEFS ABOUT LEARNERS' MATHEMATICAL ERRORS**

Alice classified learners' errors by distinguishing between unique errors, common errors, arithmetic errors and/or minor errors. She declared learners' errors to be inevitable during the learning process.

*You know, in the, ... a couple of years ago I may have been frustrated when they got things wrong, or whatever, but now I, it's part of the course, it's inevitable.*

She believed that learners' errors were essentially part of learning and had the potential to enhance learning. She regarded learners' errors as valuable learning opportunities that she purposively anticipated during classroom interaction. Alice declared learners' errors as initiators or catalysts for discussion and for learning. Hence, according to Alice, learners' errors were an expository basis from which teaching could develop.

*And I often say, you know, thank you. 'Cause without that we'll have nothing to talk about.*

*And often then I will show the class a mistake that was made. I'll show the class the step that was incorrect and then I'll say you know what should she have done? So often I'll use it as a learning to; a point for learning.*

---

<sup>1</sup> "Huh, huh" is a sound that means "no".

Alice held the opinion that learners' errors strengthened mathematical understanding through the juxtaposition of erroneous mathematical procedures with correct solutions. When asked whether she thought errors were useful as learning opportunities, she responded as follows:

*Oh, definitely, because it I think it strengthens understanding. No child is errorless; no child never makes a mistake and ... No, I think errors are good. Errors are good in that you can, sort of learn something from it. And we get that aha! light bulb thing going on, you know, which is great. I think sometimes they need to know the wrong way to appreciate the right way, you know.*

Although she preferred neat learner scripts, she realized the pedagogical disadvantage when learners attempted to record only correct procedures in their scripts following the perfection thereof during rough work.

*You know, apparently one shouldn't make the kids do rough work ... rough work and then neat in their book. They ... they should make mistakes. They can see their errors and it's part of learning, so I'm ... I'm fine with it. I like nice neat books, but they can still make mistakes.*

Alice clarified her focus of interacting with learners' errors in their written assessment tasks as the provision of correct procedures and not on probing the errors to determine the cognitive sources thereof. In resonance with her opinion on the role of errors in the teaching and learning of mathematics in the classroom, the value of learners' errors in written assessment tasks resided in the juxtaposition of learners' errors with correct procedures.

*But, I just feel that too many girls will slip through my fingers if I don't on paper address their problem. Then I know I don't need to go back and address each one individually. I put it on their paper, I've explained it, I've written a comment there. I've shown that is not equal to that and I've shown them what is correct. So, it may be quite red in the end, but I feel it's important that they could see.*

Time constraints, due to the requirements of the educational system, were cited as an inhibiting factor in the ideal approach to learner errors and to formative assessment practices.

*You know what I think is the ideal and it's always such a challenge because of time constraints and requirements. I sometimes feel those requirements come in the way of proper learning.*



Alice was of the opinion that the aims and the outcomes of prescribed portfolio assignments did not always critically assess learners' mathematical understanding appropriate for that specific time.

#### **4.4 ALICE'S INTERACTION WITH LEARNERS' MATHEMATICAL ERRORS**

##### **4.4.1 Verbally During Learning Periods**

###### ***Professed***

During the semi-structured interviews, Alice described her response to learners' errors as positive. When asked how she interacted with learners' errors during learning periods, she responded as follows:

*No, I think it needs to be dealt with in a positive way. You know I don't, ja<sup>1</sup> ... I think you need to respond to the child positively, because ... I try and make errors a good thing, you know. I often say to the kids I'm so glad you did that so that I can show the class what can go wrong and how it should, ... how we should complete the sum. So, it is not good to make errors, obviously you wanna get full marks. But I don't make it like ... oh no, how could you do that, weren't you listening! I never say that. It's more the type of thing I'm so glad you slipped up there 'cause now we can talk to the class and maybe you're not the only one. Let's sort that out. So it's for me it's something to talk about.*

She illustrated her asserted interaction with learners' errors during learning periods with the claim that she expressed appreciation and commended learners for making errors.

*Like I say, I will ... I will more often than not praise an error, you know, and ... and not just dismiss it.*

She claimed to utilize learners' errors instantaneously during classroom interaction. According to Alice, she utilized learners' errors pedagogically as an expository basis from which teaching could develop.

*I even ... sometimes the girls will make an error that is just so perfect, I actually use it.*

Alice described herself as patient and tolerant to learners' errors. She did not allow learners' errors to frustrate her. She asserted that she refrained from disparaging learners either verbally or by tone of voice. She stated that she attempted not to disapprove of learners' errors.

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<sup>1</sup> "Ja" is the Afrikaans translation for "yes".

*If I think about it, I never talk nasty, I never use a bad tone, so I think if I ... if they see I'm accepting of their error, they are going to be accepting of it as well, and I don't belittle them and I think there's a ... a nice comfortable atmosphere.*

In instances where Alice suspected learners' errors to be complex and beyond the limits of a classroom discussion, she followed up with individual attention. Alice did not make errors on the blackboard on purpose. However, she indicated a keen interest in obtaining information regarding the pedagogical value thereof. She was inclined to such an approach if the approach could enhance mathematical learning.

*I don't do it on purpose, I don't always think to do that. Is that, would that be an advisable thing? Occasionally? You don't wanna do it every lesson?*

Alice recognized the pedagogical benefits of peer involvement in the detection of learners' errors. However, she did express her concern about the effect this might have on the emotional wellbeing of the more sensitive learners in her classroom. Alice focused on creating an atmosphere in the classroom in which learners were experiencing sufficient safety and comfort to make errors. Although Alice professed to purposively creating and maintaining a safe learning environment, she was cognisant of learners who were protecting themselves from exposure to peers.

*I think pointing out the error, this is a very tricky one because, ... Well, quite frankly, whether I point out the error or the class, the learner could feel uncomfortable. But I think if it's got a good environment in the class the learner shouldn't feel bad either way. But there's a fine line between the child who made the mistake and the class seeing the mistake. One has to address it in a positive way, you know.*

### ***Enacted***

I decided to use the video-recorded classroom events as an auxiliary source of data. The teachers' facial expressions and their body language were hence not employed as potential data. However, the alternative mechanical recordings of the classroom events produced teachers' verbal interjections (see section 3.6.4) in their interactions with learners' mathematical errors. During classroom observations and on the mechanical recordings, Alice generally acknowledged learners' mathematical contributions and apparently responded to all audible contributions. Alice once used an Afrikaans exclamation that

appealed to the learner with a cautionary tone: “O, Jong<sup>1</sup>!” in response to the learner’s error.

Although Alice was inclined to procedural teaching (see section 1.6.2), she accommodated learners’ contributions and attempted to incorporate these contributions in her teaching. Her interactions with the first two learners (L1 and L2) and with learner four (L4), in the following excerpt, are illustrative of this. Alice portrayed an inclination towards paraphrasing and rephrasing learners’ mathematical contributions in order to enhance the mathematical value thereof. Alice’s interactions with the first two learners (L1 and L2) and with learner seven (L7), in the excerpt below, serve to demonstrate the tendency. The excerpt pertains to solving the equation<sup>2</sup>  $\frac{3a}{5} - a = 2\frac{1}{2}$ , eventually written to  $\frac{6a}{10} - \frac{10a}{10} = \frac{25}{10}$  and ultimately to  $-4a = 25$ .

*T: Let’s look at this: solve for x. Any ideas? Anything that you think we should do? Yes?*

*L1: We can say two times two plus five, plus one, is five over two.*

*T: You’re taking this to an improper fraction, okay? Two times two is four plus one is five, over two. It’s an improper fraction now, which is good.*

*L2: Can’t you find the lowest common multiple, I mean denominator?*

*T: Okay, for the five and the two, because we want to eliminate fractions. What would it be?*

*L2: Ten.*

*T: Okay, so what should we do with that ten? What do we do normally when we add or subtract fractions?*

Numerous examples of Alice correcting learners’ errors through transmission, were recorded. Alice usually attempted to convey to the learner why the contribution, the response or the question, was erroneous. Alice’s interactions with the third and the fifth learners (L3 and L5) in the excerpt serve as examples.

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<sup>1</sup> Oh, careful!

<sup>2</sup> The mathematical problems pertaining to case A are quoted directly from Laridon, et al. (2006). Please consult the list of references.

L3: *Can't you get all the fractions on the same side and the ...*

T: *Fractions on the same side? I thought in equations we try to get all the variables on one side and the numbers on the other. This fraction doesn't have a variable. This one does. This is actually perfect. We've got all the variables on the left and the constants on the right, even though it's a fraction. So I think let's rather go this route. Leave it the way it is.*

Alice confirmed and incorporated the fourth learner's (L4) contribution in her teaching.

L4: *Can't you make that minus a over one?*

T: *Very good, yes! Minus a over one.*

L4: *And then all over ten.*

T: *Should we make the denominators ten?*

Alice responded to the fifth learner's (L5) apparently inappropriate suggestion by transmitting procedural information regarding the difference between fractions to the learner.

T: *What do we do with the first fraction?*

L5: *Don't you have to do minus a first?*

T: *Minus a from what? Okay, be careful. We could do that. Minus three a over five minus a, we could do that, but we need to have the same denominator. There are various ways we could take from here that could work, but we can't just subtract now.*

Alice's remark to the sixth learner (L6), as cited below, was not mathematically sound. In my opinion, to multiply the balanced equation with two would not have been the preferred or the most effective strategy, but the principle was not different to multiplying the equation with ten, as suggested by the fourth learner (L4) and to which Alice seemed to be steering. The solution to such an equation would not differ from the original one.

T: *(to another learner, L6) You said we multiply by two. I don't agree with that, 'cause if you multiply by two are you doubling and then you are changing the sum. In actual fact you are making everything bigger by two.*

Alice paraphrased and rephrased the seventh learner's (L7) mathematical contribution in order to enhance the mathematical value thereof.

L7: *Can't you times the denominator and the numerator by two?*

*T: Good! By multiplying by two over two we are multiplying by one and it doesn't change. You're just making equivalent fractions, okay?*

Alice endeavoured to clarify learners' errors and misconceptions with explanatory monologues. Her interactions with learner eight (L8) and with learner nine (L9), presented below, serve as demonstrations thereof. Learner eight (L8) seemed to portray confusion between an index, the power to which some base is raised exponentially, and a coefficient; resulting in an over-generalization of an exponential rule.

*T: Next step? Yes?*

*L8: Ma'am, I think maybe we can take the ten to the top and you make it negative ten and it's gonna be six a minus ten and minus ten a minus ten.*

*T: You know what the problem is there? We're not working with exponents. Remember if you have something like this that becomes ten to the positive one? I think you're getting confused with exponents, okay? If you've got exponents, negative exponents, in the denominator that is what happens with exponents, okay? So let's not make that error there, okay? You can't do that. This is a fraction; six tenths of a.*

Learner nine (L9) plausibly experienced difficulty in relating multiplicative inverses to the procedure that Alice had demonstrated to solve  $a$  from  $-4a = 25$ . In addition to illustrating how Alice explored the learner's mathematical misconception, Alice's interaction with learner nine (L9), as cited below, serves to illustrate the learner's tendency to search for algorithms or mathematical, procedural rules.

*L9: How do you know? Why is the twenty-five the numerator? How do you know whether it's the numerator or the denominator?*

*T: (inaudible)*

*L9: But one is twenty-five and one is four?*

Alice did not initially address the learner's particular question.

*Because remember the operation here. It's minus four times a. This is your operation. It's multiplied on this side. It is twenty five divided by minus four. This is the operation change. Nothing happens to the sign. You go from multiply. If the operation is divide it's vice versa. Signs don't change.*

The learner continued with her attempt to clarify her own uncertainty.

L9: *If the one for a was on the right hand side would that be the numerator? If twenty-five was on the other side, would it be at the bottom? Is it always the right hand side on the top and the left hand on the bottom?*

Alice then realized what the learner was actually referring to, subsequent to witnessing her attempt to find an algorithm or rule for the procedure.

*The coefficient, the number at front of the a is in the denominator. You're dividing by the coefficient.*

The recorded classroom events produced a limited number of instances where Alice employed instructional scaffolding (see section 3.6.4) of individual learners through questioning or compelling learners to reflect on their own mathematical reasoning. An excerpt from one such an instance ensues. Learners had to solve the following problem: Sam needed 47 balls of wool to knit a jersey. She bought five packets and two separate balls of wool. How many balls were in each packet?

T: *Let's have a look here. Just check. Do they ask you how many packets there were or how many balls in each packet?*

L: *Yes, how many balls.*

T: *Were in each packet? But there are only 47 balls and five packets. Five times 21 is gonna be over 47. I think you need to rethink that. I'm not going to tell you. You can figure it out. Read the question carefully again.*

The impression was given that Alice attempted to put learners at ease about their competency in mathematics by classifying learners' mathematical errors as "minor problems". Alice appeared to be inclined to classifying learners' errors. However, congruent to her interaction with learners' errors in written assessment tasks, the classification portrayed, according to me, a superficial stance on learners' mathematical errors. Three excerpts ensue:

*It's normally not a major problem. It's normally just a careless mistake as well.*

*There's your mistake. So, multiplication error.*

*Your sign<sup>1</sup> is a problem.*

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<sup>1</sup> Although I take cognisance of teachers' use of this terminology, I personally do not subscribe to it and interpret the nature of such an error as considerably more intricate.

In resonance with her tendency to enhance the mathematical value of learners' mathematical contributions, as previously illustrated, Alice occasionally refrained from interacting with learners' errors for the sake of the ultimate mathematical goal or rephrased learners' contributions without emphasizing the mathematical errors. Two illustrative excerpts are displayed below. Alice posed the equation  $\frac{x}{6} + x - \frac{5}{2} = x - 2$  to learners to solve and asked for contributions from the learners of how to approach the procedure.

*T: What do you do normally when you want to get rid of the denominators?*

*L: Just cancel out.*

*T: In other words, you're multiplying each of the numbers by six.*

Another example concerning fractions in balanced equations ensues. Learners had to solve the equation  $\frac{x-2}{2} + x = 2$  which had been written to  $\frac{x-2}{2} + \frac{2x}{2} = \frac{4}{2}$ .

*L: I cancelled out all the denominators, 'cause they're all the same.*

*T: You multiplied the entire equation with two.*

A limited number of instances in which Alice entertained (see section 3.6.4) learners with their mathematical errors was recorded. Alice wrote the following equation on the blackboard:  $(x-1)(x+3) = 0$  and subsequently requested suggestions from the learners.

*L: Don't you use nought as your x?*

*T: Let's say x is nought. What is this bracket equal to?*

*L: Nought.*

The learner's peers responded in a chorus with: "minus one!"

*T: Oh, oh, minus one times three is minus three. Not gonna work.*

Infrequent instances of learners' errors that were not detected and instances in which Alice accepted and confirmed erroneous statements were recorded. Learners had to complete a table for the following problem. The cost of hiring mountain bikes at a holiday resort in KwaZulu-Natal is R25 per hour.

*T: Does the cost of hiring mountain bikes per hour (own emphasis) decrease the longer you hire the bikes?*

*L:* No, it gets more. (The rate per hour actually stayed constant.)

*T:* It gets more and it's in proportion. (Alice formulated the question inappropriately.  
The total cost increased with the time. The hourly rate stayed constant.)

### ***Comparative Synopsis***

During the semi-structured interviews, Alice professed to anticipate learners' errors purposively during learning periods and to utilize learners' errors instantaneously during classroom interaction. However, although Alice's interaction with learners' errors was observed to be routine, incidents of utilizing learners' errors for teaching were not recorded. Alice alleged to be patient and tolerant to learners' errors and to respond positively and collectedly. Her observed interaction with learners' errors confirmed her assertion. The following variety of predominant categories emerged from Alice's interactions with learners' errors during learning periods:

- Correction of learners' errors through transmission, concurrent with an explanation.
- Classification of learners' errors.
- Enhancing learners' mathematical contributions.

Alice typically augmented her correction of learners' errors through transmission with explanations of what was mathematically unacceptable. She was inclined towards clarifying learners' errors and misconceptions through monologues. She often paraphrased and rephrased learners' mathematical contributions, refraining from accentuating the learners' errors, in order to enhance the mathematical value of the learners' contributions. She classified learners' mathematical errors in a way that created the impression that she was putting learners at ease about their errors. Although Alice attempted to create an atmosphere of safety and comfort in her classroom, she was cognisant of learners who protected themselves from exposure to peers. Due to her concern for the emotional wellbeing of the more sensitive learners in her classroom, Alice preferred to refrain from the public discussion of learners' errors and from peer involvement in error discussions.

#### **4.4.2 In Writing in Assessment Tasks**

##### ***Professed***

During the semi-structured interviews, Alice affirmed her conviction about the pedagogical value of sustained revision and continuous, formative assessment. She indicated a positive



link between formative assessment, self-assessment, and a reflective learner disposition. She further expressed a strong support for reassessment and described an ideal approach to reassessment as setting and administering duplicate test papers for each term test.

*But, what the ideal is, I think, is to reassess with similar questions that test that you have discussed. In the perfect world one would then, two or three days later, give them the same test, different questions, slightly changed, but give them the same. So, the ideal is to actually set two tests every time.*

Although Alice admitted to not consciously pondering on error-triggering questions, she instantaneously identified two such questions from her own experience and was indisputably positive about the importance thereof.

*So, that's ... that's one that comes to mind now which yes, we often ... I don't sort of ponder it and look for problems like that, but I ... I will probably put one in where I know I'm gonna be testing a certain concept.*

Alice distinguished between “common”, “unique”, “arithmetic”, “minor”, etc. learners’ errors in written assessment tasks. She differentiated her interaction with learners’ errors in written assessment tasks accordingly. Learners’ errors were circled with a red pen during the process of marking. She classified learners’ errors as far as possible in the learners’ scripts with succinct, written remarks. Intricate errors, especially those that were, according to Alice, unique to a particular learner, would be corrected on the learner script with a complete, correct procedure.

*I ... I like to write a note on their test. I circle the mistake. If they have tried something, but have gone wrong I will often complete it correctly; write it next to theirs. Specially, if it's not a common problem, but a unique problem to that child. I will circle the problem and show them how they should have completed it. That type of thing, you know ...*

Alice identified arithmetic errors or calculation errors, albeit not necessarily correcting them.

*... and of course, if it's just a small arithmetic error I will method mark. They'll perhaps lose a mark there. If they've said three times two is five, I'll say wrong operation, you needed to multiply, you added. I tell them what they've done wrong and then I'll method mark the rest. So then they can see. They've followed on correctly from there.*

Alice referred to obvious errors, that she expected learners to recognize themselves, as “minor errors” and declared that she did not interact with those as such. However, Alice

gave the impression that she would highlight those errors during the memorandum discussion.

*So, if I haven't addressed it on paper when I feel it's a minor error that the learner can just see on her paper, I will use it as a starting point for my lesson to address those issues, answer questions and then move on.*

Common, prevalent learners' errors were recorded and discussed in the classroom.

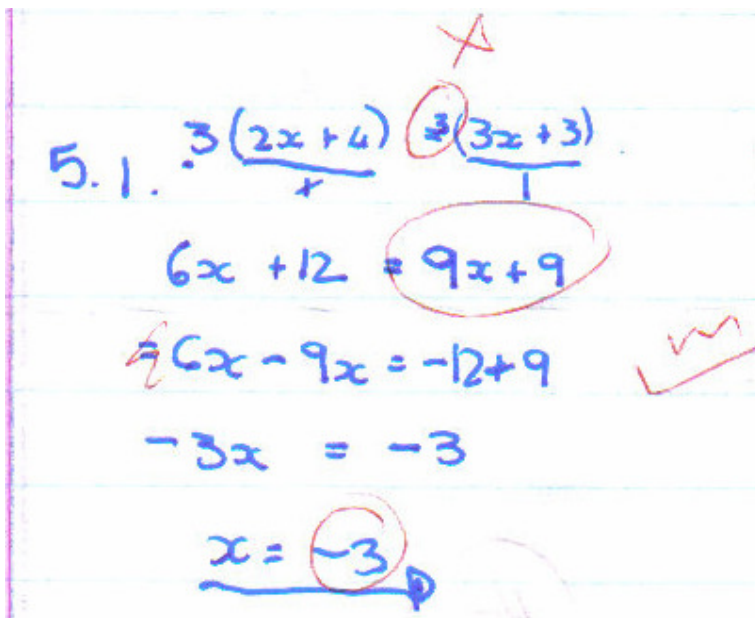
*If there's a common problem, I'll make a note of it on the memorandum that I can discuss as an overview for the whole class.*

Alice professed to an obligation to provide complete, correct procedures to learners in instances where the learners' errors were of a less general type.

*You know, I feel that you've got twenty to thirty learners in a class and they don't all have the same problem in a test. And it's difficult to address all the individual problems in the class the next day. If there's a common problem, I'll make a note of it on the memorandum that I can discuss as an overview for the whole class. But, I just feel that too many girls will slip through my fingers if I don't on paper address their problem. I put it on their paper I've explained it, I've written a comment there.*

### ***Enacted***

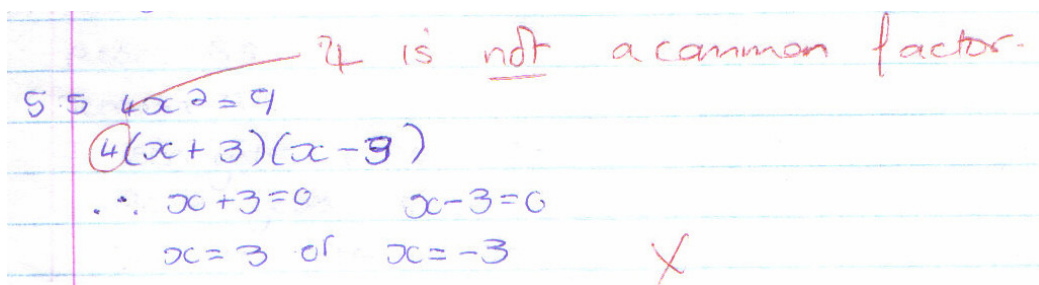
Analysis of the written assessment tasks showed that Alice used a variety of markings to indicate learners' errors in assessment tasks. In some instances, Alice refrained from supplying explanatory, written comments about the nature of the errors. In the following example, a learner expressed both sides of the equation  $3(2x + 4) = 3x + 3$  as fractions with one as the denominator. The learner then wrote three as a coefficient of the numerator on the right hand side of the equation, probably over-generalizing the approach to obtain similar denominators to numerators. Alice encircled the three, marked it wrong, encircled  $9x + 9$  the learner got as a result and encircled the final, erroneous result. Alice, however, did not show the correct solution or give any written, explicative comments. The four participating teachers regularly ignored learners' inappropriate use of the equal sign when working with equations. However, this excerpt also demonstrates an instance in which Alice indicated the incorrect syntax.



5.1.  $3(2x+4) = 3(3x+3)$   
 $6x + 12 = 9x + 9$   
 $6x - 9x = -12 + 9$   
 $-3x = -3$   
 $x = -3$

Figure 4-1 Question 5.1 (i)<sup>1</sup> of written assessment from school A

Alice often attempted to indicate the nature of their errors to learners by supplementary written comments. Alice responded with “4 is not a common factor” on the learner’s attempt to approach  $4x^2 = 9$  with  $4(x + 3)(x - 3)$ . However, Alice did not comment on the fact that the learner disregarded the right hand side of the balanced equation.



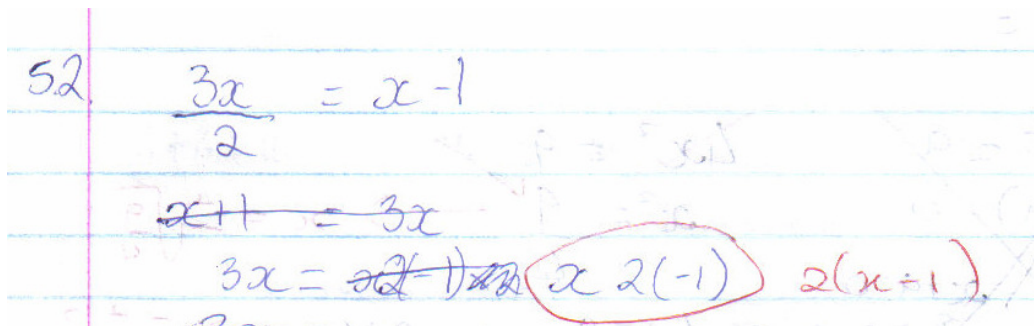
5.5  $4x^2 = 9$   
 $4(x+3)(x-3)$   
 $\therefore x+3=0 \quad x-3=0$   
 $x=3 \text{ or } x=-3$

Figure 4-2 Question 5.5 (i) of written assessment from school A

In other instances, Alice indicated an error and subsequently corrected the error, refraining from supplying the learner with a written explanation of the nature of the error. In the subsequent example the learner, whilst multiplying the balanced equation with two, failed to multiply every term with two. In addition to that, the learner merged the two terms on the right hand side and wrote them as one product. The learner rewrote  $\frac{3x}{2} = x - 1$  as

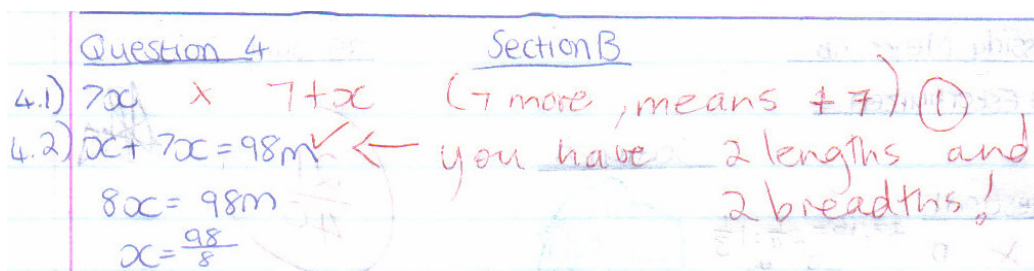
<sup>1</sup> The Roman numerals are used to indicate reference to the same question in more than one learner’s script.

$3x = x2(-1)$ . Without supplying the learner with an auxiliary, written explanation of the error, Alice encircled and corrected this with  $2(x - 1)$ .



**Figure 4-3 Question 5.2 of written assessment from school A**

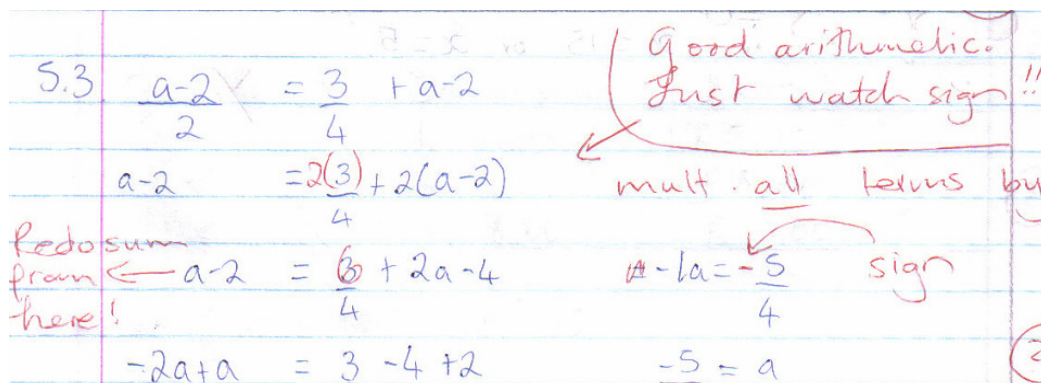
Alice regularly supplied learners with explanatory, written commentary on their scripts. In the example below the learner interpreted the relation between the length and the breadth of a rectangle incorrectly. Alice recorded the corrected value of  $7 + x$  and explained the nature of the error with the phrase: “seven more means plus seven”. The learner subsequently regarded only one length and one breadth of the rectangle in expressing the perimeter thereof in a formula. Alice indicated the nature of the error with the comment: “You have two lengths and two breadths!”



**Figure 4-4 Question 4 (i) of written assessment from school A**

In the ensuing example, a learner repeated the error of failing to multiply every term in the equation with the lowest common multiple by rewriting  $\frac{a-2}{2} = \frac{3}{4} + a - 2$  to  $a - 2 = \frac{3}{4} + 2(a - 2)$ . Alice attempted to correct the procedure and augmented her effort with the following remarks, explicating the learner’s error and the desired approach to solving the equation: “Multiply all terms by two” and “Redo sum from here!” This can serve as an example of Alice’s attempts to accommodate a learner’s approach to solving an equation. Alice refrained from improving on the learner’s inappropriate choice of two

instead of four as the lowest common multiple, attempting to facilitate the learner to continue with her (the learner's) strategy.



5.3  $\frac{a-2}{2} = \frac{3}{4} + a-2$

$a-2 = \frac{2(3)}{4} + 2(a-2)$

Redo sum from here!  $a-2 = \frac{6}{4} + 2a-4$

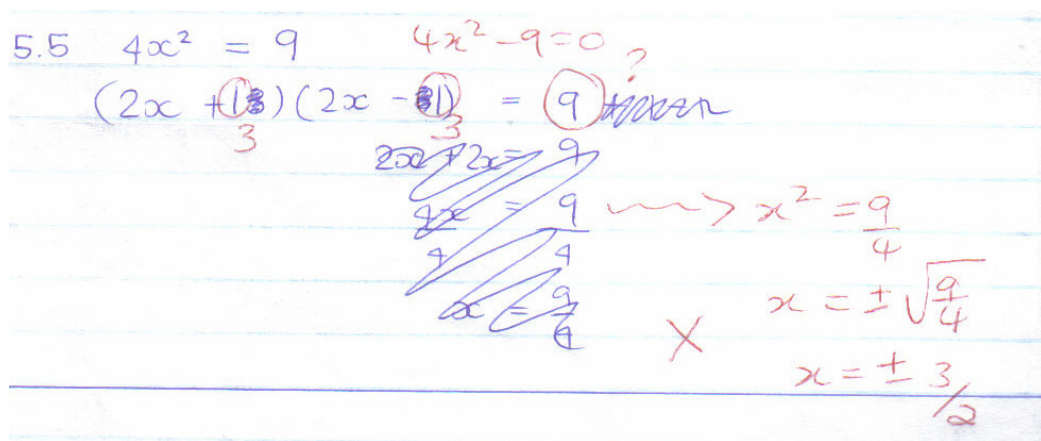
$-2a+a = \frac{3}{2} -4 +2$

$-5 = a$

Annotations: "Good arithmetic. Just watch sign!!", "mult. all terms by", "sign", "2"

Figure 4-5 Question 5.3 (i) of written assessment from school A

Below is an example of an incorrect factorization of  $4x^2 - 9$  to  $(2x + 1)(2x - 1)$ . Alice initially corrected these factors, albeit without utilizing that strategy to complete the solution. However, Alice recognized the format of an alternative strategy, that of  $x^2 = \frac{9}{4}$ , which the learner scratched out and used that to explain to the learner how to solve for  $x$ . Alice attempted to accommodate the learner's initial approach to solving the equation. She provided the learner with a complete, correct solution to the problem. This example is illustrative of several of the markings that Alice utilized to indicate learners' errors.



5.5  $4x^2 = 9$   $4x^2 - 9 = 0$  ?

$(2x + \frac{1}{3})(2x - \frac{1}{3}) = 9$  ~~is wrong~~

~~$2x + 2x = 9$~~

~~$2x = \frac{9}{4}$~~

~~$x = \frac{9}{4}$~~

$x^2 = \frac{9}{4}$

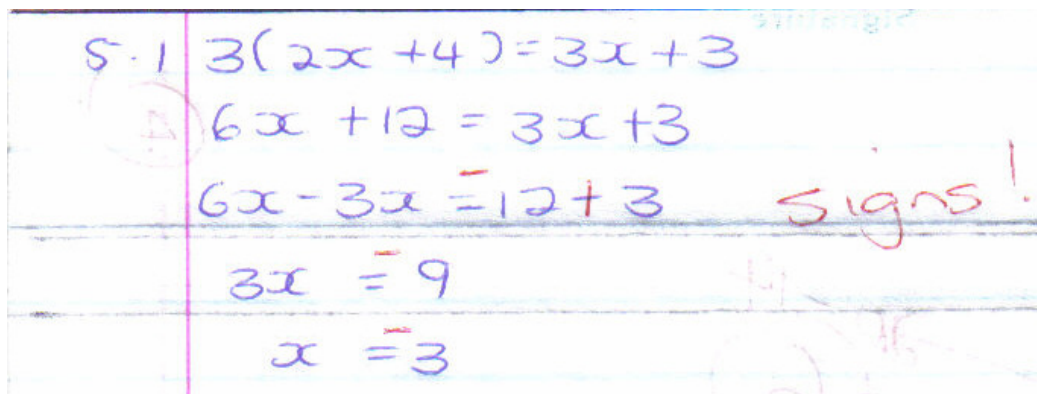
$x = \pm \sqrt{\frac{9}{4}}$

$x = \pm \frac{3}{2}$

Figure 4-6 Question 5.5 (ii) of written assessment from school A

Alice was inclined to classifying learners' mathematical errors. The observed classification included computational errors, careless errors, "sign" errors, and copied incorrectly. The following excerpt shows that the learner was not consistent in adding

number inverses in order to get zero as the identity element for addition. Confusion with the algorithm of *moving terms and changing signs*<sup>1</sup> on the right hand side of the equation seems plausible. The equation  $6x + 12 = 3x + 3$  was followed with  $6x - 3x = 12 - 3$ . Alice corrected the erroneous right hand side of the equation and classified the errors by writing: “signs!”



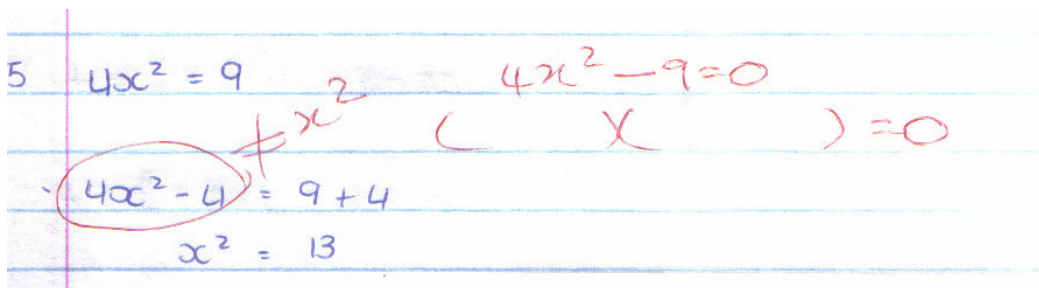
5.1  $3(2x + 4) = 3x + 3$   
 $6x + 12 = 3x + 3$   
 $6x - 3x = 12 + 3$  signs!  
 $3x = 9$   
 $x = 3$

**Figure 4-7 Question 5.1 (ii) of written assessment from school A**

In a limited number of instances, Alice supplied a learner with written comments that could potentially compel the learner to reflection and critical assessment of her own thinking. The learner erroneously performed subtraction as the inverse operation of multiplication. Alice appeared to overlook the particular misconception, but encircled the left hand side of the equation and wrote  $\neq x^2$  in response to the learner’s  $x^2 = 13$  that followed  $4x^2 - 4 = 9 + 4$ . I am of the opinion that to confront the learner with such a statement could potentially enhance critical reflection. In contrast to what I perceive as Alice’s commendable written interaction, she refrained from focusing the learner’s attention to the unbalanced equation resulting from the addition of negative four on the left hand side and plus four on the right hand side respectively.

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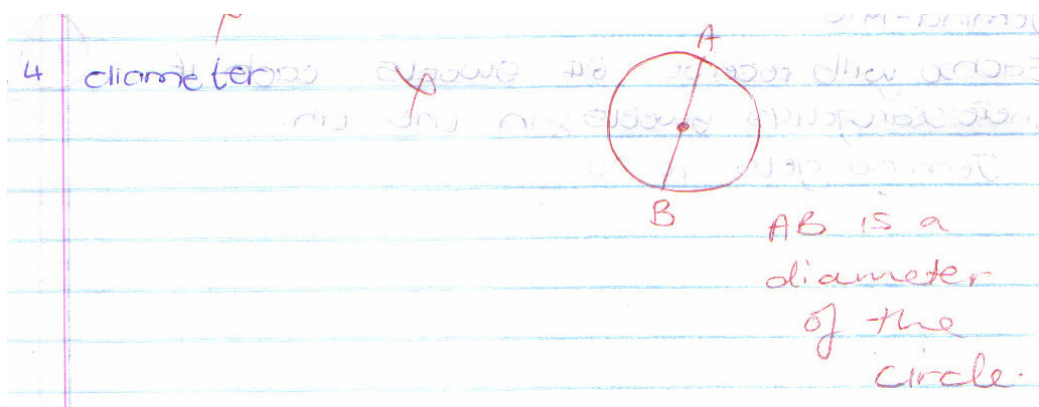
<sup>1</sup> I am aware of teachers’ inclination towards the teaching of this mathematical rule, albeit not personally subscribing to it.



5  $4x^2 = 9$   $4x^2 - 9 = 0$   
 $(x)(x) = 0$   
 $4x^2 - 4 = 9 + 4$   
 $x^2 = 13$

**Figure 4-8 Question 5.5 (iii) of written assessment from school A**

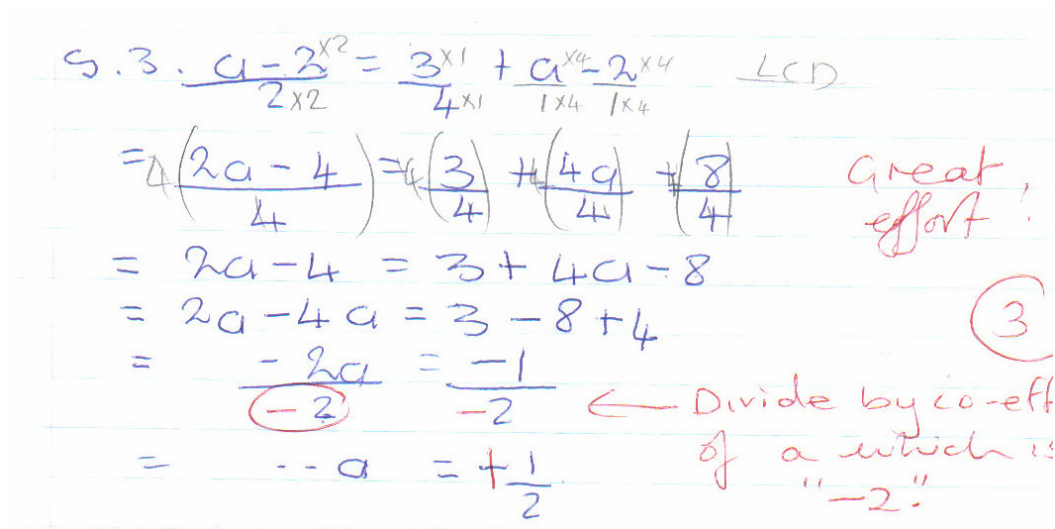
When, in the example below, the learner identified the distance around a rectangle as a *diameter* instead of a *perimeter*, Alice heuristically facilitated the learner to understanding. Alice employed a sketch of a circle that showed diameter AB. She explained: “AB is a diameter of the circle”. I sense that the implicit focus of the learner’s attention to the discrepancy between the two geometric figures could have compelled the learner to the critical assessment of her own reasoning, through reflection.



**Figure 4-9 Question 4 (ii) of written assessment from school A**

The following excerpt simultaneously serves as an example of an encouraging interjection and an instructional interjection (see section 3.6.4). The learner was unable to convert her solution of  $-a = -\frac{1}{2}$  to  $a = \frac{1}{2}$ . Alice compelled the learner to change her approach, of dividing by two, to the equation  $-2a = -1$ , by indicating division of the balanced equation by negative two instead, commenting with: “divide by coefficient of  $a$  which is  $-2$ ”. This time Alice did not accommodate the learner’s approach to solving for the variable. According to me, the suggestion to multiply the balanced equation with negative one would have been in line with the learner’s strategy. This excerpt further demonstrates an

instance in which Alice ignored the incorrect syntax of the learner's inappropriate use of the equal sign while working with an equation.



$$\begin{aligned}
 5.3. \frac{a-2x^2}{2 \times 2} &= \frac{3x^1}{4 \times 1} + \frac{a^{x4} - 2x4}{1 \times 4 \quad 1 \times 4} \quad \text{LCD} \\
 &= \frac{2a-4}{4} = \frac{3}{4} + \frac{4a}{4} + \frac{8}{4} \quad \text{Great effort!} \\
 &= 2a-4 = 3+4a-8 \\
 &= 2a-4a = 3-8+4 \quad \textcircled{3} \\
 &= \frac{-2a}{-2} = \frac{-1}{-2} \quad \leftarrow \text{Divide by co-eff of a which is "-2"} \\
 &= \dots a = +\frac{1}{2}
 \end{aligned}$$

Figure 4-10 Question 5.3 (ii) of written assessment from school A

### Comparative Synopsis

Alice indicated a differentiated approach to learners' errors in written assessment tasks, based on her perception of the nature of the particular error. She professed to interact with learners' errors in ways that varied between indicating the error to supplying the learner with a complete, intact, written solution. Content analysis of the written assessment tasks confirmed this assertion. Alice exhibited an inclination towards classifying learners' errors. A range of categories emerged from the analysis of her interactions with learners' errors in written assessment tasks. Alice persisted in recording common, prevalent learners' errors during the marking of the assessment tasks with the purpose of presenting these to the learners in the classroom during the memorandum discussion.

Content analysis of the written assessment tasks showed that Alice portrayed a relatively persistent approach to communicating with learners in writing. Some of the communication was encouraging in nature. Alice's written interaction with learners' errors in assessment tasks could conclusively be summarized as a sincere, but inconsistent attempt to explicative communication with her learners. The inconsistency of her communicative interaction is based on the emergence of a variety of categories described as follows:



- Errors indicated without an explanation and without a correction.
- Errors indicated with an explanation and without a correction, occasionally with an instructional interjection (see section 3.6.4 and appendix C).
- Errors indicated and corrected without an explanation.
- Errors indicated and corrected with an explanation.
- Errors indicated and corrected without an explanation, additionally provided with a complete, correct, written solution.
- An inconsistent indication of exact errors.
- An inconsistent indication of syntactical errors.

In addition to the preceding description of her written interaction, Alice attempted to accommodate learners' alternative approaches to solving problems, albeit these approaches were different to what I perceive as the more appropriate ones. She made limited use of instructional scaffolding, of reflective remarks and of utilizing heuristics to facilitate learners' mathematical understanding.

#### **4.5 CONCLUSION**

Alice described mathematics as a language, a way of communicating and a means of understanding the social world. Alice focused on her learners, on the learners' thinking, their prior knowledge and their active involvement while she was describing her professed approach to teaching mathematics. She focused on personal traits and a positive disposition towards learners in identifying her predominant, individual attributes, as a mathematics teacher.

Alice's professed perceptions of learners' errors were positive. According to Alice, learners' errors could potentially enhance learning and strengthen mathematical understanding through the juxtaposition of erroneous mathematical procedures with correct solutions. She subscribed to a contingency between learners' errors and correct procedures in both contexts. Her focus of interacting with learners' errors was hence on the provision of correct procedures and not on probing the errors for the sources thereof.

Expository, explanatory teaching through transmission was prevalent in Alice's classroom. Learner involvement was overtly encouraged in Alice's classroom. However, with the

exception of the cooperative pair discussions, classroom interactions were constrained to dialogues between the teacher and an individual learner. Alice often posed questions to learners in order to initiate a discussion, but instances of responding to a learner's question with a question were not recorded. Alice's questions were primarily aimed at mathematical procedures. Alice portrayed an inclination towards procedural questions, while she apparently avoided reflective questions. The responsibility for the mathematical evaluation of learner contributions seemed to reside with Alice. Although Alice frequently accommodated learners' contributions, the accommodation appeared to be conditional. Contributions concurrent with the preconceived procedure were readily accommodated.

Corresponding to her professed response to learners' errors in the classroom, Alice interacted with errors in a positive and patient way. Alice's observed and recorded interaction with learners' errors concurred with her assertion that she preferred to avoid peer involvement in error discussions. Her interaction with learners' errors predominantly occurred on a one-on-one basis. Although Alice ostensibly interacted with the majority of audible learners' errors, her observed and recorded interactions primarily resembled the correction of learners' errors through transmission, augmented with elucidative explanations of what was mathematically unacceptable. To a certain extent, her enacted interactions with learners' errors concurred with her professed utilization of learners' errors as a teaching approach. The elaborate way in which she attempted to supply learners with mathematical explanations of what their errors were, was in resonance with an expository teaching approach through transmission.

Recorded instances of how Alice interacted, in writing, with learners' errors in assessment tasks corresponded to her professed approach to learners' errors in written assessments, as obtained from the data collected through semi-structured interviews. Alice portrayed a strong inclination towards written communication with her learners. Alice routinely indicated learners' errors, frequently corrected the errors and frequently supplied learners with elucidative, written comments. Her interaction with learners' errors during learning periods demonstrated a considerable degree of correspondence to her interactions with learners' errors in written assessment tasks.

**5.1 INTRODUCTION TO BARRY**

Barry, teaching at school B (see section 3.6.2), was a white Afrikaans-speaking male with four years teaching experience. He had been teaching mathematics for four years and felt comfortable teaching mathematics up to a grade 12 level. He obtained a Baccalaureus Educationis degree with mathematics at second year level, although he completed a mathematics module on third year level. Barry omitted a personal definition of mathematics during completion of the written, structured interview. However, during the semi-structured interviews he referred to mathematics as a formal learning area, best taught traditionally. He regarded mathematics as a science that demanded discipline as a prerequisite for the learning thereof.

*It's (the outcomes-based approach and related socio-constructivist ideals underpinning the South African curriculum) not working in maths. I don't know if it's working in the other subjects, but mathematics is unfortunately one of those formal subjects that you just have to teach in the old ways. I believe mathematics is ... is quite a ... a science that needs discipline.*

Construed from the semi-structured interviews, Barry believed that mathematics was learnt by demonstrating correct solutions to learners. He supplied learners with intact mathematical examples they were expected to copy from the blackboard.

*I mostly do everything on the board for them just to copy down, just to make sure that they have everything, and that they've got the correct ones.*

Due to Barry's perception of an apathetic disposition towards mathematics among learners, he professed to revert to a teacher-centred approach to teaching mathematics as his only viable option. As the main role player, Barry accepted responsibility for the learning in his classroom.

*So I normally do it; mostly do it on a teacher-centred method, because that's the only way I'm getting them to work, 'cause we can't leave it in their hands.*

At the time of this study, he experienced the teaching of mathematics as difficult and frustrating.

Barry described his strengths as a mathematics teacher in the following way:

*I know my subject very well and also know exactly what the curriculum requires. I also exercise discipline very efficiently.*

Barry's approach to the teaching of mathematics was the exemplary stepwise provision of correct mathematical solutions on the blackboard. Barry believed that learners' mathematical understanding was improved by reproducing these examples in writing. He illustrated the way he thought mathematics was best taught as:

*... by giving as many examples in class as possible and making learners practise a lot.*

Barry made an orderly and well-groomed impression. He had an exceptionally neat handwriting and utilized the blackboard methodically. He was well organized and prepared and appeared self-confident. Barry's voice was clear and audible and he appeared attentive to not talking while he was writing on the blackboard, noticeably turning from the blackboard and making eye contact with the learners while he was teaching.

## **5.2 CLASSROOM VIGNETTE**

Barry's learners did not have to gather and line up outside the classroom prior to the commencement of the learning period. They arrived at the classroom in small groups or individually and entered the classroom upon arrival. Barry responded to learners' greeting on their arrival, but did not extend a general greeting. Learners took their seats as they entered the classroom.

Two class leaders were appointed to perform daily homework-control. The names of learners, who neglected to do their homework, were meticulously recorded. Barry often used to interrupt his teaching to enquire about the homework of those learners notorious for failing to do their homework. The class leaders submitted the debit slips they had to complete for those learners who had neglected their homework, at the end of each learning period. Barry appeared strict about his rule that the homework of those learners whose scripts were not opened at the correct page in time, was reported as not done. A second rule, regarding the status of learners' homework, was that incomplete homework was regarded as neglected homework. Barry discussed the solutions to the previous day's homework on completion of the homework-control. The entire set of homework problems was discussed. Barry explained the relevant mathematical procedures stepwise and in detail.

During the initial segment of learning periods, while homework was controlled and discussed, various disciplinary issues were addressed impromptu by Barry during the course of this two-week data collection period. Learners' absenteeism seemed to be an aspect that received much attention. Barry would interrupt his teaching to get information regarding learners he identified to be absent, from their peers. Latecomers appeared to be another subject of concern. Learners who used the mathematics period to catch up with homework in other learning areas created another apparent disciplinary problem. Barry maintained discipline during the course of the learning period by occasionally addressing the whole class to restore order. It was uncommon for him to reprimand individual learners for misconduct.

Subsequent to the homework discussion, Barry would usually introduce the next section of the mathematics curriculum with a formal heading that learners could find in their supplementary notes to take down in their scripts. Barry's preferred teaching style was to demonstrate and explain mathematical solutions on the blackboard in a detailed, stepwise, procedural manner. Numerous, similar examples of particular procedures were executed and explained on the blackboard by him. Learners copied from the blackboard while Barry was explaining solutions and procedures. His teaching regularly contained algorithms or cues, as illustrated by the following three excerpts. The first excerpt pertains to the addition or subtraction of fractions.

*People, you only subtract the numerators, not the denominators.*

The second excerpt pertains to solving equations with fractions, containing a variable in the denominators.

*And if I had a  $x$  and a  $x$  squared, what would I put in the LCD? The  $x$  squared. So for each kind of variable, you always take the one with the highest exponent. Not gonna put the  $x$  and the  $x$  squared only gonna put the  $x$  squared.*

The third excerpt pertains to the equation<sup>1</sup>  $\frac{x+3}{12} - 4 = 5$ . Barry was illustrating the erroneous procedure of writing the equation as  $\frac{x}{12} = 9 - 3$ .

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<sup>1</sup> The mathematical problems pertaining to case B are quoted directly from Nel, et al. (2006). Please consult the list of references.

*Listen, you cannot take something above a fraction line over if that thing underneath the line isn't gone yet. So you first take the thing underneath the line across before you can take anything else across.*

An apparent inclination towards “contiguity teaching” (Gage & Berliner, 1998, pp. 238, 606), teaching learners to recognize cues and clues, coupled with specific mathematical strategies and reinforced through the repetition thereof, was witnessed.

*T:  $y = \frac{2}{3}x - 2$ . Okay, what have we got there?*

*L1: A fraction.*

*T: A fraction, so is minus one, zero and one going to be the easiest values to choose?*

*L2: No.*

*T: No, so what we are going to choose; things that would ...?*

*L3: Cancel out.*

*T: Cancel with the ...?*

*L4: Three.*

*T: Three.*

*T: So we're gonna use ...?*

*L5: Minus three.*

*T: Minus three, zero and three, because those three's would cancel with the three underneath the line. Have you got that?*

During a discussion of how to draw the graph of a straight line using the so-called “table method”, learners were instructed to establish whether the gradient of the straight line was a fraction or an integer. Should the gradient be an integer, the choice of the numbers negative one, zero and one from the domain was suggested. Should the gradient be a fraction, it was suggested to choose numbers equal to the denominator from the domain.

During this segment of the learning period, Barry ensured a degree of learner involvement with his tendency to ask non-directive questions and by occasionally interrupting his teaching and allowing learners to complete the mathematical procedures they were copying from the blackboard. This segment of the learning period could be compared to a

contained, controlled tutorial. During this tutorial-type segment, Barry attended to individual learners. In instances where learners indicated a lack of understanding, Barry explained mathematical procedures elaborately and patiently. Usually not much time was allocated to these segments and Barry used to return to the blackboard to continue with the transmission of the correct solutions and intact examples. When a learner indicated a lack of understanding or portrayed a misconception during the teaching, Barry addressed the learner's problem by transmitting the relevant procedural information (see section 1.6.2). Barry used to approach the particular learner during the course of that learning period by referring back to the specific misconception, after several minutes had elapsed, while explaining another, similar example on the blackboard.

During his explanatory, expository style of teaching, Barry used to ask an abundance of questions, and did so quite frequently. He habitually asked learners trivial computational questions, creating the impression that he applied this tactic to ensure that the learners were paying attention. Three examples of these trivial teacher questions are given below.

*What is two times zero?*

*So, what is one times four?*

*How many times does five go into twenty?*

Learners stayed involved during the learning period and answered questions spontaneously, albeit without putting up their hands prior to answering and not being addressed in person. Some learners would answer simultaneously with Barry. Barry almost never addressed individual learners with questions. In some instances, he allowed momentary periods for learners to make contributions and he often confirmed these by rephrasing the learner contribution. The following excerpt serves as an example:

*T: We are going to divide the four by the two and then we get  $x$  is?*

*L: Two.*

*T: Two.*

Barry made abundant use of rhetorical questions like the examples that follow. He often referred to mathematical procedures or topics as difficult or not difficult.

*Is that difficult?*

*Have you got that?*

*Are you all with me?*

*Do you follow?*

*Can you see it?*

*Do you agree?*

During a forty-minute learning period, Barry used a variety of these questions up to thirty times. Learners usually did not reply to these questions and apparently, Barry did not anticipate responses from them. Occasionally a learner would respond positively. In instances where a learner indicated a negative response, Barry addressed this by paying individual attention to the particular learner. Barry asked a copious number of questions relating to mathematical conventions and to mathematical procedures. The following ten teacher questions are exemplary.

*Do I have to write the one next to a  $x$ ?*

*Do I have to write plus zero?*

*How many equal signs can each step have?*

*Can I get  $x$  alone if it's inside a bracket?*

*How do I get that two away from the  $x$ ?*

*How do I make a two a fraction?*

*I have to solve  $x$ , I have to work out what the  $x$  is, where do you start?*

*What does the name of this method tell us?*

*So can I now take away that minus and put it at both the one and the three?*

*Does the minus go with?*

The nature of these questions portrayed an attempt to prevent learners from making unnecessary syntactical and procedural errors. The instances where Barry allowed opportunities for learner responses were usually corresponding to lower levels of questions. Barry himself generally answered the majority of questions without delay; particularly the more challenging questions. Barry was not recorded to answer a learner's question by asking another question.



During the transmission of mathematical examples and procedures, Barry paid attention to minor details and warned learners about common errors that could cause them to lose marks in written tests and examinations. Barry placed emphasis on syntactical, mathematical issues during his teaching:

*People, you must remember your arrows, you must remember your labels, you must remember your zero's, 'cause I can assure you in the test on Friday, half of you are not going to put those things on.*

Barry seemed to possess a comprehensive knowledge of common learners' errors related to algorithmic, mathematical procedures and continuously emphasized these. An excerpt, illustrating this, is provided below. The excerpt pertains to applying the formula for the

gradient of a straight-line graph,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ .

*People, be very careful in the test for this formula, because you tend to mix them up, right? The x's above and then the y's below. Remember, y is above, x is below, okay? And do not put plusses between them, because people tend to do that as well. Remember that there is (sic) minuses between them. Be careful that you don't put them in the wrong places. Y goes in y's place and x goes in x's place.*

Barry was inclined to make deliberate mathematical errors on the blackboard occasionally to determine whether learners were actually concentrating, to compel learners to pay proper attention to what they were copying from the blackboard or to convince learners to be careful of these errors. However, Barry's deliberate mathematical errors did not always have the desired effect. On one occasion, the learners continued to copy from the blackboard without noticing the error. Barry had to draw their attention to the erroneous procedure.

The nature of the classroom discourse could be described as an abrupt dialogue between Barry and a specific learner at a time. Discussion among learners was not observed to be encouraged. Any interaction between learners was followed up and controlled by Barry by instantaneously enquiring what the conversation between the learners entailed. Learner contributions, when acknowledged, were treated on a one-on-one teacher-learner basis. Barry retained the authority to evaluate the quality and the correctness of learners' mathematical contributions. Peer involvement in mathematical discussions played no apparent role in the classroom. No whole-class discussions of learner contributions were

observed during the lessons observed. There were no incidents of learners offering alternative approaches to solving mathematical problems during the lessons observed.

Barry played a dominant role in the teaching. He exercised precise control over the course of each learning period by determining the pace of the teaching and by the seemingly negligible contribution the learners were making to the teaching. Except for occasional controlled, contained tutorial-type segments, entire learning periods were typically utilized for teaching. No time was earmarked for learners to continue with the homework assigned for the following day.

The impression was created that learners preferred to avoid exposure of their mathematical contributions to their peers. Barry noticed a learner making a presumably common learner error and decided to share this with the class.

*Huh, huh, look what she's doing, people!*

The learner protested while her peers were laughing. Barry continued and demonstrated her mathematical error on the blackboard. However, he comforted her with:

*No, nothing to be shy about. Eighty percent of the grade 9's does it anyway.*

Barry did not attempt to contextualize mathematical problems during the lessons observed. He read and explained theoretical concepts regarding straight-line graphs from the notes prior to applying these in mathematical examples.

*You'll see what I mean by that in the activity.*

The semantics of Barry's mathematical language often portrayed non-mathematical meanings, as illustrated by the three exemplary excerpts:

*I have to get this  $x$  alone, people. First, we take the one over. Then it becomes a plus one.*

*Minus times a minus is a plus, so those negatives goes (sic) away.*

*I am multiplying this eight to the  $x$ , so if I take it across, I divide, né<sup>1</sup>? Have only the  $x$  left, I also don't want the minus there, so I take the minus with the eight, and then, what happens then? It stays a minus eight.*

---

<sup>1</sup> "Né" is an Afrikaans word that means "Is it not so?"

Notwithstanding the fact that Barry and his grade 9 learners used a prescribed textbook he prepared and handed out auxiliary notes to which learners had to refer. Barry seemed to be well prepared, in control and not dependent on the textbook. However, he did follow the order of the textbook and exclusively used examples from the textbook for discussion.

### **5.3 BARRY'S BELIEFS ABOUT LEARNERS' MATHEMATICAL ERRORS**

Barry was of the opinion that learners' errors could be useful as learning opportunities during an expository correction of the error(s) executed by the teacher on the blackboard.

*Because if they thought they made the mistakes themselves, and I correct them then, then I think that they learn more out of their own mistakes than out of my mistakes.*

The pedagogical value of learners' errors resided in the recognition of similar erroneous approaches by other learners in the classroom. When asked whether he thought that errors could be useful learning opportunities he responded as follows:

*Yes, I do, 'cause if one learner makes an error then you have the opportunity to correct it while the other learners witness it, and would they might then realize that they also have that same error without even knowing it. So, I think it is a good thing then to tackle them in class, to sort out those problems.*

Barry considered that the complete and correct solutions he presented on the blackboard provided learners with sources for learning. He believed that learners required exposure to their own mathematical errors juxtaposed with the corrected mathematical procedures in order to learn mathematically.

Barry experienced the school system and the associated requirements as problematic. The fact that learners were not allowed to keep their corrected test scripts was a constraining factor in effective learning. As written assessment tasks were usually in the format of fill-in question papers, the learners did not even have access to the question papers they had been assessed with.

*So, I also go through the tests and the exams with them afterwards, after I marked the scripts, and then I let them copy down the corrections in pencil on their scripts. The only problem that I do have with the system now is that we cannot allow them to keep their scripts, which I think is a big problem. It is, ... they can then see what they did wrong, but when they study again, they have nothing; they only have their book again. They have nothing to reference, to see where they made their mistakes. That is a problem.*

Barry held the opinion that learners were not thinking critically and were lacking proper understanding of mathematics. Therefore, the learners were unable to identify mathematical errors that the teacher intentionally made on the blackboard. The learners merely accepted everything that was written on the blackboard. They mechanically copied mathematical examples from the blackboard.

*Because now they simply copy down and they don't go over and look at what they've copied down, so that they can make them, ... it their own.*

## **5.4 BARRY'S INTERACTION WITH LEARNERS' MATHEMATICAL ERRORS**

### **5.4.1 Verbally During Learning Periods**

#### *Professed*

During the semi-structured interviews, Barry admitted to getting upset when learners asked questions in the classroom. However, he qualified that questions resulting from a lack of attention upset him. He declared himself willing to assist learners who lacked conceptual understanding and experienced problems with mathematical content. Barry claimed to be attentive to recognizing and addressing learners' errors during classroom interactions.

*Well, mainly the learners would tell me if they do not understand something and then I would notice that they are making a mistake somewhere and I will correct that mistake. Sometimes I do ask specific questions to get the error from them because I know that most of them do make that error. And then sometimes I would make the error on the board and see whether they do notice that I did make the error.*

Barry explained that his response to learners' errors during classroom interaction depended on various factors including the nature of the error and the personality of the learner. Should the personality of a learner allow that, he might consider conveying to a learner that the error was a stupid one. However, he would refrain from using such an approach when the particular learner was shy or sensitive.

*It depends on what kind of error it is and, well, who made the error; it depends on the person as well. Some people are very sensitive about being corrected, or they feel shy for making the error, so it depends on the person, how you would approach it. Some people you can just say this is a stupid error and they would understand it. But then for some people you wouldn't say that.*

Barry claimed to focus on identifying learners' errors during classroom interaction and to approach the errors composedly in order to protect the more sensitive learners. He preferred to refer to learners' errors as common in order to prevent learners from shying away from asking for clarity.

*Well, I would just point out that it is a mistake and I would mainly say that it's ... it's a ... a common mistake that most people do make, so that they don't feel that they are stupid or that they gone be mads (sic) or something. I would just say most people does (sic) this mistake or do you make this mistake and then I just handle it as if it isn't actually such a big problem and point out the correct way.*

Barry identified making deliberate errors on the blackboard as a typical feature of his teaching approach. Although he was apprehensive about peer involvement in the discussion of learners' errors, he did recognize the value thereof conditionally. Barry suspected that learners were scared to ask questions overtly in the mathematics classroom, due to their peers' contemptuous attitude. Peers often made fun of learners who made errors. Barry admitted to being infuriated by this kind of conduct.

*Well, I do get that... that if someone gives a wrong answer in the class, the rest of the class do laugh at them. They still do that, very badly, although they don't know the answer themselves. So, I choose to not do that at all, because I don't think it is very good for the person that is trying, but just get it wrong and then those that are not even trying, laugh at them. And I also get quite angry.*

Consequently, Barry discouraged peer involvement in the public and whole-class discussion of learners' errors. However, those learners, who had the desire to achieve academically, were more open to the public discussion of errors and to their peers' involvement therein.

*But I do get that if; we've got this two totally separate groups in every class. There's this group that wants to work and then there's a group that doesn't want to work. The group that doesn't want to work will laugh at the group's answers. That is working, but then inside the group that is working, they would also help each other correct. They would not laugh, because they know the seriousness about this.*

### ***Enacted***

Although I made the pragmatic decision to omit the analysis of video-recordings with the purpose of employing data regarding facial expressions and body language, the alternative mechanical recordings produced verbal interjections (see section 3.6.4) as used by the

participants in their interactions with learners' errors. Barry infrequently used verbal interjections to respond to learners' errors. Two elucidative excerpts are cited below.

*T: What is zero minus three? It's minus three.*

*L: I thought it was zero.*

*T: (whistles)*

*T: You thought multiply?*

*L: Yes.*

Learners had to determine the  $x$ -intercept of the straight-line graph  $y = -\frac{1}{3}x - 6$ . The equation  $0 = -\frac{1}{3}x - 6$  had been written as  $18 = -x$ .

*T: Getting positive eighteen is equal to minus  $x$  and then you swap them to make the  $x$  positive and then eighteen negative.*

*L: No!*

*T: Yes!*

Barry usually corrected learners' errors promptly and directly, without involving peers. In some instances, he endeavoured to convey the reason for the contribution being erroneous, to the learner. However, Barry was not consistent in this. The first few excerpts are indicative of instances in which he refrained from supplying explanations to the learners. In the following example, Barry was revising the four quadrants on the Cartesian plane with the learners.

*T: In this one  $x$  is ...?*

*L: Positive.*

*T: No, negative.*

The following examples of Barry's interaction with learners' errors pertain to simple operations on integers. The examples illustrate his interactions with learners' calculation errors.

*T: Three times minus four is ...?*

*L: Twelve.*

*T: Minus twelve.*

*T: Minus six, minus another one is ...?*

*L: Minus four.*

*T: Minus seven.*

The subsequent excerpts are representative of instances in which Barry attempted to describe the nature of learners' errors or to provide learners with cues prior to correcting the errors. The excerpt below pertains to the relation between the gradients of two perpendicular lines. The gradient of one of the two lines was  $m = 5$ .

*T: If my gradient was five, the new gradient would be ...?*

*L: Minus five.*

*T: Minus one over five, because it's actually five over one, né?*

Learners had to find the equation of a straight line from a given graph, depicting the two intercepts (0;2) and (2;0).

*T: If I have to write down their coordinates, what would this coordinate point be?*

*L: Two.*

*T: Two?*

*L: Two, two.*

*T: Zero. Two, two would be there.*

*L: Oh!*

The following excerpt pertains to finding the equation of a straight line from a given graph, depicting the two intercepts (0;5) and (-3;0).

*T: What would be the first point?*

*L: Five, zero.*

*T: Minus three, zero and zero, five. People, don't write the x's in the y's place and the y's in the x's place.*

The excerpt presented below exhibits evidence of an attempt by Barry to scaffold the learner's thinking instructionally (see section 3.6.4), albeit eventually correcting the

learner's subsequent error without drawing the learner's attention to that. The example pertains to the relation between the gradients of two perpendicular lines. The gradient of one of the two lines was  $m = \frac{1}{2}$ .

*T: If this gradient of this line is a half né, what must I multiply it by to give me the answer minus one?*

*L: Minus a half.*

*T: Huh, huh.*

*T: You can see that a positive times a negative is gonna be minus, so you know that it must be negative, and how many halves make one?*

*L: Two.*

*T: So what must I multiply it by?*

*L: Two.*

*T: So my new gradient is minus two.*

The following excerpts serve to show how Barry approached significant misconceptions among the learners. In the first excerpt, a misconception about points on the Cartesian plane and coordinates was probable. Barry seemed to understand the nature of the learner's misconception.

*T: How many points do we need before we can determine the equation of a straight line?*

*L1: Two.*

*T: Two.*

*L2: Four.*

*T: It's a  $y_2$  and a  $y_1$  and a  $x_2$  and a  $x_1$ .  $Y_2$  and  $x_2$  is one point.*

*L: What, Sir?*

*T:  $x_2$  and  $y_2$  is one coordinate point.*

*L: Oh, Sir, you're confusing me!*

In the second excerpt Barry was demonstrating to learners how to find the equation of a straight line, parallel to the line  $y = -x + 3$ , through the point  $(-1;3)$ . Following substitution



of the coordinates, Barry wrote down  $3 = 1 + c$ , finding  $c = 2$  and the equation of the line to be  $y = -x + 2$ .

*L: Sir, why did the minus come back?*

Apparently Barry was under the initial impression that the learner was referring to the fact that the gradients of both straight lines equalled negative one.

*T: Because they said this line that we get now, that we have to get, is parallel to that one.*

However, the learner referred back to where Barry solved for the y-intercept,  $c$ . The learner was confused between the  $x$ -coordinate, negative one, of the point  $(-1;3)$  and the term, positive one, in the equation  $3 = 1 + c$ .

*L: Did you change the one into a positive to find  $c$ ?*

Eventually Barry realized to what the learner had been referring.

*T: Oh, this one and that one that we took across, is not the same one.*

*L: Oh!*

*T: Do you follow people?*

*T: Listen, this minus one is the gradient. There it is; okay? Then they said this thing, this other line goes through the point minus one, three  $(-1;3)$ . That's another minus one. This is not the gradient. This is a  $x$ -value. You can't have a gradient in the coordinate.*

*L: Okay!*

*T: Okay, do you follow?*

*T: This could have been a minus five. It is just a coincidence that it's exactly the same value as the gradient.*

*L: So, it's possible that it could be another value.*

*T: Yes, very likely.*

The following excerpt shows how Barry responded to a learner's suggestion that was in effect mathematically correct and descriptive of what actually happened mathematically in performing the particular procedure. However, the suggestion was different to Barry's approach. Barry erroneously conveyed to the learner that the suggestion was flawed. In addition to that, this example serves to show how Barry frequently responded to learners'

errors with repetitive, negative interjections (see section 3.6.4) and with a cautious tone.

Learners had to solve the equation  $\frac{3}{x} = \frac{1}{4x} - \frac{2}{5}$ .

*T: I have to solve x, I have to work out what the x is, where do you start?*

*L: Sir, you make the denominators the same.*

*T: No, no, no, no, no, listen, listen, be very careful. You will make the denominators the same if it's an expression, there isn't an equal.*

*L: Oh, ja!*

*T: If there's a equal, you'll find the ...?*

*L: x*

*T: The LCD<sup>1</sup>, and you multiply only the numerators by it, né?*

*T: So that you don't do your equations the same way as you do your expressions, 'cause then you're not going to get your x on its own.*

### ***Comparative Synopsis***

During the semi-structured interviews, Barry claimed to be attentive to recognizing and addressing learners' errors during classroom interactions. Observations and mechanical recordings of the classroom events confirmed Barry's assertion that he routinely interacted with learners' errors in the classroom. Barry's enacted interaction with learners' errors was in correspondence with his professed interaction of correcting learners' errors, exposed during learning periods, promptly and directly. In some instances, he attempted to describe the nature of learners' errors by conveying reasons for the contributions being erroneous. These clarifications were often cues rather than mathematical arguments. However, Barry often corrected learners' errors without supplying explanations to the learners. To make errors on the blackboard on purpose was supposedly part of Barry's prevalent teaching approach. Barry was in fact observed to make deliberate errors occasionally. Barry was apprehensive about peer involvement in the discussion of learners' errors. According to him, learners were scared to ask questions overtly in the mathematics classroom, due to their peers' contemptuous attitude. For this reason, Barry discouraged peer involvement in

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<sup>1</sup> "lowest common denominator"

the whole-class discussions of learners' errors. The vast majority of teacher-learner interactions observed in the classroom happened on a one-on-one basis.

#### 5.4.2 In Writing in Assessment Tasks

##### *Professed*

During the semi-structured interviews, Barry identified a focus on including error-triggering questions in written assessment tasks, based on his experience and knowledge of common learners' errors. He relied on the information obtained from written assessment tasks, such as tests, to evaluate learners' pedagogical needs.

*So, I would then more first assess the group and see how they would've approached the problem. So, that's why I do rely on tests more.*

Such information might direct his teaching approach. Subsequent to the marking of learners' written assessment tasks, Barry would discuss the memorandum in class and would demonstrate correct solutions to the problems on the blackboard. Learners were expected to copy these solutions from the blackboard on their scripts. During such expository discussion, Barry would highlight those learners' errors that he recalled as most common. Barry believed that learners required exposure to their own mathematical errors juxtaposed with the corrected mathematical procedures in order to learn mathematically.

*Well, when I go through the test after I marked it, then I would do the whole memorandum on the board. And then I will specifically point out the most common mistakes that I could remember. And they do have to make the corrections then in pencil on their tests. And then I also find that's quite valuable, because they could see what they did wrong. They write down the correct thing and I believe that they then would understand the problem better.*

According to Barry, proper mathematical learning could not take place unless learners had access to their written and corrected assessment tasks. Barry was under the impression that without intact, correct examples of typical solutions to mathematical questions, the learners had no resources.

*They only have their books. Which is also not such a good reference these days, because they don't, ... the quality of work is not that good. I mostly do everything on the board for them just to copy down, just to make sure that they have everything, and that they've got the correct.*

He recognized the exposition of typical assessment tasks as a pedagogical obligation. Barry therefore believed that, in preparation for term tests and examinations, it was essential to provide learners with worksheets identical to the written assessment tasks (test- or examination-papers).

*Once I set the exam paper for June, I went and I sat down with the computer with this paper next to me and for every single question there was in the paper I set a revision sheet. So, if I asked them in question one to take a recurring decimal to a fraction, then I would put four examples of that in the revision sheet. And so I went through the whole exam paper. Both papers, there was only three questions that wasn't in that revision sheet. And I went through that thing with them, step by step by step before the exam.*

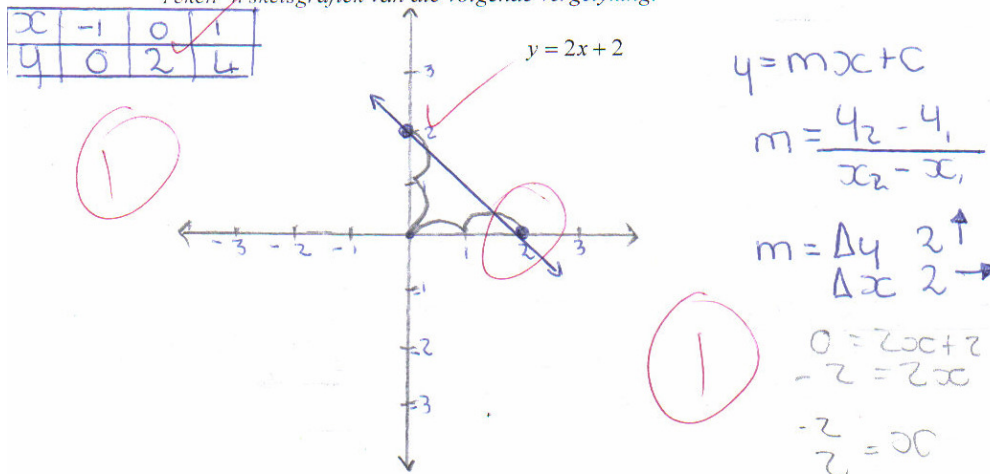
*I do give them revision sheets, before every test and exam, which I also do with them then.*

### ***Enacted***

It was illustrated through analysis of the written assessment tasks that Barry made no use of written comments or of instructional scaffolding (see section 3.6.4 and appendix C) in learners' written assessment tasks. He was consistent in merely indicating learners' mathematical errors with markings (see section 3.6.4 and appendix C). Barry often, but not always, indicated what was mathematically correct or wrong with conventional markings and awarded a mark without any written comments. In a limited number of incidents, he indicated a learner's error either by a question mark or by encircling the particular incorrect value. In the ensuing example, Barry encircled an incorrect  $x$ -intercept. Although the  $x$ -coordinate of the  $x$ -intercept was correctly calculated by the learner as  $x = -1$ , the straight line was drawn in such a way as to depict an  $x$ -intercept at  $x = 2$ . In the process, the direction or the orientation of the line was flawed. However, Barry did not draw the learner's attention to this discrepancy.

4. Draw a sketch graph of the following equation:  
*Teken 'n sketsgrafiek van die volgende vergelyking:*

LO 2

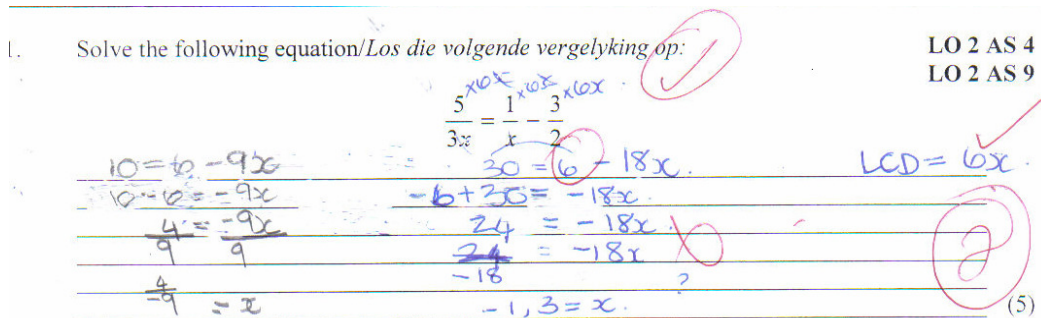


5. Draw a sketch graph of the following equation:

LO 2

**Figure 5-1 Question 4 of written assessment from school B**

In the subsequent excerpt, Barry encircled the only correct term in the balanced equation, omitting to indicate the errors in the remaining two terms. On suspected realization thereof, he encircled the correct mark as well, in order to correspond to the correct term he (accidentally?) encircled.



**Figure 5-2 Question 1 (i) of written assessment from school B**

The excerpt below shows one of an extremely limited number of instances in which Barry corrected learners' errors. The learner probably over-generalized the algorithm of *moving terms and changing signs* in order to find  $9x$  in the third line of the procedure. Barry corrected the error.



1. Solve the following equation/Los die volgende vergelyking op:

$$\frac{5}{3x} = \frac{1}{x} - \frac{3}{2}$$

Handwritten work:

$$\frac{5 \times 2x}{3x} = \frac{1 \times 2x}{x} - \frac{3 \times 2}{2}$$

$$10 = 6 - 9x$$

$$10 - 6 = -9x$$

$$4 = -9x$$

$$\frac{4}{-9} = x$$

LCD = 6x

3

Figure 5-3 Question 1 (ii) of written assessment from school B

The four participating teachers regularly ignored learners' inappropriate use of the equal sign when working with equations. The ensuing example demonstrates such an instance in Barry's interactions with learners' errors in written assessments.

Solve the following equation/Los die volgende vergelyking op:

LO 2 AS 4  
LO 2 AS 9

$$\frac{5}{3x} = \frac{1}{x} - \frac{3}{2}$$

Handwritten work:

$$\frac{5 \times 2x}{3x} = \frac{1 \times 2x}{x} - \frac{3 \times 2}{2}$$

$$= 10 = 6 - 9x$$

$$= 10 - 6 = -9x$$

$$= 4 = -9x$$

$$= \frac{4}{-9} = x$$

LCD = 6x

5

(5)

Figure 5-4 Question 1 (iii) of written assessment from school B

The following excerpt illustrates an instance in which Barry refrained from exactly pointing out several of a learner's errors. The learner separated the nominator and the denominator of the fraction  $\frac{5}{3x}$  and interpreted these numbers as the terms 3x and 5. In line three, the learner neglected to perform an operation on all the terms in the balanced equation. In addition to that, the learner interpreted  $\frac{1}{x}$  times x as zero. In the penultimate line, the inverse operation of times three (x3) was indicated and performed as minus three (-3).

1. Solve the following equation/Los die volgende vergelyking op:

$$\frac{5}{3x} = \frac{1}{x} - \frac{3}{2}$$

~~$$\frac{5}{3x} = \frac{1}{x} - \frac{3}{2}$$

$$3x = \frac{1}{x} - \frac{3}{2} - 5$$

$$3x(x) = \frac{3}{2} - 5$$

$$3x^2 = -1\frac{1}{2} - 5$$

$$3x^2 = -6\frac{1}{2}$$

$$x^2 = -6\frac{1}{2} - 3$$

$$x^2 = -9\frac{1}{2}$$~~

$$x^2 = -9\frac{1}{2}$$

$$9\frac{1}{2} = x$$

Figure 5-5 Question 1 (iv) of written assessment from school B

The ensuing example shows an instance in which Barry overlooked a learner's correct mathematical approach. The excerpt illustrates the learner's attempt on the left hand side and Barry's procedure, copied from the blackboard during the memorandum discussion, on the right hand side. The two approaches correspond up to the penultimate line of the procedures. The learner earned no marks of the possible five marks.

2. Solve the following equation/Los die volgende vergelyking op:

$$\frac{x-5}{3} + 5 = 2$$

~~$$\frac{x-5}{3} + 5 = 2$$

$$= \frac{x-5}{3} = 2-5$$

$$= x-5 = -3 \times 3$$

$$= x = -9-5$$

$$= x = -14$$~~

$$LCD: 3$$

$$10-6-9x$$

$$10-6 = -9x$$

$$4 = -9x$$

$$\frac{x-5}{3} + 5 = 2$$

$$\frac{x-5}{3} - 2 = 2-5$$

$$\frac{x-5}{3} = -3$$

$$x-5 = -3 \times 3$$

$$x-5 = -9$$

$$x = -9+5 = -4$$

Figure 5-6 Question 2 of written assessment from school B

### Comparative Synopsis

Other than mentioning his classroom discussion of common learners' errors, subsequent to the assessment, Barry made no particular reference to his interaction with learners' errors in written assessment tasks during the semi-structured interviews. He asserted that learners' errors in written assessment tasks directed the memorandum discussion. During the expository discussion of the memorandum, he would highlight those learners' errors that he recalled as most common. Except for indicating learners' errors with a variety of

markings, no other forms of written interaction emerged from the content analysis of the written assessments of Barry's learners. Barry was consistent in predominantly indicating learners' mathematical errors with markings.

## **5.5 CONCLUSION**

In identifying his predominant, individual attributes, as a mathematics teacher, Barry highlighted his mathematical knowledge and his ability to instil and maintain discipline. He described mathematics as a formal and disciplined science. In his professed approach to teaching mathematics, Barry revealed a preference for traditional teaching methods, encompassing expository teaching and drill work.

Barry relied on expository, explanatory teaching through transmission almost without exception. He often transmitted algorithms to the learners. Barry made profuse use of rhetoric and lower-level questions. He portrayed a tendency to alert learners to common, procedural and syntactical, mathematical errors. Barry's recorded mathematical language revealed considerable deviation from generally accepted semantics. A tutorial-type slot was only observed occasionally and was utilized during the teaching phase of the learning period. Barry often employed an entire learning period for teaching. Learner participation in Barry's classroom was spontaneous, but not encouraged.

Barry professed to perceive learners' errors as useful learning opportunities, in contingency with complete and correct solutions provided to learners during expository teaching. The dual advantage of learners' exposure to their own mathematical errors resided in the recognition of similar erroneous approaches by other learners in the classroom and in the juxtaposition of the errors with the corrected mathematical procedures, presented by the teacher. The complete and correct solutions Barry presented on the blackboard provided learners with sources for learning, and not their mathematical errors per se.

Barry's enacted interactions with learners' errors during learning periods concurred with his professed response to errors. He typically, almost without exception, interacted with learners' errors in the classroom. Barry persisted to interact composedly with learners' errors. However, he often corrected learners' errors abruptly and occasionally used repetitive, negative interjections (see section 3.6.4) in his response to learners' errors. Observations and mechanical recordings confirmed his professed approach of correcting learners' errors as a prevalent means of interaction. However, he occasionally attempted to



provide learners with explanations of what was mathematically unacceptable, albeit in an instrumentalist (Ernest, 1988; Skemp, 2006) way. Barry's habitual interaction with learners' errors on a one-on-one basis was in resonance with his professed reluctance to involve peers in error discussions.

Except for the variety of markings used to indicate learners' errors in written assessment tasks, Barry ostensibly refrained from employing written comments to interact with these errors. It is possible to understand the predominant absence of written comments in learners' assessments, in line with the semi-structured interview data. Barry did not refer to a preferred approach to learners' errors in written assessment tasks during the semi-structured interviews. However, he did protest about the predicament of filing learners' written assessment tasks in portfolios, hence prohibiting learners from utilizing the assessments formatively. Instead of describing his interaction with learners' errors in assessment tasks, he focused on a description of his prevalent practice of the memorandum discussion. Barry's interaction with learners' errors during learning periods showed no correspondence to his interaction with learners' errors in written assessment tasks.

**6.1 INTRODUCTION TO CHLOE**

Chloe, at school C (see section 3.6.2), was a white Afrikaans-speaking female with two years teaching experience. She had been teaching mathematics for two years and felt comfortable teaching mathematics up to a grade 12 level. However, she preferred teaching natural sciences. Chloe obtained a B Sc degree with mathematics at second year level and completed a PGCE. Chloe's personal definition of mathematics was:

*Mathematics teaches people knowledge, skills and values with which they can bring order to their lives. Mathematics is bringing order where there is chaos, i.e. taking "disorganised" data, putting it in a frequency table and drawing a graph that presents the information. Mathematics can also be seen as a language to be used in various fields.*

Chloe confirmed her personal definition of mathematics during the semi-structured interviews with her description of mathematics as a means to create order and to obtain the cognitive skills of analysis, synthesis and evaluation.

*I believe mathematics is about teaching people how to bring order in their lives. I believe it's a tool that is used, not only in work, but in your everyday life. It teaches you analysis and evaluation and synthesis of things, and then obviously to recall knowledge and whatever.*

She perceived a mathematics classroom as an environment characterized by order, discipline, and a consistent routine.

*And the main picture is to create order. So, what I believe about mathematics is that there should be an orderly environment. And that there should be discipline.*

She expressed the way mathematics was best taught as:

*Mathematics should help people bring order in their lives, it is therefore crucial to give people steps to use in order to be able to approach questions or problems with certainty. Learners should also be allowed to discover steps on their own so that they can gain confidence in their thinking ability.*

During the semi-structured interviews, Chloe confirmed her perception that mathematics was best taught by the application of stepwise procedures.

*And I'll say okay, let's go back to the example. What did we do there? Okay, step by step. What was step one? Okay, did you do step one? Good, what was step two? ... Until they start getting the pattern of what they're supposed to be doing.*

Consequently, Chloe believed that, in order to organize learners' thinking, learners should be provided with steps in their approach to solving mathematical questions.

*And then, as teaching specifically goes, people should be helped to order how they think. So, by giving them steps, especially in algebra, by showing them how, and then also by letting discover for themselves sometimes how the steps work, so that they can learn to think for themselves.*

Construed from the semi-structured interviews, Chloe held the opinion that, although she was the only person in the classroom with a set of knowledge, she was not the only person capable of explaining mathematical procedures. She argued that peers could make a valuable pedagogical contribution in explaining solutions to each other.

*And I also believe that peers can help explain in a way that's different from what I do. And sometimes they understand each other better. So I don't have a problem with that. I really don't have a problem with that. I don't feel like I'm the only one with the ability to explain in the classroom. I might be the only one with a set of knowledge, because I am the teacher, but anyone can explain. So, I believe that.*

Chloe described her role in the mathematics classroom as a normative one, determining what was mathematically sound. She was of the opinion that her learners accepted the mathematical knowledge they received from her, uncritically.

*So, as from my, from the role I'm playing, I would be like ... to be the one that sets the standard, so that they know that they can trust this. I don't make mistakes on the board on purpose.*

Chloe believed that, with the atmosphere she created in her classroom, she elicited in her learners a frankness to ask questions. She identified her strengths as a mathematics teacher as:

*... encouraging and motivating learners, not fearing difficult questions from learners, allowing learners to have different ways of understanding concepts and answering questions, a well-disciplined classroom environment*

She kindled a mathematical interest among learners by exposing learners to information regarding previous mathematicians and mathematical applications in realistic context.

## 6.2 CLASSROOM VIGNETTE

Chloe waited outside the classroom for the learners to arrive at the beginning of each learning period. Chloe's learners had to gather outside the classroom and form a neat row prior to entering the classroom. She would compel the learners to behave with repetitive interjections:

*Okay guys, line up quickly, please!*

*Okay, thank you, guys!*

When the learners had settled down outside the classroom, Chloe would allow them to enter. Learners remained standing on entering the classroom and waited at their desks to be seated. After Chloe had greeted the learners, they were directed to take their seats. Chloe appeared adamant about obtaining order prior to addressing the learners with a greeting. She would orchestrate the learners with interjections like:

*Thank you, right thank you! That is enough, thank you! That is enough, thank you!*

Latecomers were punished consistently and immediately. Learners were allowed to choose between receiving demerits or sitting on the floor during the entire learning period.

Chloe executed individual homework-control on a daily basis, albeit not controlling the scripts of the whole class each time. She had either written the solutions to homework problems on the blackboard prior to the commencement of the learning period or showed the solutions on a transparency through an overhead projector. While Chloe was examining the learners' scripts, the learners were marking their own solutions. She demanded silence during these activities.

*Quiet, absolutely quiet, as you are busy marking.*

Learners who neglected to complete their homework received demerits. A record of such learners was kept. Chloe instructed learners to work in pencil in instances where learners indicated being uncertain of the correct approach to solving a problem and she would request to see an attempt from a learner. On completion of the homework-control, Chloe explained and discussed some of the homework examples on the blackboard. She accommodated learners' individual mathematical problems by asking:

*Is there anything that I should explain again here? Put your hand up if you want an explanation. Don't yell out.*

Discussion of the homework was followed by the introduction of a subsequent topic. On introduction of a new topic, Chloe instructed the learners to draw a line underneath the previous section of work and to write the date in their scripts. During the teaching, she showed examples on the blackboard while the learners were paying attention to the mathematical elucidation. Chloe instructed the learners to pay undivided attention.

*When I'm working now and I'm explaining, you're not writing down. You're just listening to me, né? Then I'll give you chance afterwards.*

Chloe consistently resorted to expository teaching to introduce new examples for the learners to copy from the blackboard. She placed much emphasis on stepwise mathematical procedures and focused on the teaching of algorithms. Chloe told the learners:

*I want you to learn the steps, because when we get to the long things, you have to follow a pattern, a method, otherwise you're going to get stuck.*

Chloe fastidiously followed the order of the prescribed textbook and almost exclusively referred to examples from the textbook.

Chloe involved learners during her teaching by asking learners for contributions, albeit within the context of the mathematical procedure. When posing questions to learners during this segment of the learning period, Chloe predominantly focused on questions relating to mathematical procedures, as the following examples are exhibiting:

*What will I have to do to get rid of the minus two?*

*Can I times if I have a minus in between?*

*So what do we do first? First step is remove the brackets and then?*

Chloe retained the authority to evaluate the quality of learner contributions. Although Chloe asked for learner contributions, these contributions were dealt with on a one-on-one teacher-learner basis. She often approached specific individuals with a question and only she responded to these learner contributions. It happened occasionally that Chloe did not approach an individual learner with a question. Learners, who wanted to respond to the question, were instructed to put up their hands. Chloe would address one of the learners to answer the question.

*If you wanna answer, put your hand up.*

Learners were allowed to ask questions during the teaching phase. Chloe responded immediately and directly to the questions that learners were asking while she was demonstrating complete and explanatory examples on the blackboard. Chloe was not observed to encourage mutual peer participation. In comparison to the questions related to mathematical procedures that Chloe initiated, she often responded to learners' questions with reflective questions and questions related to mathematical interpretation. The excerpts below are representative of more challenging questions.

*What went through your mind to find the value of  $x$  – that's the whole point of equations?*

*What do you think it's (LCD) going to be?*

*Why can't  $x$  be one?*

*What's the difference? The result is the same, but what is the difference?*

However, when questions of a higher level were posed to learners, Chloe either answered these questions herself, or a response to these questions was not pursued. Chloe often answered her own questions promptly without allowing an opportunity for learner contributions:

*T: What we're going to do first?*

*T: We're going to remove the brackets.*

Although Chloe acknowledged alternative approaches to solving mathematical problems, learners were discouraged from pursuing these.

Chloe maintained discipline during the teaching phase in a way that resembled conducting an orchestra. Although she never raised her voice, her disciplinary style could rather be described as authoritarian than authoritative. She constantly remained aware of what learners were doing and, without interrupting her teaching, addressed individual learners with interjections:

*Thank you, you are done.*

*Will you please keep quiet?*

*Can you sit up please?*

*That is enough.*

*Pay attention.*

The semantics of the mathematical language Chloe primarily used in the classroom can be demonstrated with the following excerpts. In the first excerpt, Chloe was indicating constant terms, not containing a variable, in a balanced equation.

*So go to that thing, né? Go and encircle the thing that must move.*

During the process of solving for  $x$  from a balanced equation, Chloe isolated an aspect of the coefficient and allocated meaning to it. An excerpt, illustrative of the tendency, is presented below.

*How do I get rid of the minus in front of the  $x$ ?*

Chloe supplied an explanation for finding the identity element for addition (zero) by adding the inverse of a particular number:

*I want it to get to the other side, but I can't do it by division. There's a plus in between so I must minus, né? If you have something with plus and minus you must stay in that category. Then, because I have minus eight  $x$  plus eight  $x$ , it falls away.*

Referring to zero in the balanced equation  $-3x - 12 = 0$ , Chloe explained:

*Minus three  $x$  minus twelve equals our place holder. This is our place holder so that we don't have an open space.*

During the lessons observed, there was one incidence noted where Chloe contextualized a mathematical problem. Learners had to manipulate the formula for converting temperature in degrees Fahrenheit to degrees Celsius and vice versa.

*Forty-five degrees Celsius, where does it get as hot as that in our country? Upington. Yes, so we're in Upington, but we have American friends and they want to know how hot it is. So we're going to convert forty-five degrees Celsius to Fahrenheit.*

Chloe apparently attempted to provide learners with rules, steps or algorithms during her discussion of solving problems in realistic context. While reading a word problem, she taught learners that the word *is* was usually an indication of where the equal sign in the number sentence would be. Chloe seemingly took responsibility for the mathematical interpretation of essential terminology in the word problems and formulated the relevant equations for learners.

*Which means six  $x$  minus four  $x$  will be forty-six. How will I get that? I will have to subtract, four  $x$  from six  $x$ .*

Chloe captured the approach to solving problems in realistic context with:

*Let the number be  $x$ . You always start with this sentence; very important.*

Subsequent to her teaching, Chloe allowed learners time to copy the examples from the blackboard. The silence that Chloe demanded appeared to dissuade learners from engaging in mathematical discussion or to seek peer assistance.

*You are quietly writing down. You are not talking to anyone.*

During this phase of the learning period, while learners were copying from the blackboard, Chloe responded to learners' questions regarding the illustrated examples. Learners were encouraged to ask questions and to clarify their understanding.

Once learners had copied the examples from the blackboard, the last few minutes of the learning period were utilized as a tutorial. While the learners were allowed to continue with their homework for the following day, Chloe attended to individual problems on demand. The majority of the teacher-learner interactions that took place during the tutorial-type segment of each learning period were on a one-on-one basis.

### **6.3 CHLOE'S BELIEFS ABOUT LEARNERS' MATHEMATICAL ERRORS**

Chloe initially claimed, during the first semi-structured interview, to believe that learners' errors could be useful as learning opportunities. However, during the subsequent semi-structured interview she acknowledged doubt and apprehensiveness about utilizing learners' errors pedagogically. She perceived a focus on correct procedures as a preferred approach to the teaching and learning of mathematics. Chloe perceived a focus on learners' errors as inhibiting effective learning. She pronounced learners' errors as a possible source of confusion. Contrary to this opinion, she gave an account of an incident during which she had succeeded in relating a learner's error to a misconception.

*I'm just afraid that, ... so yes and no. I haven't made my mind up about this. I'm just afraid that sometimes, by focusing someone's attention on the error, you prevent them from actually learning what they should be doing. Because now, you're so hard busy saying what they're not supposed to do, that that's all that they remember. And then they don't focus on what they actually should do. So, the question remains, and it's worked, I've seen it worked that way and the other way around in different contexts. The question remains, should you say don't do this or should you rather enforce the right method much louder?*

Chloe held the opinion that making errors caused learners to feel stupid.



*You don't want to discourage participation, especially if you're asking something that they are not necessarily prepared for. Then you want to encourage them rather by not making them feel small and ... and silly, what's the word, ja.*

She seemed to be adamant about her decision not to deliberately make errors on the blackboard. She believed that deliberate errors on the blackboard caused confusion among learners.

*Well, I don't think I believe in doing that, not with grade 9's, 'cause they, my opinion is, they need to know what really is so. So, as from my, from the role I'm playing, I would be like, to be the one that ... that sets the standard, so that they know that they can trust this. I don't make mistakes on the board on purpose.*

*I think it confuses people. Ja, I think it confuses people.*

Chloe classified learners' errors, distinguishing between unreasonable or unacceptable learners' errors, which were made when learners had already mastered mathematical procedures and justifiable errors, which she expected when learners were in the process of mastering mathematical procedures.

*Unless if I feel that this is something that this learner has already mastered, and I think by now she should, he or she should know how to answer it. I would say I think you must try again.*

Chloe distinguished between learners' errors in response to questions with distinct answers and those with multiple answers. Plausibly, Chloe's perception of learners' mathematical errors corresponded to incorrect answers.

*With the quantitative question in maths it's really difficult to now say well, that's not, that might be right, because it's wrong or it's right. And to tell someone that three over four might be a half, or might be six over eight, then you're misleading that person.*

Chloe's perceived focus on incorrect answers as representative of learners' mathematical errors concurred with an additional comment from her own personal experience that she added in the written, structured interview.

*Most learner errors I experience come as a result of poor mathematical calculation and number ability that should have been developed in previous grades.*

During the semi-structured interviews, Chloe described her perception of the pedagogical value of assessment as a bilateral source of information. Both teacher and learners obtained information regarding learners' mathematical understanding, their level of

mastering mathematical concepts and procedures and learners' progress. Assessment assisted in identifying particular content areas in which learners' understanding was inadequate.

*It shows both the learner and the teacher whether or not a learner has grasped what he was supposed to. Or it can show where we must still work on some things.*

Chloe suggested that learners' errors might inform her teaching practice by either altering her usual classroom approach to cooperative learning, by utilizing alternative ways of explaining mathematical procedures or by choosing different exercises for learners to practise their skills.

*Yes, definitely. You would ... you would alter some of your methods and especially if it's work that you have just started with. Now if they wrote a small test and you can see that there's something wrong, you're going to try and explain it in another way. Yes, if you see that what you did that it worked. So you would alter your explanations, you would alter for example the exercises that you choose. You would alter whether you work ... let them work alone or in groups. All of that to allow in the limited time and the limited resources.*

Chloe identified the disparity between learner abilities, a lack of time and large classes as inhibiting factors to a more effective pedagogical approach to learners' errors.

## **6.4 CHLOE'S INTERACTION WITH LEARNERS' MATHEMATICAL ERRORS**

### **6.4.1 Verbally During Learning Periods**

#### ***Professed***

Chloe claimed to differentiate her responses to learners' errors during classroom interactions according to a particular learner's personality, as perceived by her.

*I think it's a very personal thing, because if you're working with an individual and you've come to know that person for a while, a few months or so, you will know how to handle that person, so it becomes very personal. There are learners for which you must say *ag nee man*<sup>1</sup>, why did you make this mistake again, you *mos*<sup>2</sup> know this and that, but some learners you can't do that with. So, I think it's personal, yes. And that would determine how you handle it.*

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<sup>1</sup> "Ag nee man" is an Afrikaans expression that translates to "Come on!" or to "Oh, no!"

<sup>2</sup> "Mos" is an Afrikaans word that means something has been dealt with before; it is beyond discussion.

During the semi-structured interviews, Chloe indicated a preference for deflecting a question to peers when a learner's error was detected during classroom interaction. She expressed a reluctance to interrogate a particular learner's thinking during classroom interaction. She was careful not to expose individual learners to contemptuous peer responses. However, her response might depend on the way she classified the learner's error.

*Okay, what I would normally do, well, would be to hear if there's someone else. What do other people, other learners think about this learner's response? Or, I would not necessarily direct it to this person, I might divert it and say let's hear from someone else. So as not to expose that person. Unless if I feel that this is something that this learner has already mastered, and I think by now she should, he or she should know how to answer it, I would say I think you must try again.*

It was revealed during the semi-structured interviews that Chloe perceived whole-class discussions most appropriate when the possibility of multiple solutions to a problem existed. She held the opinion that, considering the good chance of misleading or deceiving learners, the discussion of learners' errors in instances of distinct solutions should not be allowed. Chloe believed that she (the teacher) had to correct a learner's error immediately when that error appeared in a mathematical question with a distinct or single answer.

*I think it would depend on whether you're working with a quantitative type of question or a qualitative type of question. With the quantitative question in maths it's really difficult to now say well, that's not, that might be right, because it's wrong or it's right. And to tell someone that three over four might be a half, or might be six over eight, then you're misleading that person. So, in that sense I would think a more direct answer is better for him or her. If it's a more qualitative type of thing like in statistics, where there might be many possible answers, then that's a question where you can easily involve peers and form a discussion around it to ask if that may be the best idea to represent a set of data.*

The way in which Chloe seemed to envision interaction with learners' errors portrayed an avoidance of emphasising the error per se, concurrent with a focus on the correction thereof, through expository transmission, repetition and reinforcement.

*The question remains, should you say don't do this or should you rather enforce the right method much louder?*

During one-on-one teacher-learner interactions in the classroom, Chloe asserted to gradually guiding a learner through an example the learner had copied from the blackboard

in order to facilitate the learner comparing the example stepwise with his or her own attempt until she and the learner reached the error the learner had made. Chloe would conclude her intervention by prompting the learner to decide how to correct his or her own attempt in order to resemble the copied example.

*I would ... I would refer, ... if I come to an individual's desk then I would refer them back to the example we just did. ... And I'll say okay, let's go back to the example, what did we do there? Okay, step by step. What was step one? ... What was step two? And then check to ... so that they can break it down to see where they went wrong. And then when we get to the step where the mistake is. Then I'd say, okay, how must we change this so that it looks like that? Until they start getting the pattern of what they're supposed to be doing. Okay?*

### ***Enacted***

Chloe occasionally corrected learners' mathematical errors through transmission. An excerpt from one such a circumstance in which she corrected the learner's error immediately, is cited below.

*T: Where do natural numbers start?*

*L: Zero.*

*T: One, né? One.*

However, several situations in which she approached learners' errors with questions were recorded. Although the example that follows bears evidence to an attempt by Chloe to elicit mathematically sound contributions from a learner, Chloe, subsequent to spontaneous peer involvement, eventually produced the mathematics (Inman, 2005).

*T: If I divide by zero, what is it?*

*L: Zero.*

*T: Think again?*

*L: Zero.*

*T: No, can I divide by zero?*

*Peers: No!*

*T: What is it? It is undefined.*

Ample instances in which Chloe scaffolded individual learners instructionally (see section 3.6.4) through questioning were recorded. In the example below, learners had to solve the following problem<sup>1</sup>: The sum of three consecutive integers is  $-84$ . Find the numbers. One learner focused on selecting negative  $x$  as the first of the three integers.

L: *So, in other words, will that be minus  $x$  plus minus  $x$  plus minus  $x$ ?*

T: *Tell me, consecutive numbers, what do they do?*

L: *They're right after each other.*

T: *Okay, like?*

L: *One, two, three.*

T: *What did you do with one to get to two?*

L: *Plus.*

T: *Plus one. So what do we do with  $x$ ? Say now  $x$  is our first number. How do we get to the next consecutive number?*

L: *Also plus.*

T: *Plus one and then to the second number?*

L: *Plus one.*

In the meantime, the learner altered his or her choice of first integer to positive  $x$ . The learner continued to suggest another series of three consecutive integers, namely  $x + 1 + 2$ .

T: *But hold on, is it  $x$  plus one plus two? That's gonna be  $x$  plus three.*

L: *It's  $x$  plus one  $x$  plus two  $x$ . (Chloe did not interact with this error of suggesting  $x + 1x + 2x$  as a possible series of consecutive integers.)*

T: *Remember what we've said. If we start one, two, three, then one is  $x$ . That's where we start, so the first number is  $x$ . The second number, what did we do to get that?*

L: *Plus one.*

The following excerpt shows an instance in which Chloe refrained from interacting with a learner's errors for the sake of the ultimate mathematical goal. Chloe subsequently

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<sup>1</sup> The mathematical problems pertaining to case C are quoted directly from Seeliger (2006). Please consult the list of references.

rephrased the learner's contribution without emphasizing the learner's errors. Learners had to rewrite the subject of the equation  $t^2 = 9x$  to  $t$ .

L: *Ma'am, can't nine squared go into three x; won't nine x be three x?*

T: *The square root of nine x. I have to separate it, because I can't get the square root of x, but I can get the square root of nine. You're very close.*

L: *Three.*

T: *Three times the square root of x.*

Instances of impassiveness or unresponsiveness towards learners' errors were captured on the data collected in Chloe's classroom. At least two misconceptions are presented in the excerpt cited below. Both misconceptions were related to inverse operations and the identity elements for addition and multiplication. The subject of the formula was in fact one over t, but the first learner suggested multiplication therewith. The second learner's suggestion revealed confusion between the inverse operations for multiplication and addition. Chloe apparently took no notice of these while she persistently repeated her question she had posed to the class. Chloe seemed to be fixed on pursuing the approach of writing down the reciprocals of the terms on both sides of the balanced equation and appeared to steer the learners to her preconceived approach. Eventually Chloe transmitted her preferred approach to the class without probing any of the two misconceptions that were recorded. Learners had to rewrite the subject of the equation  $\frac{1}{t} = \frac{c}{s}$  to  $t$ .

T: *We have one over t equals c over s. How are we going to get t? Think!*

L1: *Times with one over t. Yes, you times it. (The learner referred to  $\frac{1}{t}$  as the multiplicative inverse of  $t$ .)*

T: *How can I get it to be just t? So, if I want t I have?*

L2: *Negative, negative (This learner portrayed apparent confusion between the additive inverse and the multiplicative inverse of  $t$ .)*

T: *s over c guys! I can just flip it around, but then I must flip both sides, because this is equations. What I do on one side, I have to do on the other side.*

The impression was created that Chloe was usually not inclined to accommodate learners' mathematical contributions or alternative approaches. She seemed to steer learners towards her preferred approach to solving mathematical problems. The learner, quoted in the following excerpt, appeared to be disempowered by the interruption of his or her mathematical strategy. Learners had to solve the following problem: If 12 is added to seven times a certain number the sum is 250. Find the number.

L: *Gonna be rounded off.*

T: *Why would you want to round it off?*

L: *'Cause I divided two hundred and fifty by seven, so I can figure out what seven times that number ...*

T: *Tell me, can you divide by seven if there's still another number standing there?*

L: *But I'm trying to get seven times that number plus twelve.*

T: *But what's the method that I taught you? What do we do first?*

L: *See, now I don't know, 'cause this plan of mine is not working.*

T: *So what do we do first? First step is remove the brackets and then? Move all the x's to the one side and all the numbers to the other side. Do that.*

Chloe's referral to brackets and to more than one term containing a variable in the preceding excerpt appeared to be irrelevant in the particular instance, considering the equation  $7x + 12 = 250$  as a possible algebraic description of the problem.

Although Chloe habitually clarified learners' errors while correcting these through transmission, I am of the opinion that some accounts could potentially create confusion among learners. In the following excerpt, the learners had to write down possible values for  $a$  and  $b$  if  $a(b - 1) = 0$ . In addition to putting, what I perceive to be, an irrelevant question to the learner, the mathematics was eventually produced by Chloe (Inman, 2005) during the interaction.

L: *Minus one.*

T: *Huh, huh, think again; b can't be?* (Chloe appeared to get confused between finding the constraints on a variable in a denominator, on condition that the number was real, and writing down possible values for variables from a zero product.)

L: *Zero.*

*T: Huh, huh, b minus one will be equal to zero, so what will b be ...?*

*L: a*

*T: Huh, huh, why? You're gonna have b minus one equals zero, so I'm gonna move minus one to the other side so that b will be equal to one. So those are your two possibilities.*

In the subsequent excerpt, a learner wanted to apply the “steps” Chloe had previously taught them regarding brackets, i.e. to determine the product in order to write down a series of separate terms. However, the equation they had to solve was in the form of a zero product. The learners had to write down possible values for  $a$  and  $b$  if  $a(b - 1) = 0$ . Chloe avoided reference to the zero product during her interaction with the learner’s inappropriate contribution. This excerpt serves to illustrate one incident during which Chloe refrained from interacting with a learner’s error through questioning. She interacted with negative interjections (see section 3.6.4) instead.

*T: No, you don't. You don't, okay? Why you don't, is you want to specifically; if you are going to work out the brackets, if you are going to multiply, you'll have a.b and you'll have minus a, né? Okay, now you have two situations where you have an a in. Is it possible now to get a value for a? No, it's harder to work with. What makes it easier to determine a value for a?*

*L: If it's separate.*

*T: If it's separate. So you keep it separate.*

The learner’s use of the concept “separate” in the preceding excerpt was problematic. The term “separate” is conventionally applied in relation to a series of terms; particularly resulting from multiplication or the application of the distributive law. The expansion of brackets, what Chloe discouraged the learner to do, would have resulted in separate terms. However, Chloe confirmed the use of this inappropriate concept twice.

In the next example, a learner expressed confusion about the result of a mathematical procedure. Learners had to solve the equation  $\frac{x}{4} + \frac{3}{2} = \frac{x}{2} + \frac{3}{4}$  which had been written as  $x + 6 = 2x + 3$ . The balanced equation was multiplied with negative one in order to solve for the variable. The result of  $x = 3$  was obtained from  $-x = -3$ . Chloe did not quite address the learner’s predicament and, in my opinion, Chloe’s mathematical explanation could potentially aggravate the learner’s confusion.



L: *Why did you put like three under minus three?*

T: *Why did I put three ...?*

L: *Under minus three equals to three and why didn't you leave it as minus three?*

T: *What you got here, when you move two  $x$  to the other side, you had two  $x$  (should have been  $x$ ) minus two  $x$ , okay? That was equal to three. You moved six to the other side. That gave you minus six and then if you say  $x$  minus two  $x$ . You're gonna get minus  $x$  and three minus six is minus three. If I have minus on both sides the minuses fall away and I'm left with  $x$  equals three. Is that it?*

### ***Comparative Synopsis***

During the semi-structured interviews, Chloe asserted to interact with learners' errors during learning periods by gradually guiding a learner through illustrative examples that learners had copied from the blackboard. She allegedly based her interaction with learners' errors on the comparison of a learner's mathematical attempt to the relevant, explanatory, copied procedure. Occurrences of such interactions were observed and recorded. However, this type of interaction with learners' errors was confined to the tutorial-type segment of the learning period. Chloe interacted with learners' errors in a number of different ways (see section 6.2 and above) during the teaching phases of learning periods. Chloe indicated a preference for deflecting a question to peers when a learner had made an error during classroom interaction. However, the deflection of such a question did not involve error discussion per se, due to the possibility of peer mockery. The predominance of one-on-one interactions with learners' errors during learning periods corresponded to Chloe's expressed reluctance to interrogate a particular learner's thinking during classroom interaction.

### **6.4.2 In Writing in Assessment Tasks**

#### ***Professed***

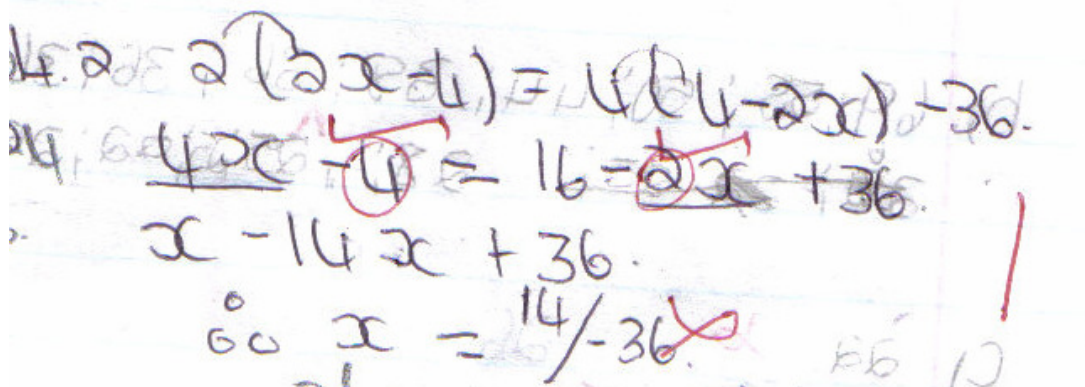
During the semi-structured interviews, Chloe focused on a description of her prevalent approach to memorandum discussions subsequent to marking assessment tasks. She did not refer to a preferred approach to learners' errors in written assessment tasks during these interviews. Chloe described an in-depth memorandum discussion as tedious and unnecessary. She preferred to direct a memorandum through the overhead projector and allowed learners time to copy correct solutions on their test scripts. Subsequently learners

were given an opportunity to ask Chloe to explain particular questions from the written assessment task.

*Okay, what I would do after a test is to go through the memorandum with them to discuss it with them. So, I would put it up (inaudible) or write it out on the board. And then the idea is that they must copy on the work that they've done, where they made mistakes, they must copy the correct answers to see whether they understand it or not. And then I give them the opportunity to ask if there's anything in specific. I've learnt that to go word for word through the whole paper is often a tedious thing, and not necessary, but rather to hear is there anything that I must explain again.*

### **Enacted**

Analysis of the written assessment tasks revealed that there were scripts on which Chloe simply used the conventional signs for right or wrong as indications of her assessment of the learners' work. On other scripts, Chloe indicated learners' errors with exclamation marks, by encircling errors, by underlining errors or with question marks. A limited number of instances where Chloe responded to learners' errors through other forms of written interaction, other than the variety of markings, emerged from the content analysis of the written assessments. The following excerpt serves to demonstrate Chloe's frequent avoidance of correcting learners' errors and from providing written comments of any nature. In this example, Chloe encircled a few of the errors, of which some might have been computational, but did not write down the correct values, did not classify the errors, or describe the nature thereof. The learner made a number of procedural errors, inter alia, regarding the distributive law, multiplicative inverses, additive inverses and similar terms.



Handwritten student work for Question 4.2(i) showing algebraic steps with errors circled in red:

$$4.2 \text{ (i) } 2(2x-4) = 4(4-2x) - 36$$

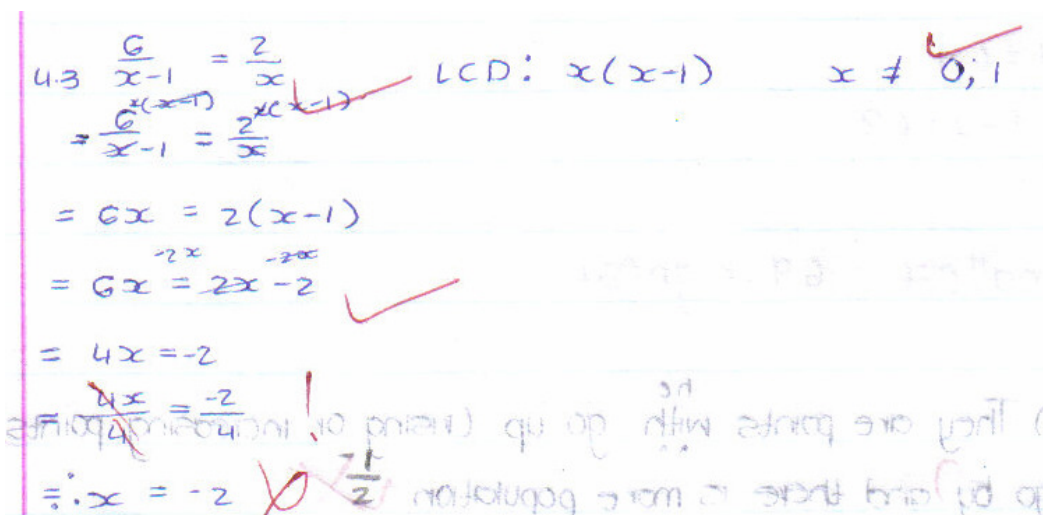
$$4 \cdot 2x - 4 = 16 - 2x + 36$$

$$x - 14x + 36$$

$$x = 14 / -36$$

**Figure 6-1 Question 4.2 (i) of written assessment from school C**

A lack of clarity in the way Chloe indicated erroneous procedures or incorrect values was apparent. Chloe often refrained from interacting with learners' errors. In other instances, Chloe indicated correct values as erroneous. In the following example, Chloe indicated a supposed error with an exclamation mark. However, what she indicated as erroneous ( $\frac{4x}{4}$ ) was actually correct. In addition to that, the example serves to show an instance in which Chloe overlooked the learner's inappropriate use of the equal sign while working with an equation.



4.3  $\frac{6}{x-1} = \frac{2}{x}$  LCD:  $x(x-1)$   $x \neq 0, 1$

~~$\frac{6 \cdot x(x-1)}{x(x-1)} = \frac{2 \cdot x(x-1)}{x(x-1)}$~~

$= 6x = 2(x-1)$

$= 6x = 2x - 2$  ✓

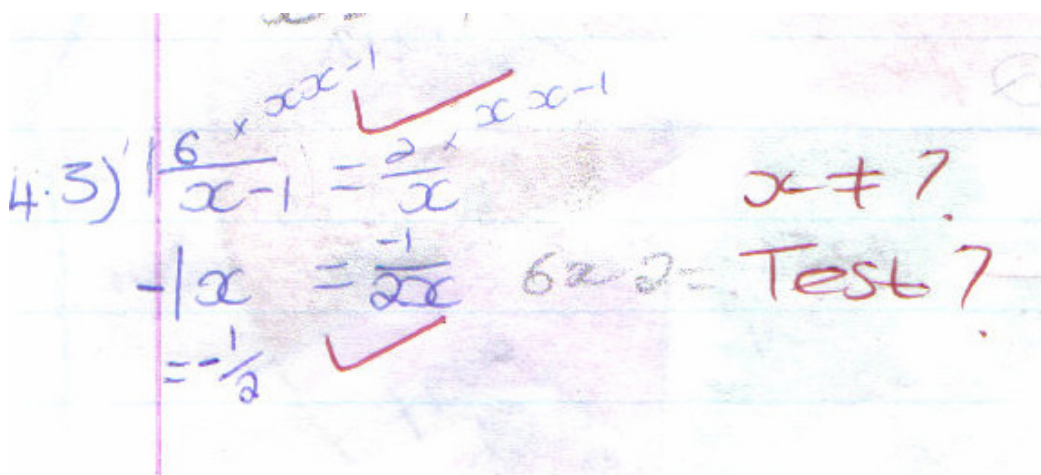
$= 4x = -2$

~~$\therefore \frac{4x}{4} = \frac{-2}{4}$~~  !

$\therefore x = -\frac{1}{2}$

Figure 6-2 Question 4.3 (i) of written assessment from school C

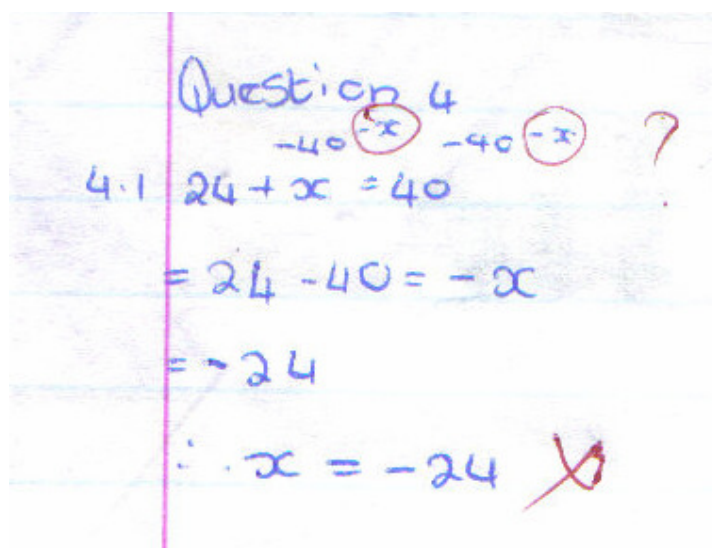
The subsequent example serves to show how Chloe refrained from penalizing a learner, irrespective of the mathematical incorrectness of the learner's preceding statement. Instead of simplifying the left hand side of the equation to  $6x$ , the learner wrote down  $-1x$  and instead of a value of  $2(x-1)$  on the right hand side, the learner wrote down  $-\frac{1}{2x}$ . The way in which the learner had depicted the lowest common denominator resembled the value  $x^2 - 1$  instead of  $x^2 - x$ . Chloe did not comment on that.



4.3)  $\frac{6 \times x x^{-1}}{x-1} = \frac{2 \times x x^{-1}}{x}$   $x \neq ?$   
 $-|x = \frac{-1}{2x}$   $6 \times 2 = \text{Test ?}$   
 $= -\frac{1}{2}$

**Figure 6-3 Question 4.3 (ii) of written assessment from school C**

In the following excerpt, a learner exactly imitated what Chloe explained on the blackboard previously. Chloe had the habit of writing the number inverse for addition a bit smaller and elevated, almost resembling the exponent of a power. The learner did precisely that. However, Chloe encircled this attempt and indicated her confusion with a question mark.



Question 4  
 $-40 \textcircled{-x} - 40 \textcircled{-x} ?$   
 4.1  $24 + x = 40$   
 $= 24 - 40 = -x$   
 $= -24$   
 $\therefore x = -24$

**Figure 6-4 Question 4.1 of written assessment from school C**

However, the possibility exists that, based on the practice<sup>1</sup> of writing variables on the left hand side of an equation, Chloe rather encircled the number inverse  $-x$  for that reason. Chloe overtly expressed her preference for writing variables on the left hand side of an

<sup>1</sup> I acknowledge experiential awareness of this practice, albeit not personally subscribing to it.

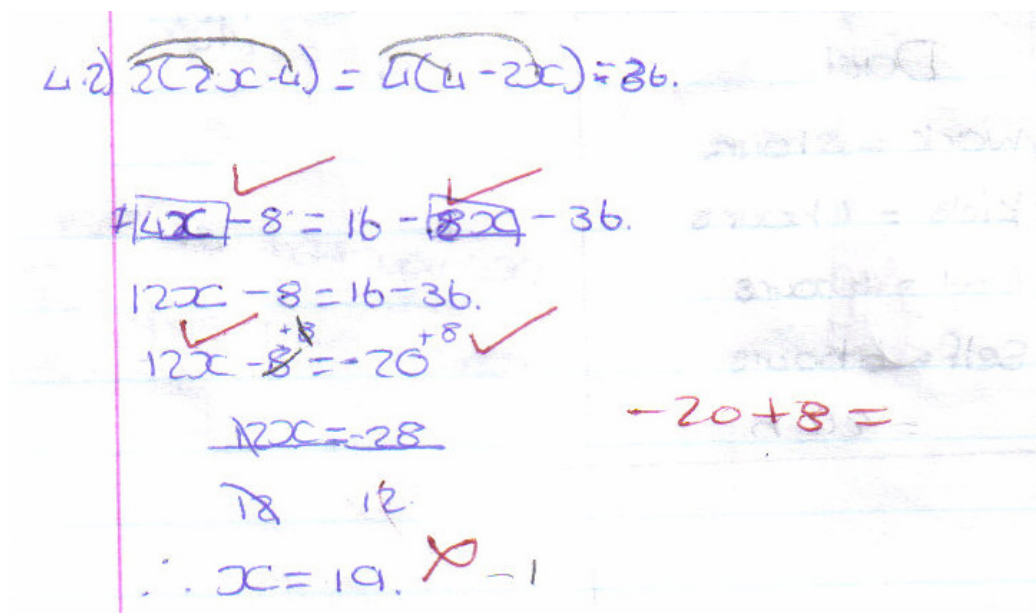
equation in the classroom once. During that occurrence, Chloe paraphrased a learner's question regarding terms, containing a variable, on the left hand side of the balanced equation:

*Would it be wrong if you move the numbers to the left hand side and the x's to the right hand side?*

She continued to answer the question herself:

*No, it wouldn't. It's the same thing, but can I, for the sake of the example, move it the other way around so that everything looks the same? Okay? Good!*

The final excerpt serves as one of an extremely limited number of instances in which Chloe augmented her assessment with written comments or explanatory symbols. The learner made a computational error. Instead of writing down the correct answer, Chloe scaffolded the learner's way of thought instructionally by copying the operation as  $-20 + 8 =$



4.2)  $4(2x-4) = 4(4-2x) = 36.$

$4x-8 = 16-36.$

$12x-8 = 16-36.$

$12x-8 = -20$

$12x = -28$

$12 \quad 12$

$\therefore x = 19. \quad \times -1$

$-20 + 8 =$

**Figure 6-5 Question 4.2 (ii) of written assessment from school C**

### **Comparative Synopsis**

Chloe refrained from describing her interaction with learners' errors in written assessment tasks during the semi-structured interviews. She focused on an account of the subsequent memorandum discussion. Content analysis of the written assessment tasks revealed that Chloe used a variety of markings to indicate learners' errors. She predominantly indicated

learners' errors without supplying elucidative, written comments. Chloe often abstained from indicating learners' exact errors. Instances where Chloe focused learners' attention to the inappropriate use of the equal sign, when working with equations, were not recorded.

## **6.5 CONCLUSION**

In her personal definition of mathematics, Chloe referred to the learning area as a language, albeit accentuating the order she associated therewith. She identified her strengths as a mathematics teacher as her mathematical knowledge and the disciplined classroom environment that she managed to maintain. Chloe gave prominence to the teaching of stepwise, correct procedures as a preferred approach to teaching and learning.

Expository, explanatory teaching through transmission took place in Chloe's classroom, almost without exception. Chloe overtly encouraged learner involvement with her questions. However, the participation of learners in Chloe's classroom was carefully managed and contained by Chloe, who was not observed to allow for spontaneous participation. Classroom interactions were constrained to dialogues between Chloe and individual learners. Chloe's recorded mathematical language portrayed considerable deviation from the generally accepted semantics of mathematics. Although Chloe recognized learners' alternative approaches, she was observed to do that without conferring mathematical status to these approaches.

Chloe perceived the discussion of learners' errors and teachers' deliberate errors as possible sources of confusion. She proclaimed a focus on learners' errors as a possible barrier to effective learning. Chloe expressed doubt and apprehensiveness about utilizing learners' errors pedagogically. Based on how she viewed herself and her role in the classroom, she refrained from making deliberate errors on the blackboard. Her perception of learners' errors seemed to be focused on incorrect answers.

Chloe portrayed ambivalence in the way she interacted with learners' errors during learning periods. She occasionally refrained from probing learners' errors. She was observed to portray a degree of impassiveness or unresponsiveness towards learners' errors, particularly when she was focusing on pursuing a preconceived approach to solving mathematical problems. However, she regularly approached learners' errors with questions and showed an unanticipated high occurrence of using questions to scaffold learners' thinking instructionally (see section 3.6.4). Although, during the semi-structured

interviews, Chloe adamantly purported to the obligation of immediately correcting learners' errors, she interacted with learners' errors through questioning in a remarkably high number of instances. My presence in the classroom and the purpose of the research could account for this discrepancy between her professed and her enacted interactions with learners' errors.

*I: Do you have anything regarding learners' errors that you became aware of during the time that I was observing or perhaps in retrospect after your interaction with me that you want to share with me?*

*T: I think I became more confident with throwing the question back at the class in a group discussion. So, if someone would answer a question and it's wrong, I became more confident in saying okay, is there someone else or what do the rest of the class think about this. So I think, ja, I think that was something that happened. I became more confident in asking them back to answer the question.*

Chloe asserted to deflecting questions to peers, following incorrect responses. However, such instances were rare. Her interaction with learners' errors during learning periods predominantly took place on a one-on-one basis. Chloe's interaction with learners' errors in written assessment tasks did not correspond to her interaction with learners' errors during learning periods. Her extremely limited use of written comments in assessments was in stark contrast to her frequent employment of questions and instructional scaffolding during learning periods.

### 7.1 INTRODUCTION TO DAWN

Dawn, at school D (see section 3.6.2), was a white English-speaking female with 17 years teaching experience. She had been teaching mathematics her entire teaching career, but only felt comfortable teaching mathematics up to a grade 10 level. She obtained a Higher Diploma in Education with mathematics at second year level. She enrolled for a Baccalaureus Educationis Honours degree in mathematics education at the beginning of the year prior to that in which the study was undertaken. The fieldwork was conducted towards the end of her final semester of study. She defined mathematics as:

*Mathematics is beautiful. It works forwards and backwards. It's the "words" of some thoughts. We use maths as a tool and as a language to understand, explain and explore our reality.*

Dawn recognized her strengths as a mathematics teacher as:

*I have a passion to enable people to do maths. I have a very broad experience: grade 0 to grade 11. I like to use manipulatives and discussion. I can be very animated.*

During the semi-structured interview, Dawn described her views on the teaching and learning of mathematics as constructivist (see section 1.6.3 and 2.2.3).

*I come from a constructivist paradigm. I think you build meaning and you build it through making it your own. I know it's lots of fancy words to say that, but unless I understand what I'm doing and I'm able to explain to you why I'm doing what I'm doing, I don't have full understanding, I don't have control over what I'm doing and so that is why.*

Dawn recognized a problem-based approach to the teaching of mathematics as most preferable. She elaborated on her suggested problem-based approach during the semi-structured interview, qualifying the mathematical problems as pertinent and socially significant to learners. In addition to her suggested problem-based approach to the teaching of mathematics, these excerpts serve to illustrate Dawn's reference to cooperative learning and to whole-class discussions.

*Start by using what is known. Have scaffolding (stories, games, models, manipulatives). Allow children to work together to solve a problem. Class atmosphere that allows for discussion and evaluation without a negative vibe. Revisit, revise, refine. ... Problem-based. By that I mean a relevant problem. As far as possible a relevant problem proposed*



*and the children should get together. I think it should be cooperative learning to solve it. ... In fact, I don't, I think maths just becomes a computational time filler if you don't have problems, 'cause you don't think. It's just a sit down, do it, get it all right, next; it's a factory. So, if you don't have problems, I don't think, ... you're not teaching maths, actually. That's my belief. I think it's part and parcel of it.*

Dawn supported the cooperative learning of mathematics. She preferred learning to occur in homogeneous, cooperative pairs to avoid the phenomena of “workhorses” and “free riders” (Nolinske & Millis, 1999, p. 32).

*I let them work together a lot even if it's ... I find the pairs at this stage works well and they've chosen who they work best with. Sometimes I have to move them around, but peers are good, because they're just ... They're edging each other on and they don't feel intimidated. But, then you'll have to choose the pairs carefully. I find that that helps, otherwise there's one workhorse and the others just copying, a free rider. So, I really like to do them, very closely link them, pair them according to ability. Every now and then I've done a situation, mixed group, mixed ability, then you have one person who leads, but that's very seldom.*

Dawn professed to engender a democratic (Wolk, 1998) classroom identified by discussion and negotiation.

*I think you should from there decide what the best way to put forward or argue your solution ... And then I think, after that we should look at the different solutions, and say fine, this one's mathematically sound, it's logical, it's reasonable. That's how I think maths should be taught.*

She believed that drill through repetitive execution of routine mathematical problems was still valuable.

*I still think there's place for drill, and routine problems, definitely. But, as far as possible, I think they should be contextualized instead of lists and rows of sums.*

Dawn believed that mathematics was learnt through construction of meaning. Understanding and meta-cognitive processes were prerequisites for mathematical learning.

*It's just, I think it's just getting them to the point where they'll think what they're doing. If this isn't working, then what? Have I got it right? I don't know. Have I understood it correctly?*

In order to learn mathematics, learners were required to develop the ability to reason logically.

*There needs to be a lot more discipline, and a focus in terms of, this is what we're doing, this is why we're discussing this. There's a lot more logical thinking, in-depth thinking in maths.*

## **7.2 CLASSROOM VIGNETTE**

Dawn's learners used to enter the classroom and take their seats. However, some learners used to remain standing behind their chairs. On the second bell, Dawn requested learners to stand. She greeted the learners and prompted them to sit down. Dawn usually utilized the first segment of the learning period for the discussion of homework problems. However, during the two-week period of fieldwork, Dawn also introduced three learning periods with pop quizzes. She subsequently discussed and marked the pop quizzes, prior to the homework discussion.

The learners received homework on a daily basis. Dawn supplied the learners with the answers to homework problems of a more complex nature concurrent with the demarcation of the homework. Learners had the opportunity to identify their mathematical errors or their lack of understanding during the subsequent classroom discussion of these homework problems the following day. Although Dawn discussed the homework at the beginning of each learning period, she did not relentlessly perform individual homework- or book-control. However, she executed a degree of homework-control by approaching individual learners for contributions during the discussion. There were no observable consequences for learners who neglected to do their homework. In instances where the nature of the homework problems allowed this, Dawn approached specific learners to supply the correct solutions to these problems orally. Dawn occasionally identified particular learners to show solutions to more intricate homework problems on the blackboard. In some instances, Dawn read the correct solutions from the teacher's guide. She performed individual book-control once during the fortnight of fieldwork.

On three occasions, a segment of the learning period was utilized for the introduction of a new mathematical topic. Dawn involved the learners in these introductions. The average speed concept was introduced with a whole-class discussion on the meaning of the word average, followed by requesting a learner to simulate a vehicle on a drive to the nearest shopping centre, using sound effects. She introduced financial mathematics with a group competition among the learners, allowing them one minute to write down as many foreign currencies as possible. The concepts simple and compound interest were introduced by

allowing learners two minutes to read the discussion of these concepts in their textbooks. On completion of that, Dawn facilitated a classroom discussion on their understanding of these concepts. All learners were usually participating and the atmosphere in the classroom seemed to be relaxed and pleasant. Dawn usually allowed time for learners to reflect on new information and to make contributions. The role Dawn played in these discussions was, however, a prominent and principal one.

Dawn's teaching approach was evident of an inclination towards problem-based teaching. The observation might be related to the specific mathematical topics that had been discussed during the two-week fieldwork period, though. On occasion Dawn reverted to conventional teaching through transmission. Learners were allowed to ask questions and to make contributions during whole-class discussions of homework or of new mathematical topics. However, Dawn retained the authority to evaluate the mathematical quality and correctness of learner contributions.

Dawn used a prescribed textbook for learning- and teaching-support. She followed the order of the textbook and she primarily referred to the textbook. She did, however, occasionally pose other problems to the learners. On introduction of financial mathematics, she brought diverse foreign currencies to the classroom and asked learners to determine the value thereof in South African Rand. A degree of contextualization of mathematical problems was evident. During their classroom discussion of direct and inverse proportion, Dawn contextualized direct proportion with a reference to cellular phone contracts and to buying groceries in bulk. However, although Dawn illustrated inverse proportion with the example of a rectangle with a constant area, much emphasis was put on the algorithmic approach of "constant product, inverse proportion; constant quotient, direct proportion". While learners were graphically representing realistic situations such as paying postage for mail delivery of parcels, no reference to the realistic situation regarding the discreteness of the graph or the constraints in terms of the domain or the range were raised.

Dawn used mathematical language appropriately. Once again, due to the nature of the mathematical topics discussed during the lessons observed at school D, opportunities for inducing inappropriate mathematical language were probably not created. Dawn focused learners' attention to mathematical syntax regarding the use of the equal sign.

Dawn often portrayed an awareness of developing learners' relational understanding (Skemp, 2006) and reflective mental skills. She asked a number of reflective questions that could potentially compel learners to meta-learning (Slabbert, et al., 2009) and critical thinking during her classroom interaction with individual learners and during whole-class discussions. However, little evidence of Dawn following up on these questions could be found. In some instances, Dawn appeared not to anticipate a response from learners. Exemplary excerpts are quoted below.

*How do you know it's wrong?*

*Can you confirm it on your graph?*

*Are you sure you've done this correctly?*

*He can't drive at a constant speed, but can I work out his average speed?*

She frequently encouraged learners to ensure that their answers were meaningful and to demonstrate their thinking in writing. She regularly asked learners whether the mathematical procedures and approaches were making sense to them.

The final segment of each learning period was predominantly utilized as a tutorial during which learners worked on allocated mathematical problems and Dawn gave individual attention to learners on request. A clear distinction existed between mathematical problems allocated for homework and those that were to be solved during the tutorial-type segment of each learning period. Learners were encouraged to work in pairs or in small groups during the tutorial-type segment. Dawn suggested to these cooperative pairs to divide the assigned problems selectively between them and to discuss their respective solutions with each other. However, the learners seemed to be apprehensive about doing an exercise partially and preferred to do everything individually. Dawn accommodated this, but recommended them to work concurrently and in consultation with each other. In contrast to that, Dawn focused on individual learners during the tutorial-type segment of the learning period and attended to learners' mathematical problems on a one-on-one basis. Dawn did not once interrupt the learners in order to address certain mathematical problems with general, whole-class discussions. She requested the assistance of three learners to act as tutors during this segment of the learning period on one occasion.

Learners had opportunities during most of the learning periods to approach Dawn with possible mathematical problems they experienced, whether with previous homework or

with a current assignment. Prior to handing in their workbooks, Dawn instructed them to encircle those examples they had to correct, to indicate any further queries with a star in the margin and to approach her with these.

Dawn was never observed raising her voice in order to instil discipline. She occasionally addressed individual learners in a calm and collected way. In some instances, she admonished the class as a whole.

### **7.3 DAWN'S BELIEFS ABOUT LEARNERS' MATHEMATICAL ERRORS**

Dawn classified learners' errors, distinguishing between conceptual errors and computational errors. Prior to classifying learners' mathematical errors, Dawn determined the origin of the erroneous procedure to be either mathematical or linguistic. A linguistic problem might be situated in insufficient reading skills or in an inadequate vocabulary. Dawn's classification of learners' errors could be indicative of a more profound perception of errors, probably infused by her exposure to scholarly literature during her postgraduate studies.

*Is this a reading error, is this a maths error, is it a computational error or is it an understanding error?*

When asked whether she thought that errors could be useful as learning opportunities, Dawn responded as follows:

*Absolutely! Absolutely! Oh yes, even categorizing your errors and teaching the children to categorize the errors is much ... It's a strategy, it's a problem-solving strategy. It's a learning strategy.*

Dawn, hence, regarded dealing with mathematical errors as a learning strategy. In order to incline her learners towards this strategy, she encouraged her learners to classify their own mathematical errors. Dawn believed that the processes of teaching and learning mathematics commenced from learners' mathematical errors through dialogue, characterized by discussion and negotiation.

*T: So, for me, errors are a starting point in terms of the teaching and the learning process.*

*I: A starting point for what?*

*T: Conversation. And with that conversation you find out how much they know, how much they don't know and you can begin to build meaning.*

Dawn held the opinion that learners' encounters with their own mathematical errors had the potential to enable learners to develop strategies to mathematical problem-solving.

*Yes, it's a problem-solving; ... it is a strategy to put forward a problem where I know they will encounter; they will come up against something that they can't do, so that they stop and think, yes.*

She expressed the opinion that learners' positive disposition towards their own errors, particularly those in written assessment tasks, and the associated public discussion of these errors, could enhance their mathematical learning.

*Whereas, I've got one or two that are quite happy to say oh! look what I did. This is what I did and it's such a stupid mistake. It's quite strange, but those are the people that are more able, because they enable themselves. They actually embrace the fact; and it's funny, though I see that those people, because they're open to that, ... There's more discussion, there's more learning going on. So, they've broken that whole cycle and it's a positive spiral.*

Time constraints were cited as inhibiting the ideal approach to learners' errors and to reassessment. The educational system required that learners' written assessment tasks were to be filed in portfolios with the implication that learners did not have access to these for formative purposes.

*Unfortunately, with the grade 9's we have to keep the scripts, which makes it a bit difficult. But they then, they work on it in class so they're allowed to correct and remark and make marks on their scripts in a different colour, usually. Because that prompts them to know what they still need to learn and where they made their mistake.*

Dawn experienced learners' mathematical performance, narrowly perceived as their achievement, expressed as a mark, to be a source of concern for the learners. According to her, learners focused on the breakdown of the marks during memorandum discussions with the purpose of increasing their marks. Their mathematical understanding was of less significance to them than the mark they had obtained. Dawn was of the opinion that learners did not appreciate the pedagogical value their mathematical errors have. For these learners errors were synonymous with losing marks.

*And at this stage I have some pupils that are very, very concerned about their marks and they want to know where the mark breakdown is. ... I think for a lot of them it's just about the marks in the end and they're not seeing the link between the mark and the concept.*

Dawn thought that learners did not possess the competence to engage in a mathematical conversation. She cited this perception as an additional obstacle to the ideal interaction with learners' errors.

*I think a lot of our children don't know how to do that conversation.*

She contributed an additional comment on learners' mathematical errors in the written, structured interview.

*Errors can be the starting blocks for learning or they can be the leak in self-esteem. There are certainly categories of errors, but an infinite number of individuals. I think that the best way to make the most of errors is to help the individual to understand the type of error he/she made or makes consistently and equip them to deal with it/them!*

## **7.4 DAWN'S INTERACTION WITH LEARNERS' MATHEMATICAL ERRORS**

### **7.4.1 Verbally During Learning Periods**

#### ***Professed***

During the semi-structured interview, Dawn professed to promote a democratic (Wolk, 1998) classroom characterized by argumentative classroom discourse and subsequent negotiation (see section 2.3.1), allowing learners to have control over the decisive processes of what constituted mathematical truth.

*I think you should from there decide what the best way to put forward or argue your solution ... And then I think after that we should look at the different solutions, and say fine, this one's mathematically sound, it's logical, it's reasonable. That's how I think maths should be taught.*

*Today we had an example where he drew the graph inaccurately and then he got something different from the graph to the logical thinking. And that is so good, because now we had two boys and he was, ... his partner was saying, but this is right, this is what I've done. Why? And now they're looking for the problem, they're looking for the error, and they're both convinced. So yes, I do believe, it's fantastic.*

Interrogation of a learner's perception on what was mathematically right or wrong was of more importance to Dawn than assessing the learner to be correct.

*I like to say, okay what have you done, where have you gone wrong? And (name) for example, in class today, oh! it's wrong, it's just wrong, but why? Why do you say that? I'm more interested in why I should believe it's wrong than whether it's right or wrong, yeah. I really think that you're constructing, you make meaning. You have to understand what you're doing to be able to use what you know.*

From her description of how she responded to learners' errors during learning periods, the impression was given that Dawn initially aimed at achieving clarity from the learner.

*What I normally do is I first ask them. I say, what is the problem and depending on what sort of answer they give me ... if they don't know, then I say let's go back to the question. What was the question? And I find out if the error is in comprehending the question. Then, from that discussion, if they've understood, then they tell me how they're gonna solve it and I say, so what is the problem?*

Following clarification, she facilitated the learner to reasoning and to constructing meaning.

*We say, why you're stuck here? Or, what is the problem here? If they don't know, okay, tell me what you have done so far? So, that's how I normally handle it, in terms of my strategy. It's just, I think it's just getting them to the point where they'll think what they're doing.*

Dawn implied that clarification on the nature of the learner's problem, the type of mathematical error or the quality of the learner's knowledge, was a prerequisite for her pedagogical decisions.

*It depends on what type of error they make. I usually try and clarify that with the learners. Is this a reading error, is this a maths error, is it a computational error or is it an understanding error.*

She distinguished between a learner who experienced difficulty with the interpretation of a mathematical question, and a learner not possessing the necessary or appropriate approach to solving the problem. Dawn expressed a positive opinion, during the semi-structured interview, about peer involvement in error discussion, but she was apprehensive of singling out particular learners. She experienced learners reacting defensively when they were individually confronted with their mathematical errors. She preferred to initiate a public and whole-class discussion of learners' errors with an anonymous example of an error. She might allow learners an opportunity to indicate who had made such an error, but preferred not to identify the learner herself. Dawn expressed being more comfortable with



involving peers in the discussion of alternative approaches to solving a specific problem than with their involvement in error discussion.

*If you say this kind of error was done like that and then I'll say, well, who did something similar? Then they're happy to say no, no, no, it's fine. But to put somebody on the spot and say this is what you did, now tell us everybody ... how ... what's wrong with this? They start off defensively and then I don't think the learning is as effective. It changes from class to class, definitely, but I wouldn't do that with this class.*

### ***Enacted***

Dawn occasionally posed erroneous responses to the learners and asked the learners what the errors could have been. In the first excerpt, the question<sup>1</sup> was to find the correct value for  $x$  if  $x:25 = 3:5$ .

*I want to know something. ... If  $x$  to 25 is the same as three is to five (3:5) what is  $x$ ? ... There was an option that said A 9 and they circled that. So, they said it was nine is to 25 (9:25). What was their mistake here?*

During another incident, Dawn wrote a false mathematical statement on the blackboard, albeit arriving at the correct answer and challenged learners to evaluate the statement. The question was to increase 48 in the ratio 3:5. Dawn imitated a common error by writing the following on the blackboard:

$$48 \div 3 = 16 \times 5 = 80$$

Dawn frequently corrected learners' errors through transmission, occasionally without augmenting the corrections with clarifications. In the following examples, she immediately corrected the learners' errors without explanation. The first excerpt pertains to the graphical representation of cases of direct proportion.

*L: My  $x$ .*

*T:  $y$*

Learners received tables with sets of numbers. They had to determine whether these sets of numbers were examples of direct proportion, inverse proportion or neither. The following excerpt pertains to that exercise.

*L: Here's a product and here's direct ...*

---

<sup>1</sup> The first two mathematical problems are quoted directly from the written assessment task.

*T: No, direct is correct, so this is wrong, quotient. When I divide it ...*

A person cycled a distance of 18 kilometres between Pretoria and Centurion<sup>1</sup>. Learners received a graphical representation of the distance from the starting point. Each hour was divided into four equal segments.

*T: How far is he from home?*

*L: Very far.*

*T: He is eighteen kilometres from home.*

The cyclist left Pretoria at 11:00 and arrived in Centurion at 12:30. This excerpt further serves to show an incidence in which Dawn interacted with a learner's error through negative interjections (see section 3.6.4).

*L: It takes him half an hour to get there.*

*T: It didn't take him half an hour, eleven 'til twelve thirty.*

*L: It's half-an-hour.*

*T: No, no, no, no, it's not, an hour and a half.*

In other instances, although Dawn promptly informed the learners that their contributions were erroneous, she clarified the nature of the errors through explanatory transmission. Learners had to draw a graph to determine the cost of sending parcels overseas, using the fact that the cost for sending a parcel was eight Rand (R8,00) per gram.

*T: What did you get?*

*L: It's five Rand (R5,00).*

*T: No, the mass of goods; so it's the mass of goods for R125,00. What is the mass of goods? So, your answer should be in kilograms.*

Dawn often attempted to scaffold learners' thinking instructionally (see section 3.6.4) with reflective questions. With these questions, she endeavoured to compel learners to reflect on their own solutions and to elicit the mathematics from the learners. However, in some instances the mathematics ultimately came from Dawn (Inman, 2005). Learners received

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<sup>1</sup> With the exception of problems from the written assessment task, the mathematical problems pertaining to case D are quoted directly from Laridon, et al. (2004). Please consult the list of references.

tables with sets of numbers. They had to determine whether these sets of numbers were examples of direct proportion, inverse proportion or neither. The following excerpt pertains to that exercise.

*T: This is an example of what type of proportion?*

*L: Indirect proportion.*

*T: Think about it. Is it a more-more relationship or a more-less relationship?*

*L: More-more; direct.*

*T: Direct, yes. You're right, okay?*

In the following excerpt, learners received a graphical representation of the distance of a moving object from its starting point. The movement was indicated by three sections AB, BC and CD. The displacement of 50 metres was completed in nine seconds. During the first phase of the movement, the object covered 20 metres in four seconds.

*L: Ma'am is it twenty kilometres?*

*T: No.*

*L: Or can I say twenty divided by four?*

*T: Yes, but what is it and what is your speed?*

*L: Five kilometres.*

*T: Look at the graph. In four seconds, so what is your speed? Per second Twenty divided by four is right. That's right, so it's five metres per second. (Dawn did not maintain scaffolding the particular learner instructionally.)*

*L: Oh?*

*T: Is it really fast?*

*L: Not really.*

In the following excerpt, Dawn interpreted and rephrased an erroneous learner contribution to a mathematically correct statement. In the process she temporarily allowed the learner's error, refraining from interrupting, in order to reach the ultimate mathematical goal of eliciting the mathematics from a learner (Inman, 2005). Learners were asked to find the correct value for  $x$  if  $x:25 = 3:5$ .

*L: They take it to the power of five and the power of three.*

*T: Well done! Well done! Do you see their mistake? They said five squared ( $5^2$ ) is 25, therefore three squared ( $3^2$ ) must be nine.*

The following excerpt serves to illustrate an incident in which Dawn involved learners in the negotiations regarding mathematical results. Learners were asked to determine the distance represented by 20 centimetres, using the given ratio of one to five thousand (1:5000)<sup>1</sup>.

*T: What is the answer to number seven L1?*

*L1: Ten thousand.*

*T: Ten thousand centimetres, which is? Which is? One kilometre, isn't that ...?. Am I right?*

*L2: It's twenty times five thousand.*

*T: It's a hundred thousand, which is? Is it ten kilometres? It's not!*

(A lot of commotion among the learners was recorded while the learners were suggesting many different answers.)

*T: Come, show me.*

### ***Comparative Synopsis***

During the semi-structured interview, Dawn alleged to promoting a democratic classroom in which learners were involved in negotiations regarding mathematical truth. However, an extremely limited number of such negotiations were observed in her classroom. Dawn asserted that she interacted with learners' errors during learning periods by initially attempting to achieve clarity from the learner. Subsequent to clarification, Dawn claimed to facilitate the learner to reasoning and constructing meaning. Dawn's observed employment of questions like those presented below, confirmed this assertion.

*Okay, all right, so what do you understand so far?*

*How do you know?*

*What does an inverse relationship mean?*

---

<sup>1</sup> The problem is quoted directly from the written assessment task.

*Does that make sense?*

*See? You've got it? Just for my own benefit, tell me again?*

Dawn showed a strong inclination towards scaffolding learners' thinking instructionally. However, she corrected learners' errors through transmission in an unanticipated high number of instances. Although she frequently clarified the nature of the errors through explanatory transmission, she was more frequently observed to correct learners' errors without augmenting the corrections with clarifications. Dawn was ostensibly positive, but apprehensive about peer involvement in error discussion. She preferred to discuss learners' errors without revealing the identity of the particular learner. In resonance with this, the majority of recorded teacher-learner interactions were on a one-on-one basis.

#### **7.4.2 In Writing in Assessment Tasks**

##### ***Professed***

Dawn did not mention a particular approach to interacting with learners' errors in written assessment tasks during the semi-structured interview. She focused on the memorandum discussion and indicated a preference for a systematic memorandum discussion. She encouraged learners during this discussion to present their alternative approaches to their peers.

*What we normally do for assessment tasks, is I hand back their scripts and we go through it question by question. ... I'm also able to say, are there alternative methods and use people's examples and say well, you've solved it this way, which way would you prefer? So there's a little bit of discussion with that.*

Dawn supported reassessment with concurrent written assessment tasks.

*And they may ask for another chance. In which case, sometimes we've said all right, this is how the test is going to be and then we set another assessment task which is very similar and it gives them a second chance. And I usually find that for the weaker ones (inaudible) it gives them confidence and they're able to do it again. But we don't do that as often as we should, probably just because of the time constraints.*

##### ***Enacted***

Analysis of the written assessment tasks revealed that Dawn often used written comments on the assessment tasks to communicate with her learners. The questions or instructions she posed to the learners in the following excerpts are illustrative of this tendency. Content

analysis provided evidence that Dawn encouraged learners to show their thinking during problem-solving and rewarded learners for clear and logical thinking. The following excerpt indicated how she penalized the learner irrespective of the learner’s correct choice from the multiple possibilities, based on the learner’s inability to convince her of how he or she arrived at the conclusion. Dawn queried this with the comments “Doesn’t match.” and “Why?” Dawn’s inclination to reflective questioning is illustrated with her use of the question “Why?” Following a question like this, through implicit referral of a learner to reflect, the potential for a learner to meta-learn exists (Slabbert, et al., 2009).

9. R10 800 shared in the ratio of 2 : 3 : 4 is

A) 2 400 : 4 800 : 3 600

**B) 2 400 : 3 600 : 4 800**

C) 4 400 : 2 800 : 3 200

D) 2 000 : 3 000 : 4 000

*Handwritten notes:* "doesn't match", "2 x 10 800 : 3 x 10 800 = 4 x 10 800", "why x?", "Divide times", "1", "2" [4]

**Figure 7-1 Question 9 of written assessment from school D**

Dawn commended and encouraged learners for their potentially viable mathematical strategies, irrespective of the crudeness thereof and despite resultant mathematical errors. In the example supplied below, a learner executed a perfect strategy to solving the problem, but was unable to interpret the result. Dawn commended the mathematical approach and urged the learner to reflect on the implication with her “But why?”

16. Which item is the better value for money?

A) 125g soap @ R3,25

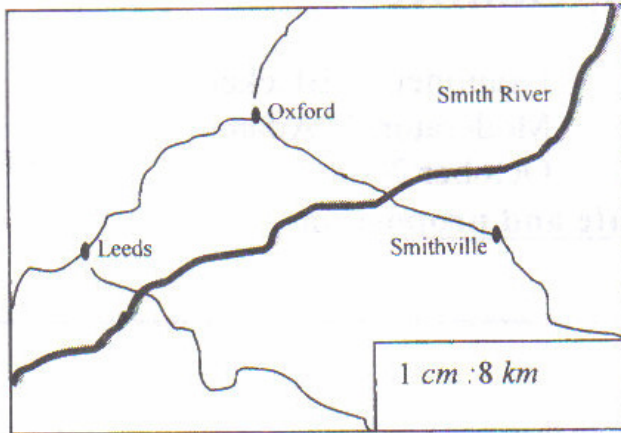
**B) 100g soap @ R2,75**

*Handwritten notes:* "125g x 4 = 500g = R13", "100g x 5 = 500g = R13,75", "Right working", "But why?", "2" [3]

**Figure 7-2 Question 16 of written assessment from school D**

The example below illustrates Dawn’s encouraging way of communicating with the learners, using the written remark: “Come on, you CAN”.

One centimeter on the map represents 8 kilometers on the land.



Come on  
you CAN

About how far apart are Oxford and Smithville on the land?

- A) 4 km   
  B) 16 km   
  C) 35 km   
  D) 24 km

**Figure 7-3 Question 3 of written assessment from school D**

No attempt to approach the comparatively serious error in the ensuing excerpt, other than the question mark and the reflective question, is apparent. The learner demonstrated uncertainty and an inability to increase a number in a given ratio. The learner attempted to divide the number by both possibilities three and five, respectively. There was not evidence that learners responded to these reflective questions or that Dawn followed up on these questions.

10. 48 increased in the ratio of 3 : 5 is

A) 60   
  B) 120   
  C) 80   
  D) 56

$$\frac{48}{3} = 16$$

$$\frac{48}{5} = 9$$

$$16 + 9 = 25$$

$$\frac{48}{3} = 16$$

$$\frac{48}{5} = 9$$

$$16 + 9 = 25$$

$$\frac{48}{3} = 16$$

$$\frac{48}{5} = 9$$

$$16 + 9 = 25$$

What are you doing here?

**Figure 7-4 Question 10 (i) of written assessment from school D**

The following two examples are exemplifying Dawn's interaction with learners' errors through instructional scaffolding. In the first excerpt, the instructional scaffolding of the learner's thinking was done by directing the learner towards correct interpretation of the question with the remark: "side:perimeter, not side:side", in addition to underlining the key words.

5. What is the ratio of the length of a side of a square to its perimeter?

A) 1:1  
 B) 1:2  
 C) 1:3  
 D) 1:4

$3:3 \rightarrow 1:1$   
Simplified.

$\text{side : perimeter}$   
 $\text{not}$   
 $\text{side : side.}$

$\text{C} \quad [1]$

**Figure 7-5 Question 5 of written assessment from school D**

In the second example, a learner portrayed a dependence on the calculator, which use was prohibited during the assessment task. In order to support the learner in calculating 15% of 1172, Dawn instructionally scaffolded the learner’s thinking with: “Try 10% increase  $\approx 117 \rightarrow 20\%$  increase  $\approx \dots$ ”.

4. Last year there were 1172 students at Beaton High School. This year there are 15 % more students than last year. Approximately how many students are at Beaton High School this year?

A) 1800  
 B) 1600  
 C) 1500  
 D) 1400  
 E) 1200

$\frac{1172}{1} \times \frac{15}{100}$   
 $= 17380$   
 $= \frac{1738}{10}$

$\text{Try } 10\% \text{ inc is } \frac{1172}{10} = 117.2$   
 $\therefore 20\% \text{ is } \dots$

$\frac{1172}{10} = 117.2$   
 $\frac{1172}{10} = 117.2$

[3]

**Figure 7-6 Question 4 (i) of written assessment from school D**

The subsequent example serves to illustrate an instance in which a learner actually solved a problem correctly, but did not succeed in interpreting the result. Dawn acknowledged the learner’s attempt and endeavoured to facilitate the learner to the interpretation of his or her solution with the heuristic use of the encircled numbers 1, 2 and 3 on the right hand side. Dawn probably attempted to indicate the correct result of three kilograms of R38,00 each to the learner.



14. At R38,00 per kg, how much cheese can Susan buy for R114,00?

- (A) 3kg
- (B) 2kg ~~X~~
- (C) 1,5kg
- (D) 2,5kg

~~R38~~ ~~R114,00~~ ~~R114,00~~ ~~R38,00~~

$114 \div 38 = 2$   
 $R38,00 = 1\text{kg}$   
 $R114,00 = 2\text{kg}$

Right idea - check your calculation.

$$\begin{array}{r} 38 \\ +38 \\ \hline 76 \\ 383 \\ \hline 114 \end{array}$$

[3]

**Figure 7-7 Question 14 of written assessment from school D**

Although Dawn portrayed an inclination to the provision of written comments or to instructional scaffolding with which to manoeuvre learners to reflection, she did, in multiple instances, merely indicate right and wrong answers with conventional markings (see section 3.6.4 and appendix C). Although the learner's confusion between proportional increase and proportional allotment is obvious in the following excerpt, Dawn merely employed a conventional marking to indicate the flawed approach. She apparently overlooked the learner's inappropriate strategy and perceived confusion.

10. 48 increased in the ratio of 3 : 5 is

- A) 60
- B) 120
- C) 80
- (D) 56

$48 \div 8 = 6$   
 $3 \times 6 = 18$   
 $5 \times 6 = 30$

[3]

**Figure 7-8 Question 10 (ii) of written assessment from school D**

Dawn infrequently provided learners with correct solutions to problems. The following example shows a particular instance in which she approached the learner's error by supplying a written explanation of the nature of the learner's error. Dawn underlined the key words to facilitate the learner to the desired interpretation of the question.

7. To mix a certain colour of paint, Alan combines 5l of red paint, 2l of blue paint, and 2l of yellow paint

What is the ratio of red paint to the total amount of paint?

- A) 5:2
- B) 9:4
- C) 5:4
- D) 5:9

Handwritten student work for Question 7:

5:2:2  
5l : 2l + 2l = 9 parts altogether  
= 5:4

⓪ [3]

Figure 7-9 Question 7 of written assessment from school D

The excerpt provided below shows how a learner determined a quotient incorrectly by writing  $\frac{8}{35} \div \frac{4}{15}$  as  $\frac{35}{8} \times \frac{4}{15}$ . Dawn responded with an exclamation mark and a question mark and by capturing the calculation correctly.

2. Divide:

$\frac{8}{35} \div \frac{4}{15}$

A)  $\frac{1}{2}$

B)  $\frac{6}{7}$

C)  $\frac{32}{525}$

D)  $\frac{7}{6}$

Handwritten student work for Question 2:

$\frac{8}{35} \div \frac{4}{15} = \frac{35}{8} \times \frac{4}{15}$

$= \frac{32}{525}$

$\frac{8}{35} \times \frac{15}{4}$

⓪ [3]

Figure 7-10 Question 2 of written assessment from school D

In the final excerpt provided, Dawn commended the learner for his or her thinking with the comment “on the right track”. However, she refrained from highlighting a possible misconception exposed during the learner’s attempts to arrive at the final solution to the question. The learner rewrote  $\frac{15}{100}$  of 1172 as  $\frac{15}{100} \times \frac{1}{1172}$ . Dawn apparently ignored or overlooked this. In addition to that, she refrained from indicating the syntactical flaw of writing 15% as  $\frac{15\%}{100}$ .

4. Last year there were 1172 students at Beaton High School. This year there are 15 % more students than last year. Approximately how many students are at Beaton High School this year?

A) 1800  
 B) 1600  
 C) 1500  
 D) 1400  
 E) 1200

~~$\frac{1172}{1} \times \frac{15}{100}$~~       ~~$\frac{1172}{1} \times \frac{100}{15}$~~

$\frac{15\% \text{ of } 1172}{100} \times 1172$       $\frac{15}{100} \times \frac{1}{1172}$      on the right track.

$\frac{1172}{117} = 10\% \text{ more}$

$$\begin{array}{r} 1172 \\ \times 117 \\ \hline 8204 \\ 12890 \\ \hline 137124 \end{array}$$

(2) [3]

**Figure 7-11 Question 4 (ii) of written assessment from school D**

**Comparative Synopsis**

Dawn portrayed a strong inclination towards communication by employing written comments in assessment tasks. She commended and encouraged learners in writing. She often compelled learners to reflect, to critically assess their thinking and to meta-learn with questions and written remarks like those presented below:

*Is this mathematically true?*

*Try to look at the whole picture!*

*Where did you get this?*

*Does not make sense; please explain!*

Dawn often scaffolded learners’ thinking instructionally, occasionally facilitating understanding with the use of heuristics. Dawn encouraged learners to explicate their thinking during problem-solving and rewarded learners for clear and logical thinking. However, Dawn was not consistent in her interactions with learners’ errors in written assessment tasks. She sporadically refrained from identifying learners’ exact errors or learners’ confusion or limited understanding as exposed by their errors. Dawn infrequently corrected learners’ errors without supplying accompanying explanations. Although Dawn refrained from describing her interaction with learners’ errors in written assessment tasks during the semi-structured interview, her inclination towards written communication was convincing.

## **7.5 CONCLUSION**

In the personal definition of mathematics that Dawn generated, she referred to mathematics as a language and recognized mathematics as an explorative and explanatory tool. She accentuated personal qualities and passion in the description of her qualities as a mathematics teacher. Dawn described her preferred approach to the teaching of mathematics as constructivist, focusing on problem-based and cooperative learning. Dawn had exposure to scholarly literature through her postgraduate studies in mathematics education.

Observations revealed that Dawn frequently refrained from expository teaching. This resulted in learning periods distinguished by dominant tutorial-type segments. Learner involvement was overtly encouraged by Dawn. However, learner participation was limited to cooperative pair discussions and dialogues between Dawn and individual learners. Attempts to accommodate learners' contributions were recorded in Dawn's classroom. The responsibility for the mathematical evaluation of learner contributions seemed to reside with Dawn, though. Dawn made abundant use of reflective questions and questions associated with mathematical interpretation, albeit adequate evidence of probing learners' responses was not observed.

Dawn expressed the opinion that the origin of learners' erroneous procedures could be either mathematical or linguistic in origin. She perceived learners' errors as valuable learning opportunities from which teaching and conversational dialogue could develop. She held the opinion that interaction with his or her own errors could develop a learner's problem-solving strategies and enhance mathematical learning. From own experience, Dawn affirmed that a learner's willingness to participate in the public discussion of his or her errors could improve mathematical learning. Dawn identified a number of factors as obstacles in the preferred interaction with learners' errors. Time constraints and the requirements of the educational system were inhibiting the ideal approach to learners' errors. In addition to that, learners found it challenging to engage in a mathematical conversation and were more concerned with their achievement expressed as a mark than with their mathematical understanding.

Dawn purported to have established a democratic classroom in which learners negotiated mathematical meaning. However, instances of negotiation, albeit observed and recorded, were limited. She professed to a concern regarding learners' mathematical meaning as

opposed to learners' correct responses. Frequent instances in which Dawn corrected learners' errors through transmission without debate or negotiation were recorded, though. Her recurrent use of reflective questions and questions aimed at scaffolding learners instructionally concurred with her professed approach of achieving clarity and facilitating learners to understanding. She did not maintain her professed interaction with learners' errors and portrayed ambivalence in her approach. She frequently corrected learners' errors directly and abruptly, without discussions, during learning periods. Her predominantly one-on-one interaction with learners was in resonance with the apprehension she professed about peer involvement in error discussion.

Dawn's interaction with learners' errors in written assessment tasks displayed a higher level of correspondence to her professed constructivist approach to teaching mathematics than her interactions during learning periods. Although content analysis of the written assessments revealed instances in which Dawn corrected learners' errors, her employment of reflective, written remarks and questions outnumbered other ways of interacting with learners' errors in assessments.

### ***8.1 INTRODUCTION AND OVERVIEW OF RESEARCH REPORT***

The purpose of this concluding chapter is to consolidate the research questions pertaining to this investigation, the research process pursued to obtain the evidence addressing the research questions, the construed findings, the recommendations and the resultant conclusions. A comparative synopsis of the four contextual narratives in section 8.2.1 is followed by a recapitulation of the three secondary research questions in section 8.2.2. A classification of the teachers' beliefs about learners' mathematical errors and the teachers' interactions with errors follows in section 8.2.3. A discussion of emerging themes is subsequently presented in section 8.3, followed by methodological and scientific reflections in section 8.4. The recommendations for teacher-training and professional development and for further research in section 8.5 precede the concluding thoughts in section 8.6.

The investigation was directed by the primary research question: *How do secondary school mathematics teachers interact with learners' mathematical errors?* The purpose of the research was to investigate the interactions of secondary school mathematics teachers with learners' mathematical misconceptions or errors. The study further sought to explore teachers' beliefs about mathematics, about learners' mathematical errors and the role errors can play in the teaching and learning of mathematics. The study is embedded in the teaching and learning of mathematics, contextualized in grade 9 classrooms. Feeding into the primary research question were three secondary research questions:

- What beliefs about mathematics, about learners' mathematical errors and about the role errors can play in the teaching and learning of mathematics do mathematics teachers have?
- How do mathematics teachers interact verbally with learners' errors during learning periods?
- How do mathematics teachers interact in writing with learners' errors in assessment tasks?

The rationale for conducting the research was argued from perspectives embracing a personal interest, gaps in the literature and the questionable state of mathematics education in South Africa. The investigation was executed in four South African secondary schools in the Gauteng province.

The research paradigm in which my theoretical perspective is located is defined as constructivist-interpretive (Mackenzie & Knipe, 2006). My ontological assumptions are described as relativist (Denzin & Lincoln, 2005; Smith & Hodkinson, 2005), my epistemology as constructivist/socio-constructivist (Ernest, 1997) and the research methodology as qualitative (Merriam, 1991). The research design is identified as an educational, descriptive, interpretive, multiple-case study (Merriam, 1991; Yin, 2003). The four participants were conveniently selected from partnership schools of the university where I lectured part-time. Alice taught at a gender-specific, all-girls' school, Barry at a focus-school, specializing in Arts and Entrepreneurship, Chloe at a co-educational school and Dawn at a private, Christian school. The research was contextualized in grade 9 classrooms. Data were collected through interviews, observations and document analyses. Transcriptions of the semi-structured interviews and the classroom events, the written, structured interviews and the learners' written assessment tasks were analysed according to the Miles and Huberman model (1994) of within-case data reduction and data display. Coding of the chunks of data was done both deductively and inductively (Miles & Huberman, 1994). The data are presented as a contextual narrative for each participant. These data presentations are demarcated along themes, inter alia the participant's professed mathematical beliefs, a classroom vignette, the participant's professed beliefs about learners' errors and the participant's verbal and written interaction with learners' errors, both professed and enacted.

The findings from this investigation show that when teachers believe that the value of learners' errors is vested in the corrections thereof, rather than using these opportunities for discussion, valuable opportunities for learners to develop and improve their meta-cognitive abilities may potentially be lost. The findings further show that a focus on the mere correction of learners' errors probably denies learners opportunities to develop a mathematical discourse. The results of the investigation illuminate a disapproving disposition towards errors as well as the emphasis on achievement during assessment as barriers to engendering a socio-constructivist learning environment in which interactions with learners' errors enhance learning and establish a negotiating mathematical

community. In addition to the preceding findings, the study reveals that the ways in which teachers interact with learners' errors do not necessarily correspond to their teaching practices or their mathematical beliefs. A comprehensive discussion of the findings ensues in sections 8.2 and 8.6.

## **8.2 OVERVIEW OF FINDINGS**

### **8.2.1 Comparative Synopsis of Contextual Narratives**

All four research participants were sufficiently qualified to teach grade 9 mathematics, with only Dawn being exposed to postgraduate mathematics education. The predominant, individual attributes, as mathematics teachers, which were identified by the participants, portrayed stark differences. Alice focused on personal traits and a disposition towards learners. Barry highlighted his mathematical knowledge and his ability to instil and maintain discipline. Although Chloe recognized her ability to motivate learners as an attribute, she concurred with Barry in emphasising her mathematical knowledge and the disciplined classroom environment that she managed to maintain. In resonance with Alice, Dawn accentuated personal qualities and her passion for mathematics teaching.

The personal definitions of mathematics that the research participants had constructed rendered dissimilarities congruent to those identified in their attributes. Alice described mathematics as a language, a way of communicating and a means of understanding the social world. The definition that Dawn generated demonstrated considerable correspondence to that of Alice. Dawn referred to mathematics as a language and recognized mathematics as an explorative and explanatory tool. Barry described mathematics as a formal and disciplined science. Although Chloe, like Alice and Dawn, referred to mathematics as a language and a tool, she was concurring with Barry in accentuating the order she associated with mathematics.

The four participants' professed approaches to teaching mathematics revealed similar irregularities and resemblances as previously discussed. Alice and Dawn focused on their learners, on the learners' thinking, their prior knowledge and their active involvement. Barry revealed a preference for traditional teaching methods, encompassing expository teaching and drill work. Chloe was in agreement with Barry in giving prominence to the teaching of stepwise procedures. Dawn was the only participant who referred to or implied constructs like constructivism, instructional scaffolding and negotiation.



The segmentation of the learning periods portrayed a degree of equivalence in all four classrooms. A segment that entailed the introduction and discussion of a subsequent topic usually followed the homework discussion. With the exception of Barry's classroom, the learning periods concluded with a tutorial-type slot during which the teacher attended to individual learners' mathematical difficulties. Although differences existed in the duration and the order of the segments, a tutorial was identified in all four of the classrooms. However, the routine in Barry's classroom differed from the other routines. The tutorial-type slot was only observed occasionally and was utilized during the teaching phase of the learning period. Barry often employed an entire learning period for teaching through transmission. Dawn frequently refrained from expository teaching, resulting in a more dominant tutorial-type segment with considerable time being allocated to it.

The introduction of a new mathematical topic with the announcement of the appropriate heading for learners to record in their workbooks was observed in the classrooms of Alice, Barry and Chloe. In contrast to this, Dawn innovatively introduced new topics and involved the learners in the process. Expository, explanatory teaching through transmission took place in Barry's classroom and in Chloe's classroom almost without exception, was prevalent in Alice's classroom, and occasionally observed in Dawn's classroom. Barry could be distinguished from the other research participants in his tendency to alert learners to common, procedural and syntactical, mathematical errors. The textbook played a fundamental role in all four classrooms. The mathematical exercises learners had to do and the mathematical problems learners had to solve were exclusively found in the textbooks. An attempt to contextualize mathematical problems was often observed in Alice's classroom and in Dawn's classroom, seldom in Chloe's classroom and not once in Barry's classroom. Albeit to a lesser degree in Dawn's classroom, the transmission of algorithms was observed in all the classrooms.

A degree of learner participation in classroom discussions was observed in all four classrooms. However, with the exception of the cooperative pair discussions that were encouraged in Alice's classroom and in Dawn's classroom, classroom interactions were constrained to dialogues between the teacher and an individual learner. The equivalent seating arrangements of rows of learners facing the blackboard in all four classrooms might have contributed to the nature of the classroom interactions. Although learner involvement was overtly encouraged by Alice, Chloe and Dawn, Chloe carefully managed and contained the learner participation in her classroom, not allowing for spontaneous

participation. Conversely, in Barry's classroom, learner participation was spontaneous, but not observed to be encouraged.

The participating teachers' apparent disposition towards learners' mathematical contributions, including learners' suggested alternatives to solving problems, revealed minor dissimilarities. The responsibility for the mathematical evaluation of learners' contributions seemed to reside with the teacher in all four of the classrooms. Attempts to accommodate learners' contributions or learners' suggested alternatives were recorded in Alice's classroom and in Dawn's classroom. However, the accommodation of learners' contributions was seemingly conditional. Adaptation of teachers' procedural approaches in Alice's classroom and in Dawn's classroom was maintained within the accepted procedural boundaries. A single instance of negotiation was observed in Dawn's classroom. Barry did not request mathematical contributions from the learners during the two-week observation period. Chloe acknowledged learners' contributions or alternative approaches, but was not observed to employ these in her teaching.

Although the nature of the classroom communities established in each of the four classrooms appeared to differ, they seemed distinctly similar in the aspect regarding the role of the teacher. It was not observed that any of the four teachers accepted the role of a participating member in the classroom community. An apparent dependence of learners on the teacher was observed in all four classrooms, with the teacher acting as the validating authority in all of the classrooms.

Interesting trends were observed in terms of the teachers' questioning. All four participants asked questions aimed at recalling knowledge, applying knowledge and mathematical interpretation. However, Dawn was dominant in her demonstration of a tendency towards questions associated with mathematical interpretation. Alice, Barry and Chloe portrayed a similar trend in their focus on procedural questions. Barry was not observed to ask reflective or open questions. Conversely, Dawn made abundant use of reflective questions, albeit adequate evidence of probing learners' thinking was not perceived. Barry made profuse use of rhetorical questions and often asked questions related to mathematical conventions or mathematical syntax.

All four participants predominantly listened to learners' contributions in an evaluative way: listening in anticipation of preconceived answers (Davis, 1997). Chloe and Dawn were observed to occasionally listen interpretively, asking questions to obtain clarity about

learners' conceptions and understanding. No instances of hermeneutic listening were observed in any of the four classrooms during the respective periods of fieldwork. In none of the four classrooms were learners' contributions observed to alter the flow and structure of the learning periods.

With the exception of Barry, the participating teachers assessed learners informally on a regular basis. However, only Dawn made use of alternative assessment techniques like pop quizzes, journal entries and mind-maps. The incidence of informal assessment in Dawn's classroom was higher than in the other classrooms. The approach to formal, written assessment was analogous in all four the classrooms. Learners wrote a standardised term test of forty marks on average, comparable to their class work. The term tests that learners at school A and school D wrote contained problems in realistic context. Conversely, the mathematical problems in the term tests that learners at school B and school C wrote were isolated from the context. Recognition of the different mathematical topics being discussed in the four classrooms during the periods of fieldwork is important. Based on this difference, the preceding comparison may be inappropriate. A reflection on the respective memorandum discussions, following the formal, written assessments, ensues in section 8.3.6.

Two views on learning, the behaviouristic view and the constructivist view, encompassing the socio-constructivist view, have been discussed in the literature review pertaining to this study (see section 2.2). Teachers' mathematical beliefs and their professed and enacted approaches towards teaching in general and towards error-handling in particular, can be mirrored in these two views on learning (Davis, 1997; Gatt & Vella, 2003; Olivier, 1992). However, these two views correspond to and are situated on a continuum of possible orientations towards mathematics, mathematics teaching and mathematics learning. The data analysis confirmed the assumption that some teachers portrayed ambivalence in their classroom practices. Chloe, for example, was convincingly behaviouristic in her observed teaching approach, but her questioning portrayed constructivist elements. Hence, to classify a teacher consistently in terms of all the relevant aspects of their practice as behaviouristic or as constructivist was unfeasible, particularly in the case of Chloe and of Dawn.

The literature review informed the structure of the contextual narratives, including the classroom vignettes (see sections 4.2, 5.2, 6.2 and 7.2). The following table, serving as a

visual and comparative synopsis of the contextual narratives, is compiled from constructs construed from the literature review. Teachers' beliefs about errors and interaction with errors are delineated separately, as a synopsis of the three secondary research questions (see section 8.2.3). The relevant constructs for the contextual narratives are the following:

- Teachers' professed beliefs about mathematics, teaching mathematics and learning mathematics (Ernest, 1988).
- Observed teaching approach (Ernest, 1988; Meyer & Turner, 2002), encompassing the prevalent presentation of the content (Bauersfeld, 1994; Karagiorgi & Symeou, 2005; Skemp, 2006; Tobin & McRobbie, 1999) and inclination towards cooperative learning (Kovalainen & Kumpulainen, 2007; Szydlik, et al., 2003).
- Observed classroom discourse (Cazden & Beck, 2003; Martens, 1992; Mehan, 1979; Tobin & McRobbie, 1999) and concurrent learner involvement (Davis, 1997; Mason, 2000).
- Observed teachers' questioning (Cazden & Beck, 2003; Davis, 1997; Hargreaves, 1984) and listening (Davis, 1997; Ruggiero, 1988).
- Observed roles and relationships including the degree of authority the teacher exercised (Askew & Carnell, 1998; Ernest, 1988; Kaldrimidou et al., 2004; Mason, 2000), the accommodation of alternative strategies (Brodie, 2008) and negotiation (Bauersfeld, 1994; Tobin & McRobbie, 1999).
- Classroom management, delineated in terms of seating arrangements (Tollefson & Osborn, 2008), general routine and segmentation of the learning periods (Davis, 1997), teaching and learning support materials (Ernest, 1988) and assessment (Baviskar, Todd Hartle & Whitney, 2009; Buhagiar & Murphy, 2008; Cross, 2009; Pegg & Panizzon, 2008; Popham, 2007).

**Table 8-1 A visual and comparative synopsis of the contextual narratives**

<b>Teacher/Aspect</b>	<b>Alice</b>	<b>Barry</b>	<b>Chloe</b>	<b>Dawn</b>
Professed beliefs about mathematics, teaching mathematics and learning mathematics	Platonist; behaviouristic with constructivist elements	Instrumentalist; behaviouristic	Instrumentalist-Platonist; behaviouristic	Problem-solving; constructivist
Observed teaching approach	Explainer; behaviouristic	Instructor; behaviouristic	Instructor; behaviouristic	Facilitator-explainer; constructivist with behaviouristic elements
Observed discourse and learner involvement	Behaviouristic with constructivist elements	Behaviouristic	Behaviouristic	Behaviouristic with constructivist elements
Observed teacher's questioning and listening	Procedural questions, evaluative listening, behaviouristic	Procedural, trivial and rhetorical questions, evaluative listening, behaviouristic	Primarily procedural questions, open, reflective questions and questions related to mathematical interpretation, evaluative and interpretive listening, behaviouristic with constructivist elements	Open, reflective questions and questions related to mathematical interpretation, evaluative and interpretive listening, constructivist with behaviouristic elements

## 8.2.2 Comparative Synopsis of Secondary Research Questions

### *Secondary Question One*

With the exception of Chloe, the participating teachers' beliefs about learners' errors were positive. Alice, Barry and Dawn acknowledged learners' errors as useful learning opportunities that they asserted they employed in their teaching. Dawn professed to a belief that learners' errors could potentially develop a learner's problem-solving strategies and enhance mathematical learning. Concurrent with this view were the beliefs expressed by Alice and Dawn, about learners' errors as initiators for discussion and learners' errors as an expository basis from which teaching and conversational dialogue could develop. However, the participating teachers predominantly recognized the value of learners' errors in the juxtaposition of erroneous mathematical procedures with correct solutions. The participating teachers usually conceded that the importance of learners' errors was inherent to a contingency with complete and correct mathematical solutions. The complete and correct solutions, transmitted to the learners through expository teaching, provided learners with sources for learning, and not their mathematical errors per se. With the exception of Barry, the participants were observed to classify learners' mathematical errors. In line with the findings of Leu and Wu (2005), the classifications were superficial, usually focusing on computational errors and so-called "sign" errors. Chloe's references to learners' errors ostensibly portrayed a tendency to confine her perceptions of errors to a primary focus on incorrect answers. Alice, Barry and Dawn identified the requirements of the educational system as a principal systemic impediment to interacting with learners' errors, while Alice, Chloe and Dawn identified time constraints as a barrier. Chloe additionally referred to large, heterogeneous classes as problematic. Barry and Dawn identified obstacles pertaining to the learners. Both of them referred to a lack of proper understanding of mathematics and the inability to think critically. Dawn recognized an incapability to engage in a mathematical conversation and a concern with achievement, expressed as a mark, rather than with their mathematical understanding, as factors impeding interaction with learners' errors. All four participating teachers identified contemptuous peer attitudes and the possibility of peer mockery as a primary impediment to interaction with learners' errors.

### *Secondary Question Two*

Alice and Barry routinely responded to all audible mathematical contributions, responses or questions, correct or incorrect, from learners. Chloe was occasionally observed refraining from verbally interacting with learners' errors during learning periods, while Dawn was recorded refraining from verbally interacting with exact errors in isolated incidences. All four teachers predominantly corrected learners' errors through transmission, without involving peers. Barry and Dawn occasionally corrected learners' errors promptly and directly, without supplying explanations to the learners. Alice was observed clarifying learners' errors with explanatory monologues, while Chloe usually explained why learners' contributions, responses or questions were erroneous or indicative of essential misconceptions. However, the majority of the teachers' accounts were related to mathematical procedures and were cues, rather than of a fundamentally mathematical nature. Although Alice and Barry employed questions to scaffold learners instructionally in isolated instances, they were not observed interacting with learners' errors through questioning. Chloe and Dawn often interacted with learners' errors through questioning and often scaffolded learners instructionally with questions, albeit varying in the degree to which the questions were reflective or implicit. Unfortunately, the teachers generally did not maintain these interactions through questioning. On many occasions the participating teachers ultimately produced the mathematics, as described by Inman (2005).

### *Secondary Question Three*

Considerable inconsistencies existed among the four participants in the way they interacted in writing with learners' errors in assessment tasks. Written interaction with learners' errors ranged from indicating errors with markings of some kind to supplying learners with complete, intact, mathematical solutions. Barry and Chloe primarily interacted with learners' errors in assessments by indicating the errors with a variety of markings. Alice and Dawn were both inclined to interact and communicate with written comments. However, variation in Alice's and in Dawn's interactions also occurred. Both of them corrected learners' errors with or without elucidative, written comments. Content analysis of the written assessments showed numerous instances in which all four of the participating teachers did not interact with learners' exact errors. The teachers often interacted in writing with obvious learners' errors, for example the so-called "sign" errors or computational errors, but refrained from probing learners' errors that were, in my opinion,

of a more intricate nature. Individual learners often repeated similar types of errors without teachers diagnosing these repetitive errors. The likelihood of a teacher's written interaction with a learner's errors in an assessment task being contingent on the learner's performance appeared to be plausible.

### **8.2.3 Classifying Teachers' Beliefs about and Interactions with Errors**

The following table serves as a visual and comparative synopsis of the three secondary research questions. I made the pragmatic decision, for comparative purposes, to classify the participating teachers' predominant interactions with learners' errors according to the same, simplified system I used for the contextual narratives. The aspects I considered during classification of teachers' verbal interactions with learners' errors during learning periods were derived from the literature review. These include the following:

- Teachers' focus on either correction of or probing of errors during their interaction with errors (Brodie, 2005; Halim & Mohd.Meerah, 2002; Heinze, 2005; Kaldrimidou et al., 2004; Leu & Wu, 2005; Santagata, 2005).
- The incidence of opportunities provided for learners to give feedback on and discuss their inappropriate understanding (Bauersfeld, 1994; Barkatsas & Malone, 2005; Beswick, 2005; Brodie 2008; Ernest, 1988; Mehan, 1979; Tobin & McRobbie, 1999).
- The nature and aim of instructional scaffolding (Martens, 1992; Santagata, 2005).
- The degree of peer involvement in error discussion (Heinze, 2005; Santagata, 2005).
- Teachers' interpretation or classification of learners' errors (Leu & Wu, 2005; Olivier, 1992; Smith, et al., 1993).

Teachers' written interactions with learners' errors in assessment tasks were classified according to:

- the occurrence of teachers' written comments (Brodie, 2008; Leu & Wu, 2005; Smith, et al., 1993);
- the formative and reflective value of the interaction (Baviskar, et al., 2009; Pegg & Panizzon, 2008; Santagata, 2005); and





- teachers’ interpretation or classification of learners’ errors (Leu & Wu, 2005; Olivier, 1992; Smith, et al., 1993).

**Table 8-2 A visual and comparative synopsis of the three secondary research questions**

<b>Teacher/Aspect</b>	<b>Alice</b>	<b>Barry</b>	<b>Chloe</b>	<b>Dawn</b>
Beliefs about learners’ errors and the role of errors	Behaviouristic with strong constructivist elements	Behaviouristic with limited constructivist elements	Behaviouristic	Constructivist
Professed verbal interaction during learning periods	Behaviouristic with constructivist elements	Behaviouristic	Behaviouristic	Constructivist
Enacted verbal interaction during learning periods	Behaviouristic	Behaviouristic	Behaviouristic with constructivist elements	Behaviouristic with constructivist elements
Written interaction in assessment tasks	Behaviouristic with constructivist elements	Behaviouristic	Behaviouristic	Constructivist with behaviouristic elements

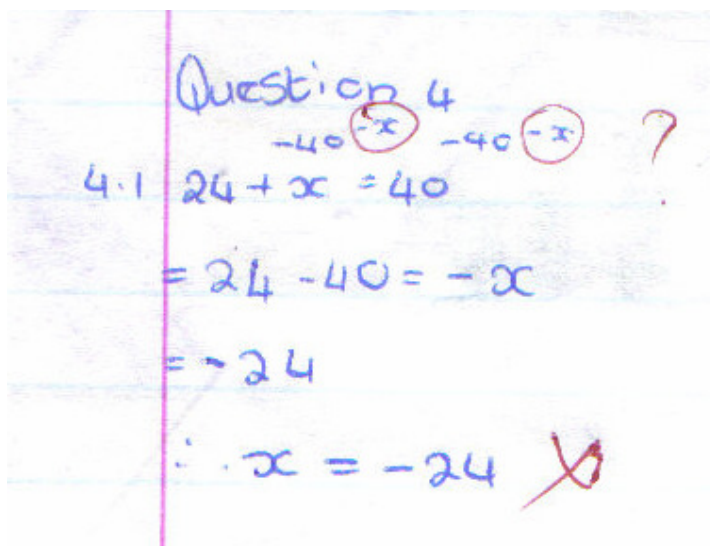
The teachers’ verbal interactions with learners’ errors during learning periods and written interactions in assessment tasks were usually inconsistent attempts to explicative communication with learners. The participants predominantly interacted verbally with learners’ errors during learning periods by correcting the errors through transmission, with an explanation and without involving peers. The participating teachers primarily interacted in writing with learners’ errors in assessments by indicating learners’ errors with various markings, albeit inconsistently indicating exact errors. Irrespective of significant, individual differences in professed and enacted teaching approaches, the general trend in interacting with learners’ errors among the four participants portrayed a considerable

degree of correspondence. A plausible classification of the participants' overall approaches could be the correction of learners' errors.

### 8.3 EMERGING THEMES

#### 8.3.1 Learning Corresponds to Teaching

Learners' misconceptions in mathematics may be a consequence of the teaching. In the following excerpt (see section 6.4.2), a learner exactly imitated what Chloe had previously explained on the blackboard. Chloe had the habit of writing the number inverse for addition a bit smaller and elevated, almost resembling the exponent of a power. The learner did precisely that.



Question 4  
 $-40^{-x} - 40^{-x} ?$   
 4.1  $24 + x = 40$   
 $= 24 - 40 = -x$   
 $= -24$   
 $\therefore x = -24$

**Figure 8-1** Question 4.1 of written assessment from school C

In the second excerpt (see section 5.4.1), from Barry's classroom, Barry was demonstrating to learners how to find the equation of a straight line, parallel to the line  $y = -x + 3$ , through the point  $(-1;3)$ . Following substitution of the coordinates, Barry wrote down  $3 = 1 + c$ , finding  $c = 2$  and the equation of the line to be  $y = -x + 2$ . The learner's mathematical understanding, as exposed by the terminology he or she employed, portrayed little mathematical substance, but considerable correspondence to Barry's semantics (see section 5.2).

*L: Sir, why did the minus come back?*

Apparently Barry was under the initial impression that the learner was referring to the fact that the gradients of both straight lines equalled negative one.

*T: Because they said this line that we get now, that we have to get, is parallel to that one.*

However, the learner referred back to where Barry solved for the  $y$ -intercept,  $c$ . The learner was confused between the  $x$ -coordinate, negative one, of the point  $(-1;3)$  and the term, positive one, in the equation  $3 = 1 + c$ .

*L: Did you change the one into a positive to find  $c$ ?*

### **8.3.2 Understanding Relationally and Thinking Critically**

Both Barry and Dawn referred to learners' inadequate mathematical understanding and deficiency in critical thinking. However, the way in which the participating teachers often interacted with learners' errors did not have the potential to improve learners' understanding, address learners' misconceptions or to enhance learning. The teachers sporadically scaffolded individual learners instructionally through questioning. The questions teachers posed to learners varied in the degree to which these questions were reflective or implicit. Some questions might have been more powerful in compelling learners to reflect on their own mathematical reasoning, while other questions were blatantly leading in nature. How can teachers interact with learners' errors in order to enhance learners' understanding and critical thinking? Kaldrimidou, et al. (2004) describe teachers' actions of cautioning or guiding learners or correcting errors themselves, as a way of retaining the monopoly over errors. Idealistically, one of the pedagogical aims in a socio-constructivist environment is for learners to become autonomous (Ernest, 1988). However, control needs to be transferred to learners in order for them to become independent thinkers (Kaldrimidou et al., 2004). The following excerpt from Alice's classroom serves to highlight an opportunity to develop learners' competence in mathematical investigations and relational understanding that emerged during the tutorial-type segment of a learning period. Learners had to complete a table for the time a certain amount of food would last, depending on the number of people there were on a camping trip. The learner continued with the pattern of subtracting the number eight, she recognized as difference between the first two values. The erroneous strategy produced correct answers for the first two options, but not for the last option. However, neither the learner nor Alice continued with the pattern to the last pair of numbers in order to evaluate the learner's approach.

Number of people	12	15	20	30	40
Number of days that the food lasts	40	32	$a$	$b$	$c$

*L: Ma'am, I went to say it's minus eight over here.*

*T: No way, there's something fishy here.*

*T: You're subtracting eight there? Ja, but who says the next one is eight less? It is, it is, but it's not a sound method. I'll have to think about that. That seems to be a coincidence then. There's no pattern with adding in the first row, so how can we assume there's a pattern in the bottom row? Stick to the inverse proportion.*

During this incident, Alice took responsibility for deciding on the mathematical appropriateness of a learner's solution. The mathematical predicament was not shared with the rest of the group. Peers were not involved in deciding on the appropriateness of this learner's mathematical attempt. No negotiation took place. Alice also did not refer the problematic situation back to the learner for further investigation. How did the way in which Alice interacted with the learner's error contribute to the development of the learner's mathematical understanding?

In the subsequent excerpt (see section 6.4.1), from Chloe's classroom, a learner wanted to apply the "steps" Chloe had previously taught them regarding brackets, i.e. to determine the product in order to write down a series of separate terms. However, the equation they had to solve was in the form of a zero product. The learners had to write down possible values for  $a$  and  $b$  if  $a(b - 1) = 0$ . In my opinion, an opportunity for the learner to develop relational understanding of equations was not utilized.

*T: No, you don't. You don't, okay? Why you don't, is you want to specifically; ... if you are going to work out the brackets, if you are going to multiply, you'll have  $a.b$  and you'll have minus  $a$ , né? Okay, now you have two situations where you have an  $a$  in. Is it possible now to get a value for  $a$ ? No, it's harder to work with. What makes it easier to determine a value for  $a$ ?*

### 8.3.3 Openness to the Discussion of Errors

All four participating teachers raised the predicament of contemptuous peer responses to learners' errors. The actuality resulted in an apprehension among the teachers to discuss learners' errors publicly. In Barry's classroom, two class leaders were appointed to exercise homework-control and to record the names of learners who did not complete their

homework. This could have aggravated the contemptuous disposition of learners that Barry described during the semi-structured interviews. However, the teachers' attitude towards errors could also impede or facilitate the spontaneous public discussion of learners' errors. How can teachers interact with errors in order to engender openness to the discussion of errors? The ensuing excerpt is illustrative of an incident in Chloe's classroom. Chloe accidentally made a mathematical error pertaining to distance, time and speed on the blackboard. She reacted vehemently on recognizing the error:

*If we want to work out the speed, we take distance and we multiply it with the time travelled. Does that make sense? No, that's nonsense, okay? Will you change this, please? Will you change it quickly, please? Before I teach you nonsense, because you know speed equals distance divide by time, okay?*

What perception regarding errors did Chloe's reaction create among her learners?

### 8.3.4 The Development of a Mathematical Discourse

Dawn raised the issue that learners were generally not proficient in mathematical dialogue. Did the way in which the participants interact with learners' errors create opportunities for learners to develop a mathematical discourse? The participating teachers infrequently interacted with learners' errors by employing questions. Unfortunately, the teachers generally did not maintain this interaction through questioning and the teacher ultimately produced the mathematics. Even in instances where the teachers interacted with learners' errors through questions, the teachers took initiative and responsibility for dealing with the errors. Instances in which teachers requested learners to exemplify their thinking or to analyze their errors were not observed. The excerpt presented below (see section 4.4.1), from Alice's classroom, is illustrative of an instance in which Alice interpreted the learner's thinking.

*T: Next step? Yes?*

*L: Ma'am, I think maybe we can take the ten<sup>1</sup> to the top and you make it negative ten and it's gonna be six a minus ten and minus ten a minus ten.*

*T: You know what the problem is there? We're not working with exponents. Remember if you have something like this that becomes ten to the positive one? I think you're getting confused with exponents, okay? If you've got exponents, negative exponents,*

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<sup>1</sup> The excerpt pertains to solving the equation  $\frac{3a}{5} - a = 2\frac{1}{2}$ , eventually written to  $\frac{6a}{10} - \frac{10a}{10} = \frac{25}{10}$ .

*in the denominator that is what happens with exponents, okay? So let's not make that error there, okay? You can't do that. This is a fraction; six tenths of a.*

Although teachers' "reaching out" (Davis, 1997, p. 364) to learners' contributions, interpreting and paraphrasing learners' contributions (Ball, 2000; Kovalainen & Kumpulainen, 2007; Maree, 2004), is acknowledged in the literature as concurrent to socio-constructivist classrooms, it is my opinion that an inappropriate degree of paraphrasing may impinge on the development of learners' mathematical discourse. In addition to the preceding observation, teachers' paraphrasing of learners' contributions, without eliciting the mathematics from the learners (Inman, 2005) may conceal possible misconceptions learners have. An excerpt from Alice's classroom (see section 4.4.1) exemplifying the way in which the participating teachers often rephrased learners' contributions to enhance the mathematical value thereof, is presented below. The teacher's approach might have inhibited the learner's opportunity to develop a mathematical discourse. Learners had to solve the equation  $\frac{x-2}{2} + x = 2$  which had been written to

$$\frac{x-2}{2} + \frac{2x}{2} = \frac{4}{2}.$$

*L: I cancelled out all the denominators, 'cause they're all the same.*

*T: You multiplied the entire equation with two.*

### 8.3.5 Probing Errors versus Correcting Errors

Corresponding to the description by Leu and Wu (2005), the participating mathematics teachers generally portrayed a tendency to classify learners' mathematical errors in a superficial way. The teachers' classifications of errors usually included computational errors, as well as so-called "sign" errors. The participants seldom sufficiently probed learners' errors to eventually relate the errors to an essentially mathematical origin. They generally expressed a belief that learners' errors could potentially enhance learning and strengthen mathematical understanding through the juxtaposition of erroneous mathematical procedures with correct solutions. The teachers' focus appeared to be on the correction of the errors and not on probing the sources thereof. In classrooms where discourse played an insignificant role and learners were involved to a minor degree, teacher-learner interactions were probably prevented from developing to a level conducive for error-handling, other than correcting the errors. Learners in Dawn's classroom had to determine how many bricks nine men could lay in one day, using the fact that six men laid

2000 bricks per day. Dawn initially attempted to probe the learner's thinking with her introductory, reflective question, but did not maintain that. She reverted to correcting the learner's statements.

*L: They say nine men can lay two thousand bricks, how many can six men lay? I know this is wrong, completely wrong, because they would lay less. (The learner confused the information with what was to be determined.)*

*T: Would they lay less?*

*L: Yes because there's nine days. (The problem explicitly stated that the bricks were laid per day.)*

*T: Not how many days.*

*L: Oh! Okay! I divided nine by three here.*

*T: You divided by two. So you said three people can lay ...*

*L: Look Ma'am, I said six divided by two thousand.*

*T: Two thousand divided by six. Does it make sense?*

*L: Yes.*

*T: Does it really?*

### **8.3.6 Learners' Expectations and Assessing Formatively**

How should the relation between assessment and learners' achievement be transformed for teachers to interact formatively with learners' errors? During the semi-structured interview that was conducted with Dawn, she raised the issue of learners' preoccupation with their marks. Popham (2007) asserts that formative assessment should preferably have no consequences for achievement. Dawn described a classroom culture that was contra-indicative to assessing formatively.

*And at this stage I have some pupils that are very, very concerned about their marks and they want to know where the mark breakdown is. And sometimes it works well that you can say well we have (inaudible) lost a mark here, because of this concept, because you've done this and it helps (inaudible) that it breaks. But I think for a lot of them it's just about the marks in the end and they're not seeing the link between the mark and the concept.*

Remarks made by the other research participants confirmed the focus on achievement and the concurrent reduction of feedback to the provision of correct procedures through transmission. Three excerpts ensue.

*T: 'Cause give her that same test three days later and she's had class practice and homework, she can actually get full marks. And I think that's a motivating factor, you know. ... So it is not good to make errors; obviously you wanna get full marks.*

*T: 'Cause we wanna see where it goes through both axes, you get marks for that.*

*L: How much?*

*T: Many.*

*T: Guys, make sure. Don't make silly mistakes and throw marks away.*

Formative assessment is a process of enhancing learning, subsequent to obtaining information regarding learners' mathematical understanding (Popham, 2007; Stiggins, 2005). An appropriate and deliberate adjustment in teaching approach is paramount in this process (Popham, 2007). However, the four participating teachers explained correct procedures to learners and provided learners with complete solutions to mathematical problems during the classroom discussions of the memorandums (see section 3.6.2). The memorandum discussions were consistent with the prevalent teaching approach during regular learning periods in Alice's, Barry's and in Chloe's classrooms. A trend towards traditional teaching through transmission was observed in Dawn's classroom, particularly during the second half of the memorandum discussion. The way in which the participating teachers gave feedback to learners, subsequent to the formal assessments, demonstrated substantive equivalence among the four participants, focusing on correct procedures and on learners' achievement. The observations concur with those of Adendorff (2007), reporting that the participants in his study portrayed limited knowledge of formative assessment and refrained from employing formative assessment.

### **8.3.7 Teachers' Pedagogical Content Knowledge**

The literature unambiguously asserts the relation between teachers' PCK and their interactions with and beliefs about learners' mathematical errors (Adler, et al., 2005; Ball, 2000; Halim & Mohd.Meerah, 2002; Magnusson, et al. 1999; Shulman, 1986; Smith, 1999; Smith & Neale, 1989; Van Driel, et al., 1998). Although I take cognisance of



this prominent factor in teachers' interactions with learners' errors, it is beyond the scope of my study to evaluate the respective levels of PCK the participating teachers maintained. However, I do regard a comparative description of certain aspects of their PCK as relevant to the study. The comparative description is delineated along the following aspects:

- Knowledge of learners' difficulties with learning (Bromme, 1994; Magnusson, et al. 1999; Van Driel, et al., 1998).
- Anticipation and identification of learners' errors (Ball, 2000; Noddings, 1990).
- Understanding, interpretation and sense-making of learners' contributions (Adler, et al., 2005; Ball, 2000).
- The nature and effectiveness of addressing learners' errors (Ball, 2000; Noddings, 1990; Shulman, 1986; Smith & Neale, 1989; Wood, et al., 1976).
- Reinforcing learners' misconceptions (Halim & Mohd.Meerah, 2002).

Barry appeared to be knowledgeable about learners' errors and was observed to anticipate learners' errors proactively. However, Barry's focus was primarily on procedural, rather than on mathematical issues. The other three participants gave the impression of interacting with learners' errors on an impromptu basis, rather than in anticipation of specific errors. Chloe occasionally did not precisely address a learner's contribution, while isolated incidences thereof were observed in Alice's classroom. The sense-making of learners' errors portrayed a general inclination among the participants towards the immediate context of the error; the procedural context, thus. It was not observed that learners' errors were probed to reveal the mathematical origin of the errors. Learners' errors were primarily addressed by correcting the errors. In line with the findings of Santagata (2005), even in instances where the participants employed instructional scaffolding, the aim of the instructional scaffolding was the correction of the errors. I am not convinced that any of the participants facilitated processes (Noddings, 1990) during which learners' knowledge was conceptually reorganized (Shulman, 1986). Instances during which learners were possibly confused and learners' misconceptions potentially reinforced, as delineated by Halim and Mohd.Meerah (2002), were observed in Chloe's classroom.

### **8.3.8 Teachers' Expected Approaches**

Of the four participating teachers, Alice and Barry portrayed considerable consistency across the spectrum of aspects considered in the classification of the participants' beliefs and actions. However, significant discrepancies were observed in Chloe's and in Dawn's beliefs and actions (see sections 8.2.1 and 8.2.2). Chloe strongly exercised control in her classroom. She primarily taught through the transmission of rules and by demonstrating intact, complete procedures. In addition to that, Chloe expressed hesitation to focus on learners' errors. Yet Chloe was observed to interact with learners' errors with questions and to scaffold learners instructionally. Although it might have been an outcome of my presence and the nature of the research, she was able to adapt to the circumstances that prevailed during the two-week observation period. In contrast to Chloe, Dawn referred to herself as constructivist. Her teaching approach deviated from the more traditional approaches. She was positive about learner involvement. She asked reflective questions and scaffolded learners instructionally. However, a high incidence of interacting with learners' errors through abrupt correction of the errors was observed in her classroom. These inconsistencies might indicate that interactions with learners' errors was a far more complicated matter than merely perceiving the interaction as corresponding to a teacher's prevalent teaching approach or to the teacher's professed or enacted mathematical beliefs.

## **8.4 REFLECTIONS**

### **8.4.1 Methodological Reflection and Limitations**

I embarked on a multiple-case study to investigate the interaction of mathematics teachers with learners' mathematical errors. The sampling was conveniently (Cohen, et al., 2005) done, with no prior knowledge of the participants' prevalent teaching approaches. In retrospect, I realize that a small-scale survey of teachers' professed beliefs about learners' errors, conducted among the staff members at the schools, might have provided sufficient insight to opt for purposive and maximal variation sampling (Creswell, 2005; McMillan & Schumacher, 2001). A significant improvement in cross-case comparisons could have been accomplished.

Cross-case comparisons are complicated by the actuality of various mathematical topics being discussed in the various classrooms during the periods of fieldwork. Although the study was not embedded in a particular learning outcome or mathematical topic, expository

teaching and the teaching of algorithms might have been more prevalent in some topics than in others. The classification of Dawn's teaching approach as constructivist, with behaviouristic elements, might have been a function of the particular topics being discussed in her classroom during the observation period. However, a considerable intersection of topics discussed in Alice's classroom and in Dawn's classroom existed.

The participating teachers were exposed to questions regarding learners' errors in the written, structured interviews. In addition to that, the focus of the semi-structured interviews was on error-handling. The exposure could have had an influence on the way teachers interacted with learners' errors during the research period. Cognisance of this influence should probably be taken in considering the discrepancies in Chloe's narrative.

I deliberately decided to commence with the data analysis more than two months after completion of the data collection process (see section 3.6.4). Idiosyncratic issues emerging from the semi-structured interviews could not be followed up with the participants. One such an example is the aspect of previous experiences with mathematics and the resultant mathematical identity of the participant (Jita & Vandeyar, 2006). This probably resulted in impoverished cross-case comparisons. The postponement of the data analysis further resulted in a limited personal insight into teachers' decisions, for example regarding their written interaction with errors in assessment tasks.

I experienced my role as interviewer a particularly challenging one. I am of the opinion that I could have capitalized more on the semi-structured interviews as a method of data collection, had I been more competent in and comfortable with interviewing.

Due to a plethora of data, resulting in mechanical recordings in excess of 2000 minutes, I found it practically unattainable to employ both video-recordings and digital voice-recordings as sources of data. My decision not to analyze the complete set of video-recordings necessitated the omission of teachers' body language and facial expressions as data analysis categories. Including these categories could have enhanced the descriptive value of the qualitative research account.

My personal narrative and my scientific academic background posed enormous challenges to me in my capacity as a qualitative researcher. I went through several cycles of designing a conceptual framework that did not portray a judgemental disposition or some degree of stereotyping. Furthermore, the actuality of my scientific and mathematical

training (Ernest, 1997) necessitated continuous reflection on the way I presented the data, consciously focusing on refraining from categorizing and evaluating participants' actions.

#### **8.4.2 Scientific Reflection**

In certain aspects, the empirical findings of this investigation correspond to those from previous studies, as reported in the literature. I concur with Heinze (2005) and Santagata (2005) on a number of observations. Teachers were primarily in charge of deciding what was mathematically right or wrong; teachers directly solved a large number of all errors; a considerable number of the errors were merely corrected without explanation and the purpose of correcting learners' errors was to continue with the planned course of the learning period. However, the findings relating to teachers' impassiveness towards errors differ from those by Heinze (2005), Leu and Wu (2005) and Santagata (2005). They found and reported that teachers ignored a considerable number of learners' errors during classroom discourse. Alice and Barry were not observed to ignore learners' errors during learning periods. Infrequent incidences of impassiveness towards learners' errors were observed in Chloe's and in Dawn's classrooms.

My impressions regarding instructional scaffolding concur with those reported by Santagata (2005) and Meyer and Turner (2002). The primary focus of instructional scaffolding was to arrive at the correct answers and not the analyses of the learners' errors. The instructional scaffolding usually did not result in learners' self-regulation due to low levels of questioning and giving cues.

The empirical findings regarding peer involvement in error discussions, as reported in the literature by Santagata (2005) and Kaldrimidou et al. (2004) correspond to what I observed in the four classrooms. Both teachers and learners were reluctant to discuss errors publicly. Teachers retained the monopoly over errors by their actions of cautioning or guiding learners or correcting errors themselves.

Alice's case fitted the descriptions in the literature comfortably. Alice's observed teaching approach, classified as explainer, corresponded to her Platonist-professed mathematical beliefs (Ernest, 1988). Her beliefs about errors as well as her observed interaction with learners' errors were behaviouristic with constructivist elements, both during learning periods and in assessment tasks. Alice portrayed an orientation towards classifying learners' errors, as described by Leu and Wu (2005). Alice usually attempted to put

learners at ease about their errors, referring to learners' errors as an indication of merely computational failure or carelessness.

Both Barry and Chloe expressed instrumentalist views (Ernest, 1988) of mathematics. Their prevalent teaching approaches were concurrent to the role of instructor (ibid.), congruent to their professed beliefs. In resonance with Ernest's anticipation of the beliefs of such teachers regarding errors, during the semi-structured interviews Chloe expressed hesitation to focus on errors. She perceived errors as negative and contra-indicative to learning. In contrast to Chloe and to what Ernest (1988) predicts, Barry was positive about the value of learners' errors, albeit conditionally. However, Chloe's observed verbal interactions with learners' errors during learning periods portrayed convincing constructivist elements. She occasionally listened interpretively (Davis, 1997) to learners' contributions and asked a limited number of open, reflective questions and questions related to mathematical interpretation. Different to what Barkatsas and Malone (2005) found, Chloe's and Barry's professed beliefs about mathematics corresponded to their classroom practices. In comparison to Barkatsas and Malone's (2005) report, an interesting contradiction is observed in Chloe's case. Chloe's professed beliefs regarding learners' errors were behaviouristic or instrumentalist, while her verbal interactions with learners' errors portrayed a constructivist orientation, albeit weak and inconsistent.

Dawn professed to problem-solving or constructivist views of mathematics. Her observed teaching approach could be classified as facilitator-explainer (Ernest, 1988) or constructivist, with behaviouristic elements. Her enacted teaching approach portrayed a convincing degree of correspondence to her professed beliefs. Dawn's beliefs about learners' errors and her professed interaction with learners' errors were constructivist. However, her observed verbal interaction with learners' errors during learning periods revealed strong behaviouristic elements and could be classified as behaviouristic with constructivist elements. Barkatsas and Malone's (2005) argument that teachers' beliefs are often articulated theoretically, in isolation of the practical context and thus do not correspond to their classroom practices or the way they interact with learners' errors, cannot account for Dawn's case. There was an acceptable degree of correspondence between her observed teaching approach and her professed beliefs. The dissonance is relevant to her verbal interaction with learners' errors during learning periods only. Considering the accord between her professed beliefs and her teaching approach,

Beswick (2005) and Op't Eynde and De Corte's (2003) argument that the demands of the discipline influence teachers' practice, cannot account for Dawn's case either.

### **8.4.3 Reflection on the Conceptual Framework**

In designing the conceptual framework for this investigation, I was guided by the literature to accept a reciprocal relation between teachers' mathematical beliefs, their classroom practices and the way they interacted with learners' errors. The blue and green arrows on the conceptual framework in section 2.4.2 depict these relations. The literature supported the actuality of teachers' interactions with learners' errors being an extension of their prevalent teaching approaches or classroom practices. A teacher's predominant teaching approach and his or her interaction with learners' errors are expected to be congruent. The purple arrow on the conceptual framework depicts this relation. Teachers' enacted classroom practices, including their interactions with learners' errors would hence reflect concurrent mathematical beliefs (Barkatsas & Malone, 2005; Bauersfeld, 1994; Beswick, 2005; Brodie, 2008; Ernest, 1988; Heinze, 2005; Leu & Wu, 2005; Mehan, 1979; Santagata, 2005). The findings of this study, pertaining to the relation between teachers' mathematical beliefs and their teaching approaches, are in resonance with those reported in the literature. The relation depicted by the blue arrows is hence confirmed. However, Chloe's case and, particularly, Dawn's case are in conflict with the relations depicted by the green arrows and the purple arrow. Chloe was classified as behaviouristic in terms of her professed mathematical beliefs and in terms of her observed teaching approach while her interaction with learners' errors portrayed constructivist elements. Dawn was classified as constructivist in terms of her professed mathematical beliefs and in terms of her teaching approach, albeit her teaching approach contained behaviouristic elements. Her interaction with learners' errors was behaviouristic, with constructivist elements. In reflection on the findings, it is hence suggested that the purple arrow and the green arrows be depicted as broken arrows to allow for these discrepancies.

## **8.5 RECOMMENDATIONS**

### **8.5.1 Recommendations for Teacher Training**

Teachers' interactions with learners' mathematical errors are complex and not necessarily predictable. Dawn's case is an appeal for concern amongst teacher educators. It cannot be accepted as a *fait accompli* that socio-constructivist teachers will interact with learners'

errors in a socio-constructivist way. Hence, error-handling should preferably be included in teacher-training and professional development courses. Chloe's case is encouraging. Chloe was able to deviate from a strong behaviouristic approach to teaching during her interactions with learners' errors. These two cases can hence serve as a motivation for officially presenting instruction on error-handling in teacher-training and professional development courses where teachers are being trained in a reform-oriented approach. It is further recommended, in resonance with the literature, that undergraduate modules or professional training courses concerned with error-handling include foci on teachers' roles in socio-constructivist classrooms (Adler, et al., 2005; Ball, 2000; Kovalainen & Kumpulainen, 2007; Morine-Dershimer & Kent, 1999), teachers' questioning skills (Cazden & Beck, 2003; Hargreaves, 1984; Mason, 2000) and teachers' listening skills (Davis, 1997; Ruggiero, 1988). By addressing the preceding aspects of classroom discourse, namely the teacher's role as supportive and specialist collaborator in interpretation and negotiation (Adler, et al., 2005; Kovalainen & Kumpulainen, 2007; Morine-Dershimer & Kent, 1999), posing appropriate (Hargreaves, 1984) high-level (Meyer & Turner, 2002), open-ended and meta-cognitive questions (Cazden & Beck, 2003; Mason, 2000) and refraining from evaluative listening (Davis, 1997), the desired outcome of teachers' discontinuance of expository teaching and the teaching of rules (Meyer & Turner, 2002) may be facilitated. Such a classroom atmosphere will be conducive to a higher degree of learner involvement, requesting feedback from learners on their understanding (Beswick, 2005) whilst placing less emphasis on merely arriving at correct solutions to mathematical problems (Martens, 1992).

In order for teachers to interact with learners' mathematical errors appropriately, so as to stimulate learners' meta-cognitive activities (Melis, 2005; Santagata, 2005) with the purpose of restructuring their cognitive frameworks (Beswick, 2005; Palincsar & Brown, 1984; Tobin & McRobbie, 1999), teachers need to be empowered with knowledge regarding learning and learners' errors. It is hence recommended, in agreement with Thijs and Van den Berg (2002), to expose pre-service teachers and professional teachers to literature concerned with constructivist views on learning and learners' mathematical errors.

However, the potential efficiency of professional development initiatives will probably only be realized once teachers recognize the limitations of their own practices. A mere awareness of reform-oriented approaches to error-handling will probably not result in a

consistent and sustained adaptation of teachers' interaction with learners' mathematical errors. In order for teachers to acknowledge their instructional limitations, they should be convinced by having proof of unsatisfactory levels of learning quality provided to them (Cross, 2009).

The prominent role teachers' mathematical beliefs play in teachers' perceptions of learning and their prevalent teaching approaches (Borasi, 1996; Cross, 2009; Ernest, 1991; Leu & Wu, 2005; Wilson & Cooney, 2003) necessitates a focus on reflection in modules or courses concerned with error-handling (Ernest, 1988). It is recommended that teachers are facilitated to reflect on their mathematical beliefs, their beliefs about learning and their interactions with learners' errors (Korthagen, 2004; Luft, 2001; Stols, et al., 2007; Zeichner, 2006), in addition to being empowered with knowledge of learning and learners' mathematical errors.

### **8.5.2 Recommendations for Further Research**

An aspect that emerged from the study is the semantics of the language used by the mathematics teachers. Skemp (2006) refers to selected instances of these as "examples of instrumental understanding" (p. 89). Further research to investigate the relation between teachers' semantics and learners' misconceptions in mathematics is recommended.

The explorative nature of the data collected through this investigation restricted conclusive evidence to further illuminate the finding of dissonance between teaching approach and interaction with learners' mathematical errors. Plausible explanations for the phenomenon could be the focus of further studies.

## **8.6 CONCLUSIONS**

Aligned with the emerging themes in section 8.3, the following conclusions have been drawn:

- Teachers' inappropriate ways of transmitting mathematical procedures to learners and teachers' improper semantics may result in or sustain learners' misconceptions in mathematics.
- The development of learners' meta-cognitive abilities, critical thinking skills and relational understanding are seriously challenged when teachers retain the authority to generate and evaluate mathematical contributions.



- The covert, negative messages teachers convey to learners regarding errors obstruct the engendering of openness towards error discussion, while the inhibiting peer influence is maintained.
- An inappropriate degree of interpreting and paraphrasing learners' contributions, thus refraining from eliciting the mathematics from the learners, denies learners the opportunity to develop a mathematical discourse.
- Teachers perceive the value of learners' errors residing in the corrections thereof. Teachers hence prefer to rather correct than to probe learners' errors.
- Assessing learners formatively, or interacting with learners' errors formatively, is enormously challenged while results and achievement are portrayed as objectives and assessment and achievement are synonymous.

The primary research question: *How do secondary school mathematics teachers interact with learners' mathematical errors?* is synoptically and conclusively answered with the ensuing remarks. Concurring with the literature (Leu & Wu, 2005; Santagata, 2005; Smith, et al., 1993), the participating teachers' focus on correct procedures resulted in the confrontation and replacement of learners' misconceptions through instructional methods during their interactions with learners' errors. Observed classroom circumstances were usually unfavourable for learners to expose the cognitive frameworks that supported their understanding (Tobin & McRobbie, 1999). The absence of high-level reasoning processes (Santagata, 2005) during the participating teachers' interactions with learners' errors probably impeded the restructuring of learners' mental structures (Palincsar & Brown, 1984). However, the most prominent conclusion of this investigation is construed from the cases of Chloe and Dawn. Teachers' verbal interactions with learners' errors during learning periods do not necessarily correspond to their prevalent teaching approach. Their professed mathematical beliefs fail to account for the discrepancy. This investigation refutes the unequivocal acceptance of a teacher's interactions with learners' errors as part-and-parcel of his or her teaching approach. The way in which a teacher interacts with learners' errors is a separate and discrete component of the teacher's practice.

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**APPENDIX A:**  
**THE DEVELOPMENT OF THE INTERVIEW PROTOCOLS**

### *Semi-structured Interviews*

Subsequent to conducting the first semi-structured interview with Alice, prior to the fortnight of fieldwork at school A, a critical focus on the research questions prompted me to omit the two questions, indicated below, in the semi-structured interviews that followed:

- How do you think, do their mathematical errors make learners feel; what is your perception of how your learners react to the errors they make?
- What do you think are the expectations that your learners have of your role in the classroom?

With these adjustments in place, I conducted the initial semi-structured interviews, prior to the respective observation periods, with the following two participating teachers, Barry and Chloe, by posing to them this basic set of questions:

- What was your perception of errors when you were a learner at school?
- Do you ever make deliberate errors in the classroom?
- Do you have a particular opinion on or strategy about how to deal with learners' errors in the classroom?
- How do you normally handle learners' errors in the classroom?
- Do these two strategies differ, and if they differ, why?
- What is your opinion about peer involvement in error discussion, i.e. the public and whole-class discussion of learner errors?
- Do you believe that errors can be useful as learning opportunities?

The following basic set of questions guided the remaining semi-structured interviews, subsequent to the respective observation periods at schools A, B and C.

- Do learners' errors inform your practice?
- What, according to you, are the main purposes of assessment?
- How do you normally interact with or perhaps use learners' errors from written assessment tasks such as tests and examinations?
- How do you believe mathematics should be taught?

## *Structured Interviews*

### INVESTIGATING THE INTERACTION OF MATHEMATICS TEACHERS WITH LEARNERS'

#### MATHEMATICAL ERRORS

The structured interview forms part of my data collection strategies. I will appreciate if you could take time to complete the following questions.

Please supply the following biographical and professional information:

- (a) Total of years teaching experience

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- (b) Total of years teaching mathematics

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- (c) Initial tertiary qualification (e.g. B. Sc.)

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- (d) Did you take mathematics as a main or as a subsidiary subject during your initial training?

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- (e) Highest qualification in mathematics (e.g. Mathematics 1)

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- (f) Postgraduate qualifications (if any)

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- (g) Were any of your postgraduate qualifications concerned with mathematics or mathematics education?

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- (h) If yes, please supply detail.

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- (i) Do you hold any formal responsibility for mathematics at your school and please describe the responsibility?

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- (j) Please mention the training sessions you have attended since 2005.

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- (k) Please state, in your own words, your personal definition of mathematics.

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(l) Describe the way you think mathematics is best taught.

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(m) Please name any academic book or article on mathematics education you have read in the last twelve months

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(n) Please add any comments you have on learners' errors, either from your personal experience or personal views you may have.

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Thank you for your cooperation and your time!



## **APPENDIX B: EXAMPLES OF TRANSCRIPTS**

## *Semi-structured Interviews*

### TRANSCRIPTION OF INTERVIEW PRECEDING OBSERVATIONS WITH BARRY AT SCHOOL B

FRIDAY 08 AUGUST 2008

*I want to start with your own school days, your own experience as a learner at school and ask you: what was your perception of errors when you were a learner at school?*

Oh! My perception of errors?

*Not necessarily perhaps your own perception, but how did ... how did you experience the way the teacher handled errors or interacted with errors when you were at school?*

Well, in the class environment, we did, ... I wasn't really in a class that asked questions, actually it was in our school, we never asked a question, don't know why. So I don't know how she would handle it, ... she would have handled it in the class. But she would always, for every single paper that she marked, she would write the whole correction in every paper for everybody.

*Individually?*

Yes.

*In other words, when you received your script back, you would have seen ...*

Yes.

*... why you got, for example, three out of seven for a ... for a solution?*

Yes, and she would also, after handing them out, give them; go through the memorandum with you again,

*Ja?*

... so that you can see what to do.

*So, in other words, her response to errors during assessment was formative, in the sense ...*

Yes.

*... that you ... you could see why you made the error, what error it was, ...*

Yes.

*... but during the normal classroom interaction, according to your experience, there was virtually no interaction.*

No, it was mostly one (inaudible).

*Ja, and ... and the response to the errors, when ... when ... when you were a learner yourself, have ... did you experience it as negative or ...*

No.

*Quite positive?*

No, it was always positive.

*Mmm*

Ja.

*Okay.*

Ja, we did not have a negative teacher.

*Yes. And now to return to your situation, your current situation, what do you think your learners in your classroom expect or believe regarding errors?*

My learners are very scared to ask questions, but some think are not, most of them are very scared because they think I get very upset when they ask questions. I do get upset sometimes.

*Mmm*

Specially if they ask me something that I have just said, in those words ...

*Mmm*

... and they just didn't listen. So, I get upset when they ask me things because they didn't listen. But if they ask me something that is really a problem, that I can see they don't understand, then I help them with that. And we also have a programme where I am available for one hour after school every week where they then also come to you if they are too scared to ask in front of their friends ...



*Mmm*

... Tell me Sir, that I couldn't do this during the week, and then I will also help them there. They don't really make use of that.

*Mmm mmm*

So, I also go through the tests and the exams with them afterwards, after I marked ...

*Mmm*

... the scripts, and then I let them copy down the corrections in pencil on their scripts. The only problem that I do have with the system now is that we cannot allow them to keep their scripts, ...

*Mmm*

... which I think is a big problem. It is ... they can then see what they did wrong, but when they study again, they have nothing; they only have their book again.

*Okay, and ...*

They have nothing to reference, to see where they made their mistakes. That is a problem.

*And the question paper? May they keep that?*

No, we mostly in the grade 8's and 9's ... we have fill-in papers.

*Mmm*

So they fill in the answers on the question sheets (inaudible).

I do give them revision sheets ...

*Mmm*

... before every test and exam which I also do with them then.

*Ja?*

But they also don't do that on their own.

*Mmm*

They don't care about that.

*Mmm*

*In other words, the tests that they do write have very little formative value.*

Mmm

*They ... they see it once ...*

Yes.

*... or for a day or so, and then they need to file it?*

Yes, that is a big problem that I've got with the system.

*Mmm*

It's ... if I refer back to when I was at school ...

*Mmm*

I've always started with my tests.

*Mmm*

After I've finished studying my tests and went through every mistake that I have made, then only I went to my book and if I've finished with my book, then I go to previous exam papers ...

*Mmm*

*... of previous years ...*

*Mmm*

*... and stuff like that and they only have their books.*

*Mmm*

Which is also not such a good reference these days, because they don't ... the quality of work is not that good. I mostly do everything on the board for them just to copy down, just to make sure that they have everything, and that they've got the correct ones.

*Mmm*

But that also doesn't work.

*Mmm*

Because now they simply copy down and they don't go over and look at what they've copied down, so that they can make them ... it their own

*Mmm*

Which is a problem.

*Ja.*

I've got a huge problem with the system ...

*Mmm*

... at this moment

*Mmm mmm mmm*

... as a whole.

*Ja?*

It's not working in maths. I don't know if it's working in the other subjects, but mathematics is unfortunately one of those formal subjects ...

*Mmm*

... that you just have to teach in the old ways.

*Mmm mmm*

*Ja, I can hear the problem, because if an error, if they make the error and they can't, like you say, follow up on the correction and understand the difference between the correct and the incorrect solution, they may probably repeat the error in the exam.*

Yes. And a problem that we have in this school is that the learners do not study at all ...

*Mmm, mmm*

... not at all (inaudible). A few years ago we accidentally had a teacher that, you know, hotel department, which is simply studying work, it is nothing to understand ...

*Mmm*

... nothing scientific at all ...

*Mmm*

... and she accidentally copied the memorandum at the back of every single question paper ...

*Mmm mmm*

... and only saw that when they handed it back after the exam; that the answers is stapled to the back and 70% of the group still failed. They couldn't see it.

*It is incredible! Ja!*

So mathematics is ... is difficult to teach these days ...

*Mmm*

... very frustrating.

*Mmm mmm*

And there are kids also still have that culture of the world owe them a lot ...

*Mmm*

... especially, which in most of the cases they ... they do just get jobs ...

*Mmm*

... which is sad, because they have nothing to encourage them to work harder.

*And ... and these children are all advantaged; they are ...*

Most of them are.

*... privileged children ...*

Ja.

*... from ... from wealthy backgrounds?*

Yes, ja, we do have some that struggle, but most of them are quite (inaudible).

*Mmm*

*But it ... it is a good thing that I know this before I start doing the observations, because that is something that I will definitely be on the look out for.*

Mmm

*How do your learners' errors, or the errors your learners make ... how do those errors make you feel?*

It makes me very frustrated, because if you take the exam paper for example. Once I set the exam paper for June, I went and I sat down with the computer with this paper next to me and for every single question there was in the paper I set a revision sheet.

*Mmm mmm*

So if I asked them in question one to take a recurring decimal to a fraction, then I would put four examples of that in the revision sheet. And so I went through the whole exam paper. Both papers; there was only three questions that wasn't in that revision sheet. And I went through that thing with them step by step by step before the exam and still about 54% of my group failed. And that makes me very frustrating. They don't attend the extra lessons. They simply don't have an interest. Many of them would directly come to you and say, but Sir, we're not here for the mathematics; we're here for the arts.

*Mmm*

So, they don't care.

*But for them, one of the mathematics learning areas is compulsory?*

Mmm, yes, it is.

*Ja?*

But most of them are in any case going to check, to choose literacy. We do have the few that will choose maths.

*Mmm*

But most of them are simply going to choose literacy.

*Mmm, ja, that ... I will understand your situation better, I suppose, by the end of my two-week observation, ja?*

*Do you ever decide to make errors in the classroom deliberately?*

Yes, I do that. And then they mostly do not even see that I have ... the error. They simply copy down. Then I'm standing there, looking at them. They don't know what I am looking at.

*Mmm mmm*

And I would say, is this correct? Yes, Sir, it is correct. But then it is a very obvious thing.

*Mmm mmm*

Which also just tells me that they are not really paying attention. Because if they were, they would have known that this was a wrong thing that I wrote down.

*Mmm*

And that also makes me quite angry, makes me very frustrated.

*And do you do that often enough for them to be on the lookout for that? I mean, has it not become part of your classroom, or your way of teaching?*

Ja, I ...

*By now, should they not be on the lookout for that?*

Ja.

*Should they not read your body language?*

They should. There are some. We have a few in each class that really want to work.

*Mmm*

They do pick it up.

*Mmm*

But most of them do not. I do ... do that quite often, not every single period.

*Hmm hmm, hmm hmm*

But, maybe two, three times a week.

*Mmm mmm*

I would do something like that. But, they just believe everything you say and go with that. Which means that they do not understand the first principles (inaudible).

*Ja, well I am definitely now very curious to experience such a ... a little happening in your classroom. And then I would like to ask you about peer involvement. If ... if a learner in your classroom makes an error, how ... what do you think, or what is your opinion about peer involvement, in other words, about the a public and a whole-class discussion on that error?*

Well, I do get that ... that if someone gives a wrong answer in the class, the rest of the class do laugh at them. They still do that, very badly.

*In other words ...*

Although they don't know the answer ...

*... it is not ...*

*... themselves.*

*... it is not constructive?*

No, it's not. So I choose to not do that at all, because I don't think it is very good for the person that is trying ...

*Mmm*

... but just get it ... get it wrong and then those that are not even trying, laugh at them.

*Mmm*

And I also get quite angry.

*Mmm*

But I do get that if ... we've got this two totally separate groups in every class. There's this group that wants to work and then there's a group that doesn't want to work. The group that doesn't want to work will laugh at the group's answers ...

*Mmm mmm*

... that is working, but then inside the group that is working, they would also help each other correct.

*Mmm mmm*

They would not laugh ...

*Mmm*

... because they know the seriousness about this.

*Ja, but what you're actually saying, is that the group as a whole, or ... or the ... the dynamics in the group, is not conducive to a whole-class discussion on an error.*

No, I think we've got too many different individuals in the class, ...

*Mmm*

... completely different.

*Mmm*

Because we have got the all sorts of arts, we have got management courses ...

*Mmm*

... hotel courses, they all come together in the mathematics class.

*Mmm*

That is not (inaudible).

*Mmm*

If we could have them ... hotel take maths ...





*Mmm*

... together ...

... and visual arts take maths together, then I think that would have been better, but it is completely mixed, which I think ...

*Mmm*

... is the main reason for this (inaudible).

## *Classroom Activities*

### TRANSCRIPTION OF CLASSROOM ACTIVITIES AT SCHOOL B

WEDNESDAY 13 AUGUST 2008

T: Listen people, the bell has gone a long time ago. At your desks open your book. You know this. Why must I always tell you? Where's (name)? People, listen, if your book is not open when the class captain comes by, your homework is not done. Come, people! Class captains, the homework is not up for discussion. It's either done or not done. Why are you sitting there?

L: I always sit here.

*(Teacher shows solutions to homework problems on the blackboard.)*

T: What was the equation there again?  $y = x - 1$ . Does yours go like that?

L(?): Yes.

T: Does yours go through there?

L(?): Yes.

T: If there's nothing written there, it's actually a ...?

L(?): One.

T: One. And my m is? One. Do you follow? ... Can you see that?

T: Where does both those graphs go through the y-axis?

*(Question asked after he had just told the learners.)*

L(?): Minus one.

T: At minus one.

T: Listen, if you come late you don't come up the stairs noisy! Where were you? Why does it take so long?

*(Comments addressed to a latecomer.)*

T: Do you follow? ... Do you follow? ... Do you follow?

T: You must have the  $y$  alone on the one side. Then you must have something and an  $x$  and then something that doesn't have an  $x$  ... your  $c$  is the one that doesn't go with an  $x$ .

*(Take note of teacher's semantics!)*

T: Where will it go through the  $y$ -axis?

L(?): *(inaudible)*

T: At minus one. And now what is my  $m$  value? Minus one. Do you follow?

T: Does yours look that way?

*(Teacher does not wait for learners to respond.)*

T: So you didn't use the method I used to draw them now, did you?

*(Learners do not yet know the particular approach.)*

L(?): No.

T: You used your dots, but yours must look that way. Is this difficult?

L(?): No.

T: Did some of your graphs not end up on straight lines?

L(?): No.

T: Who's suppose to be sitting there next to you? Where's (name)? Wasn't he in register?

T: What goes first in the coordinate point?

L(?):  $x$

T:  $x$

*(Teacher shows how to draw line meticulously, using all given points, showing how to read coordinates.)*

T: That line, people is going to be parallel to the ...?  $x$ -axis, because all those points were two units above  $x$ . Can you see it? So what do you notice?

*(T answers question himself.)*

T: That if you have the equation  $y$  is equal to something, there's no  $x$ , okay, then we can say that that would be a line parallel to the  $x$ -axis and going through the  $y$ -axis at that point that  $y$  was equal to in the equation.

T: If I draw it on a number line it's going to be a dot ... on a Cartesian plane it's a line. Do you follow?

*(Teacher warns learners about confusion with number lines.)*

T: Do you follow? Is this difficult?

T: Follow?

T: Do you follow?

T: Verstaan jy?

*(Asking an individual learner in Afrikaans whether the learner understood.)*

T: Are you with me? Who's not with me?

L(?): *(indicates)*

T: Because you were late and now I have to wait for you.

T: Do I need all these points to draw the line? No, I actually only need two points to draw a line because if I have two dots I can draw a line through them. Do you follow?

*(Teacher answers question himself.)*

*(Teacher tells them to use three points, but contradictory to the two other methods that will follow.)*

T: Do you follow?

T: And these points that I choose are which values? The  $x$ -values.

*(Teacher answers question himself.)*

T: Okay, because if I know what  $x$  is, I can work out my  $y$ . I'll have minus one plus one inside the bracket, which gives me?

L(1): Sorry Sir, what did you say?

T: If  $x$  is minus one, I'll have minus one plus one in the bracket, which gives me?  
*(Teacher stays patient.)*

L(1): Nought.

T: Nought inside the bracket. Nought times anything is?

L(1): Nought.

T: Nought, so  $y$  is nought.

T: Do you follow? Nought plus one is?

L(1): One.

T: One. One times two is?

*(Etc.)*

T: Do you agree?

T: Label your axes put your arrows and indicate the origin.

*(Repeats numerous times.)*

T: Is that difficult?

L(?): No.

T: How do I draw that? ( $y = 3$ )

*(Teacher shows solutions stepwise in greatest detail on blackboard while orally keeping up with each and every step.)*

T: How would that line look like?

*(Teacher expects learners to recognise line parallel to  $x$ -axis.)*

T: Come, people, we just did those examples.

L(2): *(Explains)*

T: Mmm mmm

T: Yes. Through?

L(2):  $x$ -axis

T: Hhuu hhuu At?

L(2): At three.

T: At three. People, this is a line parallel to the  $x$ -axis, going through the  $y$ -axis at three. Do you follow?

T: Can you see that?

T: What type of graphs are these? Okay people, they are line graphs. That is quite straightforward.

*(Teacher answers own question immediately.)*

*(Teacher explains meaning of the term “point of intersection” and links to simultaneous solutions of linear equations, relating alternative solution to accuracy. Teacher tells about equality of equations and equality of equations in the particular point of intersection.)*

T: Is where these two graphs cross each other. Are you with me?

L(?): Yes, Sir.

T: Do you follow? ...

T: Do you follow? ...

T: Do you follow? ...

T: Do you follow? ...

T: Do you follow? ...

T: Will you be able to get  $x$  alone if you see something like this? Can I get  $x$  alone if it's inside a bracket?

L(3): No.

T: No.

T: So the first thing I'm gonna do is ...?

L(3): Take it out of the bracket.

T: Gonna take it out of the bracket, expand it, multiply in. Two times  $x$  is?

L(?): Two  $x$ .

T: Two times one is ...?

L(4): Two.

L(5): One.

T: Who said one? (*impatiently*) Let's take the two across, becoming a minus two. What is three minus two?

L(?): One.

T: One.

T: How do I get that two away from the  $x$ ?

L(?): Divide.

T: Divide it on the other side. What is the  $x$  that I got there for the point of intersection?

L(?): A half.

T: A half.

T: Do you follow? Two times a half is?

L(?): One.

T: Do you follow?

T: Will you be able to calculate points of intersection? Works the same. You would always put the equations of the two graphs equal, find the  $x$  and put the  $x$  back into one of the two equations to find the  $y$ .

T: What do you notice when you compare the two equations of (b) and (c)? That it gives us the same point. Have you got that?

*(Teacher answers own question immediately.)*

*(Teacher explains approaches to solving linear equations graphically and algebraically. Teacher does not explain why learners need to determine  $x$  and  $y$  values for point of intersection.)*

T: Take out your notes.

*(Notes are used in conjunction with textbook.)*

T: How many people's homework was not done? *(to class captains)*

T: (Name) you must give me your mother's telephone number at the end of this period. Why was your homework not done? *(to an individual learner)*

L(?): Didn't understand it.

T: You what? Why don't you ask? Is it so difficult? Did you listen yesterday?

T: Do you follow?

T: Your  $m$  is a number and your  $c$  is a number. Do you follow?

*(Teacher explains how to use the formula for the gradient of a straight line.)*

T: You need two points ... you must have the same point's values first and then the same point's values second. You can't mix them up. Do you follow?

*(Teacher did not give learners opportunity to count blocks. Teacher did not discuss gradients of horizontal or vertical lines. Teacher tells parallel lines have similar gradients. Teacher tells about perpendicular lines.)*



T: Mathematically, if I multiply these gradients, it will always give me a minus one. This is theory. You study this. There are actually three methods in drawing straight lines.

T: This is about time. Why are you late again? *(to another latecomer)*

T: Do you follow?

*(Teacher tells about approaches to drawing straight line graphs; some are easier, others take more time, etc.)*

T: Have you got that?

T: Do you follow? ...

T: What does the word dual imply?

L(?): Two.

T: Two; two-intercepts method. A line will always cut a  $x$ - and a  $y$ -axis. So it cuts two axes, except if it's one of these special cases, which is parallel to one of the axes, okay?

*(A line through origin would be impossible to be drawn with this method. The information is out of context; probably means nothing to learners.)*

T: Do you follow?

*(Although the teacher is not dependent on textbook, he follows the textbook exactly.)*

T: Write the heading quickly.

T: You need to know all three the names.

T: Explain the meaning of gradients.

T: The gradient as I explained says what about your graph?

L(?): How steep will the slope be? *(No reference to change in  $y$  &  $x$ .)*

T: Yes.

T: It determines how steep the graph goes up or down. Do you follow?

*(Teacher does not explain concepts increase or decrease.)*

T: Do you follow? ...

T: Do you follow? ...

T: Do you follow? ...

T: How many points did you need to calculate that  $m$ ?

L(6): Two.

L(7): Three.

T: Two points. Two! Do you follow? ...

T: Do you follow?

*(Teacher shows example and then divides learners into groups, using different combinations of points to determine the gradient of one particular line.)*

T: Which columns do you want to use?

L(?): First and second last one.

T: Follow?

T: People, be very careful in the test for this formula because you tend to mix them up, right? The  $x$ 's above and then the  $y$ 's below. Remember,  $y$  is above;  $x$  is below, okay? And do not put plusses between them, because people tend to do that as well. Remember that there is minuses between them.

T: Now I cannot put minus minus two and two minuses next to each other. It's going to look funny. So, I have to put that one in a bracket. Do you follow?

T: Two minus five is? Minus ...? Three. Minus times a minus is a?

L(?): Plus.

T: Plus. Then I get minus three over three. Which is ...? Minus ...? One. Is this difficult? You just need to know the formula and how to apply it.

T: People we are finished with (a) now. We are busy with (b). Aren't you following?

*(Teacher focuses on how to avoid errors. He warns learners about common errors.)*

T: What? Did you get minus one? Where is your textbook?

T: What is the points; did you also get minus one? You are using ...?

*(Teacher looks at points the learners are using.)*

T: Why did you put in a plus two?

L(8): Where Sir?

T: There.

L(8): Is it a minus?

T: Yes, your  $y$  is minus two. It must just be a minus two there.

L(8): *(inaudible)*

T: I haven't worked it out yet. We'll see now.

*(Teacher decides what is right or wrong in the class.)*

T: I'm quickly going to work it out; using other points like you did.

*(Teacher makes an error with the gradient formula; immediately corrects it and tells the learners.)*

T: Huh, huh, huh, huh, look what I'm doing there now. I'm putting the  $x$ 's above the line.

*(Teacher does not explain what he is doing this time. Learners are busy on their own.)*

T: I get minus three.

L(?): Yes.

L(9): Just look here?

L(9): Is it right?

T: Yes, you got minus three.

T: Listen people, yours is not gonna look like mine 'cause we used different points.

*(Teacher explains about different points on one line; giving the same value for the gradient. Teacher does not show collinear points diagrammatically.)*

T: Have you got that?

T: What don't you understand? *(to an individual learner)*

L(10): *(inaudible)*

T: About?

T: What about it?

L(10): Sir I thought it was minus one.

T: No.

T: The m in (a) is not gonna be the same m in (b).

L(10): Sir, but how did you get *(inaudible)* the second a minus seven? In the block it's *(inaudible)*.

T: The minus is in the formula and that minus is that minus; that minus is that minus; but then it also had in the block a minus two, and that's two minuses.

*(The learners get confused with negative coefficients and the difference as indicated in the formula. Look up in the textbook!)*

T: This is now number (c). A new question. *(Learners expect to see the same result each time.)*

L(10): Is it possible to be a positive answer? *(Learners portray no relational understanding to what has previously been taught in terms of the direction/increasing/decreasing.)*

T: Mmm

T: You must get a half. Did you get a half?

*(Teacher has made a mistake.)*

L(11): Two.

T: A two?

T: Which points did you use?

*(Teacher looks at a learner's solution.)*

L(11): In this one I did it exactly as you said.

T: You are not supposed to get a half, people. People, what have I done?

L(?): Sir, you put the  $x$ 's above, not the  $y$ .

T: Yes.

T: Which is?

T: Two, you should get two. Can you see how easily you could do this mistake?

L(12): Yes.

T: And then why didn't you tell me?

L(12): *(inaudible)*

T: Am I the Sir?

L(12): Yes.

T: I also make mistakes.

T: Which points did you do?

L(?): *(inaudible)*

T: What question are you at? You used (b)'s columns. No, you must get the same answer as I did. Is this now (c)?

*(Teacher looks for errors in the learner's script.)*

T: First of all, your  $x$ 's is above the line and your  $y$ 's is below the line. Can you see that? And also, that ... oh, no, no, no, no; you minused it this way and that way ...

*(The learner has probably switched the order of the coordinates when substituting.)*

L(?): Which method is that?

*(The learners are holding on to methods.)*

T: No, no, no, this is not one of the sketching methods yet. This is only to determine the gradient.

T: If I know that lines are parallel, what do I know about their m-values?

L(?): They are minus one.

*(Learner is probably confusing gradients with relation  $m_1 \times m_2 = -1$ .)*

T: Huh huh, they are the same. Can you see this is in the format  $y = mx + c$ ?

T: What is my m in this equation ( $y = -x + 3$ )? m is minus one. So, the m in your new equation will also be ...? Minus one. Do you follow?

*(Teacher answers himself.)*

T: Do you follow? ...

T: Do you follow?

T: And if it's a one, we don't have to write the one, do we?

L(?): No.

T: What is that minus one in the coordinate point? It is a ...?

L(?): *(inaudible)*

T: What's the minus one in the coordinate point?

L(13): x

L(14): y

T: The x. And what's the three in the coordinate point? The y. Do you follow? ...

T: Can you see that?

T: So c is the only unknown thing. Can you see that?

T: Minus and minus gives me?

L(?): Plus.

T: Plus one and if I take it across ...? Then we find that  $c$  is? Two. Do you follow?

*(Teacher answers himself.)*

T: Then we go back to the standard equation and we write it down with the  $m$  and the  $c$  that we have now. What is  $m$ ? Minus one. What is  $c$ ? Two. Do you follow? ...

*(Teacher answers himself.)*

*(Learners ask questions informally and spontaneously.)*

T: Do you follow?

L(15): Why did the minus come back?

T: Because they said this line that we get now ... that we have to get, is parallel to that one.

L(15): Did you change the one into a positive to find  $c$ ?

T: Yes, yes.

L(15): Oh!

T: Oh, this one and that one that we took across, is not the same one.

L(15): Oh!

T: Do you follow people?

L(15): No.

L(16): Yes.

T: Listen, this minus one is the gradient. There it is. Okay? Then they said this thing, this other line, goes through the point minus one, three. That's another minus one. This is not the gradient. This is a  $x$ -value. You can't have a gradient in the coordinate.

L(15): Okay!

T: No.



T: Okay. Do you follow?

T: This could have been a minus five. It is just a coincidence that it's exactly the same value as the gradient.

L(15): So, it's possible that it could be another value?

T: Yes, very likely. Have you got that?

*(Except for a short tutorial-type segment, the teacher utilizes the entire double period to teach until the bell goes. Learners usually get homework.)*





## **APPENDIX C: THE DEVELOPMENT OF THE VARIOUS DATA MATRICES**

## *Interviews*

Subsequent to transcribing the mechanically recorded semi-structured interviews, the data were approached with the following deductive data categories:

- Beliefs teachers had about the pedagogical value of learners' errors.
- Beliefs teachers had about the pedagogical value of public and whole-class discussion of learners' errors, including peer involvement in error discussion.
- Beliefs teachers had about the pedagogical value of assessment.
- Beliefs teachers had about learner errors per se.
- Beliefs teachers had about teaching and learning mathematics.
- Teachers' professed responses to learners' errors during classroom interaction.
- Teachers' professed responses to learners' errors in written assessment tasks.

A number of codes emerged inductively during the initial coding process. These were the following:

- Teachers' beliefs about contextual factors influencing their responses to learners' errors.
- Teachers' beliefs about the influence peers have on their decisions concerning error-handling during classroom interaction.
- The beliefs teachers have about the learners or about learner dispositions.

When these data reduction and data coding processes were completed for each individual participant, I paraphrased the chunks of data on data matrices I created on Microsoft Word Documents. At this stage, these processes were limited to within-case data analysis. The following data matrix was designed and applied:

**Table C-1 Data matrix for the individual coding of the interview data**

<b>DATA MATRIX FOR INDIVIDUAL CODING OF INTERVIEW DATA</b>	
Participant X at school X	
Teachers' beliefs about the pedagogical value of:	
Learners' errors	
Public and whole-class discussion of learners' errors/peer involvement in error discussion	
Assessment	
Teachers' professed interactions with learners' errors:	
Verbally during learning periods	
Written in assessment tasks/tests	
Remedial actions (emerged inductively)	
Teacher and classroom culture in terms of:	
Beliefs about teaching and learning mathematics	
Teachers' beliefs about learners' errors	
Teachers' beliefs about contextual factors (emerged inductively)	
Teachers' beliefs about peer influence (emerged inductively)	
Teachers' beliefs about learners and/or learner dispositions (emerged inductively)	

## Observations

A data matrix was designed and utilized to categorize the *error moments* transcribed from the mechanically recorded classroom activities. The classification was manually performed with the data matrix in a Microsoft Word Document on my personal desk top computer.

**Table C-2 Data matrix for individual coding of classroom data**

<b>DATA MATRIX FOR INDIVIDUAL CODING OF CLASSROOM DATA</b>	
Teacher X at School X	
Teacher's verbal interactions with learners' errors during learning periods:	
Teacher's impassiveness or unresponsiveness towards learners' errors	
Interacting with learners' errors through verbal interjections or repetitive negative interjections	
Prompt correction of learners' errors through transmission without explanatory detail	
Correction of learners' errors through transmission, augmented with clarifying explanations	
Instructional scaffolding of individual learners through questioning	
Peer involvement in error discussion	
Classification of errors: <ul style="list-style-type: none"> <li>• computational</li> <li>• careless</li> <li>• copied incorrectly</li> <li>• "sign error"</li> </ul>	
Enhancing the mathematical value of a learner's contribution by paraphrasing and rephrasing the contribution without emphasizing the learner's error(s), thus temporarily allowing the error(s) to elicit the mathematics from the learner	
Entertaining learners with their mathematical errors	
Learner's error is not detected and teacher accepts and confirms the statement or teacher evaluates the learner's contribution or approach incorrectly	
Steering learners towards the teacher's preconceived approach	

DATA MATRIX FOR INDIVIDUAL CODING OF CLASSROOM DATA	
Teacher X at School X	
Teacher's verbal interactions with learners' errors during learning periods:	
Involving learners in the negotiation regarding mathematical results	
Not addressing the learner's precise error	
Employing heuristics to interact with errors	

### *Classroom Vignettes*

The following aspects relating to classroom management were considered:

- The seating and greeting ritual and seating arrangements.
- Instilling and maintaining discipline.
- Homework- and book-control.
- Utilization and segmentation of the learning period.
- Teaching and learning support materials.
- Incidence and purpose of assessment.

The following aspects relating to teaching approach were considered:

- Emphasis of teaching approach, for example procedural, algorithmic, problem-based.
- Introduction and discussion of a subsequent topic.
- Nature and contextualization of mathematical problems.
- Inclination towards cooperative learning.

The subsequent additional aspects relating to the nature of the classroom discourse were considered:

- Semantics of the mathematical language.
- Learner participation and negotiation.
- Evaluation of the mathematical quality of learner contributions.
- Inclination towards alternative approaches to solving mathematical problems.

- Teacher's questioning and listening.

### *Teachers' Questioning and Listening*

#### *Questioning*

The inductive teachers' questioning categories were as follows:

- Recalling knowledge
- Applying knowledge
- Conventional information
- Procedural information
- Trivial calculations or readings
- Reflective questions
- Mathematical interpretation

**Table C-3 Data matrix for the classification of teachers' questions**

DATA MATRIX FOR THE CLASSIFICATION OF TEACHERS' QUESTIONS				
Teacher	Alice	Barry	Chloe	Dawn
Recalling knowledge				
Applying knowledge				
Conventional information				
Procedural information				
Trivial calculations or readings				
Reflective questions				
Mathematical interpretation				

## *Listening*

Teachers' listening was classified according the categories described by Davis (1997).

**Table C-4 Data matrix for the classification of teachers' listening**

DATA MATRIX FOR THE CLASSIFICATION OF TEACHERS' LISTENING				
Teacher/Listening	Alice	Barry	Chloe	Dawn
Evaluative				
Interpretive				
Hermeneutic				

## *Documents*

I approached the process of document analysis, through data reduction, deductively with four potential data categories in mind. These potential categories were the following:

- Response to errors
- Feedback
- Instructional scaffolding
- Classification of errors

The potential categories were embedded in the conceptual framework of my study. The category *response to errors* allowed for all the possibilities of how a participating teacher interacted in writing with learners' errors in assessment tasks. The category *feedback* included written comments, encouraging or authoritative, and written, expository explanations of what went wrong with the learner's approach to or interpretation of the mathematical problem. The category *instructional scaffolding* incorporated written cognitive support, aimed at manoeuvring the learner's reasoning towards understanding. In order to compel the learner to reflect on his or her own reasoning, this type of cognitive support would not be explanatory or expository. The scope of teachers' classification of learners' errors entailed categorizing errors as computational errors, careless errors, "sign errors" or those copied incorrectly. A data matrix was designed and utilized to categorize the chunks of data from the written assessment tasks. The classification was manually performed with the data matrix in a Microsoft Word Document on my personal desk top computer.

**Table C-5 Initial data matrix for individual coding of assessment data**

DATA MATRIX FOR INDIVIDUAL CODING OF ASSESSMENT DATA	
Teacher X at School X	
Teacher's interactions with learners' errors in written assessment tasks:	
Response to learners' errors	
Feedback	
Instructional scaffolding	
Classification of errors	

On completion of the first cycle of data reduction through the assignment of appropriate codes to chunks of data, I recognized that I had categorized a significant number of these chunks of data to two categories, being *response to errors* as well as one of the remaining categories. During an exploration of the various *responses to errors*, a number of issues emerged. Through examining the written assessment tasks, I discovered certain teacher responses to learners' errors in written assessment tasks for which I initially did not define categories. These were:

- writing down the correct mathematical procedure on the learner's script;
- correcting the learner's errors; and
- marking the errors in one of a couple of ways.

These categories thus emerged inductively. I subsequently decided, based on these inductively emerging codes, to rearrange the data categories. All recorded teacher responses to learners' errors in written assessment tasks were collectively pronounced as feedback. The decision was based on the recognition of all probable teacher responses as feedback. The two categories, *response to learners' errors* and *feedback* were consequently omitted.

The resultant categories for the individual coding of assessment data were the following:

- Errors indicated with
  - a conventional marking (a tick or a cross);
  - encircling;
  - underlining;
  - an exclamation mark;

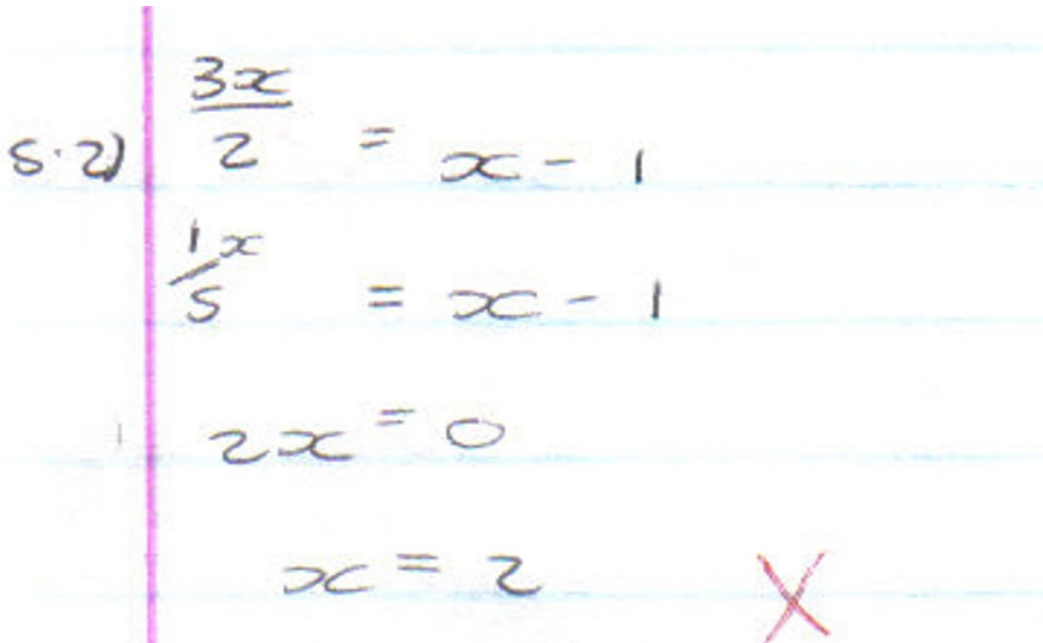


- a question mark; and
- scratching out.
- Exact errors not indicated
- An indication of syntactical errors
- Correction of learners' errors
  - without a written, mathematical explanation;
  - with a written, mathematical explanation; and
  - by providing a complete, correct, written solutions.
- Written comments were:
  - a mathematical explanation of why the learner's effort or interpretation was erroneous;
  - an explanation without a correction;
  - an explanation of what the appropriate mathematical approach would have been;
  - instructional interjections;
  - authoritative interjections;
  - encouraging interjections;
  - potentially compelling the learner to reflection and critical assessment of his or her own thinking;
  - heuristically facilitating the learner to understanding;
  - instructional scaffolding;
  - accommodating the learner's alternative approach; and
  - classification of learners' errors.

The ensuing examples illustrated the categories as set out above.

- Errors indicated with one of the possible markings

- a conventional marking (a tick or a cross)

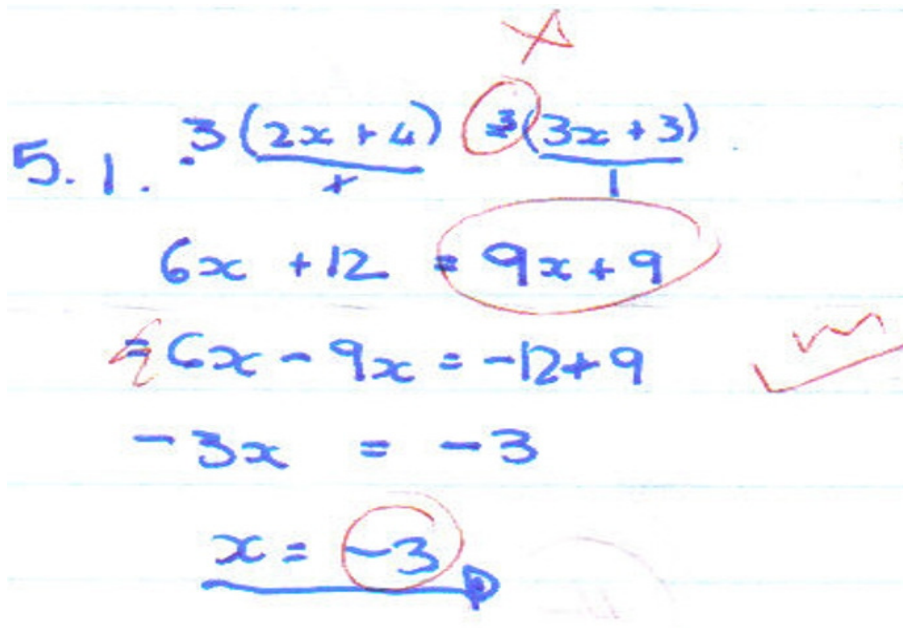


Handwritten student work on lined paper. A vertical purple line is drawn on the left side. The work consists of the following lines:

- 5.2)  $\frac{3x}{2} = x - 1$  (A small blue checkmark is written to the left of this line.)
- $\frac{1x}{5} = x - 1$
- $2x = 0$
- $x = 2$  (A large red 'X' is written to the right of this line.)

Figure C-1 Written assessments: indicating errors with conventional markings (ticks or crosses)

- encircling the learner error(s)



Handwritten student work on lined paper. The work consists of the following lines:

- 5.1.  $\frac{3(2x+4)}{1} + \frac{3(3x+3)}{1}$  (A red 'X' is written above the second fraction.)
- $6x + 12 = 9x + 9$  (The entire equation is circled in red.)
- $6x - 9x = -12 + 9$  (A red checkmark is written to the right of this line.)
- $-3x = -3$
- $x = -3$  (The number -3 is circled in red.)

Figure C-2 Written assessments: encircling error(s)



- underlining the learner error

10. 48 increased in the ratio of 3 : 5 is

- A) 60
- B) 120
- C) 80
- D) 56

$$48 \div 3 = 16 \times 5$$

Handwritten work for question 10:

- Top right:  $48 \div 3 = 16$  (circled),  $16 \times 5 = 80$  (circled), [3]
- Middle:  $3:5$ ,  $= 3+5+48$  (underlined),  $= 56$  (underlined), "why?"
- Bottom right:  $48 \div 3 = 16$  (circled),  $16 \times 5 = 80$  (circled), [3]

Figure C-3 Written assessments: underlining errors

- an exclamation mark

2. Divide:

$$\frac{8}{35} \div \frac{4}{15}$$

- A)  $\frac{1}{2}$
- B)  $\frac{6}{7}$
- C)  $\frac{32}{525}$
- D)  $\frac{7}{4}$

Handwritten work for question 2:

- Top:  $\frac{8}{35} \div \frac{4}{15} = \frac{8}{35} \times \frac{15}{4} = \frac{32}{525}$  (circled), [3]
- Middle:  $\frac{8}{35} \times \frac{15}{4} = \frac{32}{525}$  (circled), [3]
- Bottom:  $\frac{8}{35} \times \frac{15}{4} = \frac{32}{525}$  (circled), [3]

Figure C-4 Written assessments: indicating errors with exclamation marks



- a question mark

(S2)  $\frac{3x}{2} = x - 1$

$\frac{3x}{2} = \frac{2x}{2} - \frac{2}{2}$  ?

$6x = 4x - 4 \cdot (-4x)$

$6x - 4x = -4$

$\frac{2x}{2} = \frac{-4}{2}$

$x = -2$  ✓ (2)

Figure C-5 Written assessments: indicating errors with question marks

- scratching out an error

S.2.  $\frac{3x^{k1}}{2^{k1}} = \frac{x^{k2}}{1^{k2}} \frac{1}{1^{k2}}$  (LCD) one equal sign per line

~~$\frac{3x}{2} = \frac{2x}{2} - \frac{2}{2}$~~

$3x = 2x - 2$

$3x - 2x = -2$

$x = -2$  ✓ (3)

Figure C-6 Written assessments: scratching out errors

- Exact errors not indicated

1. Solve the following equation/Los die volgende vergelyking op:

$$\frac{5}{3x} = \frac{1}{x} - \frac{3}{2}$$

Figure C-7 Written assessments: exact errors not indicated

- An indication of syntactical errors (see figure C-6)
- Correction of learners' errors
  - without a written, mathematical explanation of why the learner's effort or interpretation was erroneous

Figure C-8 Written assessments: correcting errors without written, mathematical explanations

- augmented with a written, mathematical explanation

Question 4 Section B

4.1)  $700 \times 7 + x$  (7 more, means  $\pm 7$ ) (1)

4.2)  $0x + 70x = 98m$  ← you have 2 lengths and 2 breadths!

$80x = 98m$

$x = \frac{98}{8}$

Figure C-9 Written assessments: correcting errors with written, mathematical explanations

- by providing a complete, correct, written solutions

53.  $\frac{a-2}{2} = \frac{3}{4} + a-2$

$2\left(\frac{a-2}{2}\right) = 2\left(\frac{3}{4} + a-2\right)$

$a-2 = 6 + 2a-4$  (3)

$a-2a = 6+2-4$

$-a = 4$  →  $= \frac{3}{2}$

$a = -4$  ✓

---

Carry on ---

→  $-a = \frac{3}{2} - 2$

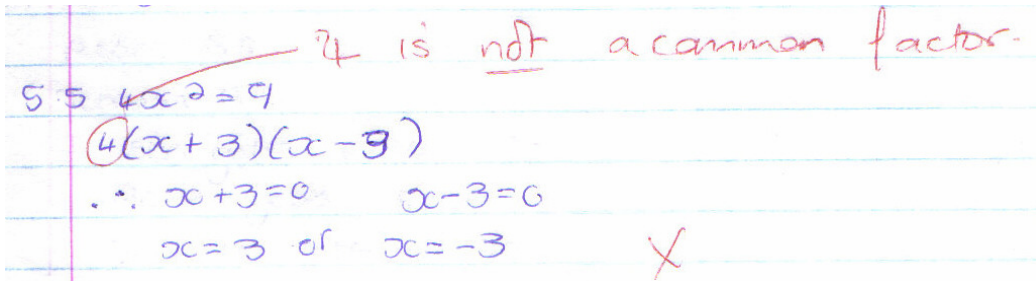
$-a = 1\frac{1}{2} - 2$

$-a = -\frac{1}{2}$

$a = \frac{1}{2}$

Figure C-10 Written assessments: providing complete, correct, written solutions

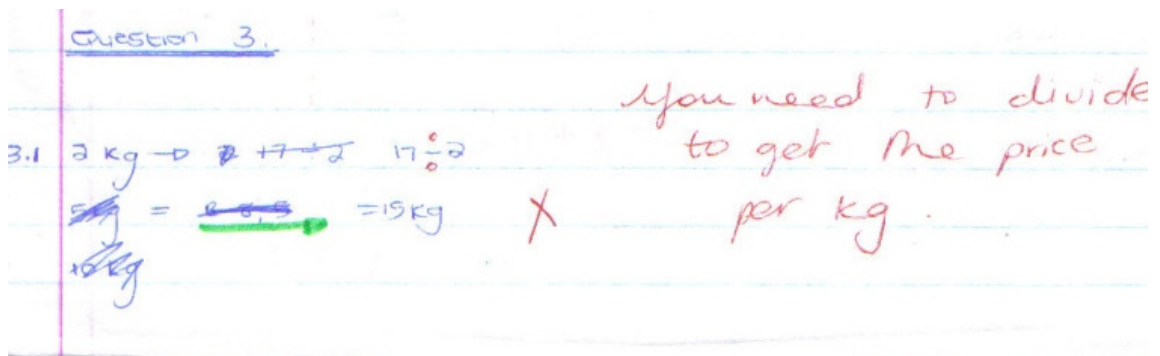
- Written comments
  - a mathematical explanation of why a learner's effort or interpretation was erroneous, as described previously in the section on correcting learners' errors (see figure C-10)
  - an explanation *without* a correction of the learner's error.



5.5 ~~4~~ $x^2 = 9$  4 is not a common factor.  
 $4(x+3)(x-9)$   
 $\therefore x+3=0 \quad x-3=0$   
 $x=3 \text{ or } x=-3$  X

Figure C-11 Written assessments: providing mathematical explanations of errors

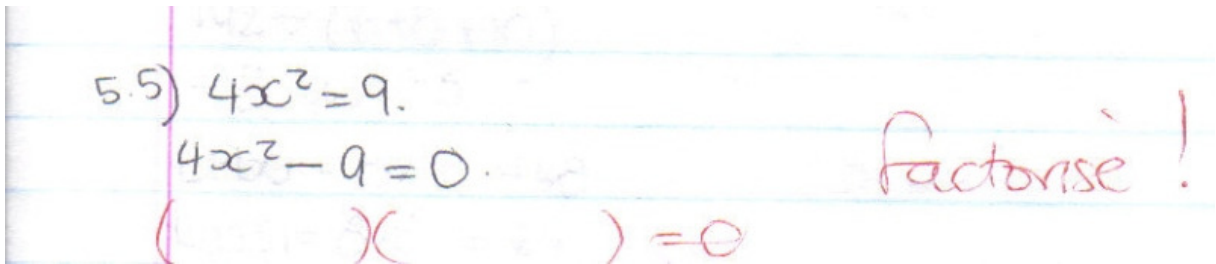
- an explanation of what the appropriate mathematical approach would have been



Question 3.  
 3.1 2 kg  $\rightarrow$  ~~17 + 2~~  $17 \div 2$   
~~17~~ = ~~17~~ = 19 kg X  
 You need to divide to get the price per kg.

Figure C-12 Written assessments: explaining the appropriate mathematical approach

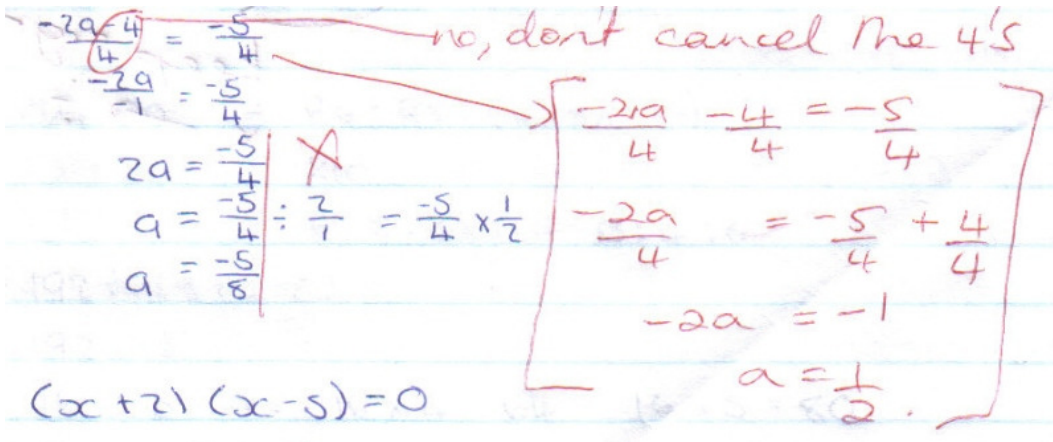
- instructional interjections



5.5)  $4x^2 = 9.$   
 $4x^2 - 9 = 0.$  factorise!  
 $( \quad ) ( \quad ) = 0$

Figure C-13 Written assessments: providing instructional interjections

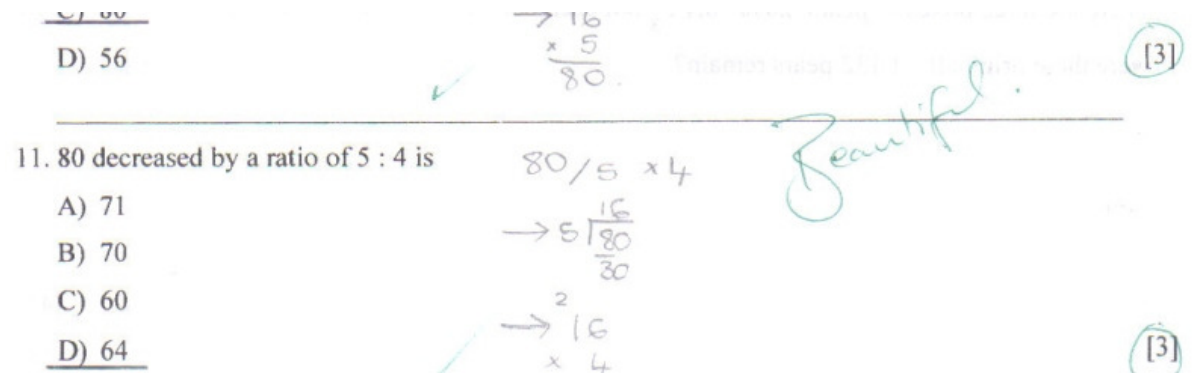
- authoritative interjections



$\frac{-2a-4}{4} = \frac{-5}{4}$  no, don't cancel the 4's  
 $\frac{-2a}{4} = \frac{-5}{4}$   
 $2a = \frac{-5}{4}$  X  
 $a = \frac{-5}{8}$  no, don't cancel the 4's  
 $a = \frac{-5}{8}$   
 $\frac{-2a}{4} = \frac{-5}{4}$   
 $-2a = -5$   
 $a = \frac{1}{2}$   
 $(x+2)(x-5) = 0$

Figure C-14 Written assessments: providing authoritative interjections

- encouraging interjections



D) 56 ✓ Beautiful. 3

11. 80 decreased by a ratio of 5 : 4 is ✓

A) 71  $80/5 \times 4$

B) 70  $\rightarrow 5 \overline{)80}$

C) 60  $\rightarrow 16$

D) 64  $\times 4$  3

Figure C-15 Written assessments: providing encouraging interjections



- potentially compelling the learner to reflection and critical assessment of his or her own thinking

9. R10 800 shared in the ratio of 2 : 3 : 4 is

A) 2 400 : 4 800 : 3 600

**B) 2 400 : 3 600 : 4 800**

C) 4 400 : 2 800 : 3 200

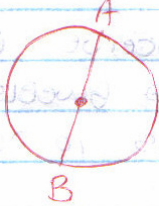
D) 2 000 : 3 000 : 4 000

*Handwritten notes:*  
 2 x 10 800 : 3 x 10 800 : 4 x 10 800  
 doesn't watch  
 why x ?  
 Divide times  
 [4]

**Figure C-16 Written assessments: compelling learners to reflection**

- heuristically facilitating the learner to understanding

4 diameters



*Handwritten text:*  
 AB is a diameter of the circle.

**Figure C-17 Written assessments: heuristically facilitating understanding**

- instructional scaffolding

4. Last year there were 1172 students at Beaton High School. This year there are 15% more students than last year. Approximately how many students are at Beaton High School this year?

1800  
 (B) 1600  
 (C) 1500  
 (D) 1400  
 (E) 1200

$$\frac{1172}{1} \times \frac{15}{100} = 17 \frac{380}{100} = 1738$$

Try 10% inc is  $\frac{1172}{10} = 117.2$   
 ... 20% is ...

Figure C-18 Written assessments: scaffolding instructionally

- accommodating the learner's alternative approach

5.5  $4x^2 = 9$       $4x^2 - 9 = 0$  ?

$(2x + \frac{3}{2})(2x - \frac{3}{2}) = 9$  ~~answer~~

~~$2x + 2x = 9$   
 $4x = 9$   
 $x = \frac{9}{4}$~~

$x^2 = \frac{9}{4}$   
 $x = \pm \sqrt{\frac{9}{4}}$   
 $x = \pm \frac{3}{2}$

Figure C-19 Written assessments: accommodating alternative approaches

- classification of learners' errors

5.3

$$\frac{a-2}{\textcircled{a}2} = \frac{3}{4} + a - 2$$

copy error. (✗)

Figure C-20 Written assessments: classifying errors

Following the preceding process, the ensuing data matrix was utilized to rearrange data categories resulting from the written assessment tasks:

**Table C-6 Final data matrix for the individual coding of assessment data**

<b>FINAL DATA MATRIX FOR INDIVIDUAL CODING OF ASSESSMENT DATA</b>	
Teacher X at School X	
Teacher's written interactions with learners' errors in assessment tasks	
Errors indicated with one of the possible markings (without an explanation and without a correction)	
Exact errors not indicated	
An indication of syntactical errors	
Correction of learners' errors:	
Without a written, mathematical explanation of why the learner's effort or interpretation was erroneous	
With a written, mathematical explanation of why the learner's effort or interpretation was erroneous	
By providing complete, correct, written solutions	
Written comments:	
A mathematical explanation of why a learner's effort or interpretation was erroneous	
A mathematical explanation of what the appropriate mathematical approach would have been	
<ul style="list-style-type: none"> <li>• instructional interjections</li> <li>• encouraging interjections</li> <li>• authoritative interjections</li> </ul>	
Potentially compelling the learner to reflection and critical assessment of his or her own thinking	
Heuristically facilitating the learner to understanding	
Instructional scaffolding	
Accommodating the learner's alternative approach	
Classification of errors: <ul style="list-style-type: none"> <li>• computational</li> <li>• careless</li> <li>• copied incorrectly</li> <li>• "sign error"</li> </ul>	



## **APPENDIX D: ETHICAL DOCUMENTS**

## *Letter Requesting Permission from Schools*



Dear Principal/Vice Principal/Head of Department

### APPLICATION FOR PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

I am a Masters candidate at the University of Pretoria in the Department of Curriculum Studies, led by Hannah Barnes. My main research interest is teacher-learner interactions in mathematics classrooms, with a focus on how mathematics teachers interact with learners' mathematical errors. The study will be of a descriptive nature and will not be evaluative.

As researcher, I shall attend a grade 9 mathematics teacher's classes on a daily basis for a maximum of two weeks, make observations, take field notes and make mechanical recordings for transcription purposes. No video-analysis will be done, no video-clips will be submitted with the dissertation and the video-recordings will not be made publicly accessible. Anonymity will not be compromised by these video-recordings. No learners will be identified. I shall need two opportunities to conduct semi-structured interviews with the research participant, one prior to and one subsequent to the observation period. The semi-structured interviews will be tape-recorded for transcription purposes. The recorded teacher-learner classroom interactions surrounding mathematical errors and the interviews will be analysed. I shall need access to learners' written assessment tasks to analyse the way feedback is done.

I wish to apply for permission to conduct research at your school. It would be greatly appreciated if I could enlist your support for this research.

Thank you for your attention.

Yours faithfully

## *Letter of Informed Consent*

Dear Participant

### INVESTIGATING THE INTERACTION OF MATHEMATICS TEACHERS WITH LEARNERS'

#### MATHEMATICAL ERRORS

This study, for the purposes of the degree Magister Educationis at the University of Pretoria, is an empirical exploration of the interaction of mathematics teachers with learners' mathematical errors in the classroom and in assessment tasks. The study will be of a descriptive nature and will not be evaluative.

As researcher, I shall attend your classes on a daily basis for a maximum of two weeks during one or two school terms, make unstructured observations, take field notes and make mechanical recordings for transcription purposes. No video-analysis will be done, no video-clips will be submitted with the dissertation and the video-recordings will not be made publicly accessible. Your anonymity will not be compromised by these video-recordings. No learners will be identified. I shall need two opportunities to conduct semi-structured interviews, preceding and concluding the fortnight of field work. The semi-structured interviews will be tape-recorded for transcription purposes. The recorded teacher-learner classroom interactions regarding mathematical errors and the semi-structured interviews will be analysed. I shall need access to learners' written assessment tasks to analyse the way feedback is done. Learners' names and marks or grades are irrelevant to the study and will not be recorded.

As participant in the research project, it will be expected of you to utilize learning periods in a standard way. You will be asked to participate in a structured interview and in two semi-structured interviews.

You are invited to participate in this research project. However, in line with the principle of voluntary participation in research, you are free to withdraw from the research at any time, including choosing not to take part right from the start.

You are offered no specific inducements to be a participant in this study.

What will you be expected to do, what information will be required and how long will your participation take for each specified task?

- The researcher will attend your classes on a daily basis, for a maximum of two weeks during one or two school terms, make unstructured observations, take field notes and make mechanical recordings for the purposes of transcribing the classroom interactions regarding learners' errors. During these classroom visits, it will be expected of you to utilize the learning period in a standard way, following the teaching style and interacting with learners according to your normal pattern.
- Two semi-structured interviews will be conducted at times which suit your timetable, one to precede and the other to conclude the fortnight of fieldwork. The semi-structured interviews will be mechanically recorded for the purposes of transcribing the data. Each interview will take up to 20 or 30 minutes of your time.
- It will be expected of you to participate in a structured interview. Information on your qualifications, professional experience and your views on mathematics and on mathematical errors will be recorded.
- You will need to make written assessment tasks available to the researcher for document analysis.

By signing this letter of informed consent I understand the following:

- My participation in this research is voluntary, meaning that I might withdraw from the research as a participant at any time.
- As a research participant I will at all times be fully informed about the research process and purposes.
- In line with the regulations of the University of Pretoria regarding the code of conduct for proper research practices for safety in participation, I will not be placed at risk or harmed in any way.

- My privacy with regard to confidentiality and anonymity as a human respondent should and will be protected at all times. No video-clips will be submitted with the dissertation and the video-recordings will not be made publicly accessible. A pseudonym will be used for me in the research report. Learners' names and marks or grades are irrelevant to the study and will not be recorded.
- The school where I teach and the district in which the school is, will not be identified.
- Transcriptions of semi-structured interviews conducted with me will be made accessible to me.
- Research information will be used for the purposes of this enquiry and will be recorded in a research dissertation. An article based on the research information may be published in an academic journal.
- The final research report will be made accessible to all the participants.
- My trust will not be betrayed in the research process or its published outcomes and I will not be deceived in any way.
- I hereby give informed consent to participation in this research.

Name of teacher	Signature for informed consent and participation	Date
Name of researcher	Signature	Date
JC (Hanlie) Verwey		
Contact details of researcher 0123612313 0824676467 hanlie.verwey@gmail.com		



## *Letter of Information*

Dear Parents/Guardians/Learners

### INVESTIGATING THE INTERACTION OF MATHEMATICS TEACHERS WITH LEARNERS'

#### MATHEMATICAL ERRORS

I am a Masters candidate at the University of Pretoria in the Department of Curriculum Studies. My main research interest is teacher-learner interactions in mathematics classrooms, in particular those interactions concerned with learners' errors.

The focus of the study is on how teachers elicit responses from learners, how teachers interact with learners' errors, and on how the teachers utilize errors as learning opportunities. As researcher, I will not interact with the learners and will not interact with the teacher during learning periods, consequently not intervening in the learning period at all. My presence at the back of the class will be as unobtrusive as possible, so as not to interfere with teaching nor hinder learning.

As researcher, I shall attend a grade 9 mathematics teacher's classes on a daily basis for two weeks, make observations, take field notes and make mechanical recordings for transcription purposes. The video-camera will be placed on a tripod at the back of the classroom and not all learners will be in the view of the camera. No video-analysis will be done, no video-clips will be submitted with the dissertation and the video-recordings will not be made publicly accessible. Anonymity will not be compromised by these video-recordings. No learners will be identified. I will have access to learners' written assessment tasks. The names of the learners and their marks or grades will not be recorded.

Approval to proceed with the research was granted to me by the Gauteng Department of Education and the relevant school districts. The proposed research was also approved by the Department of Curriculum Studies at the University of Pretoria.



I wish to inform you that I shall be attending the classes in which your child is a learner in order to collect the relevant data for my study. It would be greatly appreciated if I could enlist your support for this research.

Yours faithfully