Chapter 1

Introduction

Mathematical and computer modelling have been playing an increasingly important role in the Computer Aided Engineering (CAE) process of many products in the last 60 years. Simulation offers great advantages in the development and analysis phase of products and offers a faster, better and more cost effective way than using physical prototypes alone. The ever increasing demand for new and improved products in the vehicle industry has decreased the time available for the development of new vehicles, but at the same time the demands on quality, reliability and mass that are set for the vehicle, by both the client and the manufacturer, are becoming ever more stringent. These requirements have lead to the investigation of procedures and methodologies that will reduce the development time of new vehicles without inhibiting the quality of the vehicle.

A high level layout of a typical product development life cycle is shown in Figure 1.1. The product development cycle will start with a set of user requirements for the product. The user requirements are then translated into a set of design parameters which can be used by the designers to generate concept designs for the product. After the various concepts have been evaluated a single concept will result from the concept selection process. A detail design of the conceptual product is then performed which will result in a set of drawings which can be used to manufacture a prototype of the product. The product can then undergo various tests to verify whether the product meets the user and design requirements set out at the start of the product development process. If the prototype satisfies all requirements, mass production of the product can commence. However, if the prototype does not satisfy all requirements the short comings have to be identified and the process will either return to the conceptualization phase, the design phase or the manufacturing phase. Having gone through the entire product development process up to where a physical prototype has been built and then realizing that there is a conceptual or design flaw has great cost and time implications.

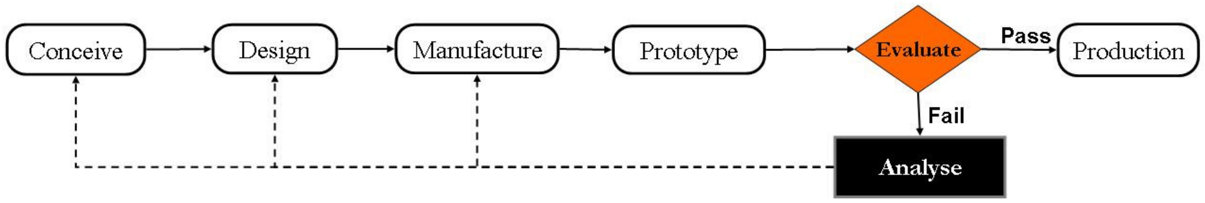

Figure 1.1. Typical product development life cycle

Figure 1.2 shows the cost as the development process of the product continues. It is clear from this figure that great savings in cost, and time, can be realised if the evaluation of the product can be performed as early as possible in the development cycle. This is where a well founded CAE process holds exceptional benefits.

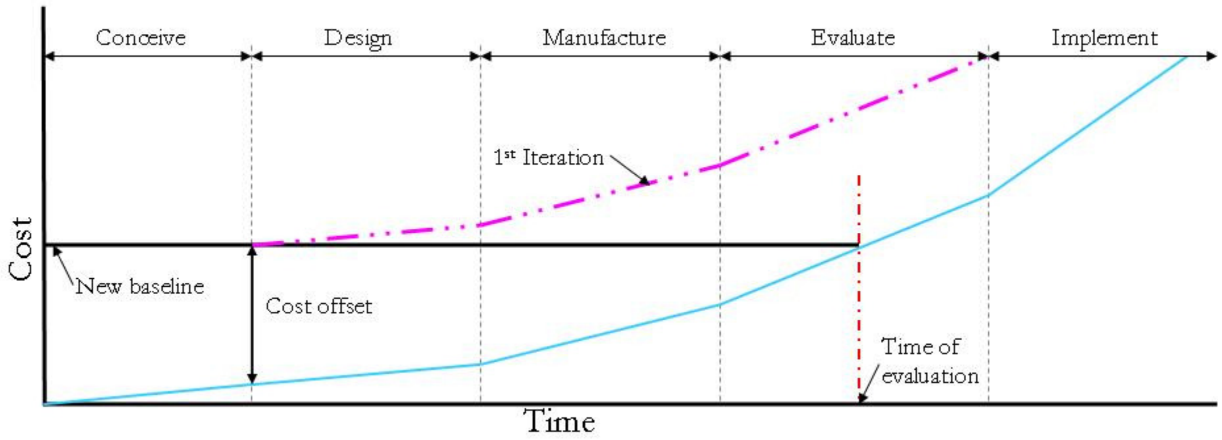

Figure 1.2. Typical product development life cycle cost

A CAE process with validated simulation models enables early evaluation of the product. Having the simulation models available implies that the evaluation of the product can be performed early on in the development process as the evaluation can be performed without the need for a physical prototype. Figure 1.3 indicates a product development cycle with numerous evaluation checkpoints. At each check point different aspects of the product can be evaluated. Take for example the development of a new vehicle. Various concepts have been generated for the suspension system and a concept selected. The suspension system's kinematics is evaluated in order to check for bump steer, suspension travel, etc. If the concept conforms to the design requirements the concept suspension moves to the design phase. After the various subsystems of the vehicle, such as the suspension system, has gone through a detail design the subsystems can be modelled by the analysts and integrated into a full vehicle simulation model that can be used to evaluate, for example, the durability of the vehicle. Again, the results will be that the product satisfies the durability requirements and it can go into production, or it has unsatisfactory performance and requires refinement. It is not advocated that the product development process is purely based on simulation. It is therefore recommended that a physical prototype still be manufactured and tested.

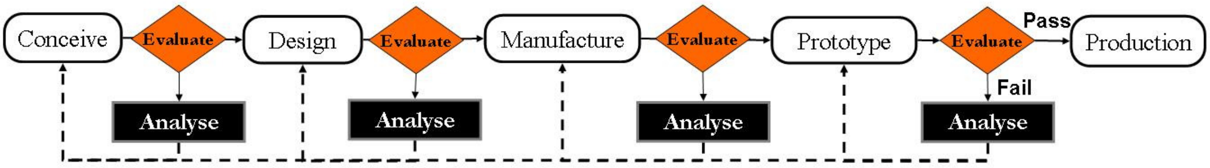

Figure 1.3. Product development life cycle with continuous evaluation

Even though the systematic evaluation of the product throughout its development process, as proposed in Figure 1.3, may lead to an increase in development cost and time, this methodology, if properly executed, has the potential to offer greater overall savings in time and cost and at the same time ensures that the product delivered to the client meets all the requirements and is of exceptional quality and design.

1. Problem statement

The development life cycle presented in Figure 1.3 requires a CAE process with validated simulation models of components, subsystems and systems. In the context of this study, components are elements such as the leaf springs, with the subsystems being the suspension system, and the system the full vehicle. This study forms part of a larger project that is concerned with obtaining a library that contains simulation models of components and subsystems that can be used to create full vehicle simulation models that can be used in the CAE process. Accurate full vehicle multi-body simulation (MBS) models are heavily dependent on the accuracy at which the subsystems, and more fundamentally, the different components that make up the subsystems, are modelled. In the commercial trailer market, at which this study is aimed, a relatively small number of "standard" suspension systems are used, which makes it feasible to develop detailed mathematical models for these and use them as building blocks in the design of new trailers. It is needless to say that an accurate model of a leaf spring is needed if an accurate subsystem model is to be created of the suspension system.

A validated model of the suspension system shown in Figure 1.4 has to be created. The primary goal set for the model is to be able to predict the forces at the attachment points where the suspension system is attached to the vehicle chassis. The model has to be validated by comparing the predicted and measured forces at the suspension attachments as the ultimate goal is to use the suspension model in full vehicle durability simulations.

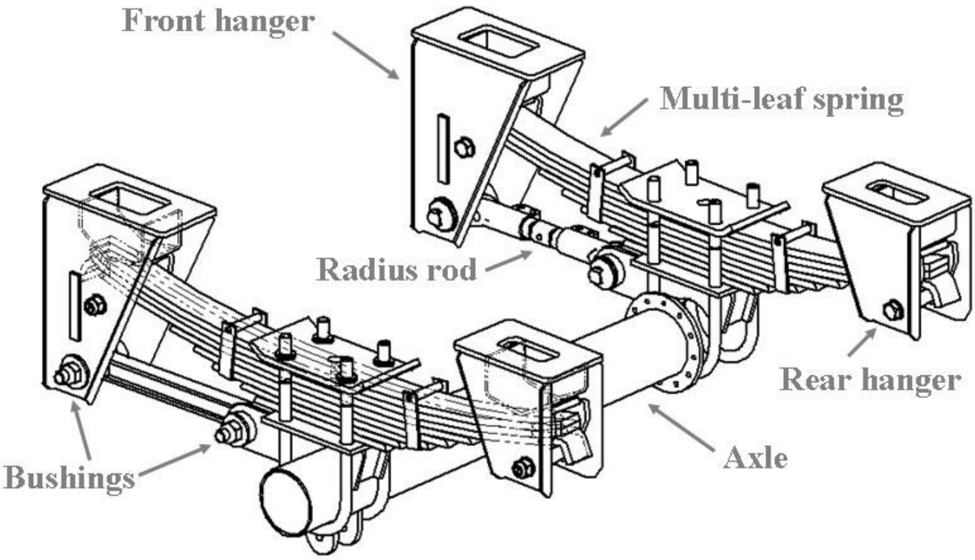

Figure 1.4. Suspension system of interest

2. Introduction to suspension system of interest

Figure 1.4 shows the suspension system that will be considered in this study. The figure shows the suspension system with a multi-leaf spring consisting of 8 blades (or leaves) having a uniform cross-section through the length of the blade. The leaf spring and radius rod constrains the axle in the vertical, longitudinal and lateral directions. The suspension system is attached to the chassis via the hangers. In this configuration the leaf spring is supported by the front and rear hangers instead of a fixed-shackled end configuration (see Figure 1.5 for an example of a fixed-shackled end configuration). In addition to the suspension system in

Figure 1.4 with the multi-leaf spring, another leaf spring will be considered that has a parabolic thickness profile along the length of the blade. This leaf spring will be referred to as a parabolic leaf spring.

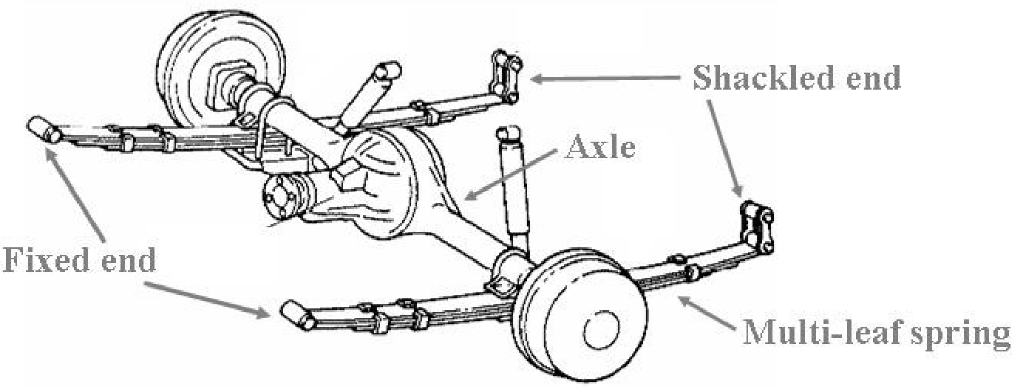

Figure 1.5. Suspension system with leaf spring in fixed-shackled end configuration (Adopted from Monroe (2011))

A systematic approach will be followed in creating a validated model of the suspension system, shown in Figure 1.4, which can be used in durability simulations. This systematic approach entails that the suspension system be broken down into smaller subsystems and the subsystems broken down into the various components (shown in Figure 1.6). Models of the components are then created and validated and then integrated into subsystems which are again validated. The subsystems are then integrated such that a model of the complete suspension is created and once again goes through a model validation process before it is used in full vehicle simulations.

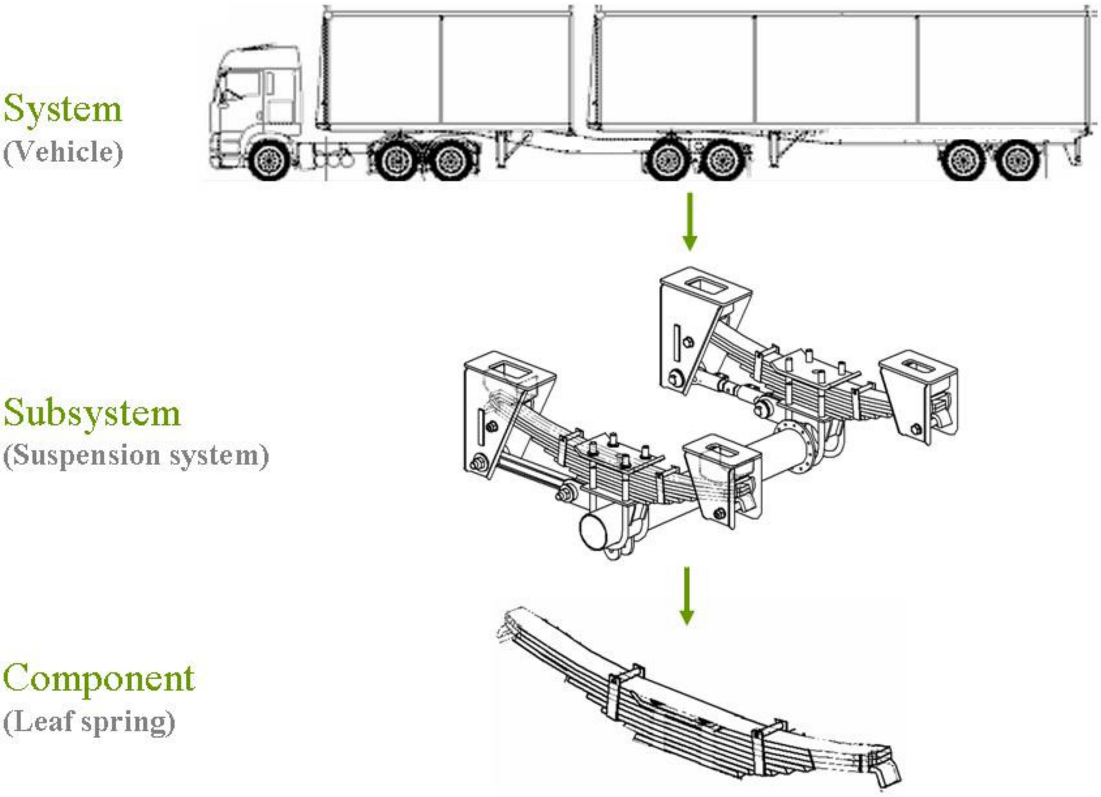

Figure 1.6. Systematic modelling approach

The component of greatest importance, when considering the vertical behaviour of the suspension system, is the leaf spring. The leaf spring has been used in vehicle suspensions for many years. It is particularly popular in commercial vehicles as it is robust, reliable and cost effective. Leaf springs are nonlinear devices which dissipate energy through inter-leaf friction and have force developing characteristics that are dependent on the static load and the amplitude of the imposed displacements. Leaf springs can exhibit highly nonlinear behaviour with hysteresis. High fidelity suspension models require that the nonlinear behaviour of the components, such as the leaf spring, be captured. Fancher *et al.* (1980) state that "since truck leaf springs are complicated nonlinear devices, involving hysteretic damping, their representation in detailed analyses of vehicle dynamic studies of ride, braking or handling is not easily accomplished using linear approximations or simplified models." With any mathematical model it is ideally the aim to develop a model that is as simple (computationally efficient) as possible and as complex (accurate) as necessary. It will obviously be the goal in this study to obtain accurate simulation models that are also computationally efficient.

In order to obtain an accurate model of the vertical behaviour of the leaf spring the model should be able to capture important aspects of the behaviour of the physical leaf spring. The force-displacement characteristic in Figure 1.7 shows the typical aspects that are present in the behaviour of the multi-leaf spring when it is compressed and extended (in tension). Note that the following convention is used concerning compression and tension: when the spring is compressed the displacement and the force is taken as negative. This convention will be used throughout the study. In general, the multi-leaf spring will seldom be in tension as this occurs only when the wheels loose contact with the road. This situation may have a higher possibility of occurring under off-road and very rough road conditions than under smooth on-road conditions. The focus will be on the compressive behaviour of the spring in this study.

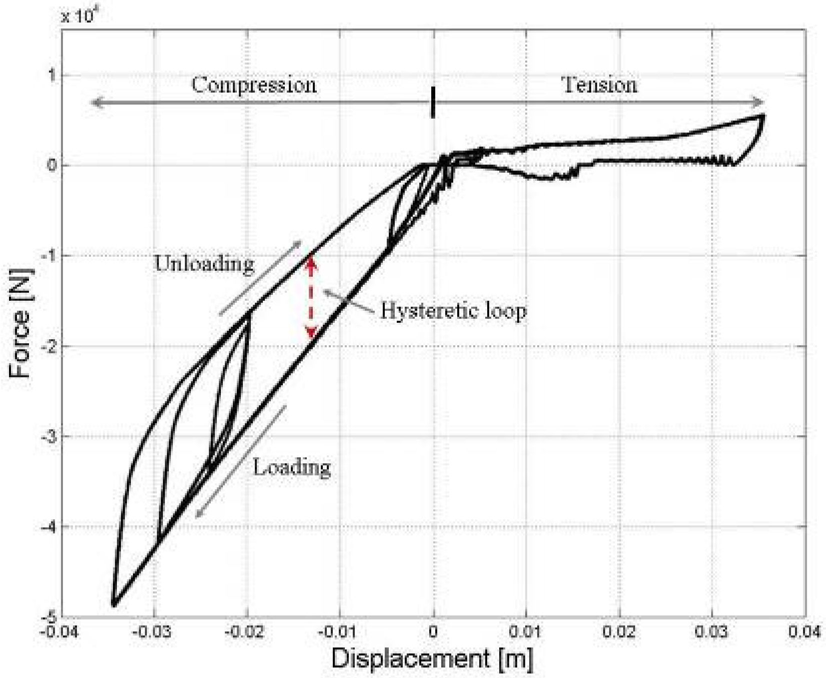

Figure 1.7. Typical force-displacement characteristic of a multi-leaf spring

From Figure 1.7, which shows the force-displacement characteristic of the multi-leaf spring, the two major aspects that a leaf spring model has to capture are identified as:

- the spring stiffness and,
- the hysteresis loop

These two aspects are dependent on the configuration of the leaf spring as well as the contact and friction processes that exists between the individual blades of the leaf spring. The stiffness of the leaf spring is affected by the configuration of the leaf spring (i.e. number of blades, geometry of blades, and loaded length of the leaf spring). The hysteresis loop is mainly governed by the friction and contact processes that exist between the individual blades. Therefore the number of blades, and the way the individual blades make contact with each other, will affect the size of the hysteresis loop. The leaf spring model should be able to capture the stiffness of the leaf spring as well as the hysteretic behaviour of the leaf spring.

After a validated model of the leaf spring has been created this component model can then be integrated into a subsystem representing the suspension system. The main requirement that is put to the leaf spring suspension model in this study is that it should be able to predict not only the spring characteristic, but also the vertical forces, which are transmitted to the chassis at the attachment points, accurately. This requirement is set, as it is required to obtain a leaf spring suspension model that can be used in full vehicle simulation models to perform durability analysis on the vehicle's structure. It is therefore essential to be able to predict the loads that act onto the chassis at the suspension attachment points. The fact that the vertical forces are the focus of this study does not mean that the lateral and longitudinal forces are not important but only that manoeuvres such as handling and performance (braking and acceleration) simulations are not the driving factors. Capturing the vertical behaviour will be the starting point from which the models can then be extended to capture longitudinal and lateral behaviour as well.

The result of a literature study, performed on the leaf spring models that exists and their use in vehicle simulations, is discussed in the next paragraph.

3. Literature study

A literature study was conducted to obtain an idea of the leaf spring models that have been developed and whether they are able to give accurate predictions of the force-displacement behaviour of the leaf spring as well as reaction forces on the vehicle attachment points. The application of the different leaf spring modelling methods in vehicle simulations is noted along with whether they were validated and for which parameters.

Sugiyama *et al*. (2006) suggests that existing leaf spring models can roughly be classified into three categories; (1) a lumped spring model, (2) a discretized model, where a number of rigid links, connected by springs and dampers, are used to account for the structural flexibility of the spring blades and (3) finite element models. Omar *et al*. (2004) reviews several techniques for modelling leaf springs. These include the use of empirical formulae and experimental testing, equivalent lumped systems, simple beam theory and finite-element methods. From the literature it would seem that there are various different leaf spring models that have been developed using different methods. The different approaches used to model leaf springs can be classified into the following broad categories:

- Beam theory,
- Analytical/Empirical models,
- Equivalent models,
- Discrete methods (or Finite segment method),
- Finite element methods (includes beam element models),
- Neural Network models,

- Lumped mass spring models (Equivalent lumped system),
- Graphical techniques and,
- Kinematic models.

Each of the different models have their own advantages and disadvantages, therefore it is expected that not all the models will give the same accuracy in different applications. In the following paragraph we will look at the use of the different leaf spring modelling approaches in various studies.

3.1 Leaf spring models in previous studies

A short review of the application of some of the different leaf spring models in previous studies will be given in this paragraph. The studies will be arranged according to the approach used to model the leaf spring.

3.1.1. Analytical/Empirical models

The analytical models use algebraic equations that are able to fit the experimentally obtained force-displacement characteristics of a leaf spring.

The objective of the research reported in the study by Fancher *et al.* (1980), was to 1) measure the force-producing characteristics of several different types of leaf springs while exciting them at various amplitudes and frequencies of oscillation about nominal loading conditions and 2) develop a means for representing the force-deflection characteristics of leaf springs in a form suitable for use in simulations of commercial vehicles. The test results showed that the leaf springs have rather unique force-deflection characteristics. Therefore, a model suitable for representing their characteristics over wide ranges of loading, deflection amplitudes, and random reversals of velocity is needed for use in vehicle dynamic simulations. Accordingly, they devised an equation to represent the characteristics of the leaf spring. They compare the predictions from this equation with test data and it shows that the model is indeed capable of representing the characteristics of the leaf spring, capturing the stiffness as well as the hysteresis loop. The model by Fancher *et al.*(1980) uses equations to represent the spring force which consist of a linear and exponential term. These equations are merely a fit to the envelope of the force-deflection characteristic of the leaf spring.

Cebon (1986) describes an experimental investigation into the behaviour of some typical leaf springs for realistic operation conditions. The accuracy of three alternative analytical spring models, suitable for use in vehicle vibration simulation, are also examined. The equation presented by Fancher *et al.* (1980) formed the basis of Cebon’s (1986) fitting procedure. The measured responses of the leaf springs are compared to simulations which use empirical descriptions of their low frequency (quasi-static) behaviour. Cebon (1986) concludes that two different empirical descriptions can be used for accurately predicting the force developed by typical road vehicle leaf springs.

Application in vehicle simulations

In a study by Cole & Cebon (1994), they describe both a 2D and 3D model of a four-axle articulated vehicle. They summarize that a 2D model may be satisfactory for predicting the tyre forces of a heavy vehicle if: (1) the vehicle speed is high enough to prevent excitation of sprung mass roll modes, and (2) the contribution of the unsprung mass roll modes to the tire forces are small. Attention is also given to modelling the tandem-axle, leaf-sprung trailer suspension. The hysteresis of the leaf spring element is modelled using the method of Francher *et al.* (1980) and Cebon (1986). The radius arms (or radius rods) were found to have a significant effect on the behaviour of the suspension. The model used in this study was validated by comparing the predicted tire forces with the measured tire forces (at the tire/road interface) and showed good correlation. The results suggest that further refinement of the trailer suspension model is needed to simulate its complex behaviour accurately.

Conclusion

The analytical model presented by Fancher *et al.* (1980) and Cebon (1986) show that it can model the stiffness and the hysteresis loop of the leaf spring accurately. However, the use of these analytical models in durability analyses are limited by the assumptions that the vertical force, that is developed by the spring, is divided equally between the front and the rear supports. This assumption may become invalid during non-symmetric loading or due to suspension configuration effects that causes the reaction forces between the front and rear not being equal.

3.1.2. Equivalent models

With this type of model the leaf spring is modelled as an equivalent system using a vertical spring (or a combination of series and parallel springs) with a damper and/or friction element. Figure 1.8 shows an example of such an equivalent model. The equivalent model aims at emulating the leaf spring by accounting for the different physical phenomena individually. For example, the stiffness of the spring is modelled by the spring and the hysteresis by the damper. An example of this modelling approach can be found in the study by Hoyle (2004) who models the leaf spring by using two springs in series, with a friction model between the two that would represent the two stiffness regimes of the leaf spring (see Figure 1.8)

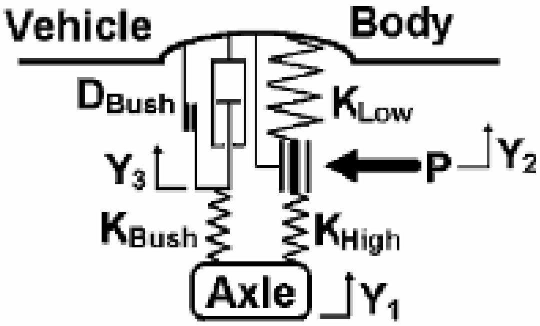

Figure 1.8. Equivalent leaf spring suspension model (Hoyle (2004))

Application in vehicle simulations

Hoyle (2004) extended his leaf spring model to include the relaxation and recovery regimes generated by the rubber bushes used in the suspension system. This was modelled as a spring in series with a damper/coulomb friction element. The study establishes the principle characteristics of the truck suspension and goes on to describe the linear and nonlinear models created to simulate the frequency response characteristics of the vehicle suspension. Comparison of the frequency response predictions with those of the actual vehicle revealed that the predictions of the nonlinear model were far better than the linear damped 5 DoF model. The model of Hoyle (2004) gives good results when the acceleration transmissibility frequency response of the suspension is analysed, giving good comparison between the sprung mass and unsprung mass natural frequencies.

The dynamic interaction between an articulated vehicle and surface undulations is investigated by ElMadany (1987) using the equivalent technique to model the leaf spring. The effects of the frictional force generated in the laminated springs, bump-stops, wheel hop, road characteristics, loading condition and vehicle speed on the ride comfort and the road safety are discussed and evaluated. ElMadany (1987) models the friction in the suspension in one of two ways: 1) A linear spring and friction damper acting in parallel (directly coupled friction damping). 2) A linear spring in parallel with an elastically friction damper (elastically coupled friction damping). No validation was done to verify that their models can indeed capture the behaviour of the suspension accurately.

Conclusion

The equivalent model may give good results when used in ride analysis, but this model would not give good results when used in durability analysis, as its load path is not correct.

3.1.3. Discrete methods (or Finite segment methods)

This method discretizes the leaf spring into rigid elements. The rigid elements are then connected by, for example, a torsion spring and damper. The characteristics of the torsion springs and dampers are then adjusted until the leaf spring model's force-displacement characteristic is the same, or within some acceptable accuracy, to the physical leaf spring's force-displacement characteristic. Figure 1.9 shows a three link model of a leaf spring with two torsion springs (the number of links refer to the amount of links used to represent the leaf spring and does not include the other links such as for the shackle).

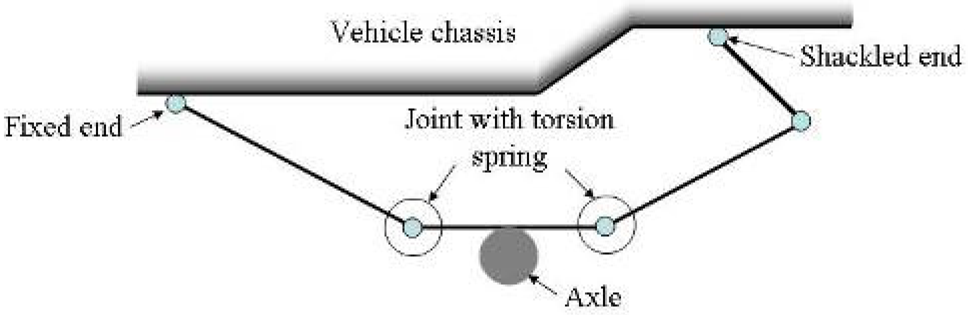

Figure 1.9. Discretized model of a leaf spring (Adapted from Huhtala *et al.*(1994))

Application in vehicle simulations

In the study by Huhtala *et al*. (1994) the aim was basically the same as that of Cole & Cebon (1994), being the prediction of the tire-road interaction forces. However, in the study by Huhtala *et al*. (1994) they model the multi-leaf spring as four links with two torsional springs, two bushings and a revolute joint. They state that when modelled in this way the model can represent the behaviour of a multi-leaf spring even when a braking force is applied to the wheel. They model the parabolic leaf spring in a similar manner. They show that the dynamic axle loads are much larger when multi-leaf springs are used compared to parabolic springs with dampers. Neither the model nor the sub-models were validated.

Yang *et al.* (2007) report a systematic methodology which is used to evaluate and improve vehicle ride comfort. The vehicle dynamics model of a tractor with tandem suspension is modelled and simulated in ADAMS. The modelling methods of nonlinear characteristic components and various road excitation inputs are introduced. These components include leaf springs, dampers and rubber bushes. In modelling the leaf spring they make the following assumptions in order to better reflect the damping characteristic of the leaf spring; (1) “Because each piece of leaf spring is a continuous flexible body, the discrete method can be used here to divide each spring into a number of quality modules, each of which can be regarded as a rigid quality and linked together with Timoshenko beams.” (2) “Adjacent leafs are clamped under normal working conditions, and only tangential direction friction happen because of the relative movement along the tangential direction”. (3) “The centres of all leafs are clamped by central bolts, so certain length of the central leaf spring can be treated as invalid length.” They showed correlation between the PSD weighted RMS of the seat acceleration, for laden and unladen cases, for different speeds. Simulation results show good agreement with the trends of experimental results but does not predict the values accurately.

Jayakumar *et al.* (2005) present a leaf spring model that can be used in road load simulations. They model the leaf spring in a similar way to the model shown in Figure 1.9. The model parameters are identified from static force-deflection test data. The advantage of this modelling method is that a simple model can be easily constructed to reproduce the kinematic and compliance properties of the actual leaf spring. They show correlation results for a static vertical test, a static longitudinal test and vertical random vibration. They also measure the vertical reaction force at the hanger bracket and shackle attachment points over severe proving ground durability events. The leaf spring model show good correlation compared to the test data even though the hysteresis loop could not be captured by the model. It may be that this spring has a very small hysteresis loop and therefore does not have an influence on the results.

Ekici (2005) compares the results of the three-link leaf spring model to test results. The geometry of this model consists of three rigid links with the leaf-spring compliance incorporated in the model through two nonlinear torsional springs at the centre-link joints. The model does not seem to be able to capture the hysteresis behaviour. They show the results of the acceleration obtained from experiments and simulation, but it is not clear from the paper where on the vehicle this acceleration measurement was taken.

Prasade *et al.* (2006) state that their experience with the 3 link leaf spring model is that it has difficulty predicting the lateral loads accurately. One of the reasons they contribute the lack of accuracy to, is that the 3 link leaf spring model cannot represent the roll behaviour of the actual suspension very well. However, Jayakumar *et al* (2005) suggests that the three-link leaf spring model can be used in durability simulations.

Conclusion

The studies that have been mentioned above, all used the discrete method to model the leaf spring. In all these studies the leaf spring had one fixed end and one shackled end. The applicability of the discrete segment method to model a leaf spring, configured as the suspension system of interest shown in Figure 1.4, is unknown and has to be investigated.

3.1.4. Finite element methods

The finite element and discrete methods are very similar. The distinction is made as the discrete method can be used directly in many rigid body dynamic software packages, whereas the finite element method requires additional software. The formulation of how the different elements are connected also differs between the two methods. Depending on the type of elements and the number of elements used the finite element method can become very computationally expensive. For details on the finite element method itself the reader is referred to the studies mentioned in the following paragraph.

Application in vehicle simulations

Often a combination of physical testing and analytical methods is used to obtain the load histories. This method is commonly called the hybrid load analysis method. Prasade *et al.* (2006) state that one of the important requirements of this method is an accurate mathematical representation of the suspension. They have a 3 link leaf spring model that has been used in various simulations. They however found that this modelling approach has difficulty predicting the lateral loads accurately. One of the reasons they contribute the lack of accuracy to, is that the SAE 3 link leaf spring model cannot represent the roll behaviour of the actual suspension very well. They use a beam element leaf spring model to address some of the limitations of the 3 link model. The 3 link and beam element model was subjected to various combinations of vertical, longitudinal and lateral loads. The two models give almost identical loads in the vertical and fore/aft direction, but the behaviour in the lateral direction is totally different. They show that the beam element model represents the roll stiffness of the actual suspension better than the 3 link model. The beam element was compared to measured forces and showed good correlation in the vertical direction and for braking events, but did not have the same good correlation for the acceleration events. They also show correlation for reaction forces in the vertical and lateral directions at the spring eye and shackle to frame attachments. It should be noted that no evidence is shown that this beam model can indeed predict the hysteresis loop correctly.

In the investigation of Sugiyama *et al.* (2006), a nonlinear elastic model of a leaf spring is developed for use in the simulation of multi-body vehicle systems. They develop a nonlinear finite element model of the leaf spring based on the floating frame of reference approach. They discuss the pre-stresses as well as the contact and friction that govern the nonlinear behaviour of the leaf spring. They conclude that their proposed leaf spring model, that includes the effect of windup, contact and friction between the spring blades, can effectively be used for assessing the dynamic stability of sports utility vehicles. No experimental model validation was performed to justify their conclusion.

Moon *et al.* (2007) developed a flexible multi-body dynamic model which can emulate the hysteretic characteristic and analyze the dynamic stress within a taper leaf spring. A finite element model of each leaf was created in MSC.Nastran which was then used to create a modal neutral file to create a flexible body of the leaf spring in ADAMS. Rigid dummy parts were attached at the places where the flexible bodies of the individual blades were in contact with one another in order to apply the contact model. This had to be done as contact could not be defined between two flexible bodies in the version of ADAMS used at the time of their study. Friction was defined in the contact model to represent the hysteretic characteristics of the leaf spring. They validated the leaf spring model by comparing the force-deflection curves for different excitation amplitudes. The results show good correlation.

Omar *et al*. (2004) state that accurate modelling of the leaf spring is necessary in evaluating ride comfort, braking performance, vibration characteristics and stability. They discuss two finite element methods that take into account the effect of the distributed inertia and elasticity, and use them to model the dynamic behaviour of leaf springs. They compare the predicted spring stiffness of their proposed model using the floating frame of reference formulation with the predictions of several other models: Equivalent lumped mass spring, equivalent beam cross-section, beam theory and the finite-element method. They state that the different techniques used to calculate the spring stiffness do not lead to the same results because different assumptions are used in each model. The assumptions made in their proposed model are as follows:

- The effect of the pre-strain due to bending of the blades during the assembly process of the spring is neglected. They believe that the pre-strains have the effect of increasing the stiffness of the leaf spring.
- The effect of the spring eyes, shackle arm, and the bushing elements are neglected.

They also discuss the importance of the number of modes that are included in the finite element model on the computational time and accuracy. Great effort was put into the model, but it was only compared to the results of other mathematical models. The ability of the proposed model to accurately represent, not only the spring stiffness, but also the hysteresis loop is not known as it was not validated against experimental data or data other than the stiffness values. Thus, there is not sufficient proof that the proposed model can indeed predict the hysteresis loop, accurately.

The paper by Li & Li (2004) presents a finite element algorithm to address the contact problem encountered in multi-leaf springs. According to them, “the most challenging part of stress analysis for the multi-leaf spring is perhaps to determine the contact status and pressure distributions between the contact faces of any two consecutive leaves”. To model the contact, a special type of interfacial element needs to be placed between adjacent blades. They state that the traditional analysis of leaf springs is based on classical beam theory due to its simplicity. However, the classic beam theory itself does not directly offer an analytical solution to the contact problems of layered members such as encountered in multi-leaf springs. They state that various approximations must be introduced to beam theory to address the contact problem. These include assumptions of concentrated load and continuous contact. The purpose of the work by Li & Li (2004) is to attempt to bridge the gap between the classical beam theory and the contact problem. As a preliminary study they ignore any frictional effects and concentrate on the distributions of the normal contact stresses. They validate their algorithm by comparing the predictions of their model with experimental results of the bending stress and the vertical load vs. deflection of the leaf spring. Good correlation was obtained for the bending stresses and the loading part of the vertical load vs. deflection results, but the model is not yet able to capture the hysteresis behaviour of the spring. Their study takes the full structure of the leaf spring into account and models it as a simply supported beam (pin and roller at the two supports respectively). It should be investigated whether the simply supported model of the leaf spring can indeed account for non-symmetric springs and more importantly for non-symmetric loads (e,g braking). Furthermore, it is unknown whether their model can account for leaf spring assemblies where the effective length changes.

Qin *et al*. (2002) presents detailed finite element modelling and analysis of a two-stage multi-leaf spring, a leaf spring assembly, and a Hotchkiss suspension using ABAQUS. Included in their models were the nonlinearities due to large deformations, the interleaf contact as well as friction. The spring and suspension characteristics such as spring rate, windup rate, roll rate, and roll steer were analyzed. The validation was done by comparing the force-deflection and strain-deflection results from experimental measurements and simulations by loading the spring in 15 steps. For the leaf spring assembly they compare the roll moment vs. roll angle and the steer angle vs. roll angle. All the comparisons showed good correlation. They did analyse the leaf spring assembly windup but did not validate it against experimental results.

Conclusion

All the studies mentioned here was concerned with modelling a leaf spring that was fixed at one end and shackled at the other end except for Li & Li (2004) who had it supported by a pin and roller. This however is equivalent to the fix and shackled end configuration. Prasade *et al* (2006) showed results indicating that the beam element model can be used in durability analysis.

3.1.5. Neural network models

Neural networks are computationally efficient mathematical models that can be trained, through input-output data sets, to emulate smooth nonlinear functions. A neural network consists of neurons that can be connected to form various types of networks. Figure 1.10 shows a simple neuron. A network of a number of these interconnected neurons is shown in Figure 1.11. The network shown is known as a feedforward neural network. A neural network is trained during which the adjustable variable weights (*w*) and biases (*b*) are adjusted until the neural network is able give the correct output for a specific input. After training, the neural network can be used to emulate the function which it has been trained with. More detail on neural networks will be given in Chapter 3.

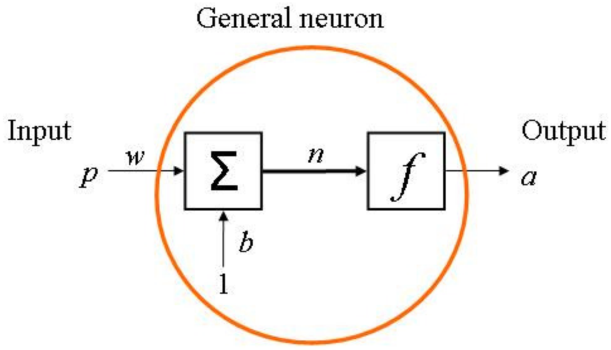

Figure 1.10. Simple neuron

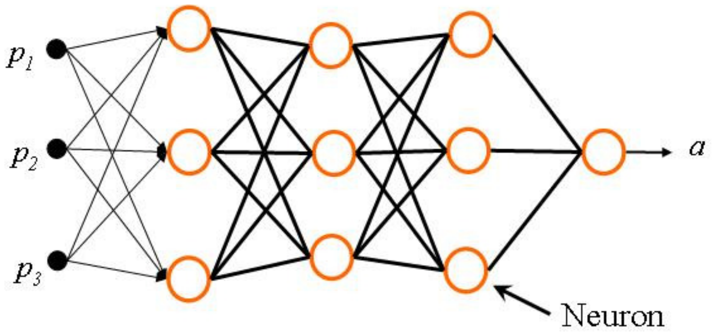

Figure 1.11. Feed forward neural network

Application in vehicle simulations

Leaf springs are known to have nonlinear and hysteresis behaviour. Ghazi Zadeh *et al*. (2000) state that this makes their mathematical modelling difficult and susceptible to a considerable amount of estimation errors. Ghazi Zadeh *et al* (2000) state that the force-deflection curves that characterizes a leaf spring is very difficult to emulate using neural networks. They state that a neural network approach is successful when a smooth function is emulated and when a set of data points, that are evenly scattered over the entire working space of the function variables, are available. This set of data points are required to construct a set of input-output data points which can be used to train the neural network with. They show that the recurrent neural network is able to emulate the leaf spring behaviour accurately after it is taught with a set of input-output data points. They showed that the neural network emulates the leaf spring well by comparing the neural network and their analytical model's results in both the time and frequency domains. They compared the force-displacement and the spectral density functions of the tire force, spring force and acceleration of the unsprung mass with the results obtained from an analytical model.

Conclusion

The study of Ghazi Zadeh *et al.* (2000) showed that a neural network can be trained to accurately emulate a leaf spring. It should be noted that in order for the neural network to accurately emulate the leaf spring a comprehensive set of data is required to train the neural network. This has to be kept in mind when the neural network is to be used but limited data is available on the leaf spring.

3.2. Summary of leaf spring modelling techniques

A summary of the different approaches that exists to model the leaf spring along with some of the advantages and disadvantages of each method as well as an indication of the validation that have been done for the particular approach, is given in Table 1.1.

Table1.1. Summary of leaf spring modelling techniques

Approach	Variations on approach	Advantages	Disadvantages	Validation
Discrete method	• Connect elements with Timoshenko beams (Yang et al., 2007)			• Correlation between the PSD weighted RMS of the seat accelerations for laden and unladen for 4 different speeds shows good agreement in trend but does not predict the values accurately. (Yang *et al,* 2007)
	• Connect segments with springs and dampers (Rill *et al*., 2003), (Milliken and Milliken, 2002), (Huhtala *et al*. ,1994)	• Able to predict behaviour under braking conditions (Huhtala *et al*., 1994)		
	• SAE three-link model (Jayakumar *et al*., 2005)	• Most common model according to Prasade *et al.* (2006) • Three-link leaf-spring model is easy to construct (Jayakumar *et al*., 2005) • Three-link model accurately represents the kinematic and kinetic behaviour of the physical leaf-spring (Jayakumar *et al*., 2005) • Three-link model is very simple and has fewer degrees-of-freedom (Jayakumar *et al*., 2005) • Three-link model simulations are very efficient and easy to perform without encountering any numerical difficulties by the ADAMS solver. (Jayakumar *et al*., 2005)	• Lateral loads very inaccurate (Prasade *et al*., 2006) • Cannot represent the roll behaviour very well (Prasade *et al*., 2006) • Inter-leaf friction not always included	• Correlation for static vertical test, a static longitudinal test and random vibration. Also measure the vertical reaction force at the hanger bracket and shackle attachment points. All these tests show good correlation. (Jayakumar *et al.*, 2005)
	• Extension of the SAE three-link model is to use nonlinear vertical spring in parallel to account for the nonlinear force-deflection behaviour in the vertical direction (Jayakumar *et al*., 2005) , (Ekici, 2005)		• Introduces spurious load path (Jayakumar *et al*., 2005)), (Ekici, 2005) • Likely misrepresentation of longitudinal behaviour makes this model unsuitable for application in road load simulation (Jayakumar *et al*., 2005), (Ekici, 2005)	
		• Requires only rigid body modelling capabilities that are available in most existing general-purpose multi-body computer codes (Sugiyama *et al*., 2006)	• Large number of discretized bodies in order to achieve accurate solutions. Leads to a large number of degrees of freedom (Sugiyama *et al*., 2006)	
			• No systematic and generally acceptable procedure for determining the number and properties of the discrete bodies, springs and dampers (Sugiyama *et al.*, 2006)	
Finite Element Method	• Full FEM model with contact and friction		• Computationally expensive • Use of these models impractical in multi-body vehicle simulations (Sugiyama *et al*., 2006)	

Approach	Variations on approach	Advantages	Disadvantages	Validation
	• Floating frame of reference (to model stiff leaf springs that experience small elastic deformations) (Omar *et al.*, 2004)	• Leads to a reduced order model that includes all significant deformation modes (Sugiyama *et al.*, 2006) and (Omar *et al.*, 2004)		
	• Absolute nodal coordinate formulation (to model soft leaf springs that experience large deformations) (Omar *et al.*, 2004)	• Enables more detailed finite-element models for the large deformation of very flexible leaf springs (Omar *et al.*, 2004)		
	• Beam element models	• Considered state-of-the-art (Jayakumar *et al.*, 2005), (Ekici, 2005) • Model gives good results in all directions (exception see disadvantages) (Prasade *et al.*, 2006)	• In situations where it is subjected to high fore/aft acceleration on high reverse braking events results not good (Prasade *et al.*, 2006) • Results in a very large, and extremely nonlinear model with a high number of degrees-of-freedom (Jayakumar *et al.*, 2005). • It showed accurate prediction in jounce condition but not in roll condition. (Tavakkoli, 1996)	• Shows correlation of vertical and lateral reaction forces at the spring eye and shackle attachment points (Prasade *et al.*, 2006)
	• FlexBody in Adams		• Contact can not be applied between flexible bodies in ADAMS (Moon *et al.*, 2007). Have to add dummy parts	• Compare force deflection curves for different excitation amplitudes. Show good correlation. (Moon *et al.*, 2007)
Simple Beam theory	• Simple beam theory has been extended to include large deflections, dual rate springs, stiffness modification due to shackles, initial camber and constant cross section (single leaf) design. (Cebon, 1986)	• Provide designers with estimates of large deflection spring rates	• Neglect interleaf friction	
Lumped Mass Spring Model			• This approach is too simple to take into account the effect of different deformation modes of the leaf spring in vehicle suspensions since all the spring characteristics are modelled by an equivalent spring constant (Sugiyama *et al.*, 2006) • The effect of the distributed inertia and stiffness of leaf springs are neglected (Sugiyama *et al.*, 2006) • The nonlinear characteristics of leaf springs due to contact and friction between blades cannot be captured (Sugiyama *et al.*, 2006)	

Approach	Variations on approach	Advantages	Disadvantages	Validation
Analytical model (Empirical model)		• Easy to implement • Can capture stiffness and hysteresis loop (Fancher *et al.*, 1980)	• Parameter values for the closest fit are strongly sensitive to the displacement amplitude. Not possible to fit equation to all hysteresis loops using single set of parameters. (Cebon, 1986) • Assumes reaction forces are equally divided between the front and rear supports	• Compared to experimental force-displacement data Fancher *et al.*, 1980) • Used in model to predict tire forces. Showed good correlation (Cole & Cebon, 1994)
Equivalent model			• Spurious load path	• Frequency response predictions showed good correlation with experimental data (Hoyle, 2004)
Neural Network models		• No need to model complex physical phenomenon's (for example friction or contact)	• Non-physics based model	

3.3. Conclusion

From the literature study it can be concluded that there is not a clear best leaf spring model. Different models exist and various studies have shown that these models can indeed represent aspects of the leaf spring accurately and give good results when used in simulations. It would seem that the type of model that can be used to give accurate predictions depend on the kind of parameters that are to be predicted. Thus, these models are very particular to the problem they are used in, and they may not give the same results in a different application, or when other sets of parameters are to be predicted.

Take for instance the model developed in the study by Hoyle (2004). They derive a model for the leaf spring that can take into account the different stiffness regimes, hysteresis, as well as the rubber bush relaxation. This model gives good results when the acceleration transmissibility frequency response of the suspension is analysed giving good comparison between the sprung mass and unsprung mass natural frequencies. However, this model is not adequate for predicting the forces that are transmitted to the chassis through the leaf spring suspension as this model does not have the same load path as the real leaf spring. Thus, this model is adequate when looking at the frequency response of the vehicle but when the forces between the suspension and the chassis need to be analysed the model becomes inadequate. This implies that there may be a number of ways to model a leaf spring but that only some of them may be useful in certain cases, thus making the leaf spring models problem dependent (see Figure 1.12). However, it should be kept in mind that a model should be as simple as possible and as complex as necessary, implying that the model be able to predict the forces acting on the chassis may be too complex to be used for predicting the frequency response of the vehicle, or vice versa. This may imply that it would be better to use one model for a specific problem and another for a different problem. This has the advantage that a model can be selected for a specific problem that gives the best computational efficiency and accuracy.

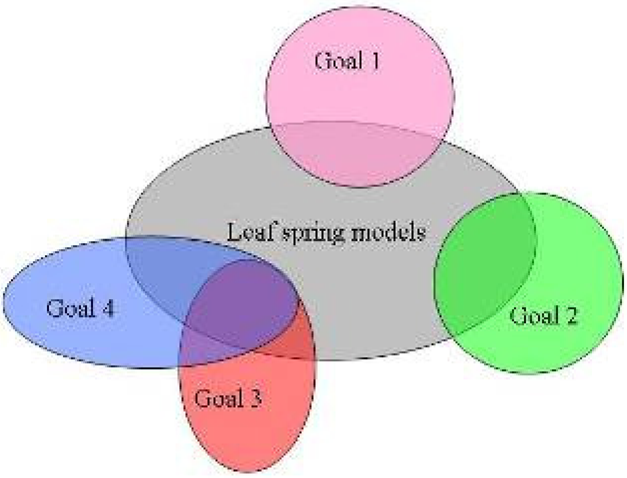

Figure 1.12. Applicability of models to simulation goals

The minimum requirements that are set for the leaf spring model in this study are that it has to be able to capture the spring stiffness and hysteresis loop of the leaf spring. In addition to these requirements, the model should preferably be able to account for changes in the load length of the leaf spring which affects the stiffness of the leaf spring (the loaded length and its effect on the stiffness of the leaf spring will be discussed in Chapter 2). As already stated the model of the leaf spring should be accurate yet computationally efficient. Unfortunately, the accuracy and computational efficiency of each model is not known and a modelling technique cannot be selected according to these two criteria. It is however expected that the finite element method might be the least computationally efficient model. The majority of the leaf spring models in the literature study considered the leaf spring configuration where the leaf spring is attached to the vehicle chassis using the fixed-shackled end configuration. The suspension system used in this study does not use the fixed-shackled end configuration to attach the leaf spring to the chassis, instead the leaf spring is supported by the hangers, as was shown in Figure 1.4. The ability of the models used to model a leaf spring in the fixed-shackled end configuration might not be able to capture the load length change effect. Chapter 3 will address the modelling of the leaf spring in detail.

4. Overview of study

A brief overview of the study is given here with the layout of the study shown in Figure 1.13.

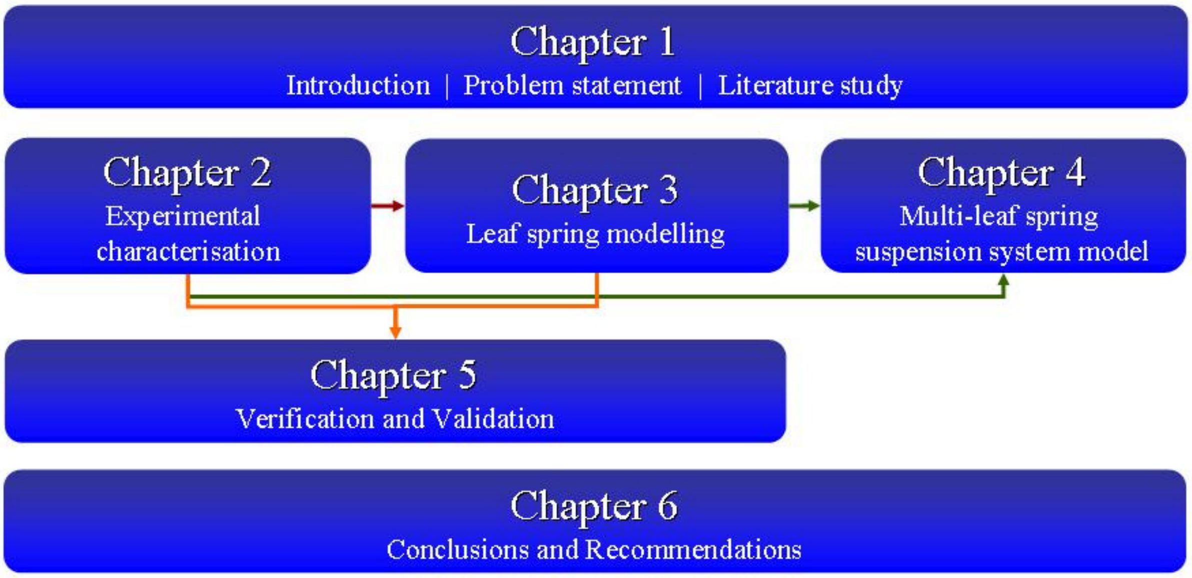

Figure 1.13. Overview of study

In this chapter, Chapter 1, the reader was introduced to the problem. A literature study was conducted that showed that many different modelling techniques exist that can be used to model a leaf spring. Because this study is concerned with obtaining a validated model of the leaf spring and of the suspension system, two primary elements are of concern i.e. the physical system of interest and the simulation model. The process of obtaining a validated model implies that the mathematical model has to be created and the experimental data gathered which is then used to parameterise and validate the model. The experimental characterisation that was performed to collect the required experimental data for model validation is presented and discussed in Chapter 2.

Chapter 3 is concerned with leaf spring modelling. Two fundamentally different modelling approaches is used i.e. a physics-based and non physics-based modelling approach. For the physics based model a novel model was developed that uses a macro modelling approach. Two formulations are presented for the elasto-plastic leaf spring model that are parameterised by merely extracting three or four parameters from experimental data depending on the formulation used. The elasto-plastic leaf spring model is used to model both the multi-leaf spring and the parabolic leaf spring. A method is also proposed that can account for the changes in the stiffness of the leaf spring due to changes in the loaded length. This method can be used together with the elasto-plastic leaf spring model which results in a model that can capture the stiffness, the hysteresis as well as the changes in loaded length of the leaf spring. In additional to the physics-based elasto-plastic leaf spring model a non physics-based method was used that uses neural networks to emulate the leaf spring. The neural network was only used to emulate the multi-leaf spring in this study but can be used to emulate the parabolic leaf spring and is expected to give similar results.

The validated elasto-plastic leaf spring model is used to model a simplified version of the suspension system in Chapter 4. This model of the simplified suspension system is created to verify whether the subsystem model using the elasto-plastic leaf spring model is able to predict the forces that act onto the chassis. This subsystem model is validated using the experimental data obtained during the experimental characterisation performed in Chapter 2.

Chapter 5 discusses the verification and validation process and presents a new validation metric that can be used in a quantitative validation process. The validation metric is then used to calculate the accuracy of the elastic-nonlinear formulation of the elasto-plastic leaf spring model and the neural network model. The accuracy as well as the efficiency of the two modelling techniques are presented and compared. The study is concluded in Chapter 6 with the final conclusions and recommendations for future work.