CHAPTER 4

CONTROLLER DESIGN METHODS

The design methods for multi-loop as well as multivariable controllers for the LTI model of the Steckel hot rolling process are discussed in this chapter. First the design method of multi-loop PID and PI control of the plant is given. The design methodology of a multivariable controller is discussed in the second part of this chapter. As a multivariable control method an H_∞ controller was used.

4.1 PID/PI Controllers

4.1.1 Introduction

The application of SISO-PID/PI control to close the loop around the transfer functions on the diagonal of the transfer function matrix of the linear model in Eq. 3.1 was considered as an option for a controller. This was done because the inputs to these transfer functions have a more direct influence on the corresponding output than the other input variables of the MIMO model have on the same outputs. This is apparent from the simulations for SID purposes in chapter 3. The transfer functions on the diagonal of the linear model in section 3.4.1 are of two different forms. A PID synthesis using an affine parameterization was chosen to design the SISO controller for the transfer function $g_{11}(s)$ in the first LTI model. For the transfer functions $g_{22}(s)$ and $g_{33}(s)$ PI synthesis by using pole assignment was applied to design SISO controllers. In subsections of 4.1 these methods are briefly discussed.

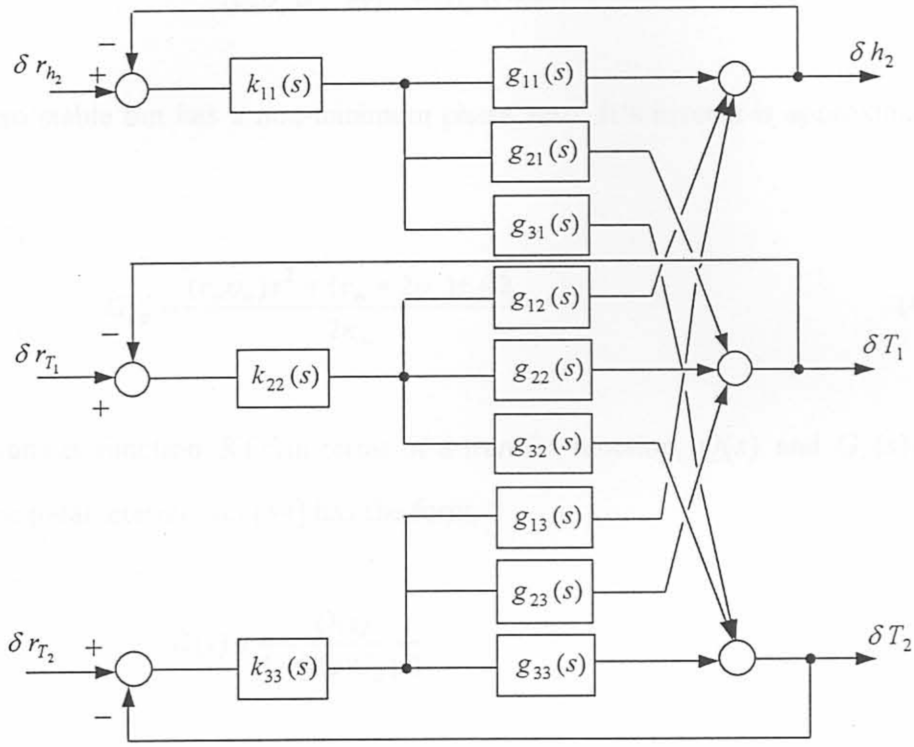

Figure 4.1: Blockdiagram for diagonal PID/PI control.

4.1.2 Controller Design Method for $g_{11}(s)$

4.1.2.1 Controller for Step Input

The time delay of $g_{11}(s)$ was replaced by a first order Pade approximation, given by,

$$e^{-s\tau_o} \approx \frac{2 - s\tau_o}{2 + s\tau_o} . \tag{4.1}$$

With this approximation the transfer function $g_{11}(s)$ which is of the form,

$$g_{11}(s) = G_o(s) = \frac{\kappa_o e^{-s\tau_o}}{\upsilon_o s + 1} , \tag{4.2}$$

became

$$G_o \approx G_{op}(s) \equiv \frac{(-\tau_o \kappa_o)s + 2\kappa_o}{(\tau_o \upsilon_o)s^2 + (\tau_o + 2\upsilon_o)s + 2}. \tag{4.3}$$

This model is also stable but has a non-minimum phase zero. It's inverse is approximated by,

$$G_{op}^i = \frac{(\tau_o \upsilon_o)s^2 + (\tau_o + 2\upsilon_o)s + 2}{2\kappa_o}. \tag{4.4}$$

The controller transfer function $K(s)$ in terms of a transfer function, $Q(s)$ and $G_o(s)$ and based on an affine parameterization [34] has the form,

$$K(s) = \frac{Q(s)}{1 - Q(s)G_o(s)}. \tag{4.5}$$

With $Q(s)$ biproper an F_Q with relative degree 2 is chosen as,

$$F_Q = \frac{1}{\alpha_2 s^2 + \alpha_1 s + 1}. \tag{4.6}$$

The unity feedback controller then results as,

$$K_p(s) = \frac{Q(s)}{1 - Q(s)G_{op}(s)} = \frac{F_Q(s)G_{op}^i(s)}{1 - F_Q(s)G_{op}^i(s)G_{op}(s)} = \frac{(\tau_o \upsilon_o)s^2 + (\tau_o + 2\upsilon_o)s + 2}{(2\kappa_o \alpha_2)s^2 + (2\kappa_o \alpha_1 + \tau_o \kappa_o)s}. \tag{4.7}$$

For the PID controller of the form,

$$K_{PID}(s) = \kappa_p + \frac{\kappa_i}{s} + \frac{\kappa_d s}{\tau_d s + 1}, \tag{4.8}$$

the PID controller parameters become:

$$\kappa_p = \frac{2\tau_o \alpha_1 + 4\upsilon_o \alpha_1 + \tau_o^2 + 2\tau_o \upsilon_o - 4\alpha_2}{4\kappa_o \alpha_1^2 + 4\kappa_o \alpha_1 \tau_o + \tau_o^2 \kappa_o}, \tag{4.9}$$

$$\kappa_i = \frac{2}{2\kappa_o\alpha_1 + \tau_o\kappa_o}\ , \tag{4.10}$$

$$\kappa_d = \frac{(2\kappa_o\alpha_1 + \tau_o\kappa_o)^2(\tau_o\upsilon_o) - (2\kappa_o\alpha_2)(2\kappa_o\alpha_1 + \tau_o\kappa_o)(\tau_o + 2\upsilon_o) + 8\kappa_o^2\alpha_2^2}{(2\kappa_o\alpha_1 + \tau_o\kappa_o)^3}\ , \tag{4.11}$$

$$\tau_d = \frac{2\alpha_2}{2\alpha_1 + \tau_o}\ . \tag{4.12}$$

4.1.2.2 Compensator for Ramp Input

A compensator connected in series with the PID controller $K(s)$, as can be seen from Fig.4.2, was used to make the control system a type 2 system which is able to give a zero steady state error on the output for a ramp input. The purpose of the fast zero in the compensator is to draw the additional root locus branch, which is caused by the pole at $s = 0$, into the stable left half s-plane (LHP).

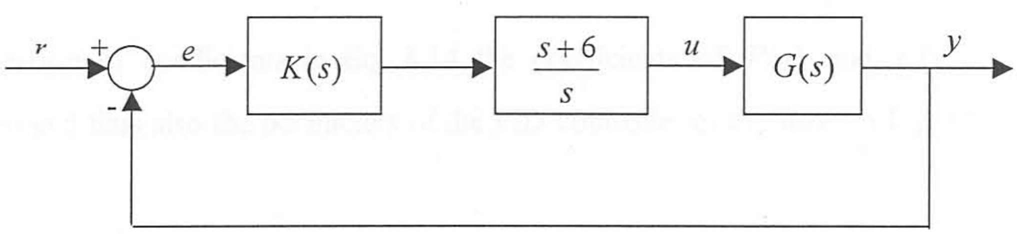

Figure 4.2: SISO feedback configuration with compensator.

4.1.3 Controller Design Method for $g_{22}(s)$ and $g_{33}(s)$

A SISO pole assignment technique, based on a polynomial approach [34], was used to find the PI controller parameters for the controllers for $g_{22}(s)$ and $g_{33}(s)$.

If the controller and model transfer functions are respectively,

$$K(s) = \frac{P(s)}{L(s)} \quad \text{and} \quad G_o(s) = \frac{B_o(s)}{A_o(s)}\ , \tag{4.13}$$

then for the PI synthesis of this approach $A_{cl}(s)$ can be an arbitrary (chosen) polynomial of degree $n_c = 2n-1$ and the polynomials $P(s)$ and $L(s)$, with degrees $n_p = n_l = n$, such that,

$$A_o(s)L(s) + B_o(s)P(s) = A_{cl}(s)\ . \tag{4.14}$$

$A_{cl}(s)$ is a polynomial, of which the roots represent the closed loop poles. More on the choice of the closed loop poles is said in chapter 5 where the controller design is discussed.

For the PI synthesis the polynomials $P(s)$ and $L(s)$ were chosen as,

$$P(s) = p_1 s + p_0\ , \tag{4.15}$$

$$\overline{L}(s) = l_1 s + l_0\ , \tag{4.16}$$

$$L(s) = s\overline{L}(s)\ . \tag{4.17}$$

By comparison of coefficients in Eq. 4.14 the coefficients of $P(s)$ and $\overline{L}(s)$ can be determined and thus also the parameters of the PID controller of the form in Eq. 4.8 such that,

$$\kappa_p = \frac{p_1 l_0 - p_0 l_1}{{l_0}^2}\ , \tag{4.18}$$

$$\kappa_i = \frac{p_0}{l_0}\ , \tag{4.19}$$

$$\kappa_d = \frac{-p_1 l_0 l_1 + p_0 l_1^2}{l_0^3}\ , \tag{4.20}$$

$$\tau_d = \frac{l_1}{l_0}. \tag{4.21}$$

For the PID controller of the form

$$K_{PID}(s) = \kappa_c \left(\frac{\tau_I s + 1}{\tau_I s} \right) \left(\frac{\tau_D s + 1}{\alpha \tau_D s + 1} \right), \tag{4.22}$$

the parameters in terms of the polynomial coefficients of $P(s)$ and $\overline{L}(s)$ become

$\kappa_c = \frac{p_o}{l_o}$, $\tau_I = 1$, $\tau_D = 0$ and $\alpha = \infty$

thus a PI controller of the form

$$K_{PI}(s) = \kappa_c \left(\frac{\tau_I s + 1}{\tau_I s} \right) \tag{4.23}$$

results.

4.2 H$_\infty$ Controller

4.2.1 The H$_\infty$ Control Problem

Since a MIMO control method is discussed in this section, small letters and large letters used in the figures here denote vectors and matrices respectively. The H$_\infty$ controller design method, used to compute a multivariable controller, is confined to the H$_\infty$ space. The norm of the space, H$_\infty$, is defined as,

$$\|G\|_\infty := \sup_{\omega \in \Re} \overline{\sigma}(G(j\omega)), \tag{4.24}$$

i.e. the least upper bound of G [28].

When computing an admissible controller, $K(s)$, the H$_\infty$ space norm of the closed loop transfer function $H_{\tilde{y}_1 \tilde{u}_1}$ (see Fig. 4.3) has to be minimized such that, given $\gamma > 0$, where $\bar{\sigma}$ denotes the maximum singular value, ω_c the crossover frequency and S and C, the sensitivity and complementary sensitivity functions of the standard feedback configuration without weights (Fig. 4.5) respectively .

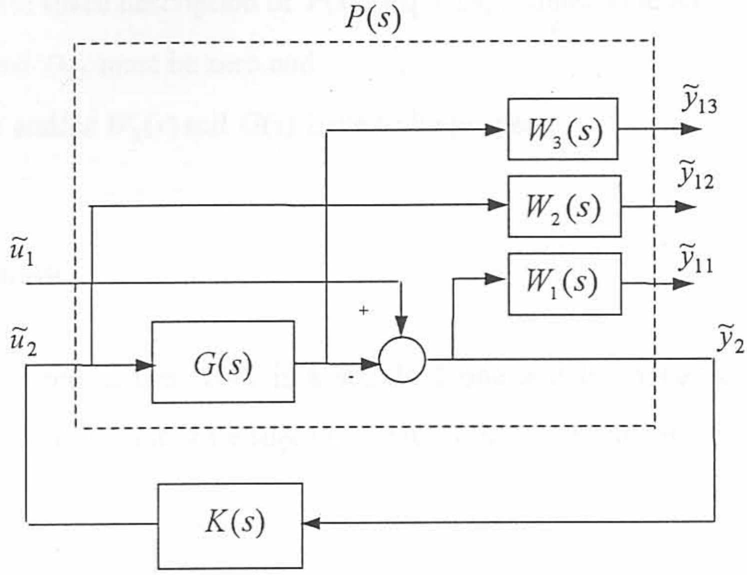

Figure 4.4: Standard feedback configuration with weights.

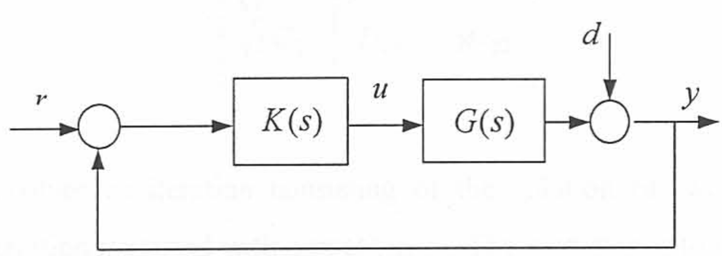

Figure 4.5: Standard feedback configuration

In Fig. 4.4 $\tilde{u}_1 = u$ is the exogenous input vector and $\tilde{y}_{13} = y$ is the output vector. Let plant $P(s)$ be given in its state space realization as

$$P(s) := \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{4.28}$$

$$\left\| H_{\tilde{y}_1\tilde{u}_1} \right\|_\infty \leq \gamma \qquad (4.25)$$

with

$$\min_{K(s)} \left\| H_{\tilde{y}_1\tilde{u}_1} \right\|_\infty = \gamma_{optimal} \quad \text{with } \gamma_{optimal} < \gamma$$

The fixed parameter type controller obtained will thus be sub-optimal.

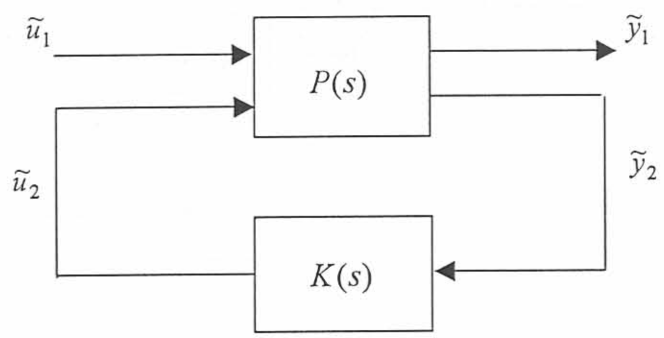

Figure 4.3: Synthesis block diagram

4.2.2 Weights for Plant Augmentation

As shown in Fig. 4.4 plant $G(s)$ was in this problem augmented by weighting functions $W_i(s) = c_i(s) \cdot I_{nxn}$ $(i = 1,2,3)$ to form plant $P(s)$. Two bounds were taken into account when selecting weights: The performance bound given by

$$\bar{\sigma}[S(j\omega)] \leq \bar{\sigma}[W_1^{-1}(j\omega)] \ , \qquad \omega < \omega_c \qquad (4.26)$$

and the robustness bound given by

$$\bar{\sigma}[C(j\omega)] \leq \bar{\sigma}[W_3^{-1}(j\omega)] \ , \qquad \omega > \omega_c \qquad (4.27)$$

To be able to apply the H$_\infty$ synthesis successfully, $P(s)$ must have the following properties:

i) the state space description of $P(s)$, Eq.4.28, is detectable and stabilizable,

ii) D_{11} and D_{22} must be zero and

iii) $W_2(s)$ and/or $W_3(s)$ and $G(s)$ have to be proper.

4.2.3 H$_\infty$ Synthesis

The H$_\infty$ synthesis used in this work is a standard one and is applied to the augmented plant, $P(s)$, with the following state space realization scaled by the iteration parameter, γ:

$$\widetilde{P}(s) = \left[\begin{array}{c|cc} A & \frac{1}{\sqrt{\gamma}}B_1 & \sqrt{\gamma}B_2 \\ \hline \frac{1}{\sqrt{\gamma}}C_1 & \frac{1}{\gamma}D_{11} & D_{12} \\ \sqrt{\gamma}C_2 & D_{21} & \gamma D_{22} \end{array}\right] \tag{4.29}$$

The synthesis involves an iteration consisting of the solution of two modified Riccati equations. The iteration is started with a guess in γ. The next step is to determine X_∞ and Y_∞, denoting the unique, real, symmetric solutions of the following two algebraic Riccati equations (ARE).

For X_∞ the ARE is:

$$(A - B_2 D_{12}^T C_1)^T X_\infty + X_\infty (A - B_2 D_{12}^T C_1) - X_\infty (B_2 B_2^T - B_1 B_1^T) X_\infty + \widetilde{C}_1^T \widetilde{C}_1 = 0 \tag{4.30}$$

with

$$\widetilde{C}_1 = (I - D_{12} D_{12}^T) C_1 \ .$$

For Y_∞ the ARE is:

$$(A - B_1 D_{21}^T C_2)Y_\infty + Y_\infty (A - B_1 D_{21}^T C_2)^T - Y_\infty (C_2 C_2^T - C_1 C_1^T) Y_\infty + \widetilde{B}_1^T \widetilde{B}_1 = 0\,, \qquad (4.31)$$

with

$$\widetilde{B}_1 = B_1 (I - D_{21}^T D_{21})\,.$$

The parameter γ is increased stepwise for every iteration until the ARE solutions are positive semi-definite, i.e.

$$X_\infty \geq 0 \;, Y_\infty \geq 0 \text{ and } \overline{\lambda}(Y_\infty X_\infty) \leq 1$$

where $\overline{\lambda}$ denotes the maximum eigenvalue.

The controller finally obtained in its state space description is

$$\hat{K}(s) = \left[\begin{array}{c|c} A - K_F C_2 - B_2 K_C + Y_\infty C_1^T (C_1 - D_{12} K_C) & K_F \\ \hline K_C & 0 \end{array}\right]$$

where

$$K_F = (Y_\infty C_2^T + B_1 D_{12}^T)\,,$$
$$K_C = (B_2^T X_\infty + D_{12}^T C_1)(I - Y_\infty X_\infty)^{-1}\,.$$