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Finite Element Developments and Applications in Structural Topology Optimization

by

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Summary

Title:	Finite element developments and applications in structural topology optimization
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In this two-part study, developments in finite element technology and the application thereof to topology optimization are investigated. Ultimately, the developed finite elements and corresponding topology optimization procedures are aimed at, but not restricted to, aiding the design of piezoelectrically driven compliant mechanisms for micropositioning applications. The objective is to identify and exploit existing, or to develop new, finite element technologies to alleviate the numerical instabilities encountered in topology optimization. Checkerboarding and one-node connected hinges are two commonly encountered examples which can directly be attributed to inadequacies or deficiencies in the finite element solution of structural problems using 4-node bilinear isoparametric finite elements (denoted Q4). The numerical behaviour leading to checkerboard layouts stems from an over-stiff estimation of a checkerboard patch of Q4 elements. The numerical model of a one-node connected hinge using Q4 elements, on the other hand, possesses no (or very little) stiffness in rotation about the common node.

In the *first* part of the study, planar finite elements with in-plane rotational (drilling) degrees of freedom are investigated. It is shown that the skew-symmetric part of the stress tensor can directly be used to quantitatively assess the validity of the penalty parameter γ, which relates the in-plane translations to the rotations. Thereafter, the variational formulations used to develop these planar finite elements with drilling degrees of freedom are extended to account for the piezoelectric effect. Several new piezoelectric elements that include in-plane rotational degrees of freedom (with and without assumed stress and electric flux density) are implemented, evaluated and shown to be accurate and stable.

Furthermore, the application of alternative reduced order integration schemes to quadratic serendipity (Q8) and Lagrangian (Q9) elements is investigated. Reduced or selective reduced integration schemes are often used to enhance element accuracy by 'softening' higher order deformation modes. However, application of reduced integration schemes to Q8 and Q9 elements is usually accompanied by element rank deficiencies. It is shown how the application of five and eight point modified integration schemes preserve the accuracy benefits of reduced integration, while preventing element rank deficiencies.

In the *second* part of the investigation, the salient features of elements with drilling degrees are utilized in two schemes to prevent, or improve the modelling of, one-node connected hinges. In principle, the first scheme uses the rotations computed at interior nodes to detect excessive rotations at suspect nodes. The second scheme essentially replaces planar elements forming a one-node hinge, where appropriate, with a more realistic beam model of the material layout while other elements in the mesh are modelled using planar elements as usual.

Next, the dependence of optimal topologies on element formulation is demonstrated. Attention is especially paid to plate and shell applications. It is shown that Mindlin-Reissner based elements, which employ selective reduced integration on shear terms, are not reliable in topology optimization problems. Conversely, elements based on an assumed natural strain formulation are shown to be stable and capable of reproducing thin plate topology results computed using shear-rigid elements. Furthermore, it is shown that an *ad hoc* treatment of rotational degrees of freedom in shell problems is sensitive to the related adjustable parameter, whereas optimal topologies, using a proper treatment of drilling degrees of freedom are not.

Finally, the use of reduced order integration schemes as a strategy to reduce the stiffness of a checkerboard patch of elements is considered. It is demonstrated that employing the five and eight point integration schemes, used to enhance the accuracy of Q8 and Q9 elements, also significantly reduce the stiffness of a checkerboard patch of elements, thereby reducing the probability of observing checkerboard layouts in optimal topologies.

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To **Michele**, my wife, my love and my best friend.

> Let me not to the marriage of true minds
> Admit impediments. Love is not love
> Which alters when it alteration finds,
> Or bends with the remover to remove.
> O no, it is an ever fixed mark
> That looks on tempests and is never shaken;
> It is the star to every wand'ring barque,
> Whose worth's unknown although his height be taken.
> Love's not time's fool, though rosy lips and cheeks
> Within his bending sickle's compass come;
> Love alters not with his brief hours and weeks,
> But bears it out even to the edge of doom.
> If this be error and upon me proved,
> I never writ, nor no man ever loved.
> – William Shakespeare

Shell, these words are as true now as they were the day I first recited them to you, over 13 years ago. I truly love you and I dedicate this thesis to you.

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