

**Evaluation of Earth gravity field models used for precise  
satellite orbit determination through applications of  
Satellite Laser Ranging data**

by

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Evaluation of Earth gravity field models used for precise satellite orbit determination through applications of Satellite Laser Ranging data

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## **Abstract**

One of the applications of the Satellite Laser Ranging (SLR) technique is the derivation of gravity field models; these models have various geophysical and geodynamical applications. Gravity field modelling has reached a new era where the latest satellite missions (CHAMP, GRACE and GOCE) are thought to provide significant improvement of global gravity field information in terms of quality and spatial resolution. In particular, the recent satellite missions carry on-board Global Navigation Satellite System (GNSS) receivers, accelerometers, K/Ka-band microwave system (e.g. in GRACE) and gradiometers (e.g. in GOCE) allowing measurements of gravity field with unprecedented accuracy in contrast to the unsteady and fragmented orbit tracking by unevenly distributed SLR ground stations.

Numerous gravity field models have been derived based on the newly available data sets by various research groups globally. Due to the availability of high quality SLR and satellite data, some of the older gravity field models are being updated as new models with higher degree and order are developed. Notwithstanding the significant progress in gravity field modelling, research focusing on assessing the accuracy and precision of the existing gravity field models has largely remained insufficient. The difference between the observed and computed satellite orbit (which is often expressed as the O-C range residuals) is used as a parameter for Precise Orbit Determination (POD) of satellites. Furthermore, O-C range residuals computed during SLR analysis are used as proxy parameters for evaluating the accuracy of gravity field models.

The work presented in this thesis firstly reviewed and evaluated the accuracy of gravity field models released between 1990 and 2008. The accuracy of the gravity field models was examined by analysing the O-C residuals computed from LAGEOS 1 and 2 data analysis based on a set of twelve gravity field models. The results demonstrated that in general, there has been an improvement in the accuracy of gravity field models released between 1990 and 2008 by a factor of 2 based on improvements in the O-C residuals. Additionally, the influence of SLR tide parameterization (the IERS 2010 solid Earth and pole tide models) on the O-C residuals across five gravity field models has been assessed and results illustrate that the solid Earth and pole tides parameterization influence on the O-C residuals is dependent on the type of gravity field model. In order to ascertain the significance of mean differences in the Standard Deviations (SD) of O-C residuals based on the tide parameterization options, the student's *t*-test was used. Results suggest that in general the O-C residuals derived from SLR LAGEOS 1 data have insignificant mean SD differences across the tide parameterizations. On the other hand analysis of SLR observations of LAGEOS 2 resulted in statistically significant mean SD differences in the O-C based on EIGEN-CG03C, EGM2008 and AIUB-GRACE01S gravity field models. The  $J_2$  coefficient forms part of the SLR Data Analysis Software (SDAS) package output products and was investigated in this thesis due to its role in understanding mass-redistribution within the Earth system (i.e. the equatorial bulge due to centrifugal force and rotation). In particular, the  $J_2$  coefficient computed from SLR analysis of LAGEOS 1 and 2 data sets and based on the four selected gravity field models were compared with a priori  $J_2$  coefficients from the four models and those published in the literature. The results indicated that the  $J_2$  coefficients computed from the SDAS package were in agreement with the published coefficients. For geophysical applications, the relationship between the  $J_2$  parameter and LOD and AAM was investigated by use of data adaptive analysis methodology (the empirical mode decomposition). The results demonstrated that some degree of synchronization exists between the signal components of  $J_2$  and LOD and  $J_2$  and AAM.

**Keywords:** Satellite Laser Ranging tracking, Earth's gravitational field, gravity field models, orbit sensitivity analysis, orbit parameter estimation,  $J_2$  spherical harmonic coefficient.

### List of publications

The following contributions have been published and/or in/to various peer review journals as part of this work.

1. Investigating the effect of tide parameterization on the accuracy of gravity field models (Submitted).
2. Analysis of Earth's oblateness and geophysical excitation functions related to polar motion (Submitted).
3. Botai, M.C. and Combrinck, L. Global geopotential models from Satellite Laser Ranging data with geophysical applications: A review. *South African Journal of Science*. 2012:108(3/4); 1-10.
4. Botai, M.C. and Combrinck, L. Investigating the accuracy of gravity field models using Satellite Laser Ranging data. *South African Journal of Geology*, 2011; 114.3-4:539-544.
5. Botai, M.C. and Combrinck, L. Investigating the variability of Earth gravity field's  $J_2$  spherical harmonic coefficient using Satellite Laser Ranging data. 11<sup>th</sup> SAGA Biennial Technical Meeting and Exhibition, Swaziland, 2009:603-606.

## Declaration

I, Mihloti Christina Botai, hereby declare that the work on which this thesis is based, which I hereby submit for the degree Doctor of Philosophy, Faculty of Natural and Agricultural Sciences at the University of Pretoria, is my own work except where acknowledgements indicate otherwise. This work has not previously been submitted by me for another degree at this or any other tertiary institution.

.....

7 February 2013

## **Dedication**

I dedicate my dissertation work to my wonderful family, specially my husband Dr. Joel Botai for his unconditional love, wisdom, patience and support during my PhD research and to our precious children (particularly our daughter, Mong'are Palmira whose little life journey began during the course of my research), you are the joy of our lives. A special dedication goes to my mother and grandfather who passed away during the course of my doctorate work. I also dedicate this dissertation to my supervisor Prof. Ludwig Combrinck for believing in me, without his support and guidance none of this work would have been possible.

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## Acronyms

AAM	: Atmospheric Angular Momentum
BE-B	: Beacon Explorer-B
CDDIS	: Crustal Dynamics Data Information System
CGS	: Centro de Geodasia Spaziale
CHAMP	: CHALLENGING Minisatellite Payload
CoM	: Centre-of-Mass
CRF	: Celestial Reference Frame
DORIS	: Doppler Orbitography and Radiolocation Integrated by Satellite
EIGEN	: European Improved Gravity model of the Earth by New techniques
EEMD	: Ensemble Empirical Mode Decomposition
EMD	: Empirical Mode Decomposition
EOP	: Earth orientation parameters
GEM	: Goddard Earth Models
GGM	: Global Geopotential Model
GFZ	: GeoForschungs Zentrum
GLONASS	: Global Orbiting Navigation Satellite System
GNSS	: Global Navigation Satellite Systems
GRACE	: Gravity Recovery and Climate Experiment
GSFC	: Goddard Space Flight Center
GOCE	: Gravity field and steady-state Ocean Circulation Explorer
GPS	: Global Positioning System
HartRAO	: Hartebeesthoek Radio Astronomy Observatory
IAG	: International Association of Geodesy
IERS	: International Earth Rotation and Reference System
ILRS	: International Laser Ranging Service
IMF	: Intrinsic Mode Functions
JCET/GSFC	: Joint Centre for Earth System Technology/Goddard Space Flight Centre
JGM	: Joint Gravity Models



LAGEOS	: LAsEr GEODynamics Satellite
LOD	: Length-Of-Day
MSL	: Mean Sea Level
NCAR	: National Center for Atmospheric Research
NCEP	: National Centers for Environmental Prediction
NIMA	: National Imagery and Mapping Agency
O-C	: Observed minus Computed
OSU	: Ohio State University
PGR	: Post-Glacial Rebound
POD	: Precise Orbit Determination
PPN	: Parameterized Post Newtonian
SBA	: Special Bureau for the Atmosphere
SDAS	: SLR Data Analysis Software
SLR	: Satellite Laser Ranging
TAI	: International Atomic Time
TEG	: Texas Earth Gravity
TOF	: Time-Of-Flight
UTC	: Universal Time
VLBI	: Very Long Baseline Interferometry
ZD	: Zenith Delay