Chapter 6

Conclusion

We have presented an investigation into the theoretical and practical usefulness of the LULU operators on multidimensional arrays and the resulting Discrete Pulse Transform. An initial summary of the grounding theory of the LULU operators was presented in Chapter 2. In Section 3.4 we provided an in depth look at the theoretical soundness of the nonlinear decomposition obtained via the recursive application of the LULU operators over increasing window or neighbourhood size, that is, the Discrete Pulse Transform. This work is original and new and has been written up as an article. Chapter 4 provides an extensive look at the concept of a scale-space which has become a very prominent phenomenon is image analysis literature. We delve into a history of scale-spaces and specifically the development of the most common scale-space, the Gaussian scale-space. In Section 4.6 we provide new theory on a missing link in the theory of scale-spaces, namely an axiomatic definition of a scale-space operator and the derived scale-space from this operator. In Section 4.7 we define the LULU scale-space related to the DPT and show that it aligns with the axiomatic theory in Section 4.6. Sections 4.8.2 and 4.8.3 as well as Chapter 5 investigate the practical significance of the LULU scale-space in feature detection, image segmentation and improving image quality, respectively, to ensure its usefulness in the fundamental requirements of image analysis. The work in these sections is original and new. Specifically the new results are:

- The consistent decomposition property of a nonlinear decomposition (Section 3.4.3).
- Measuring the ability of the LULU smoothers to smooth an array in

the sense expected (Section 3.4.5).

- Definition of a scale-space operator and scale-space (Section 4.6).
- The derivation of the LULU scale-space (Section 4.7).
- The relationship between sharpening an image and the DPT (Section 5.2).
- Best approximation theory for the LULU operators (Section 5.3).
- Noise removal ability of the LULU operators (Section 5.4).
- Useful feature point detection using the DPT (Section 4.8.2).
- Segmentation with the LULU scale-space (Section 4.8.3).

We have a number of publications from this work namely [8], [56], [53], [6], [7] and [54], and an article under review, [55]. In addition the following research is still open to investigation

1. In [244], it is described that any image can be represented as a linear combination of some basis. If a method can be determined such that most coefficients are zero when adjusted in some way the result is a compression. The bulk of the DPT pulses are in the lowest scales so by compressing these scales in some manner a useful LULU compression may be possible.
2. A faster, hopefully real-time, 1-, 2- and 3D implementation of the DPT is already underway.
3. The connectivity concepts discussed in Section 3.2 should also be investigated for improved image analysis with the DPT, specifically the multi-scale connectivity presented therein. An implementation of the DPT allowing for alternating connections will allow for investigation into ideas such as this.
4. Another idea for improving image quality as discussed in Section 5 is to add the pulses of the DPT back in order of some measure on the pulses instead of simply adding back full scale levels for partial image reconstruction. An optimal value for the level of smoothing n can be found with further research.

5. As mentioned in Chapter 4.8.2 the scale-space life-time and signatures may provide an indication of whether a pixel is noise, texture, small detail, large detail etc. Image segmentation taking this into account may prove fruitful.

6. A sound theoretical study of the distributional properties of the LULU operators in two and higher dimensions is still open for investigation. The method introduced in [33] for deriving the distributions of the output of the LULU operators on sequences will be investigated for an extension to $\mathbb{Z}^d$.

7. Pattern recognition and background modeling using the pulses of the DPT will also prove a fruitful exercise.

8. All items listed above should include comparisons to current state-of-the-art techniques in the various applications to fully ground LULU theory and the DPT within the image processing field.