# **Analysis of Music with Information Theory**

by

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#### Summary

Information Theory is an important element in many sciences and technologies. The aim of this research is to investigate the application of Information Theory as a method of analysing aspects of music with the purpose of developing models of selected characteristics in compositions.

The candidate developed computer software to apply the principles of Information Theory to analyse pitch distribution, interval distribution, rhythmic content, and a combination of pitch and rhythm. Results obtained with the analyses were applied to develop statistical and graphical models of music. The programmes comprise three independent programmes: a graphical interface for entering the information, a second programme calculates the entropy values by applying analytical routines to the musical information. A third programme generates the tables and graphical models from the information obtained with the analyses programmes, in a variety of formats. Although each programme may be used independently, they are mainly designed for use as a single application in which the individual programmes are totally transparent.

Twenty-two compositions, categorised into three stylistic groups, were analysed with the computer programmes. The three groups are Contemporary Popular songs, Classical Art songs and Art songs of the twentieth century. The selection of the first two groups was based on their continued popularity ratings as indicated by the availability of recordings. In the case of the Contemporary Popular songs the availability of printed scores was also a criterium. Unfamiliar songs were selected for the third group.

After the generation of the entropy values of the songs, a combination of the resultant entropy values was compiled in both tabular and graphical formats.

Each of the individual songs generated unique entropy and stochastic values — an indication of their unique characteristics. To obtain the average, and maximum and minimum values for each group of compositions, the entropy and stochastic results for the individual compositions in each group were combined to develop further tabular and graphical models. The minimum and maximum entropy and stochastic values differ significantly for each of the groups. The contemporary popular songs show the largest numbers of stochastic orders and the highest redundancy values; the contemporary art songs generated the lowest number of stochastic orders and the lowest redundancy values. With one exception — Franz Schubert's Ave Maria — the classical art songs generated entropy values and stochastic orders that, with some overlapping, fall between the highest value of the contemporary popular songs and the lowest values of the contemporary art song group. Ave Maria, which has found considerable popularity as a contemporary popular song, generated values that overlap with both that of the popular song group as well as that of the classical art song.

This research shows that statistical models of specific compositions as well as music styles may be generated with Information Theory. Using the methods described in this dissertation the characteristics of individual compositions may be compared with models representing specific musical characteristics and styles.

analysis, computer, information, intervals, melody, model, music, pitch, popular, predictability, rhythm, statistics, style, unpredictability

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#### Opsomming

Informasie Teorie het noodsaaklike toepassings gevind in vele tegnologieë en wetenskappe. Met hierdie navorsing word die toepassing van Informasie Teorie, om sekere aspekte van musiek te analiseer, ondersoek. Die doel is om vas te stel of modelle van kenmerkende aspekte van musiek daarmee ontwikkel kan word.

Rekenaarprogrammatuur is deur die kandidaat ontwikkel om die beginsels van Informasie Teorie op analise van toonhoogtes, intervalle, en ritmiese inhoud, asook n kombinasie van toonhoogte en ritme van musiek toe te pas. Die resultate van die toepassing is gebruik vir die skep van statistiese en grafiese modelle. Die programmatuur bestaan uit drie selfstandige rekenaarprogramme: 'n grafiese koppelvlak, waarmee die musikale inligting in die databank van die rekenaar gevoer word, 'n tweede program bereken die entropiewaardes deur die toepassing van die analitiese algoritmes op die musikale inligting. Die derde program is verantwoordelik vir die daarstelling van die tabulêre en grafiese modelle in 'n verskeidenheid formate. Die genoemde programme kan onhafhanklik gebruik word, maar is ontwerp om hoofsaaklik as 'n eenheid gebruik te word.

Twee-en-twintig komposisies, wat aan drie verskillende stilistiese groepe behoort, is gebruik vir die ontleding met die rekenaarprogrammatuur. Die drie stylgroepe is: hedendaagse populêre musiek, kunsliedere uit die Klassieke tydperk, en kunsliedere van die twintigste eeu. Die seleksie van die eerste twee groepe was volgens populariteit soos aangedui deur die aantal opnames op plaat en CD wat daarvan beskikbaar is. In die geval van die hedendaagse populere musiek is die beskikbaarheid van gepubliseerde musiek ook in ag geneem. Die derde groep is algemeen onbekende werke.

Nadat die entropiewaardes van elke komposisie bereken is, is dit in tabulêre en grafiese formaat saamgestel.

Elk van die individuele komposisies het unieke entropie-resultate gelewer. Om die gemiddelde, maksimum, en minimum waardes van elk van die groepe te verkry, is die entropiese en stochastiese waardes van die individuele komposisies in elke groep, gekombineer en verdere tabulêre en grafiese modelle geskep. Die minimum en maksimum entropiese en stochastiese waardes van elk van die groepe verskil kenmerkend van mekaar. Die hedendaagse populêre musiek toon die hoogste aantal stochastiese reekse met die hoogste oortolligheidswaardes; die groep hedendaagse kunsliedere het die laagste aantal stochastiese reekse gegenereer asook die laagste oortolligheidswaardes. Met een uitsondering — Franz Schubert se *Ave Maria* — het die Klassieke kunslied stochastiese reekse en entropiewaardes getoon wat, met 'n mate van oorvleuling, tussen die hoogste waardes van die hedendaagse populêre group en die laagste waardes van die hedendaagse kunslied-groep lê. *Ave Maria*, wat ook baie bekendheid verwerf het as populêre musiek, het waardes getoon wat oorvleuel met die waardes van beide die hedendaagse populêre groep en die Klassieke kunslied.

Hierdie navorsing toon aan dat statistiese modelle van individuele komposisies sowel as van musikale style met behulp van Informasie Teorie ontwikkel kan word. Die toepassing van die metodes wat in hierdie dissertasie beskryf word, maak dit moontlik om stylkenmerke van individuele komposisies te vergelyk met modelle wat spesifieke karaktertrekke van musiek verteenwoordig.

analise, informasie, intervalle, melodie, model, musiek, onvoorspelbaarheid, populêr, rekenaar, ritme, statistiek, styl, toonhoogte, voorspelbaarheid,

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#### INTRODUCTION

# 1.1 Music analysis and musicology

Since Italian scholars began studying musical style during the early years of the seventeenth century, music analysis has become the backbone of musical study of many different kinds. Guido Adler (1855 - 1941) formulated the modern idea of style analysis about 100 years ago and it has become an integrated part of most branches of musicology. Analysis is an invaluable tool by means of which music can be assimilated, stylistically classified and identified, learned, and its techniques understood. The results of analytical studies are the subject of countless books on harmony, form, interpretation, composition, and music history. In more recent times music analysis has also become an important aspect of comparative musicology. It is an effective method of learning and describing the style and characteristics of one musical tradition by comparing it with another. Most people will be able to recognise that there is a difference between a composition by Bach and Mozart, or between a work by Schoenberg and a Zulu song, but to describe such differences it is necessary to use a process of analysis, even if this is done mentally.

The Harvard Dictionary of Music defines musical style as follows: -

In a musical composition, 'style' refers to the methods of treating all the elements — form, melody, rhythm, etc. In practice, the term may be applied to single works; ... to composers; ... to types of compositions; ... to media; ... to methods of composition; ... (Apel: 1983, pp. 811-12)

The products of musical analysis are those measurable elements that contribute to the specific characteristics of musical style. For many years most stylistic analyses have been analogous, which means that descriptive models are established that in turn may be used for rough comparisons and to develop a descriptive terminology to label the results of subsequent analyses. The use of models and vague terminology (usually idiosyncratic to music) often leads to misuse or is often inadequate. The results of such research is sometimes vague and misleading, especially when applied to contemporary music. To overcome these problems various researchers have endeavoured to devise and develop new methods, of which the Schenkerian system of analysis and Hindemith's classification of chords are only two examples. Some of these attempts were more successful than others but all pale by the progress made in other fields of scientific research with the help of modern technology.

#### 1.1.1 The role of technology in music analysis

During the last twenty years electronic technology has forged ahead at an intimidating pace. It seems that mainly creative musicians were able to keep up — and only just — by incorporating the most up to date technology in the creative process. Many composers and performers have at their disposal all the benefits of the increasing powers of synthesisers and computers and the music industry has thrived equally. It therefore seems peculiar that analytical musicologists seemingly have progressed little further than the use of recording equipment and word processors, and continue to rely exclusively on traditional, and often archaic, methods of stylistic analysis. Except a handful of progressive researchers — who are often regarded with some degree of suspicion — musicology has virtually been left behind. This is, however, not always due to a lack of interest among researchers but is mostly the result of a lack of training in newly established and innovative avenues of research. Very often, institutions that train musicologists and researchers, are themselves not properly equipped, or lack the necessary trained personnel to broaden the horizons of research methodology.

In those cases where computer technology is being used for research in stylistic analysis, there is a tendency to base the research on established traditional methods. Computers are merely applied as an aid and for the sake of expediency. As a result, those aspects of analysis that may still benefit from a degree of subjective interpretation — for the purpose of scholarly discourse — become devoid of any interpretation. Obviously, a degree of subjective interpretation can be incorporated by the programmer which would provide a degree of predetermined subjectivity to the results.

Some experiments that involve music have been done to make better use of the computer's ability to process information very rapidly and effectively. These include the application of fractal principles, and several statistically derived calculations, all with varying degrees of potential for practical application. Although these experiments have shown varied results most of them have not found any real, substantial or wide, acceptance, mainly because of the lack of co-operation between musicologists and technologists. Since the first generations of personal computers were mainly graphically or text

orientated rather than sound orientated (most are unable to produce more than a basic beep), music applications require that the elements of music (pitch, rhythm, etc.) are first translated into graphical symbols or numbers. This requires a sound knowledge of the intricacies of computer graphics. Many of the more innovative music applications therefore require a good knowledge of music as well as proficiency in computer programming. More recently computer designers and software developers have begun to integrate extensive sound and graphical capabilities as basic components of standard computer equipment and at prices that are within the reach of most musicians. The improved accessibility of computers, in both a technical and material sense, now allows for increased research and applications of computers to music.

#### 1.1.2 Information Theory

The essential assumption of Information Theory, as applied to Communication Science, is that it provides statistical information about recurring structural elements in any form of communication. In most forms of communication as well as other applications, the application of Information Theory has become a very complex but effective tool with many uses.

Information Theory has been applied to many different fields of study and, as shall be shown in the following chapters, it has also been applied to music. However, during the 1950s to 1970s, when most of the experiments with music were done, only mainframe computers had the computational power required for the calculations, and only a few researchers had access to these — many with a limited background in music. As a result the research was time-consuming and often costly and could hardly be pursued extensively. With the event of the greatly improved and faster personal computers and simplified computer languages, the possibility has arisen for non-computer specialists to make use of these facilities to a greater extent. Most personal computers today have a greater capacity and operate faster than earlier mainframe computers. Because they are also relatively inexpensive most serious researchers now have computers at their disposal.

Early experiments with Information Theory and music were limited to using single elements such as pitch counts or rhythmic values. These elements are the most obvious and may readily be counted manually. However, calculations done by hand with hand-held calculators are time consuming and prone to errors. To apply Information Theory most effectively and to obtain more interesting and significant results, it is desirable to apply it extensively to as many structures of music as possible. As an analogy of this process the following example may be used: counting single notes in a melody exclusively and establishing the stochastic processes involved will give only a limited amount of information about the melody. Alternatively, if ever increasing groups of notes (melodic units, semi-phrases, phrases and periods) are statistically compared, the outcome will provide much more information about the music. Furthermore, it is also important that the collection of information and

subsequent calculations are as accurate as possible, as even slight errors will produce erroneous results.

#### 1.2 Aims of this study

With this research the candidate aims to show that, by means of specifically developed computer programmes, the principles of Information Theory may be applied to elements of music that were not previously included in research with Information Theory. Most of the research done in the past was limited to single dimensions of music, such as pitch or rhythm. In this dissertation, the analysis is extended to include interval sequences, rhythmic sequences and a variety of multi-dimensional combinations. Due to computational limitations of the past, analyses of subsequent structural elements were usually limited to the second or third order<sup>1</sup>. With the computer programmes developed for this research all the possible orders may theoretically be generated.

The information obtained by the analyses are combined in graphs to show that it is possible to produce models of particular music styles or characteristics. Using models of music analysed in this manner the proposed hypothesis is that particular music or music that exhibits specific characteristics should therefore also generate similar results or that the results should at least be confined to specific limits.

Furthermore, the results of the analyses are used to establish whether the information content of music has a possible bearing on its popularity with the listening public. The candidate's hypothesis is that there is a direct link between the propensity of music to become generally popular and the entropy (information content) of the music. As the entropy of music increases, its general popularity or tendency to become generally popular should therefore decrease. Conversely, this implies that repetitious music with regular and repeated rhythms and within conventional tonal harmonic constraints would be less entropic (more redundant or predictable) and will therefore produce higher order values and should therefore be more popular or should at least have a greater propensity to become popular.

Intuitively most people would agree that certain types of music are more accessible or more popular than others. For example, why would Beethoven's Symphony No. 5 be singled out for popularisation above any of his other compositions, by adding rock beats and other devices to it? Similarly many listeners would prefer to repeatedly listen to an aria of a 17th century opera, than to a recitative. Why is it that certain composers are more popular amongst the general public than others (e.g. Tchaikovsky vs. Stravinsky). Often ardent lovers of 'classical' music argue that J. S. Bach's music is too 'heavy' while they would endlessly listen to the ballet music of Tchaikovsky. The most obvious elements that may be an indication of relative popularity are the repetitious quality of certain elements, such as motives and phrases. Rock musicians of the last thirty to forty years have extensively ex-

The term 'order' refers to the length of the structural elements. For example, a rhythmic sequence comprising 5 note values is of the 5th order.

ploited the element of repetition to popularise their music. Certain melodic phrases ('hooks'), are repeated endlessly, over a basic and standard harmonic pattern, very often with little or no variation and often with even less lyrical content. Added to this is the steady, rhythmical drumbeat, emphasised by the bass guitar. This is only interrupted by 'breaks', usually between repeats or at the end of major phrases.

Record producers who know what ingredients are necessary to make a record 'successful', will consciously or subconsciously, select those 'songs' which will appeal most to their purchasing public. Admittedly, there are probably many factors that influence the success of a 'hit', including promotional finesse, a group's image and fame, current public preferences, and other socio-economic factors. This candidate's argument is, however, that their selection is to a large extent based on the information content of such a piece of music.

This study will attempt to show that the information content of music, with specific reference to melody, is an important underlying factor in the popularity of music.

#### 1.3 Research methods

The first four chapters of this dissertation deal with the methods, history and background of Information Theory. These chapters provide a background to the second half of the thesis that contains detailed descriptions and interpretations of the analyses. Chapter five is a technical discussion of the computer programmes developed for the analyses of the music in this dissertation. Chapter six is a detailed discussion of the results of the analyses of the music selected for this research. The latter is summarised and interpreted in Chapter seven while the final conclusions are made in Chapter 8. All the data values obtained with the analyses are collected in the appendices at the end of this volume and are presented in tabular as well as in graph format.

Research for this dissertation began with research in the principles, history and methods of Information Theory. This was followed by investigating the applications of Information Theory, not only as it had been applied to music but also how it is currently applied to a variety of sciences and fields of research. In the process of this research three basic hypotheses were formulated. These are:

- 1. Information Theory allows for the generation of statistical models that reflect the characteristics of specific elements of music.
- The general popularity of music directly relates to its redundancy (predictability) and is reflected by the amount of information as reflected in the statistical models.
- 3. Statistical models of music may be used as basic criteria for the selection of music that needs to be applied for specific purposes or is required to conform to typical characteristics.

Twenty-two songs, in three categories, were selected to form the basis for the analyses. For the purpose of comparison the songs had to be of different character, style, and composed during different character.

ent periods. One set of songs was selected from the modern popular repertoire and the criteria for their selection was consistent popularity; the number of recordings that were made of them over a period of twenty years, and whether they were published as scores. The second group of songs were selected from the Art song tradition of the Classical period. Again the songs' popularity was decisive in their selection and the eight most frequently recorded songs (obtained from recording catalogues) were chosen. To provide adequate contrast, the third group of songs were selected from the twentieth century Art song repertoire and the criterion for their selection was that they had little or no proven record of general popularity. There is a detailed list of all the individual songs in Chapter 6 and a complete set of the scores of the melodies of the songs in Appendix I.

To obtain the desired data to confirm (or negate) the hypotheses discussed above, a number of computer programmes were developed by the candidate to generate the entropy values for a variety of musical aspects. They are: the entropy values for the melodic elements, pitch, intervals, rhythm, and a combination of pitch and rhythm. Due to the complexities involved the development and testing of the software proved to be rather time consuming. To keep up with improved hardware and software the programmes required continuous upgrading to suit the more powerful processors and expanded memory capabilities of the computers applied for this research. Since the actual calculation of the entropy uses extensive processing time, it was deemed preferable to regularly upgrade the programmes to faster machines as they became available.

After extensive testing of the programmes for accuracy and efficiency, the information of the selected music was fed into the database. The results were compiled and subsequently interpreted.

As a final note it should be mentioned that there is an abundance of current sources available about Information Theory as a statistical method. There are also many books and articles about the application of Information Theory to a large variety of other sciences. However, published results of research done with music dried up during the late 1970s. Although aspects of Information Theory are taught at some Universities in the USA and Great Britain, it would seem that little if any progress has been made to expand the application of Information Theory to music. This would explain why so little has been written about the subject during the last 20 years.

9

#### MUSIC AS COMMUNICATION MEDIUM

Information Theory as a science emerged from studies and research aimed at increasing the efficacy of communication channels. It is primarily concerned with finding answers to what information is, how it can be measured and how communication devices could be designed to transmit information most efficiently. Chapter 2 surveys the research and events that lead to Information Theory becoming an independent study. This chapter mainly deals with music as a communication process and the most important elements involved.

The term 'information' has various connotations and is commonly used as a synonym for facts. In the context of this study, however, the term information refers to the quantifiable parameters of the carrier of a message, or the signal. In other words, it is a measurement of the mode by which information is conveyed rather than its semantic substance.

The measured quantity of information in a message is referred to as Entropy. As the quantity of information in a message increases, its entropy also increases. In a perfect communication system<sup>1</sup> — in which the encoding is optimal—the entropy of the received message will always be the same as the entropy of the message before transmission.

Most systems of communication, however, are not that efficient and do not transmit messages that are free of interference. Such systems tend to increase the entropy of the received message. Due to

A perfect communication system is a system in which the received message is exactly the same as the original message, without interference from an outside source.

imperfections in a system, the effects of noise and interference has a disruptive influence on the resulting message, thereby increasing the entropy.

#### 2.1 Music as communicative process

Music, like other forms of art, is a rather complex communication system by which a composer communicates musical information to the listener by means of a musical performance. The processes involved satisfy the requirements of the classical model of a unidirectional communication process involving a source, channel and a receptor:

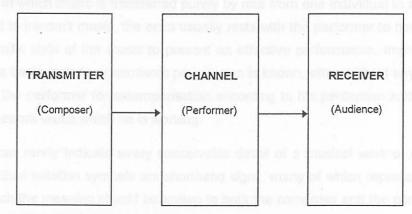


Figure 2-1. Unidirectional communication model

The model above is a simplified representation of music as communication medium. The model does not reflect several interactive factors and restrictions that are inherent in musical performance and that are significant to the effectiveness of the transmission process. The latter ultimately affect the outcome of the musical message by increasing its entropy, or unpredictability. Three of the more important components of a musical performance that may have an important influence on the outcome of a performance are:

- 1. Notation, the customary method by which composers of the Western world communicate or imply their creative intentions. Inherent limitations in the notation system as well as printing and editorial errors may contribute to the masking of the original idea of the composer. In societies where music is transmitted from memory through generations of musicians, the ultimate musical results may be considerably different from the originally conceived musical idea. It would therefore potentially have maximum entropy.
- Interpretation, the contribution of the performer to the creative process in which knowledge of style, experience, mood, and musical ability normally plays an important role.
- Acoustical conditions, which may act as an important element of modification in the musical communication process. The sounds produced in a performance may be significantly changed by the time it reaches the audience.

Below is a discussion of the more salient features of the components listed above, as well as the way they may affect the outcome of a performance.

#### 2.1.1 Notation as musical code

Composers of Western Music normally use the symbols of music notation to represent musical sounds and other directions on paper. There are also musical traditions that have no tradition of notated music and in which music is transferred purely by rote from one individual to another. Whatever the method used to transmit music, the onus usually rests with the performer to have a sound knowledge of the specific style of the music to present an effective performance. Improvised Jazz is an example where a basic melody or harmonic progression is known, often without any notation and that is then used by the performer for extemporisation according to his perception and understanding of the stylistic framework within which he is working.

Music notation can rarely indicate every conceivable detail of a musical work or of the composer's intentions. Individual notation symbols are shorthand signs, many of which represent comprehensive concepts, of which the meaning should be known to both the composer and the performer. An example is the figured bass that is frequently found in music of the Baroque period. To extemporise a figured bass effectively, an extensive knowledge of the harmonic implication as well as other performance principles is required. Merely taking the bass line and melody at face value, without prior knowledge of what the figuration actually means and how it should be interpreted, will inevitably result in a distorted rendition.

## 2.1.2 The role of the performer

Whereas a composer or editor uses music notation to encode a musical work, a performer has to decode it again to realise the music into actual sounds. The interpreter, which in terms of Communication Science, is an active element in the transmission process, is limited by the completeness of the provided code, the extent of his knowledge and understanding of the period style of the music, as well as the meaning of the codes with which he works.

Frequently, the performer himself is also part of a bi-directional and interpersonal mode of communication involving the audience which may itself also be susceptible to prevailing conditions and influences. Consequently, it is quite likely that during coding and decoding, the resulting product may differ from that which the composer originally had in mind. Because of the interpretative role that the performer has to fulfil, he normally contributes — by virtue of his own understanding of, and interpretative disposition to the music — additional information to the original message. A secondary process

is thus set in motion in which the composer's original intentions may be modified by to the performer's mood, technical ability, and understanding of the style. In terms of the originally notated score the performance of a composition can subject it to interference thereby increasing its entropy and causing the musical message to become unclear or distorted in other words unpredictable in terms of the original score.

#### 2.1.3 Performance interference and its effect on the transmitted message

Several additional factors that may be classified as interference, tend to affect the outcome of a musical performance. These include acoustical conditions, geographical location, instrumentation, also the psychological orientation of both the performer and the audience. What an audience normally hears is a combination of the composer's intention plus the various influences referred to. In Communication Science this is called noise.

Most communication models, especially those referring to electronic media, allow for the effect of noise on the message. As far as music is concerned, factors such as notational and interpretative obfuscation and the effects of acoustic conditions may be regarded as interference factors that could mask the original message as conceived by the composer. These factors are included in the adapted communication model below.

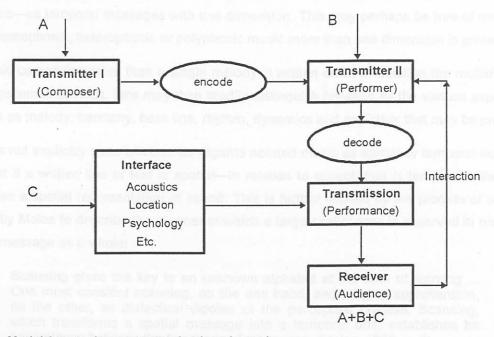


Figure 2-2. Model for music as communication channel

The model illustrated in Figure 2-2, is an expanded version of the original communication model (Figure 2-1) to allow for the dual role of the performer as channel between the composer, as well as

his role as secondary transmitter of information. Also shown are other noise factors (C) over which neither the performer nor the composer has any control.

From the above certain conclusions may be drawn:

- a) In music, as with any other transmitted message, the amount of information (entropy) contained therein always tends to increase and become less predictable it can never become more predictable or contain less information than the composer had intended. It is significant that in this respect music as communication is analogous to the laws of electronic message transmission.<sup>2</sup>
- b) More authentic results should be obtained if a message is studied as close to its origin as possible and before many interference factors have had effect. In the case of a musical message this would be the score, preferably before any editing has been effected.

Most analyses by musicologists are therefore based on scores that allow the most authentic details of the musical elements to be readily and visually available.

#### 2.2 Music as temporal art

According to Abraham Moles, music in sound (as opposed to music in score) is a modulation of duration and thus a temporal art as opposed to the spatial messages of the visual arts (Moles: 1966, pp. 8-9). Moles further classifies messages according to dimensions and classifies speech—and by implication music—as temporal messages with one dimension. This may perhaps be true of monodic music but in homophonic, heterophonic or polyphonic music more than one dimension is present.

When music comprising more than a single melody is written down in notation the multidimensional character becomes obvious. One may then readily distinguish between all the various aspects of the music such as melody, harmony, bass line, rhythm, dynamics and any other that may be present.

Moles does not explicitly state whether he regards notated music as spatial or temporal but one may deduce that if a written line of text is spatial—in relation to speech that is temporal,—then notated music is also a spatial representation of sound. This is further implied by the process of scanning, a term used by Moles to describe the manner in which a large spatial area is observed in order to take in a visual message as a whole:

Scanning gives the key to an unknown alphabet at the time of learning ... One must consider scanning, on the one hand, and integral apprehension, on the other, as dialectical dipoles of the perceptual process. Scanning, which transforms a spatial message into a temporal one, establishes an equivalence between the two types of messages. (Moles: 1966, p. 9)

In other words, Moles implies that a temporal message could be represented spatially but because of the necessity of scanning to learn its contents, it remains temporal, even if in a visual form. Since the

Chapter 3 contains a more detailed description of these principles.

score has until recently been the most common method by which a composer could permanently store and transmit his work to the performer, it should contain a large number of components of his musical style. As such, a score is a carrier of direct or implied information about the stylistic features of that music.

#### 2.3 Music notation as source of information

In general terms, and as shown earlier in this chapter, a message is transmitted from the sender to the receiver by means of codes, the structure of which should be known to both sender and receiver to make it intelligible. Examples of information codes include hand signals, structured sounds (e.g. language and music), electrical impulses, writing and notation.

The structure of a message is its source of information and is defined by Jagjit Singh as follows:

It produces messages by successively selecting discrete<sup>3</sup> symbols from a given stock such as letters of an alphabet, words in a dictionary, notes on a musical scale, colons in a spectrum, or even the mere dash-dot twin of telegraphy. In other words, the message actually transmitted is a selection from a set of possible messages formed by a sequence of symbols from its own repertoire. (Singh: 1967, p. 12)

The value of the score as a source for analytical and stylistic study has always been regarded as indispensable by scholars. It is the most convenient and often the most accurate means of representation of music. This is shown by the fact that even in ethnomusicological research continuous attempts are made to modify traditional notation so that music may be graphically presented. Even when traditional notation is totally inadequate, new graphical systems are designed with the principal purpose of producing a visual representation of the music.

A musical score therefore serves as the coded form of the musical message as transmitted by the composer and as such should contain most of the stylistic information as well. However, the score of a composition usually implies more information than that which is immediately available from the notation itself. The date of a Baroque composition, for instance, also implies the kind of ornamentation that should apply, while the country of origin could indicate how *notes inégales* should be performed. Consequently, the conclusion may be drawn that the stylistic information of a composition primarily involves two distinct sources: the information embraced by the notation itself; and historical or circumstantial information that is largely interpretative and often based on erudition or intuitive deduction.

#### 2.4 Semantic and aesthetic information of music

Of the two types of information discussed in the preceding section, the information provided by notation is usually fixed and therefore more reliable for stylistic evaluation than interpretative information

The term 'discrete' means separate or individual.

that may be eccentric and subjective and possibly vary from one performance to another. The many recordings and performances of compositions by different performers, that differ in interpretation without necessarily being incorrect, shows that interpretative information may vary greatly but that the information provided by the notation remains unaffected.

Referring to the two types of information mentioned above, Abraham Moles distinguishes between semantic and aesthetic information and defines the two categories as follows:

(a) Semantic information, having a universal logic, structured, articulable, translatable into foreign language, serves in the behaviourist conception to prepare actions. (b) Instead of to a universal repertoire, esthetic information, which is untranslatable, refers to the repertoire of knowledge common to the particular transmitter and particular receptor ... One may liken it to the concept of personal information. (Moles: 1966, p. 129)

Those elements in music that are dictated by conventions and that are common to a genre, style period, or specific composer, constitute semantic information because they are translatable into another language (for example, technical terminology), and are normally not subjected to interpretative variations<sup>4</sup>. Semantic information also includes those attributes in a composition that are inherent to the composer's personal style and that distinguishes one composition from another.

Those elements that are subject to influence by personality, mood, acoustical surroundings, quality of instruments and size of ensemble constitutes aesthetic information. A significant portion of aesthetic information, Moles points out, is made available by the limitations of the score and the manner in which music is written down, (Moles: 1966, p. 138), while ...

Above all, there are the differences or latitudes in interpretation which arise from the instrument (or orchestra) and the performer (or conductor). In the case of the ... orchestra, the latitudes can attain such fantastic values ... that it is impossible to speak of a symphonic work without at least referring to a particular performer, if not a particular performance. (Moles: 1966, p. 139)

Although Moles only mentions the orchestra, the same argument is obviously true of any musical performance.

Two of the more obvious sources of aesthetic information are that of tempo and volume, while elements such as attack, tone colouring and instrumentation are further examples. All these elements are subject to fluctuations not only because of the performer's interpretation but also because of environmental conditions such as the size of a concert hall, acoustical properties, size and type of audience, temperature and geographical location.

The distinction between semantic and aesthetic information has important implications for this study, as semantic information is measurable or, in other words, quantifiable. Aesthetic information usually

Two keyboard players will probably perform a figured bass differently but stylistically correct, each providing their own instinctive aesthetic information to the performance. However, should the actual chord progressions be changed in the process of extemporisation, perhaps out of character for the period, composer or specific piece, the semantic information is changed.

does not allow translation into quantifiable units and is normally too mutable for accurate measuring.<sup>5</sup> However, the fact that these elements are not quantifiable by virtue of their variability and indeterminacy does not necessarily exclude them from a stylistic study, especially when they specifically have a bearing on, or are an inherent part of the musical style. It is imperative that such elements are included or kept in mind. However, this is predominantly the domain of traditional analysis.

The relationship between semantic and aesthetic information in music is never fixed or constant and may depend on the style period, composer, individual composition and may even differ between sections of the same work. In essence it depends on the ratio between those musical elements that are fixed (for a style period, composer, or composition) and those that are left optional for a specific performance. For instance, the interchangeability of instruments of some Baroque instrumental compositions, increases the aesthetic information of such works because it adds to those elements that are uncertain. In a work written for a specific instrument this knowledge is included with semantic information. Whether semantic or aesthetic, the information provided (or not provided) by the composer tells one something about his intentions, be they dictated by period convention or personal choice.

Generally semantic information is largely dependent on those elements in music that are fixed by convention, the composer, or is present in a specific work. Serial or synthesised music by a twentieth century composer could include more fixed and controlled elements, such as tempo, volume and envelope shape that would result in a greater amount of semantic information than in a composition in which these elements are subject to the many performance influences described above. As such, the ratio between semantic and aesthetic information is an important factor that should be kept in mind in a statistic study of music.

# 2.4.1 Originality of musical information

The information content of a musical message is directly related to the degree of originality<sup>6</sup> contained in the composition. A composition that contains a greater quantity of original material will also contain more information. The reason for this is that a smaller portion of the information is reduced to basic sets therefore generating a greater number of sets. In works that contain less original information there will be fewer sets of larger dimensions. For example, the harmonic information of a piece of music with a regular and repeated harmonic structure will contain more redundant (repeated) information than a piece of the same length in which harmonic progressions are not repeated. Aleatoric music in which only a limited number of parameters are fixed, for instance, could contain a high degree of aesthetic information with near maximum originality. The semantic elements will be those restrictions that are fixed by the inventor although the results themselves could be totally unpredictable.

The modern technique of digital recording in which sounds are represented by number sequences may present new possibilities for analysis.

A serial composition, on the other hand, in which almost all the parameters are subjected to serial treatment could have a high originality factor with a high semantic content as well as a low predictability factor.

The model below is an adapted model based on one by Moles (Moles: 1966, p. 141) and shows how the relationship between semantic and aesthetic information and the possible degree of information (entropy) and banality (redundancy) of a composition may be illustrated. The rectangle on the left indicates semantic information content, while the rectangle on the right represents the aesthetic information content. Portions above the horizontal line show the originality factor for both the aesthetic as well as semantic information of a composition:

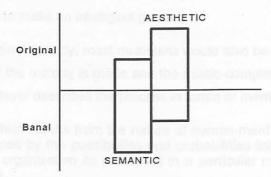


Figure 2-3. Mole's model of semantic and aesthetic Information

The example shows that the ratio between aesthetic and semantic information may change depending on the number of parameters that are fixed. It also shows that the ratio between the predictable or banal elements of each may vary as well.

Of the two aspects of music, aesthetic and semantic, this study is primarily concerned with the measurable part or semantic information of the model above—in effect the ratio between originality and banality of the musical message. Some confusion could arise, however, because of the term 'semantic' and some clarification is required.

#### 2.4.2 Semantic information and meaning in music

Normally the word 'semantic' is a linguistic term that refers to 'meaning' in language. In the context in which the term is used here, there is no attempt to attach any connotation of meaning to music.

In terms of music, 'meaning' remains an elusive and controversial subject. As a form of expression, music nevertheless embodies much information that is normally described and discussed in musical terminology such as, tonality, metre, genre, period, form, monophony, polyphony, and homophony. In respect of these parameters, the information contained in music does have a degree of semantic

<sup>&</sup>lt;sup>6</sup> The term 'originality' is here used to describe the degree to which new material is introduced in a composition, and does not refer

meaning but mainly by the inherent processes the terminology describes. These processes imbue a composition with its stylistic characteristics. Music thus contains 'meaning' purely by virtue that it embraces those elements that provide information about the style that characterises it.

#### 2.5 Expectation and information in music

By merely listening to an unknown piece of music it is possible to identify the style period, genre or even the composer; some people are even able to identify the conductor or performer. Through continuous listening and studying, those elements that are stylistically idiomatic to a period, composer or performer are learnt and assimilated. This is made possible by the information that is made available to the listener, thus allowing him to make an intelligent guess.

Having listened to part of an original melody, most musicians would also be able to complete it in the same style. A mental analysis of the melody is made and the music completed according to parameters previously learnt. Leonard Meyer describes the process in terms of mental conditioning:

...the expectation which results from the nature of human mental processes are always conditioned by the possibilities and probabilities inherent in the materials and their organization as presented in a particular musical style. (Meyer: 1957, p. 44)

The mental processes involved are beyond the scope of this study but a less obvious implication of the phenomenon of recognition is the probability or predictability element that is often taken for granted. To identify a composition or the style of a melody it is necessary to assume that it would continue in the style that it began. If this were not so, all music would be totally disparate in structure and style and it would be impossible to distinguish genres and forms or even vaguely categorise them. The fact that some entertainment musicians sometimes dress up a trivial tune in a variety of styles such as that of Bach, Liszt, and Jazz, supports the idea that musical style is a source of information that conforms to expectation.

#### 2.6 Elements of musical information

Musical terminology abounds with nomenclature that provides information about music and that implies elements of predictability. An extreme example of predictability in music is the concept of scales that consist of a fixed and predetermined sequence of intervals. It would, for instance, be inconceivable to consider a major scale that does not comprise two identical tetrachords each with two whole tones and a semitone and there is 100% certainty that all major scales will have the same predictability value. Therefore, in some situations (for example the major or minor scale) musical terminology implies maximum redundancy. More often a degree of unpredictability is implied, however. Some musical forms fall in the latter category: a piece in sonata form deviates from the theoretical model

but is nevertheless classified as a sonata form because there are general similarities to the predetermined model.

Much of what is referred to today as 'general musicianship' is no more than the assimilation and dissemination of musical information. Traditional music analysis — be it harmonic, melodic, or structural — is a process of gathering information about music and frequently demands a measure of interpretation to allow for the limitations of musical terminology.

The nomenclature used in musical analysis serves as carrier of information about a composition and simultaneously is an indicator of the conventions that may apply to the music. John Fiske explains that ...

We are always checking the accuracy of any message we receive against the probable; and what is probable is determined by our experience of the code, context and type of message, in other words, by our experience of convention and usage. Convention is a major source of redundancy, and thus of easy decoding. (Fiske: 1982, pp. 11-12)

Fiske uses the word 'convention' in reference to information that is already known by previous experience. As such it excludes originality which is a source of new information and thus unpredictability.

Some confusion may arise about the use of the term information. A natural deduction is that the more information that is known about a specific composition, the easier it should be to identify it. However, it is important to realise the difference between information about a composition and the information inherently generated by a composition. Information about a composition serves to identify a work, while information contained in a piece of music represents the level of tonal, harmonic and rhythmic structural organisation. An increase in structural complexity is directly related to the increase in the level of unpredictability (entropy). For example, a melody that contains only diatonic notes in two regular and symmetrical phrases, both of which are repeated contains less information and is more predictable than a through-composed melody of the same length that contains all twelve notes of the scale and no repeated phrases and therefore contains more information.

The process of reaching a decision based on prior experience may readily be illustrated by the decision-making tree of Figure 2-4 by which the key of a piece of music with, for instance, two sharps may be established.

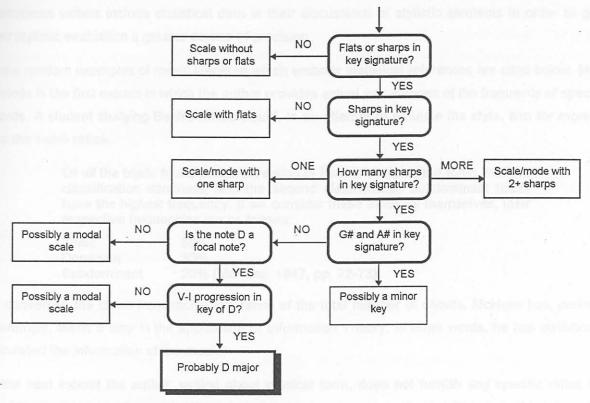


Figure 2-4. Decision-making tree

Many musicians, for whom the recognition of a key has virtually become a matter of intuition the process shown above would clearly seem long-winded and perhaps simplistic. Through continuous use it becomes an unconscious process; one by which various possibilities are systematically eradicated; a consistent narrowing of choices until a final conclusion is arrived at. The fact that a musician has the confidence to state a key is in itself an indication that a certain set of circumstances will always be predictable and generate similar results.

#### 2.7 Music analysis and statistics

Usually, music analysis is more than merely a process of making single decisions and a good deal of intuition and interpretative judgement is often necessary. Many books on musical style present interpretative information in descriptive phrases and classify musical elements in broad terms that often allow for a degree of tolerance within specific limitations. The use of the term sonata form has already been mentioned, and other examples that may be mentioned are: final cadence, ascending or descending melodic curve, and varied repetition, to name but a few.

To make their analysis more relevant and precise some authors endeavour to describe musical elements in more detail. For instance, two forms of perfect cadence are sometimes referred to as 'perfect authentic cadence' and 'imperfect authentic cadence', to distinguish perfect cadences with chords in root position from those containing inverted chords.

Sometimes writers include statistical data in their discussions of stylistic elements in order to give their stylistic evaluation a greater degree of precision.

Some random examples of music literature which embody statistical references are cited below. Most obvious is the first extract in which the author provides actual percentages of the frequency of specific chords. A student studying Bach's music would, in an attempt to emulate the style, aim for more or less the same ratios:

Of all the triads found in first inversion in Bach's chorales the tonic, the first classification dominant, and the second classification subdominant triads have the highest frequency. If we consider these triads by themselves, their respective frequencies are as follows:

Tonic

50%

Dominant

30%

Subdominant

20% (McHose: 1947, pp. 72-73)

By converting the chord frequencies into ratios of the total number of chords, McHose has, perhaps unwittingly, taken a step in the application of Information Theory. In other words, he has statistically calculated the information of the music.

In the next extract the author, writing about musical form, does not furnish any specific ratios but provides a measure of predictability or probability by phrases such as 'often enough', 'occur frequently', 'unlike', and 'unique':<sup>7</sup>

Other forms occur often enough to be grouped into other formal categories. Frequently, however, disclosure of the tonal structure and design of a composition results in the discovery that the form is unlike that of other known compositions. Such pieces are said to have free, or more aptly, unique forms. Our approach will be to consider those forms that occur frequently before directing our attention to the unique ones. (Green: 1979, p.5)

Analysis of the following lines by Leon Dallin in his book, *Techniques of Twentieth Century Composition* (Dallin: 1975, pp. 6-7), can, with some exceptions, be translated into absolute values:

Stepwise motion is predominant [51% at least]<sup>8</sup> in the melody, and scale-line motion in the opposite direction invariably [100%] follows the descending fifths. There is a balance [about 50%] between the notes above and below the starting pitch. A climactic effect is lacking [0%], because the highest note comes in both the first and third phrases [100% of the two phrases] ... This device occurs in five of the nine measures [55%]. In each instance [100%], besides occurring on different pitches, it is subtly altered ... (Dallin: 1975, pp. 6-7)

Italics by the candidate

Figures in brackets were added.

The fact that the information in the three extracts were first gleaned from the music and then in some way or other quantified in order to be represented, either by means of figures or adjectives implying absolute and relative quantities, is an indication that there are elements in music which are naturally suitable for the application of the methods of Information Theory.

#### 2.8 Redundancy and musical style

As will be illustrated in the next chapter, redundancy refers to the predictability in a message and in music refers to the banality or predictability of the stylistic elements. It is inversely related to entropy. In Information Theory, redundancy, is also used to refer to probability or the structure of a message. The same argument may be used in music. A composition which adheres totally to a theoretical model is totally predictable and may be said to have a redundancy of 100% as it provides no new information.

A composition written according to the style of a specific model — for instance by a student in composition — would, by implication show definite similarities. Such a composition will also show certain characteristics peculiar to the composer and stylistic period. As far as style is concerned, the entropy value of a composition would thus give an indication of the individual style compared to an overall model with the fixed elements represented by the redundancy.

One of the nebulous abstractions that is sometimes used in conversations about music is the concept of 'understanding' music, and it would seem that the 'understanding' of music is closely dependent on the amount of information that a composer has vested into a musical work, in other words the relationship between the redundancy and predictability of music. Pop-songs which have clearly defined, repetitive rhythms with a steady tempo, employ a limited repertoire of harmonies, and only have one or two melodic periods which are continuously repeated, clearly have a greater redundancy. Sales charts show that music with these characteristics have a greater general appeal than most contemporary academic music in which variety and originality of musical information is a major factor.

Redundancy also helps solve problems associated with the audience. If we wish to reach a large, heterogeneous audience we will need to design a message with a high degree of redundancy. A small, specialist homogeneous audience, on the other hand, can be reached with a more entropic message. Thus popular art is more redundant than highbrow art. (Fiske: 1982, p.12)

This may well be the reason why music by composers such as Schoenberg, Hindemith and so many other twentieth century composers has found less general acceptance in contrast to music by composers such as Tchaikovsky. It is probably also the reason why specific works of a particular composer are more popular than others and why it often happens that compositions which are less conventional are often surreptitiously included in concert programmes which predominantly include 'well known' or popular works with a high redundancy.

Traditional methods of music analysis make it possible to obtain a generally detailed and total picture of a musical work. Usually the various aspects and elements of the music are studied and analysed in isolation after which the results may then be viewed as a whole. Complete separation of the various elements is not always possible as there are always interactions at work, for example, a melody and the harmony which underlies it. By first extracting information from the individual elements and then studying the results in relation to each other, a descriptive understanding of the work becomes available. The results are usually loosely worded descriptions referring to form, melodic shape, harmonic complexity, textures, and other musical elements contained in the analysis. A description of this kind is useful in discussing the style of a composition by itself or in comparison with other works, but has major shortcomings when a single model is to be devised which could represent most of the salient as well as the less obvious aspects of the music. By no means does this imply that it would be of any value to attempt to reduce the style of a composition to a single numerical value. This would merely mask the significance of the stylistic qualities of each element which contribute to the uniqueness of the composition. Not only would important stylistic elements be lost, but by combining the various aspects into a single value, those that are peculiar to a specific style would be obscured.

A single element also serves very little purpose in classifying a complete work stylistically. Deryck Cooke provides a number of clear examples in his book, *The Language of Music*, in which he classifies melodies according to their sequential intervals and continues by giving each melodic pattern an emotive value (Cooke: 1959, pp. 113-167). Cooke recognises sixteen different kinds of melodic patterns and it is significant that each pattern represents melodies selected from a repertoire spanning more than five hundred years. One of Cooke's examples is the descending stepwise minor melody starting on the tonic, 8-7-6-5 (minor), for which he lists eighteen examples by composers which include Ockeghem (c 1480) to Benjamin Britten (b 1946). (Cooke: 1959, pp. 163-164) If these melodies were only to be judged by the sequence of the pitches they embrace, it would be virtually impossible to classify them stylistically. However, when the rhythmical and harmonic properties of such selected sections are taken into account, two additional dimensions or stylistic indicators are added to the study, thereby making it possible to obtain a clearer idea of the stylistic properties of the work. As the musical dimensions are increased in an analysis so the identification of the stylistic elements are given greater depth and become more accurate. However, the number of elements that can be taken into account may vary greatly and depend on the style of music in question.

#### 2.9 Conclusion

Music as a form of human communication shares a number of similarities with other forms of communication. Of primary importance in the study of musical style is the fact that composers through the years have imposed certain restrictions and points of reference on their music which makes it recognisable. Most of these structural elements, be they harmonic, melodic, rhythmic or formal, can be measured and expressed in terms of numerical data— usually by way of ratios. For the purpose of the application of Information Theory this is ideal because it can only be applied to quantifiable elements.

As a communication medium, music is more complex than many other forms of communication, especially when a performer contributes to the artistic rendition of the composer's creation by way of aesthetic information. Other factors also have an influence on the final product the listeners hears and which will add or subtract from both aesthetic and semantic information. It is for this reason that the score is the best source for entropy analysis because it presents the semantic information of the music in its purest form.

Before the application of the statistical principles of Information Theory is demonstrated, the next chapters will cover the mathematical concepts and historical background of Information Theory.

3

# INFORMATION THEORY: DEFINITIONS AND HISTORICAL BACKGROUND

# 3.1 Towards a theory of information - a historical survey

Although the statistical principles applied in Information Theory are similar to those applied to the relatively older science of thermodynamics, which dates back to the middle of the nineteenth century, it took nearly a hundred years before these principles were actually developed into a theory to measure modes of communication.

As far as the measuring of information in a communication system is concerned, some initial research was done by H. Nyquist (1924), R. A. Fischer (1925), and R. V. L. Hartley (1928). The science of Information Theory is, however, mostly indebted to the work of V. A. Kotel'nikov (1947), N. Wiener (1948) and C. E. Shannon (1948). Shannon with his publication, *A Mathematical Theory of Communication*, is regarded as being responsible for establishing the foundations of Information science as it is known today.

In the following pages a short historical survey of Information Theory is provided. First the development of the concepts of entropy in physics and thermodynamics is discussed. This is followed by a

Bell Systems Technical Journal, Vol. 27, pp. 379-423, 623-656. Republished in collaboration with W.W. Weaver as The Mathematical Theory of Communication, Urbana: Univ. of Illinois, 1949.

more detailed account of how similar theories were subsequently employed in the development of the science of communications and telecommunications.

## 3.1.1 Entropy, disorder and the movement of heat molecules

In 1854, the German scholar, Rudolf J. E. Clausius (1822-1888) published a scientific paper<sup>2</sup> in which he formulated what has since become the second law of thermodynamics.<sup>3</sup> In essence the formulation states that in an isolated system

... no process is possible whose sole result is the transfer of heat from a colder to a hotter body. (Kendall: 1973, p. 59)

The above law is closely allied to the first law of thermodynamics which states that when two bodies with unequal temperatures are brought into contact, a process of equalisation will take place. Entropy, which is the measure of disorder in a system, reaches its maximum value as the system reaches equilibrium. According to these two laws heat can only transfer from a hotter body to a colder body until the temperature of both is equal. Entropy or disorder in an isolated system can, in other words, only increase and never decrease.

According to Kendall the British physicist, James C. Maxwell (1831-1879), was one of the first to approach the concept of entropy from a probabilistic point of view. (Kendall: 1973, p. 59) Maxwell formulated an equation by which the distribution of velocities of gas molecules may be calculated<sup>4</sup>.

Further research in the statistical properties of gas molecules and heat was undertaken by the Austrian physicist Ludwig Boltzmann (1844-1906):

It was he who linked the thermodynamic concept of entropy with the statistical concept of disorder. (Kendall: 1973, pp. 59 -60)

In his book, Vorlesungen über die Gastheorie,<sup>5</sup> Boltzmann provides a formula which expresses the logarithm of the resulting probabilities of the distribution of molecular velocities as a ratio of:

#### Equation 3-1. Boltzmann's formula

#### $n\log_e n$

This has since become an important component of statistical physics but one which will also recur frequently in this discussion on the application of Information Theory to music.

Clausius subsequently published his findings in a book: (1864) Abhandlungen über die mechanische Wärmtheorie, Braunschweig: Friedriech Vieweg.

A branch of physics which describes the physical properties of matter and energy.

Funk & Wagnalls, 1983: vol. 17, James Clerk: 'Maxwell'.

Further research with the concept of entropy and statistical physics continued to be done by a number of investigators such as J. von Neumann and L. Szilard.

It was during the late 1920s that several communication engineers began investigating the possibility of applying the concepts of entropy to communication.

## 3.1.2 Entropy and communications

The first public telegraph line was installed in Britain in 1843. The Italian, Guglielmo Marconi (1874-1937) announced the discovery<sup>6</sup> of wireless telegraphy in 1895. Nearly twenty years before, in 1876, Alexander G. Bell's invention, the telephone, had successfully transmitted human speech. But none of these systems were yet perfect and during the first decade of the twentieth century continuous research relating to the problems that still plagued electronic communication was conducted.

Distortion, signal noise, and inter-symbol interference necessitated techniques to overcome these difficulties and ensure transmission of intelligible messages. This research was predominantly done at the Bell Telephone Laboratories in America where H. Nyquist (1924, 1928) and R.V.L. Hartley (1928) were actively working on these problems. Most of their findings were published during the 1920's in articles such as Hartley's 'Transmission of Information'. More or less at the same time, similar research was done by Norbert Wiener who specialised in the biological application of the transmission of information as, for example, in the nervous system. (Shannon: 1949, p. 3)

#### 3.1.2.1 R.V.L. Hartley

From the sources<sup>8</sup> on the subject it would appear that of the research done at the Bell Telephone Laboratories, it was Hartley's work that stimulated further thought on the application of the concept of entropy in Information and Communication Science the most. In fact it was in the article mentioned above that Hartley expressed the theory that the quantity of information could be measured. Hartley based his theory on a principle simultaneously formulated by Nyquist and the German, Kopfmuller in 1924 which ...

... states that for transmitting telegraph signals at a given rate a definite—frequency bandwidth is required. (Reza: 1961, p. 11)

Leipzig: Barth, 1896.

The U.S.A. Supreme Court pronounced in 1943 that Nikola Tesla was the inventor of the radio (Cheney: 1981, pp. 176-184).

<sup>&</sup>lt;sup>7</sup> 1928. Bell System Technical Journal, vol. 7, pp. 535-564.

A more detailed account of the historical background of information theory is provided in E.C. Cherry's article, 'The Communication of Information', *American Scientist*, October, 1952.

Nyquist and Kopfmuller's work was further refined by D. Gabor (1946) and D. M. Mackay (1948), whose contribution to the measurement of information was important to Hartley's work. (Reza: 1961, p. 11)

Hartley's theory is founded on the concept that the information content of a message depends on the successive selection of symbols from a specific set of symbols. In terms of music the seven notes of a diatonic major scale may be used as an example. In an imaginary melody of eleven notes the notes may occur in 7<sup>11</sup> (7 to the power 11) different ways. In more general terms, where N is the number of notes in a melody with length L, there are N<sup>L</sup> possible different combinations in which the notes could occur.

Furthermore, Hartley continues by showing that the information in a message needs to be calculated from the actual symbols used in a message against the capacity of the system which transmits it and not from the total possible symbols available. (Kendall: 1961, p. 61) If the imaginary melody is again taken as example, this means that the melody of eleven notes in length, represents the capacity of the melody. If this melody should comprise only five different tones out of the possible seven of the major scale, the selected five represent the capacity of the melody, and not the seven notes of the scale.

#### 3.1.3 Quantifying information.

The formula that Hartley arrives at to express the maximum information of a set with **n** symbols and with **K** being a constant, is:

Equation 3-2. Maximum information of a set of symbols

$$I = K \log n$$

To demonstrate how Hartley arrives at this equation the seven notes (n = 7) of a major scale may again be used. Presuming that a random generator generates one note at a time and with equal probability, the seven notes may be represented as the set,  $\{x_1, x_2, x_3, \ldots x_7\}$ , each of which has an equiprobable chance of being heard next. The amount of information that the selection of a particular note generates may be expressed as a function of the seven notes, thus:

Equation 3-3. Information of a single note from a set of seven notes

$$I(n_x) = f\left(\frac{1}{7}\right) .$$

where I is the information content and  $n_x$  any one of the seven notes in the set. One of the possible ways of expressing the function above is by using logarithms, thus the amount of information associated with each element in the set is:

Equation 3-4. Information of a single note of seven using logarithms

$$I(n_x) = -\log\left(\frac{1}{7}\right)$$

thus:

$$I(n_x) = \log(7)$$

#### 3.1.4 Calculation of information with logarithms base two

Hartley's original suggestion was to calculate information using logarithms on base ten. Nevertheless, calculations of information are often done with logarithms with a base of two, in which case the amount of information is expressed as *bits*<sup>8</sup>. The following paragraphs illustrate why binary units<sup>9</sup> are often preferred for calculations of information contents.

The fact that a single element can be expressed with two equal possibilities (i.e. on/off, sound/silence, yes/no), and as such is the minimum amount of information a system can generate, <sup>10</sup> makes the use of binary counting especially appropriate. As an example the choice of playing a note on a musical instrument may be used: at any moment a performer is presented with the choice of playing a note or not playing a note, a set of two possibilities. The silence may be represented by the number '0' for a note not played (state not true), or '1' for a played note (state true). At any subsequent moment this possibility is repeated, thus the performer is continuously confronted with a situation of two possibilities with a 50% (0.5) probability. In base ten notation the calculation of the information will be:

Equation 3-5. Information of a single element using natural logarithms

$$I(x) = -\log(\frac{1}{2})$$
$$= \log(2)$$
$$= 0.3010$$

When natural base logarithms are used the units of information are expressed in nats, when base ten is used the information content is expressed in Hartleys.

In contrast to the decimal system which uses ten as its base, the binary system uses two as its base. Compare the following decimal numbers and their binary equivalents: DECIMAL: 2 5 9 3 4 10 BINARY: 01 10 11 100 101 1000 1001 1010 110 111

As with the entropy of thermodynamics which can never have a negative value, a communication system can never have a negative information value - it is impossible to have a value smaller than zero.

However, if base two logarithms<sup>12</sup> are used the result of the same calculation is:

Equation 3-6. Calculation of information using binary logarithms

$$I(x) = -\log_2(\frac{1}{2})$$
$$= \log_2(2)$$
$$= 1$$

The parity of the information content for a single note on a score, as a rudimentary system, may thus be expressed as a single binary unit, customarily contracted into the word 'bit'. Using binary logs as base, Hartley's calculation for x number of symbols is written as:

Equation 3-7. Information for x number of symbols in Hartley's

$$I_{\text{max}} = -\log_2 x$$
$$= -\log_2 x^{-1}$$
$$= \log_2 x$$

All the calculations thus far are based on the premise that at any given moment each symbol of a system is equally probable. Such static systems are also called systems without memory—memoryless. Communication systems, including music, are rarely all that simple as there are usually structural principles involved. In music, for instance, various conventions dictate the use of subsequent chords or their inversions in a chordal progression.

According to Kendall, Hartley's formulation had little impact, at first, outside the realm of electronic communication and it was not until ...

... Shannon took it up and extended it in a paper (1949) which may be regarded as the effective starting-point of the current interest in the subject. (Kendall: 1973, p. 62)

This was also the point at which the concept of Information Theory as an independent field of investigation began. Since its relatively recent genesis, Information Theory has become an important and active component of Communication Engineering and Cybernetics. The latter refers to the manner in which information is moved, controlled and responded to by living organisms and machines. Because of its extensive and successful application in these sciences it has found extensive application

Conversion from a logarithm of a base ten number to the logarithm of a base two number is done with the formula (where n represents the number of elements):  $\log_2 n = \frac{\log n}{\log 2}$ 

Cybernetics emerged as a science in the late 1940's. The auto-pilot of an airliner is an example of the application of Cybernetics. (Longman Dictionary: p. 275)

in other disciplines as well, and has since become an autonomous mathematical science of communication.

## 3.2 Claude Shannon and the stochastic process

In 1948 Shannon, who at the time was mainly concerned with cryptographic systems, conceived a formula based on the one by Hartley which allows for the unequal distribution values of elements from a discrete source<sup>13</sup> (as apposed to a continuous source) with a finite set of elements. He referred to it as a *stochastic process*, and it is expressed as a ratio of maximum entropy. (Shannon & Weaver: 1949, p. 179) A discrete source implies a limited set of symbols and is described by Shannon as:

... as system whereby a sequence of choices from a finite set of elementary symbols \$1 . . . \$n can be transmitted from one point to another ... We can think of a discrete source as generating the message, symbol by symbol. It will choose successive symbols according to certain probabilities ... (Shannon: 1948)

According to Reza an important but independent contribution to the formulation of Information Theory was made by N. Wiener with his two books, *Cybernetics*<sup>14</sup> and *Extrapolation, Interpolation, and Smoothing of Stationary Time Series*<sup>15</sup>. Reza writes:

N. Wiener was one of the first scientists who clearly described the stochastic nature of communication problems. Wiener put in focus the fact that communication of information is primarily of a statistical nature. That is, at a given time a message is drawn from a universe of possible messages according to some probability law. At the next moment another message from this universe will be transmitted. (Reza: 1961, p. 375)

The Stochastic process Reza refers to, indicates some important differences in the concepts of Hartley's theory and that of Shannon and Wiener. Whereas Hartley conceived his theory on static systems with any symbol of a set occurring at any given moment with equal probability, Shannon worked with dynamic systems with memory which allows for,

- the presence of a formal structure in a message in which certain successions of symbols may be excluded, limited or required; and
- the possibility that each symbol may occur with different relative frequencies (Kendall: 1973, p. 62). This may be explained using a melody as an example in which
- the appearance of a specific interval may according to convention or choice, preclude it to be followed by another specific interval; and certain intervals will be more predominant than others.

Shannon also identifies continuous systems, in which the message is treated as a continuous function, and mixed systems, which is a combination of a discrete and continuous system. (Shannon: 1948)

<sup>14</sup> Cambridge: Technology Press. 1948.

New York: John Wiley. 1949.

## 3.2.1 The Markov<sup>17</sup> process

Point no. 3 above is a special case or subclass of the stochastic process and is known as a *Markov* chain or a dependent stochastic process. The theory that serves as the basis of the Markov chain is that the occurrence of a symbol in a time-dependent message depends on the previous symbol or that certain symbols may dictate or exclude the use of certain other symbols. Shannon explains the process as follows:

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A system which produces a sequence of symbols (which may, of course, be letters or musical notes, say, rather than words) according to certain probabilities is called a *stochastic process*, and the special case of a *stochastic process* in which the probabilities depend on the previous events, is called a Markoff process or a Markoff chain. (Shannon: 1968)

In other words, the Markov chain refers to the application of 'rules' or conventions in the inherent structure of a message by which it is made coherent, and makes allowances for the relationship between symbols and not only their individual ratios.

Markov processes are also found in conventional music practices, for instance in melodic construction where certain intervals or chords, according to convention, require to be succeeded by other specific intervals or chords or, conversely, preclude certain subsequent intervals or chords. Even in some of the more current musical systems the stochastic process, and especially the Markov chain is strongly in evidence. An example is strict serial music in which the elements of a complete series are dependent on and dictated by preceding elements.

Through the years composers and theorists have continuously relied on systems or conventions to serve as the basis for their compositions. The traditional system of harmony is one example in which the I - IV - V - I chord progression is only too familiar. Arnold Schoenberg's dodecaphonic technique is another example in which preconceived limitations and so-called 'rules' play an important role. For a melody to make sense, its note sequences are usually bound and influenced by structural and tonal elements complemented by idiosyncrasies of convention and 'style'.

Common examples of traditional melodic theory that are essentially stochastic in character include the tendency of the leading tone to resolve to the tonic, or for a melodic leap to be followed by a note within the leap; consecutive leaps in tonal melodies tend to outline specific chords and melodies tend to outline harmonic progressions. Many similar examples may be cited which tend to indicate limitations or precepts of the choice of notes and intervals imposed by convention or so-called 'rules'.

This is a phonetic spelling, the name is also spelled Markoff.

#### 3.2.2 Ergodic processes

Perhaps a more difficult concept is the ergodic process, a distinct form of the Markov chain. Weaver describes it as follows:

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Although a rigorous definition of an ergodic process is somewhat involved, the general idea is simple. In an ergodic process every sequence produced by the process is the same in statistical properties. Thus the letter frequencies, digram frequencies, etc., obtained from a particular sequence, will, as the length of the sequence increases, approach definite limits independent of the particular sequence. (Shannon and Weaver: 1986, pp. 45-46)

In simple terms the ergodic process hypothetically means that the relative redundancy values of sections of a message approaches or has the same value as the message as a whole.

Starting with the frequency distribution of single elements in a message, Information Theory, also allows for the calculation of the relationship between the various elements and the manner in which they are organised. Its most valuable application to music is that it calculates those aspects that are conventionally or stylistically fixed as a ratio of the maximum number of possibilities that such a system allows. A composer imposes his personal choice in selecting, a) the parameters such as rhythm, and chromaticism, and, b) allows himself artistic freedom within the chosen system and parameters. This means that the ratio between entropy or redundancy and maximum redundancy can actually represent a measure of the style of music. Hence Entropy is a measure of the creativity and originality applied in a composition within a stylistic framework.

## 3.3 Entropy

Using Hartley's equation, Shannon demonstrated that the quantity of information produced by a Markov process can be measured, and referred to this quantity as entropy:

The quantity which meets the natural requirements that one sets up for 'information' turns out to be exactly that which is known in thermodynamics as entropy. (Shannon: 1968, p. 16)

According to Shannon the quantity of information can only be stochastically measured if certain conditions are met. Stochastic entropy is represented by the formulation  $H(p_1, p_2, \ldots, p_n)$  where  $p_i$  is the probability of the i-th element. The conditions are <sup>18</sup>:

 Continuity of H in the p<sub>i</sub>. If the probabilities of the events change slightly, the measure of information (H) should similarly vary slightly.

Based on Shannon's paper, 'A Mathematical Theory of Communication', 1948, and Reza's An Introduction to Information Theory, 1961, pp. 80-81.

2. Symmetry of the H function in every  $p_k$ . This means that a change in the order of events should not result in a difference of measurement, or

Equation 3-8. Symmetry of the H function in every pk

$$H(p_1, p_2, \dots, p_n) = H(pi_1, pi_2, \dots, pi_n)$$

where  $i_1, i_2, ..., i_n$  is any permutation of 1, 2, ..., n

3. Extremal properties. When the probabilities of an event are equal, the H function should have its maximum value for that set. In other words, when all events are equally probable there is maximum uncertainty:

Equation 3-9. Extremal properties

$$H_{\text{max}}(p_1, p_2, \dots, p_n) = H(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{2})$$

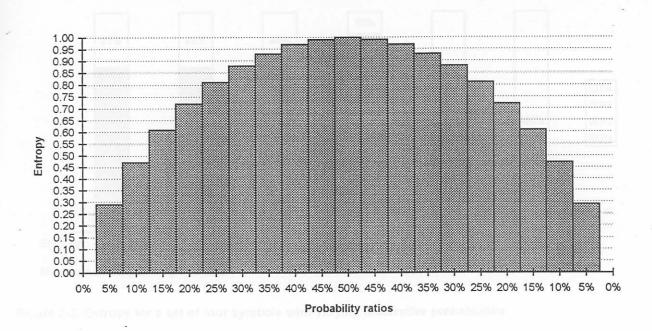
4. Additivity. In a system which contains a choice which itself comprises two choices, the original value of H is equal to the weighted sum of the H values of the individual possibilities.

The formula that Shannon arrived at and which satisfies the above criteria is:

Equation 3-10. Shannon's formula for information in a stochastic process

$$H = -K \sum_{i=1}^{n} p_i \log p_i$$

Figure 3-1 illustrates the entropy value in bits for probabilities of **p** (shaded areas) and **1-p** (white areas); note that as the probability of either **p** or **1-p** reaches its maximum value—meaning that the outcome of a choice becomes more certain—the entropy tends towards 0. When both **p** and **1-p** are equally probable, the entropy reaches its maximum value of 1 bit.



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Figure 3-1. Entropy for a set of two symbols with changing probabilities

An important implication of a set with total parity is pointed out by Singh:

For when all choices are equally probable and none prevails over any other, a person is, of course, completely free to choose among the various alternatives. As soon as one or more messages become more probable than any other, the freedom of choice is restricted and the corresponding information measure ... must naturally decrease. (Singh: 1967, p. 17)

The same principle applies to a set with a greater number of elements with varying probabilities. A message comprising four elements (p1, p2, p3, p4) with varying probabilities is demonstrated in Figure 3-2. Note how the summated entropy changes from maximum entropy when the four elements are equally probable to continuously lower entropy values as the probabilities become more unequal with some elements becoming more probable and others less probable. The first bar in the graph represents equal probability for the four elements, which become progressively more unequal.

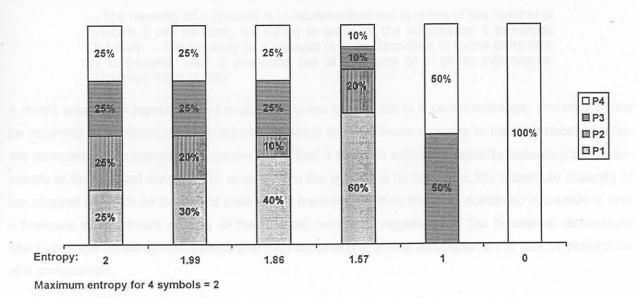


Figure 3-2. Entropy for a set of four symbols with varying illustrative probabilities

When the four elements of the example above are equally probable their entropy is at the maximum (2) and the quantity of information is at its maximum. As soon as the probability of some of the elements increases (reducing that of the other elements) the quantity of information they contain is reduced. Minimum information is generated as soon as one of the elements of the four has a probability of 100%.

#### 3.3.1 Maximum entropy

From the foregoing examples it is clear that when parity exists between the elements of a set of finite symbols in a message there is a definite relationship between the number of symbols and the sum of the information generated by each. As the number of symbols in a message increases the maximum entropy also increases. Using Hartley's formula (Equation 3-2) it is possible to calculate the maximum entropy a symbol-set is capable of generating and, ignoring any noise that may be present, this value would also be the capacity of a transmitting system. This value is expressed by the formula Log<sub>2</sub>n, where n represents the number of elements. As shown in Figure 3-1, a two-element message set has a maximum information value of 1 bit, while a four-element message set has a maximum value of 2 bits (see Figure 3-2). As freedom of choice decreases due to disparity of elements, the amount of information also diminishes and the influence of structural design becomes more prominent.

Shannon makes some important comments in this respect which are equally true when Information Theory is applied to music:

... The capacity of a channel is to be described not in terms of the number of symbols it can transmit, but rather in terms of the information it transmits. Or better, ... the capacity of a channel is to be described in terms of its ability to transmit what is produced out of a source of a given information. (Shannon: 1968, p. 16)

A music score, that represents the music by means of symbols is a coded message, and may hence be regarded as a communication channel of which the maximum capacity to transmit codes is virtually impossible to measure. The reason being that it tends to adjust its capacity according to the demands of the musical message. If, according to the quotation by Shannon, the maximum capacity of the channel depends on the actual code that is transmitted, then the code contained in a score is also a measure of maximum entropy of the musical message, regardless of the theoretical dimensions and size of the music-symbol palette that may be available to the composer at the time of conception of a composition.

#### 3.3.2 Relative entropy

The formulae thus far illustrated the calculation of the actual entropy of a message as well as the calculation of the maximum entropy that such a message could generate (when all the symbols have parity value). As such, these independent values merely reflect specific information about the message itself as a discrete instance and the results would not be suitable for comparative study because each different message would produce different maximum entropy values. To make the results more meaningful and to allow it to be used for comparative evaluation the entropy of the message may be expressed as a ratio of the maximum entropy of the elements of the set.

The relationship between the entropy and the maximum entropy of a message is expressed as a percentage calculated with the following formula, where **H** denotes entropy:

Equation 3-11. Relative entropy

$$H_{rel} = \frac{H}{H_{max}} (100)$$

## 3.3.3 Absolute and relative redundancy

When the information contents or entropy of a message reaches its maximum value, its predictability decreases diametrically because the selection of the symbols belonging to the set becomes increas-

ingly more equal. Whereas entropy refers to unpredictability or information of a message, predictability is referred to as *redundancy*.

The redundancy of a message is calculated by subtracting the entropy of a message from the maximum entropy. For instance, if H(x) represents the entropy of a message with n symbols, the absolute redundancy is:

Equation 3-12. Absolute redundancy

$$R_{abs} = \log_2 n - H(x)$$

As with relative entropy, if redundancy is to be used for comparative purposes, a common unit base needs to be used. For this purpose relative entropy is subtracted from 1 to obtain relative redundancy. (If relative redundancy is expressed as a percentage, relative entropy is subtracted from 100):

Equation 3-13. Relative redundancy

$$R_{rel} = \frac{\log_2 n - H(2)}{\log_2 n}$$
$$= \frac{1 - H(x)}{\log_2 n}$$

As the information set becomes more ordered, it becomes more predictable with an increasingly smaller information content until maximum redundancy is reached. Any major scale is an example of an element with maximum redundancy as each subsequent note in the scale is always fixed and fully predictable, at least as far as subsequent intervals are concerned. Major scales have a maximum entropy of log<sub>2</sub>7, or 2.81 bits. However, a melody in a major key would not have equal distribution of the seven notes of the scale and could be expected to have an entropy value of less than 2.81 bits.



# A HISTORICAL SURVEY OF APPLICATIONS OF INFORMATION THEORY IN MUSIC

The principles of Information Theory as the statistics of information in communication was formulated during the late 1940s. Besides Thermodynamics and Communication Engineering (see Chapter 3), many applications in a diversity of disciplines have benefited from the possibilities Information Theory presents. Some of the most important applications include cybernetics, psychology, seismology, oceanography, automation, mathematical logic, neurophysiology, biochemistry, linguistics, computer science, economics, and artificial intelligence. In some applications, the theory is mainly used to establish models based on existing data, which in turn may be used to make intelligent predictions. Other applications use the theory for comparative studies in which specific instances may be compared with global possibilities.

For reasons expressed in the introduction and perhaps, because of the artistic association of music, it has had relatively little exposure to the methods of Information Theory, especially during the last 25 years. A degree of proficiency in mathematics and computers is obviously required, and because it means venturing into disciplines traditionally considered to be outside the realms of music, musicologists have been hesitant to make use of the possibilities it offers. Established methods of analysing music have for many years proven themselves quite adequate in many respects and alternative methods are perhaps not regarded as an urgent need.

Nevertheless, as shown in Chapter 1, music is also a mode of human communication which could benefit from the innovations and discoveries of modern technology. Many disciplines such as music therapy, marketing, broadcasting, and accelerated learning—fields that are not always directly related to music—apply music functionally. These applications often require music of a specific character or style that has shown to be suitable to that particular application. Conventional music analysis is rarely used to select music for these applications and most of the selection is done intuitively or by trial and error.

Traditional analytical methods, because of the familiar parameters they present, remain invaluable for the study of music and its properties, technically and artistically. Because of its technical orientation it is an important method by which the art of music making, interpretation, and composition can be understood and taught. Traditional methods usually allow for a large measure of flexibility of interpretation, and is suitable for what is sometimes referred to as 'fuzzy matching'. In music this kind of imprecise comparison has proven to be very useful and through the years a complete technical vocabulary has been established which continues to expand. Yet, many of the terms used have a wide margin of meaning to allow diversity within a basic framework.<sup>2</sup>

The fact that analysts have resorted to general terminology to describe generic but diverse elements of music is an indication of the unpredictability of the art and therefore the creative forces that are at work. But although the traditional methods of analysis have an important role in the academic study of music, it is also often of a highly subjective nature which allows different analysts to arrive at different conclusions. However, by definition there is a difference between the technical features and the informational properties of music. If music is required for a specific application, that depends on its underlying communicative properties, a different approach or method is necessary to illustrate these properties.

Alternative methods of analysis are from time to time devised, many of which attempt to reduce the various structural levels of music to more manageable and concise values. The Schenkerian system of analysis is a good example of a system which, by reducing redundant elements, attempts to arrive at the underlying structure of a composition. In spite of the obvious merits of such new analysis methods and the fact that they are often recognised and accepted, they usually remain the domain of the specialists and are rarely accepted as part of the everyday study of music where the traditional methods remain firmly entrenched.

Progressive contribution to the knowledge about music very often comes from individuals who have less than a professional interest in music and whose field of specialisation is the technology which

In an attempt to be more precise, some authors invent adjectives which qualify specific musical procedures. Robert W. Ottman in his book *Elementary Harmony* (Prentice-Hall, 1983) uses the terms **Perfect authentic** and **Imperfect authentic cadence**, depending on the chord inversions and voice leading. By using these terms he tries to pinpoint specific characteristic features of

cadences which would otherwise resort under the generic term Perfect cadence.

Fuzzy matching is part of the science of Artificial Intelligence and allows for the comparison of parameters which have a broad base of structural similarity in general without being identical. Musical form is a common example in which fuzzy matching is applied in music analysis; theoretically there are a relatively small number of prototypes (sonata form, binary form, ternary form, and rondo form) to which a large quantity of music may be compared.

they use in the development of their new theories and techniques. This situation also applies to the research of the application of Information Theory to music which, to date, has mainly been done by researchers who, although specialists in communication technology or statistics, often have no more than a basic technical knowledge and a love for music.

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Most of the important research done to date by researchers is, with some exception, limited to specific aspects of music. Usually the aspects that are musically most obvious such as rhythm and pitch are singled out for analytical treatment with Information Theory. There are some researchers, however, who have ventured into more complex analyses but who, during the early years, were still hampered by technological limitations. In the Introduction (Chapter 1) it was pointed out that very little research has been done or at least was published since the 1970s. It seems that the ascendance of powerful personal computers and the possibilities they offer drew much of the attention of researchers away from the analytical possibilities. This is shown by the tremendous amount that has been published about computer applications in music during the last twenty years, much of which is devoted to methods of representing information of music in computer languages. The remainder of this chapter is a synoptic overview of researchers and their work on Information Theory applied to music.

## 4.1 Music analysis by means of information theory

An early reference to the possibility that Information Theory could be applied to music appeared in 1946 or 1949<sup>3</sup> when Shannon and Weaver published in their book, *The Mathematical Theory of Communication*. Although their treatise was not directly concerned with music the authors inferred the possible application of Information Theory to music when they stated that:

When we have an information source which is producing a message by successively selecting discreet symbols (letters, words, musical notes, spots of a certain size, etc.) the probability of choice of the various symbols at one stage of the process being dependent on the previous choices (i.e. a Markoff process), what about the information associated with this procedure? The quantity which uniquely meets the natural requirements that one sets up for 'information' turns out to be exactly that which is known in thermodynamics as entropy. (Shannon & Weaver: 1968, p.12)

Shannon only mentions music incidentally as a source of information and continues to describe the manner in which the principles of Information Theory are applied to information sources in general.

Nearly ten years passed before the validity of Shannon and Weaver's comments were realised. Although not addressing Information Theory as such, an important study of music as a communication medium and information source was done in 1956 by Leonard B. Meyer in *Emotion and Meaning in Music*. Approaching the subject from a psychological point of view, Meyer expresses the opinion that

There seems to be disparity about the date of this publication and according to the sources consulted the date of publication of this book varies between 1946 to 1962. The edition of Shannon and Weavers book used for this research is dated 1968 and is probably a reprint. A further reprint may be found in a collection of their most important papers on information theory, edited by David Slepian in *Key Papers in the Development of Information Theory*, IEEE Press, 1973.

musical style is part of a predictability pattern which becomes internalised and on which textbooks are subsequently based. (Meyer 1956: Chapters 1 and 2) He links emotional expectation to musical experience without venturing further into the province of information content as such. Not only did Meyer himself use many of the ideas contained in his book as basis for further research in Information Theory, but a number of subsequent researchers took note of his ideas.

The first reports about actual analytical studies with Information Theory also appeared during the 1950s. 'Information Theory and Melody' is the title of an article by Richard Pinkerton published in *Scientific American*. (1956, pp. 77-87) Pinkerton's research consisted of counting the pitches of thirtynine nursery melodies to obtain certain values which he subsequently used as a model to create melodies of his own. The method he applied to compose the melodies is discussed in the second half of this chapter.

Meyer continued his discourse a year later with an article, 'Meaning in Music and Information Theory', in the *Journal of Aesthetics and Art Criticism*. (1957, p. 412-424) In this article Meyer links 'meaning' in music with the information content of music. He refers to his book mentioned above (*Emotion and Meaning in Music*), when he says:

In that [book's] analysis of musical experience many concepts were developed and suggestions made for which I subsequently found striking parallels indeed equivalents in information theory. Among these were the importance of uncertainty in musical communication, the probabilistic nature of musical style, and the operation in musical experience of what I have since learned to be the Markoff process. (Meyer: 1957, p. 412)

In essence, Meyer's article deals with the correlation of musical meaning, value and information and arrives at the conclusion that these three elements are different but

... related experiential realisations of a basic stochastic process governed by the law of entropy. (Meyer: 1957, p. 424)

Meyer did not do any analyses himself, and limited his discussion to the arguing of hypotheses and intuitive concepts. Some two years later, in 1958, Joseph E. Youngblood published the results of his musical analysis in an article with the title, 'Style as Information', in the *Journal of Music Theory* in which he also expresses the belief that Information Theory could be suitably used to identify musical style. (Youngblood: 1958, pp. 24-35)

Youngblood's analysis is tentative as he only works on the pitch distribution of various melodies by Schubert, Schumann and Mendelssohn as well as four Gregorian chants. Although his work is important from a historical point of view, he does confuse information generated by stylistic conventions at the source (the composer and the score) and information as received by the listener. In his discussion

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of maximum entropy of the chants he rationalises the use of both a base of twenty seven<sup>4</sup> as well as twelve notes:

...modern listeners are prepared to respond to twelve divisions of the octave, and consequently, maximum uncertainty is for them represented by log12 and not by log27. (Youngblood: 1958, p. 31)

The problem with Youngblood's viewpoint is that he ignores the fact that a composition should be regarded as a closed discrete system in which the total number of symbols contained in the message dictates the maximum amount of information (in this case uncertainty). This was discussed in Chapter 3 and is a basic premise stipulated by Hartley (see page 3-4). What a listener is ready to respond to has very little bearing on the inherent style of a composition and the information it generates. Any limitation or freedom of choice a composer displays in his composition should be seen against the possible choices that are made available to him by convention, the system which serves as a frame of reference, and the actual limitation he consciously or unconsciously imposes on himself. Only a small portion of a very large range of possibilities available to the composer may be present in a specific composition and only those elements used by the composer represent the information content of the composition.

In the conclusion to his article Youngblood admits that other factors would have to be brought into consideration as well but confirms the validity of Information Theory as a method of stylistic analysis:

Most musicians can at present either intuitively or on the basis of certain vague generalisations identify at least five or six historical styles. It seems, however, that it would be useful to find a means of identifying and quantifying the characteristic features of style, as well as measuring the differences between styles, if for no other reason than to provide a basis for understanding and evaluating contemporary music. (Youngblood: 1958, p. 31)

One of the most exhaustive and thorough books on information and the arts was published by Abraham Moles in 1958, under the title *Théorie de l'information et perception esthétique*. Addressing aesthetic information of the arts in general, Moles heuristically explores the role and application of Information Theory to communication from the perceivers point of view. Chapters two and three refer extensively to Moles' research and conclusions. Following the publication of Moles' book, researchers on the subject were inspired by the ideas propagated by him and often rely extensively on definitions and descriptions contained in his book. It is unfortunate that the subject about which Moles wrote so thoroughly has not earnestly been pursued by other writers, and that his treatise which is now nearly 40 years old, remains one of the few extant works on this subject.

Youngblood's statement seems to be incorrect fundamentally, as he implies that listeners only respond to the twelve chromatic tones of Western music. He thereby implies that listeners would not respond to quarter tones or smaller intervals.

<sup>&</sup>lt;sup>4</sup> It is not clear why Youngblood refers to 27 notes, neither does he provide any reason. If he refers to enharmonicism, the total number of notes theoretically available is 35 (5 accidentals multiplied by the 7 diatonic pitch names).

Translated from French into English in 1965 by Joel E. Cohen as *Information Theory and Esthetic Perception* (University of Illinois Press).

'Information as a Measure of Structure in Music', is the title of an article by Edgar Coons and David Kraehenbuehl which was also published in a 1958 edition of the *Journal of Music Theory.* (1958: pp. 127 -161) Their approach is somewhat different to that of Youngblood in that they calculate the information of musical form in relation to human expectation with the general aim of finding a connection between musical and non-musical symbols:

Once the facts regarding the nature of pattern can be stated rigorously, it will become possible to develop a general theory of formal process in music. Since information is a measure of formal effectiveness that is independent of the specific nature of the elements composing the pattern, exact structural comparisons between musical and non-musical experiences will become feasible, paving the way to a sound theory of the symbolic processes in music. It seems reasonable to assume that the musical symbol and the reality it symbolizes have in common nothing more than their structural properties. (Coons & Kraehenbuehl: 1958, p.151)

Although hypothetically correct, Coons and Kraehenbuehl's theory is no more than just that; in music repetitions are seldom identical (except when music is repeated by means of repeat signs). According to traditional methods of structural analysis, slight variations in musical repetition will have little effect on the final structure. Information Theory on the other hand will identify such variations as being different rather than mere repeats.

Various scholars, stimulated by the challenges the new research in Information Theory presented, became involved in the science and presented their findings – usually of a tentative nature – in journal articles. Analysis with Information Theory, mainly on the rhythmical contents of melodies, was done by John Brawley in 1959 as part of research towards a Master's thesis at the University of Indiana, while a more general approach was taken by Joel Cohen in 'Information Theory in Music'. (Cohen: 1962, pp. 137-162)

However, the most important research during the 1950's and 1960's was done by Lejaren Hiller who contributed a number of articles and a book in the following years up to 1967. Hiller's main interest was in the application of Information Theory in composing music, which is treated at greater length later in this chapter. His book, *Information Theory and Musical Analysis*, (1962) is the first extensive work to elaborate on the application of the theory to music. In co-operation with Calvert Bean an analysis of four sonata expositions was done and the results published in the *Journal of Music Theory*. (Hiller & Bean: 1966, pp. 96-137)

A year later, in 1967, Hiller and Ramon Fuller published the results of an analysis of Webern's symphony op. 21, also in the *Journal of Music*. (Hiller & Fuller 1967: pp. 60 -115) This work by Hiller and Fuller is significant because conventional analysis is compared and combined with Information Theory analysis in an attempt to show how the two may complement each other. Part of the contents of this article is based on Fuller's thesis for a Doctorate of Musical Arts at the University of Illinois, *An Information Theory Analysis of Anton Webern's Symphonie*, Op. 21. (unpublished: 1965)

The conventional analysis of Hiller and Fuller's is straightforward and deals mainly with the pitch structure of the various appearances and transpositions of tone rows, the canonic structure, instrumentation, and rhythm. The entropy analysis is more extensive, in fact one of the most comprehensive until then. Four dimensions of entropy analysis are included: pitch, four different types of interval relationships, rhythm, and pitch and rhythmic combinations. The results of their computations are recorded on a large number of graphs.

In 1964 analytical research on Karnatic music was done by Gift Siromoney and K. R. Rajagopalan and the results published in an article with the title, 'Style as Information in Karnatic Music'. (Siromoney & Rajagopalan 1964: pp. 267-272) Applying similar criteria as Youngblood, Siromoney and Rajagopalan also limited their analysis to the counting of pitches. This analysis could have shown some interesting results because Karnatic music is based on a completely different tuning system than that of Western music and provides for seven *svaras* and twenty-two *sruthis* to the octave with a maximum entropy of 2.807 bits and 4.459 bits respectively.

For no apparent reason the two researchers limited the number of notes:

In this paper, we shall however represent the notes of the Karnatic system by their nearest notes in the Western scale. (Siromoney & Rajagopalan: 1964, p. 268)

The calculations are further incomplete because the particular scales peculiar to each *raga* type were not taken into account.

In 1966, L. A. Hiller and C. Bean attempted to solve problems of the analysis of musical structures by applying Information Theory to subdivisions of larger sections. (Hiller & Bean: 1966, pp. 60-115) The results of their investigation was published in the *Journal of Music Theory*, under the title 'Information Theory and Analyses of four Sonata Expositions'.

These studies mainly have in common the fact that they have either concentrated on the frequency of individual pitches or on rhythm only, and that pitch relationships and interval relationships were ignored. Establishing pitch frequencies is important in musical analysis with Information Theory in order to obtain an idea of note distribution, but to base an entire analysis only on this aspect of music without regard for any other, contributes little to the analysis of musical style. Pitch names by themselves merely outline intervals. Musical activity is largely dependent on the movement of intervals. The characteristics of a melody is therefore dependent on its intervallic structure and not the pitch names.

Some of the problems that Information Theory presents in the stylistic analysis of music were investigated in 1971 by Norman Dale Hessert in a dissertation towards a D. Phil. degree, *The Use of Information Theory in Musical Analysis*. He confirms that pitch frequencies are less valid by themselves and believes that interval relationships are more important:

- Using twelve melodic intervals from the unison through the major 7th makes possible a standard alphabet by which to analyse melodies of any style period.
- Analysing the interval content of a melody avoids the problems of determining tonal centres and exact points of modulation.
- 3. Intervals more clearly reflect the elements of redundancy and entropy (unity and variety) than do scale degrees. For example, using the scale as basis of analysis, the repetition of a motive at a different pitch level is reflected in the numerical results as an element of variety rather than unity.
- 4. Analysis at the level of first-order transition probabilities (single notes) is probably not very meaningful. It would be easy to show that two melodies which are dissimilar in style can have the same pitch content. (Hessert: 1971, p. 44)

Thus far most of the analytical research has tended to be inconclusive for the following reasons:

- Much of the research has concentrated on a single dimension of music such as melodic pitches, rhythm or form. Musical style is multifaceted and a single element can only represent one such facet.
- Even where multidimensional analysis was done, some researchers have tried to express the
  results of the analysis as a single numerical figure, instead as a model of multiple figures, of
  which each figure would represent one of the aspects of musical style.
- 3. The Markov Process (see Chapter 2), which is important for the indication of the tonal and structural coherency of musical style, has rarely been applied consistently. This has resulted in a simplistic application of Information Theory.
- The representation of entropy values purely as numbers usually does not allow for a clear overview of the results and therefore could tend to be meaningless and unfamiliar to many musicologists.

Hessert concludes his Ph.D. dissertation with the comment that ...

Certainly no application of information theory has thus far been able to reflect either the existence or the effects of the Markov process in its numerical results. And, since it is the Markov process which is the basis of this particular theory of messages and communication, it would seem that those who have worked in this area have been unsuccessful in obtaining any really meaningful results in their application of information theory to the analysis of music. (Hessert: 1971, p. 85)

Hessert's criticism is concerned with the way Information Theory had been applied and not with the theory itself. In fact, he specifically refers to the simplistic application of Information Theory to single aspects of music, often with disregard for the interrelationships which exist, not only between the various aspects, but also between smaller and larger elements of specific aspects. In all fairness it

should be added that most of the researchers who have laid the foundations of Information Theory and music were not musicians themselves which may be why they mainly concerned themselves with the most obvious musical elements; melody and rhythm. Musicians who venture into research with Information Theory usually did not have access to the technological resources that their colleagues in the Communications sciences had, and which was a restricting factor on the extent of their work.

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Not only has the value of Information Theory been recognised as an analytical tool, there were also a number of composers — professional and amateurs — who realised that Information Theory could be applied to the compositional process. The next section discusses some of aspects of composing with the help of the principles of Information Theory.

## 4.2 Music compositions based on information theory

Although composition is not the subject of this research some of the techniques used by the composers mentioned here address some of the arguments that are discussed. Composers have applied a variety of principles and methods of Information Theory to create their music that tend to support some the arguments that are expounded in this dissertation. Because of this some of the more important composers and their works as well as the principles they applied are discussed here.

Since the 1950s when computers and electronic musical instruments began their fast rise to general application, a number of composers have been working towards emulating the processes of musical compositions by using electronic equipment. Artificial intelligence, of which Information Theory is an important feature, has become an important element in composing music with the aid of computers. Artificial intelligence attempts to emulate human thought processes but requires that large amounts of information be programmed into a database which the program has access to. The information referred to here is synonymous with the 'rules' or parameters which need to be applied to sound in order to create acceptable music. Because computers are very effective for the manipulation of numbers, the application of Information Theory, which translates conventions into number values, may serve as an excellent vehicle for 'musical rules' to create music with computers.

Music composed with the aid of computers can play an important role in attempts to discover the factors that make up musical style. An artificial computer composition based on synthesised information may be compared with music composed by humans to give an indication of those elements which are required in the original computer programmes. Stochastic principles offer a method by which composers of computerised music may incorporate parameters into their compositions.

Pierce and Shannon were among the first researchers to compose music with the help of a computer which was programmed with information obtained with Information Theory. In 1949, they completed a chorale-like piece of music by selecting diatonic chords from a preconceived list of chords. However, the music they created was not yet totally generated by the computer. Besides the list of chords, they also manually manipulated the phrase structures and cadences. A rule which they included in the pa-

rameters was that each subsequent chord should have a note in common with the preceding chord and in the same voice (Pierce & Shannon: 1949) — an example of the stochastic principle at work.

The results were stylistically not very 'human' as is evident from Hessert's commentary:

The weaknesses of the composition... are many — the use of second inversion triads, the three consecutive ascending melodic fourths, the impossibility of root movement by second (sic) — but, in terms of information theory, the problems are that the probabilities for the occurrence of the individual chords are not taken into consideration and that the Markoff chain is not part of the compositional process. (Hessert: 1971, p. 88)

Hessert's observations concern the absence of common musical conventions which could have been resolved by including additional information in the generating programme. His criticism is therefore mainly based on his expectancy of a certain style of music in which certain conventions may be found. There was, in other words, too much entropy or information present in the composition generated by Pierce and Shannon to satisfy Hessert's preference. It is revealing that Hessert's objections are subjectively based on well known traditions and conventions. This is an important observation as far as this research is concerned, since he compares a composition which had been composed according to its own inherent criteria to musical style of his own personal preference. This seems to support the argument that Information Theory is able to extract and make prominent those elements that are regarded as preferential or necessary for a specific purpose. Pierce and Shannon's music has a high entropy value with the result that it is too unpredictable to fit within a preconceived style of music. The parameters which they included in their composition are, in other words, too sparse to create music of a recognisable style.

Some composers prefer to apply the principles of Information Theory without attempting to incorporate traditional conventions. Important work in this respect was done by lannis Xenakis whose composition, *Metastasis* (1953-1954) was created according to stochastic principles. In an article of 1966, *The Origins of Stochastic Music*, Xenakis, who is also a mathematician, explains that the creation of, what he calls probabilistic music in the twentieth century, was a natural outflow of serialism:

... in 1954 a music constructed from the principles of indetermination was developed from, amongst other things, the impasse of serial music; two years later I baptized this music 'musique stochastique'. It was as a musical necessity that the laws pertaining to the calculation of probabilities found their way into composition. (Xenakis: 1966, p.12)

Although Xenakis refers to indetermination in the quotation above, his application of stochastic principles, in reality implies a method of composition in which indetermination is controlled by basic predetermined factors. In fact his reference to the 'calculation of probabilities' confirms that he uses basic 'rules' within which indetermination is allowed to work. The basic techniques and theories on which

This is a translated and abridged version of the French article, 'Les musiques formelles', which was published in Revue Musicale (no. 253-254:1966). The translator is G.S.N. Hopkins.

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Xenakis based his compositions are contained in a series of articles and essays which appeared in a variety of French and German journals and a book published between 1955 and 1968.8

Further experiments with musical compositions generated by means of Information Theory, but in which the Markov chain features to some extent were done by David Slepian in 1951 and Richard Pinkerton some five years later.

Slepian's technique consisted of letting a colleague listen to the first portion of a bar of music. The colleague was then required to complete the bar. A second colleague listened to the portion completed by the previous colleague, and so on until the composition was considered complete. By applying this method Slepian ensured that each portion of music was always connected to what preceded it.

Pinkerton's approach differed radically and was based on the analysis of a group of nursery melodies according to the methods of Information Theory. He calculated the first and second transition probabilities (orders) for the seven notes of the diatonic scale plus an additional symbol which represented the rest or lengthened note. Using this information in a binary framework in which some notes had to be followed by others while other notes were followed by a choice of two notes, Pinkerton was able to conceive elementary melodies.

The most successful composition which incorporated stochastic principles was the *Illiac Suite* for String Quartet by L. A. Hiller and L. M. Isaacson. Composed in 1957 with help of a computer which was programmed with compositional rules and selections based on the Monte Carlo technique. Hiller and Isaacson's technique was rather complex and they based the suite on a combination of Hindemith's interval sequence (Series I) and that particular interval's distance from the unison (proximity value).

The numerical values they obtained are shown in Table 4-1 and were manipulated in a variety of ways to obtain different results and to ultimately generate the suite. 'Stylistic' variation in the *Illiac* Suite was achieved by increasing and decreasing the entropy value of the individual notes in relation to the whole.

The journals referred to here include *Gravesaner Blätter* (1955-1965), *La Nef* (1967), and *Revue d'Esthètique* (1968). The book from which certain portions were used is *Musiques Formelles* (Richard Masse, 1963).

A copy of the 'Illiac suite' was reprinted with corrections in the authors' publication, Experimental Music, (1959, pp. 182-197).
 The Monte Carlo technique is based on the laws of chance by which choices are made from sets of random numbers and made to conform to statistical controls.

Interval	Interval number	Hindemith Series I	Proximity value	Combined value
	V	X	Υ	Z=X+Y
Unison	0	13	13	26
Octave	12	12	1	13
Fifth	7	11	6	17
Fourth	5	10	8	18
Maj 3rd	4	9	9	18
Min 6th	8	8	5	13
Min 3rd	3	7	10	17
Maj 6th	9	6	4	10
Maj 2nd	2	5	11	16
Min 7th	10	4	3	7
Min 2nd	1	3	12	15
Maj 7th	11	2	2	4
Tritone	6	1	7	8

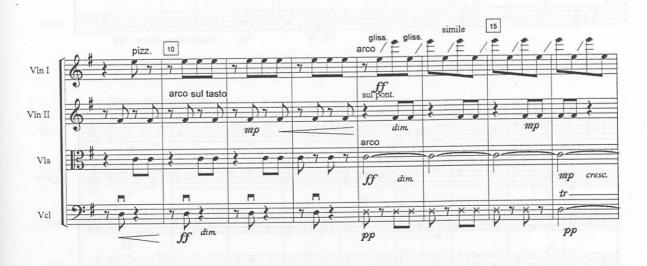
Table 4-1. Calculation of the note values for the *Illiac Suite* by L. A. Hiller and L. M. Isaacson

The following examples are extracts of the four movements or 'experiments', as the composers referred to them. A comparison of the first number of bars of each movement gives an idea of the degree of variation that the composers managed to achieve by using the basic range of notes shown in the table together with a varied manipulation of the stochastic processes.





Example 4-1. Illiac Suite: Experiment I, L. A. Hiller and L. M. Isaacson





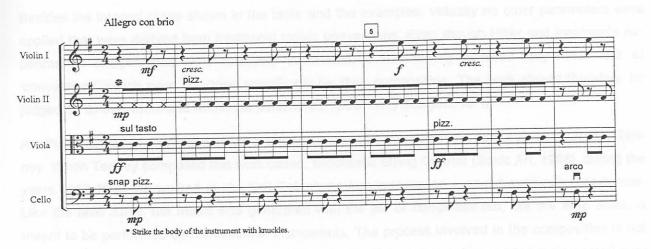
Example 4-2. Illiac Suite: Experiment II, L. A. Hiller and L. M. Isaacson







Example 4-3. Illiac Suite: Experiment III, L. A. Hiller and L. M. Isaacson







Example 4-4. Illiac Suite: Experiment IV, L. A. Hiller and L. M. Isaacson

Besides the interval range shown in the table and the examples, virtually no other parameters were applied that were derived from traditional music conventions. Even though Hiller and Isaacson's experimental compositions were relatively successful, it was mainly because they made use of 'conventions' which were designed specifically for their composition. The work should therefore be judged on its own merit and within the parameters specifically designed for it.

Another example of stochastic music is shown on the next page and is a composition by James Tenney. When Tenney composed this work called, Stochastic String Quartet (Sonic Art, 1988), during the years 1962-1963, he worked at the Bell Telephone Laboratories on a sound synthesis programme. Like the *Illiac Suite*, the music was generated with the aid of computers but, like the *Illiac Suite*, is meant to be performed by conventional instruments. The process involved in the composition is not unlike that used by Hiller and Isaacson, and Xenakis but the composer has exerted more control over the general structure of the piece as well as over the note values. Like the *Illiac Suite* the texture is predominantly contrapuntal.



Example 4-5. Stochastic String Quartet, James Tenney

#### 4.3 Conclusion

Some of the research applications in music that has been done since the formulation of Information Theory, were mentioned in this chapter. Little of this research has resulted in any dramatic changes in the approach to music analysis, neither has it had any marked affect on the traditional methods of music analysis. Investigation of this science has shown that although many other disciplines have benefited from the principles of Information Theory, little progress has been made in its application to music during the last two decades. Some of the applications have had an indispensable and far reaching influence on the development of technology, to such an extent that Information Theory has become an important branch of the science of statistics.

Besides the analyses of music that has been described in this chapter, other applications are mainly limited to composition in which a number of composers have used various aspects of Information Theory.



## COMPUTER PROGRAMMES FOR STOCHASTIC MUSIC ANALYSIS

Music analysis with Information Theory requires the processing of large quantities of information and a computer is an ideal medium for this task. In fact, when large compositions are comprehensively analysed a computer is indispensable and has some important benefits:

- 1. large quantities of information may be stored on a variety of storage mediums which may in turn be used with different systems and software packages;
- 2. access to, and accurate manipulation of information is greatly enhanced;
- representation of information and processed data may be extensively varied by using, among others, numeric, descriptive, and graphic formats;
- provided that the application programmes work properly and the information fed into the computer is correct, accurate calculations and results are ensured;
- results are relatively free of subjective information, barring those that are incorporated into the programme; and
- provided that the correct information is fed into the computer, experiments will be repeatable, even by different operators. In other words because of the fixed methodology that the programmes apply, the results should always be the same.

For the reasons mentioned above the author used computers and computer software extensively to collect and process information. All the programmes used for the analyses, calculations and presentation of the entropy values described in this thesis were designed and developed by the candidate. Although the initial prototypes and test programmes were written in Pascal, the programming languages used for the final versions of these programmes are a combination of C and Visual Basic<sup>®1</sup>

The programmes were written with specific criteria in mind:

- Entry of music data should be as uncomplicated as possible. Preferably by means of a graphical interface with a pointing device—the use of a keyboard for data entry is more prone to errors.
- 2. The programme should be able to identify as many errors as possible. For example, excessive note values in a bar.
- Errors in the database should be easy to rectify and by means of the same graphical interface as the entry phase.
- 4. The database should be able to contain the 'raw' information of the music as well as the results of the calculation so that these need not be re-entered or recalculated.
- 5. Results of the entropy calculations should be presented in a number of possible ways, i.e. numerical or graphical.

Although not specifically designed for entropy analysis, many software packages are currently available that could be used to do basic entropy calculations. However, although these programmes are sufficient to do the necessary calculations, the basic music information needs to be entered in numeric or alphabetic format—this is very time-consuming and prone to errors. The most useful type of programme to use for this purpose is one of the many available spreadsheet programmes, while some of the database programmes on the market also include the necessary mathematical functions. Two useful packages that were experimented with are EXCEL®, a spreadsheet programme which runs under windows®, and which has numerous functions suitable for calculating entropy values, and ACCESS®,² a database programme which is capable of interacting with EXCEL®, but which could also be used separately.

## 5.1 The music analysis programme

The programme developed for this research essentially comprises three separate programmes. Although the three programmes are capable of working independently they were designed to work to-

VISUAL BASIC is a new generation of the older programming language, Basic. This version is predominantly 'event driven' and relies to a large extent on graphical elements. VISUAL BASIC is published by Microsoft.

Both ACCESS and EXCEL are trademarks of Microsoft.

gether and to interact seamlessly. In other words each of the programmes is capable of working by itself without requiring that the other two programmes are loaded. The three programmes are linked by a number of databases which should be mutually available. The databases are:

#### Criteria database

This database is transparent<sup>3</sup> and not editable. It mainly contains all the information required to interpret the notation and in reality consists of various smaller databases.

#### Composition information

The identifying information of each composition is stored in this database. The fields<sup>4</sup> contained in this database are:

Composition identification number

Title of Composition

Composer's name

Date of composition

Composition category (defined by programmer)

Additional Information (for comments and other information)

#### Music Information.

All the music information for each of the compositions is stored in this database. The most important fields for the purpose of this research are (a large number of additional fields contain additional information used for other applications such as printing):

Composition identification number

Entry identification

Bar number

Pitch representation (numerical/alphabetical format: octave, pitch name, accidental)

Pitch number (allowing for octave range and accidentals)

Note value (note length expressed as a function of 128)

A transparent program function in the background and is not obvious to the operator.

Each field in a database represents a bit of information. A predetermined set of fields make up a record, while a group of records form the database.

#### **Entropy Information**

The results obtained by the entropy analysis routines are added to this database which has the following fields:

Composition Identification Number

Order Number

Entropy - pitch

Entropy - note duration

Entropy - pitch and note duration

Entropy - pitch ratios

Entropy - intervals

It will be noted that the three last databases have one field in common, the *Composition identification* number. This field serves as a link between the three databases for any of the compositions and allows the databases to contain the information of more than one composition at a time. To work with the information of a specific composition, its identification number is entered. Only that composition's information is then made available and database management is thus more effective.

The manner in which the three programmes interact and mutually have access to the databases described above is shown in the schematic illustration below. The dotted line connecting the three databases represents the *composition identification number*. The direction of the arrows indicates whether a programme only has access to data or whether it can also modify data in a particular database. For example, the interface for data entry has access to and can modify both the *composition information* database as well as the *music information* database but can only access the *criteria and rules* database without changing; the entropy analysis programme only has access to the *composition information* database and the *music information* database but can only modify the *entropy information* database:

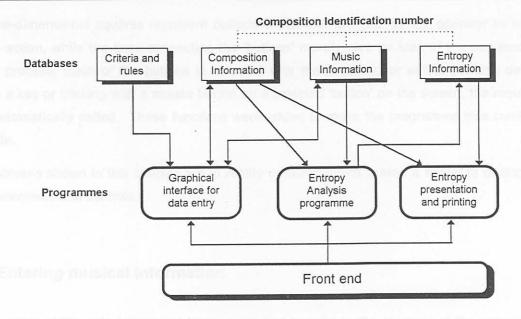


Figure 5-1. Databases and primary programmes for music analysis

The box marked 'Front end' represents a short controlling routine, or user-interface which calls<sup>5</sup> the three main programmes. This routine comprises the main screen of the programme and allows one to choose the composition for analysis or to enter a new composition. From this main screen the operator therefore controls the other programmes. The illustration below is a copy of the main screen which also shows why it is called a 'graphical interface'—many of the elements are represented by pictures some of which appear three-dimensional.

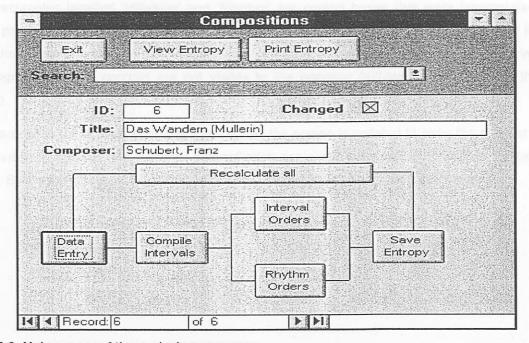


Figure 5-2. Main screen of the analysis programme

Computer terminology used when one computer programme or routine, branches to another programme or routine which then takes control of the processor.

The three-dimensional squares represent buttons which are 'pressed' by the operator to select the required action, while the lines connecting the 'buttons' merely give an idea of the sequence of the analysis process. Each of the buttons is selected with the keyboard or with a pointing device. By pressing a key or clicking with a mouse button on a selected 'button' on the screen, the required routine is automatically called. These functions were added to make the programme less cumbersome to operate.

All the screens shown in this chapter are in reality coloured which makes it easier to distinguish the various elements and controls.

## 5.1.1 Entering musical information

Because most of the calculations are transparent and happen in the memory of the computer, only two elements of the programme, the entry of information and the presentation of calculation results are visible. The following illustration shows the screen that is used to enter the musical information into the database.

As with the main screen, a pointing device or the keyboard is used to select each of the functions on the screen, a copy of which is shown below. First a pitch is selected, then the octave number and the accidental, followed by the note value. The note value is selected by pointing at one of the values shown in the right hand box and the selected symbol then changes colour from black to red. The small rectangles labelled 'HRvalue' and 'Rhythm' are control boxes that were incorporated at the testing phase of the programme and shows the converted value for each of the pitches. The list to the left of the screen is a copy of the database in table format and is also a control. In the final version of the programme these controls are not visible to the operator (although still present in the background).

After each entry, or after an entry has been edited the screen is updated to show the number of notes that have been entered, the currently entered pitch name and note value, as well as a listing of the entries. By means of cursor keys, the entries may be perused for alteration or insertion.

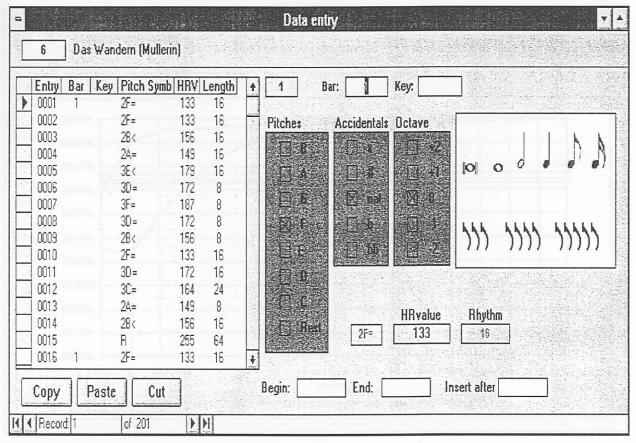


Figure 5-3. Data entry screen

#### 5.1.2 Displaying or printing the calculated information

Once all the calculations have been made the analysis programme is activated. These calculations are also transparent and the only indication that the program is running is a counter which indicates the programme's progress. The entropy that has been generated by the analysis programme may be displayed or printed in a variety of formats. If printed to the screen or to the printer, a choice may be made to print out the whole calculation process or just a summary in which only the final entropy values are printed. The following illustration shows a series of entropy values in graph form as it appears on the screen (in colour) and as it will look in printed form (in grey-scale). Details of the various parts of the graph and their interpretation is discussed in the next chapter. In addition to the graph, the screen below also shows the title of the composition, its identification number and the composer of the work:

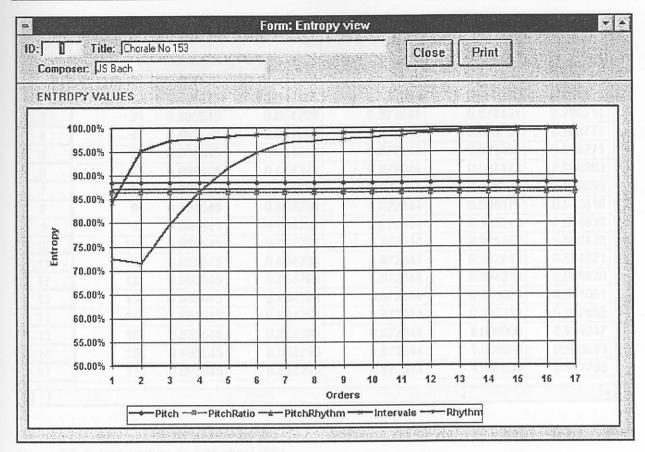


Figure 5-4. Entropy values in graph format

In the example above the straight horizontal lines represent the pitch, pitchratio and pitch-rhythm entropies. Since there is only one entropy value for each of these, the lines are drawn across the graph purely for the sake of clarity. The curved lines connect the respective entropy values for each of the orders of the stochastic processes of the intervals and the rhythm. The same formats mentioned above may be printed to a file on a storage medium such as a diskette or a hard disk in ASCII format. In this case the information may be incorporated in word-processing packages or, after suitable conversion, may be accessed by programmes such as spreadsheets from where it may be further manipulated.

The next illustration shows the same values as those of the previous graph. Note that the values in the first three columns are the same for all the order values. This ensures that a horizontal line for each specific value is drawn across the graph. Also note that the last three values (orders 15 - 17) for the Interval entropies are 100%, these are also automatically included by the programme to allow for the greater number of orders found in the Rhythm entropies:

Form: Entropy view - Datasheet							
		A	- B - C		-D →	E -*	
	Order	Pitch	PitchRatio	PitchRhythm	Intervals	Rhythm	
1	1	0.885645	0.864288	0.872044	0.843093	0.724332	
2	2	0.885645	0.864288	0.872044	0.952258	0.714624	
3	3	0.885645	0.864288	0.872044	0.972431	0.796714	
4	4	0.885645	0.864288	0.872044	0.976731	0.865777	
5	5	0.885645	0.864288	0.872044	0.983460	0.916271	
6	6	0.885645	0.864288	0.872044	0.987422	0.948083	
7	7	0.885645	0.864288	0.872044	0.988238	0.968699	
8	8	0.885645	0.864288	0.872044	0.989175	0.973416	
9	9	0.885645	0.864288	0.872044	0.990243	0.976633	
10	10	0.885645	0.864288	0.872044	0.991452	0.980430	
11	11	0.885645	0.864288	0.872044	0.992812	0.984821	
12	12	0.885645	0.864288	0.872044	0.994333	0.989820	
13	13	0.885645	0.864288	0.872044	0.996029	0.991061	
14	14	0.885645	0.864288	0.872044	0.997913	0.992465	
15	15	0.885645	0.864288	0.872044	1.000000	0.994047	
16	16	0.885645	0.864288	0.872044	1.000000	0.995819	
17	17	0.885645	0.864288	0.872044	1.000000	0.997798	

Figure 5-5. Entropy values in data-sheet view

The database programme that is used to gather the information and calculate the entropy values, is capable of exporting its information in a variety of formats to other commercially published software packages. The analysis results which appear in this dissertation were exported in a format suitable for importation to EXCEL, a spreadsheet programme. EXCEL allows further manipulation of the data for the creation of graphs and tables, and was used to incorporate the illustrations into this text (see chapter 6 and 7). The benefits of the exporting capability is that none of the data needs to be transported manually to word processors or other programmes, thereby excluding the possibility of any errors.

## 5.2 Aspects of computer programming for music applications

Some procedures and methods are explained here as they are common to all the computer programmes used for the analysis of this research.

To make the most effective use of the capabilities of a computer an important consideration when a score is translated into numerical information, is that a traditional music score represents sounds in a symbolic rather than in a logical and graphical manner. A single note represents various bits of information and the performer has to establish the pitch, accidental, volume and time value by interpreting it. Computer applications need to translate all this information into a workable system of numbers and the programmer needs to find a balance between what should be done manually, and what should be

left to the computer. Obviously, if most of the calculations are done by the computer the likelihood of errors is reduced, while otherwise time-consuming calculations are much faster. Entering musical information by using pitch names and numbers for note length means that the programmer has to do the translation from music symbols to the numbers the computer uses for its calculations. If, on the other hand, the programmer is able to enter music symbols by means of a graphical interface, and the computer does the translation into numbers, the process of data entry should be more efficient and less prone to errors.

However, to present music graphically on a computer screen requires extensive programming and large quantities of processing power. Because of the limitations of earlier personal computers the initial programmes, written in **Pascal**, required the programmer to translate the notes into a format suitable for the calculations. With the arrival of event-driven, graphical programming languages, the limitations of the older languages were largely eradicated. It, therefore became more efficient to use a graphical interface for data entry.

In designing the graphical interface many criteria had to be kept in mind. For example, pitch does not only imply a pitch name, but also an octave range as well as possible key signatures and accidentals. For a musician it is obvious that the sharps or flats of a key signature affect all of the same notes in all octaves (unless cancelled by a natural sign). This is, however, not so obvious to a computer and a great deal of programming is required to arrive at a value that reflects the pitch accurately. A similar situation occurs when a note is altered within a bar by means of an accidental. Traditional convention has it that all subsequent occurrences of the same note should be altered with the proviso that the note does not occur in a different octave. Some editors and composers tend to ignore this convention which further complicates the process of computer translation. Each time a new note is encountered either the programmer or the computer has to allow for the implied characteristics of a pitch.

Traditional music theory and convention also imply a distinction between enharmonically equivalent notes. Most sophisticated music systems allow for thirty-five different pitch names, which accommodate the five different pitch configurations that accidentals can generate, for example,  $C^{bb}$ ,  $C^{b}$ , C,  $C^{\#}$ , and  $C^{x}$ . As it should also be possible to apply information theory analysis to other systems, such as the dodecaphonic technique, and for that matter, also systems that may use quarter tones or smaller, the programmes had to be designed to allow for as many tuning systems as possible.

A similar situation of symbol interpretation arises concerning note duration and rhythm which, in traditional notation, is represented by specific notes and rest shapes in conjunction with a time signature and bar lines. The complexity of the interpretation increases with all additional symbols that affects the duration of a note. For example a fermata compounds the intricacies of computer interpretation of a note's duration. Note values in a score are symbolic representations of ratios of note lengths and relative accents but require extensive interpretation by a predetermined set of 'rules' incorporated in the conversion programme.

## 5.3 Computer representation of musical data

Most computer applications in music aim at incorporating as many elements of music as possible often seeking to provide for archaic and modern notation requirements as well. Usually the purpose of these programmes is to give an accurate graphic rendition of the score on the screen or in a printed format. Very often such programmes are also used as a tool for composition that requires it to be able to perform a composition with the help of synthesisers and other electronic equipment. This kind of application would require control over as many aspects of music as possible, including tone colours, attack, and envelope shape. There are a limited number of such programmes available e.g. MUSIC WRITER +®, SCORE®, PERSONAL COMPOSER®, and FINALE®. Research at many institutions on various applications of computers in music is being done continuously and a recent addition to the range of products is SIBELIUS®, currently one of the most powerful programmes available.

The complexity and dimensions of a software programme increases exponentially as the quantity of information demanded by it to work properly becomes greater. Especially the graphical representation of music demands intricate calculations and programming routines. Fortunately some of the facilities that the commercial programmes have, were not needed for the purpose of this research and much of the computer's resources could be used for the actual calculations. These functions—mainly MIDI compatibility—could nevertheless be added later.

Pitch and note values were the most important elements that needed careful consideration and the following sections mainly deal with the calculations of values for these elements respectively.

### 5.4 Pitch representation

## 5.4.1 Numerical pitch representation

Each of the pitches of a composition is represented by a single number. Since these pitch numbers are also used to calculate intervals between two adjacent pitches, the numbering system used for this purpose is designed in such a way that intervals are calculated by subtracting one pitch number from another. The pitch representation system developed for this research is based on a system formulated by Walter Hewlett and described by Ann K. Blombach. (ADCIS: 1989, pp. 50-58) Some minor alterations were made to make the number allocation more suitable to the requirements of the programmes. The Hewlett system allocates a specific and unique number to each letter name. As shown in Table 5-1, the value allocated to each of the pitch names was 2 greater than the Hewlett numbers. The reason for this change is that when a flat or a double flat (see Table 5-3) is added to the pitch name 'C', of which the Hewlett number is '0', the resulting value, in Hewlett numbering is -1 or -2 respectively. By increasing all the values by two, negative numbers are avoided:

Pitch Name	Pitch Number	Hewlett Number
С	2	0
D	10	8
E	18	16
F	25	23
G	33	31
Α	41	39
В	49	47

Table 5-1. Hewlett pitch values

Accidentals are also given a numerical value. The third column of the table below illustrates the value of each accidental; the second column shows the symbols used for each accidental:

Accidentals	Computer symbols	Accidental values
double flat	<<	-2
flat	<	-1
natural	space	0
sharp	>	1
double sharp	>> 10	2

Table 5-2. Numerical values of accidentals

By adding the value of the accidental that is attached to a note, to the value of the note itself, each note obtains a unique value, or HR<sup>6</sup> number:

Pitch	HR	Pitch	HR	Pitch	HR	Pitch	HR	Pitch	HR
Cpp	. 0	C <sub>p</sub>	1	С	2	C#	3	C×	4
Dpp	8	Dp	9	D	10	D#	11	D <sup>x</sup>	12
Epp	16	Eb	17	Е	18	E#	19	Ex	20
Fbb	23	Fb	24	F	25	F#	26	F <sup>x</sup>	27
G <sup>bb</sup>	31	G⁵	32	G	33	G <sup>#</sup>	34	G <sup>x</sup>	35
Abb	39	Ab	40	Α	41	A <sup>#</sup>	42	Ax	43
Bpp	47	B⁵	48	В	49	B#	50	B <sup>x</sup>	51

Table 5-3. Pitches and their representative HR numbers

Using these values any melody may be represented as an array of numbers, for instance the G major scale:

Pitch	G	Α	В	С	D	E	F#	G
HR number	33	41	49	56	64	72	80	87

Table 5-4. HR numbers for the G major scale

The numbers in Table 5-3 are for one octave only and 54 is added or subtracted from the pitch number for higher or lower octaves respectively. For example, the G above middle C has an HR value of 33. The number of G two octaves above middle C is therefore  $33 + (2 \times 54) = 141$ . To avoid negative numbers that would be generated by octaves below middle C, the numbering system used for the

<sup>6</sup> HR is a mnemonic derived from 'Hewlett System of Pitch Representation'.

programmes under discussion begin with the C that lies three octaves below middle C. The HR number of middle C, for the purpose of this research, is therefore  $2 + (3 \times 54) = 164$ .

#### 5.4.1.1 Meantone tuning and equal temperament

With traditional theoretical practice in mind, the pitch values described above are ideal as they give a different value for otherwise enharmonic intervals and pitch representation. Obviously for music that is not composed according to the theory of the old tuning systems, as for example much of twentieth century music, alternative methods could be devised. In twelve tone music there are only twelve tones, as opposed to the thirty five of the older convention, and composers are forced to make use of the archaic note representation purely because of the lack of viable alternatives and not because of the possible structural or tonal implications such a note symbol might have.

For a stylistic analysis of this nature, a single procedure in which no allowance is made for the tonal and harmonic limits, would produce erroneous results. If a true stylistic evaluation is to be achieved, it should be kept in mind that composers who were active when equal temperament prevailed, worked under totally different acoustic conditions than those who composed with the acoustical principles of equal tuning. Equal temperament allows a choice of 12 notes, each of which, because of enharmonicism, can be written in a number of different ways. Meantone<sup>7</sup> tuning allows for 35 different tones, only some of which may be selected depending on the tonal centre, mode and instrument. The dissimilarity in the basic theoretical principles that underlie the two tuning systems needs to be facilitated in the stylistic analysis.

A method was devised that allows analysis according to both tuning systems. For equal temperament the enharmonic pitches and intervals are simply equalised and a second array created in the database so that each melody is represented by two numerical arrays, one for meantone pitches and intervals and one for equal temperament. Pitch names for the equal temperament array use only the letter names and accidentals which result in perfect, major, and minor intervals, and in the case of fourths and fifths, also augmented intervals.

#### 5.4.1.2 Numerical interval representation

The modified HR numbering system discussed above provides a quick and simple method to obtain numerical values for intervals between two pitches. Interval values are obtained by subtracting the HR value of the lower note from the HR value of the upper note; the perfect fifth,  $E^b$  -  $B^b$ , for example, has an interval value of 48 - 17 = 31. Negative values are made positive by adding 54. For ex-

The fact that there are a variety of ways in which the intervals of meantone tuning may be calculated, is of little significance here. Only the fact that the meantone differentiates between tones which are enharmonically interchangeable within the system of equal temperament are of importance.

ample, the value for the a perfect fifth,  $B^b$  - F = 25 - 48 = -23. By adding 54 to the latter, the value of 31 (perfect fifth) is again obtained. The following table is a list of the various intervals and their number values. The order of the intervals in the table is not based on the actual musical sizes but rather on the numerical value allocated to each interval.

Interval	Value	Interval	Value
unison	0	diminished fifth	30
augmented unison	1	perfect fifth	31
diminished second	6	augmented fifth	32
minor second	7	diminished sixth	37
major second	8	minor sixth	38
augmented second	9	major sixth	39
diminished third	14	augmented sixth	40
minor third	15	diminished seventh	45
major third	16	minor seventh	46
augmented third	17	major seventh	47
diminished fourth	22	augmented seventh	48
perfect fourth	23	diminished octave	53
augmented fourth	24	octave	54

Table 5-5. Interval values derived from HR numbers

It should be stressed that the HR interval values do not reflect, in terms of semitones, the actual acoustical values of the intervals they represent. They are merely a method by which intervals are calculated in the computer programmes and have properties which expedite a variety of manipulations. One property which is rather useful is the symmetrical properties of the numbers. An interval inversion, for example, is calculated with the formula: ABS(interval value - 54)<sup>8</sup>. For instance the inversion of a major sixth (39) has a numerical value of 39 - 54 = -15, with an absolute value of 15 - 40 minor third.

The system has the added benefit in that each number may be stored as a single ASCII<sup>9</sup> code in the database. To simplify the computer programming, ASCII characters were also used for data and string manipulation. This aspect is described later in this chapter. ASCII codes are a standardised series of letters, numbers and symbols, each of which has a specific number (0-256). The symbols are referred to as characters. Since each character always has the same number, it may be used as a mnemonic to represent number sequences.

# 5.4.2 Music rhythms

Programmers who write computer programmes for musical application have a variety of approaches to translating note values and rhythms into computer language. The most common approach is to

ABS means 'absolute value' and converts all numbers to a positive value.

ASCII is a mnemonic which stands for 'American Standard Code for Information Exchange', and ensures effective communication between different electronic media.

allocate a number to a specific note value, with the smallest note value being allocated the number 1, and each successive note value sequentially designated a higher number. Another method, and one which is often used in computer programmes, is to allocate letter symbols to the various note values. Two such systems are shown below in Table 5-6; the DARMS<sup>10</sup> system makes use of mnemonics while the MUSTRAN<sup>11</sup> system uses numerical representation. (Wittlich, Schaffer & Babb, pp. 20-23)

NOTE VALUE	DARMS	MUSTRAN
whole note	W	1-1-1-1
half note	Н	2
quarter note	Q	4
eighth note	E	8
sixteenth note	S	16
thirty-second note	To a second	32
sixty-fourth note	X	64
one hundred twenty- eighth note	Y	128
dot	ed and that he	
tie	F (prefixed)	F (before second value)
rest	R (prefixed)	R (after value)

Table 5-6. DARMS and MUSTRAN note duration codes

The DARMS system is the least complicated and very suitable if mathematical manipulation is not required. However, this approach becomes very complicated when dealing with dotted and double dotted notes, tied notes, and irregular subdivisions such as triplets. For the application of Information Theory an adapted form of the MUSTRAN system is much more suitable for the calculation of ratios, i.e. the expression of duration as a percentage of all the note values combined in the section of music being analysed. By inverting the sequence of numerical values that is allocated to each of the note values in the table above, algorithmic manipulation becomes relatively easy. The range of available values allows for 128<sup>th</sup> notes. The latter is very rare in music but allows for more accurate calculations.

The values used for the programmes of this research are shown below. Each of the note values is given a number which is directly related to the actual value of the note expressed as a multiple of 128.

NOTE VALUE	MUSTRAN
whole note	128
half note	64
quarter note	32
eighth note	16
sixteenth note	8
thirty-second note	4
sixty-fourth note	2
one hundred twenty- eighth note	1

Table 5-7. Adapted MUSTRAN note value system

The DARMS code was originally called the 'Ford-Columbia' code and was developed by Stefan Bauer-Mengelberg.

Jerome Wenker developed the MUSTRAN code at the Indiana University for application in ethnomusicological research.

To obtain the relative note values, each note is expressed as a ratio of the total duration of the complete composition. The latter is obtained by simply adding all the note values together. The bar division, which dictates the position of accentuated note gradations, or beats, form the subdivisions of the overall composition. A piece of music without any bar lines is regarded as comprising a single bar due to the absence of any regular metrical subdivision of the music. After a note is entered into the database, the note value is converted to a multiple of 128. This means, for instance, that a dotted quaver has a value of 24 (16 + 8), while a quaver triplet has the value, 21,33. In contrast to the pitch number, which is stored in ASCII code, the note value is stored as a number because these calculations may result in fractions as is the case of the quaver triplet. ASCII code cannot be presented by fractions.

Another approach that was considered was to express a note value in terms of real time, in other words as note value per second or per minute. However, this approach requires that the tempo indications and variations are constantly kept in mind and that note values are continuously calculated in relation to the tempo of the music. Unless, as in the case of some types of electronic music, it is expressly composed with specific real time criteria and limits in mind, using real time as a measuring unit is not an effective or expedient method.

# 5.5 The computer algorithms

Each of the main programmes discussed in this chapter comprises a number of different routines, some of which are not exclusive to a single programme. For the sake of clarity each of the routines are demonstrated here as if they are totally independent.

In the flow charts that follow the arrows indicate the direction of the flow of the programme and the branching is done according to basic Boolean logic. The boxes with the light borders represent the routines where branching takes place, while the rectangles with the heavier borders represent the functions that are called by the branching routines.

# 5.5.1 Algorithms for melody analysis

Analysis of the melodic data comprises different steps and can be done according to the following parameters controlled by the operator:

- 1. Pitches only
- 2. Pitches by ratio
- Pitches and note values combined
- 4. Stochastic analysis of rhythm

#### 5. Stochastic analysis of intervals

As the melodic information is stored in an array it is relatively easy to perform string manipulations and readily allows comparison between various sections of the array. The algorithms for each of the types of analysis listed above are described below with the flowcharts.

### 5.5.1.1 Pitch and rhythm analysis

Because of their similarity, the algorithms for the calculation of single pitches and note values are discussed simultaneously. In principle a shadow array<sup>12</sup> is created to hold the various values, be they pitches or note values. Looping through the codes of the melody, each new character is added to the shadow-array and a counter<sup>13</sup> for the number of different codes (C)<sup>14</sup> as well as a counter for that specific code (SC) is increased. If a code already exists in the shadow-array, only the counter for that specific code (SC) is increased. For every code encountered a counter for the total number of codes (TC) counted is increased as well.

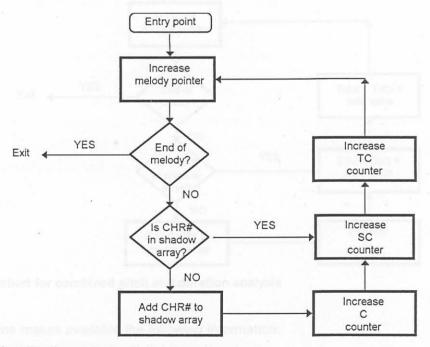


Figure 5-6. Flowchart for the analysis of pitch or note values

Once the end of a melody is reached, three different totals become available:

1. The total number of different codes encountered (C);

Shadow arrays are similar to normal arrays but only stay in the computer's memory temporarily. Once it is no longer needed it is removed from memory. Usually it contains duplicate or temporary data for comparison or manipulation.

Counter usually begin with a value of 0 and are increased each time a specific action occurs.

These are mnemonics used as variables in the programme. Variables represent registers which can hold a variety of changing data.

- 2. The number of times each code occurs (SC); and
- 3. The grand total number of codes in the melody array (TC).

Another algorithm uses these results to calculate the various entropy and redundancy values. Towards the end of this chapter the algorithms for entropy calculations are discussed.

#### 5.5.1.2 Pitches combined with duration

The flowchart for the routine to calculate the values of pitches based on their duration values is very similar to the one that merely calculates the frequencies of pitches. It differs in that duration is used as the measuring unit in multiples of 128<sup>th</sup> notes. Each time a note code is encountered, the note value associated with it is added to the counter:

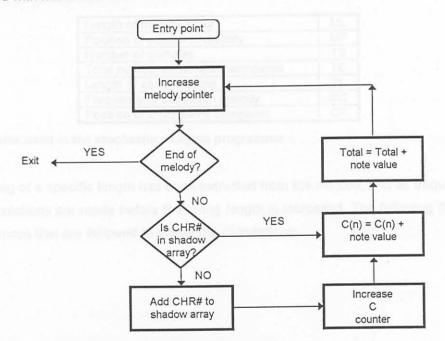


Figure 5-7. Flowchart for combined pitch and duration analysis

The above routine makes available the following information:

- The total number of pitch codes in values of 128<sup>ths</sup> (Total);
- the occurrence of the different pitches, also in values of 128<sup>ths</sup> (TC[n]);
- the total number of different codes encountered (C).

# 5.6 Stochastic analysis of intervals and rhythm

The programme routines for the stochastic analysis of melodies are somewhat more complex as the frequency of increasingly larger portions of the melody have to be counted and compared. In order to

achieve this a two-dimensional interval array, containing the interval values for both equal temperament as well as meantone tuning, is created. As with the pitches the intervals are converted to ASCII codes, which make comparisons easier to manage. The second dimension of the array contains the direction of the interval and is indicated by a mnemonic; '>' for a descending interval, and '<' for an ascending interval.

In essence the programme consists of various nested loops. In the principal loop the string length for comparison is continuously lengthened until it has the same length as the melody array. A second loop, which is nested inside the first loop, runs through the melody array, first compiling the sample and then counting the frequency of its occurrence by stepping through the melody from beginning to the end. Other loops in the program are dependent on certain conditions being met.

The variables<sup>15</sup> used in the programme and flowchart are shown in the following table:

Length of melody array	ML
Position of pointer in melody	MP
Number of samples	TS
Total number of samples compared	TC
Length of sample string	SL
Frequency of sample in melody	SC
Position of array being compared	CP

Table 5-8. Variables used in the stochastic analysis programme

Each time a string of a specific length has been extracted from the melody, and its frequency counted the entropy calculations are made before the string length is increased. The following flowchart illustrates the sequences that are followed to obtain the calculations:

Variables are labels given to pieces of information. A counter is a variable which is incremented or decremented every time a condition is met. Fixed variables cannot be changed and usually serve as a mnemonic for a specific numerical value.

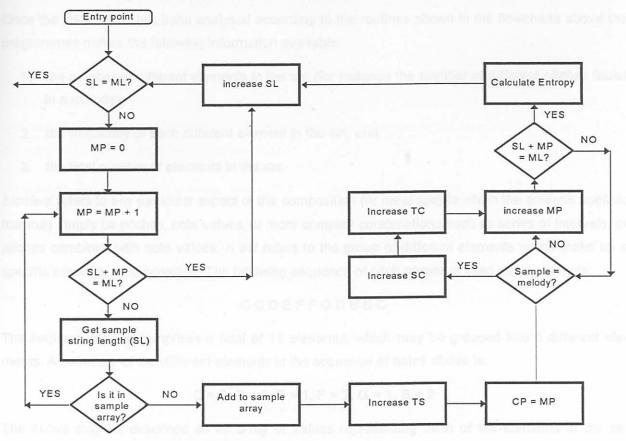


Figure 5-8. Flowchart for stochastic melody analysis

The flowchart for the stochastic analysis of rhythm is virtually the same, except that it only needs a one-dimensional array because there is no need to allow for any interval differentiation.

### 5.6.1 Calculation of entropy values

Once all the necessary data has been collected in the database it may be analysed using some basic algorithmic calculations. The computer used for the analysis in this research, an IBM compatible personal computer, is fitted with a mathematical co-processor and has a precision of seven decimal digits and a dynamic range of 10-38 to 10+38. The co-processor increases the speed of calculations and is more accurate but is not a prerequisite.

One problem that was encountered was that some calculations result in extremely small fractions, smaller than seven decimal places, operations on these tend to result in an error of < 0.0000001. Because results of analyses are only calculated to the nearest sixth decimal place, this error has no significant effect on the results. However, it causes summations which are placed within a continuous loop, and which can only be exited when two values are the same, to continue ad infinitum. The only way to overcome this was to round off any number to the nearest sixth decimal place.

Once the information has been analysed according to the routines shown in the flowcharts above the programmes makes the following information available:

- the number of different elements in the set (for instance the number of different pitches found in a melody);
- 2. the frequency of each different element in the set; and
- 3. the total number of elements in the set.

Element refers to any particular aspect of the composition (or message) to which the analysis applies; this may simply be pitches, note values, or more complex combinations such as series of intervals, or pitches combined with note values. A set refers to the group of different elements which make up a specific aspect of a composition. The following sequence of pitch names is used as an example:

#### CCDEFFGDBBC

The sequence above comprises a total of 11 elements, which may be grouped into 6 different elements. A summery of the different elements in the sequence of notes above is:

$$C = 3$$
,  $D = 2$ ,  $E = 1$ ,  $F = 2$ ,  $G = 1$ ,  $B = 2$ 

The above may be described as an *array* of values representing each of the elements in the sequence. An array is thus a summary of the different elements in a set. In computer language the shown example would be contained in a two-dimensional array — dimensioned as: *Array*(6, 2) — to contain:

- 1. the number of different elements (6 in this case) and
- the quantity of each specific element in the array.

A table illustrates the structure of the array more clearly:

	Counter:	1	2	3	4	5	6
1	Pitch:	С	D	Е	F	G	В
2	Quantity:	3	2	1	2	1	2

Table 5-9. Illustrative values in an element array

If the array, represented by A contains the frequency of each element of a set of n elements, the total number of elements would be expressed as:

Equation 5-1. Total elements in an array

total elements = 
$$A_1 + A_{2+} \dots A_n$$
  
=  $\sum A_n$ 

With the value of the total set as well as the values of each of the elements now available, the relative frequencies of each element may be calculated by dividing the value of each element by the total set value. This is shown in the equation below:

Equation 5-2. Element proportions of total elements

$$pA_n = \frac{A_n}{\sum A_n}$$

A simple calculation at this point checks for any calculation error by adding all the relative frequencies together. If the calculations were done correctly, the summation should add up to 1.

The next step is to calculate the entropy values of each of the different elements in the set, A, and summate them to obtain the actual entropy value, H, of the set as a whole. In order to obtain the information contents of the set in bits, the binary logarithms of the relative frequencies are weighted (multiplied) by the relative frequencies, p, and summated:

Equation 5-3. Calculation of the entropy of a set of values

$$H(pA_1, pA_2, ..., pA_n) = -(pA_1 \log pA_1 + pA_2 \log pA_2 + ... pA_n \log pA_n)$$
or
$$H = \sum_{i=1}^{n} pA_1 \log_2 pA_1$$

To achieve the information (entropy) value of the set, a simple loop is set up that operates on the relative frequencies value of each element and adds it to a variable called entropy\_total<sup>16</sup>. The flow-chart below illustrates the steps the programme runs through to achieve this. As PASCAL has no direct function to do conversion from logarithms on base 10 to binary logarithms, it was necessary to include an algorithm in the programme to do this conversion. This algorithm is also shown in the following flowchart. The variable EV is used to hold the entropy value:

onger mnemonics for variables consisting of more than one word are connected with underscores.

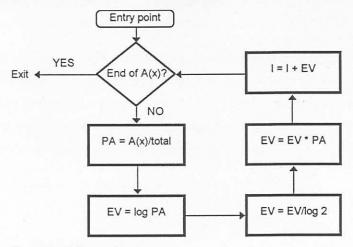


Figure 5-9. Flowchart for the calculation of entropy

Maximum entropy is somewhat simpler to calculate as it requires no loop and the operation is only done on the number of elements in the set, where N represents the total number of elements in the array being investigated:

Equation 5-4. Calculation of Maximum Entropy

$$H_{\text{max}} = \frac{bin \ N}{bin \ 2}$$

With the entropy value and maximum entropy values in two separate variables a third variable is used to calculate the relative entropy as a percentage value:

Equation 5-5. Calculation of relative Entropy

$$H_{rel} = \frac{H}{H_{\text{max}}} 100$$

A fourth variable is used to hold the redundancy value:

Equation 5-6. Calculation of relatively Redundancy

$$R_{rel} = 1 - H_{rel}$$

## 5.7 Conclusion

In the development of the analysis programmes a structured approach was adopted, in which sections of each of the three main programmes were divided into smaller routines. Each phase was thoroughly tested to assess the effectiveness and accurateness of each routine and where possible error traps were included to further ensure accuracy of the results.

6

# ANALYSIS OF SELECTED COMPOSITIONS

This chapter is devoted to the entropic analysis of a number of selected compositions, the results of which are summarised in Chapter 7. The first part of this chapter deals with the more fundamental aspects of entropy calculations—in other words, sets comprising single elements such as pitch distribution. In the second part, compound calculations, those comprising more than one order, are shown and discussed. These are pitch analyses based on ratio, and analyses based on pitch combined with note values.

For the sake of reference, a distinction is made between note values and rhythm. When referring to note values, the duration of a note (or rest) as an independent unit is implied, with the smallest unit being 128<sup>th</sup>. For instance, a quarter note has a value of 32, while a whole note has a value of 128. Rhythm refers to the grouping of notes (and rests) according to patterns of note sequences. In other words, whereas note value refers to a note's temporal property, rhythm refers to its recurrent position in a linear sequence of notes.

#### 6.1 Selection of the music

The melodies of twenty-two compositions analysed were selected from three categories of music: seven from the popular repertoire of the last twenty years, eight from the Art song tradition of the 18<sup>th</sup> century, and seven songs from the repertoire of Art songs of the 20<sup>th</sup> century. The reason the Classical Art song group has one more composition than the other groups is that one composition,

Schubert's Ave Maria shows characteristics that are similar to those of the Popular Music group. As will be made clear in the following section, this composition is treated separately.

In the following discussion, tables, and graphs the abbreviations 'P', 'S' and 'M' are used to refer to the categories 'Popular songs', 'Serious Art songs' and 'Modern Art Songs' respectively. These labels are not intended to imply any qualitative attributes to any of the categories, but are merely used for the sake of expediency and identification. They distinguish between three styles of music which are different in character, style, and in many ways, in purpose.

Appendix I contains the scores of the melodies of the twenty-two songs.

# 6.1.1 Popular contemporary music

The selection of the popular music for analysis proved to be more difficult than had been anticipated. Originally the intention was to request sales statistics from the larger recording companies and publishers. For many years, weekly or monthly charts were made available by these companies indicating the 'Top Ten' or 'Top Twenty' most popular recordings for that week or month. Because of the complex preference for different kinds of music in South Africa, these popularity charts are no longer made available on a regular basis, the reason being that only sales of Western popular music used to be represented in these charts, disregarding the popularity and sales of township music which often exceeds the sales of Western pop music in this country.

A number of record companies were nevertheless prepared to provide the candidate with the sales figures for 1994, but is was soon realised that by limiting the selection of items from this list would in reality limit the choice to a specific and limited period. This could mean that a song appeared on the list of top sellers because of greater promotional efforts by the publishers or exposure by the media. That an artist may already enjoy a degree of popularity with his audience could have an important influence on the sales of a new recording. However, many of the songs that appear on popularity lists do not maintain their popularity for extended periods and are soon forgotten.

An additional problem was that very few of the compositions that appear on popularity lists are immediately available as sheet music. Transcriptions are usually only made available after a piece has proven its popularity over an extended time, and depending on popular demand. The information provided by the record companies was of the most popular albums sold during 1994. Sales figures of record albums do not necessarily give any indication which individual song (or songs) in the collection is the most popular. The decision therefore was to select songs based on their consistent popularity. In other words, songs that essentially have become 'classics' in their own right. That the more consistently popular items usually come out in print, already suggested which items to choose from the repertoire available. Obviously, the choice had to be limited to only a small selection from this large repertoire. In a sense, based on the arguments mentioned, the music publishing industry has made the

selection of popular music possible by its own dynamics<sup>1</sup>. Periodically, collections of compositions of the most popular items are being made available in both recorded and printed form. Since these collections are expensive to produce, it is obvious that mainly those compositions that have a proven record of popularity over a longer period are included in such collections.

A variety of CD catalogues and guides were used to ascertain the most popular recordings and which appeared most frequently in the recording guides. The sheet music for these items were also readily available, most often in albums or collections that confirmed the continuing popularity of the chosen items.

Items included in the popular music selection all became popular during the 1970s and the 1980s and therefore have a proven popularity history of between one and two decades. They are:

- Benny Andersson & Björn Ulvaeus (music and lyrics), Thank you for the music, London: Bocu music, 1977. Made popular by the Swedish group, ABBA.
  - Phil Collins (music and lyrics), One more night, London: Hit and Run Music, 1984. Recorded on Virgin Records by Phil Collins himself, and one of the singer's most popular compositions to date.
  - John Denver (music and lyrics), Annie's song, London: Winter Hill Music, 1974. Recorded on RCA Records by the composer.
  - Claudio Gizzi (music), Summer love, Johannesburg: EMI-Brigadiers Music, 1976. Another instrumental piece, made popular by the Pan-flute player Zamfir on Philips (TOS 1072). The cover of the score states that this piece was '16 weeks on the Springbok Top 20'.
  - Johnny Pearson (music), Sleepy shores, theme from the BBC TV series Owen M.D., Johannesburg: Bandstand Publications, 1971. This is not a song; it is a piano solo of which the melody is still very popular today.
  - 6. Stevie Wonder (music and lyrics), You are the sunshine of my life, Hollywood: Stein & van Stock and Black Bull Music, 1972. Recorded on Tamla Records. This item was made popular by Stevie Wonder himself and is one of his songs that helped to make him popular.
  - 7. Victor Young (music) and Edward Heyman (lyrics), Love letters, Woodford Green: Warner Bros/IMP, 1988. Originally published in 1945. Although this piece was composed fifty years ago, it was included because of the fact that it was revived during the 1980s. It would therefore be interesting to see how it compared with the more recent compositions.

All the items in the list above are by different composers, and were made popular by different artists. Because of their lasting popularity amongst the general public, these items may be regarded as representative of the popular music that appeared over a twenty year period. It seems a fair deduction

In this case Market forces.

that they therefore contain those elements that are appealing to the listeners.

# 6.1.2 Classical Art Song selection

Since most art songs are available in print and have been on the market for many years, the popularity of music from the 'serious music' repertoire could not be judged by the availability of published albums. A different method had to be used to establish the popularity of this category of music. Here the selection was based on the inclusion in *The Gramophone Good CD Guide 1994*. Those compositions that were recorded most frequently by a variety of performers was the primary criterion for the selection of this list. Performers of this category of music are usually highly professional and know the preferences of their audiences. Recording companies are also more likely to invest in recordings that—because of the music or the fame of the performer—ensures maximum financial returns.

#### The items included in this list are:

- Johannes Brahms, 'Nachtigall', Op. 97, No. 1, 15 Selected Songs, Book II, London: Alfred Lengnick, 1931, p. 10.
- Johannes Brahms, 'Liebestreu', Op. 3, No. 1, 15 Selected Songs, Book II, London: Alfred Lengnick, 1931, p. 21.
- Franz Schubert, Die schöne Müllerin: 'Halt', Schubert First Vocal Album, New York: Schirmer, 1895, p. 12.
- 4. Franz Schubert, Die schöne Müllerin: 'Das Wandern', *Schubert First Vocal Album*, New York: Schirmer, 1895, p. 3.
- Franz Schubert, 'Rosamunde', Schubert First Vocal Album, New York: Schirmer, 1895, p. 292.
- 6. Franz Schubert, 'Ave Maria', Schubert First Vocal Album, New York: Schirmer, 1895, p. 258.
- 7. Robert Schumann, Dichterliebe: 'Ich will meine Seele tauchen', Op. 48, *Norton Critical Scores*, edited by Arthur Komar, New York: Norton, 1971, p. 22.
- 8. Robert Schumann, Dichterliebe: 'Das ist ein Flöten und Geigen', Op. 48, Norton Critical Scores, edited by Arthur Komar, New York: Norton, 1971, p. 31.

One of the items in the list, Schubert's *Ave Maria*, proved a particularly interesting choice. It is not only a well know Arts song, but featured on various popularity charts during the 60s and 70s. In this chapter, continuous reference is made to this fact and the effects it has on the results of the analysis.

<sup>&</sup>lt;sup>2</sup> Christopher Pollard (Ed.), Harrow: General Gramophone Publications, 1994.

This composition is included in the S-group because of its historic placement in the Classical period.

# 6.1.3 20th Century Art Song selection

Whereas the compositions of the Popular Music group and the Classical Art Song group were selected because of their proven popularity, the 20th Century Art Song group are compositions that have had relatively little exposure as recordings. That no, or few recordings of these songs are available on the market, tends to indicate that they are generally unknown and possibly less popular. An important factor is that these songs all demonstrate contemporary tonal or rhythmic elements, which distinguishes them from the other two categories as well.

The eight items selected for this purpose are:

- Alban Berg, 'Nun ich der Riesen', No. 3 from Four Songs, Op. 2, Anthology of Twentiethcentury Music, edited by Mary H. Wennerstrom, Englewood Cliffs: Prentice-Hall, 1969, pp. 30-31.
- Lennox Berkeley, 'How love came in', A Heritage of 20<sup>th</sup> Century British Song, Vol. 2. Boosey & Hawkes, 1977, pp. 6-8.
- Arthur Bliss, 'Being young and green and green', A Heritage of 20<sup>th</sup> Century British Song, Vol.
   Boosey & Hawkes, 1977, pp. 9-11.
- Benjamin Britten, 'Since she whom I loved', A Heritage of 20<sup>th</sup> Century British Song, Vol. 2. Boosey & Hawkes, 1977, pp. 50-51.
- Martin Dalby, 'Cupid and my Campaspe', A Heritage of 20<sup>th</sup> Century British Song, Vol. 2. Boosey & Hawkes, 1977, pp. 59-61.
- Charles Ives, 'In Flanders fields', Norton Anthology of Western Music, edited by Claude V. Palisca, New York: Norton, 2nd edition, 1988, pp. 719-721.
- Peter Warlock, 'Whenas the rye', A Heritage of 20<sup>th</sup> Century British Song, Vol. 2. Boosey & Hawkes, 1977, pp. 211-213.

# 6.2 Entropy analysis

Five elements of each composition in the three groups were subjected to analysis with Information Theory:

1. Entropy values for pitch distribution. The results for this method are the easiest to obtain and could be done manually, although it is laborious and prone to errors. In essence each pitch name constitutes a single element.

- Entropy values for pitch distribution and note values combined. For these calculations the
  pitches and duration values combined to establish the elements of a set. A single element
  therefore has two dimensions, pitch and duration. A quarter note G, for example, is a different
  element than an eighth note G.
- 3. Temporal ratios of pitches. Unlike the two criteria above, which are based on the frequency of each element expressed as a ratio of the total number of elements, the temporal ratios of pitches are derived from the total duration that these pitches are heard as a ratio of the total duration of the music.
- 4. Stochastic interval values. The principle behind this method is that specific intervals or sequences of intervals may influence the selection of subsequent intervals, which in turn may again influence the next interval or groups of intervals. To obtain these values, a combination or order of interval sequences of increasing length are isolated as elements. The process begins with a single interval (order 1).
- Stochastic rhythmic values. In essence the calculation of these values is similar to those for stochastic interval values, except that rhythmic values are used for the basis of the calculations instead of intervals.

The scores of each of the melodies are collected in Appendix I and may be used as reference in the following discussion. Appendix II contains a complete summary of all the entropy values, together with graphs for each of the compositions mentioned.

As mentioned earlier in this chapter, Schubert's *Ave Maria*, presents a unique situation. Although composed by a Romanticist, it has become very 'popular' and has been recorded by pop musicians and performers of Art songs alike. In fact this piece of music is one that has become very popular amongst various popular vocalists and instrumentalists of the last fifty years. The problem was, therefore, to which category it belonged. Eventually, the decision was made that it uniquely supports the hypothesis of this thesis. Hence it was treated as an individual item, a decision that proved invaluable to link the values of the Popular music group with those of the Serious music group. In the following tables and graphs the results of the analyses of this item are usually separated from the other compositions of the S-group and given additional attention throughout this chapter.

In the process of entering data in the database, all note values and pitches were used as they appear on the score. In traditional methods of analysis, repeats in music that are indicated by repeat signs are often ignored. For this research all repeats were included in the database, except when a composition as a whole is repeated, in which case there would be no change in its entropy. The reason is that repeats form an essential part of a composition's overall structure and contribute to the quantifiable elements of the music: number of pitches, groups of rhythmic and interval sequences as well as overall duration. Since repeated sections are an essential part of the character or style qualities of a composition it is important that they are included in an analysis of this nature.

## 6.2.1 Pitch entropy values

As was pointed out in Chapter 4, the study of Information Theory as applied to music began with the study of pitch distribution, and since this is also the least complex to calculate, it seems an appropriate point of departure. The following table lists all the pitch entropy values for the three groups of compositions described above. Column 3 shows the number of different pitches used in each composition. Pitch in this sense also refers to silences, in other words rests.

The entropy values of this analysis indicate how the composer chose his pitches. A lower entropy (or higher redundancy) indicates that is there less equality in the distribution of the pitches or alternatively, that the composer has shown a predilection for certain pitches. Perhaps the similarity of the entropy values is predictable, since most of the pieces are tonal and would, because of the inherent characteristics of tonal music, produce a similar pitch distribution in which certain degrees of the scale have greater tonal weight than others. It is noteworthy that the entropy value of *Ave Maria* is the lowest of all the items of the list. Alban Berg's *Nun ich der Riesen*, a dodecaphonic composition has the highest entropy.

From the table it is clear that the entropy values for the different groups show relatively little difference. The S-group shows a higher entropy than the P-group, and the average entropy of the M-group is slightly higher than the S-group. The difference between the lowest (*Ave Maria*) and highest entropy value is nearly 13.08 points.

Group	Title	Pitches	Pitch Entropy
P	Annie's song	9	82.54%
P	You are the sunshine of my life	13	83.87%
P	Summer love	21	84.40%
P	Love letters	13	85.77%
P	One more night	14	87.02%
P	Sleepy shores	29	89.11%
Р	Thank you for the music	17	90.16%
	more strength the end, comma	Average	86.12%
S	Ave Maria	13	81.42%
S	Ich will meine Seele	7	84.04%
S	Das ist ein Flöten	11	87.57%
S	Liebestreu	19	88.38%
S	Halt	10	89.46%
S	Rosamunde	11	90.33%
S	Das Wandern	9	91.90%
S	Nachtigall	18	93.02%
	morned tiers; the actual period th	Average	89.24%
M	How love came in	17	87.76%
M	Whenas the rye	23	88.43%
M	Cupid and my Campaspe	26	89.06%
M	In Flanders fields	13	89.69%
M	Being young and green and green	18	91.66%
M	Since she whom I loved	21	91.79%
M	Nun ich der Riesen	18	94.50%
		Average	90.41%

Table 6-1. Pitch entropies for the three composition groups

As was mentioned in Chapters 2 and 3, calculations based exclusively on pitch names are not sufficient to make any conclusive deductions. Table 6-1 seems to support this argument, especially since there is so much overlapping among the three groups. This suggests that unless all the notes in the music are of equal length, pitch quantity by itself does not accurately reflect the true information of the pitch distribution in a piece of music. For example, the sequence of notes below contains 4 As, 2 Bs and 1 C, a ratio of 57.14%, 28.58% and 14.28% respectively:



If the same sequence is considered according to the total time that each note is heard, in other words if the temporal properties are also considered, there is a radical change in ratios: A = half note, B = half note and C = whole note. The ratios then are 25%, 25% and 50% respectively. Since the maximum entropy for the three notes remains the same, regardless of the type of calculation, it is obvious that the two calculations would produce widely disparaging relative entropy values. The calculations that incorporate the temporal values seem to be more accurate since it reflects the actual duration that each note is heard as a ratio of the duration of the whole piece.

For the sake of comparison and completeness, and where applicable, the tables that follow include the entropy values for pitches.

# 6.2.2 Pitch-ratio entropy values

In Communication Science entropy is usually measured in terms of time, it seems logical that the temporal element should somehow feature in the calculations. Table 6-2 illustrates the entropies of pitches calculated as a ratio of duration. The last column in the table contains the entropy values of Table 6-1 to facilitate comparison.

The effect of calculating pitches by their ratios, instead of by merely counting them, is already obvious by the different position of each of the compositions in the table. For instance, *Sleepy shores* has the sixth highest entry for the P-group in Table 6-1 but moves to the position with the highest entropy for the S-group in Table 6-2. *Nachtigall*, which has the highest entropy value in the preceding table now moves to the second position overall. The reasons for these changes were explained in the preceding section and are confirmed here; the actual period that a pitch is sounded may be much shorter or longer than might be suggested by the frequency of pitches.

The entropy value of *Ave Maria* maintains its low position in the list, and shows an even lower entropy value than in Table 6-1, confirming that it is not only the most predictable as far as the pitch distributions is concerned, but also when the entropy of the pitches are calculated in respect of their temporal values. In fact the temporal pitch entropy suggests a predictability of just under 25% (75.5% entropic).

Group	Title	Pitches	Pitch ratio Entropy	Pitch only
Р	You are the sunshine of my life	13	78.42%	83.87%
P	One more night	14	82.70%	87.02%
P	Summer love	21	83.46%	84.40%
Р	Annie's Song	9	83.87%	82.54%
Р	Thank you for the music	17	85.95%	90.16%
Р	Love letters	13	86.73%	85.77%
Р	Sleepy shores	29	88.15%	89.11%
	. IIII	Average	84.18%	86.12%
S	Ave Maria	13	75.70%	81.42%
S	Nachtigall	18	83.79%	93.02%
S	Liebestreu	19	87.32%	88.38%
S	Rosamunde	11	87.45%	90.33%
S	Ich will meine Seele	7	88.27%	84.04%
S	Das ist ein Flöten	11	88.21%	87.57%
S	Halt	10	88.93%	89.46%
S	Das Wandern	9	94.21%	91.90%
		Average	88.31%	89.24%
М	Cupid and my Campaspe	26	83.81%	89.06%
M	How love came in	17	84.27%	87.76%
M	In Flanders fields	13	87.17%	89.69%
M	Whenas the rye	23	88.60%	88.43%
M	Being young and green and green	18	89.47%	91.66%
M	Nun ich der Riesen	18	90.66%	94.50%
M	Since she whom I loved	21	90.93%	91.79%
		Average	87.84%	90.41%

Table 6-2. Pitch ratio entropies for the three composition groups

The difference between the lowest and highest entropy values has now increased to 18.51 points, an indication that the duration of the pitches has made a dramatic difference to the pitch distribution. On average, the S-group (excluding *Ave Maria*) has a slightly higher entropy than the M-group.

A graph illustrates the tendency of higher entropy values for the S-group and the lower tendencies of the values for the P-group. There is, however, an area where the higher values of the P-group and S-group overlap with the lower ranges of the S-group and M-group, indicated by the dotted rectangle:

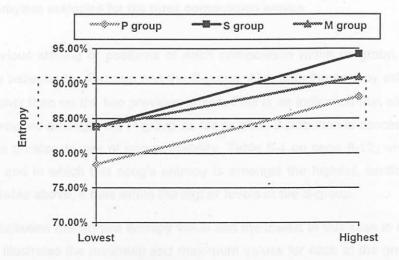


Figure 6-1. Maximum and minimum ranges for the pitch-ratio entropies

# 6.2.3 Pitch-Rhythm entropy values

A third method of calculating the entropy values of pitches is to combine them with their rhythmic values (in contrast to duration). In other words, a note, C, with the rhythmic value of a quarter note differs from a C with a value of an eighth note. This approach effectively combines the predictability (redundancy) of the pitch distribution with that of the first order rhythmic entropy. The entropy values are shown in the next table. The entropy values shown in Table 6-1, and Table 6-2 are shown again for the sake of comparison.

University of Pretoria etd - Koppers M H A (1995)

Group	Title	Pitches	Pitch/Rhythm Entropy	Pitch Ratio Entropy	Pitch only
Р	Summer love	21	82.96%	83.46%	84.40%
Р	Love letters	13	84.76%	86.73%	85.77%
P	Annie's song	9	86.57%	83.87%	82.54%
Р	You are the sunshine of my life	13	87.22%	78.42%	83.87%
Р	Thank you for the music	17	88.95%	85.95%	90.16%
Р	Sleepy shores	29	89.70%	88.15%	89.11%
Р	One more night	14	90.42%	82.70%	87.02%
	high of Figure 6-1 where the 6-9	Average	87.23%	84.18%	86.12%
S	Ave Maria	13	90.81%	75.70%	81.42%
S	Ich will meine Seele	7	87.94%	88.27%	84.04%
S	Halt	10	88.89%	88.93%	89.46%
S	Liebestreu	19	91.32%	87.32%	88.38%
S	Das Wandern	9	93.06%	94.21%	91.90%
S	Das ist ein Flöten	11	94.10%	88.21%	87.57%
S	Rosamunde	11	94.66%	87.45%	90.33%
S	Nachtigall	18	95.42%	83.79%	93.02%
		Average	92.20%	88.31%	89.24%
М	Since she whom I loved	21	90.88%	90.93%	91.79%
M	Cupid and my Campaspe	26	91.31%	83.81%	89.06%
М	How love came in	17	92.24%	84.27%	87.76%
M	Whenas the rye	23	93.05%	88.60%	88.43%
M	In Flanders fields	13	93.06%	87.17%	89.69%
M	Being young and green and green	18	94.88%	89.47%	91.66%
M	Nun ich der Riesen	18	97.94%	90.66%	94.50%
		Average	93.34%	87.84%	90.41%

Table 6-3. Pitch-rhythm entropies for the three composition groups

Besides the obvious shifting of positions of each composition within its group, there is now also a greater disparity between the P-group and the S-group. Also note the entropy value of *Ave Maria* that is now much higher than on the two previous tables. This is an indication that although this particular piece may be grouped amongst the P-group as far as pitch distribution is concerned, its rhythmic entropy indicates a greater degree of unpredictability. Table 6-4 on page 6-12, which contains the first order entropies and in which this song's entropy is amongst the highest, confirms this observation. Likewise in the table above, it falls within the higher levels of the S-group.

The difference between the highest entropy value and the lowest in this case is nearly 16 points. The following graph illustrates the minimum and maximum values for each of the groups. The dotted rec-

tangle indicates the area where minimum and maximum entropy levels of the three groups overlap:

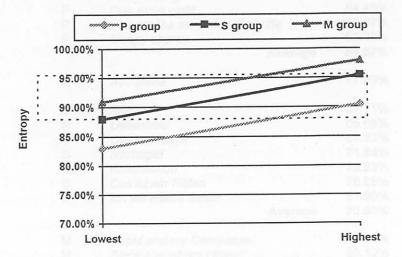


Figure 6-2. Maximum and minimum ranges for the pitch-rhythm entropies

Unlike the graph of Figure 6-1 where the S-group is higher than the M-group, Figure 6-2 shows that the positions have reversed. From Table 6-2 and Table 6-3 it is obvious that the Popular music selection (P-group) generally has a higher redundancy rate than the Serious Art music group (S-group), but that the lesser known 20<sup>th</sup> century songs (M-group) are essentially similar to the Serious Art song selection. That entropies which take the rhythms in consideration are more redundant in the case of Popular songs, is an indication that these pieces are rhythmically more stagnant, more repetitive and less varied; in other words more predictable.

## 6.2.4 Rhythm entropy values

The difference in the distinct rhythmical character of each of the groups of songs is even more evident by isolating the rhythms for entropic analysis. The following table only shows the first order of rhythmic grouping. Later in this chapter all the rhythmic orders are shown.

From the table below it is obvious that the rhythmic coherency or predictability is especially marked in the P-group and M-group. Important is the fact that—compared to the entropy values for pitches only, and entropy values for the pitch ratios—the rhythmic entropy values for all the compositions are significantly lower. One of the items, *Thank you for the music*, is below 50%, indicating that its rhythmic structure is more than 50% predictable. This seems to indicate that rhythmic coherency is an important factor in music's appeal to the listener. The Classical group of songs are rhythmically much more complex than the Popular group and, to a lesser extent, than the 20<sup>th</sup> century group.

Group	Title	Rhythmic	
	un ranges for the first order thy	Entropy	
P	Thank you for the music	48.24%	
P	Summer love	50.85%	

Р	Love letters	57.66%
P	Annie's song	61.57%
P	One more night	64.40%
P	You are the sunshine of my life	68.07%
P	Sleepy shores	68.30%
Julya	Average	59.87%
S	Ave Maria	77.89%
S	Halt	54.71%
S	Liebestreu	65.68%
S	Das Wandern	68.33%
S	Nachtigall	71.84%
S	Rosamunde	73.63%
S	Das ist ein Flöten	78.08%
S	Ich will meine Seele	81.90%
	Average	70.60%
М	Cupid and my Campaspe	53.73%
M	Since she whom I loved	59.12%
M	How love came in	63.38%
M	In Flanders fields	66.56%
M	Whenas the rye	70.45%
M	Being young and green and	74.40%
	green	84.63%
M	Nun ich der Riesen	67.47%
	Average	01.41%

Table 6-4. Rhythmic entropies for the first order for the three composition groups

The difference between the highest and lowest values in this case is 46.39 points. The graph below shows the maximum and minimum entropy values for the first order rhythmic entropy values of each group. The dotted rectangle indicates where there is overlapping of maximum and minimum values:

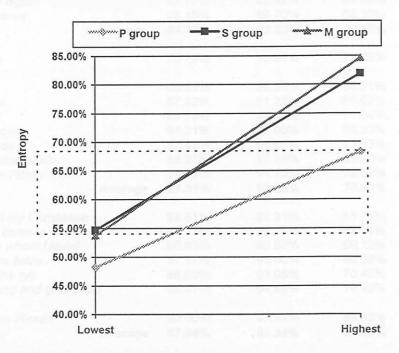


Figure 6-3. Maximum and minimum ranges for the first order rhythm entropies

In the discussion of the interval orders later in this chapter, it will be shown that the composers of the modern pieces have relied more on interval variation, and that they possibly resorted to a greater rhythmic redundancy to achieve musical coherency. The difference between the pitch entropy and the rhythmic entropy of *Ave Maria*, may be a similar balancing factor in which the greater rhythmic entropy compensates for the predictability of the pitch set.

At this stage it is clear that the selected compositions are mainly differentiated in respect of their temporal pitch distribution (Table 6-2), rhythmic entropy and a combination of both (Table 6-3). It has also been shown that pitch entropy by itself has little bearing on the intrinsic stylistic character of the music but that pitch-entropy calculations only become effective when note values and rhythm are taken into account. Considering the tables shown so far, many of the calculations for exclusive pitch distribution actually contradict the entropies calculated for the combination of pitch and rhythmic values.

The entropy values illustrated in Table 6-2, Table 6-3, and Table 6-4 are summarised in the table below. Although there is some overlapping of values, the averages of the pitch-ratio, pitch-rhythm and rhythm entropy values already provide a good pointer to the stylistic entropy values of the various genres. The entropy values for pitch distribution are not included for the reasons already expounded on earlier in this chapter.

Group	Title	Pitch ratio	Pitch/Rhythm	Rhythmic	Average
		Entropy	Entropy	Entropy	entropy
P	Summer love	83.46%	82.96%	50.85%	72.42%
P	Thank you for the music	85.95%	88.95%	48.24%	74.38%
Р	Love letters	86.73%	84.76%	57.66%	76.38%
Р	Annie's song	83.87%	86.57%	61.57%	77.34%
P	You are the sunshine of my life	78.42%	87.22%	68.07%	77.90%
Р	One more night	82.70%	90.42%	64.40%	79.17%
P	Sleepy shores	88.15%	89.70%	68.30%	82.05%
	Average	84.18%	87.23%	59.87%	77.09%
S	Ave Maria	75.70%	90.81%	77.89%	81.47%
S	Halt	88.93%	88.89%	54.71%	77.51%
S	Liebestreu	87.32%	91.32%	65.68%	81.44%
S	Nachtigall	83.79%	95.42%	71.84%	83.68%
S	Das Wandern	94.21%	93.06%	68.33%	85.20%
S	Rosamunde	87.45%	94.66%	73.63%	85.25%
S	Ich will meine Seele	88.27%	87.94%	81.90%	86.04%
S	Das ist ein Flöten	88.21%	94.10%	78.08%	86.80%
	Average	88.31%	92.20%	70.60%	83.70%
М	Cupid and my Campaspe	83.81%	91.31%	53.73%	76.28%
M	How love came in	84.27%	92.24%	63.38%	79.96%
M	Since she whom I loved	90.93%	90.88%	59.12%	80.31%
M	In Flanders fields	87.17%	93.06%	66.56%	82.26%
M	Whenas the rye	88.60%	93.05%	70.45%	84.03%
М	Being young and green and green	89.47%	94.88%	74.40%	86.25%
M	Nun ich der Riesen	90.66%	97.94%	84.63%	91.08%
	Average	87.84%	93.34%	67.47%	82.88%

Table 6-5. Summary of pitch and rhythm entropy values

A graph illustrates the entropic differences between the maximum and minimum values of the three

groups and shows that there is a degree of overlapping of the maximum values of the P- and M-groups and the minimum values of the S-group.

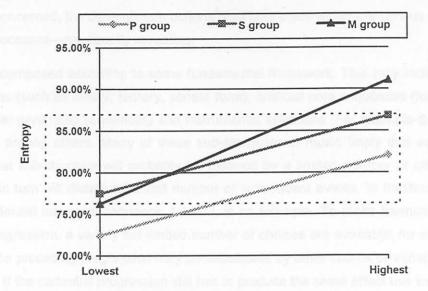


Figure 6-4. Maximum and minimum range of average entropy values

The results thus far indicate that:

- 1. both pitch distribution and rhythmic organization is more complex in the Classical Art songs (S-group) than in the other two groups; and
- 2. there seems to be a lower entropy limit for both pitch and rhythmic distribution. It appears that the lower limit for pitch distribution is around 80% while that for rhythm is around 60%. These figures are approximations and not fixed and may even be lower for other genres of music or individual pieces. Children's songs and certain types of folk songs, which centre on the pentatonic scale would probably have lower limits and lower entropy values.

Intuitively one may suspect that much popular music relies on rhythmic simplicity and repetitiveness for its popularity. Should the repetitive accentuation of beats, so often found in popular music, be taken into account along with the indicated redundancy levels, the overall entropy values of this music would drop dramatically. Some years ago, various recordings of some of the better known 'Classics' were released in which the beats are strengthened by percussion instruments and bass guitars that enhance the basic harmonic progressions. These recordings became very popular possibly because the entropy of both the rhythm (percussion) and the harmony (reinforced bass line) were lowered.

It is conceivable that music that generates entropy values below a certain limit may be equally unpopular as music of which the entropy is too high. Music with entropy values that are too low could then be described as monotonous and boring, while music with very high entropy values would be described as being too 'heavy'. Nevertheless, this is probably a matter of individual preference with different people having different entropy tolerances.

## 6.3 Stochastic music analysis

Although the figures obtained so far have already provided some significant information about the compositions concerned, the dynamic interactions<sup>3</sup> that take place within the various aspects of music —stochastic processes—are equally revealing.

Most music is composed according to some fundamental framework. This may include tonality, preconceived forms (such as binary, ternary, sonata form), artificial note sequences (for example serialism, or computer generated sequences) and instrumental selections (For example SATB choir, string quartets, etc.), among others. Many of these sub-structures in music imply that each new musical event within that sub-structure will probably be followed by a limited number of other and different events, which in turn will dictate a limited number of subsequent events. In traditional harmony, the well-known cadential harmonic sequence, I-IV-V-I, is an example. To make harmonic sense at each stage of the progression, a variety but limited number of choices are available; for example, some of the chords in the preceding progression may be substituted by other chords or variations of the same chord: I-II-V<sup>7</sup>-I. If the cadential progression still has to produce the same effect this kind of substitution of chords cannot be done at random but is controlled by convention. The keyword here is 'progression', which in itself implies that there are certain self-generating dynamics within the tonal system. Even the most rudimentary handbooks on harmony make this abundantly clear. Similar 'rules' apply to many other aspects of music, including the progression of melodic intervals, or what is also referred to as voice leading.

Nevertheless, within the mentioned structures (melodic, harmonic, rhythmic) there is also a factor of randomness. The mentioned examples have a single common denominator—random selection from a limited set of possibilities. Amongst Information scientists this is referred to as stochastic<sup>4</sup> processes.

# 6.3.1 Stochastic interval entropy

Stochastic analysis of the interval contents of in music is the next step in the entropic analysis of the music for this research. To obtain the results shown in the following pages, ever increasing series (orders) of interval sequences were compared. The results obtained, essentially reflect the frequency at which specific events—in this case intervals—are followed by other specific events, in ever increasing complexity and expressed as a ratio of the overall number of events of the same order.

The number of orders generated by each composition varies and is specific and directly related to the complexity of the intervals used. In the charts that follow the generation of orders was halted when the entropy values reached 100%, or when the order number reached 70. Once the entropy has

<sup>3</sup> Structural principles that are inherent in voice leading and rhythmical coherency.

Derived from the Greek word 'stochos' which means to aim for or to target.

reached 100% each order thereafter also has an entropy of 100%. Essentially this means those interval sequences that belong to these orders only occur once and, in other words, have reached maximum distribution or randomness. The software used to generate these values could not accurately manipulate character strings that exceeded 256 characters. Since each interval is represented by 3 characters (size, octave and direction) the number of intervals per order was limited to 85. For the sake of accuracy the number of orders was reduced to a maximum of 70. This was not seen as a major drawback since only four of the compositions used for this study exceeded the 70<sup>th</sup> order, while the next highest was 54 orders (*Rosamunde*).

All the interval orders for the compositions under discussion are shown below:

Group	Title Title	Interval
		Orders
P	Summer love	22
P I	Love letters	34
Р	Annie's song	39
p 3	You are the sunshine of my life	40
P	Thank you for the music	70
P	One more night	70
P	Sleepy shores	70
	Average	49.29
S	Ave Maria	70
S S	Nachtigall	4
S	Das ist ein Flöten	14
S	Das Wandern	15
S	Halt	18
S	Ich will meine Seele	19
S	Liebestreu	25
S	Rosamunde	54
	Average	21.29
M	In Flanders fields	7
M	Whenas the rye	8
M	Being young and green	3
M	Nun ich der Riesen	3
M	Cupid and my Campaspe	9
M	Since she whom I loved	9
M	How love came in	20
	Average	8.43

Table 6-6. Comparative Interval-Order quantities

Keeping in mind that the extent of the interval orders is directly related to the inherent structural arrangement of the intervals of a composition, the table above is rather revealing. The P-group of songs on average has the largest number of orders with three of the individual items generating more than 70 orders.

Excluding *Ave Maria*, the S-group produced orders that are-on average less than half that of the P-group, indicating a greater overall interval complexity, fewer repeated sequences, and greater variety—in other words, a lower degree of predictability. Again, *Ave Maria*, which also produced more than 70 orders, is the exception for the S-group. This suggests that, besides the redundancy of the pitch

distribution, this song's popularity may also be ascribed to its predictable interval relationships.

Whereas, the M-group indicated little to distinguish in the comparison of the pitch, pitch-ratio, pitch-rhythm, and rhythmic entropies, it features far fewer interval orders than the other two groups. This indicates that the interval sequences of these songs are even less predictable than those of the S-group, further supporting the argument that the composers of these songs relied more on rhythmic unity, and—in some instances—pitch distribution to provide musical coherency, and that they used interval variation to provide musical interest.

However, the quantity of interval orders is not the most important aspect of the interval analysis. Each order is associated with an entropy value as well. The most convenient manner of illustrating these entropy values is by presenting them as graphs. The following series of graphs show the curves of the change in entropy values of each composition in the order they are listed in the table above. Since the first number orders show the transition between orders most clearly and to ensure similar visual representation, only the first 15 orders of each of the songs are shown. See Appendix II for a complete list of orders and entropies for each of the songs.

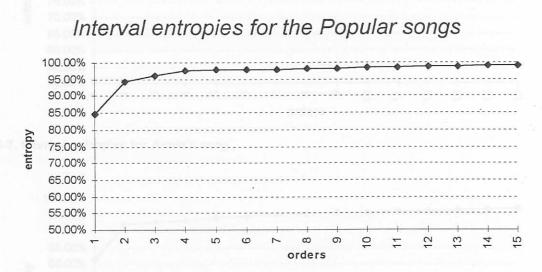


Figure 6-5. Interval entropies for Summer love



Figure 6-6. Interval entropies for Love letters



Figure 6-7. Interval entropies for Annie's song



Figure 6-8. Interval entropies for You are the sunshine of my life



Figure 6-9. Interval entropies for Thank you for the music

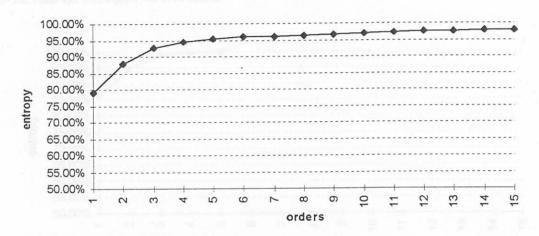


Figure 6-10. Interval entropies for One more night

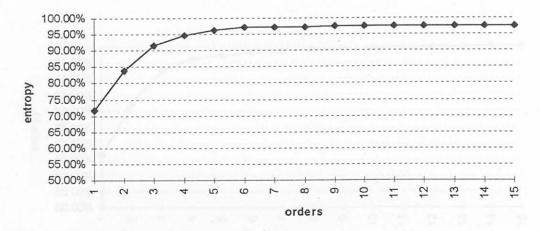


Figure 6-11. Interval entropies for Sleepy shores

# Interval entropies for the Classical Art Songs

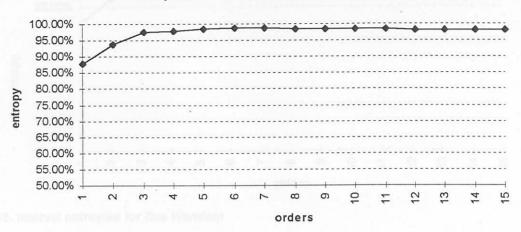


Figure 6-12. Interval entropies for Ave Maria

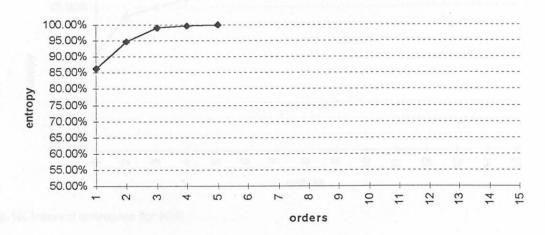


Figure 6-13. Interval entropies for Nachtigall

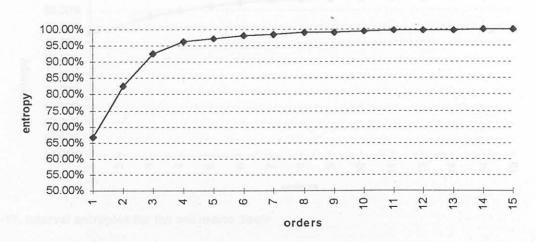


Figure 6-14. Interval entropies for Das ist ein Flöten

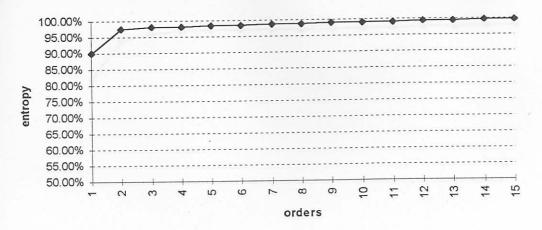


Figure 6-15. Interval entropies for Das Wandern

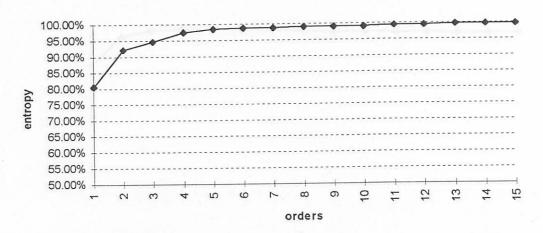


Figure 6-16. Interval entropies for Halt

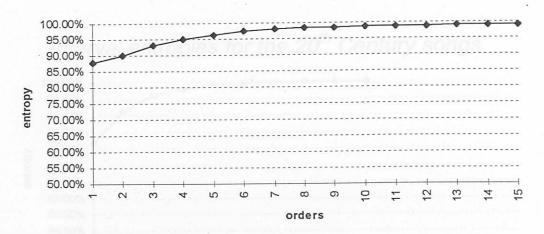


Figure 6-17. Interval entropies for Ich will meine Seele

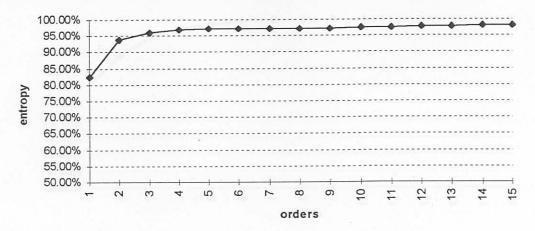


Figure 6-18. Interval entropies for Liebestreu

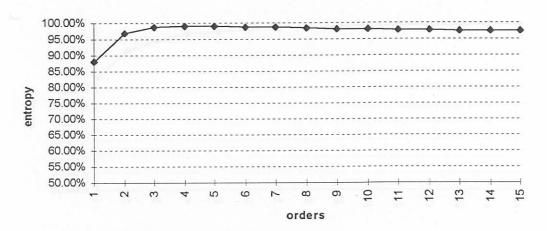


Figure 6-19. Interval entropies for Rosamunde

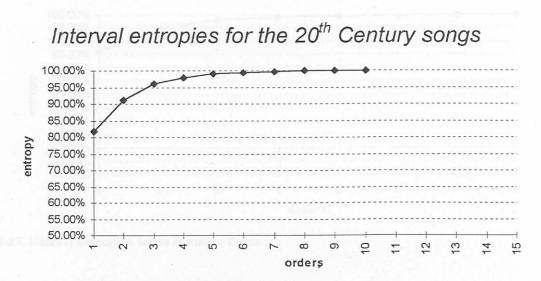


Figure 6-20. Interval entropies for Cupid and my Campaspe



Figure 6-21. Interval entropies for Since she whom I love



Figure 6-22. Interval entropies for How love came in

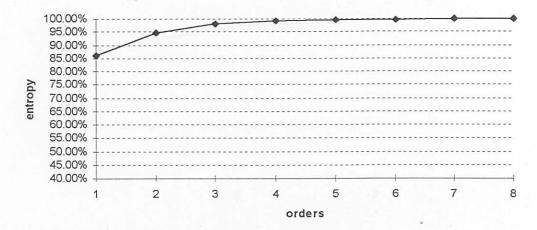


Figure 6-23. Interval entropies for In Flanders fields

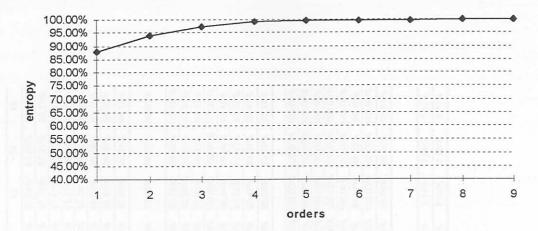


Figure 6-24. Interval entropies for Whenas the rye

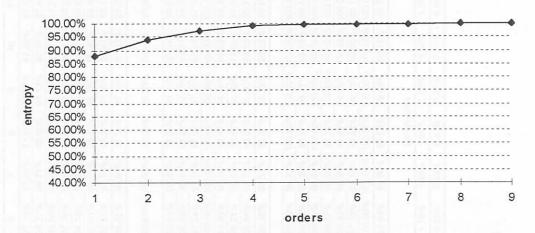


Figure 6-25. Interval entropies for Being young and green

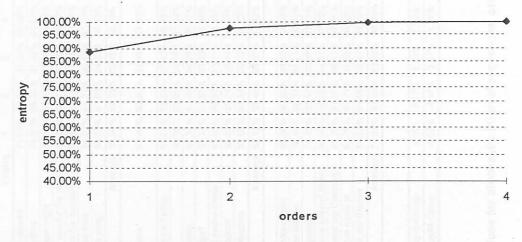


Figure 6-26. Interval entropies for Nun ich der Riesen

To facilitate a sensible explanation of the preceding graphs the values for the first fifteen orders are shown in tabular form on the following page.

Orders	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Sleepy shores					96.13%	97.16%	97.20%	97.27%	97.37%	97.50%	97.53%	97.62%	97.58%	97.55%	97.51%
Love letters	74.31%	85.96%	91.89%	94.01%	95.80%	96.37%	96.71%	96.96%	97.13%	97.27%	97.41%	97.57%	97.73%	97.91%	98.10%
One more night	79.24%	87.78%	92.65%	94.63%	95.59%	95,96%	96.20%	96.49%	96.74%	97.01%	97.23%	97.46%	97.67%	97.88%	98.02%
Thank you for the music	80.75%	89.72%	95.42%	96.77%	97.46%	97.76%	97.86%	97.94%	98.03%	98.12%	98.22%	98.22%	98.21%	98.21%	98.22%
You are the sunshine	81.57%	92.09%	92.78%	93.43%	94.06%	94.59%	95.21%	95.51%	95.63%	95.81%	95.99%	96.18%	96.39%	96.61%	96.83%
Summer love	84.83%	94.27%	96.11%	97.52%	97.81%	97.91%	98.01%	98.13%	98.26%	98.40%	98.55%	98.72%	98.90%	99.08%	99.24%
Annie's song	88.03%	91.65%	94.02%	95.18%	95.92%	96.51%	97.19%	97.37%	97.60%	97.87%	98.17%	98.49%	98.69%	98.80%	98.92%
Average	80.06%	89.33%	93.49%	95.19%	96.11%	96.61%	96.91%	97.10%	97.25%	97.43%	97.59%	97.75%	97.88%	98.01%	98.12%
Ave Maria	87.99%	93.80%	97.46%	97.93%	98.53%	98.67%	98.62%	98.59%	98.50%	98.42%	98.32%	98.23%	98.15%	98.07%	98.01%
Das ist ein Flöten	66.81%	82.61%	92.43%	96.28%	97.34%	98.01%	98.38%	98.96%	99.21%	99.44%	99.61%	99.70%	99.79%	99.89%	100.00%
Halt	80.60%	92.05%	94.76%	97.54%	98.41%	98.69%	98.87%	98.98%	99.12%	99.21%	99.32%	99.43%	99.56%	99.65%	99.74%
Liebestreu	82.34%	93.80%	96.08%	96.97%	97.25%	97.24%	97.25%	97.29%	97.35%	97.44%	97.56%	97.69%	97.85%	98.03%	98.23%
Nachtigall	86.32%	94.80%	98.99%	99.68%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Ich will meine Seele	87.88%	89.87%	93.10%	94.86%	96.11%	97,39%	98.18%	98.47%	98.56%	98.69%	98.76%	98.85%	98.94%	99.04%	99.16%
Rosamunde	88.02%	96.74%	98.86%	99.07%	99.05%	98.81%	98.60%	98.42%	98.26%	98.05%	97.86%	97.70%	97.55%	97.43%	97.39%
Das Wandern	89.89%	97.57%	98.22%	98.23%	98.44%	98.54%	98.73%	98.91%	99.00%	99.10%	99.21%	99.34%	99.48%	99.64%	99.81%
Average	83.12%	92.49%	96.07%	97.52%	98.09%	98.38%	98.57%	98.72%	98.79%	98.85%	98.90%	98.96%	99.03%	99.10%	99.19%
In Flanders fields	66.56%	80.89%	89.30%	94.69%	98.03%	99.05%	99.72%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Whenas the rye	70.45%	82.27%	87.21%	90.91%	94.42%	95.83%	97.38%	98.57%	98.97%	99.25%	99.41%	99.59%	99.79%	100.00%	100.00%
Being young	74.40%	85.57%	93.96%	97.21%	98.31%	99.20%	99.84%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Since she whom I loved	80.95%	88.99%	94.61%	97.23%	98.43%	99.27%	99.65%	99.79%	99.89%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Cupid and my Campaspe	81.78%	91.26%	96.11%	97.91%	98.96%	99.40%	99.66%	99.87%	99.93%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
How love came in	83.76%	91.71%	94.55%	96.59%	97.31%	97.76%	98.15%	98.55%	98.79%	99.05%	99.11%	99.17%	99.24%	99.32%	99.40%
Nun ich der Riesen	84.63%	91.99%	98.47%	99.79%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Average	77.51%	87.53%	93.46%	96.33%	97.92%	98.64%	99.20%	99.54%	99.66%	99.76%	99.79%	99.82%	99.86%	99.90%	99.91%
				- 5				Differen	ces						
P-Group/S-Group	3.06%	3.17%	2.58%	2.32%	1.98%	1.78%	1.66%	1.62%	1.53%	1.42%	1.32%	1.21%	1.14%	1.09%	1.07%
S-Group/M-Group	-5.62%	-4.97%	-2.61%	-1.18%	-0.16%	0.26%	0.63%	0.82%	0.87%	0.91%	0.88%	0.86%	0.84%	0.80%	0.72%

Table 6-7. Entropy values for stochastic interval sequences, order 1 - 15

Because each of the graphs also represents a different composition, it is obvious that all the curves would be quite different. This then also confirms the difference in character between each composition—if the graphs were similar or had appeared similar, they would have sounded the same or similar as well. The distinctiveness of each of the songs' interval graphs can be used as one component of a complex graph that identifies each song. Examples of such complex graphs are shown towards the end of this chapter.

However, if the average figures in Table 6-7 are used in conjunction with the graphs, the entropy value of the S-group (including *Ave Maria*) at each order is clearly higher than those of the P-group. At the first order the difference between the entropies of the two groups is more than 3 points but as the orders progress the difference in values gradually decreases; at order 5 the difference is nearly 2 points; at order 10 the difference is 1.4 points at order 15 it is about 1 point.

Similarly, there is a difference between the average entropies of the S-group and the M-group but the order 1 difference is -5.62; order 5 is -0.16; order 10 is 0.91 and order 15 has a difference of 0.72.

To summarise the above, the S-group on average begins with a higher entropy for the stochastic interval values, than both the P-group and the M-group. The S-group maintains its higher values compared to the P-group, but this difference gradually decreases as the order number increases. Whereas the M-group starts with a lower value than the S-group, at order 5 the entropy value of the M-group is higher than that of the S-group, reaches its greatest difference at order 10. The difference gradually decreases as the order numbers increase. The difference in entropy values at each order, would suggest that the predictability of the interval sequences of the S-group generally reduces faster than those of the S-group, which in turn reduces faster than those of the P-group. This information together with the number of orders each composition generates provides the overall predictability of each composition as well as for each group. The interval entropies of the Popular songs are relatively low while the number of orders indicates that the structural dynamics of these songs stretch over a greater length of the music.

In the case of the Modern songs, maximum entropy is reached comparatively fast, hence the steeper curve. This indicates that the music of the M-group does not rely as much on interval coherency than either the Popular songs or the Art songs.

# 6.3.2 Stochastic rhythm entropy

The same methods applied to obtain the stochastic interval entropies are applied to the rhythms of each of the songs and the values thus obtained may be illustrated similarly. Below is a table of the number of the rhythmic orders generated by each of the compositions.

Group	Title		Rhythmic Orders
Р	Summer love		22
P	You are the sunshine of	mv life	41
P	Annie's song		50
Р	Love letters		69
Р	Thank you for the music		70
Р	One more night		70
P	Sleepy shores		70
1/	cicopy circina	Average	56
S	Ave Maria		70
S	Nachtigall		13
S	Ich will meine Seele		13
S	Das ist ein Flöten		15
S	Das Wandern		15
S	Halt		18
S	Liebestreu		22
S	Rosamunde		54
		Average	21.43
M	Nun ich der Riesen		4
M	In Flanders fields		7
M	Being young and green		7
М	Since she whom I loved		11
M	Cupid and my Campasp	е	12
M	How love came in		12
М	Whenas the rye		13
	The state of the s	Average	9.43

Table 6-8. Comparative rhythm-orders

As with the values obtained for the rhythmic elements of the songs in the first part of this chapter, the stochastic character of the rhythm – in terms of the numbers of orders generated – again shows a marked difference among the three groups.

Keeping in mind that those compositions with orders of 70 could possibly generate even higher orders, the average of the P-group and *Ave Maria* would, in other words, be even higher than shown. However, the figures provided in Table 6-8 adequately indicate the difference among the three groups. In essence the relative values indicate how soon the stochastic processes at work in the rhythms of each group of songs reach maximum entropy. In the case of the pieces under discussion, the S-group reaches maximum entropy more than twice as fast than the P-group, while the M-group reaches maximum rhythmic entropy nearly twice as fast as the S-group. The argument stated earlier in this chapter, that the acceptability of a piece of music by certain sectors of the listening public is largely dependent on the rhythmic structure, seems to be reinforced by the figures shown above.

The following series of charts illustrate the curves of the rhythmic entropies of the songs to the 15<sup>th</sup> order.

# Rhythm entropies for the Popular songs

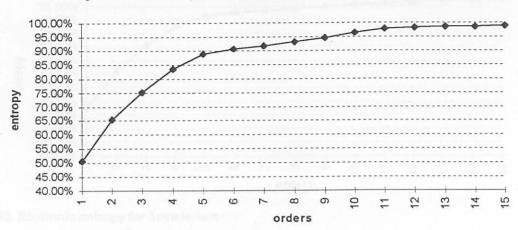


Figure 6-27. Rhythmic entropies for Summer love

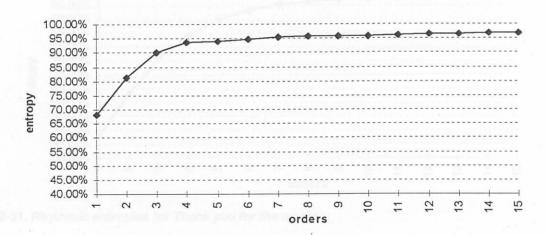


Figure 6-28. Rhythmic entropies for You are the sunshine of my life



Figure 6-29. Rhythmic entropies for Annie's song

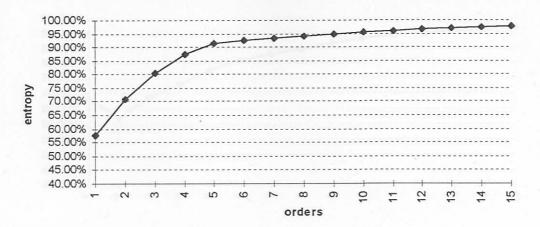


Figure 6-30. Rhythmic entropy for Love letters



Figure 6-31. Rhythmic entropies for Thank you for the music



Figure 6-32. Rhythmic entropies for One more night

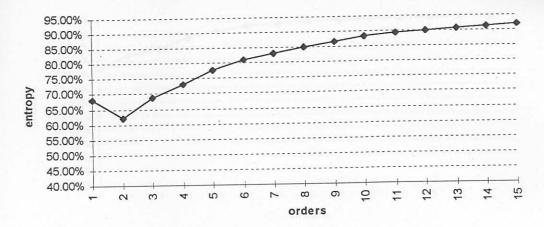


Figure 6-33. Rhythmic entropies for Sleepy shores

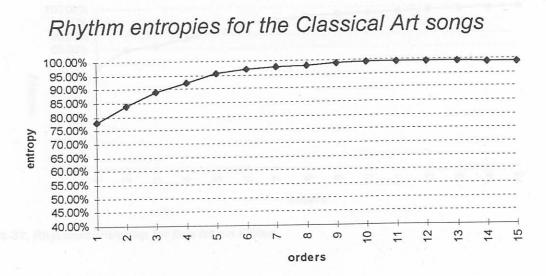


Figure 6-34. Rhythmic entropies for Ave Maria

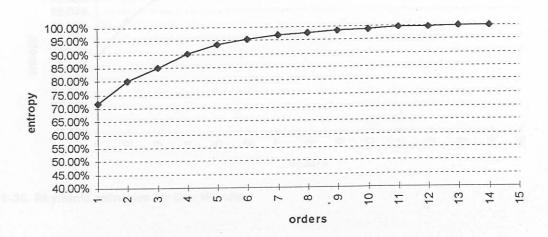


Figure 6-35. Rhythmic entropies for Nachtigall

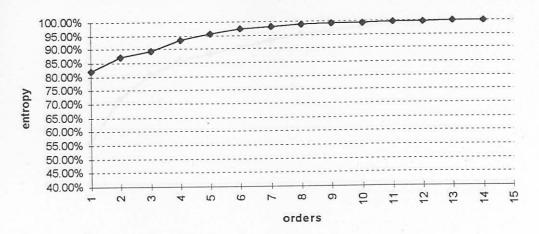


Figure 6-36. Rhythmic entropies for Ich will meine Seele

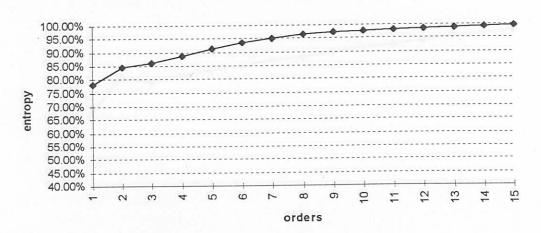


Figure 6-37. Rhythmic entropies for Das ist ein Flöten

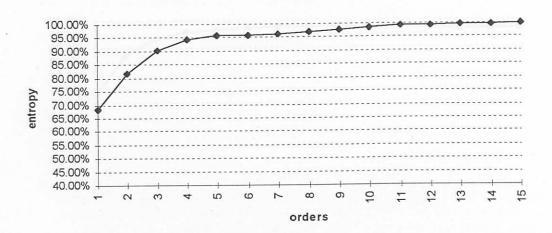


Figure 6-38. Rhythmic entropies for Das Wandern

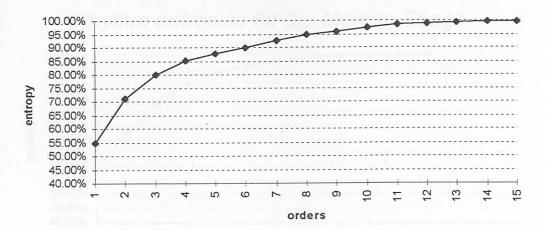


Figure 6-39. Rhythmic entropies for Halt



Figure 6-40. Rhythmic entropies for Liebestreu

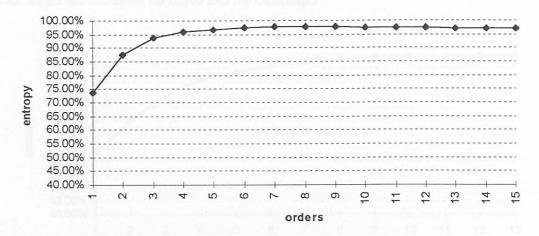


Figure 6-41. Rhythmic Entropies for Rosamunde

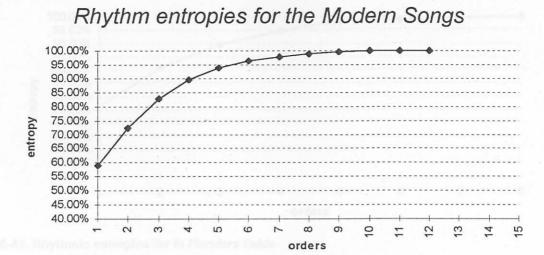


Figure 6-42. Rhythmic entropies for Since she whom I loved

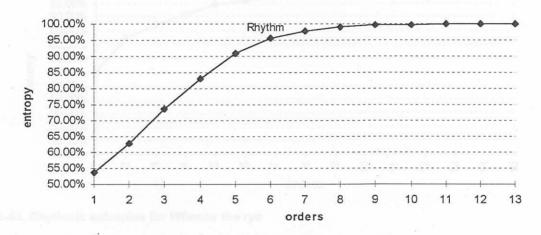


Figure 6-43. Rhythmic entropies for Cupid and my Campaspe

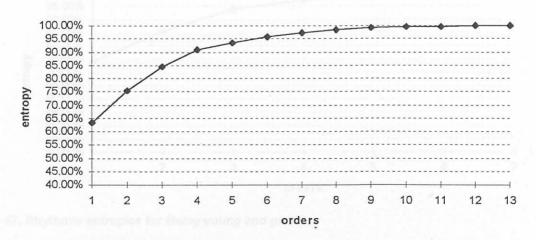


Figure 6-44. Rhythmic entropies for How love came in

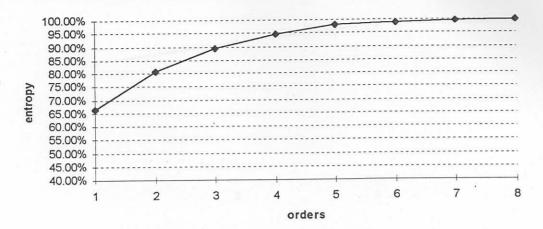


Figure 6-45. Rhythmic entropies for In Flanders fields



Figure 6-46. Rhythmic entropies for Whenas the rye

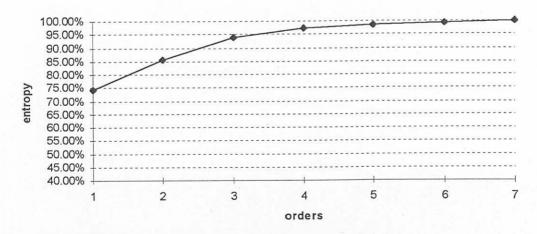


Figure 6-47. Rhythmic entropies for Being young and green

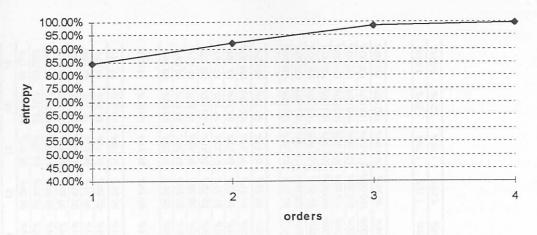


Figure 6-48. Rhythmic entropies for Nun ich der Riesen

The first 15 orders of the rhythmic entropies are shown on the following page:

******************************	Orders	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Thank you for the music		48.24%		76.14%	84.49%	89.93%	93.09%	95.16%	96.36%	97.07%	97.54%	97.89%	98.00%	98.07%	98.14%	98.17%
Summer love		*********	65.35%	75.31%	83.59%	88.81%	90.80%	91.75%	93.11%	94.70%	96.43%	97.85%	98.24%	98.48%	98.70%	98.96%
Love letters		57.66%	71.02%	80.49%	87.47%	91.48%	92.71%	93.46%	94.08%	94.91%	95.51%	96.06%	96.56%	96.95%	97.34%	97.62%
Annie's song		61.57%	74.31%	81.93%	87.04%	89.95%	92.06%	92.91%	93.68%	94.67%	95.72%	96.84%	97.80%	98.49%	98.74%	98.74%
One more night		64.11%	80.33%	88.98%	93.49%	95.04%	95.76%	96.04%	96.36%	96.67%	97.01%	97.31%	97.47%	97.66%	97.72%	97.78%
You are the sunshine		68.07%	81.32%	90.14%	93.48%	93.88%	94.74%	95.50%	95.62%	95.76%	95.92%	96.11%	96.31%	96.53%	96.75%	96.99%
Sleepy shores		68.30%	62.30%	68.80%	73.16%	77.87%	81.05%	83.15%	85.19%	86.85%	88.31%	89.31%	90.00%	90.71%	91.34%	91.97%
	verage	59.83%	71.15%	80.26%	86.10%	89.57%	91.46%	92.57%	93.49%	94.38%	95.20%	95.91%	96.34%	96.70%	96.96%	97.18%
Ave Maria		77.89%	83.89%	89.17%	92.45%	95.44%	97.04%	97.64%	98.26%	98.85%	99.09%	99.14%	99.19%	99.11%	99.07%	99.02%
Halt		54.71%	70.97%	80.01%	85.27%	87.66%	90.08%	92.61%	94.65%	96.09%	97.41%	98.54%	99.01%	99.39%	99.56%	99.74%
Liebestreu		65.78%	80.01%	78.26%	80.10%	82.50%	84.05%	85.65%	87.06%	88.60%	90.11%	91.48%	92.93%	94.35%	95.51%	96.69%
Das Wandern			81.86%		94.57%	95.87%	95.99%	96.35%	96.93%	97.70%	98.55%	99.10%	99.36%	99.49%	99.65%	99.82%
Nachtigall		***********	80.00%	~~~~~~~~~~	90.37%	93.71%	95.39%	96.94%	97.73%	98.39%	99.00%	99.54%	99.76%	99.87%	.100.00%	100.00%
Rosamunde	*********		87.47%	*********	95.90%	96.85%	97,46%	97.78%	97.93%	97.73%	97.56%	97.43%	97.32%	97.23%	97.15%	97.09%
Das ist ein Flöten		78.08%	84.78%	85.99%	88.80%	91.35%	93.66%	95.15%	96.68%	97.20%	97.85%	98.31%	98.62%	98.97%	99.36%	99.79%
Ich will meine Seele	•••••	81.90%	87.36%	89.41%	93.46%	95.62%	97.40%	98.34%	98.84%	99.15%	99.39%	99.53%	99.67%	99.83%	100.00%	100.00%
	verage	70.61%	81.78%	86.09%	89.78%	91.94%	93.43%	94.69%	95.69%	96.41%	97.13%	97.70%	98.10%	98.45%	98.75%	99.02%
***************************************								07.050/	00.000/	99.57%	99.74%	99.87%	99.93%	100.00%	100.00%	100.00%
Cupid and my Campaspe			62.78%		83.19%	90.94%	95.68%	97.95%	99.08%	99.69%	99.89%	99.95%	100.00%	100.00%		100.00%
Since she whom I love		59.12%	************	***********	89.45%	93.74%	96,49%	98.01%	98.63%	99.09%	99.53%	99.80%	99.90%	100.00%		
How love came in		***********	75.60%		90.78%	93.65%	95.72%		100.00%	100.00%	100.00%	***********	100.00%	100.00%		100.00%
In Flanders fields			94.78%	************	99.36%	99.66%	99.79%	99.93%	99.90%	100.00%	100.00%	100.00%	100.00%		100.00%	
Whenas the rye			93.91%	************	99.09%	99.45%	99.62%	99.80%		100.00%	100.00%	100.00%	100.00%	100.00%		100.00%
Nun ich der Riesen				99.79%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%		100.00%
Being young		89.59%	97.90%	99.57%	100.00%	100.00%	100.00%						***************************************			
	Average	75.46%	84.97%	90.81%	94.55%	96.78%	98.19%	99.00%	99.52%	99,79%	99.88%	99.94%	99.98%	100.00%	100.00%	100.0076
									Difference	es				************	000000000000000000000000000000000000000	000000000000000000000000000000000000000
Difference P-Group - S-Group	**********	10.78%			3.68%	2.37%	1.98%	2.12%	2.20%	2.03%	1.92%	1.79%	1.75%	1.75%	1.79%	1.84%
Difference S-Group - M-Group	***************************************	4.85%	3.19%	4.72%	4.77%	4.84%	4.75%	4.31%	3.83%	3.38%	2.76%	2.24%	1.88%	1.55%	1.25%	0.98%

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Table 6-9. Combined rhythmic entropies for orders 1 - 15

Compared to the stochastic analysis of the interval sequence, the entropies for the rhythm sequences generally begin much lower. Most of the compositions also indicate a shallower curve toward 100% entropy. It is also noteworthy that, with some exceptions, the S-group's curve is even shallower. This is supported by the differences of the values of the P-group and the S-group (see Table 6-9). The difference starts with 10.78 points, decreases to 2.37 points at the 5<sup>th</sup> order and then gradually decreases to 1.92 points and 1.84 points at the 10<sup>th</sup> and 15<sup>th</sup> order respectively.

Two of the gradients of the M-group are very similar, although they begin with different 1<sup>st</sup> order entropies. Both reach maximum entropy at the 13<sup>th</sup> order. However, all seven compositions belonging to the S-group have steeper curve gradients than any of the other songs under scrutiny. Compared to the S-group, their average is lower with a difference of 4.85 points at the first order. In fact, their beginning entropies are very close to those of the P-group. At the 5<sup>th</sup> order their average is above that of the S-group with 4.84 points. It reaches a difference of 2.76 points at the 10<sup>th</sup> order and a difference of .98 points at the 15<sup>th</sup> order.

As was mentioned earlier in this chapter (see page 6-10), and as is now again evident, the greatest differences in the entropy of the songs are in the rhythmical characteristics of the music. The stochastic entropies of the rhythm are also directly related to aspects of the formal structure of the music, such as rhythmic units, motifs, sub-phrases, phrases, periods, as well as larger sections. Lower entropies with shallower curves indicate a greater rhythmic coherency. Repeats, sequences and similar devices—even though the pitches and intervals may differ—all contribute to a greater predictability (greater redundancy and lower entropy) of a composition.

Some significant rhythmic characteristics need specific mentioning. *Sleepy Shores* (P-Group, Figure 6-33 on page 6-30) has an interesting curve in which the second order entropy is more that 5 points lower than the first order, after which it begins its gradual upward curve. This indicates a high degree of rhythmic unity of consecutive rhythmic values. A similar deviation from the general shape of the curves is found in *Liebestreu* (Figure 6-40, page 6-32) where the entropy drops by nearly 2 points at the 3<sup>rd</sup> order and then gradually rises. The score supports this tendency by the frequently repeated pattern of two eighth notes followed by a quarter note. Also note the virtually stagnant entropy values of *Rosamunde* (Figure 6-41, page 6-32) which, after levelling off at the 7<sup>th</sup> and 8<sup>th</sup> order, decreases slightly in entropy, before it gradually rises again toward maximum entropy at the 50<sup>th</sup> order (see complete listing of entropies in Appendix II). Another song which shows a similar tendency is *Ave Maria* (Figure 6-34, page 6-30), in which there is a gradual rise to an entropy of about 99 points at the 10<sup>th</sup> order, with a gradual decrease up to the 30<sup>th</sup> order and then a very gradual rise to maximum entropy beyond the 70<sup>th</sup> order. In this case, however, the curve evens out at a higher entropy level than that of *Rosamunde*. It is interesting that these two songs are amongst the best known songs of the Classical period (S-group).

The results obtained with the stochastic analyses of the compositions indicate that a major factor in all the songs is the rhythmic coherency. The P-group shows a more pronounced and consistent redun-

dancy pattern than the other groups. Because the P-group and the S-group were both chosen from lists that indicate their popularity, it may be argued that the rhythm, or its predictability may be a contributing factor—if not the most predominant factor.

As with all the other analyses demonstrated in this chapter, each of the compositions also has its own unique graphs, confirming that each of the songs has unique rhythmic characteristics.

## 6.4 Stylistic models of music

The typical entropy characteristics of the three groups of compositions were alluded to in the discussion of the entropies of the individual songs. It was shown that each of the individual songs of each of the three groups generated minimum and maximum values that lie within broad limits for the group to which they belong. This means that graphical models for each of the groups can be developed to illustrate the general characteristic and stylistic features of each group. An example of such a model is shown as Figure 6-4 on page 6-14 and illustrates the average minimum and maximum entropy values for all the 1<sup>st</sup> order calculations for each group. The same principle may be applied to create composite models containing the averaged extremes of all the analyses that were done.

All the calculations discussed in this chapter are summarized in the three graphical models that follow, one for each of the groups. The average maximum and minimum entropy values for the stochastic processes (intervals and rhythm) are indicated by the curves, while the single order entropy limits are represented by the rectangles. The horizontal positions of the latter are of no consequence and do not indicate any information pertaining to orders; they are merely placed in a clear horizontal area for the sake of visibility. Note that the number of orders shown are different in each graph; but that the horizontal axis of each graph has a maximum of 70 to maintain equal visual proportions of the curves.

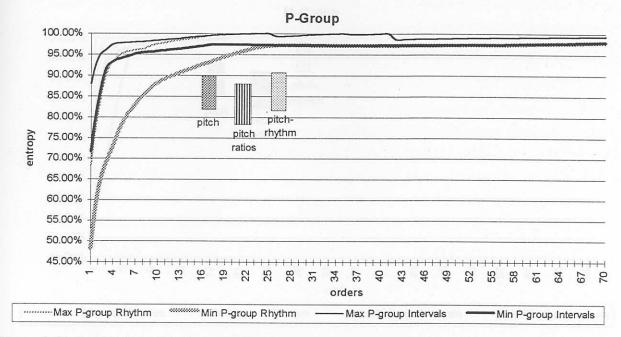


Figure 6-49. Graphical model of the combined entropy values of the P-group

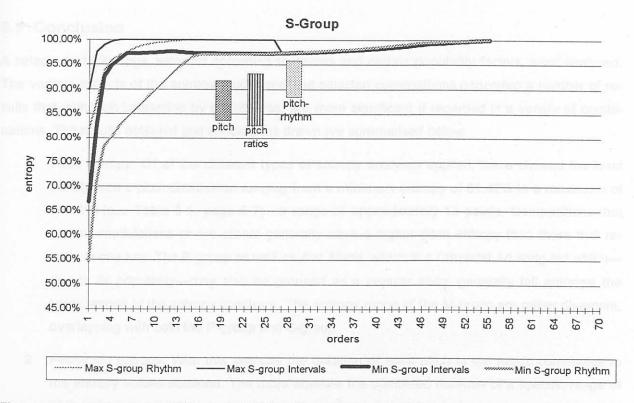


Figure 6-50. Graphical model of the combined entropy values of the S-group

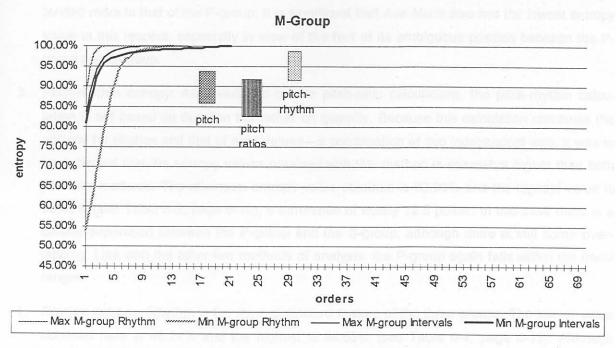


Figure 6-51. Graphical model of the combined entropy values of the M-group

#### 6.5 Conclusion

A selection of 22 songs, selected according to genres and certain popularity factors, were analysed. The various aspects of the entropy analyses of the selected compositions generated a number of results that, although interesting by themselves, are more significant if regarded in a variety of combinations. The results obtained and conclusions drawn are summarised below.

- 1. Pitch entropy. Of all the different types of entropy analyses applied, these showed the least variety with a pitch distribution ranging from a minimum entropy of 81.42% to a maximum of 94.50% (see Table 6-1, page 6-7)—a range of approximately 13 points. Compositions that contain modulations or are atonal generally have a higher pitch entropy than those that remain in one key. The P-group as well as Ave Maria, which is a Classical Art song but which—due to its popularity—may also be grouped as a popular song, generally fall amongst the lower ranges of the entropy spectrum. The entropy range of the M-group are rather divergent, overlapping with both the P-group and S-group.
- 2. Pitch-ratio entropy. With this analysis the duration of each pitch in the music contributes to the entropy values obtained. The more equable the combined duration of a specific range of pitches in a composition is, the higher the entropy would be. Lower entropic results would therefore indicate that the composer dwelt longer on some pitches than on others. With a minimum entropy of 75.70% and a maximum of 94.21% (see Table 6-2, page 6-9), the average difference between the P-group and the S-group increased (18.5 points), compared to that of the pitch entropy (13 points). The entropies of the M-group proved inconclusive, and

tended more to that of the P-group. It is significant that *Ave Maria* also has the lowest entropy value in this respect, especially in view of the fact of its ambiguous position between the P-group and S-group.

- 3. Pitch-rhythm entropy. As a variation on the pitch-ratio calculations, the pitch-rhythm calculation is not based on duration but rather on quantity. Because this calculation combines the entropy for pitches and that of note values—a combination of two independent sets, it was to be expected that the entropy values obtained with this method is somewhat higher than both previous methods. The minimum entropy value obtained is 82.96% and the highest value is 95.42% (see Table 6-3, page 6-10), a difference of nearly 12.5 points. In this case there is a clearer separation between the P-group and the S-group, although there is still some overlapping. Like with the other two methods of analysis, the P-group again falls within the lower ranges.
- 4. Rhythm entropy. Rhythm is the most distinctive feature of the three groups. The lowest value obtained here is 48.24% and the highest is 84.63% (see Table 6-4, page 6-12). Although there is still some overlapping among the three groups, the separation is now much more marked. Again the P-group ranks predominantly amongst the lower values and the S-group amongst the higher values. Interestingly, where the values of the M-group in the three previous analyses were similar to that of the higher values of the S-group, the rhythmic entropy of the M-group is now similar to the low values of the P-group. This indicates that the predictability of these pieces is mainly due to their rhythmic redundancy. Ave Maria that, with the previous types of analyses, ranks amongst the lower values of the P-group, now ranks amongst the higher values of the S-group.
- 5. Combined average of the entropy analyses (Table 6-5 and Figure 6-4, page 6-14). A combination of the pitch-ratio entropy, pitch rhythm entropy and rhythm entropy was used to obtain the average for each of the groups (because of the proximity of the pitch entropies these were not included). Although there is some overlapping between the average entropy values of the three groups, the results clearly indicate that the S-group has the widest range and both its highest and lowest values are higher than the P-group and M-group. The range of the P-group is somewhat smaller and that of the M-group is the smallest.
- 6. Stochastic interval analysis. Since most music is conceived according to inherent structural patterns (except in the case of intentional aleatoric music), it is to be expected that these could be quantified by stochastic analysis of certain aspects. Interval analysis is important in this respect, and for two main reasons; the dynamics<sup>5</sup> of melodic structures and the dynamics of tonal orientation. It should be stressed that Information Theory cannot show or measure tonality as such, but that it can be used to measure the degree of tonal orientation. Tonality, as

The term 'dynamics' in this context does not refer to its musical connotation of 'loudness' but rather to the movement of the intervals governed by specific conventions and voice leading.

used here, should be regarded in its widest meaning, including dodecaphony, and other synthesised scales. In other words the degree to which the music adheres to a specific sequence of predetermined pitches and, by implication, intervals.

Each composition generated a number of entropy values based on the length of sequences, or orders. The number of orders generated before a composition reaches maximum entropy indicates the structural predictability. Compositions with a lower number of orders are much less predictable than a composition that generates a higher number of orders. Of all the results obtained thus far, the orders of the interval entropy proved the most revealing (see Table 6-6, page 6-16). Except for *Nachtigall* (S-group, 4 orders), the seven pieces of the M-group generated the lowest orders (3) and the P-group the highest (70+). The P-group also showed the largest number of average of the orders (49.29), while the S-group and M-group showed an average of 21.29 and 8.43 orders respectively. Again, *Ave Maria* was an interesting case since it firmly belonged to the P-group with 70+ orders. A graph of the interval entropies of each of the compositions (page 6-17 to 6-23) shows that the rate at which the S-group reaches its maximum entropy of 100% is higher than that of the P-group. The M-group has an even higher increase of entropy with each of the orders. The curves of the graphs also show that each composition has its own unique shape to distinguish it from the other compositions.

7. Stochastic rhythm entropy. As for the entropies generated by the intervals, the number of orders generated by this analysis, proved equally significant. The highest orders is found in the P-group where three songs have 70+ orders, while the S-group generated orders between 13 and 54 respectively. Rosamunde, which is a favourite for many listeners generated the 54 orders, which is 32 higher than the next highest in the group. The lowest orders were generated by the M-group with a range between 3 and 13. The P-group averaged 56 orders, the S-group 21.43 orders and the M-group 9.43 orders (see Table 6-8, page 6-27). Again except for Ave Maria (70+ orders) the three groups are clearly separated. The curves of the entropy values for this analysis show that the rate at which 100% entropy is reached is slower than with the stochastic interval analysis, but as with the latter the rate of change is the highest for the M-group to a significant degree, while that of the P-group is the lowest. The graphs also show the unique character of each of the compositions, even more so than is the case with the stochastic interval analysis.

It would appear that entropic and stochastic analyses of single aspects are not sufficient to indicate the overall characteristic traits of a musical style. The same argument also applies when Information Theory is used to ascertain those factors in music that could contribute to its general popularity with the listener. Music is a complex combination and interaction of, amongst others, a variety of pitches, silences, rhythms, and dynamics. A comprehensive approach is required to identify the entropic and stochastic elements that may contribute to the popularity or acceptance of a piece of music, and

therefore contribute to the style of the music as well.

The elements of the music subjected to Information Theoretical analysis, provided sufficient information to identify specific characteristics and style elements that contribute to the stylistic nature of the music and point to the factors that contribute to its accessibility or even popularity.

Accepting that the P-group of songs are generally amongst the most popular pieces of music today, that the S-group are also popular but among a smaller selection of people, and that the M-group are virtually unknown—accept amongst connoisseurs, some definite tendencies are evident:

- Music that generally enjoys greater popularity, generates lower entropy values, especially rhythmically.
- Pieces with lower entropy values generally produce a larger number of orders. Stochastically, the popular pieces reach 100% entropy at a much slower rate than the lesser known pieces.
   One piece, Ave Maria, which shows tendencies of both extremes seems to confirm this argument.
- 3. The lesser known 20<sup>th</sup> century pieces essentially have a much shorter order distribution for the stochastic analyses, and may therefore be less palatable to the general public, even though they are more predictable as far as pitch distribution is concerned. This indicates that although pitch and rhythm distribution may be important indicative factors of the acceptance of a piece of music, they are not exclusive factors. The inherent structural dynamics for interval and rhythmic structure, balanced by the careful selection of pitch and note values, seem to be important in establishing whether a piece of music has the possibility of being accepted or even becoming popular or a classic.

The overlapping values between the results of each group tend to support the suggestion that this type of analysis may be done free from period bias. Especially in the 20<sup>th</sup> century, music has become rather eclectic, and there are many composers who compose in any of the historical styles. Furthermore, much of what today is called 'popular', shows similarities with some of the older styles of music, and is often written with just this purpose in mind. To ignore these facts and maintain the traditional system of historical divisions and classification of music would therefore contradict the aims of entropy analysis which does not claim to provide historical information of any kind. Much of traditional music analysis, is based on comparative methods for which set and preconceived models serve as point of departure. Entropy analysis, in contrast, considers the inherent dynamics of music without any specific reference to outside models. However, entropic analysis is also capable of establishing the similarities in information content of a specific period of music, or even of an individual composer.



# STOCHASTIC MODELS OF MUSIC AND THEIR APPLICATIONS

In the previous chapter, aspects of Information Theory were used to analyse music selected from three different styles. Some of the selected compositions have a proven record of popularity, while one group is relatively unknown. The aim of the analysis was to discover common entropic factors within each group. Similarities between the various compositions were used to establish broad norms and predictability factors common to three different musical styles.

Chapter 6 also dealt with developing models of the characteristics and style of a group of compositions, which may then be compared to find similarities or differences between the various different styles. However, the range of applications of Information Theory to music may be increased if individual pieces can also be compared with such stylistic models that are derived from a large quantity of music. This chapter deals with the entropy values of an individual piece of music. It shows how the entropy values of a specific composition may be used for comparison with a stylistic model, thereby establishing the degree of similarity (or difference) between the selected composition, and style and characteristics of a pre-determined group of compositions.

Even though there are some significant similarities between the different groups, each composition also has its own unique and distinct characteristics that identify it from the other compositions. The results of the analysis may therefore also effectively be applied to create a uniquely identifying composite image or model for comparison with a global stylistic model.

Most of the generated results were illustrated by means of both tables as well as graphs. The latter are no more than grades on a scale with a maximum of 100, that indicate the contents of information embodied in a composition. The fact that this scale is based on percentage values, and therefore implies relative values, makes it an ideal vehicle by which the individual characteristics of compositions may be compared. Traditional methods of comparison in music frequently use comparative methods whereby a specific piece of music is compared with a theoretical model. An example of this type of analysis is found in musical form of which examples have been given in earlier chapters. While many compositions are said to be in sonata form, very few conform completely to the standard textbook model—usually composers use such a model merely as a general guide, thereby imbuing their music with creative and artistic elements.

Comparison of composed music with an imaginary model requires much description and discourse to show how and where it deviates or conforms to the model. If two or more similar compositions are compared with each other, as well as with the model, the increase in quantity of descriptive material increases likewise.

The method suggested here creates a single model which may include or exclude as many elements as needed, and which immediately provides one with as much statistical information about a piece of music as is required. Although the results so obtained may be used to compare different compositions, there is no need for imaginary or synthetic models for reference.

# 7.1 Creating an entropy model of music

The additive process involved in creating a graphical identification of music is illustrated below.

For this discussion three of the compositions listed at the beginning of Chapter 6 were randomly selected—one from each group. All the entropic and stochastic values used in this chapter were also taken from the tables in Chapter 6:

- Stevie Wonder: You are the sunshine of my life (P-group)
- 2. Robert Schumann: Das ist ein Flöten (S-group)
- 3. Benjamin Britten: Since she whom I loved (M-group)

# 7.1.1 One-dimensional entropy combinations

In Chapter 6, a number of single dimensional entropy values for four related music elements were obtained for each of 22 compositions. These are in order of complexity:

Pitch entropy

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- 2. Rhythm entropy
- 3. Pitch rhythm entropy
- 4. Pitch ratio entropy

As mentioned before, the easiest element to calculate is pitch entropy; the three entropy values of each are:

You are the sunshine of my life	Das ist ein Flöten	Since she whom I loved
83.87%	87.57%	91.79%

Table 7-1. Entropy of pitch distribution for the three selected songs

The differences among the three compositions are obvious and may effectively be illustrated on a graph:

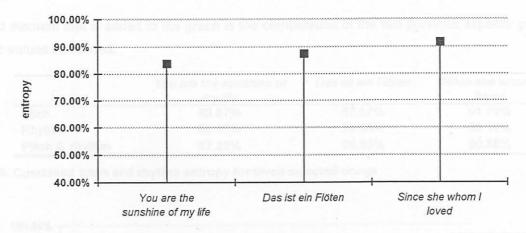


Figure 7-1. Comparison of pitch entropies for three selected songs

The figures shown above clearly show the difference between the pitch distribution of the three compositions expressed as relative entropy, and may already be used as part of a comparative study.

Rhythmic entropy is also relatively straight forward to calculate and may now be added to further enhance the graphical representation:

	You are the Sunshine of My Life	Das ist ein Flöten	Since She Whom I loved
Pitch	83.87%	87.57%	91.79%
Rhythm	68.07%	78.08%	59.12%

Table 7-2. Rhythm entropy for three selected songs

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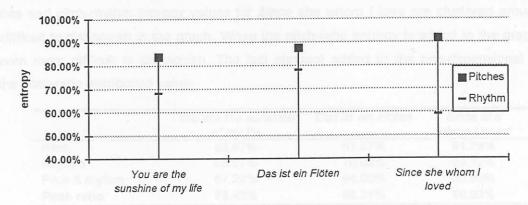


Figure 7-2. Comparison of pitch and rhythm entropies for three selected songs

A second dimension has now been added changing the visual impact of the graph, and the differences among the three compositions are self-evident. Although the pitch entropy of the third song is much higher than that of the first, the rhythm entropy is considerably lower.

The third element that is added to the graph is the combination of the two previous aspects: pitch and rhythmic values combined:

	You are the sunshine of my life	Das ist ein Flöten	Since she whom I loved
Pitch	83.87%	87.57%	91.79%
Rhythm	68.07%	78.08%	59.12%
Pitch & rhythm	87.22%	94.00%	90.88%

Table 7-3. Combined pitch and rhythm entropy for three selected songs

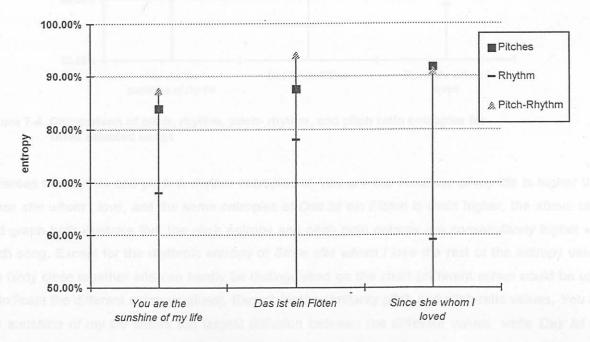


Figure 7-3. Comparison of pitch, rhythm and pitch-rhythm entropies for three selected songs

The pitches and pitch-rhythm entropy values for *Since she whom I* love are clustered around 90% and are difficult to distinguish in the graph. When the pitch-ratio entropy is added to the graph it becomes even more difficult to distinguish. The last element added to the one-dimensional entropy chart is the pitch-ratio distribution value:

a convincing arti	You are the sunshine of my life	Das ist ein Flöten	Since she whom I loved
Pitch	83.87%	87.57%	91.79%
Rhythm	68.07%	78.08%	59.12%
Pitch & rhythm	87.22%	94.00%	90.88%
Pitch ratio	78.42%	88.21%	90.93%

Table 7-4. Pitch ratio entropy for three selected songs

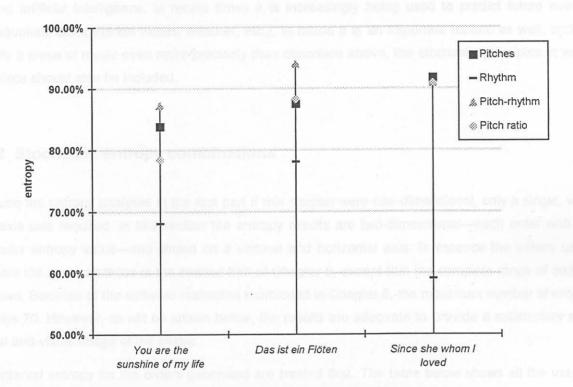


Figure 7-4. Comparison of pitch, rhythm, pitch- rhythm, and pitch ratio entropies for three selected songs

Whereas the rhythm and pitch & rhythm entropies of You are the sunshine of my life is higher than Since she whom I love, and the same entropies of Das ist ein Flöten is even higher, the above table and graph both illustrate that the pitch entropy and pitch ratio entropy are consecutively higher with each song. Except for the rhythmic entropy of Since she whom I love the rest of the entropy values are fairly close together and can hardly be distinguished on the chart (different colour could be used to indicate the different entropy values). Except for the similarity pitch and pitch-ratio values, You are the sunshine of my life shows the largest diffusion between the different values, while Das ist ein Flöten is fairly evenly distributed. In respect of pitch and pitch-ratio values Das ist ein Flöten and Since she whom I love also show similarities.

At first it was considered to add the average of the four values described above. However, since different aspects of the music (especially pitch and rhythm which are quite divergent elements) are being demonstrated, averaging the values would diminish the distinctiveness of the graphic representation. It would thus not furnish an accurate assessment (for example if the rhythmic and pitch entropy values of two songs are 80% and 40%, and 55% and 75% respectively, the average would be 60% and 65% — not a convincing or significant difference).

For identification or classification of music the basic procedures described so far may be sufficient for certain applications. However, the graphs shown above have only one dimension. Throughout this thesis some stress has been placed on the role of the stochastic dynamics in communication. This applies to many of the arts and speech, and is extensively applied in technologies such as cybernetics and artificial intelligence. In recent times it is increasingly being used to predict future events (earthquakes, stock market trends, weather, etc.). In music it is an important feature as well, and to identify a piece of music even more precisely than described above, the stochastic principles at work in a piece should also be included.

### 7.1.2 Stochastic entropy combinations

Because the entropy analyses in the first part if this chapter were one-dimensional, only a single, vertical axis was required. In this section the entropy results are two-dimensional—each order with its particular entropy value—and shown on a vertical and horizontal axis. In essence the values used here are the same as those in the second half of Chapter 6, except that the complete range of orders is shown. Because of the software restriction mentioned in Chapter 6, the maximum number of orders remains 70. However, as will be shown below, the results are adequate to provide a satisfactory statistical and visual image of the music.

The interval entropy for the orders generated are treated first. The table below shows all the values generated for the three compositions:

Order	You are the sunshine of my life	Das ist ein Flöten	Since she whom I
1	81.57%	66.81%	80.95%
2	92.09%	82.61%	88.99%
3	92.78%	92.43%	94.61%
4	93.43%	96.28%	97.23%
5	94.06%	97.34%	98.43%
6	94.59%	98.01%	99.27%
7	95.21%	98.38%	99.65%
8	95.51%	98.96%*	99.79%
9	95.63%	99.21%	99.89%
10	95.81%	99.44%	100.00%
11	95.99%	99.61%	
12	96.18%	99.70%	
13	96.39%	99.79%	

Order	You are the sunshine of my life	Das ist ein Flöten	Since she whom I loved
14	96.61%	99.89%	
15	96.83%	100.00%	
16	97.07%		
17	97.32%		
18	97.58%		
19	97.85%		
20	98.14%		
21	98.37%	Name of the last	
22	98.61%		
23	98.77%		
24	98.93%		
25	99.00%		
26	99.08%		
27	99.13%		
28	99.17%		
29	99.21%		
30	99.26%		
31	99.31%		
32	99.37%		
33	99.42%		
34	99.48%		
35	99.55%		
36	99.61%		
37	99.68%		
38	99.76%		
39	99.83%		
40	99.91%		
41	100.00%		

Table 7-5. Stochastic interval entropies for three compositions

To make the comparison between the curves more clearly visible, the values of Table 7-5 are combined in a single graph below. The lowest starting point of each curve lies above 50%, and the lowest entropy value has also been limited to 50% to make the curves more distinct. Ultimately each composition should be drawn on individual graphs, showing the full range from 0% to 100% and the individual curves may be given different colours. Important characteristics of the graph are the starting point of each curve (order 1 values), differences in the length of the curves (orders), the gradient and the general shape of each curve.

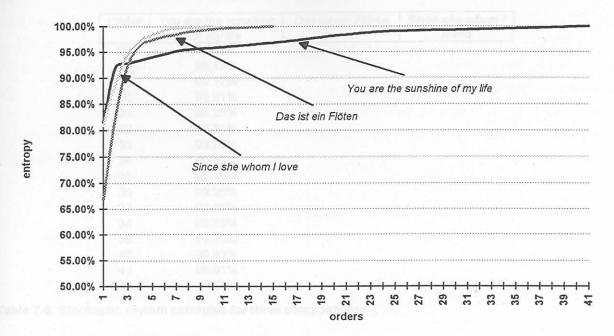


Figure 7-5. Interval entropies of the three compositions

The same principle applied previously for the intervals is also applied to the rhythmic values of the three songs. First the values are shown in tabular form, followed by a graphical rendition of the same values.

Order	You are the sunshine of my life	Das ist ein Flöten	Since she whom I loved
1	68.07%	78.08%	59.12%
2	81.32%	84.78%	72.17%
3	90.14%	85.99%	82.58%
4	93.48%	88.80%	89.45%
5	93.88%	91.35%	93.74%
6	94.74%	93.66%	96.49%
7	95.50%	95.15%	98.01%
8	95.62%	96.68%	99.04%
9	95.76%	97.20%	99.69%
10	95.92%	97.85%	99.89%
11	96.11%	98.31%	99.95%
12	96.31%	98.62%	100.00%
13	96.53%	98.97%	
14	96.75%	99.36%	
15	96.99%	99.79%	
16	97.24%	100.00%	
17	97.49%		
18	97.76%		
19	98.04%		
20	98.33%		
21	98.63%	vident, each fadica	
22	98.78%		
23	98.95%		
24	98.98%		
25	99.01%		
26	99.05%		

Order	You are the sunshine of my life	Das ist ein Flöten	Since she whom I loved
27	99.08%	about the contract of	State Surger recover
28	99.13%		
29	99.17%		
30	99.21%		
31	99.26%		
32	99.31%		
33	99.37%		
34	99.42%		
35	99.48%		
36	99.55%		
37	99.61%		
38	99.68%		
39	99.76%		
40	99.83%		
41	99.91%		
42	100%		

Table 7-6. Stochastic rhythm entropies for three compositions

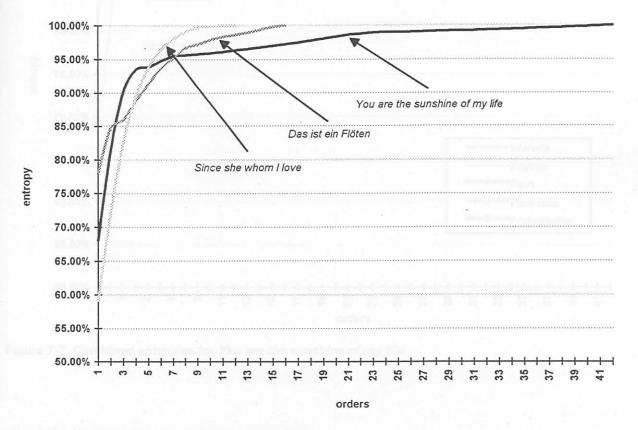


Figure 7-6. Rhythm entropies of three compositions

The differences between the curves are self-evident, each indicating different rhythmic processes at work within the structure of the compositions.

However, to demonstrate the unique properties of each of the three compositions, their individual characteristic entropies may separately be combined into a single graph. This means that the ele-

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ments peculiar to each of the songs are extracted from Figure 7-4, Figure 7-5 and Figure 7-6 and combined into a single graph. The individual graphs for each song are shown below. To maintain the same proportions between the graphs, the number of orders on the x-axis has been kept the same throughout, even where the higher orders have no entropy values. The short horizontal lines represent entropy values of single dimensions and should, in reality, only be indicated on the vertical axis as a short line or dot. They were made longer to make them more visible and do not imply any specific number of orders.

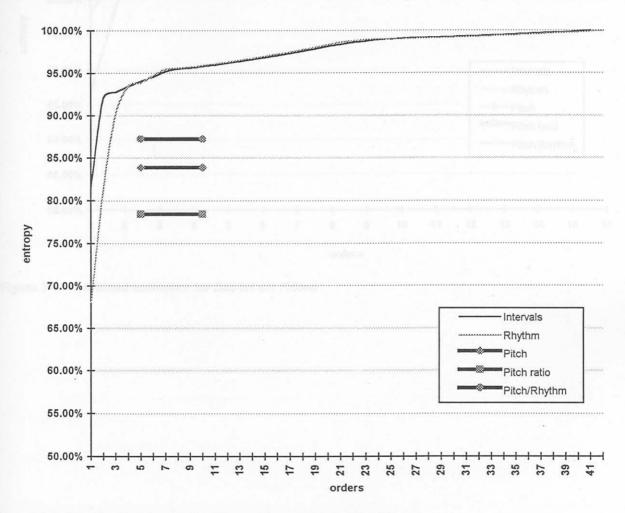


Figure 7-7. Combined entropies for You are the sunshine of my life

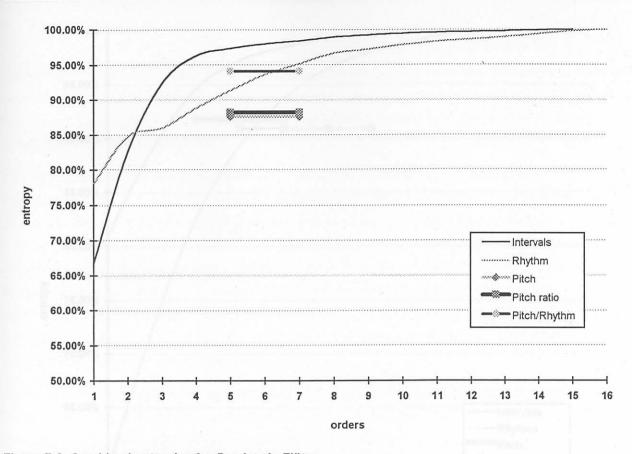


Figure 7-8. Combined entropies for Das ist ein Flöten

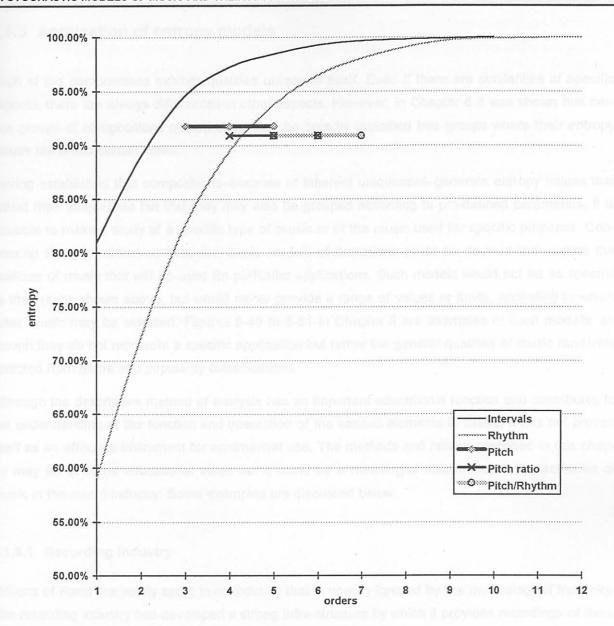


Figure 7-9. Combined entropies for Since she whom I loved

A complete set of graphs, together with tables of entropy values, for each of the 22 compositions is found in Appendix II.

Even a cursory perusal of the three graphs above, illustrates that each of the compositions is unique in many ways. Even if two compositions closely resembled each other, there would still be certain aspects where differences could be discerned unless, of course, the works were identical. By comparing the graphs of individual compositions, as shown above, with the models for a larger group of music, similarities and differences may be identified. Depending on the objectives of the comparison, this aspect may be applied to single aspects, such as rhythm, or to a variety of combinations of aspects.

### 7.1.3 Application of entropy models

Each of the compositions exhibits qualities unique to itself. Even if there are similarities of specific aspects, there are always differences in other aspects. However, in Chapter 6 it was shown that certain groups of compositions nevertheless may be broadly classified into groups where their entropy values fall within certain limits.

Having established that compositions—because of inherent uniqueness—generate entropy values that reflect their uniqueness but that they may also be grouped according to pre-defined parameters, it is possible to make a study of a specific type of music or of the music used for specific purposes. Considering the information so obtained, basic models of limitations could be devised that dictate the qualities of music that will be used for particular applications. Such models would not be as specific as the results shown above, but would rather provide a range of values or limits, according to which other music may be selected. Figures 6-49 to 6-51 in Chapter 6 are examples of such models, although they do not represent a specific application but rather the general qualities of music randomly selected from genre and popularity classifications.

Although the descriptive method of analysis has an important educational function and contributes to the understanding of the function and interaction of the various elements in music, it has not proven itself as an effective instrument for commercial use. The methods and results described in this chapter may be of some educational value but it could be a meaningful instrument for the selection of music in the music industry. Some examples are discussed below.

#### 7.1.3.1 Recording industry

Millions of Rand are yearly spent in an industry that is usually ignored by the musicological fraternity. The recording industry has developed a strong infra-structure by which it provides recordings of thousands of popular musicians to the public. In the employment of many of these large corporations are specialists who know what qualities are required to make a best-seller; which qualities are preferred for the specific echelon of the population for which they cater. This ability, or 'feel' by talent hunters and promoters is usually acquired by many years of close contact with the types of music in which they specialise.

It is suggested here that the learning processes involved in obtaining this experience are essentially an acquired ability to recognise the entropic properties of music—the quantity of information that is generated and the rate at which it is generated, in other words the predictability of the music. However, the trial-and-error method is not fail-safe, and thousands of recordings are made that never become well known and are quickly forgotten. Few people ever know about these failures, because active promotions of such records are quickly ceased to save on the costs.

To help reduce the number of failures that are produced regularly, the system described in this chapter could be used to develop basic models for specific types of music that have proven to be commercially viable. New additions to the repertoire could be compared to ascertain if they entropically fall within the set limits indicated by the models.

It is not implied that this is a fail-proof method, since the promotional efforts involved in bringing a musician or group of musicians to fame is an important factor. Prior popularity of artists also plays an important role but even the most popular artists regularly produce music that is quickly forgotten. Only those songs that have the required quality to maintain the public's interest eventually become 'classics', a quality that can solely be ascribed to the musical characteristics.

#### 7.1.3.2 Copyright controversies and litigation

Every so often, there are controversies concerning plagiarism in the world of popular music. Since the traditional methods of analysis are often vague and open to interpretative manipulation, conclusive decisions are rare. Especially when such cases end up in court it often results in costly and inconclusive flascos.

Information Theory, and particularly the methods illustrated in this thesis, could effectively be applied to help solve copyright contraventions and controversies about plagiarism; perhaps even before these reach the courts. The degree of entropic similarity between two compositions may easily be measured using stochastic analysis. Since certain elements, such as the drum rhythms and chord progressions are common to many types of popular music, these could be ignored while the essential melodic qualities are isolated for scrutiny.

#### 7.1.3.3 Specific applications of music

The last fifty years has seen a steady increase in the use of music for specific applications, very often related to psychological matters. In other words, music is often used to influence, directly or subliminally, the behaviour of people. Some examples are:

- 1. Music Therapy: specific types of music used by therapists working with mentally disadvantaged children, the depressed; music used by dentists to calm their patients, and many more. Once a model of the required type of music has been developed additional music, of which the characteristics fall within ranges specified by the model, may be selected with relative ease.
- Accelerated learning: ever since accelerated learning has become popular there is a constant search for music that conforms to specific qualities. Although much of this research is based on hit or miss results, stochastic analysis could generate models of the music that has proven effective for this use and to select music that adheres to the model.

- 3. Music in Commerce: an example of commercial application of music is the music that is sub-liminally piped through speakers in shopping centres to increase purchasing by the public. Models of the most effective music may be constructed using the methods described herein and used as model for selection of the most suitable music. Obviously, the kind of music that is suitable, first needs to be established by research but many marketing institutions have already done research in this direction.
- 4. Advertising: most television and radio advertisements are accompanied by music to achieve specific effects or to influence the public somehow by associative processes. The musical characteristics that are most effective under specific circumstances may be developed as a statistical model to ensure that the best results are obtained.
- 5. Market research in broadcasting: broadcasting houses constantly need to asses their ratings with their listening public. Depending on the kind of broadcaster such market research is often based on the musical tastes for which they cater. Having established the preference of their listening public, stochastic models may be devised and the music that is being broadcast compared with the models.

#### 7.2 Conclusion

Each different composition generates unique entropy values that can be shown in a graph of two dimensions. The results so obtained could serve as a kind of 'finger-print' unique only to that composition. In chapter 6 methods were illustrated on how characteristics of stylistically similar compositions could provide the information to create a model specific to the characteristics of that group. Individual pieces may then be compared with the pre-defined model to establish to what extent the piece conforms to the group-model.

The process shown is accumulative in that any number of elements may be included in the creation of an identifying chart and therefore also in the comparison. The simplest aspect being the quantification of pitch or rhythm and their factors of distribution. As additional elements are added to the model, so the model becomes more complete and allows for more precise comparisons.

Having established the possibilities that stochastic analysis offers, it is possible to apply the methods in commerce and areas where music plays an active role in mood and behaviour modification.



### CONCLUSION

The result of this research shows that Information Theory may effectively be applied to analyse music by measuring the amount of information it contains and therefore also its rate of predictability. It was shown that the degree of popularity or appeal of music may be a reflection of the measure of predictability contained in a combination of different elements of music. Different levels of predictability in music may therefore cater for the preferences of different audiences.

Three groups of songs were analysed and the results compared. Two of the groups of songs were selected on the basis of their popularity among two essentially different listening audiences. One group comprises seven songs from the current popular repertoire; songs that have proven to be modern day classics. The second group consists of eight Art Songs from the Romantic period (Robert Schumann, Franz Schubert, and Johannes Brahms) and were also selected because of their popularity amongst an audience who generally have a different musical taste. A third group consists of seven 20th Century songs. Few of these have ever been recorded even though some of the composers are relatively well known.

One of the Classical Art songs, Schubert's *Ave Maria*, proved very useful in support of the hypothesis of this study. This song was a link between the results obtained for both the Popular and the Classical Art songs. During the last thirty years or so, *Ave Maria*, has often featured on the popularity charts—with the help of rhythmic manipulation and additions from the pop-artists—and is often included on recordings by many different music groups and soloists.

CONCLUSION 8-2

The results obtained for each of the three groups of songs were significantly different and were demonstrated by means of tables and graphs. Below is a summary of the most important conclusions of this research:

- Each of the three groups generated a characteristic range of predictability levels for six aspects of the analysis. Generally the Classical Songs were found to be less predictable than the Popular Songs. Especially the interval analysis of the Modern songs shows that the predictability of these songs is much lower than that of the Classical songs.
- Where the predictability of specific musical aspects of the different groups overlaps, the overlapping values are usually compensated for by greater differences in the predictability of other aspects of the music.
- 3. The aspect in the music that showed the largest differences in predictability, both static and structural was rhythm. The songs belonging to the Popular group of songs are much more predictable (about 50%), and are structurally much more repetitive than the Classical songs. Although the Modern songs showed similar initial predictability as the Popular songs, they showed to be structurally much less cohesive than any of the other songs in the other groups.
- 4. Interval predictability was rather lower in all three groups than had been expected. However, there was also a marked difference among the three groups, with the Classical songs and Modern songs being noticeably less predictable than the popular songs.
- A combination of the results obtained showed that the predictability factor of each group fell
  within specific upper and lower limits. This information was applied to develop a graphical
  model for each of the groups.
- 6. From the results obtained, it is clear that the melodies which are more predictable, especially rhythmically more predictable, are predominantly the more popular items (including Ave Maria and Rosamunde). In the structural analyses (stochastic entropy) these songs also showed a greater degree of structural cohesion or repetition.

In the last chapter, a proposed approach to developing specific models for music of any kind was demonstrated. A variety of possible practical applications in the music industry and other fields in which music features prominently, were discussed.

The results obtained in this research show that Information Theory certainly can practically classify music in ways that are impossible by traditional methods of analysis. Most of the research done to date has mainly revolved around the numbers stochastic analysis produces, while practical applications have largely been ignored. As has been shown, this form of analysis makes many practical and beneficial applications possible.

BIBLIOGRAPHY B-1

## Bibliography

The following list of books and articles were used in the preparation and research for this dissertation. Information Theory is a well-established science and there are many current books available on this subject. Many of these are firmly in the domain of statistics and contain subject material that lies beyond the scope of this thesis. However, to trace the history and development of the application of Information Theory, many older sources had to be consulted. Sources that deal with the application of Information Theory to music are especially rare after the late 1970s.

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Appendices

## Appendix I: Melodies of 22 selected compositions

Melodies are listed according to groups and then in alphabetical order.

Popular music (P-group)	
Annie's song, John Denver	A-3
Love letters, Victor Young	A-4
One more night, Phil Collins	A-5
Sleepy shores, Johnny Pearson	A-7
Summer love, Claudio Gizzi	A-8
Thank you for the music, Benny Andersson & Björn Ulvaeus	A-9
You are the sunshine of my life, Stevie Wonder	A-10
Classical Art songs (S-group)	
Ave Maria, Franz Schubert	A-11
Das ist ein Flöten und Geigen, Robert Schumann	A-12
Das Wandern, Franz Schubert	A-13
Halt, Franz Schubert	A-14
Ich will meine Seele tauchen, Robert Schumann	A-15
Liebestreu, Johannes Brahms	A-16
Nachtigall, Johannes Brahms	
Rosamunde, Franz Schubert	A-18
20th Century Art songs (M-group)	
Being young and green, Arthur Bliss	A-19
Cupid and my Campaspe, Martin Dalby	A-20
How love came in, Lennox Berkeley	A-21
In Flanders fields, Charles Ives	A-22
Nun ich der Riesen, Alban Berg	A-23
Since she whom I love, Benjamin Britten	A-24
Whenas the nie Peter Warlock	Δ-25

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# Appendix II: Tables and graphs of entropies of 22 Selected compositions

Compositions are listed according to groups and then alphabetically by titles. Analysis data is shown in two formats: first in table format followed by a graph representing the same values. The vertical bars in the graphs indicate the entropy for one-dimensional elements and are not an indication of any specific orders.

P	opular music (P-group)	
	Annie's song, John Denver	A-27
	Love letters, Victor Young	A-28
	One more night, Phil Collins	A-29
	Sleepy shores, Johnny Pearson	A-30
	Summer love, Claudio Gizzi	
	Thank you for the music, Benny Andersson & Björn Ulvaeus	A-32
	You are the sunshine of my life, Stevie Wonder	A-33
C	lassical Art songs (S-group)	
	Ave Maria, Franz Schubert	A-34
	Das ist ein Flöten und Geigen, Robert Schumann	. A-35
	Das Wandern, Franz Schubert	. A-36
	Halt, Franz Schubert	. A-37
	Ich will meine Seele tauchen, Robert Schumann	. A-38
	Liebestreu, Johannes Brahms	. A-39
	Nachtigall, Johannes Brahms	. A-40
	Rosamunde, Franz Schubert	. A-41
20	Oth Century Art songs (M-group)	
	Being young and green, Arthur Bliss	. A-42
	Cupid and my Campaspe, Martin Dalby	
	How love came in, Lennox Berkeley	. A-44
	In Flanders fields, Charles Ives	
	Nun ich der Riesen, Alban Berg	
	Since she whom I love, Benjamin Britten	
		۸ ۸ ۸

## Popular Music Group (P-Group)

Title: Composer: Annie's song Denver, John

Category:

84.40%

Pitch

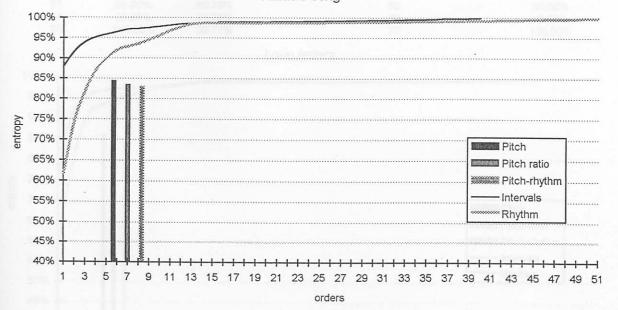
Entropies Pitch ratio 83.46%

Pitch/Rhythm 82.96%

Order	Intervals	Rhythm
1	88.03%	61.57%
2	91.65%	74.31%
3	94.02%	81.93%
4	95.18%	87.04%
5	95.92%	89.95%
6	96.51%	92.06%
7	97.19%	92.91%
8	97.37%	93.68%
9	97.60%	94.67%
10	97.87%	95.72%
11	98.17%	96.84%
12	98.49%	97.80%
13	98.69%	98.49%
14	98.80%	98.74%
15	98.92%	98.74%
16	98.97%	98.75%
17	98.99%	98.76%
18	99.02%	98.77%
19	99.04%	98.78%
20	99.07%	98.79%
21	99.10%	98.80%
22	99.13%	98.81%
23	99.16%	98.83%
24	99.20%	98.84%
25	99.23%	98.86%
26	99.27%	98.88%

Order	Intervals	Rhythm
27	99.31%	98.90%
28	99.35%	98.93%
29	99.39%	98.95%
30	99.43%	98.97%
31	99.48%	99.00%
32	99.53%	99.03%
33	99.58%	99.06%
34	99.63%	99.10%
35	99.69%	99.13%
36	99.75%	99.17%
37	99.81%	99.21%
38	99.87%	99.25%
39	99.93%	99.29%
40	100.00%	99.33%
41		99.38%
42		99.43%
43		99.48%
44		99.54%
45		99.60%
46		99.66%
47		99.72%
48		99.78%
49		99.85%
50		99.93%
51		100.00%

## Annie's song



Titel: Composer:

Love letters Young, Victor

Category:

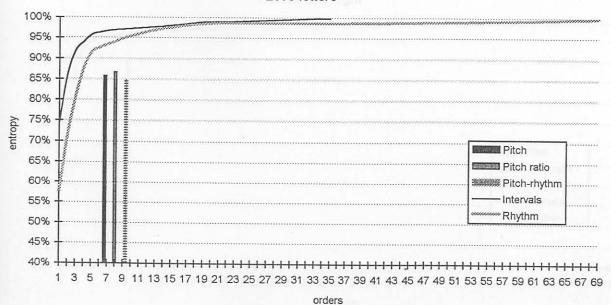
Pitch 85.77% Entropies Pitch ratio 86.73%

Pitch/Rhythm 84.76%

Order	Intervals	Rhythm
1	74.31%	57.66%
2	85.96%	71.02%
3	91.89%	80.49%
4	94.01%	87.47%
5	95.80%	91.48%
6	96.37%	92.71%
7	96.71%	93.46%
8	96.96%	94.08%
9	97.13%	94.91%
10	97.27%	95.51%
11	97.41%	96.06%
12	97.57%	96.56%
13	97.73%	96.95%
14	97.91%	97.34%
15	98.10%	97.62%
16	98.30%	97.91%
17	98.51%	98.22%
18	98.73%	98.37%
19	98.96%	98.53%
20	99.01%	98.69%
21	99.05%	98.75%
22	99.10%	98.75%
23	99.16%	98.74%
24	99.22%	98.74%
25	99.28%	98.74%
26	99.35%	98.74%
27	99.42%	98.74%
28	99.50%	98.74%
29	99.58%	98.74%
30	99.66%	98.74%
31	99.75%	98.75%
32	99.85%	98.75%
33	99.90%	98.76%
34	99.95%	98.76%
35	100.00%	98.77%

Order	Intervals	Rhythm
36	THE LONG	98.78%
37		98.79%
38		98.80%
39		98.82%
40		98.83%
41		98.84%
42		98.86%
43		98.88%
44		98.90%
45		98.92%
46		98.94%
47		98.96%
48		98.98%
49		99.01%
50		99.04%
51		99.07%
52		99.10%
53		99.13%
54		99.16%
55		99.20%
56		99.24%
57		99.28%
58		99.32%
59		99.36%
60		99.41%
61		99.46%
62		99.51%
63		99.56%
64		99.62%
65		99.67%
66		99.73%
67		99.80%
68		99.86%
69		99.93%
70		100.00%

#### Love letters



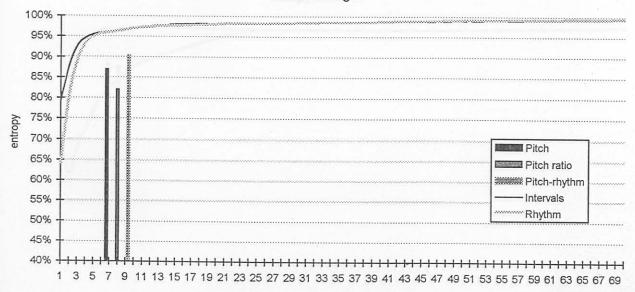
Title: Composer: One more night Collins, Phil

Category

Pitch 87.02% Entropies
Pitch ratio Pitch/Rhythm
82.70% 90.42%

Order	Intervals	Rhythm	Order	Intervals	Rhythm
1	79.24%	64.11%	36	98.50%	98.60%
2	87.78%	80.33%	37	98.53%	98.63%
3	92.65%	88.98%	38	98.55%	98.67%
4	94.63%	93.49%	39	98.58%	98.70%
5	95.59%	95.04%	40	98.61%	98.74%
6	95.96%	95.76%	41	98.63%	98.77%
7	96.20%	96.04%	42	98.66%	98.81%
8	96.49%	96.36%	43	98.69%	98.85%
9	96.74%	96.67%	44	98.72%	98.89%
10	97.01%	97.01%	45	98.76%	98.93%
11	97.23%	97.31%	46	98.79%	98.97%
12	97.46%	97.47%	47	98.82%	99.01%
13	97.67%	97.66%	48	98.86%	99.05%
14	97.88%	97.72%	49	98.89%	99.10%
15	98.02%	97.78%	50	98.93%	99.14%
16	98.15%	97.85%	51	98.96%	99.19%
17	98.17%	97.92%	52	99.00%	99.23%
18	98.19%	98.00%	53	99.04%	99.24%
19	98.21%	98.09%	54	99.05%	99.25%
20	98.24%	98.14%	55	99.07%	99.26%
21	98.26%	98.18%	56	99.08%	99.27%
22	98.27%	98.22%	57	99.10%	99.28%
23	98.27%	98.24%	58	99.11%	99.29%
24	98.28%	98.26%	59	99.13%	99.30%
25	98.29%	98.28%	60	99.15%	99.31%
26	98.30%	98.30%	61	99.17%	99.32%
27	98.32%	98.33%	62	99.19%	99.33%
28	98.33%	98.36%	63	99.21%	99.34%
29	98.35%	98.39%	64	99.23%	99.36%
30	98.37%	98.42%	65	99.25%	99.37%
31	98.39%	98.44%	66	99.27%	99.38%
32	98.41%	98.47%	67	99.30%	99.39%
33	98.43%	98.50%	68	99.32%	99.40%
34	98.46%	98.54%	69	99.34%	99.42%
35	98.48%	98.57%	70	99.37%	99.43%

### One more night



orders

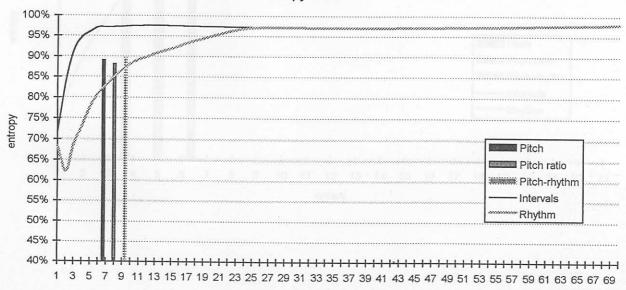
Title: Composer: Sleepy shores Pearson, Johnny

Category:

Pitch 89.11% Entropies
Pitch ratio Pitch/Rhythm
88.15% 89.70%

Order	Intervals	Rhythm		Order	Intervals	Rhythm
1	71.68%	68.30%		36	97.15%	97.13%
2	83.82%	62.30%		37	97.15%	97.13%
3	91.54%	68.80%	· 100.57%	38	97.15%	97.13%
4	94.80%	73.16%		39	97.15%	97.13%
5	96.13%	77.87%		40	97.16%	97.14%
6	97.16%	81.05%		41	97.16%	97.14%
7	97.20%	83.15%		42	97.17%	97.15%
8	97.27%	85.19%		43	97.18%	97.16%
9	97.37%	86.85%		44	97.19%	97.17%
10	97.50%	88.31%		45	97.20%	97.18%
11	97.53%	89.31%		46	97.22%	97.20%
12	97.62%	90.00%		47	97.23%	97.21%
13	97.58%	90.71%		48	97.25%	97.23%
14	97.55%	91.34%		49	97.27%	97.25%
15	97.51%	91.97%		50	97.29%	97.27%
16	97.48%	92.62%		51	97.31%	97.29%
17	97.45%	93.28%		52	97.34%	97.31%
18	97.42%	93.94%		53	97.36%	97.34%
19	97.39%	94.51%		54	97.39%	97.36%
20	97.36%	95.05%		55	97.42%	97.39%
21	97.33%	95.59%		56	97.45%	97.42%
22	97.31%	96.09%		57	97.48%	97.45%
23	97.29%	96.57%		58	97.52%	97.48%
24	97.27%	96.86%		59	97.55%	97.52%
25	97.25%	97.09%		60	97.59%	97.56%
26	97.23%	97.22%		61	97.63%	97.59%
27	97.21%	97.20%		62	97.67%	97.63%
28	97.20%	97.19%		63	97.71%	97.67%
29	97.19%	97.18%		64	97.75%	97.72%
30	97.18%	97.16%		65	97.80%	97.76%
31	97.17%	97.16%		66	97.84%	97.81%
32	97.16%	97.15%		67	97.89%	97.85%
33	97.15%	97.14%		68	97.94%	97.90%
34	97.15%	97.13%		69	97.99%	97.95%
35	97.15%	97.13%		70	98.05%	98.01%





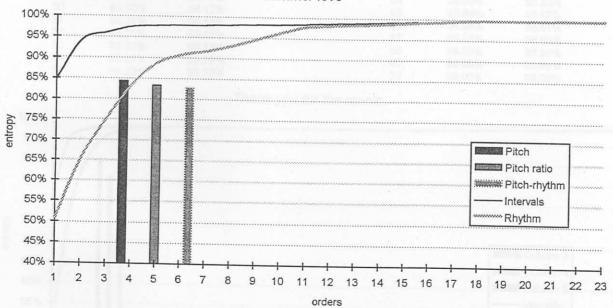
Summer Iove Gizzi, Claudio

Composer: Category:

Pitch Pitch ratio Pitch/Rhythm 84.40% 83.46% 82.96%

Order	Intervals	Rhythm
1	84.83%	50.85%
2	94.27%	65.35%
3	96.11%	75.31%
4	97.52%	83.59%
5	97.81%	88.81%
6	97.91%	90.80%
7	98.01%	91.75%
8	98.13%	93.11%
9	98.26%	94.70%
10	98.40%	96.43%
11	98.55%	97.85%
12	98.72%	98.24%
13	98.90%	98.48%
14	99.08%	98.70%
15	99.24%	98.96%
16	99.40%	99.22%
17	99.56%	99.52%
18	99.73%	99.74%
19	99.78%	99.79%
20	99.84%	99.84%
21	99.89%	99.89%
22	99.94%	99.94%
23	100.00%	100.00%

### Summer love



Thank you for the music Andersson & Ulvaeus

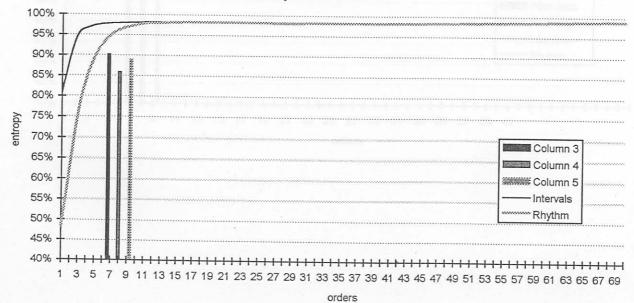
Category:

Entropies

Pitch Pitch ratio Pitch/Rhythm 90.16% 85.95% 88.95%

Order	Intervals	Rhythm	Order	Intervals	Rhythm
1	80.75%	48.24%	36	98.24%	98.20%
2	89.72%	63.44%	37	98.25%	98.21%
3	95.42%	76.14%	38	98.26%	98.22%
4	96.77%	84.49%	39	98.27%	98.23%
5 6 7	97.46%	89.93%	40	98.28%	98.24%
6	97.76%	93.09%	41	98.30%	98.26%
	97.86%	95.16%	42	98.31%	98.27%
8	97.94%	96.36%	43	98.33%	98.28%
9	98.03%	97.07%	44	98.34%	98.30%
10	98.12%	97.54%	45	98.36%	98.32%
11	98.22%	97.89%	46	98.38%	98.33%
12	98.22%	98.00%	47	98.40%	98.35%
13	98.21%	98.07%	48	98.42%	98.37%
14	98.21%	98.14%	49	98.44%	98.39%
15	98.22%	98.17%	50	98.47%	98.41%
16	98.22%	98.20%	51	98.49%	98.44%
17	98.23%	98.24%	52	98.51%	98.46%
18	98.24%	98.23%	53	98.54%	98.49%
19	98.24%	98.22%	54	98.57%	98.51%
20	98.23%	98.21%	55	98.60%	98.54%
21	98.22%	98.20%	56	98.63%	98.57%
22	98.21%	98.19%	57	98.66%	98.60%
23	98.21%	98.19%	58	98.69%	98.63%
24	98.21%	98.18%	59	98.72%	98.66%
25	98.20%	98.18%	60	98.75%	98.69%
26	98.20%	98.18%	61	98.79%	98.72%
27	98.20%	98.17%	62	98.82%	98.76%
28	98.20%	98.17%	63	98.86%	98.79%
29	98.20%	98.17%	64	98.90%	98.83%
30	98.20%	98.17%	65	98.94%	98.87%
31	98.21%	98.18%	66	98.98%	98.91%
32	98.21%	98.18%	67	98.98%	98.95%
33	98.22%	98.18%	68	98.99%	98.99%
34	98.22%	98.19%	69	98.99%	98.99%
35	98.23%	98.20%	70	99.00%	99.00%

#### Thank you for the music



You are the sunshine of my life

Composer:

Wonder, Stevie Category:

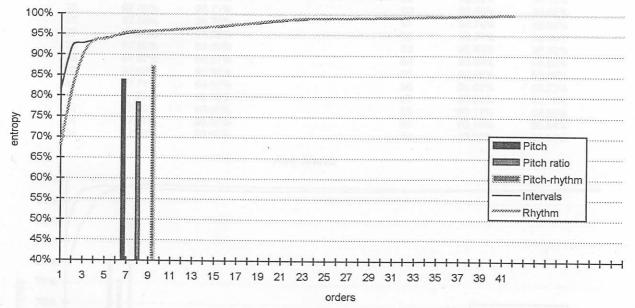
Entropies

Pitch 83.87% Pitch ratio Pitch/Rhythm 78.42% 87.22%

Order	Intervals	Rhythm
1	81.57%	68.07%
2	92.09%	81.32%
3	92.78%	90.14%
4	93.43%	93.48%
5	94.06%	93.88%
6	94.59%	94.74%
7	95.21%	95.50%
8	95.51%	95.62%
9	95.63%	95.76%
10	95.81%	95.92%
11	95.99%	96.11%
12	96.18%	96.31%
13	96.39%	96.53%
14	96.61%	96.75%
15	96.83%	96.99%
16	97.07%	97.24%
17	97.32%	97.49%
18	97.58%	97.76%
19	97.85%	98.04%
20	98.14%	98.33%
21	98.37%	98.63%

Order	Intervals	Rhythm
22	98.61%	98.78%
23	98.77%	98.95%
24	98.93%	98.98%
25	99.00%	99.01%
26	99.08%	99.05%
27	99.13%	99.08%
28	99.17%	99.13%
29	99.21%	99.17%
30	99.26%	99.21%
31	99.31%	99.26%
32	99.37%	99.31%
33	99.42%	99.37%
34	99.48%	99.42%
35	99.55%	99.48%
36	99.61%	99.55%
37	99.68%	99.61%
38	99.76%	99.68%
39	99.83%	99.76%
40	99.91%	99.83%
41	100.00%	99.91%
42		100.00%

You are the sunshine of my life



## Classical Art Songs

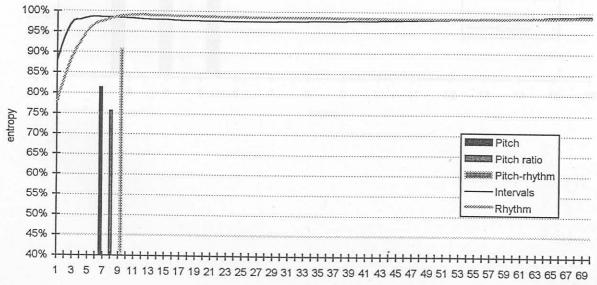
Entropies

Title: Ave Maria Composer: Schubert, Franz Category:

Pitch Pitch/Rhythm Pitch ratio 81.42% 75.70% 90.81%

Order	Intervals	Rhythm		Order	Intervals	Rhythm
1	87.99%	77.89%		36	97.80%	98.63%
2	93.80%	83.89%		37	97.81%	98.61%
}	97.46%	89.17%	10.00	38	97.83%	98.58%
	97.93%	92.45%		39	97.85%	98.56%
	98.53%	95.44%		40	97.87%	98.54%
	98.67%	97.04%		41	97.89%	98.52%
7	98.62%	97.64%		42	97.91%	98.50%
3	98.59%	98.26%		43	97.94%	98.48%
9	98.50%	98.85%		44	97.97%	98.47%
10	98.42%	99.09%		45	97.99%	98.46%
11	98.32%	99.14%		46	98.03%	98.45%
12	98.23%	99.19%		47	98.06%	98.45%
13	98.15%	99.11%		48	98.09%	98.44%
14	98.07%	99.07%		49	98.13%	98.44%
15	98.01%	99.02%		50	98.17%	98.44%
16	97.94%	98.98%		51	98.21%	98.44%
17	97.89%	98.94%		52	98.25%	98.45%
18	97.84%	98.90%		53	98.29%	98.45%
19	97.80%	98.87%		54	98.34%	98.46%
20	97.76%	98.83%		55	98.38%	98.47%
21	97.73%	98.81%		56	98.43%	98.48%
22	97.70%	98.78%		57	98.48%	98.50%
23	97.68%	98.75%		58	98.54%	98.52%
24	97.67%	98.73%		59	98.59%	98.53%
25	97.66%	98.71%		60	98.65%	98.56%
26	97.65%	98.70%		61	98.70%	98.58%
27	97.66%	98.68%		62	98.77%	98.60%
28	97.66%	98.67%		63	98.83%	98.63%
29	97.67%	98.67%		64	98.89%	98.66%
30	97.69%	98.66%		65	98.96%	98.69%
31	97.71%	98.66%		66	99.02%	98.73%
32	97.73%	98.65%		67	99.09%	98.76%
33	97.76%	98.65%		68	99.17%	98.80%
34	97.77%	98.66%		69	99.18%	98.84%
35	97.78%	98.66%		70	99.20%	98.84%





Das Wandern Schubert, Franz

S

Category:

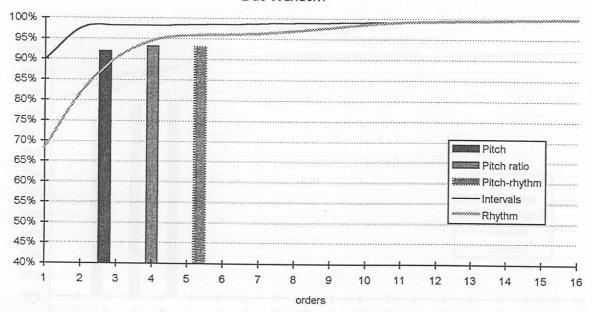
Pitch

91.90%

Entropies
Pitch ratio Pitch/Rhythm
94.21% 93.06%

Order	Intervals	Rhythm
1	89.89%	68.33%
2	97.57%	81.86%
3	98.22%	90.36%
4	98.23%	94.57%
5	98.44%	95.87%
6	98.54%	95.99%
7	98.73%	96.35%
8	98.91%	96.93%
9	99.00%	97.70%
10	99.10%	98.55%
11	99.21%	99.10%
12	99.34%	99.36%
13	99.48%	99.49%
14	99.64%	99.65%
15	99.81%	99.82%
16	100.00%	100.00%

#### Das Wandern



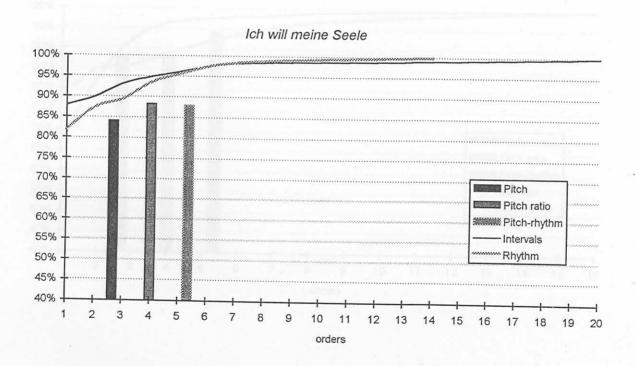
Title: I

Ich will meine Seele Schumann, Robert

Category: F

Pitch Pitch ratio 84.04% 88.27% Pitch/Rhythm 87.94%

Order	Intervals	Rhythm
1	87.88%	81.90%
2	89.87%	87.36%
3	93.10%	89.41%
4	94.86%	93.46%
5	96.11%	95.62%
6	97.39%	97.40%
7	98.18%	98.34%
8	98.47%	98.84%
9	98.56%	99.15%
10	98.69%	99.39%
11	98.76%	99.53%
12	98.85%	99.67%
13	98.94%	99.83%
14	99.04%	100.00%
15	99.16%	
16	99.30%	
17	99.45%	
18	99.61%	
19	99.80%	
20	100.00%	



Das ist ein Flöten Schumann, Robert

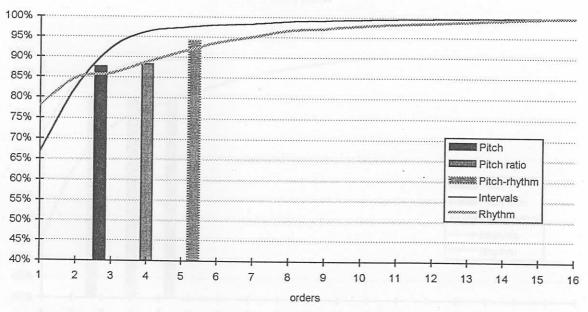
Composer: Category:

R

	Entropies	
Pitch	Pitch ratio	Pitch/Rhythm
87.57%	88.21%	94.10%

Order	Intervals	Rhythm
1	66.81%	78.08%
2	82.61%	84.78%
3	92.43%	85.99%
4	96.28%	88.80%
5	97.34%	91.35%
6	98.01%	93.66%
7	98.38%	95.15%
8	98.96%	96.68%
9	99.21%	97.20%
10	99.44%	97.85%
11	99.61%	98.31%
12	99.70%	98.62%
13	99.79%	98.97%
14	99.89%	99.36%
15	100.00%	99.79%
16		100.00%

#### Das ist ein Flöten



Halt

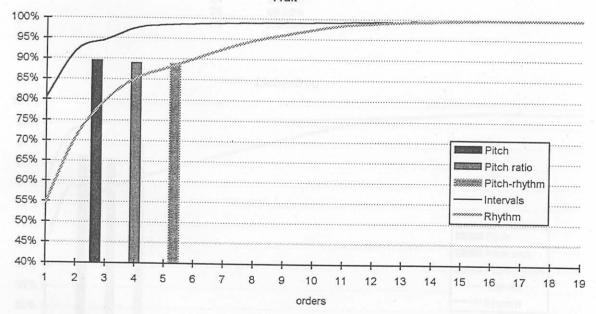
Composer: Schubert, Franz

Category:

Entropies Pitch ratio Pitch Pitch/Rhythm 89.46% 88.93% 88.89%

Order	Intervals	Rhythm
1	80.60%	54.71%
2	92.05%	70.97%
3	94.76%	80.01%
4	97.54%	85.27%
5	98.41%	87.66%
6	98.69%	90.08%
7	98.87%	92.61%
8	98.98%	94.65%
9	99.12%	96.09%
10	99.21%	97.41%
11	99.32%	98.54%
12	99.43%	99.01%
13	99.56%	99.39%
14	99.65%	99.56%
15	99.74%	99.74%
16	99.84%	99.84%
17	99.89%	99.89%
18	99.95%	99.95%
19	100.00%	100.00%



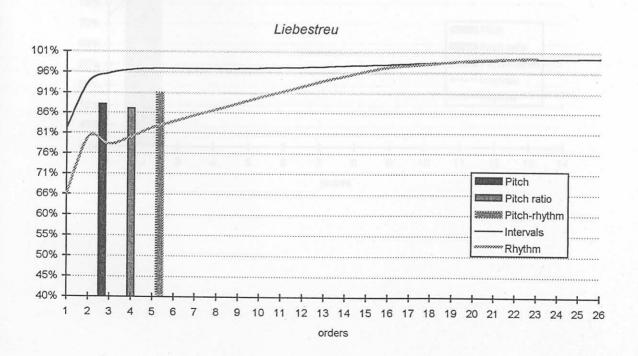


Liebestreu Brahms, Johannes

Category:

Entropies Pitch ratio Pitch Pitch/Rhythm 88.38% 87.32% 91.32%

Order	Intervals	Rhythm
1	82.34%	65.78%
2	93.80%	80.01%
3	96.08%	78.26%
4	96.97%	80.10%
5	97.25%	82.50%
6	97.24%	84.05%
7	97.25%	85.65%
8	97.29%	87.06%
9	97.35%	88.60%
10	97.44%	90.11%
11	97.56%	91.48%
12	97.69%	92.93%
13	97.85%	94.35%
14	98.03%	95.51%
15	98.23%	96.69%
16	98.41%	97.76%
17	98.60%	98.15%
18	98.79%	98.57%
19	99.00%	99.01%
20	99.22%	99.47%
21	99.45%	99.70%
22	99.70%	99.85%
23	99.77%	100.00%
24	99.84%	
25	99.92%	
26	100.00%	



Nachtigall Johannes, Brahms

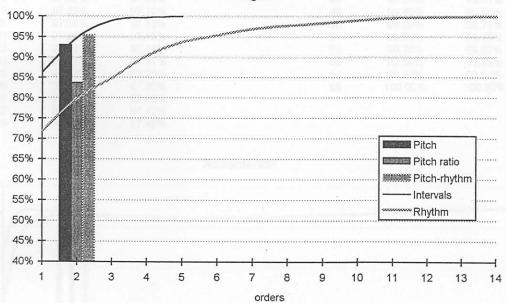
Category:

S

Entropies				
Pitch	Pitch ratio	Pitch/Rhythm		
93.02%	83.79%	95.42%		

Order	Intervals	Rhythm
1	86.32%	71.84%
2	94.80%	80.00%
3	98.99%	84.94%
4	99.68%	90.37%
5	100.00%	93.71%
6		95.39%
7		96.94%
8		97.73%
9		98.39%
10		99.00%
11		99.54%
12		99.76%
13		99.87%
14		100.00%





Title: Rosamunde Composer: Schubert, Franz

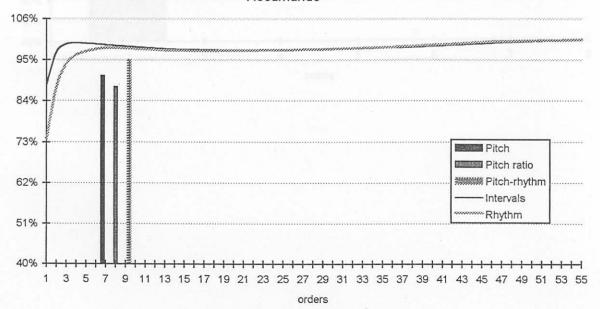
Category:

Entropies Pitch ratio Pitch/Rhythm Pitch 90.33% 87.45% 94.66%

Order	Intervals	Rhythm
1	88.02%	73.63%
2	96.74%	87.47%
3	98.86%	93.65%
4	99.07%	95.90%
5	99.05%	96.85%
6	98.81%	97.46%
7	98.60%	97.78%
8	98.42%	97.93%
9	98.26%	97.73%
10	98.05%	97.56%
11	97.86%	97.43%
12	97.70%	97.32%
13	97.55%	97.23%
14	97.43%	97.15%
15	97.39%	97.09%
16	97.35%	97.08%
17	97.31%	97.08%
18	97.29%	97.08%
19	97.27%	97.09%
20	97.26%	97.10%
21	97.25%	97.12%
22	97.25%	97.15%
23	97.25%	97.18%
24	97.27%	97.21%
25	97.28%	97.25%
26	97.31%	97.30%
27	97.34%	97.36%
28	97.37%	97.42%
29	97.42%	97.48%

Order	Intervals	Rhythm
30	97.46%	97.55%
31	97.52%	97.63%
32	97.58%	97.71%
33	97.65%	97.80%
34	97.72%	97.90%
35	97.80%	98.00%
36	97.89%	98.10%
37	97.98%	98.22%
38	98.08%	98.33%
39	98.19%	98.46%
40	98.30%	98.59%
41	98.42%	98.73%
42	98.54%	98.87%
43	98.68%	99.02%
44	98.81%	99.18%
45	98.96%	99.34%
46	99.11%	99.52%
47	99.27%	99.56%
48	99.36%	99.61%
49	99.45%	99.66%
50	99.55%	99.71%
51	99.65%	99.76%
52	99.76%	99.82%
53	99.88%	99.88%
54	99.94%	99.94%
55	100.00%	100.00%

#### Rosamunde



## 20th Century Songs

Title:

Being young and green Bliss, Arthur

Composer:

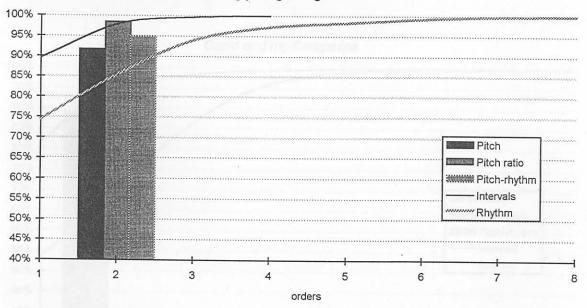
Category:

Entropies Pitch ratio

Pitch 91.66% 98.47% Pitch/Rhythm 94.88%

Order	· Intervals	Rhythm
1	89.59%	74.40%
2	97.90%	85.57%
3	99.57%	93.96%
4	100.00%	97.21%
5		98.31%
6		99.20%
7		99.84%
8		100.00%

#### Being young and green



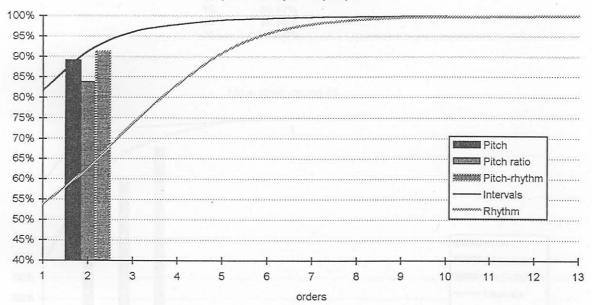
Cupid and my Campaspe Dalby, Martin

Composer: Category:

	Entropies	
Pitch	Pitch ratio	Pitch/Rhythm
89.06%	83.81%	91.31%

Order	Intervals	Rhythm
1	81.78%	53.73%
2	91.26%	62.78%
3	96.11%	73.84%
4	97.91%	83.19%
5	98.96%	90.94%
6	99.40%	95.68%
7	99.66%	97.95%
8	99.87%	99.08%
9	99.93%	99.57%
10	100.00%	99.74%
11		99.87%
12		99.93%
13		100.00%

## Cupid and my Campaspe



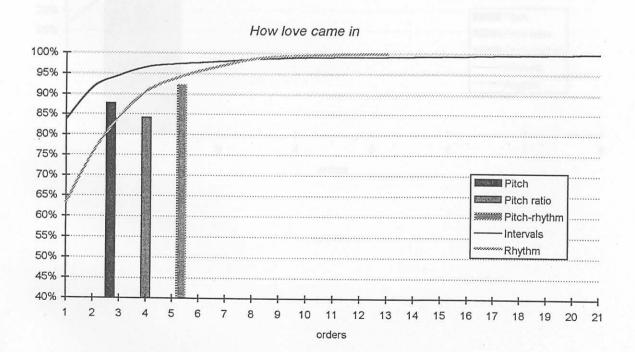
How love came in Berkeley, Lennox

Category:

M

	Entropies	
Pitch	Pitch ratio	Pitch/Rhythm
87.76%	84.27%	92.24%

Order	Intervals	Rhythm
1	83.76%	63.38%
2	91.71%	75.60%
3	94.55%	84.58%
4	96.59%	90.78%
5	97.31%	93.65%
6	97.76%	95.72%
7	98.15%	97.28%
8	98.55%	98.63%
9	98.79%	99.29%
10	99.05%	99.53%
11	99.11%	99.80%
12	99.17%	99.90%
13	99.24%	100.00%
14	99.32%	
15	99.40%	
16	99.48%	
17	99.57%	
18	99.67%	
19	99.77%	
20	99.88%	
21	100.00%	

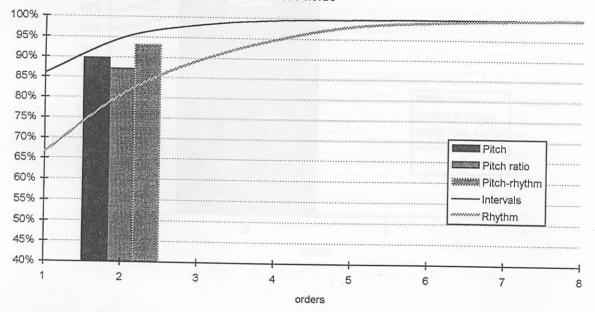


Title: In Flanders fields
Composer Ives, Charles
Category M

Pitch Pitch ratio Pitch/Rhythm 89.69% 87.17% 93.06%

Order	Intervals	Rhythm
1	86.02%	66.56%
2	94.78%	80.89%
3	98.05%	89.30%
4	99.36%	94.69%
5	99.66%	98.03%
6	99.79%	99.05%
7	99.93%	99.72%
8	100.00%	100.00%





Title: Composer Category

Pitch

94.50%

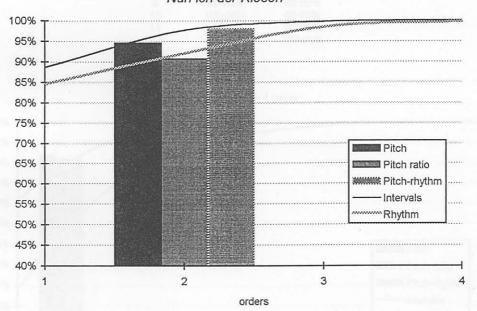
Nun ich der Riesen Berg, Alban

Entropies

Pitch/Rhythm 97.94% Pitch ratio 90.66%

Order	Intervals	Rhythm			
1	88.63%	84.63%			
2	97.63%	91.99%			
3	99.79%	98.47%			
4	100.00%	99.79%			

#### Nun ich der Riesen

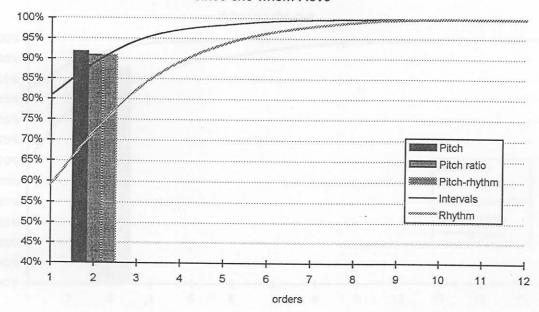


Title: Composer Category Since she whom I love Britten, Benjamin M

	Entropies	
Pitch	Pitch ratio	Pitch/Rhythm
91.79%	90.93%	90.88%

Order	Intervals	Rhythm
1	80.95%	59.12%
2	88.99%	72.17%
3	94.61%	82.58%
4	97.23%	89.45%
5	98.43%	93.74%
6	99.27%	96.49%
7	99.65%	98.01%
8	99.79%	99.04%
9	99.89%	99.69%
10	100.00%	99.89%
11		99.95%
12		100.00%

#### Since she whom I love

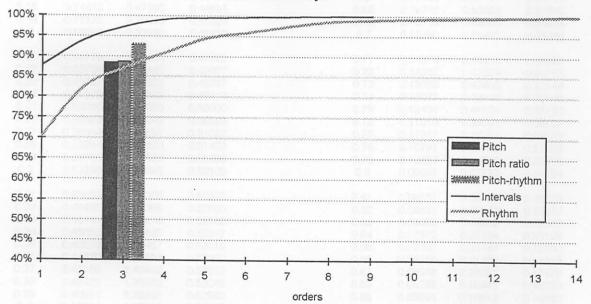


Title: Whenas the rye
Composer Britten, Benjamin
Category M

Pitch Pitch ratio Pitch/Rhythm 88.43% 88.60% 93.05%

Order	Intervals	Rhythm	
1	87.78%	70.45%	
2	93.91%	82.27%	
3	97.26%	87.21%	
4	99.04%	90.91%	
5	99.45%	94.42%	
6	99.62%	95.83%	
7	99.80%	97.38%	
8	99.90%	98.57%	
9	100.00%	98.97%	
10		99.25%	
11		99.41%	
12		99.59%	
13		99.79%	
14		100.00%	

#### Whenas the rye



## Appendix III: Tables of entropy values for P < 1

The two tables below may be used to calculate entropy values. The left hand column (p) is the percentage value for a set, the right hand column show the weighted entropy value. Once the ratios of the various sets have been calculated their entropies may be looked up and summated with the help of this table, Maximum Entropy values may similarly be looked up on the table in Appendix IV.

P	log P	log <sub>2</sub> P	P(log <sub>2</sub> P)	P	log P	log <sub>2</sub> P	P(log <sub>2</sub> P)
0.01	2.00000	6.64386	0.06644	0.51	0.29243	0.97143	0.49543
0.02	1.69897	5.64386	0.11288	0.52	0.28400	0.94342	0.49058
0.03	1.52288	5.05889	0.15177	0.53	0.27572	0.91594	0.48545
0.04	1.39794	4.64386	0.18575	0.54	0.26761	0.88897	0.48004
0.05	1.30103	4.32193	0.21610	0.55	0.25964	0.86250	0.47437
0.06	1.22185	4.05889	0.24353	0.56	0.25181	0.83650	0.46844
0.07	1.15490	3.83650	0.26856	0.57	0.24413	0.81097	0.46225
0.08	1.09691	3.64386	0.29151	0.58	0.23657	0.78588	0.45581
0.09	1.04576	3.47393	0.31265	0.59	0.22915	0.76121	0.44912
0.1	1.00000	3.32193	0.33219	0.6	0.22185	0.73697	0.44218
0.11	0.95861	3.18442	0.35029	0.61	0.21467	0.71312	0.43500
0.12	0.92082	3.05889	0.36707	0.62	0.20761	0.68966	0.42759
0.13	0.88606	2.94342	0.38264	0.63	0.20066	0.66658	0.41994
0.14	0.85387	2.83650	0.39711	0.64	0.19382	0.64386	0.41207
0.15	0.82391	2.73697	0.41054	0.65	0.18709	0.62149	0.40397
0.16	0.79588	2.64386	0.42302	0.66	0.18046	0.59946	0.39564
0.17	0.76955	2.55639	0.43459	0.67	0.17393	0.57777	0.38710
0.18	0.74473	2.47393	0.44531	0.68	0.16749	0.55639	0.37835
0.19	0.72125	2.39593	0.45523	0.69	0.16115	0.53533	0.36938
0.2	0.69897	2.32193	0.46439	0.7	0.15490	0.51457	0.36020
0.21	0.67778	2.25154	0.47282	0.71	0.14874	0.49411	0.35082
0.22	0.65758	2.18442	0.48057	0.72	0.14267	0.47393	0.34123
0.23	0.63827	2.12029	0.48767	0.73	0.13668	0.45403	0.33144
0.24	0.61979	2.05889	0.49413	0.74	0.13077	0.43440	0.32146
0.25	0.60206	2.00000	0.50000	0.75	0.12494	0.41504	0.31128
0.26	0.58503	1.94342	0.50529	0.76	0.11919	0.39593	0.30091
0.27	0.56864	1.88897	0.51002	0.77	0.11351	0.37707	0.29034
0.28	0.55284	1.83650	0.51422	0.78	0.10791	0.35845	0.27959
0.29	0.53760	1.78588	0.51790	0.79	0.10237	0.34008	0.26866
0.3	0.52288	1.73697	0.52109	0.8	0.09691	0.32193	0.25754
0.31 0.32 0.33 0.34 0.35 0.36 0.37 0.38 0.39 0.4	0.50864 0.49485 0.48149 0.46852 0.45593 0.44370 0.43180 0.42022 0.40894 0.39794	1.68966 1.64386 1.59946 1.55639 1.51457 1.47393 1.43440 1.39593 1.35845 1.32193	0.52379 0.52603 0.52782 0.52917 0.53010 0.53062 0.53073 0.53045 0.52980 0.52877	0.81 0.82 0.83 0.84 0.85 0.86 0.87 0.88 0.89	0.09151 0.08619 0.08092 0.07572 0.07058 0.06550 0.06048 0.05552 0.05061 0.04576	0.30401 0.28630 0.26882 0.25154 0.23447 0.21759 0.20091 0.18442 0.16812 0.15200	0.24625 0.23477 0.22312 0.21129 0.19930 0.18713 0.17479 0.16229 0.14963 0.13680
0.41 0.42 0.43 0.44 0.45 0.46 0.47 0.48 0.49 0.5	0.38722 0.37675 0.36653 0.35655 0.34679 0.33724 0.32790 0.31876 0.30980 0.30103	1.28630 1.25154 1.21759 1.18442 1.15200 1.12029 1.08927 1.05889 1.02915 1.00000	0.52738 0.52565 0.52356 0.52115 0.51840 0.51534 0.51196 0.50827 0.50428 0.50000	0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99	0.04096 0.03621 0.03152 0.02687 0.02228 0.01773 0.01323 0.00877 0.00436	0.13606 0.12029 0.10470 0.08927 0.07400 0.05889 0.04394 0.02915 0.01450	0.12382 0.11067 0.09737 0.08391 0.07030 0.05654 0.04263 0.02856 0.01435

# Appendix IV: Tables of entropy values for P >= 1

This table may be used to look up maximum entropy. The value for which the entropy is required is found in the left column. The second column shows the entropy derived with log base 10, while the right column provides the entropy derived with log base 2.

P 1 2 3 4 5 6 7 8 9 10	log P 0.00000 0.30103 0.47712 0.60206 0.69897 0.77815 0.84510 0.90309 0.95424 1.00000	Log2(P) 0.00000 1.00000 1.58496 2.00000 2.32193 2.58496 2.80735 3.00000 3.16993 3.32193			51 1.52 1.53 1.54 1.55 1.55 1.566 1.57 1.58 1.59 1.59	og P 70757 71600 72428 73239 74036 74819 75587 76343 77085 77815	Log2(P) 5.67243 5.70044 5.72792 5.75489 5.78136 5.80735 5.83289 5.85798 5.88264 5.90689
11 12 13 14 15 16 17 18 19 20	1.04139 1.07918 1.11394 1.14613 1.17609 1.20412 1.23045 1.25527 1.27875 1.30103	3.45943 3.58496 3.70044 3.80735 3.90689 4.00000 4.08746 4.16993 4.24793 4.32193		6 6 6	2 1.7 3 1.7 4 1.8 5 1.8 6 1.8 7 1.8 8 1.8 9 1.8	78533 79239 79934 80618 81291 81954 82607 83251 83885 84510	5.93074 5.95420 5.97728 6.00000 6.02237 6.04439 6.06609 6.08746 6.10852 6.12928
21 22 23 24 25 26 27 28 29 30	1.32222 1.34242 1.36173 1.38021 1.39794 1.41497 1.43136 1.44716 1.46240 1.47712	4.39232 4.45943 4.52356 4.58496 4.64386 4.70044 4.75489 4.80735 4.85798 4.90689		7 7 7 7 7 7 7 7 7 8	2 1.8 3 1.8 4 1.8 5 1.8 6 1.8 7 1.8 8 1.8	35126 35733 36332 36923 37506 38081 38649 39209 39763 30309	6.14975 6.16993 6.18982 6.20945 6.22882 6.24793 6.26679 6.28540 6.30378 6.32193
31 32 33 34 35 36 37 38 39 40	1.49136 1.50515 1.51851 1.53148 1.54407 1.55630 1.56820 1.57978 1.59106 1.60206	4.95420 5.00000 5.04439 5.08746 5.12928 5.16993 5.20945 5.24793 5.28540 5.32193		83 83 84 85 86 87 88 88 90	2 1.9 3 1.9 4 1.9 5 1.9 7 1.9 8 1.9 9 1.9	0849 1381 1908 2428 2942 3450 3952 4448 4939 5424	6.33985 6.35755 6.375504 6.39232 6.40939 6.42626 6.44294 6.45943 6.47573 6.49185
41 42 43 44 45 46 47 48 49 50	1.61278 1.62325 1.63347 1.64345 1.65321 1.66276 1.67210 1.68124 1.69020 1.69897	5.35755 5.39232 5.42626 5.45943 5.49185 5.52356 5.55459 5.58496 5.61471 5.64386		91 92 93 94 95 96 97 98	1.96 1.98 1.98 1.98 1.98	5904 6379 6848 7313 7772 3227 8677 9123	6.50779 6.52356 6.53916 6.55459 6.56986 6.58496 6.59991 6.61471 6.62936