CHAPTER 7

COMPARING MODELS AND FORECASTS OF THE LEVEL AND TURNING POINTS OF THE SOUTH AFRICAN STOCK MARKET

7.1 INTRODUCTION

The cointegration model of the South African stock market developed and estimated in chapter six made a contribution to the literature by establishing the factors that determine the level of the stock market in both the long-run and the short run. However, this model can also be used to forecast the stock market. This will enable investors and policy makers to simulate the impact of changes in macroeconomic indicators on the future course of the stock market and accurate forecasts of the stock market could be used by economists to forecast other macroeconomic indicators that lag the stock market such as consumption and investment[1]. In addition, forecasts of the stock market will predict the future direction of share prices and can hence be used by investors to construct profitable trading rules.

In this chapter the accuracy of the cointegration model in chapter six will be compared to other stock market models. This comparison will be done separately for the in-sample and forecast periods[2]. First the models' accuracy in modeling the level of the stock market will be compared. Then the models will be used to develop trading rules in order to compare their profitability and accuracy in modeling the direction of the stock market.

[1] Gallinger (1994) gives three reasons why share prices are leading consumption and investment. First, changes in share prices are synonymous with changes in wealth, which influence the future demand for investment goods and consumption (Barro 1990). Second, the stock market is a leading indicator of the economy and reflects information about real activity before it occurs. Finally, an increase in real economic activity increases the demand on the existing production capacity, which increases the return on assets and therefore induces increases in future capital investment.

[2] Granger (1992) points out that only the out-of-sample evaluation of stock price models is relevant for several reasons including the possibility of small sample in-sample biases of coefficients that give overly encouraging results. This was also shown by Nelson and Kim (1990).

7.2 MODELING THE LEVEL OF THE STOCK MARKET

7.2.1 The Stock Market Models

The modeling and forecasting accuracy of three stock market models, namely the cointegration model from chapter six, a random walk and a Fully Modified vector autoregressive (FM-VAR) model, will be compared. The specifications of the cointegration and error-correction models are presented in equations 7.1 and 7.2 respectively (see sections 6.4 and 6.5):

$$\log(JSE_t) = -6.584897 + 0.865694955*Log(GDP_t) - 0.0119161469*Discount_t \quad (7.1)$$

$$\Delta Log(JSE_t) = 0.3089012926 * \Delta log(JSE_{t-1}) - 0.1864008165 * Residual_{t-1} - 0.1290787154 * (Residual_{t-1} * S_t) + 0.1768269797 * \Delta(Log(Gold_t)) + 0.8690841507 * \Delta(Log(SP500_t)) - 0.04438600119 * Risk_t + 0.04178532045 * Risk_{t-1} + 0.3497508004 * \Delta log(R\$_{t-1}) + 0.0198328801 - 0.02534437603 * \Delta(Log(R_{S,\ t-1})) - 0.04484239067 * S_t - 0.041370202 * DUM98_t - 0.1455592312 * DUM00_t + 0.05524827626 * DUM94_t. \quad (7.2)$$

The explanations of the variables are given in table 7.1. Equations 7.1 and 7.2 can be combined as follows:

$$\Delta Log(JSE_t) = 0.3089012926 * \Delta log(JSE_{t-1}) - 0.1864008165 * (log(JSE_{t-1}) – (-6.584897 + 0.865694955*Log(GDP_{t-1}) - 0.0119161469*Discount_{t-1})) - 0.1290787154 * ((log(JSE_{t-1}) -6.584897 + 0.865694955*Log(GDP_{t-1}) - 0.0119161469*Discount_{t-1}) * S_t) + 0.1768269797 * \Delta(Log(Gold_t)) + 0.8690841507 * \Delta(Log(SP500_t)) - 0.04438600119 * Risk_t + 0.04178532045 * Risk_{t-1} + 0.3497508004 * \Delta log(R\$_{t-1}) + 0.0198328801 - 0.02534437603 * \Delta(Log(R_{S,\ t-1})) - 0.04484239067 * S_t - 0.041370202 * DUM98_t - 0.1455592312 * DUM00_t + 0.05524827626 * DUM94_t. \quad (7.3)$$

Table 7.1 List of Variables

Variable	Explanation
JSE	JSE allshare index
GDP	Gross domestic product
Discount	Constructed discount rate
Gold	Gold price
SP500	Standard and Poor's 500 Index (S&P500)
S	State of the business cycle dummy variable constructed in chapter five
R$	Rand-$US exchange rate
R_S	Short-term interest rate (three-month bankers' acceptance rate)
Risk	Risk premium, defined as difference between long-term interest rates of South Africa and the US (the yields on 10-year government bonds)
Residual	Residual from estimated long-run equation (see equation 7.1)

If the actual values of the explanatory variables during the forecasting period are used it gives the economic model an unrealistic benefit. Therefore, a very conservative approach will be followed with respect to the economic model whereby only observations that are available at the time of the forecast will be used. Instead of using the actual values of the explanatory variables during the forecasting period, the latest available values at the time of the forecast will be used. This implies that the explanatory variables are forecasted with a random walk where necessary. In other words, if only lagged values of a particular variable enters the stock market model in equation 7.3, then the actual value of this variable will be used in the forecast since it is available to the forecaster at the time of the forecast. For example, the rand-US$ exchange rate only enters the model in the transformation $\Delta\log(R\$_{t-1})$ which is available at the end of period t-1 to make a forecast of period t, the actual value will be used in the forecast. However, variables such as the first difference of the logarithmic transformation of the gold price enter the model contemporaneously, so that a forecast of this variable in period t is necessary for the forecast of the stock market in period t. The change in the logarithmic value of the gold price is forecasted with a random walk, in other words the first difference of the logarithmic value of the gold price in period t-1 is used as forecast for the variable in period t. This is a very conservative approach and any improvement in the forecasts of the explanatory variables should obviously improve the forecasting ability of the cointegration model for the stock market.

The second model, the random walk, is specified as follows:

$$JSE_t = JSE_{t-1} + \varepsilon_t \tag{7.4}$$

where ε_t is a random error term. This model essentially forecasts no change from the previous period's observation. This naïve model may seem like a weak challenge, but McNees (1992) has showed that it performs very well in predicting many economic variables. One of the advantages of this model is that only lagged variables is used to explain the stock market, which means that actual values are available for a one-period ahead forecast.

The third model is an FM-VAR. The vector autoregression (VAR) modeling technique is an effective means of characterizing the dynamic interactions among economic variables by reducing dependence on the potentially inappropriate theoretical restrictions of structural models. The general VAR specification can be written as follows:

$$X_t = A_0 + A_1 X_{t-1} + A_2 X_{t-2} + ... + A_k X_{t-k} + e_t \tag{7.5}$$

where X_t is a $(n \times 1)$ vector containing each of the n variables included in the VAR, A_0 is a $(n \times 1)$ vector of intercept terms, A_i is a $(n \times n)$ matrix of coefficients and e_t is a $(n \times 1)$ vector of error terms. As described by Phillips (1995), fully modified (FM) estimation of the VAR model should improve the OLS results in the presence of non-stationary regressors, I(1) processes and even cointegrating relationships. In addition, the FM-estimation procedure is valid without pre-testing for the exact cointegrating relationships or even the number of unit roots in the system. The FM-procedure specifically takes into account the possible serial correlation and endogeneities of the system. The variables of the cointegration equation (see equation 7.1[3]) were included in the FM-VAR.

Individual autoregressive (AR) models are estimated for each variable included in the VAR and the maximum order of the individual AR models is used as the order of the FM-VAR. The Akaike (AIC) and Schwartz model selection criteria were used to determine the order of the VAR. The results are presented in table 7.2 and the preferred AR model according to each model selection criteria is printed in bold. A VAR of order one was estimated since autoregressive models of order one were preferred by both criteria for all three individual models.

Table 7.2 Model Selection Criteria for Individual AR Models

Variable:	**JSE**		**Discount rate**		**GDP**	
Criteria:	**AIC**	**Schwartz**	**AIC**	**Schwartz**	**AIC**	**Schwartz**
VAR order						
0	7.49	7.52	3.42	3.45	24.96	24.99
1	**3.89**	**3.94**	**1.13**	**1.18**	**16.86**	**16.91**
2	3.93	4.01	1.14	1.23	16.89	16.97
3	3.94	4.05	1.14	1.26	16.92	17.03
4	3.99	4.13	1.18	1.32	16.95	17.10

The results of the FM-VAR model are given in table 7.3, with standard errors reported below in parenthesis. T-statistics constructed with these standard errors are asymptotically valid. Significant variables (based on the cut-off value of 1.96) are indicated in bold print. The Parzen kernel is used for the non-parametric estimation required by the FM-VAR.

[3] The variables in the ECM were not included in the FM-VAR due to insufficient degrees of freedom.

Table 7.3 Results of the FM-VAR Estimation

	JSE	Discount rate	GDP
ΔJSE_{t-1}	**-0.10**	**0.09**	-86
	(0.02)	(0.00)	(7650)
ΔDiscount $rate_{t-1}$	**-0.83**	**0.26**	-339
	(0.20)	(0.01)	(98295)
ΔGDP_{t-1}	**0.00**	**0.00**	**0.21**
	(0.00)	(1.63)	(0.01)
JSE_{t-1}	**0.86**	**0.01**	84.6
	(0.01)	(0.00)	(3927)
Discount $rate_{t-1}$	0.03	**0.97**	160
	(0.02)	(0.00)	(12167)
GDP_{t-1}	**0.0002**	**0.00**	**0.99**
	(2.14)	(1.35)	(0.00)
Constant	1.43	0.67	-240
	(8.28)	(0.52)	(4090997)

According to the results in table 7.3, the JSE can be presented by the following equation:

$$JSE_t = -0.10*\Delta JSE_{t-1} - 0.83*\Delta Discount_{t-1} + 0.0002*\Delta GDP_{t-1} + 0.86*JSE_{t-1} + 0.03*Discount_{t-1} + 0.00*GDP_{t-1}. \quad (7.6)$$

The results of the FM-VAR estimation presented in table 7.3 differs from the standard output of a VAR, since it includes not only lagged variables but also the first differences of the lagged variables[4]. However, these results can easily be rewritten to

[4] Like in the case of the random walk, this model has the advantage that only lagged variables is used to explain the stock market, which means that actual values are available for a one-period ahead forecast.

be in the same format as the standard VAR, which is easier to interpret. Table 7.4 represents the results of the FM-VAR in the format as a standard VAR[5].

Table 7.4 Reparametarized Results of the FM-VAR

	JSE	Discount rate	GDP
JSE_{t-1}	0.76	0.19	-1.79
JSE_{t-2}	-0.10	0.09	-86
Discount $rate_{t-1}$	-0.8	0.27	-179
Discount $rate_{t-2}$	-0.83	0.26	-339
GDP_{t-1}	0.00	0.00	1.64
GDP_{t-2}	0.00	0.00	0.21
Constant	1.43	0.67	-240

Figures 7.1 to 7.4 present the three stock market models graphically. These graphs highlights several differences between the different models. The FM-VAR and random walk both includes the lagged dependent variable (JSE_{t-1}) in the specification. Consequently, both these models closely follow the movements and trends in the stock market but this happens with a lag. In other words, these models pick up all the turning points in the stock market but always with a lag and never contemporaneously. For example, the stock market turning point in the third quarter of 1986 is only reflected by the FM-VAR and moving average in the fourth quarter of 1986. The cointegration model, on the other hand, sometimes deviates more than the FM-VAR and random walk from the actual stock price index, but there is no significant lag between the cointegration model and the actual stockmarket. For example, the cointegration model deviates quite substantially from the actual stock market index during 1996, while the deviations between FM-VAR and random walk models and the actual stock market are much smaller. The cointegration model and actual index peaked simultaneously in the third quarter of 1986, while the FM-VAR and random walk only peaked in the fourth quarter of 1986. Similarly, both the

[5] The FM-VAR specification is $y_t=\beta_0+\beta_1 y_{t-1}+\beta_2\Delta y_{t-1}$. This can be written as $y_t=\beta_0+\beta_1 y_{t-1}+\beta_2(y_{t-1}-y_{t-2}) = \beta_0+(\beta_1+\beta_2)y_{t-1}+\beta_2 y_{t-2}$ which is the same format as the standard VAR.

cointegration model and the actual stock price index have a trough in the third quarter of 1998, while the FM-VAR and random walk models only start their upswings in the fourth quarter of 1998.

Figure 7.1 Stock Market Models

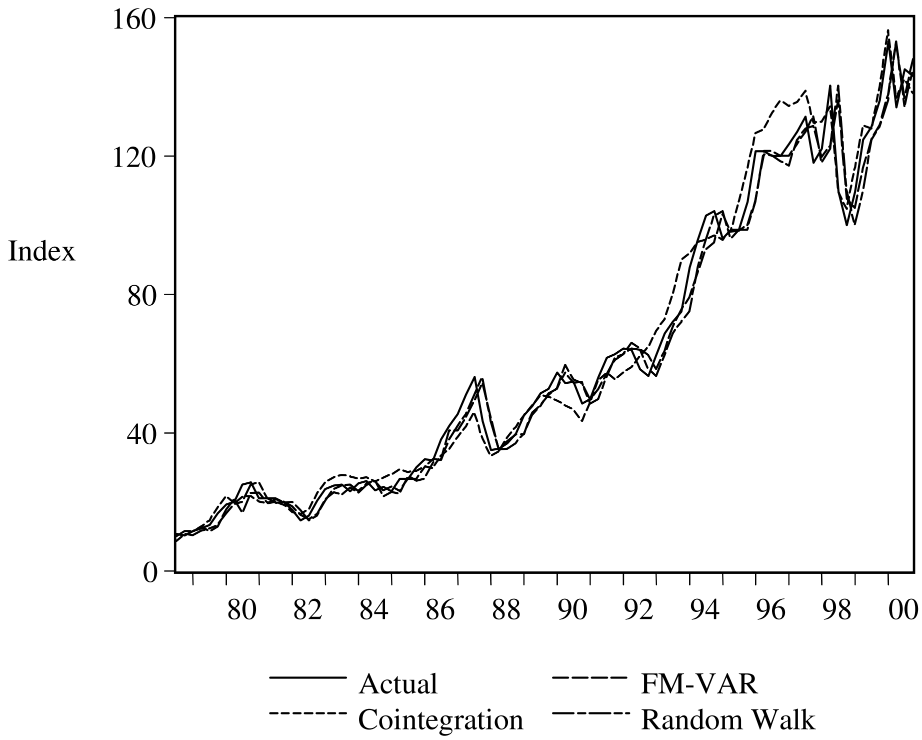


Figure 7.2 The Cointegration Stock Market Model

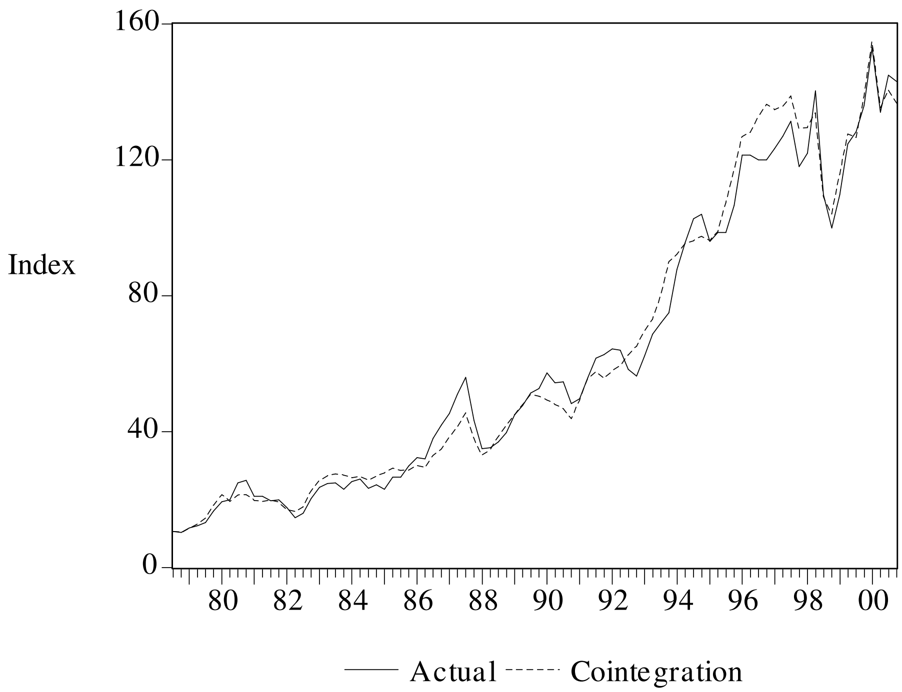


Figure 7.3 The Random Walk Stock Market Model

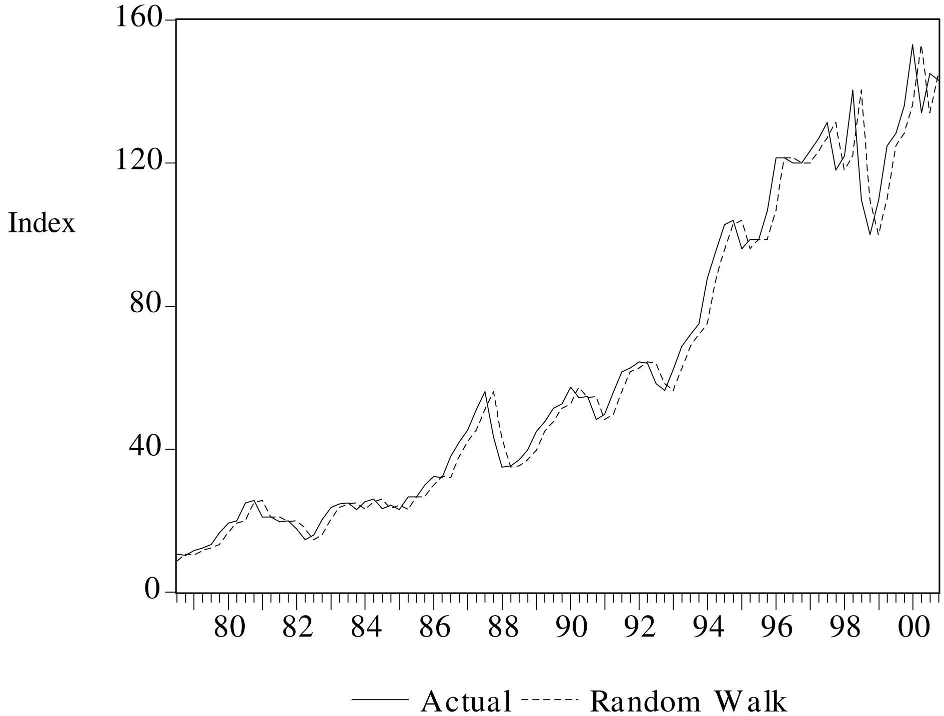
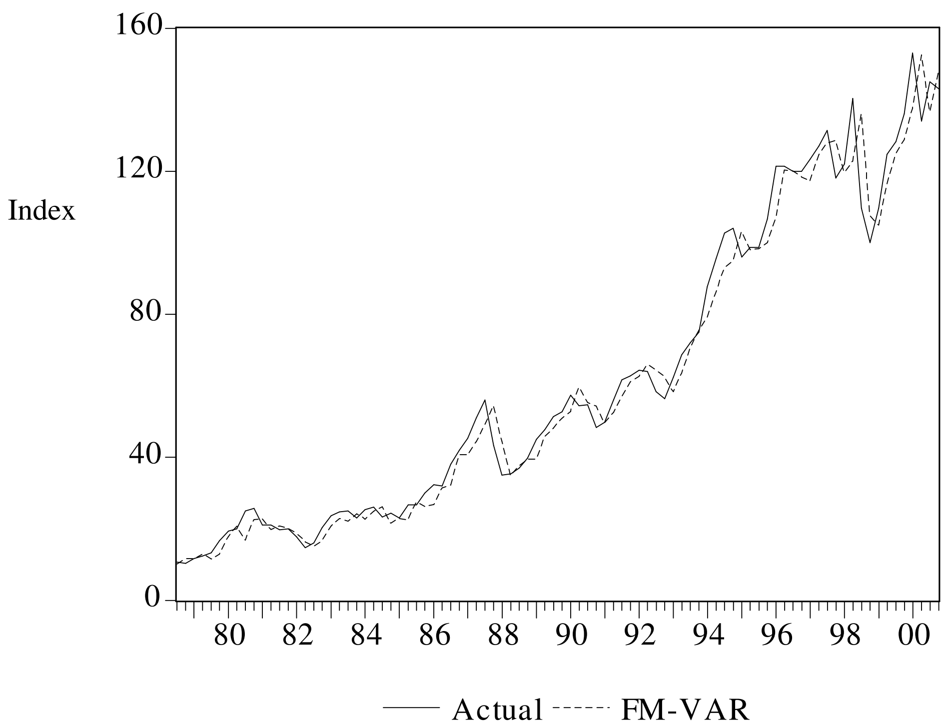


Figure 7.4 FM-VAR Stock Market Model



7.2.2 Evaluating the Stock Market Models

(i) Evaluation criteria

The performance of the three models for the sample and forecast periods will be evaluated and compared on the basis of the root mean squared error (RMSE), the root mean square percentage error (RMSPE) and Theil's inequality coefficient (U) across the observations for every period. These are defined as follows:

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(Y_t^s - Y_t^a)^2} \tag{7.7}$$

$$\text{RMSPE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{Y_t^s - Y_t^a}{Y_t^a}\right)^2} \tag{7.8}$$

$$U = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(Y_t^s - Y_t^a)^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(Y_t^s)^2} + \sqrt{\frac{1}{T}\sum_{t=1}^{T}(Y_t^a)^2}} \tag{7.9}$$

where Y_t^s is the simulated value of Y_t, Y_t^a is the actual value and T is the number of periods in the simulation (Pindyck and Rubinfeld 1991:338, 340). The RMSE is the criterion most frequently used to evaluate forecast performance, but the other two criteria have certain advantages over the RMSE. The RMSPE is similar to the RMSE, but compares each error with the magnitude of the actual value. Theil's inequality coefficient (U) is based on the RMSE, but it is scaled in such a way that U will always fall between 0 and 1. If U = 0, then $Y_t^s = Y_t^a$ for all t, and the model is a perfect fit. On the other hand, if U = 1, then the forecasting ability of the model is as bad as it possibly could be. In other words, the best forecasting model will be the one with the minimum RMSE, RMSPE and U.

Theil's inequality coefficient (U) can be decomposed into three parts as follows:

$$U^M = \frac{\left(Y^s - Y^a\right)^2}{\frac{1}{T}\sum\left(Y_t^s - Y_t^a\right)^2} \tag{7.10}$$

$$U^S = \frac{\left(\sigma_s - \sigma_a\right)^2}{\frac{1}{T}\sum\left(Y_t^s - Y_t^a\right)^2} \tag{7.11}$$

$$U^C = \frac{2(1-\rho)\sigma_s\sigma_a}{\frac{1}{T}\sum\left(Y_t^s - Y_t^a\right)^2} \tag{7.12}$$

where σ_a and σ_s are the standard deviations of the actual and simulated series respectively and ρ is their correlation coefficient. The proportions U^M, U^S and U^C are called the bias, variance and covariance proportions respectively. The bias proportion, U^M, is an indication of systematic error since it measures the extent to which the average values of the simulated and actual series deviate from each other. The variance proportion, U^S, indicates the ability of the model to replicate the degree of variability in the variable of interest. A large value of U^S means that the actual series has fluctuated considerably while the simulated series showed little fluctuation, or vice versa. The covariance proportion measure unsystematic error, in other words it represents the remaining error after deviations from average values have been accounted for. The ideal distribution over the three sources is therefore $U^M = U^S = 0$ and $U^C = 1$ (Pindyck and Rubinfeld 1991:341).

The abovementioned model selection criteria can be used to rank the performance of the different models, but it does not test whether the differences between the models' performances are statistically significant. Diebold and Mariano (1995) have suggested two tests[6] for the null hypothesis of equal accuracy of two competing forecasts[7]. Let the two rival forecasts of the time series $\{y_t\}_{t=1}^T$ be $\{\hat{y}_{it}\}_{t=1}^T$ and $\{\hat{y}_{jt}\}_{t=1}^T$, with

[6] They have also suggested an asymptotic test for the null of no difference in the accuracy of two rival forecasts, but the exact finite sample tests are preferred in the small sample context.
[7] Other tests such as an F-test for equal forecast error variances, the Morgan-Granger-Newbold test and the Meese-Rogoff test for testing the null of equal accuracy of two forecasts also exist. However, these tests are only strictly valid if several strong assumptions hold. The most important virtues of the Diebold and Mariano (1995) tests are that they are valid for a very wide class of loss functions, which need not be symmetric or continuous. In addition, the forecast errors do not have to be Gaussian or have a zero mean, and they can even be contemporaneously correlated. See Diebold and Mariano (1995) for a detailed discussion of the advantages of their tests over the other existing tests.

associated forecast errors $\{e_{it}\}_{t=1}^{T}$ and $\{e_{jt}\}_{t=1}^{T}$. The null hypothesis of equal accuracy of $\{\hat{y}_{it}\}_{t=1}^{T}$ and $\{\hat{y}_{jt}\}_{t=1}^{T}$ is:

$$H_0: E[d_t=0] \tag{7.13}$$

where $d_t \equiv [g(e_{it})-g(e_{jt})]$ and g() is the loss function applicable to the forecast. In general, the loss function, g, does not have to be a direct function of the forecast error, but can be a function of the actual and predicted values. In this case, $d_t \equiv [g(y_t, \hat{y}_{it})-g(y_t, \hat{y}_{jt})]$.

The first test suggested by Diebold and Mariano (1995) is the sign test, which tests the null hypothesis of a zero median loss differential between the two forecasts:

$$H_0: med(g(e_{it})-g(e_{jt}))=0. \tag{7.14}$$

The test statistic is

$$S_1 = \sum_{t=1}^{T} I_+(d_t) \tag{7.15}$$

where

$$I_+(d_t) = \begin{cases} 1 & \text{if } d_t > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{7.16}$$

The test statistic, S_1, is distributed binomial with parameters T (the sample size) and 0.5 under the null hypothesis. The studentized version of the test statistic is distributed standard normal in large samples:

$$S_{1a} = \frac{S_1 - 0.5T}{\sqrt{0.25T}} \overset{a}{\sim} N(0,1). \tag{7.17}$$

The second test for the null of equal forecast accuracy is Wilcoxon's signed-rank test. Unlike the sign test, this test requires symmetry of the loss differential. However, this test is more powerful than the sign test in the case of a symmetric loss differential (Diebold and Mariano 1995). The test statistic is:

$$S_2 = \sum_{t=1}^{T} I_+(d_t)\text{rank}(|d_t|) \tag{7.18}$$

where rank($|d_t|$) is the rank of the $|d_t|$ when $|d_t|$ is ordered from small to large. Like in the case of the sign test, the studentized version of the test is asymptotically distributed standard normal:

$$S_{2a} = \frac{S_2 - \frac{T(T+1)}{4}}{\sqrt{\frac{T(T+1)(2T+1)}{24}}} \overset{a}{\sim} N(0,1)\,. \tag{7.19}$$

(ii) In-sample performance

Table 7.5 presents the calculated values of the RMSE, RMSPE and Theil's inequality U as well as the decomposition of Theil's U for each of the three stock market models for the sample period.

Table 7.5 Evaluation of the In-Sample Performance of the Models

Criterion	Random Walk	Cointegration	FM-VAR
RMSE	6.883	**5.492**	6.322
RMSPE	0.977	**0.751**	0.937
U	0.046	**0.036**	0.043
U^M	0.047	**0.020**	0.048
U^S	0.006	0.104	**0.003**
U^C	0.958	0.887	**0.959**

According to the results in table 7.5, the cointegration model performs relatively well in modeling the stock market. It has the lowest root mean squared error (RMSE), root mean squared percentage error (RMSPE) and Theil's inequality coefficient (U). The cointegration model therefore outperforms the other two models in terms of these three criteria. However, when Theil's inequality coefficient is decomposed, the cointegration model has the lowest bias proportion (U^M) but the FM-VAR has the lowest variance proportion (U^S) as well as the highest covariance proportion (U^C) and is therefore preferred to the cointegration model according to these two criteria. This comparison should be seen in perspective. The cointegration model has a lower inequality coefficient with a less desirable decomposition. On the other hand, the FM-VAR has a higher inequality coefficient with a more desirable decomposition. Therefore the cointegration model is still preferred to the FM-VAR model since it has the *lowes*t inequality coefficient, which is arguably more important than the *composition* of the inequality coefficient.

Although the RMSE, RMSPE and U can be used to rank the performances of the models, it cannot be used to test whether the differences between the models are statistically significant. Therefore Diebold and Mariano's (1995) sign (S_{1a}) and Wilcoxon signed-rank (S_{2a}) tests will be used to test whether the models' accuracy is statistically different. These tests require the specification of a loss function. The following loss functions were used:

$$\text{L1: } g(e_t) = e_t \tag{7.20}$$

$$\text{L2: } g(e_t) = e_t^2 \tag{7.21}$$

$$\text{L3: } g(e_t) = \frac{\beta}{\alpha^2}\{\exp(\alpha e_t) + \alpha e_t - 1\} \text{ where } \alpha\text{=-1, }\beta\text{=1} \tag{7.22}$$

$$\text{L4: } g(e_t) = \frac{\beta}{\alpha^2}\{\exp(\alpha e_t) + \alpha e_t - 1\} \text{ where } \alpha\text{=-0.5, }\beta\text{=1} \tag{7.23}$$

$$\text{L5: } g(e_t) = \frac{\beta}{\alpha^2}\{\exp(\alpha e_t) + \alpha e_t - 1\} \text{ where } \alpha\text{=-2, }\beta\text{=1} \tag{7.24}$$

L6: $g(e_t) = \frac{\beta}{\alpha^2}\{\exp(\alpha e_t) + \alpha e_t - 1\}$ where α=-3, β=1 (7.25)

L7: $g(e_t) = \frac{\beta}{\alpha^2}\{\exp(\alpha e_t) + \alpha e_t - 1\}$ where α=-4, β=1. (7.26)

Loss functions L1 and L2 are standard, symmetric loss function that minimizes the errors and squared errors respectively. Loss functions L3 to L7 are linex loss functions[8] which are asymmetric. In these loss functions the parameter α determines the degree of asymmetry[9]. If $\alpha>0$, then the losses are approximately linear for $e<0$ and approximately exponential for $e>0$. By defining the error (e) as the actual value less the simulated value, positive values of α corresponds to the case in which underpredictions are more costly than overpredictions. Negative values, on the other hand, corresponds to the case where the function is exponential to the left of the origin and linear to the right. Furthermore, the closer α is to zero, the closer the function approximates the standard quadratic case. As explained in chapter six, overpredictions are more dangerous to investors than underpredictions, and therefore negative values of α are used in this study so that overpredictions are more costly than underpredictions[10].

In table 7.6, $\{e_{Rt}\}$, $\{e_{Ct}\}$ and $\{e_{Vt}\}$ are the error series of the random walk model, the cointegration model and the FM-VAR model respectively. The signed-rank test requires a symmetric loss function and is therefore not applied to the asymmetric loss functions L3 to L7 (see section 7.2.2). According to the results in table 7.6 all the models' accuracy are statistically different for loss function L1 according to both the sign and signed-rank tests. However, using any of the other loss functions there are no statistically significant differences in the accuracy of the models.

[8] The linex loss function was introduced by Varian (1974) and Zellner (1992).

[9] The parameter β in the linex loss function is a scaling factor, which does not influence the results. This is illustrated in Appendix 2 where the results of the test of equal accuracy are presented for different values of β. The results show that different values of β do not influence the results.

[10] See Appendix 2 for the results in the counterintuitive case of positive values of α, in other words when underpredictions are more costly than overpredictions. The results show that none of the models' accuracy is significantly different for loss functions with positive values of α.

Table 7.6 Equal Accuracy Tests for In-Sample Performance

Loss function:	L1		L2		L3	L4	L5	L6	L7
Test statistic:	S_{1a}	S_{2a}	S_{1a}	S_{2a}	S_{1a}	S_{1a}	S_{1a}	S_{1a}	S_{1a}
H_0: $med(g(e_{Rt})-g(e_{Ct}))=0$	-3*	-3*	-1.1	0.48	-1.5	-0.6	-1.5	-1.9	-1.9
H_A: $med(g(e_{Rt})-g(e_{Ct}))\neq 0$									
H_0: $med(g(e_{Rt})-g(e_{Vt}))=0$	2.5*	2.9*	1.1	1.87	1.05	1.05	0.84	0.84	0.63
H_A: $med(g(e_{Rt})-g(e_{Vt}))\neq 0$									
H_0: $med(g(e_{Ct})-g(e_{Vt}))=0$	9.5*	8.2*	-0.6	-0.1	1.26	0.21	1.90	1.90	1.90
H_A: $med(g(e_{Ct})-g(e_{Vt}))\neq 0$									

* Significant on a 1% level of significance.

(iii) Forecasting performance

The forecasting accuracy of the three stock market models are compared using the RMSE, RMSPE and Theil's inequality coefficient (U) from the first quarter of 2001 quarter until the first quarter of 2003. The results are presented in table 7.7. The preferred model according to each of the criteria is printed in bold.

Table 7.7 Evaluation of the Forecasting Performance of the Models

Criterion	Random Walk	Cointegration	FM-VAR
RMSE	14.577	**8.423**	14.690
RMSPE	0.086	**0.051**	0.088
U	0.044	**0.026**	0.044
U^M	0.007	0.136	**0.005**
U^S	0.009	**0.032**	0.000
U^C	0.984	0.832	**0.995**

According to the results in table 7.7, the cointegration model performs relatively well in forecasting the stock market. It has the lowest root mean squared error (RMSE), root mean squared percentage error (RMSPE) and Theil's inequality coefficient (U). The cointegration model therefore outperforms the other two models in terms of these three criteria. However, when Theil's inequality coefficient is decomposed, the cointegration model has the lowest variance proportion (U^S) but the FM-VAR has the lowest bias proportion (U^M) as well as the highest covariance proportion (U^C) and is therefore preferred to the cointegration model according to these two criteria.

In addition to the RMSE, RMSPE and U criteria, Diebold and Mariano's (1995) sign (S_{1a}) and Wilcoxon signed rank (S_{2a}) tests are used to test whether the models' forecasting accuracy is statistically different. The results are presented in table 7.8.

Table 7.8 Equal Accuracy Tests for Forecasting Performance

Loss function:	**L1**		**L2**		**L3**	**L4**	**L5**	**L6**	**L7**
Test statistic:	S_{1a}	S_{2a}	S_{1a}	S_{2a}	S_{1a}	S_{1a}	S_{1a}	S_{1a}	S_{1a}
H_0: $med(g(e_{Rt})-g(e_{Ct}))=0$	4	16*	6	32*	6	6	6	6	6
H_A: $med(g(e_{Rt})-g(e_{Ct}))\neq 0$									
H_0: $med(g(e_{Rt})-g(e_{Vt}))=0$	2	9*	4	23*	4	4	4	4	4
H_A: $med(g(e_{Rt})-g(e_{Vt}))\neq 0$									
H_0: $med(g(e_{Ct})-g(e_{Vt}))=0$	4	17*	2	7*	1*	1*	0*	0*	0*
H_A: $med(g(e_{Ct})-g(e_{Vt}))\neq 0$									

* Significant on a 10% level of significance.

According to the results in table 7.8, the null hypothesis that the random walk and cointegration model are equally accurate in forecasting the stock market is rejected against the alternative that they are not equally accurate if the loss function is symmetric[11]. Likewise, the forecasting accuracy of the random walk and FM-VAR

[11] The results of the sign (S_{1a}) and signed-rank (S_{2a}) are contradictory for loss functions L1 and L2. However, the signed-rank test is more powerful in the case of symmetric loss functions such as L1 and

differs significantly in the case of symmetric loss functions but not in the case of asymmetric loss functions. The null hypothesis of equal forecasting accuracy of the cointegration and FM-VAR models is rejected for the symmetric as well as the asymmetric loss functions. To summarize, the forecasting accuracy of any pair of models is statistically different for symmetric loss functions, in other words when over- and under-predictions are equally costly to investors. However, in the case of asymmetric loss functions in which over-predictions are more costly than under-predictions, only the cointegration and FM-VAR models differ significantly in terms of forecasting accuracy.

7.3 MODELLING TURNING POINTS IN THE STOCK MARKET

7.3.1 The Turning Point Models

The modelling and forecasting accuracy of the models in the previous section, namely the cointegration, FM-VAR and random walk models, are compared in modelling the direction of the stock market. The simulated values of these models are used to calculate the implied predicted direction of the stock market. In addition, they are compared to one of the most popular models used by technical analysts, a moving average[12].

One of the most popular averages used to identify major stock market trends is the 200-day (or 30-week) moving average (Jones 1991:438). The moving average line is used to create a basic trend line of stock prices. A general sell (buy) signal is created when the actual stock price index fall below (rise through) the moving average line. The following are specific signals of a sell signal (i.e. an upper turning point) (Jones 1991:438):

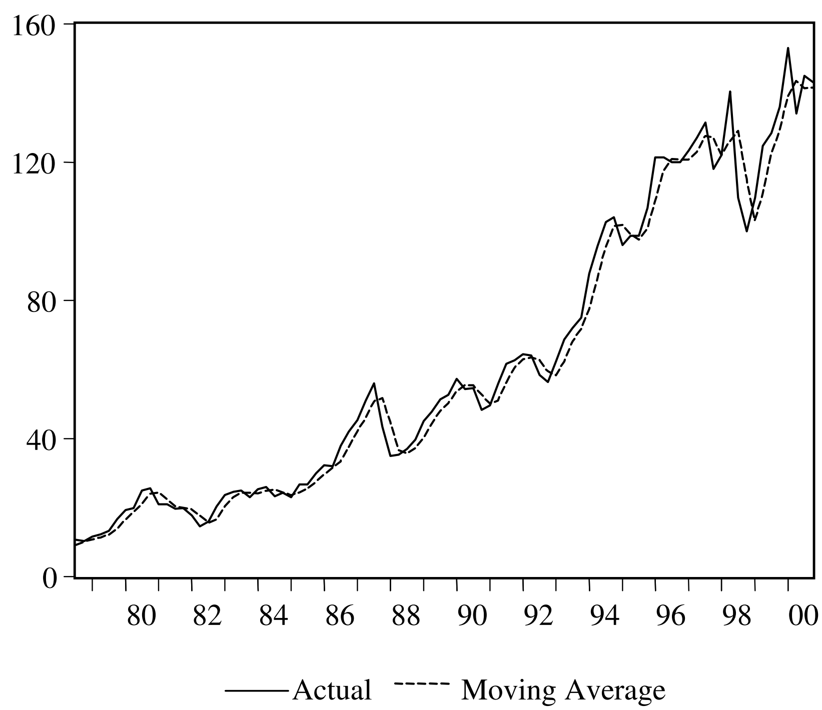
L2 and therefore the results of the signed-rank test are interpreted rather than that of the sign test (Diebold and Mariano 1995).

[12] Technical trading rules are designed to signal when to buy or sell shares and not to model the level of share prices. Therefore the moving average was only used to model the direction and not the level of the stock market, since this is consistent with its general purpose.

- The actual share price index is approaching the moving average from below, but does not cross the moving average line before it starts to fall again.
- The moving average declines after a rise and the actual share price index crosses it from above.
- The actual share price index rises above the moving average line when the average is still falling.

In this study a 30-week (or equivalently 7-month) moving average is constructed as technical trading rule. The moving average is calculated using monthly data and is then converted to quarterly data before the implied turning points are calculated. The calculated 7-month moving average of the JSE is presented in figure 7.5. The graph highlights the lag between movements of the moving average and the actual stock price index. For example, the stock market had a peak in the fourth quarter of 1980, while the turning point predicted by the moving average (i.e. when the actual index intersects with the moving average) only follows in the first quarter of 1981. Likewise, the stock market troughs in the second quarter of 1982 and the first quarter of 1988 are followed by turning point signals that are lagged by one quarter. However, the moving average seems to pick up all the peaks and troughs in the stock price index.

Figure 7.5 A Moving Average Model of the JSE



7.3.2 Evaluating the Turning Point Models

Investors are investing in the stock market to maximize their profits following a basic strategy of buying when share prices are low and selling when they are high. In order to evaluate the usefulness of the cointegration model for investors, the profitability of the different stock market models will be compared following this strategy of selling when share prices reach their predicted upper turning point and selling when share prices reach their predicted lower turning point. It is assumed that investors receive the short-term interest rate on their money while they do not hold the all-share index and that the returns are reinvested according to the same strategy as the original investment[13]. This will be compared to the returns of a buy-and-hold strategy over the sample period as well as receiving the short-term interest rate[14] on their money over the sample period. Following Heathcotte and Apilado (1974), a commission of 0,5 percent was charged on each trade[15]. Dividends were excluded from the analysis and any taxes were ignored.

(i) The in-sample profitability of the stock market models

Table 7.9 contains the results of these strategies for an initial investment of R100 at the beginning of the sample period. The second column presents the quarterly rate of return of the investment at an annual rate. The third and fourth columns contain the number and percentage of times that the specific model predicted a different direction than the actual realization of the stock market.

According to the results in table 7.9, trading according to the cointegration model would have yielded a return of 24.39 percent, which is higher than the return on the buy-and-hold strategy. In fact, the return yielded by the cointegration model is higher than that of all the other models except the moving average model which would have yielded a return of 26.71 percent. The cointegration model also outperforms all the models except the moving average in terms of the number or percentage of times that it correctly predicts the direction of the stock market.

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[13] Dividends are not included.
[14] The yield on three-month bankers' acceptances was used throughout the study.
[15] This is consistent with the rate charged by PSG, an investment services firm in South Africa (www.psg-online.co.za).

Table 7.9 In-sample Profitability of Different Trading Strategies

Model	Final value of investment[16]	Annualized rate of return[17]	Wrong predictions	% Wrong predictions
Buy-and-hold	835.380	9.89%		
Interest rate	1702.90	13.43%		
Cointegration	13564.6	24.39%	12	13.33%
FM-VAR	1268.60	11.95%	35	38.89%
Random Walk	1251.00	11.88%	35	38.89%
Moving Average	20584.7	**26.71%**	**11**	**12.22%**

(ii) The forecasting profitability of the stock market models

Table 7.10 contains the results of these strategies for an initial investment of R100 at the beginning of the forecast period. The forecast period was from the first quarter of 2001 until the second quarter of 2003.

According to the results in table 7.10, the cointegration model outperformed all the stock market models in terms of return on investment. However, it was as accurate as the moving average in terms of the number of times that it predicted the wrong direction for the stock market. Despite the good performance of the cointegration model in predicting the stock market, an investor would have been better of by simply investing in interest-bearing instruments during this particular period. However, it has to be kept in mind that dividends were not included in the calculation of these returns

[16] The final value of the investment refers to the value at the end of sample period of the R100 invested at the beginning of the sample period if the investment strategy was to invest the money in share if the model predicted that share prices will increase while the money was invested in short-term bearing instruments when the relevant model predicted that share prices would decline. With the buy-and-hold and interest rate strategies, it is assumed that the money was kept in share or interest-bearing instruments respectively for the full sample period.
[17] The annualized rate of return is calculated as the percentage increase in the original R100 investment expressed at an annual rate.

and that any taxes on the returns were ignored. Table 7.11 replicates the results in table 7.10 but includes the dividends.

Table 7.10 Forecasting Profitability of Different Trading Strategies

Model	Final value of investment	Annualized rate of return	Wrong predictions	% Wrong predictions
Buy-and-hold	84.01	-6.73%		
Interest rate	127.12	10.07%		
Cointegration	121.99	8.27%	3	30%
FM-VAR	102.02	0.81%	4	40%
Random Walk	106.41	2.51%	4	40%
Moving Average	106.94	2.72%	3	30%

Table 7.11 Forecasting Profitability Including Dividends

Model	Final value of investment	Annualized rate of return	Wrong predictions	% Wrong predictions
Buy-and-hold	91.08	-3.67%		
Interest rate	127.12	10.07%		
Cointegration	124.87	9.29%	3	30%
FM-VAR	106.46	2.53%	4	40%
Random Walk	110.75	4.17%	4	40%
Moving Average	111.30	4.38%	3	30%

The ranking of the models remain the same when dividends are included in the calculation of the rates of return. However, the difference in the returns of the stock market models and the interest-bearing scenario shrinks when dividends are included in the analysis. The yield on the cointegration model increases from 8.27 percent to 9.29 percent, while the yield on the FM-VAR model increases from 0.81 percent to 2.53 percent when dividends are added to the analysis. The returns on the moving average model increases from 2.72 percent to 4.38 percent, while the returns on the buy-and-hold scenario increases from –6.73 percent to –3.67 percent.

7.4 CONCLUSION

In this chapter the accuracy of the cointegration model developed and estimated in chapter six was compared to other stock market models. The comparison was done separately for the in-sample and forecast periods. First the models' accuracy in modeling and forecasting the level of the stock market were compared. Then the models were used to develop trading rules in order to compare their profitability and accuracy in modeling and forecasting the direction of the stock market.

The accuracy of the cointegration model developed in chapter six was compared to that of a random walk and a Fully Modified Vector Autoregressive (FM-VAR) model. The performance of these models for both the sample and forecast periods was evaluated and compared on the basis of the root mean squared error (RMSE), the root mean square percentage error (RMSPE) and Theil's inequality coefficient (U) across the observations for every period.

According to the results, the cointegration model performed relatively well in modeling the stock market within the sample period. The cointegration model outperforms the other two models in terms of the RMSE, RMSPE and U. Diebold and Mariano's (1995) sign and Wilcoxon sign rank tests are used to test whether the models' accuracy is statistically different. According to the results the accuracy of all the models differ significantly if the minimized loss function is simply the errors. However, using any of the other symmetric or asymmetric loss functions there are no statistically significant differences in the accuracy of the models.

The cointegration model also performs relatively well in forecasting the stock market, as it is preferred to the other models according to the RMSE, RMSPE and U.

According to the results, the null hypothesis that the random walk and cointegration model are equally accurate in forecasting the stock market is rejected against the alternative that they are not equally accurate if the loss function is symmetric. Likewise, the forecasting accuracy of the random walk and FM-VAR differs significantly in the case of symmetric loss functions but not in the case of asymmetric loss functions. The null hypothesis of equal forecasting accuracy of the cointegration and FM-VAR models is rejected for the symmetric as well as the asymmetric loss functions. To summarize, the forecasting accuracy of any pair of models is statistically different for symmetric loss functions, in other words when over- and under-predictions are equally costly to investors. However, in the case of asymmetric loss functions in which over-predictions are more costly than under-predictions, only the cointegration and FM-VAR models differ significantly in terms of forecasting accuracy.

The models used to model and forecast the level of the stock market is also used to model and forecast the direction of the stock market. The simulated values of these models are used to calculate the implied predicted direction of the stock market. In addition, they are compared to one of the most popular models used by technical analysts, a 30-week moving average. According to the results, trading according to the cointegration model would have yielded a higher return than the returns yielded by a buy-and-hold strategy. In fact, the return yielded by the cointegration model is higher than that of all the other models except the moving average model. The cointegration model also outperforms all the models except the moving average in terms of the number or percentage of times that it correctly predicts the direction of the stock market.

To summarize, in terms of both accuracy and profitability the cointegration model is preferred to other stock market models in modelling and forecasting the level as well as the turning points of the stock market.