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OPTIMISATION TECHNIQUES FOR COMBUSTOR DESIGN

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ABSTRACT

TITLE: OPTIMISATION TECHNIQUES FOR COMBUSTOR DESIGN

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For gas turbines, the demand for high-performance, more efficient and longer-life turbine blades is increasing. This is especially so, now that there is a need for high-power and low-weight aircraft gas turbines. Thus, the search for improved design methodologies for the optimisation of combustor exit temperature profiles enjoys high priority. Traditional experimental methods are found to be too time-consuming and costly, and they do not always achieve near-optimal designs. In addition to the above deficiencies, methods based on semi-empirical correlations are found to be lacking in performing three-dimensional analyses and these methods cannot be used for parametric design optimisation. Computational fluid dynamics has established itself as a viable alternative to reduce the amount of experimentation needed, resulting in a reduction in the time scales and costs of the design process. Furthermore, computational fluid dynamics provides more insight into the flow process, which is not available through experimentation only. However, the fact remains that, because of the trial-and-error nature of adjusting the parameters of the traditional optimisation techniques used in this field, the designs reached cannot be called "optimum". The trial-and-error process depends a great deal on the skill and experience of the designer. Also, the above technologies inhibit the improvement of the gas turbine power output by limiting the highest exit temperature possible, putting more pressure on turbine blade cooling technologies. This limitation to technology can be overcome by implementing a search algorithm capable of finding optimal design parameters. Such an algorithm will perform an optimum search prior to computational fluid dynamics analysis and rig testing. In this thesis, an efficient methodology is proposed for the design optimisation of a gas turbine combustor exit temperature profile. The methodology involves the combination of computational fluid dynamics with a gradient-based mathematical optimiser, using successive objective and constraint function approximations (Dynamic-Q) to obtain the optimum design. The methodology is tested on three cases, namely:

(a) The first case involves the optimisation of the combustor exit temperature profile with two design variables related to the dilution holes, which is a common procedure. The combustor exit temperature profile was optimised, and the pattern factor improved, but pressure drop was very high.

(b) The second case involves the optimisation of the combustor exit temperature profile with four design variables, one equality constraint and one inequality constraint based on pressure loss. The combustor exit temperature profile was also optimised within the constraints of pressure. Both the combustor exit temperature profile and pattern factor were improved.

(c) The third case involves the optimisation of the combustor exit temperature profile with five design variables. The swirler angle and primary hole parameters were included in order to allow for the effect of the central toroidal recirculation zone on the combustor exit temperature profile. Pressure loss was also constrained to a certain maximum.

The three cases show that a relatively recent mathematical optimiser (Dynamic-Q), combined with computational fluid dynamics, can be considered a strong alternative to the design optimisation of a gas turbine combustor exit temperature profile. This is due to the fact that the proposed methodology provides designs that can be called near-optimal, when compared with that yielded by traditional methods and computational fluid dynamics alone.

Keywords: combustor exit temperature profile, computational fluid dynamics, mathematical optimisation, gradient-based optimisation algorithm, successive approximation algorithm, temperature profile, design methodology

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PUBLICATIONS IN JOURNALS AND CONFERENCES

Refereed journal papers

(a) Motsamai, O. S., Visser, J. A., and Montressor, R. M, “Multi-Disciplinary Design Optimization of a Combustor”, *Engineering Optimization*, vol. 40, No. 2, pp. 137-156, 2008.

Journal papers submitted for publication

- Motsamai, O. S., Snyman, J. A., and Meyer, J. P., Optimisation of Combustor Mixing for Improved Exit Temperature Profile, HTE, submitted.

Papers in refereed conference proceedings

5 Motsamai, O. S., Snyman, J. A., and Meyer, J. P., “Mixing of multiple reacting jets in a gas turbine combustor” Proceedings of the Sixth International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics, 30 June-3 July, Pretoria, South Africa, Paper No. MO1, 2008.

6 Motsamai, O. S., Visser, J. A., Morris, R. M., and De Kock, D., “An efficient Strategy for the Design Optimization of Combustor Exit Temperature Profile”, Proc. of ASME Turbo Expo, Paper No. GT2006-91325, Bacerlona, Spain, 2006.

7 Motsamai, O. S., Visser, J. A., Morris, R. M., and De Kock, D., “Using CFD and Mathematical Optimization to Optimize Combustor Exit Temperature Profile”, Proc. of ASME Power, Paper No. PWR2006-88120, Georgia Atlanta, USA, 2006.

8 Motsamai, O. S., Visser, J. A., Morris, R. M., and De Kock, D., “Combustor Optimization with CFD and Mathematical Optimization” Proc. of Fifth SACAM, Cape Town, South Africa, 2006.

9 Motsamai, O. S., Visser, J. A., Morris, R. M., and De Kock, D., “Validation Case for Multidisciplinary Design Optimization of a Combustor”, Proc. of Botswana Institution of Engineers, 2005.

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NOMENCLATURE

Symbols:

Variable	Description	Unit
a	Approximated curvature of the objective subproblem	-
A	Approximated Hessian matrix of the objective function	-
A	Combustor casing area	m^2
A_a	Annulus area	m^2
A_d	Area of the dilution hole	m^2
AFR_{avg}	Average air-fuel ratio	-
b	Impact parameter	-
b_{cr}	Critical impact parameter	-
B_D	Transfer number	-
b_j	Approximated curvature of $g_j(\boldsymbol{x})$ subproblem	-
B_j	Approximated Hessian matrix of $g_j(\boldsymbol{x})$ subproblem	-
C	empirical constant	-
C_μ	Empirical constant for k-ε turbulence model	-
$C_{\varepsilon 1}$	Empirical constant for k-ε turbulence model	-
$C_{\varepsilon 2}$	Empirical constant for k-ε turbulence model	-
C_D	Particle drag coefficient	-
c_k	Approximated curvature of $h_k(\boldsymbol{x})$ subproblem	-
C_k	Approximated Hessian matrix of $h_k(\boldsymbol{x})$ subproblem	-
c_p	Specific heat at a constant pressure	J/kg K
C_{pg}	Specific heat capacity of gas	J/kg K
C_{pk}	Specific heat at a constant temperature for species k	J/kg K

♦ Problem-dependent

Variable	**Description**	**Unit**
D	Diffusion coefficient	-
	Drop diameter	m
D_{CR}	Critical drop diameter	m
d_h	Dilution hole diameter	m
d_j	Diameter of the jet	m
D_k	Diffusion coefficient of species k	-
E	Empirical constant for the log-law mean velocity	-
	Energy	J
f	Any quantity	♦
f', f''	Fluctuating component of quantity f	♦
$\overline{f}, \tilde{f}$	Mean component of quantity f	♦
$f(\boldsymbol{x})$	Objective function	♦
f_k	Volume force acting on species k	-
$f_{k,j}$	Volume force acting on species k in the direction j	-
g	Gravitational acceleration	m/s^2
$\mathrm{g}_j(\boldsymbol{x})$	j-th inequality constraint function	♦
G_k	Production of kinetic energy	-
H	Hessian matrix	-
$\mathrm{h}_k(\boldsymbol{x})$	k-th equality constraint function	♦
h_s	Enthalpy	J/kg
$h_{s,k}$	Enthalpy of species k	J/kg
$\boldsymbol{I}$	Identity matrix	-
J	Momentum flux ratio	-
$\hat{k}$	Lower bound	-
k	Turbulent kinetic energy	m^2/s^2
	Species	-

♦ Problem-dependent

Variable	Description	Unit
$\check{k}$	Upper bound	-
k_g	Gas thermal conductivity	W/m K
k_{opt}	Optimum ratio of flame tube area to casing area	-
k_p	Turbulent kinetic energy at point p	m^2/s^2
L_e	Length scale of ε	m
L	Reference Length	m
L_d	Latent heat of vaporization	J/kg
$\dot{m}_{fuel}$	Fuel mass flow rate	kg/s
$\dot{m}_j$	Jet mass flow rate	kg/s
$\dot{m}_{bp}$	Mass flow at the base plate	kg/s
$\dot{m}_{po}$	Mass flow at the port	kg/s
$\dot{m}_{cool}$	Mass flow of cooling air	kg/s
m	Mass of mixture	kg
	Number of inequality constraints	-
$\dot{m}_r$	Recirculating mass flow	kg/s
$\dot{m}_{air}$	Air mass flow rate	kg/s
$\dot{m}_g$	Air mass flow rate	kg/s
m_k	Mass fraction of species k	-
n	Local coordinate normal to the wall	-
	Number of design variables	-
	Number of holes	-
N	Number of species	-
n_{opt}	Optimum number of holes	-
Nu	Nusselt number	-

♦ Problem-dependent

Variable	Description	Unit
Nu_D	Nusselt number of the droplet	-
p	Pressure	Pa
$p(\mathbf{x})$	Penalty function	-
$p(z)$	pdf of mixture fraction variance z	-
P_3	Air pressure at the combustor inlet	Pa
P_4	Exhaust gas pressure at the combustor exit	Pa
P_k	Source term	-
Pr_D	Prandtl number of the droplet	-
$\dot{Q}$	External heat source term	W
q	Heat flux	W
Q	External heat source term	J
Q_D	Rate of heat conduction to the droplet surface per unit area	W
r	Radius	m
r_D	Radius of the droplet	m
r_{D1}	Initial radius of the droplet	m
r_{D2}	Final radius of the droplet	m
Re_D	Droplet Reynolds number of the droplet	-
R^n	n-dimensional real space	-
Sc_D	Droplet Schmidt number	-
Sc_k	Schmidt number of species k	-
Sh_D	Droplet Sherwood number	-
S_m	Mass source	kg/m^3s
t_R	Droplet transit time	s
T_F^0	State temperature of fuel	K
T_O^0	State temperature of oxidiser	K

♦ Problem-dependent

Variable	**Description**	**Unit**
T	Temperature	K
t	Time	s
T_0	Reference temperature	K
T_3	Temperature of air at the combustor inlet	K
T_{4avg}	Average temperature at the combustor exit	K
T_{4max}	Maximum individual temperature at the combustor exit	K
T_{4peak}	Maximum temperature in average radial profile	K
T_D	Droplet temperature	K
t_D	Viscous damping time	s
T_g	Gas temperature	K
T_w	Wall shear stress	Pa
u	Velocity in the x-direction	m/s
U^*	Dimensionless mean velocity	-
u_A	Velocity of air	m/s
u_D	Velocity of droplet	m/s
u_i	Instantaneous velocity in the i-th direction	m/s
$\mathrm{u_i}$	Three-dimensional velocity field	m/s
u_i'	Fluctuating part of velocity in the i-th direction	m/s
U_j	Velocity of the jet	m/s
V_i^c	Correlation velocity V^c in direction i	m/s
V_k	Component of diffusion velocity of species k	-
$V_{k,j}$	Diffusion velocity of species k in the direction j	m/s
$\dot{w}_k$	Reaction rate of species k	W
$\dot{w}_T$	Reaction rate	W
w	Velocity difference between product and parent droplet	m/s

♦ Problem-dependent

Variable	**Description**	**Unit**
w_k	Reaction rate of species *k*	-
$\boldsymbol{x}$	Design vector	♦
$\boldsymbol{x}^*$	Optimum design variable	♦
y	Droplet distortion	-
	Coordinate from the wall	m
Y_F^0	State mass fraction of fuel	-
Y_O^0	State mass fraction of oxidiser	-
y^+	Wall unit for law-of-the-wall	-
Y_D	Fuel vapour mass fraction	-
Y_{Ds}	Fuel vapour mass fraction at the droplet's surface	-
Y_F	Mass fraction of fuel	-
Y_k	Mass fraction of species *k*	-
Y_{max}	Maximum jet penetration	
Y_O	Mass fraction of oxidiser	-
y_p	Distance from point *P* to the wall	m
z	Mixture fraction variance	-

Greek symbols

α	Penalty function parameter for inequality constraint	-
β	Penalty function parameter for equality constraint	-
β_k	Penalty parameter	-
δ_j	Specified move limit for *i*-th design variable	♦
Δf_{norm}	Normalised step size	-
δ_i	Move limit on *i*-th design variable	-

♦ Problem-dependent

Variable	Description	Unit
$\Delta P_{overall}$	Change in overall pressure drop	-
Δx_i	Step size for *i*-th design variable	♦
Δx_{norm}	Normalised step size	-
ε	Rate of dissipation of turbulence	-
θ	Angle	degrees
μ_i	Penalty parameter	-
μ_t	Turbulent viscosity	Pa s
μ	Kinematic viscosity	m^s/s
μ_l	Liquid viscosity	Pa s
μ_g	Gas kinematic viscosity	m^s/s
υ_t	Kinematic viscosity	m^2/s
ρ	Density	kg/m^3
ρ_j	Penalty parameter	-
ρ_3	Density of air at the combustor inlet	kg/m^3
ρ_D	Droplet density	kg/m^3
ρ_g	Average gas density	kg/m^3
ρ_i	Density of the mixture	kg/m^3
ρ_k	Density for each species *k*	kg/m^3
σ	Surface tension	N/m
σ_D	Droplet surface tension coefficient	-
σ_t	Effective turbulent Prandtl number	-
τ	Stress	Pa
τ_B	Characteristic breakup time	s
τ_D	Droplet relaxation time	s
τ_E	Eddy life time	s

♦ Problem-dependent

Variable	**Description**	**Unit**
We	Weber number	-
τ_{ij}	Stresses in the *i*-th and *j*-th direction	m/s
τ_{TR}	Transit time scale	s
ω	Turbulent vorticity	-
	Oscillatory frequency	1/s

Subscripts

3	Combustor inlet
4	Combustor exit
A	Air
B	Breakup
D	Droplet
F	Fuel
g	Gas
i	Index
	Index
INT	Interaction
j	Index
k	Species
	Source
m	Mass source
max	Maximum
$norm$	Normalised
O	Oxidiser
opt	Optimum

♦ Problem-dependent

Subscripts

P	Point
s	Surface
t	Turbulent
TR	Transit

Abbreviations

CFD	Computational fluid dynamics
RANS	Reynolds-averaged Navier-Stokes equation
RSM	Reynolds stress method
LES	Large eddy simulation
DNS	Direct numerical simulation
SMD	Sauter mean diameter

♦ Problem-dependant