



UNIVERSITEIT VAN PRETORIA  
UNIVERSITY OF PRETORIA  
YUNIBESITHI YA PRETORIA

# **OPTIMISATION TECHNIQUES FOR COMBUSTOR DESIGN**

Oboetswe Seraga Motsamai

Submitted in partial fulfilment of the requirements for the degree  
PHILOSOPHIAE DOCTOR in Mechanical Engineering  
in the  
Faculty of Engineering, Built Environment and Information Technology  
University of Pretoria  
Pretoria

Supervisor: Prof J. P. Meyer  
Co-Supervisor: Prof J. A. Snyman

September 2008



---

## ABSTRACT

---

**TITLE:** OPTIMISATION TECHNIQUES FOR COMBUSTOR DESIGN

**AUTHOR:** O. S. Motsamai

**SUPERVISOR:** Prof J. P. Meyer

**CO-SUPERVISOR:** Prof J. A. Snyman

**DEPARTMENT:** Mechanical and Aeronautical Engineering

**UNIVERSITY:** University of Pretoria

**DEGREE:** Philosophiae Doctor (Mechanical Engineering)

For gas turbines, the demand for high-performance, more efficient and longer-life turbine blades is increasing. This is especially so, now that there is a need for high-power and low-weight aircraft gas turbines. Thus, the search for improved design methodologies for the optimisation of combustor exit temperature profiles enjoys high priority. Traditional experimental methods are found to be too time-consuming and costly, and they do not always achieve near-optimal designs. In addition to the above deficiencies, methods based on semi-empirical correlations are found to be lacking in performing three-dimensional analyses and these methods cannot be used for parametric design optimisation. Computational fluid dynamics has established itself as a viable alternative to reduce the amount of experimentation needed, resulting in a reduction in the time scales and costs of the design process. Furthermore, computational fluid dynamics provides more insight into the flow process, which is not available through experimentation only. However, the fact remains that, because of the trial-and-error nature of adjusting the parameters of the traditional optimisation techniques used in this

field, the designs reached cannot be called “optimum”. The trial-and-error process depends a great deal on the skill and experience of the designer. Also, the above technologies inhibit the improvement of the gas turbine power output by limiting the highest exit temperature possible, putting more pressure on turbine blade cooling technologies. This limitation to technology can be overcome by implementing a search algorithm capable of finding optimal design parameters. Such an algorithm will perform an optimum search prior to computational fluid dynamics analysis and rig testing. In this thesis, an efficient methodology is proposed for the design optimisation of a gas turbine combustor exit temperature profile. The methodology involves the combination of computational fluid dynamics with a gradient-based mathematical optimiser, using successive objective and constraint function approximations (Dynamic-Q) to obtain the optimum design. The methodology is tested on three cases, namely:

- (a) The first case involves the optimisation of the combustor exit temperature profile with two design variables related to the dilution holes, which is a common procedure. The combustor exit temperature profile was optimised, and the pattern factor improved, but pressure drop was very high.
- (b) The second case involves the optimisation of the combustor exit temperature profile with four design variables, one equality constraint and one inequality constraint based on pressure loss. The combustor exit temperature profile was also optimised within the constraints of pressure. Both the combustor exit temperature profile and pattern factor were improved.
- (c) The third case involves the optimisation of the combustor exit temperature profile with five design variables. The swirler angle and primary hole parameters were included in order to allow for the effect of the central toroidal recirculation zone on the combustor exit temperature profile. Pressure loss was also constrained to a certain maximum.

The three cases show that a relatively recent mathematical optimiser (Dynamic-Q), combined with computational fluid dynamics, can be considered a strong alternative to the

design optimisation of a gas turbine combustor exit temperature profile. This is due to the fact that the proposed methodology provides designs that can be called near-optimal, when compared with that yielded by traditional methods and computational fluid dynamics alone.

**Keywords:** combustor exit temperature profile, computational fluid dynamics, mathematical optimisation, gradient-based optimisation algorithm, successive approximation algorithm, temperature profile, design methodology



---

## ACKNOWLEDGEMENTS

---

I am indebted to Prof J. A. Visser for his guidance, wisdom and the many things learnt from him, which have not only been restricted to the subject of combustor design optimisation. It has been a privilege to work with such an excellent mentor and researcher.

I wish to specially thank my supervisor, Prof J. P. Meyer, and co-supervisor, Prof J. A. Snyman, for their helpful advice, which enabled me to successfully complete my work. The dedication and commitment of Prof Meyer to my work motivated me, especially during the final stage.

I would also like to thank Mr R. M. Morris and Dr D. J. de Kock for their technical support and friendship. I much appreciate the contribution of Dr V. B. Lunga for editing the language and grammar.

Lastly, I thank my wife and son for their encouragement, and my parents and other family members for their prayers, and the ALMIGHTY GOD of S.t Engenas for everything He has done for me.

---

## PUBLICATIONS IN JOURNALS AND CONFERENCES

---

### Refereed journal papers

- (a) Motsamai, O. S., Visser, J. A., and Montessor, R. M., “Multi-Disciplinary Design Optimization of a Combustor”, *Engineering Optimization*, vol. 40, No. 2, pp. 137-156, 2008.

### Journal papers submitted for publication

- Motsamai, O. S., Snyman, J. A., and Meyer, J. P., Optimisation of Combustor Mixing for Improved Exit Temperature Profile, HTE, submitted.

### Papers in refereed conference proceedings

- 5 Motsamai, O. S., Snyman, J. A., and Meyer, J. P., “Mixing of multiple reacting jets in a gas turbine combustor” Proceedings of the Sixth International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics, 30 June-3 July, Pretoria, South Africa, Paper No. MO1, 2008.
- 6 Motsamai, O. S., Visser, J. A., Morris, R. M., and De Kock, D., “An efficient Strategy for the Design Optimization of Combustor Exit Temperature Profile”, Proc. of ASME Turbo Expo, Paper No. GT2006-91325, Bacerlona, Spain, 2006.
- 7 Motsamai, O. S., Visser, J. A., Morris, R. M., and De Kock, D., “Using CFD and Mathematical Optimization to Optimize Combustor Exit Temperature Profile”, Proc. of ASME Power, Paper No. PWR2006-88120, Georgia Atlanta, USA, 2006.
- 8 Motsamai, O. S., Visser, J. A., Morris, R. M., and De Kock, D., “Combustor Optimization with CFD and Mathematical Optimization” Proc. of Fifth SACAM, Cape Town, South Africa, 2006.
- 9 Motsamai, O. S., Visser, J. A., Morris, R. M., and De Kock, D., “Validation Case for Multidisciplinary Design Optimization of a Combustor”, Proc. of Botswana Institution of Engineers, 2005.



---

# TABLE OF CONTENTS

---

<b>LIST OF FIGURES.....</b>	<b>xi</b>
<b>LIST OF TABLES.....</b>	<b>xiii</b>
<b>NOMENCLATURE.....</b>	<b>xiv</b>
<b>CHAPTER 1: INTRODUCTION.....</b>	<b>1</b>
1.1 BACKGROUND.....	1
1.2 REVIEW OF RELATED LITERATURE.....	4
1.3 NEED FOR THE STUDY.....	11
1.4 AIM OF THE PRESENT WORK.....	12
1.5 ORGANISATION OF THE THESIS.....	13
<b>CHAPTER 2: BACKGROUND TO GAS TURBINE COMBUSTION.....</b>	<b>15</b>
2.1 PREAMBLE.....	15
2.2 THE BASIC FEATURES OF A GAS TURBINE COMBUSTOR.....	15
2.2.1 Diffuser.....	16
2.2.2 Liquid fuel injection.....	16
2.2.3 Swirler.....	17
2.2.4 Cooling air.....	18
2.2.5 Primary zone.....	18
2.2.6 Intermediate (secondary) zone.....	19
2.2.7 Dilution zone.....	20
2.3 OPTIMISATION PARAMETERS FOR COMBUSTOR EXIT TEMPERATURE PROFILE.....	20
2.4 CONCLUSION.....	23

<b>CHAPTER 3: NUMERICAL COMBUSTION .....</b>	<b>25</b>
3.1 PREAMBLE.....	25
3.1.1 Conservation of momentum.....	25
3.1.2 Mass conservation.....	26
3.1.3 Conservation of energy (sum of sensible and kinetic energies).....	28
3.2 TURBULENT NON-PREMIXED COMBUSTION.....	28
3.3 TURBULENT NON-PREMIXED COMBUSTION APPROACHES.....	29
3.3.1 RANS in turbulence modelling.....	29
3.3.2 Unclosed terms in Favre-averaged balance equations.....	32
3.3.3 Classical turbulence models for the Reynolds stresses.....	33
3.3.4 Standard k- $\epsilon$ turbulence model.....	34
3.3.5 Strengths and weaknesses of the standard k- $\epsilon$ turbulence model.....	35
3.4 TURBULENCE CHEMISTRY INTERACTIONS.....	36
3.4.1 Primitive variable method.....	37
3.4.2 Reaction rate method.....	38
3.5 NEAR-WALL TURBULENCE MODELING.....	38
3.5.1 Wall functions.....	39
3.6 FLAME AND WALL INTERACTION.....	41
3.7 SPRAY MODELLING.....	43
3.7.1 Particle-source-in-cell model (PSICM), or discrete-droplet model.....	45
3.7.2 Liquid phase equations.....	46
3.7.2.1 Turbulent dispersion.....	47
3.7.2.2 Drop breakup.....	49
3.7.2.3 Turbulence modification.....	49
3.7.2.4 Drop collisions and coalescence.....	50
3.7.2.5 Heat and mass transfer.....	51
3.8 CONCLUSION.....	52



<b>CHAPTER 4: THE DYNAMIC-Q OPTIMISATION METHOD.....</b>	<b>53</b>
4.1 PREAMBLE.....	53
4.2 MATHEMATICAL OPTIMISATION.....	53
4.2.1 Constrained optimisation.....	53
4.2.2 Dynamic-Q method.....	54
4.2.3 Constructing the successive approximate spherical quadratic subproblems.....	56
4.2.4 Gradient approximation of objective and constraint functions.....	59
4.2.5 Particular strengths of Dynamic-Q.....	61
4.2.7 Approximation of derivatives for noisy functions.....	62
4.3 CONCLUSION.....	62
<b>CHAPTER 5: DESIGN OPTIMISATION METHODOLOGY.....</b>	<b>63</b>
5.1 PREAMBLE.....	63
5.2 MODEL VALIDATION.....	63
5.2.1 Validation error control.....	64
5.2.2 Test case.....	65
5.2.3 Case set-up.....	67
5.2.4 Boundary conditions.....	68
5.2.5 Results and discussions.....	68
5.2.6 Conclusions.....	73
5.2.7 Results discrepancies.....	74
5.3 IMPLEMENTATION OF DESIGN OPTIMISATION METHODOLOGY.....	75
5.4 NUMERICAL TOOL FOR FLOW ANALYSIS.....	76
5.5 GEOMETRIC MODEL.....	77
5.6 BOUNDARY CONDITIONS.....	80
5.7 FUEL SPRAY INJECTION MODEL.....	80
5.7.1 Experimental errors and equipment limitations.....	83
5.8 CONSISTENCY AND CONVERGENCE.....	84

5.9 COMBUSTOR NUMERICAL FLOW FIELDS.....	84
5.10 OPTIMISATION PROBLEM DEFINITION AND FORMULATION.....	87
5.10.1 Numerical integration.....	87
5.10.2 Mathematical optimisation.....	89
5.11 CONCLUSION.....	92
<b>CHAPTER 6: RESULTS.....</b>	<b>93</b>
6.1 PREAMBLE.....	93
6.2 OPTIMISATION CASE STUDIES.....	93
6.3 TWO DESIGN VARIABLES (Case 1).....	94
6.3.1 Results for Case 1.....	95
6.4 FOUR DESIGN VARIABLES (Case 2).....	98
6.4.1 Results for Case 2.....	99
6.5 FIVE DESIGN VARIABLES (Case 3).....	103
6.5.1 Results for Case 3.....	104
6.6 CONCLUSION.....	110
<b>CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS.....</b>	<b>112</b>
7.1 CONCLUSIONS.....	112
7.2 RECOMMENDATIONS.....	114
7.2.1 Improvement of simulations capabilities.....	115
7.2.2 Further development of optimisation capability.....	115
7.2.3 Extension of design optimisation method process.....	115
<b>REFERENCES.....</b>	<b>117</b>
<b>APPENDIX A.....</b>	<b>A1</b>
<b>APPENDIX B.....</b>	<b>B1</b>

---

## LIST OF FIGURES

---

Figure 2.1: The basic features of a gas turbine combustor.....	15
Figure 2.2: Explanation of terms in exit temperature profile parameters.....	21
Figure 3.1: Interactions between walls, flames and turbulence [70].....	42
Figure 5.1: Two-dimensional view of a Burner.....	66
Figure 5.2: Meshed volume of the Burner.....	66
Figure 5.3: Axial velocity at 27 mm from the quarl exit.....	69
Figure 5.4: Axial velocity at 109 mm from the quarl exit.....	70
Figure 5.5: Axial velocity at 343 mm from the quarl exit .....	70
Figure 5.6: Flow field showing temperature (K) contours.....	71
Figure 5.7 Temperature at 27 mm from the quarl exit .....	72
Figure 5.8: Temperature at 109 mm from the quarl exit .....	72
Figure 5.9: Temperature at 343 mm from the quarl exit.....	73
Figure 5.10: Three-dimensional model of the combustor.....	78
Figure 5.11: Computational grid of the combustor.....	78
Figure 5.12: Flow diagram of FLUENT coupled to optimiser.....	79
Figure 5.13: The modified Rossin-Rammler plot of spray.....	82
Figure 5.14: Combustor velocity (m/s) vectors.....	84
Figure 5.15: Combustor temperature (K) contours.....	85
Figure 5.16: Target and actual combustor exit temperature profile.....	86
Figure 6.1: Target, non-optimised and optimised combustor exit temperature profile for Case 1.....	95
Figure 6.2: Optimisation history of the objective function for Case 1.....	96
Figure 6.3: Optimisation history of design variables for Case 1, where $x_1$ = number of dilution holes and $x_2$ = diameter of dilution holes .....	97
Figure 6.4: Temperature (K) contours on the centre plane (left side) and exit plane (right side) of the combustor for the non-optimised case (a) and optimised Case 1 (b).....	97
Figure 6.5: Target, non-optimised and optimised combustor exit temperature profile for Case 2.....	100

Figure 6.6: Optimisation history of the objective function for Case 2.....101

Figure 6.7: Optimisation history of the design variables for Case 2, where  $x_1$  = diameter of secondary holes,  $x_2$  = number of secondary holes,  $x_3$  = number of dilution holes and  $x_4$  = diameter of dilution holes.....102

Figure 6.8: Optimisation history of constraints for Case 2.....102

Figure 6.9: Temperature (K) contours (exit plane) for non-optimised and optimised for Case 2.....103

Figure 6.10: Target, non-optimised and optimised combustor exit temperature profile for Case 3.....105

Figure 6.11: Optimisation history of the objective function for Case 3.....106

Figure 6.12: Optimisation history of design variables for Case 3, where  $x_1$  = diameter of primary holes,  $x_2$  = number of primary holes,  $x_3$  = number of dilution holes,  $x_4$  = diameter of dilution holes and  $x_5$  = swirler angle.....107

Figure 6.13: Optimisation history of inequality constraint for Case 3.....108

Figure 6.14: Temperature (K) contours (exit plane) for non-optimised and optimised for Case 3.....108

Figure 6.15: Swirl velocity at 30 mm from the dome face for non-optimised case and Case 3.....109

Figure 6.16: Axial velocity at 30 mm from the dome face for non-optimised case and optimised Case 3.....109

Figure 6.17: Temperature (K) contours of optimised Case 3 on the symmetrical plane.110



---

## LIST OF TABLES

---

Table 3.1: Classes of turbulence models.....	34
Table 5.1: Wall thermal conditions.....	68
Table 5.2: Inlet flow boundary conditions.....	68
Table 5.3: Mole fractions in the inlet.....	68
Table 5.4: Boundary conditions at various inlets.....	80
Table 5.5: Discretised fuel spray data.....	81
Table 5.6: Prescribed cone fuel nozzle pattern.....	83
Table 6.1: Optimisation parameters for Case 2.....	99
Table 6.2: Optimisation parameters for Case 3.....	104



---

## NOMENCLATURE

---

### Symbols:

Variable	Description	Unit
$a$	Approximated curvature of the objective subproblem	-
$A$	Approximated Hessian matrix of the objective function	-
$A$	Combustor casing area	$m^2$
$A_a$	Annulus area	$m^2$
$A_d$	Area of the dilution hole	$m^2$
$AFR_{avg}$	Average air-fuel ratio	-
$b$	Impact parameter	-
$b_{cr}$	Critical impact parameter	-
$B_D$	Transfer number	-
$b_j$	Approximated curvature of $g_j(\mathbf{x})$ subproblem	-
$B_j$	Approximated Hessian matrix of $g_j(\mathbf{x})$ subproblem	-
$C$	empirical constant	-
$C_\mu$	Empirical constant for $k-\varepsilon$ turbulence model	-
$C_{\varepsilon 1}$	Empirical constant for $k-\varepsilon$ turbulence model	-
$C_{\varepsilon 2}$	Empirical constant for $k-\varepsilon$ turbulence model	-
$C_D$	Particle drag coefficient	-
$c_k$	Approximated curvature of $h_k(\mathbf{x})$ subproblem	-
$C_k$	Approximated Hessian matrix of $h_k(\mathbf{x})$ subproblem	-
$c_p$	Specific heat at a constant pressure	J/kg K
$C_{pg}$	Specific heat capacity of gas	J/kg K
$C_{pk}$	Specific heat at a constant temperature for species $k$	J/kg K

---

◆ Problem-dependent

---



Variable	Description	Unit
$D$	Diffusion coefficient	-
	Drop diameter	m
$D_{CR}$	Critical drop diameter	m
$d_h$	Dilution hole diameter	m
$d_j$	Diameter of the jet	m
$D_k$	Diffusion coefficient of species $k$	-
$E$	Empirical constant for the log-law mean velocity	-
	Energy	J
$f$	Any quantity	◆
$f', f''$	Fluctuating component of quantity $f$	◆
$\bar{f}, \tilde{f}$	Mean component of quantity $f$	◆
$f(\mathbf{x})$	Objective function	◆
$f_k$	Volume force acting on species $k$	-
$f_{k,j}$	Volume force acting on species $k$ in the direction $j$	-
$g$	Gravitational acceleration	m/s <sup>2</sup>
$g_j(\mathbf{x})$	$j$ -th inequality constraint function	◆
$G_k$	Production of kinetic energy	-
$H$	Hessian matrix	-
$h_k(\mathbf{x})$	$k$ -th equality constraint function	◆
$h_s$	Enthalpy	J/kg
$h_{s,k}$	Enthalpy of species $k$	J/kg
$I$	Identity matrix	-
$J$	Momentum flux ratio	-
$\hat{k}$	Lower bound	-
$k$	Turbulent kinetic energy	m <sup>2</sup> /s <sup>2</sup>
	Species	-

---

◆ Problem-dependent

---



Variable	Description	Unit
$\check{k}$	Upper bound	-
$k_g$	Gas thermal conductivity	W/m K
$k_{opt}$	Optimum ratio of flame tube area to casing area	-
$k_p$	Turbulent kinetic energy at point $p$	$m^2/s^2$
$L_e$	Length scale of $\epsilon$	m
$L$	Reference Length	m
$L_d$	Latent heat of vaporization	J/kg
$\dot{m}_{fuel}$	Fuel mass flow rate	kg/s
$\dot{m}_j$	Jet mass flow rate	kg/s
$\dot{m}_{bp}$	Mass flow at the base plate	kg/s
$\dot{m}_{po}$	Mass flow at the port	kg/s
$\dot{m}_{cool}$	Mass flow of cooling air	kg/s
$m$	Mass of mixture	kg
	Number of inequality constraints	-
$\dot{m}_r$	Recirculating mass flow	kg/s
$\dot{m}_{air}$	Air mass flow rate	kg/s
$\dot{m}_g$	Air mass flow rate	kg/s
$m_k$	Mass fraction of species $k$	-
$n$	Local coordinate normal to the wall	-
	Number of design variables	-
	Number of holes	-
$N$	Number of species	-
$n_{opt}$	Optimum number of holes	-
$Nu$	Nusselt number	-

---

◆ Problem-dependent

---





Variable	Description	Unit
$Nu_D$	Nusselt number of the droplet	-
$p$	Pressure	Pa
$p(\mathbf{x})$	Penalty function	-
$p(z)$	pdf of mixture fraction variance $z$	-
$P_3$	Air pressure at the combustor inlet	Pa
$P_4$	Exhaust gas pressure at the combustor exit	Pa
$P_k$	Source term	-
$Pr_D$	Prandtl number of the droplet	-
$\dot{Q}$	External heat source term	W
$q$	Heat flux	W
$Q$	External heat source term	J
$Q_D$	Rate of heat conduction to the droplet surface per unit area	W
$r$	Radius	m
$r_D$	Radius of the droplet	m
$r_{D1}$	Initial radius of the droplet	m
$r_{D2}$	Final radius of the droplet	m
$Re_D$	Droplet Reynolds number of the droplet	-
$R^n$	$n$ -dimensional real space	-
$Sc_D$	Droplet Schmidt number	-
$Sc_k$	Schmidt number of species $k$	-
$Sh_D$	Droplet Sherwood number	-
$S_m$	Mass source	kg/m <sup>3</sup> s
$t_R$	Droplet transit time	s
$T_F^0$	State temperature of fuel	K
$T_O^0$	State temperature of oxidiser	K

---

◆ Problem-dependent

---



Variable	Description	Unit
$T$	Temperature	K
$t$	Time	s
$T_0$	Reference temperature	K
$T_3$	Temperature of air at the combustor inlet	K
$T_{4avg}$	Average temperature at the combustor exit	K
$T_{4max}$	Maximum individual temperature at the combustor exit	K
$T_{4peak}$	Maximum temperature in average radial profile	K
$T_D$	Droplet temperature	K
$t_D$	Viscous damping time	s
$T_g$	Gas temperature	K
$T_w$	Wall shear stress	Pa
$u$	Velocity in the $x$ -direction	m/s
$U^*$	Dimensionless mean velocity	-
$u_A$	Velocity of air	m/s
$u_D$	Velocity of droplet	m/s
$u_i$	Instantaneous velocity in the $i$ -th direction	m/s
$u_i$	Three-dimensional velocity field	m/s
$u_i'$	Fluctuating part of velocity in the $i$ -th direction	m/s
$U_j$	Velocity of the jet	m/s
$V_i^c$	Correlation velocity $V^c$ in direction $i$	m/s
$V_k$	Component of diffusion velocity of species $k$	-
$V_{k,j}$	Diffusion velocity of species $k$ in the direction $j$	m/s
$\dot{w}_k$	Reaction rate of species $k$	W
$\dot{w}_T$	Reaction rate	W
$w$	Velocity difference between product and parent droplet	m/s

---

◆ Problem-dependent

---



Variable	Description	Unit
$w_k$	Reaction rate of species $k$	-
$\mathbf{x}$	Design vector	◆
$\mathbf{x}^*$	Optimum design variable	◆
$y$	Droplet distortion	-
	Coordinate from the wall	m
$Y_F^0$	State mass fraction of fuel	-
$Y_O^0$	State mass fraction of oxidiser	-
$y^+$	Wall unit for law-of-the-wall	-
$Y_D$	Fuel vapour mass fraction	-
$Y_{Ds}$	Fuel vapour mass fraction at the droplet's surface	-
$Y_F$	Mass fraction of fuel	-
$Y_k$	Mass fraction of species $k$	-
$Y_{max}$	Maximum jet penetration	-
$Y_O$	Mass fraction of oxidiser	-
$y_p$	Distance from point $P$ to the wall	m
$z$	Mixture fraction variance	-

### Greek symbols

$\alpha$	Penalty function parameter for inequality constraint	-
$\beta$	Penalty function parameter for equality constraint	-
$\beta_k$	Penalty parameter	-
$\delta_j$	Specified move limit for $i$ -th design variable	◆
$\Delta f_{norm}$	Normalised step size	-
$\delta_i$	Move limit on $i$ -th design variable	-

---

◆ Problem-dependent

---

Variable	Description	Unit
$\Delta P_{overall}$	Change in overall pressure drop	-
$\Delta x_i$	Step size for $i$ -th design variable	◆
$\Delta x_{norm}$	Normalised step size	-
$\varepsilon$	Rate of dissipation of turbulence	-
$\theta$	Angle	degrees
$\mu_i$	Penalty parameter	-
$\mu_t$	Turbulent viscosity	Pa s
$\mu$	Kinematic viscosity	m <sup>2</sup> /s
$\mu_l$	Liquid viscosity	Pa s
$\mu_g$	Gas kinematic viscosity	m <sup>2</sup> /s
$\nu_i$	Kinematic viscosity	m <sup>2</sup> /s
$\rho$	Density	kg/m <sup>3</sup>
$\rho_j$	Penalty parameter	-
$\rho_3$	Density of air at the combustor inlet	kg/m <sup>3</sup>
$\rho_D$	Droplet density	kg/m <sup>3</sup>
$\rho_g$	Average gas density	kg/m <sup>3</sup>
$\rho_i$	Density of the mixture	kg/m <sup>3</sup>
$\rho_k$	Density for each species $k$	kg/m <sup>3</sup>
$\sigma$	Surface tension	N/m
$\sigma_D$	Droplet surface tension coefficient	-
$\sigma_t$	Effective turbulent Prandtl number	-
$\tau$	Stress	Pa
$\tau_B$	Characteristic breakup time	s
$\tau_D$	Droplet relaxation time	s
$\tau_E$	Eddy life time	s

---

◆ Problem-dependent

---



Variable	Description	Unit
$We$	Weber number	-
$\tau_{ij}$	Stresses in the $i$ -th and $j$ -th direction	m/s
$\tau_{TR}$	Transit time scale	s
$\omega$	Turbulent vorticity	-
	Oscillatory frequency	1/s

### Subscripts

3	Combustor inlet
4	Combustor exit
$A$	Air
$B$	Breakup
$D$	Droplet
$F$	Fuel
$g$	Gas
$i$	Index
	Index
$INT$	Interaction
$j$	Index
$k$	Species
	Source
$m$	Mass source
$max$	Maximum
$norm$	Normalised
$O$	Oxidiser
$opt$	Optimum

---

◆ Problem-dependent

---



## Subscripts

$P$	Point
$s$	Surface
$t$	Turbulent
$TR$	Transit

## Abbreviations

CFD	Computational fluid dynamics
RANS	Reynolds-averaged Navier-Stokes equation
RSM	Reynolds stress method
LES	Large eddy simulation
DNS	Direct numerical simulation
SMD	Sauter mean diameter

---

◆ Problem-dependant

---