Format and long-term effect of a technique mastering programme in first year Calculus

by

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DECLARATION

I, the undersigned, hereby declare that the dissertation submitted herewith for the degree Magister Scientiae to the University of Pretoria contains my own, independent work and has not been submitted for any degree at any other university.

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Abstract

The research on the format and long-term effect of a technique mastering programme in the first year Calculus course involves a group of first year engineering students at the University of Pretoria. Apart from conceptual understanding these students are also expected to master a certain amount of basic knowledge and rote skills in the Calculus course. The process of acquiring and assessing basic knowledge and rote skills (also referred to as must knows and techniques, respectively) is known as the technique mastering programme at the University of Pretoria.

This study addresses two research questions. The first question deals with the issue as to whether the paper-based assessment format for the technique mastering programme in first year Calculus can be replaced by computer-based assessment without a significant difference in performance. The second question deals with the long-term effect of the techniques mastering programme and the study investigates which and how much of the knowledge and skills embedded by the technique mastering programme in the first year is retained after a further two years of study.

In answer to the first question, the study shows that statistically there is no significant difference in performance in the technique mastering tests when the paper format is replaced by an online format. Yet, for a large group of students the logistics are formidable and the change to the online format under investigation is not practically feasible. The second part of the study shows that, in general, there is a disappointing decline in performance over a period of two years. There are, however, areas in which students performed better after the elapsed period. The research is of diagnostic value in determining the future of the technique mastering program with regard to both its format and contents.

Format and long-term effect of a technique mastering programme in teaching Calculus

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Chapter 1

Introduction

1.1 Setting the scene

The mainstream first semester calculus course at the University of Pretoria covers, as at most other universities, standard topics such as limits, continuity, differentiation and integration. In 2001, when the research for this study was conducted, approximately 900 students were enrolled for the course of which the majority (65%) were engineering students with the other students majoring in science subjects such as mathematics itself, physics, chemistry or information technology.

The group under discussion is typical of a first semester calculus group of any year in its diversity in preparedness for the course. On the one end of the scale are students who took Additional Mathematics as an additional subject in Grade 12, a subject of which the content overlaps with the first year calculus course. On the other end of the scale are students from educationally disadvantaged backgrounds who come to university ill prepared and laden with misconceptions regarding basic principles. There are also students who passed Grade 12 mathematics with distinction and students who come from other countries with a different school syllabus.

Devlin (1991) describes the students on the privileged side of the spectrum in a then future vision as:

> Imagine then the kind of person coming into our graduate schools, if not today, then certainly tomorrow. Brought up from early childhood on a diet involving MTV, Nintendo, graphical calculators packed with algorithms, Macintosh-style computers and, in the not-too-distant future, hypermedia educational tools as well. Such a person is going to enter mathematics with an outlook and a range of mental abilities quite

different from their instructors.

On the other side of the spectrum are students who have no experience of technology at all, even without basic technological appliances at home.

When dealing with such a diverse group and in addition to that a full curriculum and limited contact time, one has to be innovative in maximising students' learning experience and to stimulate these students in their journey of discovery.

After graduation, when students enter the job market, these students will most probably be required to utilise various resources connected to mathematics such as the internet, software programs, new textbooks, etc. They will be expected to analyse new material and apply and synthesise this new knowledge in their problem solving strategy. One of the lecturer's tasks then certainly is to expose students to resources other than the prescribed textbook (Stewart, 1999). This is feasible but time-consuming.

Although one of the main objectives of the calculus course under discussion is to equip students with problem solving skills, one cannot ignore the necessity of acquiring basic knowledge and rote skills before embarking on problem solving. This **basic knowledge** and **rote skills** are referred to as **must knows** and **techniques**, respectively. The process of acquiring and assessing the must knows and the techniques is known at the University of Pretoria as the **technique mastering programme** (**TM** programme for short).

Mastering of the must knows and techniques requires training and a fair amount of repetition from the student's side. These skills then need to be assessed, often more than once, until the required level of expertise is achieved. Technique mastering is thus time consuming and labour intensive for the student as well as for the lecturer.

Because of a high content volume for each contact period, with only enough time to teach and illustrate concepts and theory for a particular topic, there is little time to incorporate practice activities for the technique mastering programme into the contact sessions.

The time constraints imposed by the challenges mentioned above were the reason for the decision to separate the technique mastering programme from the formal contact lecture programme and to investigate the possibility of incorporating computer-based assessment into this programme. In doing so students can independently master the necessary techniques by repetitively writing tests without unnecessary external intervention. Lecturers are spared the drudgery of grading repetitive technique mastering tests and so gain valuable hours.

This study describes the implementation of computer-based assessment, in particular web-based assessment, into the technique mastering programme and compares results of the computer-based versus paper-based assessment.

In addition, the long-term effect of the technique mastering programme is investigated. A sample group of the 2001 first year students are assessed again in 2003 in their third year of study to determine how ingrained the must knows and techniques still are.

1.2 Research questions

The research questions of this study are formulated as follows:

1. Can paper-based assessment for the technique mastering programme in first year calculus be replaced by computer-based assessment without a significant difference in performance?

2. Which and how much of the knowledge and skills embedded by the technique mastering programme in first year calculus is retained after a further two years of study?

1.3 Structure of the dissertation

The dissertation consists of six chapters and three appendices.

Chapter 1 is introductory, describing the setting for the research. The research questions are formulated, followed by an exposition of the structure of the dissertation and a statement of the significance of the dissertation.

Chapter 2 gives an overview of technique mastering in general and of related programmes at other institutions. Assessment of technique mastering is discussed and a literature review is conducted on experiences with similar programmes as well as on the long-term retention of core knowledge and basic skills.

Chapter 3 puts the University of Pretoria under the spotlight. A short history of computer-based education at the Mathematics Department up to the present is given, after which the situation in 2001, when this study was conducted is described.

Chapter 4 deals with computer-based assessment. A literature review is conducted, followed by a discussion of the advantages and disadvantages of computer-based assessment. The chapter concludes by focussing on *WebCT* as a platform for computer-based assessment.

Chapter 5 endeavours to answer the first research question on whether the paper assessment in the technique mastering programme can be replaced by online assessment. The methodology is described and the collected data is statistically analysed to reach a conclusion.

Chapter 6 deals with the second research question on the long-term effect of the technique mastering programme. Both a quantitative and a qualitative investigation and conclusions are described. A topical comparison sheds light on which topics should be considered as essential knowledge.

1.4 Significance of this research

Universities are at the knowledge forefront, with the result that any university functions within an ever-changing environment. As new knowledge becomes available through research it is incorporated into the teaching programme. Lately, not only the importance of incorporating new knowledge is emphasised but also the importance of incorporating new innovative teaching methods. The 90's decade has seen technology blossom, offering new possibilities for the classroom in particular. Visualisation and ease of computation are two of the major advantages offered by technology. The internet became commonly used only in the last part of the nineties but is rapidly becoming an integral part of the education world. It would be foolish not to embrace the possibilities that technology and the internet offer. Levine, Mazmanian, Miller and Pinkman (2000) is of the opinion that

> The teaching of mathematics, and all subjects, for that matter, must undergo constant change if students are to be prepared to enter a rapidly-evolving technological world. It is no longer a question of whether to use technology in the teaching and learning experience. It is now the question of what technology to use and how and when to use it.

Calculus is a subject taught across the world and the concerns at the University of Pretoria in this regard are by no means unique. Yet, our particular student composition, our facilities, our curriculum and our approach may be somewhat different from that of other universities, especially those abroad. Although a number of other universities have embarked on similar technique mastering programmes and computer-based assessment, none of these programmes can be repeated exactly in our context. It is important to develop a programme that is tailor-made for this particular situation. It is also important to establish what the success of such a programme is. It was decided to turn to computer-based assessment for the technique mastering programme in the calculus course at the University of Pretoria and if deemed successful it could be used as an example for other universities and also for convincing the few remaining sceptics in our own department. It is also important to list concerns and problems encountered, especially in the relatively new field of technology in education.

The long-term success of the programme is perhaps even more significant than the outcome of the switch to computer-based assessment. If the long-term effect is below expectation we need to seriously reconsider our strategy. Yet, irrespective of the outcome, results should assist in addressing two issues. The first issue is the validity of what we consider as must knows. Are what we consider as must knows the basic knowledge necessary for passing the first year course or does it have long-term value, even beyond university? Are we selecting the appropriate must knows? It would be of interest here to investigate the long-term performance in the different categories of must knows and techniques. The second issue to be addressed is the method of deployment of the technique mastering programme. What were the concerns and problems encountered and where is there room for improvement? Do students benefit maximally from the programme?

An informed opinion, if not answers to these questions, will be valuable in directing the future of the technique mastering programme and improving on a programme that is a necessary and integral part of one of the most significant courses in the department.

Chapter 2

Technique mastering

2.1 Terminology and examples

As stated in the introduction there is a core of basic knowledge and rote skills necessary for venturing into deeper conceptual mathematics and problem solving, the basic knowledge referred to as **must knows** and the rote skills as **techniques**, both prerequisites for success. The process of acquiring the necessary must knows and the techniques as well as the assessment of these is collectively referred to as the technique mastering programme at the University of Pretoria, as stated before. It is not always easy to distinguish between a must know and a technique and often what is considered basic knowledge by one person could be seen as a rote skill be another. We expand on the terminology for clarification.

2.1.1 Must knows

When knowledge is vital for some activity and hopefully becomes so ingrained that it can be recalled effortlessly it is a must know. Real-life examples are facts such as that you need to drive on the left-hand side of the road in South Africa; that there are 100 cents in a Rand; that the boiling point for water is 100^0C. In mathematics the most basic must knows are, for example, facts such as that after 1 comes 2 followed by 3 etc.; that 1 plus 1 make 2; that if you divide by 0 you run into trouble.

Examples of relevant must knows:

1. The properties and shapes of the graphs of basic functions such as

$$f(x) = mx + c,\ f(x) = e^x,\ f(x) = \ln x,\ f(x) = \sin x$$

are considered as basic knowledge, must knows.

2. When solving an equation where the unknown variable is in the exponent you often need the following basic knowledge:

$$\text{If } y = e^x \text{ then } x = \ln y.$$

This is knowledge that needs to be ingrained and is therefore classified as a must know.

3. The identity

$$\sin^2 x + \cos^2 x = 1$$

is used in differentiation, integration and many other places and so is a must know.

4. When solving a quadratic equation, for example, one often makes use of:

$$ab = 0 \text{ if and only if } a = 0 \text{ or } b = 0.$$

This is a must know that is frequently encountered in problem solving.

2.1.2 Techniques

When a process is repeated so often that it becomes second nature it is classified as a technique. Real life examples include driving a car, eating with cutlery and reading. In mathematics the most basic of these techniques are counting and simple addition.

Examples of relevant techniques are:

1. Factorisation of polynomials, solving simple equations.

2. Techniques by which to determine the intercepts of a function with the axes, the domain, the range and the asymptotes.

3. The basic rules of differentiation such as the product rule, the quotient rule and the chain rule.

4. Sketching of $y = f(x + a)$ by using the graph of $y = f(x)$ and moving a units to the left on the $x-$axis.

5. 'Multiplying by one' for instance to help find indeterminate limits of the form $\left(\frac{0}{0}\right)$ such as

$$\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1} = \lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1} \cdot \frac{\sqrt{x} + 1}{\sqrt{x} + 1}$$

2.2 Technique mastering, gateway testing and assessment

What is called technique mastering testing at the University of Pretoria is similar to what is referred to as gateway testing in some American universities, also known as barrier testing. According to the Oxford dictionary (2003), gateway means

> an opening that can be closed by a gate or an entrance with or opening for a gate.

The purpose of most gateway tests is that it allows a student to pass through a gate once he/she obtains the necessary knowledge or skills to use as the key to unlock the gate when entering into a new cognitive environment. In the USA the term gateway test is used to describe a test that typically covers some collection of routine mechanical skills, rather than concepts. The purpose of gateway tests is stated by Megginson (1994) as

> to assure that some particular collection of skills has actually been learned.

Common practice is that if a student fails to master particular skills he/she is forced to relearn these skills well enough to pass the gateway test. Gateway tests can be taken during a current course or near the beginning of a follow-up course, to assure that students either have mastered routine skills taught in the present course or the prerequisite skills needed for the new course.

Megginson(1994) describes the main difference between traditional and gateway tests as follows.

> The difference between traditional testing and gateway testing is that if a student performs poorly on a gateway test over a particular collection of skills, the student cannot compensate by learning *other* collections of skills well enough to overcome the effect of the poor performance on the gateway test. Instead, the student *must* go back and relearn the particular collection of skills well enough to pass the gateway test. The test does not go away until the student has learned the material well enough to pass it.

Gateway learning, or in our case technique mastering, is categorised in the two lowest levels of Bloom's Taxonomy (1956), that of knowledge and comprehension and do not acquire entering the higher levels of application, analysis, synthesis and evaluation. The philosophy behind technique mastering is that a learner cannot construct meaning to what he/she has learnt and cannot venture into problem solving before he/she has mastered the basic knowledge and skills.

Technique mastering is embedded in the broader philosophy of **mastery learning**. The basic principles of mastery learning in a course as described in the Educational Psychology Interactive: Mastery Learning (Huitt, 1996b) include

- enough time to demonstrate mastery of objectives,
- breaking the course into manageable units of instruction and
- requiring students to demonstrate mastery of objectives for a unit before moving on to other units.

Mastery learning is further explained by the Abaetern Academy (Huitt, 1996a) as

> ... a teaching philosophy that assumes the student can and will master course objectives if the proper instruction and guidance is offered, and if the time required to learn is available.... In short, this means everyone masters the objectives of a course before they go on to the next course. This enables them to take on the next level of learning with confidence... Mastery learning is not new, but the oldest learning model we have. Would our parents encourage us to drive if we hadn't yet mastered walking?

Although the quote above refers to course objectives, the material allocated to a technique mastering programme will be a subset of the course material, yet the principles of mastery learning as stated above apply.

The Kumon method of learning mathematics, initiated by Toru Kumon in 1954 (The Kumon Philosophy, 2003), is a good example of a mastery learning programme, practised worldwide on primary and secondary school level. Kumon was convinced that his son could solve problems if the skills necessary to understand advanced level mathematics were taught one step at a time and so he created a series of worksheets to help his son. The Kumon method relies on splitting knowledge into manageable proportions, self-paced instruction and nurturing the basic reading and mathematical skills in order

> to *master*, step by step, the *skills* and *knowledge* they need for success in higher level math and reading comprehension.

An example of mastery learning is presented by the University of Nebraska-Lincoln in their Keller plan courses (Titsworth, 1997) replacing the traditional lecture-listen-test model commonly used at many institutions.

> Students are tested over the same information multiple times until they achieve "mastery" of the content.

Although mastery learning has a large component of drill and practice, this cannot be conducted blindly. According to Rissmann-Joyce (2002) in a study of the educational system and practices of Japan, it is not enough to 'drill-and-kill' specific skills, unless the underlying mathematical principles are understood. She also emphasises that mastery learning can be used for mastering critical thinking skills, referring to the Japanese situation where

> students are routinely practising the complex reasoning and critical thinking skills.

Mastery learning need therefore not be merely the mindless practising of problems but could lead to constructing new knowledge and strategies out of elementary problems, in order to venture into solving more complex problems.

Engelbrecht (1990) describes the necessity of mastery learning in mathematics. He compares the lack of even a small piece of mathematical knowledge, to a weak spot in a wall (of mathematical knowledge) causing the whole wall to tumble.

A weak point in any gateway testing or technique mastering programme is the subjectivity of the assessor in determining the content and pass criteria, referred to by Engelbrecht and Harding (2003b):

> the judgement of what concepts or techniques are assessed and what constitutes a criterion of satisfactory performance is in the hands of the assessor.

In order for any technique mastering programme to be successful students need to be repeatedly exposed to the must knows and the techniques. They need to work through lists and lists of similar practice problems and they need to be motivated to do so. In his process they need to be formatively and continuously assessed. Common practice is to set a high 'pass mark' of at least 80% and make available a series of tests. A student writes the first test and if he/she fails, he/she needs to prepare more, write the second test etc. until a satisfactory level of expertise is reached.

For whatever way gateway testing is assessed, it seems to be common practice to have sample tests or handouts available for students to practise. Tests are often available on the institutions' web sites or as handouts so that students have ample opportunity to practise for a gateway test.

Students know exactly what is expected of them, and in many cases students are rewarded for passing the gateway programme or penalised if failing. Students know beforehand what the rewards for passing the test or penalty for failing are. A reward for passing the programme could be a higher symbol in the total course grading, whereas a penalty for failing could be a maximum symbol for the actual course. In some cases students fail the actual course because of failing the gateway programme and have to retake the gateway programme during the next semester before they pass the actual course.

2.3 Long-term retention of core knowledge and basic skills

Concern regarding the long-term retention of the core knowledge and basic skills acquired by students in the first year is not unique to the University of Pretoria. It is especially disconcerting when success in follow-up courses is hampered by a lack of recall of core knowledge. It is also not uncommon for teachers of third year modules to complain that students either cannot do the mathematics of the first year any longer or, worse still, claim never to have encountered it.

Although literature in this regard is scanty, a study by Anderson, Austin, Barnard and Jagger (1998) investigates the issue of long-term retention of core knowledge when they examine the extent to which certain core first year material is retained and understood. The study involves 155 students, mostly volunteers, at fifteen different institutions in the UK, all in their third year of study. Seven questions were posed of which one was on the application of the definition of differentiability, the only question related to the TM programme at UP. It should be noted that this question was on a slightly higher level than questions typically included in the TM programme at UP. Other questions covered a spectrum wider than calculus. The material of the test was carefully selected to be representative of most first year undergraduate mathematics courses in the

UK.

Anderson et al (1998) report that

> ... only about 20% of the responses were substantially correct and almost 50% did not contain anything that could be deemed to be minimally 'credit-worthy'. This suggests that a considerable amount of what is taught to mathematics students in general as 'core material' in the first year is poorly understood or badly remembered. ... the retention of first year material is demonstrably weak and suggests some cause for concern among those who teach them.
>
> It is as if the experience of students, attending one module after another, is such that they tend to 'memory-dump' what they have had in previous modules, rather than retain it and build it into a coherent knowledge structure. In many instances, there was little that the students actually could recall, even in cases where they had gone on to futher study in the same topic later.

Miller, Mercer and Dillon (1992) in a paper on acquiring and retaining mathematics skills, conducting a study on elementary school children, mostly with learning disabilities, caution that memorisation does not implicate the ability to solve problems (moving from the concrete to the semi-concrete and the abstract).

> Rote memorization of math does not teach students to understand mathematical concepts.

The aim of the TM programme at UP surely is not only to secure students' core knowledge but to create understanding as well. If students do not think creatively enough and rather rely on memory, it could fail them in the long run.

Steyn (2003) claims that retention of information is linked to the way that it is taught. It is the responsibility of the lecturer to guide students in this process of learning and retention. Steyn proposes

> . . . that new information is more easily understood and retained when it can be related to existing information.

Weinstein (1999) also refers to the important role of the teacher when he states that

> facilitators (lecturers) can have a tremendous impact on helping students to develop a useful repertoire of learning strategies. Students should have the opportunity to reflect

> on their learning (strategies) and teachers should not only ask students what they think but also how they think.

Gagné (1977) views long-term retention of knowledge as an essential part of learning when saying that

> Learning is a change in human disposition or capability, which persists over a period of time, and which is not simply ascribable to processes of growth. ... The change must have more than momentary permanence; it must be capable of being retained over some period of time ... it must be distinguishable from the kind of change that is attributive to growth.

Linked to long-term retention of knowledge is the ability to integrate knowledge between different subjects and different years of study. Knowledge obtained in the first year of study of mathematics could reappear in a physics course, for example, perhaps in a different notation but should ideally be recognised as essentially the same knowledge.

The lack of ability to integrate knowledge is a widely recognised problem at universities. The University of Massachusetts Dartmouth currently addresses this issue in their IMPULSE programme (short for The Integrated Math Physics, Undergraduate Laboratory Science and Engineering Program) (IMPULSE, 2004).

> The IMPULSE faculty chose to integrate math, physics and engineering in order to connect concepts, applications, and methods. In the traditional classroom setting, these subjects are typically taught in an isolated manner, leaving students the responsibility to draw the connections and relationships between them. The end result is that, while students spend hours learning derivatives, for example, they are unable to use them in a different context.

According to John Dowd (IMPULSE, 2004) a physics lecturer at UMD, students benefit largely from the IMPULSE programme.

> Integration - the fact that students are learning in a cross-disciplinary kind of way - is a key goal. So that when they're doing math, they're also doing physics. They realize that these subjects somehow connect to each other. In the past, I've had students come to me who know how to take the derivative of x squared; it's $2x$. I ask them: "What's the derivative of t squared?" "I don't know; we didn't study that." They're doing

> mechanics. Many of the derivatives and integrals are over time. They don't make the conceptual jump, going from one symbol to another. They don't realize the symbol can stand for anything.

Schattschneider (2004) addresses the issue of knowledge recall in successive courses when she reports on dealing with the fact that the precalculus course at Movarian College did not meet the expectations of students and lecturers alike.

> In the calculus course, many students will insist that they have never seen or used certain techniques simply because the context is so different. (For example, solving simple linear equations certainly is covered in precalculus, but linear equations there were never solved for something called $\frac{dy}{dx}$ or $y\prime$. Inevitably, teachers need to review still again the non-calculus skills that are essential to solve calculus problems. There is often a high degree of frustration on both the part of the students and of the teachers; in the precalculus course and (for the survivors) in the calculus course that follows it, there is low morale in both camps.

Instead of the precalculus course, they successfully introduced a course called Calculus I with Review, a one-year course covering both Calculus I and the precalculus course

> but at a slower pace, integrating the review of precalculus concepts and skills as they were needed.

Important is the fact that time was spent on identifying student's weaknesses and how to address it.

In conclusion, we quote from a report on the Capstone Experience, a programme at the University of Nebraska Kearny with the purpose of integrating 'real-life' experiences into problem solving

> Many ideas we teach in the calculus sequence and in Foundations of Mathematics are abstract and difficult to fully grasp; even for students who eventually earn doctorates in mathematics, many ideas (Newton quotients and Riemann sums, for example) may take years to internalize (The Capstone Experience, 2003).

2.4 Gateway testing at other institutions

Although gateway testing is more commonly used in the USA, related programmes are also encountered elsewhere. In the United Kingdom, for example, diagnostic testing is used for almost the same purpose as gateway testing. The report titled Measuring the Mathematics Problem (Savage and Hawkes, 2000) describes the necessity to

> assess the knowledge and skills of individual students, and to identify strengths and weaknesses of whole groups.

In the USA there is an increasing emphasis on the importance of fundamental mathematical concepts and essential skills that ideally would ensure a foundation that would give all students solid preparation for work and citizenship, positive mathematical dispositions, and a conceptual basis for further study. In the state of Tennessee, for example, students are expected to pass gateway tests in a variety of subjects including mathematics, language, arts and science in order to accomplish the goals set by the State of Tennessee namely that all children should be able to read well and reach a certain level of competence in mathematics and science. Details are contained in *The Master plan for Tennessee Schools, Preparing for the 21st Century (*2003).

We highlight one case of gateway testing in particular, that of the Department of Mathematics, University of Michigan (Megginson, 1994) where gateway testing is used extensively to achieve either, or both, of the following objectives:

- *To assure that students have the prerequisite mathematical skills needed for the course.* At the university of Michigan, the gateway tests serve to indicate to students that their preparation is inadequate for them to continue with the present or follow-up courses. These tests can be given at the start of a new semester or at the end of the present one.
- *To assure that students have mastered routine skills taught in the present course.* Megginson describes some of the advantages such as more time available during lectures for concepts, multi-step problems and applications and assurance that the skills are mastered.

We compare gateway testing programmes at a few institutions in the USA in Table 2.1

Common factors present in all institutions' gateway testing include:

- no partial credit
- high pass mark
- each student has to at least master 70% of the core knowledge before he/she can attempt a follow-up test
- preparation tests are available

- limited time to write the actual test
- students know what is expected of them.

	Institutions	NAU	UMich	UWM	UTK	BSU	APICS
Course	Pre-Calculus		x			x	x
	Calculus I	x	x	x			
	Calculus II	x			x		
Preparation test available	Online sample tests	x	x	x	x		x
	Online interactive tests		x				
	Topics outlined		x			x	
	Handouts			x			
	Linked to other institutions	x					
Number of retakes	Limited	5		3	2x p.w.	3	
	Unlimited		x				
	With penalty			x	x	x	
Contribution to course grade		0%	0%	40%	0%	0%	0%
Pass criteria		$\frac{6}{7}$	$\frac{20}{25}$ or $\frac{6}{7}$	80%	70%	80%	$\frac{8}{10}$
No partial credit		x	x	x	x	x	x
Time limit for completion		x	x	x	x	x	x
Format	Pen-and-paper	x	x			x	
	Multiple choice		x				
	Online		x				
Content	Differentiation rules	x	x	x	x		
	Integration	x			x		
	Basic Algebra		x	x		x	x
	Limits		x		x		
	Trigonometry		x	x			x
	Basic math skills		x	x		x	x
Abreviations							
NAU	Northern Arizona Unversities	UMich	University of Michigan				
UWM	University of Wisconsin-Milwaukee	UTK	University of Tenessee, Knoxville				
BSU	Ball State University, Indiana	APICS	Atlantic Provinces Council on the Sciences				

Table 2.1: Comparison of a number of USA institutions' gateway testing programmes

Chapter 3

Technique mastering at the University of Pretoria

3.1 Computer-based education in mathematics at the University of Pretoria

When electronic calculators became available in the 1970's, the mathematics department at the University of Pretoria was quick to purchase a number of these and to encourage application in the classroom. Ever since it has become a priority in the department to keep abreast of technology and its applications in education. The 1980's saw the introduction of the personal computer (PC) but it was only in the 1990's that these became a common sight in offices. The concept of a computer laboratory for students also became a reality in the 1990's and computer aided instruction became a topic of research in the department (Engelbrecht, 1995). Simultaneously, graphical calculators became available.

The 1990's also saw the birth of the Reform Calculus movement where more emphasis is placed on visualisation and verbal interpretation. This movement had its impact on the mathematics teaching approach at this university and technological aid became even more important. In the USA a number of institutions started prescribing graphical calculators or supplying these in a laboratory situation for student usage. Locally these were too expensive to prescribe. For a while each lecturer in the department had a graphical calculator and as a group they shared an overhead device connected to a graphical calculator. This was not viable because of the logistics involved but also because students were only observers and not participants. A decision had to be taken whether to go for graphical calculators or for PCs. After much discussion, the department decided on a computer laboratory equipped with PCs. Initially one laboratory with six computers was supplied but subsequently another laboratory has been added with an additional 30 PCs. On campus there are presently various computer laboratories, totalling close to 2000 PCs.

The first project involving computer-based mathematics teaching ran in 1993 and involved a selected group of students with the software package *Mathematica* as a teaching aid. These students used the interactive feature of *Mathematica* to communicate with their lecturer. The experiment proved to be successful but too expensive and labour intensive to expand to the large group of first year students (Engelbrecht, 1995).

Another online education project, undertaken by the department, was a bridging course for ill prepared students, initiated in the early 1990's. The purpose was to bridge the gap between school mathematics and first year university mathematics, especially for students who had missed a year between Grade 12 and the first year of university. The software package *SERGO* was used, this package being a mastery learning programme. Theory was followed by practice exercises, followed by an evaluation test with immediate feedback. The underlying principle is that repetition builds confidence and reinforces the must knows necessary for success in the fist year calculus course. The programmerandomly chooses questions from a databank according to the level of difficulty set by the instructor. This was the department's first experience with online assessment. Because of major changes to the syllabus, the bridging course was discontinued since only a small part of the course was relevant to the new syllabus.

3.2 The current situation at the University of Pretoria

Technology has become an integral part of the teaching programme. Almost every staff member is techno literate and has his/her own PC and all students have access to a PC.

Matlab is a software package that is frequently used in the department at present. The programming and graphical features of *Matlab* are especially useful in problem solving and visualisation, respectively. In the first year calculus course, these features are explored by students in projects, an integral part of the course. Engineering students frequently use this software package to solve more complex problems in their later years. *Matlab* is available in all the engineering computer laboratories as well as in the department of mathematics. This powerful software package is a standard tool both for classroom demonstrations and for practical session explorations. *Matlab* is a computational tool, not an assessment tool.

Other frequently used software packages include *Scientific Workplace* (a type-setting programme combined with *Maple*, a powerful computer algebra system) and its smaller version *Scientific Notebook*. Both these programmes are used by some students for mathematical manipulations and presentations of projects, although the software is not available in the laboratories. Lecturers use these for mathematical purposes and also for typing articles and examination papers. *Mathematica*, (as discussed above) perhaps offers the highest mathematical capabilities and is used by researchers and students alike for problem solving, but is unfortunately also not available in the laboratories.

Engelbrecht and Harding (2001a, 2001b) describe the entrance of the department into the new telematic era with the introduction of the first web-based mathematics course in 2000. The university subscribes to *WebCT* as a platform, as previously stated. There are now a number of fully online mathematics courses and even more hybrid courses running. The university supplies the infrastructure in terms of computer laboratories and support and the department adds its knowledge and know-how. An increasing number of students have internet access at home and so this medium is becoming increasingly viable and appealing to students.

3.3 The technique mastering programme at UP in 2001

In 2001 the technique mastering programme had been running for a number of years in a fairly simple format. Appropriately categorised problems, requiring techniques rather than insight, are listed in the study guide (Appendix A). These problems are somewhat repetitive and provide ample practice opportunity. Students have access to all the problems from day one. Students are motivated to systematically work through these lists of problems to reinforce the techniques. They work on their own and seek help if necessary, either at their own lecturer, a tutor or elsewhere. Two rounds of TMT (Technique Mastering Testing) takes place per semester, preferably during the week preceding a semester test such that students can benefit from the feedback. Up to the year 2001 students were assessed on paper, repeatedly, until a satisfactory level of performance (80%) was obtained. The tests consisted of a subset of the exact questions as listed in the study guide. The fact that the system allowed a student to write consecutive TMTs until he/she has reached a satisfactory level of performance meant that assessment was a labour intensive process for the graders.

The basic format was retained in 2001 but with one significant difference. After one year's experience of running online courses using *WebCT* (Engelbrecht and Harding, 2001a, 2001b), it was decided to investigate the possibility of running the technique mastering assessment on *WebCT*.

The problems listed in the study guide were shaped into *WebCT* format to create a question databank. A number of quizzes were set to accommodate students who had to write more than once. Students were briefed on how to access the website and how to go about accessing the online quizzes. Certain groups of students did their TMTs online, certain groups maintained with the paper testing and the research sample of students were exposed to both the paper and online versions of the TMTs.

Chapter 4

Computer-based assessment

4.1 Definitions and terminology

Computer-based assessment (**online assessment** for short), in contrast to pen-and-paper assessment (**paper assessment** for short), makes use of some software package to grade the student's input, which in most cases is fed into the computer via a keyboard. On the lowest level are multiple choice questions where students shade answers by pencil on an answer sheet, assessed by an optical reader. On the other end of the scale are courses that are presented totally online in a web environment and where online assessment forms an integral part of the course. It is also possible to use only selected sections of a web environment, such as only the assessment tool. Such a course with face-to-face tuition but with one or more components on the web is referred to as a **hybrid course**. *WebCT* and *Blackboard* are examples of virtual learning environments that create a web environment for educational purposes. Such extensive software packages offer a variety of assessment options including different question types, the option to import graphics into questions, availability settings for tests, an expandable question databank, a databank for student records and many more. In the project under discussion, online assessment was used in a web environment. An extensive report on web-based learning and assessment can be found in Engelbrecht and Harding (2003b).

4.2 Online assessment

Assessment innovation, including web-based assessment is the topic of a number of projects. Examples include the Innovation Assessment Program (United Inventors Association, 2003), the

Research on Assessment Practices of the Freudenthal Institute (2003) in the Netherlands and work done at the Indiana University, Center for Innovation in Assessment (2003) and The Math-Skills Discipline Network (2003) in the United Kingdom. Engelbrecht and Harding in a paper on online assessment (2003a) take an overview of the topic. We quote loosely from this paper while simultaneously expanding on the topic.

Changing teaching approach without due attention to assessment is not sufficient. A number of authors emphasise the importance of readdressing assessment practices while teaching practices evolve. Gretton and Chalis (1999) ask:

> Assessment has various purposes. Is it for grading and sorting students? Is it for encouraging learning? The answer is yes to both, but when both technology and students' skills are evolving so rapidly, then assessment style must also evolve to ensure it continues to fulfil these objectives.

Smith and Wood (2000) link assessment and learning:

> ... appropriate assessment methods are of major importance in encouraging students to adopt successful approaches to their learning.

The answer to the question why one should venture into online assessment probably lies in the fact that more and more situations are presently created where online assessment seems the obvious route to take. Not only are computers in common use in most teaching environments, but internet access is rapidly following the same route. We are currently experiencing what Miner and Topping (2001) describe as The Web Revolution with major impact on society:

> The World Wide Web is nearly a decade old now and its effect on math and science communication has already been amazing. An enormous amount of information has been made available on the Web. Increasingly, cross-referenced research articles and abstracts are available in searchable online archives. In education, engaging interactive materials are widespread, and Web-based course management tools are becoming frequent virtual companions to traditional pedagogical tools.

When deciding on online assessment in a course it is necessary to decide what aspect of the assessment will be done online. Keeping normal term tests in a paper format and opting for online assessment for the technique mastering programme seems to fall in line with the experience of Wise (1997) in a study of Maryland's functional tests for high school graduates in mathematics, reading and citizenship:

> The CAT (Computerised Adaptive Test) versions have augmented rather than supplanted the paper-and-pencil versions: they are typically used with transfer students and students who are retaking one or more test.

In a study done by Patelis (2000) on an overview of computer-based testing he urges educators to consider online assessment

> The age of the number-two pencil in standardised assessment is far from over, but CBT (Computer-Based Testing) is becoming more popular ... educators ... are often unaware of the options available to them.

Gateway or technique mastering testing, as discussed in a previous chapter, is particularly suited to online assessment. It is important to note, however, that online assessment is not only used for testing the must knows. The traditional perception is that MCQs can only be used for testing lower level cognitive skills. This is not true, according to Hibberd (1996)

> ... they can be implemented to measure deeper understanding if questions are imaginatively constructed.

An important advantage of online testing is that students can work any time, any place and are not limited to teacher organised opportunities. In a report on *The Pros and Cons of Online Assessment,* issued by Customised Training Development (CTD for short) (2003), an independent research firm of San Francisco, it is maintained that immediate feedback is essential in formative assessment.

Kamps and Van Lint (1975) found that a traditional paper calculus test can be replaced with a similar multiple choice test provided the questions are carefully selected so that the same concepts are tested. They comment on the large amount of time necessary to set such a test, but also mention the advantage of an accumulated question-bank over a period of time.

In a successful project at the University of Wolverhampton (Thelwall, 2000), computer-based tests were designed to replace paper tests and shows that it is feasible on a large scale.

> Each test generates an exam randomly from a possible 80 000 variations and then delivers the exam, marks it and gives feedback. The project delivers around 2 000 assessments per year in various forms: fully automated tests, automatically generated randomized web tests with automatically generated marking schemes, web self-assessments with JavaScript, and automated feedback marking schemes with word macros.

Boyce and Ecker (1995) in their research on the computer-orientated calculus course at Rensselaer Polytechnic Institute see a widening scope of possibilities as a benefit of computerising routine tasks

> As some computational tasks are routinized, greater diversity of applications can be considered.

From the teacher's perspective the value of decreased grading time when assessing online should not be underestimated. When working with large groups of a hundred and more students the grading load impacts on valuable research time and affects staff budgets. Developing the tests is time consuming but a question databank can be developed that eventually saves time.

Online assessment has the further advantage of enabling the teacher to readily obtain question-by-question profiles. Subsequent refinement of questions and tests can be carried out. The empirical data that becomes available makes online testing a valuable diagnostic instrument. The objective of every MCQ should be clearly understood and a careful selection of the distracters can itself be utilised to provide diagnostic information (Engelbrecht and Harding, 2003a).

Online testing also has disadvantages. At the moment, one of the biggest disadvantages of online mathematics is symbolic representation.

> ...the Web can be a powerful vehicle for communicating mathematics and science in general, just as it is revolutionizing communication in other disciplines and fields of endeavor. In spite of this, it is nonetheless true that there are still significant obstacles to publishing mathematics on the Web that other disciplines do not face. As has long been the case in print, authoring and publishing mathematical notation can be a complicated business (Miner and Topping, 2001).

MCQs are also lacking to a certain extent. First of all there is no partial credit for these questions. MCQs with distracters based on misconceptions can enforce such misconceptions unless immediate feedback is available (Engelbrecht and Harding, 2003a). Furthermore one should always remember that guessing has to be taken into account in some or other way.

Turning to the present study: When students need to do online tests in a secure environment, the logistics of organising of computer laboratory sessions for a large group of about 900 students are formidable. A sound infrastructure with enough workstations is a prerequisite before embarking on online assessment for a large group. It is also time consuming to develop questions for the databank.

Scepticism amongst staff regarding the validity of assessing mathematics online is also still common and there is a resistance to change among less computer literate staff members. An aspect to keep in mind when converting to online assessment is that it could possibly be a new experience for students and probably a new experience for some lecturers.

It is important to keep in mind that changes of this magnitude, in any field, are usually accompanied by anxiety and 'fear of the unknown'. Research in computer-based testing indicates that once individuals have taken a computer-based test, they become more comfortable with the new testing environment and find that the benefits associated with the test offset those aspects of the test that are new and unfamiliar. (Yopp, 1999)

Because of the 'fear' experienced by learners' first time experience of online testing, it is preferable for teachers to introduce practice tests. Students can practise quizzes in their own time to familiarise themselves with the syntax of mathematical symbols and functions.

According to CTD (2003) online assessment is not always considered by educators as a viable option.

> Online assessment has proved to be a contentious area for educators. The common image of online assessment is that of computer generated tests and many teachers will immediately discard this notion seeing it as inappropriate for their subject area.

Other than self-check quizzes and online tests, online assessment can take many other forms such as

- submission of assessment items via email or a subject delivery system
- contributions to a bulletin board, either individually or in groups and
- peer mentoring and commenting on a bulletin board.

Netshapala (2001) discusses the advantages and disadvantages of using computers in the mathematics classroom. Amongst the advantages of uses for the classroom is the fact that learners may repeat the same (similar) test without feeling personal rejection from their instructor. The facelessness of the computer can be an advantage to a student with low self-esteem. There is no fear of exposure. An individual student need not feel rejection from his/her classmates either, since each individual can continue at his/her own pace. There is also value in re-writing a test since a student has to prepare and rethink the same technique again and hopefully master it in the end.

4.3 Examples of online assessment

On investigating different models, that of the Old Dominion University in Virginia in the USA stands out as an example of how students can practise skills interactively. Old Dominion University introduced their Reform Calculus project in 1992 (Bogacki, Melrose and Wohl, 1995). Their Interactive Tutorials and Tests, available on the Internet, are impressive. The use of these software packages enables a student to type his solution using prescribed notation such as sqrt(x), sin(x), /, * etc. The student's input can be pre-viewed and confirmed before submitting to avoid typing errors, processed by the software application *Mathcad.* A student can ask for a hint to solve a problem for the price of a few marks. This model also incorporates the computer algebra system *Maple* that enables students to draw graphs, a skill that could prove to be useful for future careers. Currently anyone is allowed to use the software and it is available to any institution as share-ware, for non-commercial use, provided permission is granted from the authors. Unfortunately this model is not compatible with *WebCT*.

DIAGNOSYS is a software package widely used in the UK (Appleby, 1997).

> It uses a deterministic expert system, that is, inferences drawn from a student's answers are considered to be definite. By careful design of both the network of skills and of the questions that test those skills, useful conclusions can be obtained. It should be noted, however, that such expert systems require extensive design and validation, and can still produce highly questionable results (e.g. in medical diagnosis).

Respondus and *Questionmark* are two powerful and widely used software packages for online assessment in Mathematics. Both these packages offer a variety of question types and an excellent equation editor. *Respondus'* compatibility with *WebCT* makes it an appealing option.

WebCT is an extensive software package that provides a platform for online education via the internet. When designing a course with *WebCT* there are a variety of tools to choose from such as the Calendar tool for posting the schedule of events, the Discussion tool that offers a bulletin board for posting messages by both lecturer and students, the *WebCT* Mail tool for personal communication, the Chatroom facility for synchronous communication, Content pages for posting study material in, a Whiteboard facility for freehand writing, the Quiz tool for assessment and My Record for a record of the student's marks to name but a few. *WebCT* is just one example of a virtual learning environment, others include *Blackboard* and *eCollege*.

The powerful Quiz tool provides a variety of question types - multiple choice, matching, single answer, calculated (parameter dependent) and paragraph questions - as well as a Question Databank facility. The designer sets a quiz, determines the availability settings and sets an optional password. Once the quiz is graded an abundance of statistics becomes available.

Because the University of Pretoria subscribes to *WebCT* all lecturers have the option to make use of this platform. The assessment on which this study reports was conducted via the Quiz tool of *WebCT*.

4.4 Comparing online and paper assessment

Engelbrecht and Harding (2003a) compare performances in online assessment and paper assessment in their model of presenting a course via the internet. They conclude that although there is no significant difference in performance it is advisable to combine online and paper assessment modes.

> There has never been any "disturbing" difference between the online and paper sections... Students do seem to perform slightly better in the online section in general although this is marginal in most cases ...

In this study students show a slight preference for online testing but this is again marginal.

In a study at the North Carolina State University, written homework problems were replaced by online problems, assessed by the software programme WebAssign (Bonham, Beicher and Deardorff, 2001). Student performance was similar between the paper and web sections. Although students performed slightly better in the online version, the difference was not statistically significant.

A comparison between online and paper testing in the gateway programme of the University of Michigan (discussed before) are given by LaRose and Megginson (2003). For the purpose of their study the online entrance gateway test for Calculus II was replaced by a paper test. During the courses only online testing was used. Student survey results demonstrate students' perception that the online test's effectiveness is equivalent to that of the paper version for Calculus I. The Calculus II students somewhat preferred the paper gateway.

Chapter 5

Comparison between paper and online TMTs

5.1 Scope of the study

The study reports on research with a convenience sample consisting of a group of 103 students taught by the author. The group consists of Afrikaans speaking students, the majority studying computer engineering. Table 5.1 gives the distribution of the Grade 12 marks for mathematics for the sample group as well as for the whole first year group of engineering students. From this table it is clear that from an academic point of view, this group can be considered as representative of the whole first year group of engineering students. Although most of the students have an strong academical background in mathematics, the group also contains students in the five-year-plan (extended program) who are as a rule less well-prepared for university study in mathematics.

Scheduling of activities in the technique mastering programme was simplified by the fact that the entire sample group followed the same timetable. Since the students were all reasonably computer literate, working on the web posed no problem.

As mentioned earlier, this group of students are all Afrikaans speaking and are all studying in the same programme - making them more of a homogeneous group of students than the rest of the Calculus students. Because of the homogeneity of the group, it would be dangerous to generalise the results obtained from this group of students, despite the fact that academically they form a representative sample of the entire group.

	Sample group	2001 first year Calculus students
Symbol	%	%
A	32.04	33.42
B	22.33	20.73
C	33.98	26.76
D	11.65	17.46
E	0.0	1.38

Table 5.1: Distribution of grade twelve mathematics symbols

5.2 Research methodology

Students wrote two rounds of TMTs during the semester, the first (TMT_1) was written in the week before the first major semester test, about five weeks into the semester and the second (TMT_2) one month later, in the week before the second semester test.

In both cases, all students did two similar TMTs during the same lecture period, one a traditional paper test and the other an online test. The questions and standard in these tests were similar, but the online tests consisted mainly of multiple-choice and matching questions. See Appendix B for the online versions of the TMTs. The questions used in the paper TMTs are included in the discussion in Chapter 6. The weights of the different topics in the online version of TMT_1 were not exactly the same as that in the paper TMT_1. This was corrected for TMT_2. Here the questions were carefully selected to test the same skills for both versions of the tests.

Students did the online tests under supervision during which time assistance on the technical side was available.

As mentioned in Section 3.3, students who did not get 80% in either of the online or paper TMTs, had to rewrite. These rewrites were done online only and do not form part of this study.

In this chapter the results of the online and paper TMTs are compared empirically for both TMT_1 and TMT_2.

Prior to the date set for the second TMT a questionnaire was issued on students' attitude concerning the two modes of assessment and on the technique-mastering programme in general.

5.3 Research results

Table 5.2 displays the distribution of the marks, number of students that participated, the percentage of the students that passed, the median and the mean of the two TMTs. For both the TMTs we also include a column indicating the results for the best of the two tests. The reason for this is that in order to pass the test, it was required that a student pass only one of the online or paper tests.

Marks distribution	TMT_1			TMT_2		
	Paper	Online	Best of Paper and Online	Paper	Online	Best of Paper and Online
Number of students	100			97		
Number of students that passed	53	60	83	54	53	66
% of students that passed	53	60	83	55.7	54.6	68.0
Mean	7.4	7.4	8.6	7.2	7.4	8.1
Median	8	8	8	8	8	9
Standard deviation	2.0	2.2	1.4	2.4	2.3	2.1
Pearson correlation coefficient	0.016			0.604		
Spearman correlation coefficient	0.011			0.558		
Sign test for differences: p-value	0.747			0.428		

Table 5.2: Comparison between the results of the online and paper TMTs

Although the averages for both TMTs correspond favourably it should be verified statistically that there is no difference in average performance between the paper and online versions of the TMTs. Because the data recorded on performances in the four tests is not normally distributed, a t-test is not appropriate and the non-parametric sign test for differences is applied. The p-values of 0.747 for TMT_1 and 0.428 for TMT_2 point to no significant difference in average performance on a 5% level of significance between the paper and online versions in either of the two TMTs.

The Pearson correlation coefficient $r = 0.016$ and Spearman correlation coefficient $r = 0.011$ for TMT_1 indicate poor correlation between the paper and online versions of the tests. Table 5.3shows the student performance distribution (percentage-wise) split into three categories, for the results of the paper version of TMT_1 compared to the online version. The cells away from the main diagonal of this table are heavily loaded, supporting the poor correlation between the two versions of TMT_1.

The low correlation between the paper and online versions of TMT_1 is reason for concern. Although the averages for the two versions are comparable it appears that it is not the same students who do well in both. The question of reliability immediately comes to mind. Ideally a reliability coefficient should be calculated to draw a statistically valid conclusion regarding the consistency with which students respond to the paper and to the online tests. The low correlation seems to point to a possible problem regarding the reliability. Unfortunately the questions in the two versions were not matched and neither are question by question results available for the online version, negating the possibility of calculating a reliability coefficient. For TMT_2 care was taken to match the questions in the paper and online versions and there the correlation is much higher as will be discussed shortly, although it would be wrong to conclude that the former (matching the questions) implies the latter (higher correlation coefficient). Considering the extenuating factors involved, TMT_1 of 2001 should probably be regarded as a trial run and a learning curve for both teachers and students. Students were unfamiliar with this mode of testing and experienced difficulty with the technical side, teachers were still familiarising themselves with the art of posing online questions and the problems of symbolic notation etc. Another possible explanation for the low correlation could be that in order to pass, students had to pass only one of the two tests. It is possible that once they passed the paper test they had little motivation for passing the online version of the test while students who did not pass the paper version performed better in the online version. This could explain why the averages are comparable but the correlation is low.

From the data collected on TMT_1 alone it would be unfair to conclude that the paper mode of testing could be replaced with online testing without loss of generality and the first research question would remain unanswered.

We investigate the results of TMT_2. Only a small percentage (30%) of students passed both the paper and online versions of the test. It should be noted, however, that this 30% represents 56.7% of the students that passed the paper version and 50% of the students that passed the online version.

The Pearson correlation coefficient $r = 0.604$ and Spearman correlation coefficient $r = 0.558$ for TMT_2 indicate stronger correlation between the paper and on-line versions of TMT_2. Table 5.4 shows the student performance distribution (percentage-wise) split into three categories, for

TMT_1		Paper →			
	Marks	0 - 5	6 - 7	8 - 10	Total
Online	0 - 5	2	7	8	17
↓	6 - 7	4	4	22	30
	8 - 10	10	13	30	53
	Total	16	24	60	100

Table 5.3: Student performance distribution (percentage-wise) comparing the results of the Online and Paper TMTs for TMT_1

the results of the paper version of TMT_2 compared to the online version. In this case the diagonal cells are more heavily loaded.

For TMT_2, the percentage of students that passed both the paper and online tests increased from 30% (TMT_1) to 42.7% and represents 75.9% of the students that passed the paper version and 77.3% of the online version, respectively.

The percentage of students that passed either of the two tests decreased from 83% (TMT_1) to 68.6% (TMT_2). A possible reason for this could be that students realised that the marks did not contribute to their final mark, but this again is mere speculation and beyond the scope of this study.

TMT_2		Paper →			
	Marks	0 - 5	6 - 7	8 - 10	Total
Online	0 - 5	14.6	5.2	4.2	24.0
↓	6 - 7	8.3	3.1	8.3	19.7
	8 - 10	4.2	9.4	42.7	56.3
	Total	27.1	17.7	55.2	100

Table 5.4: Student performance distribution (percentage-wise) comparing the results of the online and paper TMTs for TMT_2

In Figure 5.1, we compare distribution of the marks for the online and paper versions of each of TMT_1 and TMT_2. It is difficult to draw any conclusion from these figures except that in both tests the best of the two marks (paper and online) is significantly higher than any of the two, which seems to indicate that students have different assessment preferences.

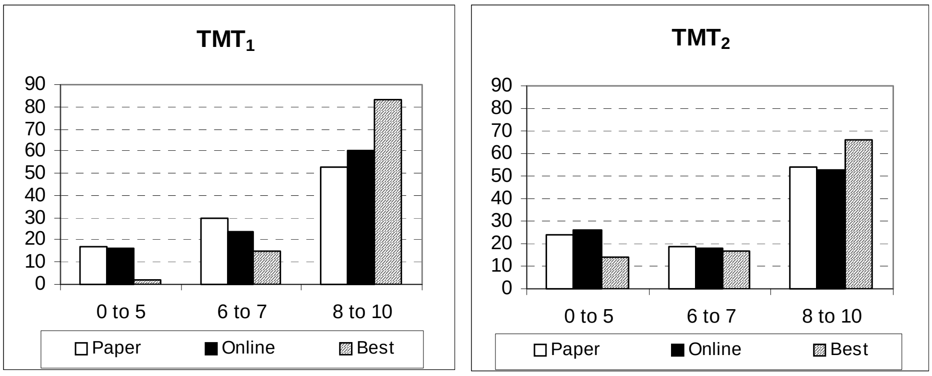

Figure 5.1: Comparison of distribution of marks for TMT_1 and TMT_2

5.4 Responses to the questionnaire

In the period between TMT_1 and TMT_2, 92 students completed a questionnaire (Appendix C). Apart from certain questions on demographics, students were asked to give reasons for their performance in TMT_1, and to indicate (and explain) their preference between the paper and online tests. The questionnaire survey reveals results that could explain some of the empirical results.

About 86% of the students who completed the questionnaire, passed TMT_1 in a first attempt, either paper or online. Of those who failed the paper test a small percentage (6%) are of the opinion that they failed because of the fact that their presentation of the answers was not mathematically correct, while a larger percentage (14%) admit that they were not sufficiently prepared for the test.

Students who failed the online TMT_1 offer the following reasons for their performance:

- They were not properly prepared.
- They guessed the answers.
- They blame the computer - they knew the answers but did not type it correctly,
- and a few had technical problems.

Of the 28 students who were not successful in their first attempt in either the paper or online tests, 5 wrote the online test twice in order to pass and 23 needed 3 attempts to pass the online test. Only 9 of these students consulted their lecturer oe a tutor for assistance in preparing for the retake.

A large percentage (69%) of the students prefer the online version of the test to the paper version. They offer the following reasons:

It is user friendly.

You do not have to draw the graphs yourself, you can just choose one.

One can work backwards toward the question.

I like multiple choice questions.

I feel comfortable in front of a PC.

Students who prefer the paper version explain their preference as follows:

One can get partial credit.

I like to write out the answer.

I think while I write.

I don't feel so pressed for time.

It is difficult to concentrate on the screen.

The given answers/notation for the online test confuse me.

Some general observations from the questionnaire may be of value in future.

- A large percentage of the students prefer the online tests and found the tests easier than the paper tests, although this is not reflected in the results in Table 5.2
- A number of students comment that when failing the first attempt, they would sit for the second attempt without preparing in advance.
- A number of students confess that they guessed the correct answer in the multiple-choice questions rather than working towards a solution.
- A suggestion arising from the results of the questionnaire is that there should be more than two TMTs during the semester, each covering a smaller part of the syllabus content.

5.5 Concerns and problems

A number of concerns related to the *WebCT* software package have been expressed, the first being the difficulty with mathematical symbols on the web. In 2001 it was somewhat difficult to get mathematical symbols on the web. *WebCT* has since introduced an equation editor that makes it easier. Another option is to link a software package such as *Respondus* to *WebCT*. Nevertheless, in 2001 there were still only a limited number of symbols available and students had to use notation such as sqrt(x) for $\sqrt{x}$, int(x^2+1)dx for $\int (x^2+1)dx$ and d/dx(x-1) for $\frac{d}{dx}(x-1)$. In spite of the fact that this notation was explained, students were not familiar with typing mathematical symbols in this manner and students commented in the questionnaire that this was a problem.

Security is a problem when students do online tests. Students could access tests from their home computer or use any computer laboratory at the university. Although a password was required, there is no guarantee that a student wrote his or her own test. To address this problem, supervised sessions were scheduled in the large computer laboratories to enable a bigger number of students to write simultaneously but a number of logistical problems were experienced during these sessions.

During informal discussions with students, it became clear that the objectives that many students have with their studies, differ somewhat from ours as teachers. Our goal is that students should master the mathematical content on a long-term basis. Some students do not value the importance of basic mathematical skills for future use. Their primary objective is to pass the course (short term). Students prefer arguing for extra marks in order to get to the required 80% minimum for a TMT, rather than prepare again and attempt a rewrite, that would benefit them in the long run.

Students also do not like the repetitive nature of the programme. They get bored with rewriting a TMT and many confess to not preparing at all before a second attempt at a TMT, just hoping that they will guess a sufficient number of answers correctly when doing the TMT online. This is counterproductive and another approach should be considered. If we could bring students to realise what the true aim of the TM programme is, they would perhaps be more prepared to put effort into the programme. The structure of the TM programme is not challenging enough for students to encourage total engagement in the programme - they do it because they have to. The system should be modified in such a way that students take part in the programme with greater motivation.

The fact that the results of the TMTs do not contribute towards their final mark, may be an important reason for the lack of enthusiasm for the TM programme. It is also difficult to develop a mechanism to *force* students to write the TMTs. The possibility to change to a system where TMTs become part of regular (contributing) tests was considered and implemented in 2002.

5.6 Conclusions

The aim of the research reported on in this chapter is to compare the online and paper versions of the technique-mastering tests in order to determine whether the paper TMTs can be replaced by an online assessment programme. The empirical comparison between the two modes of testing indicates no significant difference in average performance but low correlation between the paper and online versions for TMT_1 raises uncertainty regarding reliability. The results of TMT_1 on its own are therefore inconclusive and the first research question remains unanswered. However, results of TMT_2 not only show no significant difference in performance between the paper and online versions but also strong correlation. Based on these results the question as to whether paper testing can be replaced by online testing in the technique mastering programme can be answered affirmatively.

There are indications, however, that it is not necessarily the same students who do well in the online TMTs that do well in the paper TMTs. Students have different assessment preferences and the paper assessment format could perhaps be enhanced by adding an online component rather than being entirely replaced by the online tests. Different modes of assessment can provide for this spectrum of assessment preferences. This conclusion is supported by the preferences expressed by students in the questionnaire, indicating that the majority of the students prefer the online assessment mode but that a substantial number of students still prefer conventional paper testing.

There are advantages and disadvantages to computerising the TMTs and a number of logistical problems. Time saving on marking is a definite advantage. A disadvantage is that some students as well as some teachers experience the initial internet exposure as new and different. Most of the students are familiar with the internet but they have had little experience in using the internet in their learning. Since this is the only section of the course presented on the internet, some students experience this as overwhelming. A session was scheduled before the first test to give students an opportunity to familiarise themselves with the procedure but this was still a very new experience for many students. For many teachers, on the other hand, running an assessment programme on the internet by posting tests on the web is also a new and overwhelming experience and some teachers are reluctant to acquire the new expertise necessary for running such a programme.

WebCT tests are only used in the technique mastering programme and are not used in the general assessment of the course, simply because of the logistical problems when dealing with a large number of students. As could be expected, the teachers involved in the course decided in 2003 to use the best of the two worlds and to change to a system of using multiple choice questions that are answered by the students in pencil on paper and then marked by an optical reader. This system has a smaller marking load than conventional paper tests but is found to be logistically easier to run by the teachers than when using the internet.

It has to be emphasised that in this study we do not attempt to do a full investigation into the problems of online assessment but rather investigate the possibility of using online tests in our technique mastering programme. Since this study was done, Engelbrecht and Harding (2003a and 2003b) have done further studies on the use of online assessment in undergraduate mathematics courses at the University of Pretoria.

Chapter 6

Longer-term effects of the TM programme

6.1 Scope of the study

For the research on the longer term effects of the TM programme we focus on a sample group of third year students of 2003. The research consists of three parts - a quantitative investigation, a topical comparison and a qualitative investigation based on interviews with selected students. In the quantitative investigation we compare the performance in the two TMTs of 2001 ($TMT_1$01 and $TMT_2$01 for short) with that of the two identical TMTs of 2003 ($TMT_1$03 and $TMT_2$03 for short), respectively. In the topical comparison we discuss the 2001 and 2003 performances per question. In the qualitative investigation we look at perceptions of students on the value of the TM programme, recorded from interviews with individual students.

6.2 Research methodology

The investigation in 2003 was done towards the end of the first semester. At that time only 43 of the original sample group of 103 students were available. These were the same computer engineering students who were now doing their final mathematics module, Stochastic Processes. By then they had completed three semesters of Calculus, one semester each of Linear Algebra, Differential Equations and Numerical Methods and had nearly completed one semester of Stochastic Processes in the mathematics department. Some of the other engineering courses such as Computer Programming, Circuits, Mechanics, Physics and Digital Systems also require mathematical knowledge. One can safely assume that these students will have been exposed to a substantial amount of mathematics in the various mathematics modules as well as in related subjects.

Without prior notice, the sample group was asked to write both TMTs in the same session. The tests were in paper format only and were exact replicas of the TMTs of 2001. The TMTs written in 2003 were then graded using the same criteria as in 2001. An overall comparison is done by comparing the final marks for the TMTs of 2001 and 2003. The comparison between 2001 and 2003 focuses on this group of 43 students. The question by question results of 2003 are then compared to those of the 2001 tests to determine the longer term effect of the technique mastering programme.

To get individual feedback, and as a qualitative measure, 14 students were selected from the group of 43 students, selected on their 2001 final mark for the first semester, first year Calculus course. There were 6 students with a final mark of more than 75%, 4 students with a final mark of between 60% and 75% and 4 students with a final mark of between 50% and 60%. These students were interviewed to report on their perceptions of the TM programme of 2001. We report in detail on four interviews and quote other students. The interviews consisted of a number of structured questions concerning the technique mastering programme in general as well as questions concerning their personal performance in the TMTs of 2001 compared to that of 2003.

6.3 Quantitative investigation

We look at the correlation between the different tests and compare the means of the TMTs of 2001 and 2003. The purpose of the comparison is to use the results diagnostically to determine a strategy for similar future programmes. Table 6.1 reflects the results of the two TMTs of 2001 with that of the TMTs written in 2003.

For the 43 students in question, the average mark for $TMT_1$01 of 7.3 out of 10 is followed by an average mark of 6.1 for $TMT_1$03. For $TMT_2$01 the average mark is 8 out of 10, dropping to 6.3 for $TMT_2$03. From this data alone it is not possible to establish whether there is a significant difference in average performance between the paper tests written in 2001 and the paper test written in 2003, respectively. Again the non-parametric Sign Test for differences seems appropriate because not all data appears to be normally distributed. The p-values, obtained from this test, of 0.0008 for TMT_1 and 0.0001 for TMT_2 point to a significant difference in average performance between the

Marks distribution	TMT_1		TMT_2	
	2001	2003	2001	2003
Number of students	43		43	
Number of students that pass	20	7	31	17
% of students that pass	46.5	16.3	72.1	54.6
Mean	7.3	6.1	8	6.3
Median	7	7	8	6
Standard deviation	1.9	1.6	2.1	2.5
Pearson correlation coefficient	0.440		0.446	
Sign test for differences: p-value	0.0008		0.0001	

Table 6.1: Comparison between the results of the paper TMTs in 2001 and the paper TMTs 2003.

TMTs on both a 1% and a 5% level of significance. The drop in average performance in both TMTs from 2001 to 2003 is therefore statistically verified.

The Pearson correlation coefficient $r = 0.440$ between $TMT_1 01$ and $TMT_1 03$ indicates a moderately strong correlation between performance in the two tests. Table 6.2 shows the student performance distribution (percentage-wise) split into three categories, for the results of the paper version of TMT_1 for the two different years. Most of the cells on the main diagonal of this table are loaded, supporting the moderately strong correlation between $TMT_1 01$ and $TMT_1 03$. A relatively large percentage (34.8%) of the students that performed well in 2001 did not pass the same test in the 2003. It should also be noted that only a very small percentage of students (4.7%) who failed TMT_1 in 2001 managed to pass in 2003. Further investigation is done by looking at performance in individual questions and also by conducting interviews with a selected group of students, reported on in later sections.

The Pearson correlation coefficient $r = 0.446$ between $TMT_2 01$ and $TMT_2 03$ indicates a moderately strong correlation between the tests. Table 6.3 shows the student performance distribution (percentage-wise) split into three categories, for the results of TMT_2 for the two different years. A disturbing figure is the 41.8% of students that performed well in 2001 did not pass the same test in the 2003. Note also that the percentage of students that passed $TMT_2 03$ is more than double that of $TMT_1 03$ (39.5% compared to 16.3%).

TMT_1		2003 →			
	Marks	0 - 5	6 - 7	8 - 10	Total
2001	0 - 5	14.0	7.0	0.0	21.0
↓	6 - 7	9.3	18.6	4.7	32.6
	8 - 10	11.6	23.2	11.6	46.4
	Total	34.9	48.8	16.3	

Table 6.2: Student performance distribution (percentage-wise) comparing the results of the paper TMTs of 2001 and the paper TMTs of 2003 for TMT_1

TMT_2		2003 →			
	Marks	0 - 5	6 - 7	8 - 10	Total
2001	0 - 5	4.7	2.3	2.3	9.3
↓	6 - 7	4.7	7.0	7.0	18.7
	8 - 10	25.6	16.2	30.2	72.0
	Total	35.0	25.5	39.5	

Table 6.3: Student performance distribution (percentage-wise) comparing the results of the paper TMTs of 2001 and the paper TMTs of 2003 for TMT_2

Figure 6.1 represents a Venn-diagram for the pass percentages for the $TMT_1$01 and $TMT_1$03. This diagram shows that a relatively small percentage of 11.6% passed both $TMT_1$01 and $TMT_1$03 and that almost half of the students in this group (48.9%) did not pass any of the two tests.

Only 46.4% (34.8% plus 11.6%) of this group initially reached the requirement of 80% to pass $TMT_1$01. The reader should be reminded that students had to write $TMT_1$01 repeatedly until they passed the test, and as a result 90.7% of the large group (described in Chapter 5) eventually obtained the minimum mark of 80%. The percentage of students that scored at least 80% for $TMT_1$03 is a mere 16.3% (11.6% plus 4.7%). Possible reasons for this poor performance will be discussed in more detail (question-by-question) in Section 6.4.

Figure 6.2 represents a Venn-diagram for the pass percentages for $TMT_2$01 and $TMT_2$03 and shows a larger percentage (than that for TMT_1) of 30.2% that passed both $TMT_2$01 and $TMT_2$03 and a much smaller percentage (18.7%) that did not pass either of these two tests.

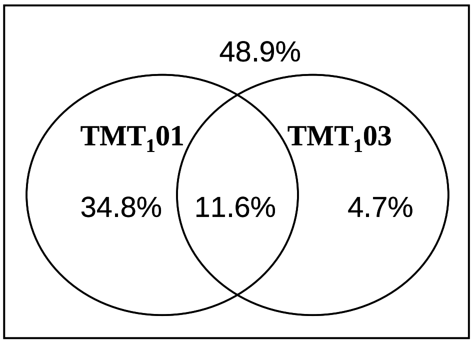

Figure 6.1: Diagram comparing the pass percentages of TMT_101 and TMT_103

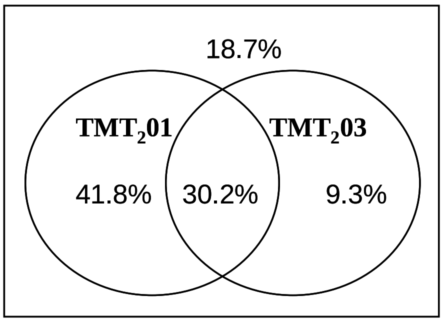

Figure 6.2: Diagram comparing the pass percentages of TMT_201 and TMT_203

A slightly larger percentage of students 72.0% (41.8% plus 30.2%) initially reached the required 80% to pass TMT_201. The percentage of students that scored at least 80% for TMT_203 is only 39.5%, but it should be mentioned that some students did not attempt to answer all the questions for TMT_203. During the interviews a few students claimed that they did not have enough time to finish both the tests, and explained that they would have put more effort into the tests if the marks were to have contributed towards their final mark. This was not a general response, however, and on closer investigation of the papers and from interviews it appeared not to be a factor that had a serious impact on the results.

6.4 Topical comparison

In this section we compare the question by question results of the 2001 and 2003 technique mastering tests. Unfortunately it is not possible to use a statistical test such as the χ^2-test for comparison purposes, due to the small sample size of 43 and the fact that in comparison tables such as those given for the questions below, the maximum of 20% zeroes, a restriction imposed by the χ^2-test, is often exceeded. The comparison is done descriptively but supported by the collected data.

Since most of the students are Afrikaans-speaking, responses are translated. Pseudonyms are used in reporting on interviews and when quoting students.

6.4.1 Comparison of $\mathrm{TMT}_1$01 and $\mathrm{TMT}_1$03

A question by question comparison is given of the first technique mastering test written in both 2001 and 2003.

Question 1

Let $f(x) = \sqrt{x}$ and $g(x) = x - 1$. Determine $\frac{f}{g}$ and $f \circ g$. You may assume that in each case the domains of f and g consist of all numbers x for which $f(x)$ and $g(x)$ make sense.

		2003 $\rightarrow$				
	Marks	0	1	2	3	Total
2001	0	0.0	0.0	0.0	0.0	0.0
$\downarrow$	1	0.0	0.0	0.0	2.3	2.3
	2	7.0	16.3	34.9	2.3	60.5
	3	4.7	4.7	27.9	0.0	37.2
	Total	11.6	20.9	62.8	4.7	

Table 6.4: Student performance distribution (percentage-wise) for TMT_1 Question 1

Means: 2.3 for $\mathrm{TMT}_1$01 and 1.6 for $\mathrm{TMT}_1$03.

Objectives: To test basic knowledge and comprehension concerning combinations of functions.

Observations: Table 6.4 shows that the percentage of students who scored two or more for the question dropped considerably from 97.7% in 2001 to 67.5% in 2003. One explanation could lie in the fact that one mark was awarded to giving the domains of the two composite functions (although the question did not explicitly ask for it) and while much emphasis was placed on domains of functions in the first year, students may have forgotten this two years later. During the interviews it became evident that these students most probably would have been able to write down the restrictions, had they explicitly been asked to do so.

In general, students interviewed claim that they last encountered composite functions in their first year and have not used it since.

> Things like these [composite functions] are rather rare [in computer engineering courses] (Charles)

The quantitative results, however, show that a reasonable percentage of students could still deal with composite functions. They have most probably used it indirectly somewhere along the line, perhaps not using the formal notation.

> ...what does $f \circ g$ mean? (Eugene on his answer sheet)
>
> Memorising is not part of my learning pattern, if I don't use it often, I tend to forget the definitions. I am better at figuring out things. (Eugene explaining his written answer)

Conclusion: Although students do not seem to have encountered the composite function notation since first year, the underlying knowledge still seems reasonably intact. It is probably a matter of not recognising the notation. They would have encountered composite functions in other mathematics courses if not in their other subjects since most functions are composite e.g. $f(x) = \sin(x + 2)$ or $f(x) = \sqrt{x-1}$.

Question 2

Solve for x : $\frac{1}{2}\ln x = 1 - \ln 2$.

		2003 →		
	Marks	0	1	Total
2001	0	18.6	4.7	23.3
↓	1	30.2	46.5	77.7
	Total	48.8	51.2	

Table 6.5: Student performance distribution (percentage-wise) for TMT_1 Question 2

Means: 0.8 for $TMT_1$01 and 0.7 for $TMT_1$03.

Objective: To test whether students can use logarithmic properties to solve basic logarithmic equations, specifically for the natural logarithm ln.

Observations: Although a reasonable high percentage (77.7%), scored full marks in 2001, only 51.2% did so in 2003, a fair drop. A number of students are surprised at their failure to solve the equation.

> I don't know what I did. I know ln, I suppose I forgot the rules. (Charles on viewing his paper)

Students claim that although they no longer solve logarithmic equations such as in this particular problem, they often use logs to solve equations, mainly making use of a calculator to compute whatever they need to. Most students seem confident of their knowledge.

> In general logs and ln are not a problem. (Conrad)

A relatively high percentage of students (18.6%) could not solve this equation in either of the two years indicating that they either never mastered basic logarithmic laws in the first place or fail to connect ln with $\log_{10}$ and $\log_2$, not an uncommon occurrence.

Conclusion: It is disappointing that the percentage of students that could solve a logarithmic equation dropped by about a third, considering that logarithmic laws are first encountered in high school. This is an important basic skill. Logarithmic laws should then be seen as knowledge that was not well-founded, neither in school nor at university level. This is a somewhat disturbing situation and needs to be addressed.

Question 3

Sketch the graph of $f(x) = |2x + 1|$.

Means: 1.4 for $\text{TMT}_1 01$ and 0.9 for $\text{TMT}_1 03$.

Objective: To test sketching of graphs of basic functions by performing vertical and horizontal shifts and stretching.

Observations: Students generally feel that they no longer have to sketch this kind of graph, but that they do make use of absolute values when solving differential equations, for example.

The number of students in 2001 who could sketch the graph correctly (65.1%) decreased to almost half of that (37%) in 2003. Furthermore, more than half of the students (51.2%) could not

		2003 $\rightarrow$			
	Marks	0	1	2	Total
2001	0	20.9	0.0	4.7	25.6
$\downarrow$	1	0.0	2.3	7.0	9.3
	2	30.2	9.3	25.6	65.1
	Total	51.2	11.6	37.2	

Table 6.6: Student performance distribution (percentage-wise) for TMT_1 Question 3

sketch this graph at all in 2003. On investigation it appears that most students tend to stick to one method, normally the one that was used at school where they had to memorise the properties of such graphs. When their memory fail them, they do not try different approaches such as reflecting the negative part of $y = 2x + 1$ about the x-axis or even calculating and plotting values. In the first year Calculus course one specific method is rarely enforced, it is expected of students to use their initiative and resources to solve problems. This does not appear to be very successful.

On the positive side David, one of the few students who did show some initiative, explains that he knew the general form and then computed a few values to confirm his results. When asked why he had the correct $x-$intercept in contrast to most other students, he replies

> I always check the x- and y-intercepts for all graphs.

Thomas, a student with an incorrect graph, shows insight when analysing his mistakes. He is an example of a student whose knowledge is slightly rusty.

> I possibly confused it [the question] with linear systems, but I realise my mistake now. I moved the 1 (one) outside the thing [absolute value]. We do not actually work with things like these [absolute value graphs] in other subjects. I know the concept though ... graphically you move everything that is negative to positive.

William is an example of someone who elegantly used the definition of absolute value to split the function into its two branches, a model student. He used exactly the same method in 2001.

Conclusion: As a result of the poor performance in this question in 2003 one is tempted to conclude that sketching functions of the form $f(x) = a|bx + c| + d$ should not be considered as a must know. However, one of the main objectives of first year Calculus course is for students to be able to sketch different graphs, whether by manipulation of well-known functions or by using

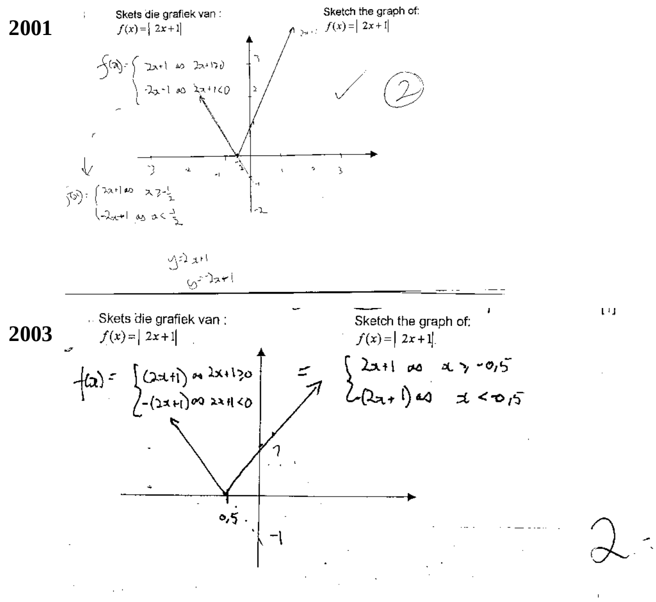

Figure 6.3: The graph of $f(x) = |2x + 1|$ as done by William

tables. Students can usually sketch $f(x) = |\sin x|$ by just reflecting the negative part of $y = \sin x$ about the $x-$axis, but they fail to use the same technique for this graph. In general students find it difficult to see connections between different sections of the work and often do not think wider than one specific method. Such skills should be cultivated more strongly in the first year.

Question 4

Sketch the graph of $h(x) = \ln(-x)$. Also give the domain of h.

		2003 $\rightarrow$			
	Marks	0	1	2	Total
2001	0	9.3	2.3	2.3	14.0
$\downarrow$	1	2.3	7.0	2.3	11.6
	2	18.6	25.6	30.2	74.4
	Total	30.2	34.9	34.9	

Table 6.7: Student performance distribution (percentage-wise) for TMT_1 Question 4

Means: 1.6 for $\text{TMT}_1 01$ and 1.1 for $\text{TMT}_1 03$.

Objective: To test horizontal reflection of a graph of a basic function such as $f(x) = \ln x$.

Observations: In 2001 74.4% of students scored full marks for this question, whereas less than half of that, only 34.9%, scored full marks in 2003, a considerable drop in performance. Students lost one mark if they had either omitted to give the domain, which was explicitly asked in this case, or did not indicate the $x-$intercept even if the shape of the graph was correct. This could explain the drop in performance.

A typical error made by students is given in the quote below

> $\ln(-x)$ is not defined since $\ln$ is defined for positive values of x only. (Sean)

An example of how well-founded knowledge can be retained is given in Figure 6.4

2001

2003

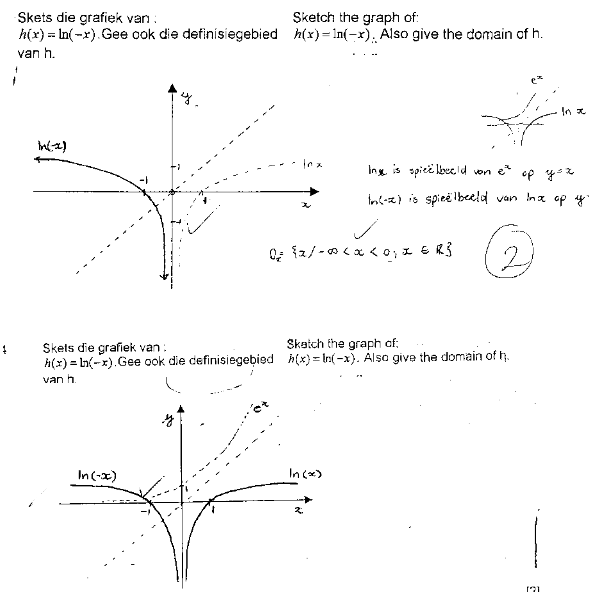

Figure 6.4: The graph of $f(x) = \ln(-x)$ drawn by one of the students in the two consecutive tests.

Conclusion: In 2003 a number of students did not answer the second part of the question and this may be one of the reasons for the drop in performance. Even though students did not score particularly well in this question, a large group (almost two thirds) of students did get the shape of the graph right indicating that they can still sketch logarithmic graphs. Properties of symmetry, as in this case a reflection about the y-axis, are important and useful for sketching graphs. There does not seem to be reason for concern here.

Question 5

Determine the following limit (if it exists): $\lim_{x\to\infty} \frac{x^2+x+1}{x^3+2x^2+1}$.

		2003 $\rightarrow$			
	Marks	0	1	2	Total
2001	0	0.0	2.3	32.6	34.9
$\downarrow$	1	0.0	0.0	11.6	11.6
	2	2.3	4.7	46.5	53.5
	Total	2.3	7.0	90.7	

Table 6.8: Student performance distribution (percentage-wise) for TMT_1 Question 5

Means: 1.2 for $TMT_1$01 and 1.9 for $TMT_1$03.

Objective: To test asymptotic behaviour of functions.

Observations: Quite surprisingly 90.7% of the students scored full marks for this question in 2003 compared to 53.5% in 2001. The mean for $TMT_1$03 is also considerably higher than that of $TMT_1$01. Furthermore 32.6% had this question completely wrong in 2001 but scored full marks in 2003. These figures indicate that limits at infinity should definitely be considered as must knows for this group of students. Quite a number of students mention the use of limits in their other subjects.

Impressive also is the fact that students use three different methods to find the limit. Many students use the method of eliminating the highest factor in the numerator and denominator which was initially taught to use in the case of the indeterminate form $\left(\frac{\infty}{\infty}\right)$ and quite a few use L'Hospital's rule, taught during the second semester of first year Calculus. A third method was used by Charles only, who explains his answer by simply stating that $x^3 >> x^2$ when $x \rightarrow \infty$, showing a fair amount of insight.

> It comes from an engineering way of thinking. (Charles)

Students confirm their continued exposure to this type of limit and application of it.

> We use it all the time like for instance in the processing of signals. I can remember it well since we did a lot of it in sequences and series [in second year Calculus]. (Sean)

We often simulate a product with a function and then you want to establish the life span of such a product. (Oliver)

Conclusion: The results of this question are significant. It is the only question of this test in which students performed better after two years. The increase in performance was not necessarily due to the technique mastering programme in the first year but to the fact that students were repeatedly exposed to this knowledge in their field of study. It was important to lay a firm foundation in the first year but due to repeated exposure over a longer period of time knowledge was not only retained but also improved upon.

6.4.2 Comparison of $TMT_2$01 and $TMT_2$03

A question by question comparison is given of the second technique mastering tests written in 2001 and 2003.

Question 1

Differentiate the function: $f(x) = \sin x(\sin x + \cos x)$

		2003 →			
	Marks	0	1	2	Total
2001	0	2.3	0.0	2.3	4.7
↓	1	2.3	7.0	32.6	41.9
	2	0.0	4.7	48.8	53.5
	Total	4.7	11.6	83.7	

Table 6.9: Student performance distribution (percentage-wise) for TMT_2 Question 1

Means: 1.5 for $TMT_2$01 and 1.8 for $TMT_2$03.

Objective: To test the product rule for differentiation.

Observations: A remarkable 83.7% of students scored full marks for this question in 2003 compared to only 53.5% in 2001, a large improvement. The mean for $TMT_2$03 was also higher than that of $TMT_2$01. These figures are pleasing but not unexpected. Students who did not score full marks in 2001 were mostly confused with the signs of the derivatives of $\sin x$ and $\cos x$, which could indicate some confusion between differentiation and anti-differentiation.

In general, students feel that differentiation techniques are definitely a must know in almost all of their subjects, although they can find derivatives and rules in tables and do not necessarily have to memorise these. Top students such as Oliver and Brian commented that they memorise rules anyway (even if formulae and rules are supplied in tests in the engineering courses), since one needs to know where and when to use each rule and formula and it saves time.

Conclusion: The results of this question are significant, especially because although students claim that they may use tables to look up rules and formulae most students are so familiar with the product rule for differentiation that they know it by heart. Results show again that the continuous use of rules enforces the basic must knows of the first year Calculus.

Question 2

Differentiate the function: $f(x) = \ln\left(\frac{x-1}{x^2+1}\right)^3$.

		2003 →			
	Marks	0	1	2	Total
2001	0	2.3	2.3	0.0	4.7
↓	1	0.0	7.0	14.0	20.9
	2	20.9	9.3	44.2	74.4
	Total	23.3	18.6	58.1	

Table 6.10: Student performance distribution (percentage-wise) for TMT_2 Question 2

Means: 1.7 for $TMT_2$01 and 1.4 for $TMT_2$03.

Objectives: To test the properties of logarithmic functions and use of the chain rule for differentiation.

Observations: In this question 58.1% of the students scored full marks in 2003 compared to 74.4% in 2001, a fair drop. In 2003 only a small percentage of students used the properties of logarithmic functions for simplification before differentiation. Most of the students focused on $\frac{x-1}{x^2+1}$ instead, recognising it as a quotient and wanting to go the quotient rule route. Students feel that since they do not use the quotient rule as often as the product rule, they cannot recall the rule and had no references available. Some of those that tried to recall the quotient rule, confused the order of the terms in the numerator, and as a result made a sign error.

Conclusion: In this question the lack of exploring different approaches is disturbing. While most of the students had no problem in differentiating the ln −function itself, they could not differentiate $y = \frac{x-1}{x^2+1}$. Students could not remember the quotient rule and did not think of simplifying by using logarithmic properties or by writing $y = \frac{x-1}{x^2+1} = (x-1)(x^2+1)^{-1}$ and then applying the product rule and the chain rule. More efforts should be spent on fostering innovative and wider thinking amongst students.

<u>Question 3</u>

y is implicitly defined as a function of x. Determine $\dfrac{dy}{dx}$ in terms of x and y if $\sqrt{xy} + 1 = y$.

		2003 →			
	Marks	0	1	2	Total
2001	0	9.3	2.3	4.7	16.3
↓	1	11.6	4.7	0.0	16.3
	2	27.9	11.6	27.9	67.4
	Total	48.8	18.6	32.6	

Table 6.11: Student performance distribution (percentage-wise) for TMT_2 Question 3

Means: 1.5 for $TMT_2$01 and 0.9 for $TMT_2$03.

Objective: To test the technique of implicit differentiation.

Observations: In 2001 only about two-thirds of students (67.4%) scored full marks for this question and this figure more than halved to 32.6% in 2003, a poor overall performance. Students feel that they rarely encounter functions that are implicitly defined in their study field.

> ... we do not use it [implicit differentiation] (Charles)

During the interviews it became evident that students cannot recall what to do because they have not used it for so long. They also seem to rely too heavily on tables of rules for differentiation techniques.

> ... [in engineering courses] we became lazy, since we just look up the formulae (Anja and Dirk)
>
> ... often it [looking up a formula] is the first step in a very long solution. (Anja)

Conclusion: Perhaps this is one of the techniques that is not essential for students in computer engineering and could only be considered as a must know in terms of their mathematical foundation in differentiation. Whether students would have been able to find $f'(x)$ if y were to be replaced by $f(x)$ is debatable. The low full score percentage in 2001 points to the fact that more time should be spent on the mastering of this particular technique.

Question 4

Find the most general form of a function f satisfying the condition $f'(x) = x(1+x^3)$.

		2003 $\rightarrow$			
	Marks	0	1	2	Total
2001	0	2.3	4.7	2.3	9.3
$\downarrow$	1	9.3	2.3	11.6	23.3
	2	4.7	9.3	53.5	67.4
	Total	16.3	16.3	67.4	

Table 6.12: Student performance distribution (percentage-wise) for TMT_2 Question 4

Means: 1.6 for $TMT_2$01 and 1.5 for $TMT_2$03.

Objective: To test basic anti-derivatives.

Observations: In first year Calculus, questions like this one are used to introduce integration. It is interesting that the same percentage, namely 67.4%, scored full marks in both 2001 and in 2003. The means are also nearly the same in the two tests. The 23.3% for 2001 and 16.3% for 2003 of students who scored 1 out of 2 , lost one mark because they neglected to mention the integration constant. Students who did not get any credit, attempt to integrate without simplifying first. The interviewed students feel that integration as such is definitely a must know.

Sean claims, justly or not, that in their engineering subjects they do not have to mention the integration constant

> ... we just keep in mind that there is a constant involved but do not compute it.

Conclusion: Although this question is fairly simple, it is still necessary to simplify before integrating. The fact that such a large percentage of students could integrate this basic function shows that they knew at least that the chain rule ($\int [f(x)]^n f'(x)dx$) does not apply in this case. One can conclude that integration is a valuable skill and is definitely a must know.

Question 5

Find the solution of the differential equation $\frac{d^2y}{dx^2} = e^x$ with the initial conditions $\frac{dy}{dx}(1) = e$ and $y(1) = -4e$

		2003 →			
	Marks	0	1	2	Total
2001	0	16.3	0.0	0.0	16.3
↓	1	0.0	0.0	0.0	0.0
	2	34.9	11.6	37.2	83.7
	Total	51.2	11.6	37.2	

Table 6.13: Student performance distribution (percentage-wise) for TMT_2 Question 5

Means: 1.7 for $TMT_2$01 and 0.9 for $TMT_2$03.

Objective: To test solving of basic differential equations with initial conditions.

Observations: The low performance of students in 2003 is reflected by the means and the fact that more than half of the students (51.2%) could not do this question at all in 2003. The 83.7% of students who scored full marks in 2001 dropped to 37.2% in 2003. This may be explained by the fact that these students had since done a course in Differential Equations in which far more sophisticated differential equations were dealt with. They may have found this problem somewhat confusing. This notion is supported by the fact that during the interviews, when students had a second look at the question most of them could explain how to do it. As in Question 4 they often omitted the integration constant, in which case they could not solve the equation fully.

Conclusion: This is clearly not the type of problem that this group of students encounters regularly. Yet the underlying knowledge seems to be intact. Students found the question somewhat unfamiliar and for this reason failed to answer it well. It is doubtful whether there is reason for concern.

6.5 Qualitative investigation

We discuss students' perceptions of the TM programme by addressing the structured questions in the interviews.

6.5.1 Responses to structured questions

After initial discussion to set the scene and to make sure the student recalls the technique mastering programme, the first of a number of questions was addressed. What follows is a report on their responses to three prominent questions.

Did the fact that you had to score 80% for such a test without it contributing towards your semester mark, influence your performance in such a test?

Most of the students cannot recall that the TMTs did not contribute to their final mark. Students remark that in an engineering programme the workload is high and therefore they spent little if any time in preparing for such tests. They do what is required of them in order to pass and it is not considered as important as some assignments and tests in mathematics or other subjects.

Martin says that he did not study as hard for a TMT as for tests that would contribute to the final mark, but he did prepare for the TMTs and commented that

> ... the TMTs were not difficult since one did not need any 'tricks' to be able to do it.

Sean mentions that since the course allowed for frequent evaluation in the form of class tests that did contribute towards the final mark, he did not mind that the TMTs did not contribute towards his semester mark.

Eugene remembers that he was nervous about the 80% pass mark at first, but later relaxed and saw it as a way to evaluate his level of knowledge.

> I did it only because I had too ... I did not prepare in advance, ... you already knew the work . . . it is reassuring to know that you can do the basic stuff.

Anja feels that it was a valuable exercise even if it did not contribute towards the final mark.

> You rewrite a TMT until you get your 80%, it is the only way to master it [skills].

Conrad feels that even if the TMTs did not contribute towards the final mark, all assessment is valuable.

Summary: Students generally feel positive towards the technique mastering programme and do not feel that the requirement of 80% but not contributing towards their semester mark influenced them negatively.

Did you find the TMTs meaningful?

Most of the students claim that they wrote the tests simply because it was required of them and at the time did not give any thought to the usefulness of the particular skills involved. It was just another task, something they had to do. Later on, in other mathematics modules, they realised that the TMTs reflect essential knowledge.

Although some students did not specifically prepare for the TMTs they feel that the formative assessment helped them to determine whether they had mastered certain mathematical skills. Oliver explains:.

> ... by the time we wrote the tests we had already covered that part of the content ... I used it [results of TMTs] to see whether I've at least mastered the basic knowledge. If not, I knew I had to work harder.

Lynn speculates on the necessity of TMTs and replies:

> ... perhaps they are necessary, then you know you know the basic stuff ... you don't think it is necessary in first year but later on you realise you need these [basic skills]. You have to know the basic rules and skills in order to succeed.

Anja comments that many engineering students are a little sceptical about whether the TM programme is necessary at all, since engineering students use tables to look up the formulae they need, but she personally thinks it is necessary to know the basics.

> ... you have to do it [TMTs] in first year, otherwise you won't understand some things later ... even if we look up formulae in engineering, it is still important to understand.

Charles also feels unsure about the effect of the TM programme because of the low frequency of tests and the small range of problems. He would have liked more basic practice exercises in order to construct his own knowledge.

> At that stage it helped to see whether you knew the work, but I needed more exercises ... it was such a small part of the work that I am not sure that it made any impact at all.

Conrad explains that the TM programme fitted into his idea of doing mathematics:

> Yes, this is how I learn, by practising ... it is like a certain level you have to obtain ... at a certain stage you have to know enough, and understand enough to continue ... when you pass [TMT], you are on a higher level.

Dirk comments that his friends who had to work harder to obtain the minimum requirements for a TMT, benefited in the end. He continues to say that students that had not mastered the basic principles earlier on, now encounter problems in different subject areas.

> In the third year, students with gaps in their knowledge start to fall out ... TMTs help you to see whether you have conceptual problems ... if you get the basics right, the rest will be OK.

Only four students, Dirk, Oliver, Brian and William (11.3% of the sample group) passed all four tests (both $TMT_1$01 and $TMT_1$03 as well as $TMT_2$01 and $TMT_2$03) at their first attempt. Dirk is a top student, Brian and Oliver each obtain distinctions in about half of their subjects and William has increased his average from 60% in the first year to more than 70% in the third year. According to these students it is more convenient to know some of the basic rules and definitions, than to look it up in the given tables. These students also referred to the necessity of understanding basic concepts and judging from their responses a programme such as the technique mastering programme is definitely meaningful.

Summary: Students generally feel that the technique mastering programme contributed to their basic skills and find the programme meaningful. One should conclude that students benefited from obtaining a set of must knows that they are able to use in all areas.

Do you have any recommendations regarding the technique mastering programme?

Dirk comments that although he has no problem with the current system he suggests that

> TMTs should contribute towards the final mark in order to force students to work harder.
>
> Introduce a similar programme in second and third year mathematics as it forces students to master every concept. Increase the frequency to weekly tests.

Dirk also comments that he likes the immediate response of the online tests but that he thinks the tests should not consist of multiple choice questions only.

Charles explains how he needed more exercises to practise the basic rules and techniques, and felt it would help students if more problems were included in the programme.

Pete would like to have a summarised version of must knows available to use in other subjects, even beyond first year.

Oliver suggests that the variables are varied i.e. in integration and differentiation not to stick as much to x as a variable but regularly use other variables such as t as well.

Conclusion: Few students have recommendations regarding the TM programme. Those that offer comments would like more practice exercises and a summary of the must knows.

6.5.2 Case studies

We include four case studies of students representing different profiles of students.

Oliver (the high performing student)

Most first year engineering students start university directly after completing their twelve years of school. Oliver came to the university after completing an extra year of study at a technical institution to help him prepare for his study at our university. The curriculum he had followed there included a fair amount of the content of the first semester engineering mathematics. He is a hard working student with a positive attitude and good results (final marks for the three first year mathematics modules are 81%, 78% and 71%). Oliver was able to score more than 80% in 2003 for both TMTs. In TMT_1 he was one of few students that included the restrictions for the functions $\frac{f}{g}$ and $f \circ g$.

Oliver's case is an example of how superficial knowledge cannot be retained over a period of time. Although Oliver managed to sketch the graph of the absolute value function $f(x) = |2x + 1|$ in the first year test, he failed to do so in the follow-up 2003 test. He claims not to use it regularly any more and says:

> I understood it while in the first year but we did not use it immediately after that. I am not very good at memorising; things get rusty after a while. If I read about it in textbooks, I remember vaguely something about multiplying with a minus on the left and right, and the signs interchange, but I get confused with what it really is. It is definitely more difficult than factorising a quadratic or algebraic expression.

It is clear that he never understood the basic concept clearly and is used to following a recipe that he can only vaguely recollect now. A further comment substantiates this.

> I realise that $|x|$ means that the inside becomes positive, but I don't know how to solve for the x inside, since you normally get two answers. I can't remember clearly, do you have to multiply by a minus inside and outside? Especially with a complicated function, I cannot think in reverse what will x be if the inside should stay positive.

He further claims that his inability stems from a lack of understanding in secondary school and a confusion that developed because of two different methods followed in secondary school and at university. He also points to the importance of the initial encounter with a concept.

> The first time we encountered absolute values was in high school and there we just memorised it. When we learned the other method in the first year (i.e using the definition of absolute values to write a function as a branch function) I understood it. I realised it is a better way to deal with absolute values, but once the technique faded, I stepped back to the way we did it in high school. I guess first impressions last.

On a question whether a misconception can reappear after a period of time and whether this is because of faulty first impressions even though being convinced otherwise along the way, Oliver replies:

> Well, it all depends on how hard I work to get rid of a misconception. If I understand it better I keep on doing it right, but some of the concepts get mixed up again if I don't use them regularly. After a while I can't remember which of the two methods is correct.

In the question on logarithms Oliver performed well on both occasions and seems confident of his knowledge. He compares his ingrained knowledge of logarithms with his somewhat shaky knowledge of the absolute value concept.

> I guess I understood ln from the start. There was never anything about ln that bothered me. I think that with absolute values it was different. The first time in high school I did not understand it well. When I do something right from the start, I never have any problems further on.

This comment again stresses how important it is to grasp a concept firmly at the onset to obtain a firm foundation and how difficult it is to rid yourself of misconceptions. It appears that the technique mastering programme was not successful in this latter aspect for Oliver.

In TMT_2 it was quite clear that Oliver had mastered the different differentiation rules well. When asked why his knowledge on differentiation is still so firmly in place, he once again refers to his secondary schooling but also points to the value of repetition and how concept understanding develops over a period of time.

> Since high school days, we continually use differentiation. Even difficult differentiation problems do not bother me at all. At the moment we particularly use differentiation and integration a lot. Integration is not as easy as differentiation for me. Sometimes I look at an integration problem and feel unsure of how to tackle it. I think that with integration I also did not understand it immediately. It takes a while for me to master something, but once I understand a concept I try to solve a problem by reasoning. I like to know my new environment before I start. I often read ahead so that the new concepts are not totally new to me. If the lecturer then explains the new concepts in class and I can ask questions when I do not understand, I am able to master the new work.

In summary, Oliver's interview highlights:

- The importance of understanding a concept properly the first time round.
- The value of repetition in ingraining knowledge and concepts.
- The improvement of concept understanding by exposure to it over an extended period.

Anja (the hard-working, average student)

Anja is an average student that had difficulty in coping with first year calculus and the heavy workload of the computer engineering course. She feels at a disadvantage with her fellow students that took Additional Mathematics as an extra subject at secondary school level. She managed to pass the first semester, first year calculus, but during the second semester struggled with integration. She spent many extra hours studying calculus and as a result, neglected the parallel linear algebra course and failed it. She repeated the linear algebra course online and passed it the next semester. Since then she has not failed any subject and will probably complete the engineering degree in the scheduled four years.

Anja has a good self-image, and in general has no problem seeking help from her lecturers and fellow students when she does not understand a concept. However, Anja has a mental block regarding her first year mathematics. Anja's case is an example of how students who are not confident about their knowledge, fail to be creative and do not use their own initiative to solve problems. When we discuss her answer sheets for the TMTs, it becomes clear that she feels uncomfortable to be suddenly confronted with first year calculus problems.

> When we had to do these tests, it was impossible to remember all the small detail. None of us could do it ... many of the students just sat and stared at $f \circ g$ not recalling the definition.

She feels that the only content one remembers is the part that you regularly use.

> We don't use it [compositions of functions] at all. I last saw it in first year. It is only because we don't use it often now that I could not do it.

Anja is one of the 9.3% (see Table 6.5) of the students that could not solve the logarithmic equation (Question 2 TMT_1) in 2001 and made similar mistakes in 2003. She compares the two tests (during the interview), laughs and remarks

> ... it is obvious that I could not do this ... I have not practised it for a while. Although I battled with ln, in 2001 I could do it eventually [for the exam], but now [in 2003] I forgot the rules ... if you don't use them often, you lose it along the way.

She cannot explain what a logarithm is, and although she claims that she remembers the principles, she clearly cannot apply the principles to natural logarithms.

> I can use logs in problems. ln was a totally new concept for me in first year. I remember the principles and can use the calculator to compute logs, but we only use $\log_{10}$ and $\log_2$ in Signal Processes. We use logs to calculate decibels ... We use it often now to change regular functions to decibels to draw certain graphs ... I'm just rusty at this stage, haven't done it for a while.

Anja needs time and practice. She could do the limit problem and credits second year Calculus for that.

> I have practised these over and over ... did twenty problems of a kind in my second year, I will be able to do it in my sleep. I can't say we use these limits a lot, but sometimes it is part of the solution of a problem.

Concerning differentiation, Anja says it is not necessary to memorise the rules.

> They [the engineering department] made us lazy since we use the tables in our textbook to look up the formulae. It is much quicker. Sometimes it [differentiation] is the first step in a long solution. Nobody works it out, we just use the formulae. There are examples of all the rules in the appendices of our textbooks, so we just look it up. I don't say it is not necessary to learn it in first year. It is important to learn it otherwise you won't understand where it fits in. Anyway, for all follow-up math modules you have to know it.

Anja's answer to Question 4 in TMT_2 on implicit differentiation shows that she lacks understanding. She did not score any marks on either occasion. Looking at her answer, she remarks.

> Again I could not do it. Normally I try to remember how I did it previously.

In summary, Anja's interview highlights:

- The importance of having a core knowledge like basic rules, techniques and skills that cultivates confidence.
- The value of repetition in ingraining knowledge and concepts.

William (the improved student)

William describes himself as always having been an average student, even at school level. He explains that at school he had no motivation to study, since he was uncertain of the future. According to him the school he attended did not expose students to the outside world locally or internationally. He is an introvert and keeps to himself. He struggled with first year Calculus basically because he was not able to manage his time properly. Eventually he failed the first semester, first year Calculus, repeated the course online and passed it with distinction.

He says that the engineering department regularly exposes them to the outside world. This exposure has broadened his horizons, although he still is uncertain of the future and what lies ahead.

William's case is an example of how stimulation and motivation can change a person's future. William's present academic record is a testimonial of success. Part of William's success is his perseverance. He wants to know more and reads about the subject. He claims that (after he failed first semester mathematics) he started to prepare his mathematics in advance, since he discovered that he is then better able to understand the new concepts.

> I force myself to prepare ahead. I work every day. Sometimes I just read through the work. Even if I do not understand what I read then, it helps if I have seen it previously and it is then explained in class. I did not do it in my first year.

Upon examining William's tests, it becomes clear that he has a deep understanding of the concepts. William considers it as very important to understand the concepts well.

> I understand the concepts now, I just have a gut feeling for it ... If you want to continue with math, you need to understand the basic principles, otherwise you will not be able to understand new concepts [built on the basic ones]. You won't be able to do higher level math unless you understand the basics.

In summary, William's interview highlights:

- The importance of intrinsic motivation.
- The value of working regularly.
- The value of thoroughly understanding basic principles and concepts.

Sean (the disadvantaged student)

Sean is a five-year-plan student (he follows the option of spreading the four year course over five years), and has an eye-sight disability. He was one of two students in class that could not read well on the blackboard or when slides were used in class. To help him (and the other student) cope, the lecturer supplied them with written notes (or copies of the slides) before the lecture started. They could then follow the discussion in class with the help of a friend that supplemented their notes with class examples. Sean struggled to keep up with the pace despite the fact that he had fewer subjects to complete in his first year. He started off with low marks for his first TMT, had to rewrite it a few times, but managed to pass it eventually. In spite of his difficulties, he managed to pass his first year mathematics on schedule.

Sean learned to cope through working regularly. He never missed any class or opportunity to learn, and always asked questions whenever he was uncertain of any concept. Sean's case is again an example of how repeated exposure can help to reinforce concepts and skills.

> I did not prepare for the TMTs since by the time we wrote the tests, we had done the work in class, but once I get my script back, I made sure that I mastered whatever I could not do in the test.

Due to Sean's positive attitude, he rarely complains about the way tests are graded and even if he fails a test, also uses it as a learning opportunity.

> The tests were just a way to force one to go through the work, which is a necessary exercise to prepare for semester tests and exams.

In his TMTs of 2003, he did not sketch the graph of $f(x) = \ln(-x)$, but was one of the few that could recall the graph of $y = \ln x$ and its domain.

The two other questions that he could not do, were on implicit differentiation and differential equations. It became clear (during the interview) that he had not mastered these two skills.

> I can't recall implicit differentiation. Some of these we last saw in the first year and in the differential equations course. I saw it only once in the differential equations course.

In summary, Sean's interview highlights:

- The importance of perseverance.
- The value of a positive attitude.
- The value of regular practice.

6.6 Conclusions and Recommendations

A number of factors that pertain to the technique mastering programme emerge from this study.

The first factor is the notion that a large percentage of students cannot integrate the knowledge of the different areas of one single course, let alone integrate that knowledge into other areas. Students tend to categorise subjects and easily conclude '*we don't use it often*' or '*we don't use it at all',* without recognising the common core knowledge. Some students think of the TM programme as some external exercise belonging to the first year Calculus programme ('*something we had to do'*) that had little to do with follow up courses. This problem is by no means unique as was discussed in the review of long-term retention of knowledge. The University of Massachusett's IMPULSE progamme for addressing this problem serves as an example. Whilst implementing a separate programme is not necessarily feasible, we should familiarise ourselves with the course material in other subjects and perhaps view examination papers and one should include examples related to other fields. Notation should ideally be standardised and where not possible students should be made aware of corresponding notations.

The study also casts light on what part of what we consider as must knows is encountered regularly in later studies. For example, the fact that knowledge of limits is so vital for this group of students came as an eye-opener. We should revise the content of the technique mastering programme regularly by taking cognisance of what happens in later years *and in other subjects*.

From interviews it transpired how important the motivational factor is. Often motivation for studying a subject stems from its perceived usefulness. It is a task of the lecturer to put the TM programme in context and to explain and demonstrate its usefulness beyond the first year.

Although at this stage great care is taken at the University of Pretoria to supply the mathematics content required in other subjects, we should also take note of the different strategies used in other departments concerning the use of the same content. If students are allowed to look up formulae and rules from tables in other subjects, it decreases the necessity to memorise formulae that are not frequently used. We are by no means advocating that memorisation should be abandoned but this issue has to be discussed by the staff involved to review the policy.

What is the long-term effect of the technique mastering programme? It is difficult to determine quantitatively how much of the basic knowledge and rote skills imparted in the first year is still retained after two years and we did not attempt to do so, the approach was more of a qualitative nature. It is perhaps wishful thinking that the knowledge imparted in the TMTs will be retained indefinitely. It can be expected that students will have forgotten some of this knowledge and skills. Yet, there is no doubt that the performance in the follow-up technique mastering tests of 2003 is disappointing in general. This is similar to the conclusion of Anderson et al (1998) that 'a considerable amount of what is taught to mathematics students in general as 'core material' in the first year is poorly understood or badly remembered.' It also supports their idea that students tend to " 'memory-dump' what they have had in previous modules, rather than retain it and build it into a coherent knowledge structure." In cases where students had exposure in later years to the knowledge gained in the first year, this knowledge certainly seems to be firmer in place. In general, the must knows and techniques are not retained to a sufficient extent over a period of time.

Although the concept of a technique mastering programme is viewed favourably and to be meaningful by students, this study shows that the impact of the programme is not strong enough. There certainly is room for improvement. A suggestion is that an online, interactive mastery programme should be initiated that provides students with the opportunity to work as often and as long as they want, even where they want to in order to master the basic skills and knowledge.

Such a programme can also provide in the need of students with a poor grounding in secondary school mathematics who need more exposure than students who, for example, did Additional Mathematics at secondary school. This is also motivation for keeping the TM programme separate from the lectures unlike the situation at Movarian College (Schattschneider, 2004) where such a programme was incorporated into the lectures. It has to be emphasised that the homogeniety of their students warranted such an inclusion.

The purpose of the TM programme should be explained more clearly at the onset. One could consider making the TM programme a prerequisite for follow-up courses such as the gateway tests used at some American universities. Such a prerequisite would ensure that students are forced to revise the content of a TM programme and have the benefit of core knowledge ready and available to start a new semester. If a similar programme can be introduced in all consecutive courses, students will encounter certain basic skills and knowledge repeatedly.

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Appendix A

TECHNIQUE MASTERING QUESTIONS: WTW 114

FUNCTIONS

1. Determine the largest possible domain of f and in each case give the value of f at the given point a:
   - **a.** $f(x) = \frac{1}{\sqrt{x-1}}$, $a = 5$
   - **b.** $f(x) = \frac{3}{|x-4|}$, $a = 1$
   - **c.** $f(x) = x^4 + x^3 + 1$, $a = 3$
   - **d.** $f(x) = \frac{1}{x^2+x+1}$, $a = 0$
   - **e.** $f(x) = \frac{1}{\sin x}$, $a = 5\pi/2$
   - **f.** $f(x) = e^{\frac{1}{x}}$, $a = \ln 3$
   - **g.** $f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1}$, $a = 2$
   - **h.** $f(x) = \tan x$, $a = -\pi/4$
   - **i.** $f(x) = \frac{|x|}{x}$, $a = -1$
   - **j.** $f(x) = \frac{x^2+2x+1}{x^2-1}$
   - **k.** $f(x) = \frac{\ln x}{x-1}$
   - **l.** $f(x) = \ln(1 + e^x)$
2. For the given functions f and g, determine $f + g$, fg, $\frac{f}{g}$ and $f \circ g$. You may assume that in each case the domain of f and g consists of all numbers x for which $f(x)$ and $g(x)$ make sense:
   - **a.** $f(x) = 2x + 5$, $g(x) = x^2$
   - **b.** $f(x) = x^2$, $g(x) = \frac{1}{x-1}$
   - **c.** $f(x) = e^x$, $g(x) = \ln x$
   - **d.** $f(x) = \cos x$, $g(x) = \frac{1}{x}$
   - **e.** $f(x) = \sqrt{x}$, $g(x) = x - 1$
   - **f.** $f(x) = \sqrt{x}$, $g(x) = 2^x$

ALGEBRAIC OPERATIONS WITH EXPONENTIALS, LOGARITHMS ETC

1. Express the following as a power of 2:
   - **a.** $\frac{2}{\sqrt{2}}$
   - **b.** $\sqrt[4]{8}$
   - **c.** 2.4^{2x}
   - **d.** $2^{x+y} \div 2^{x-y}$
   - **e.** $(2^x)^x$
   - **f.** $\frac{2^5 \cdot 2^{1/5} \cdot 2^{3/5}}{2^{-1/5}}$
2. Simplify:
   - **a.** $3a^4b^3 \cdot 2a^3b^{-2}$
   - **b.** $\frac{12x^{2n+3}}{4x^{n-1}}$
   - **c.** $(3x^2y^{-3}z^0)^{-4}$
   - **d.** $(3^{x+1})^{x+1}$
   - **e.** $(x^{1/y})^{1-y}$
   - **f.** $\sqrt{9x^2}$
   - **g.** $(4a^{-2}b^6)^{-\frac{1}{2}}$
   - **h.** $\left(\frac{\sqrt{x}}{\sqrt{x^{-5}}}\right)^{-\frac{1}{2}}$
   - **i.** $\frac{7^{-1}x^{1/4}}{3^{-2}y^{-1}} \div \frac{2(3^{-1/2})^2}{(49y)^0}$
   - **j.** $\frac{(2^{n-1})^{n-1}}{(2^{n+1})^{n+1}}$

- **k.** $\left(\frac{16a^3}{81a^{-1}}\right)^{-\frac{3}{4}}$
- **l.** $\frac{x^{8n-3}(xy^{1/2})^{-6n}x^3}{(xy)^{-4n}(x^{-1})^n}$
- **m.** $\frac{12^n \times 8^{n-1} \times 3^{n+1}}{24 \times 6^{n-2} \times 2^n}$
- **n.** $\sqrt[3]{a} \cdot \sqrt{a}$
- **o.** $\frac{(3^n)^{1-n}}{6^{-1}\ 3^{-n-1}} \div \frac{(3^{1-n})^{n+1}}{2^{-1}\ 9^{-n-1}}$
- **p.** $\frac{9^n - 3^{2n+1}}{(3^n)^2 - 3^{n+2}\ 3^n}$
- **q.** $\frac{5^x - 3^2 5^{x-2}}{5^{x+1} + 3.5^x}$
- **r.** $(a^{1/2} - b^{3/2})^2$
- **s.** $\frac{a^{-1}}{x^{-1} - a^{-1}}$
- **t.** $\left(\frac{(3y)^{-2}}{3y^{-2}}\right)^{\frac{2}{3}}$

3. Express each logarithm as the sum or difference of simpler logarithms:

- **a.** $\log\left(\frac{xy}{z}\right)$
- **b.** $\log\sqrt{\frac{x}{y}}$
- **c.** $\log(x^2y^3)$
- **d.** $\log\left(\frac{1}{x^2y}\right)$
- **e.** $\ln\left(\frac{xy^2}{x+y}\right)$
- **f.** $\ln\sqrt[3]{x^2+1}$
- **g.** $\ln\sqrt[3]{x^2y}$
- **h.** $\ln\sqrt{\frac{x^3}{y}}$
- **i.** $\log\left(\frac{x}{yz}\right)$
- **j.** $\ln\left(\frac{(x+y)^3}{xy^2}\right)$

4. Express each statement as a single logarithm with coefficient 1:

- **a.** $3\log x$
- **b.** $3\log x - \frac{1}{2}\log y$
- **c.** $\frac{1}{2}(\log x - 3\log y)$
- **d.** $2\log x + \log(x+y)$
- **e.** $2\log x - 3\log y + \log z$
- **f.** $\log(y-2) + \log y - 2\log x$
- **g.** $\frac{1}{2}[\log x - 5(\log y - 3\log z)]$
- **h.** $\frac{1}{2}[(\log x - 5\log y) - 3\log z]$
- **i.** $\log 5 + 3\log x - 2\log 4 - \log y$
- **j.** $3\log(xy) - 2\log x - \log y$

5. If $\log_a 2 = x$ and $\log_a 3 = y$ express each of the following in terms of x and y

- **a.** $\log_a 27$
- **b.** $\log_a \frac{2}{9}$
- **c.** $\log_a \sqrt[3]{6}$
- **d.** $\log_a \sqrt{12a}$

6. If $f(x) = 2^x$ determine the equations of g, h and k if

- **a.** g is symmetrical to f about the X-axis
- **b.** h is symmetrical to f about the Y-axis
- **c.** k is symmetrical to f about the line $y = x$

7. If $\log_a b = 3$, find $\log_b a$.

8. Show that if $y = \frac{r}{1+ce^{-at}}$ then $t = \frac{1}{a}\ln\frac{cy}{r-y}$.

9. If $f(x) = log_b x$ find $f(\frac{1}{b})$ and $f(\sqrt{b})$.

10. If $log_b(\frac{1}{10}) = -\frac{1}{2}$, find b.

11. Simplify:

- **a.** $4(e^x)^4 + (x^e)^4$
- **b.** $\frac{1}{2}e^{2x} \cdot 4e^{3/x}$

c. $\frac{14e^{3x}}{7e^{x}}$
d. $\sqrt{e^x} \cdot \sqrt[3]{e^{-3x}}$
e. $\ln(e^{3x})$
f. $e^{\ln 7x}$
g. $\ln(9e^x \cdot 10e^{2x})$
h. $e^{\ln(\frac{x^2}{y})}$
i. $3\ln x + \ln\frac{1}{x}$
j. $\ln e^2 + e^{-\ln x}$

12. Solve for x:(without a calculator)
a. $\ln e^{4x} = 12$
b. $e^{4x} = 12$
c. $\ln 4x = 12$
d. $e^{2\ln 3} = x$
e. $\ln x = 3\ln 2 - 2\ln 3$
f. $\frac{1}{2}\ln x = 1 - \ln 2$

GRAPHS

1. Sketch the graphs of the following functions:
a. $f(x) = 2x + 3$ on $[-4,3]$
b. $f(x) = |2x - 1|$
c. $f(x) = |x| + 1$
d. $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ -x + 2 & \text{if } x > 1 \end{cases}$
e. $f(x) = x^2 - x - 6$
f. $f(x) = \sqrt{9 - x^2}$ on $[-3,3]$
g. $f(x) = 2\sin x + 1$ on $[-2\pi,\pi]$
h. $f(x) = \cos\frac{x}{2}$ on $[0,2\pi]$
i. $f(x) = \tan 2x$ on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

2. Graph each function. Determine the domain of each function:
a. $f(x) = \ln x$
b. $g(x) = \ln(-x)$
c. $h(x) = \ln|x|$
d. $l(x) = |\ln x|$
e. $m(x) = \ln(1 + x)$
f. $m(x) = 1 + \ln x$

3. Graph each function. Determine the range of each function:
a. $f(x) = e^x$
b. $g(x) = e^{-x}$
c. $h(x) = e^{|x|}$
d. $l(x) = |e^x|$
e. $m(x) = e^{1+x}$
f. $n(x) = 1 + e^x$

LIMITS

Determine the following limits whenever they exist.

1. $\lim_{x\to 0}(x + \frac{1}{x})$
2. $\lim_{x\to 1}\frac{x^2-1}{x^2-2x+1}$
3. $\lim_{x\to 3}\frac{x^2-9}{x-3}$
4. $\lim_{x\to 0}\frac{x^2-x}{x}$
5. $\lim_{x\to 2}\frac{\pi}{2}$
6. $\lim_{x\to 0}|x|$
7. $\lim_{x\to e}\ln x^2$

8. $\lim_{x \to \frac{\pi}{2}} \tan x$
9. $\lim_{x \to \frac{\pi}{2}} (1 - \sin x)$
10. $\lim_{x \to 2} (x^3 + 4x + 1)$
11. $\lim_{x \to \infty} (x + \frac{1}{x})$
12. $\lim_{x \to \infty} \frac{x^2 + x + 1}{x^3 + 2x^2 + 1}$
13. $\lim_{x \to 0^+} \ln x$
14. $\lim_{x \to 3^-} \frac{-1}{\sqrt{3 - y}}$
15. $\lim_{x \to 1^+} \frac{x^2 - 3x + 2}{x^2 - 2x + 1}$

DIFFERENTIATION

Differentiate the following functions:

1. $f(x) = 2x^4 - 3x^2 + 5x + 2$
2. $f(x) = x^{-1/2} + x^{1/2}$
3. $f(x) = 3x^2 + \frac{1}{3}x^{-2} + x$
4. $f(x) = x^2 - 3x^{7/3}$
5. $f(x) = x + 1 + \frac{1}{x^2}$
6. $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$
7. $f(x) = \frac{12}{x} - \frac{4}{x^3} + \frac{1}{x^4}$
8. $f(x) = \sqrt[3]{x} + \frac{1}{x^3}$
9. $f(x) = x^4 - 8x^3 + 2x^2 - x + 1$
10. $f(x) = 7x^3 - 5x^2 + 3x - 17$
11. $f(x) = 9x^{-3} + 2x^{-2} - 14$
12. $f(x) = -2x^4 + x^{-2} - 3x^{3/4}$
13. $f(x) = 12x^4 + 3x^3 + 5x^{-2} - 4$
14. $f(x) = 4x^{-2} - 7\sqrt{x} + 8x^3 + 5$
15. $f(x) = 3x^3 + 2x^2 - x + 1$
16. $f(x) = 4x^3 - 7x^2 + 8x - 6$
17. $f(x) = x^3 - 3x - 2x^{-4}$
18. $f(x) = x + 3 + \frac{4}{x}$
19. $f(x) = -3x^{-3} + 4x^2 + \frac{1}{x^2}$
20. $f(x) = x\sqrt{x} + \frac{1}{x^2\sqrt{x}}$
21. $f(x) = (x^3 + 7x^2 + 5)^5$
22. $f(x) = \sqrt{x^4 + x^2 + 2}$
23. $f(x) = (\sqrt{x + 1})(x^2 + 1)$
24. $f(x) = \sqrt{x^2 + 2x + 3}$
25. $f(x) = x^2(x^3 - 1)$
26. $f(x) = (x^3 - 3x)^4$
27. $f(x) = x\sqrt{1 - x^2}$
28. $f(x) = (2x - 4)(3x^2 + 2)$
29. $f(x) = (2x - 3)^3(4x + 2)^2$
30. $f(x) = (7x + 3)^2(3x^2 - 14x + 5)$
31. $f(x) = (8x^3 - 2)(3x^2 - 5x + 10)^2$
32. $f(x) = (2x^3 - 3)^{2/3}$
33. $f(x) = (3x^2 - 2x + 1)^{1/2}$
34. $f(x) = (2x - 3)(3x + 4)$
35. $f(x) = (2x^3 - 1)(x^4 + x)$
36. $f(x) = (\frac{1}{x} + 3)(x^2 - 5)$
37. $f(x) = (2x + 1)^2(x^2 + 2)^3$
38. $f(x) = \sqrt{3x^2 + 1}$
39. $f(x) = (2x - 5)^3$

40. $f(x) = (6x - 5)^{-3}$
41. $f(x) = \frac{x^2+x+1}{x^2+1}$
42. $f(x) = \frac{2x+3}{3x+2}$
43. $f(x) = \frac{2x+1}{3x-5}$
44. $f(x) = \frac{x^2+5x-1}{x^2}$
45. $f(x) = \frac{(x+1)(x+2)}{(x-1)(x-2)}$
46. $f(x) = \sqrt{\frac{x^2+1}{x^2+4}}$
47. $f(x) = \frac{x+x^3}{\sqrt{x}}$
48. $f(x) = \left(\frac{x+1}{x-1}\right)^2$
49. $f(x) = \frac{10}{\sqrt{x}+4}$
50. $f(x) = \frac{8}{4+x^2}$
51. $f(x) = \frac{4x}{x^2+1}$
52. $f(x) = \frac{3}{\sqrt{2x+1}}$
53. $f(x) = \frac{3x^2-2x+3}{4x^2-5}$
54. $f(x) = \frac{(2x-4)(3x+5)}{2x^2+7}$
55. $f(x) = \frac{2x-3}{x^2+2x}$
56. $f(x) = \frac{4-2x+3x^2}{x^2+2}$
57. $f(x) = 2x^3 + \frac{2-x}{x^3}$
58. $f(x) = \frac{x-1}{x+1}$
59. $f(x) = \frac{x^2}{x^2+1}$
60. $f(x) = \frac{1}{x^4-2x+1}$
61. $f(x) = \sin(x^2 + 2)$
62. $f(x) = \sin(3x)$
63. $f(x) = \cos(3x)$
64. $f(x) = \cos(x^4 + 7)$
65. $f(x) = \tan(\sin x)$
66. $f(x) = \tan(x^2 + 5)$
67. $f(x) = \sin(3x + 2)$
68. $f(x) = 2\sin x - \tan x$
69. $f(x) = 1 + x - \cos x$
70. $f(x) = \frac{\sin x}{x}$
71. $f(x) = \tan(2x - x^3)$
72. $f(x) = \sin(3x^2 - 2x + 1)$
73. $f(x) = 3\sin(x^2) + 2\sin(x) - \sin(3)$
74. $f(x) = 3\sin x - 2\cos x$
75. $f(x) = -\cos(\pi x - 1)$
76. $f(x) = \cos(3x) - \tan(3x)$
77. $f(x) = x^2 \sin x$
78. $f(x) = \cos(\tan x)$
79. $f(x) = \sin(2x + 3)$
80. $f(x) = \sin(2\pi x)$
81. $f(x) = \sin\left(\frac{1}{\sqrt{x}}\right)$
82. $f(x) = \sin\left(\frac{1}{x}\right)$
83. $f(x) = \cos(-x)$
84. $f(x) = \tan\pi(\frac{1}{2} - x)$
85. $f(x) = \tan(x^2 + 1)$
86. $f(x) = \tan(5x)$
87. $f(x) = x\cos x$
88. $f(x) = \frac{\sin x}{1+x}$

89. $f(x) = 4\sin^2(3x)$
90. $f(x) = \frac{\cos x}{\sin x}$
91. $f(x) = (x^2 + 3)\sin x$
92. $f(x) = \sqrt{\sin x}$
93. $f(x) = \cos^2(x^3)$
94. $f(x) = x\cos(5x)$
95. $f(x) = \sin(2x)\cos(3x)$
96. $f(x) = \sin^2(3x) + \cos^2(5x)$
97. $f(x) = x^2\tan\left(\frac{1}{x}\right)$
98. $f(x) = (\sin x - x\cos x)^{-1}$
99. $f(x) = \frac{\sin x + \cos x}{\tan x}$
100. $f(x) = \sin^2\left(x + \frac{1}{x}\right) + \cos^2\left(x + \frac{1}{x}\right)$
101. $f(x) = \frac{x^2}{\tan x}$
102. $f(x) = \sqrt{\cos(2x)}$
103. $f(x) = \sin^2(x - 3)$
104. $f(x) = \cos^3(x^2 - x)$
105. $f(x) = \sin x\cos^2 x$
106. $f(x) = \sin^{1/3}(2x)$
107. $f(x) = 4\sin^7(2 - 4x)$
108. $f(x) = 2\sin x + 3\sin^2 x$
109. $f(x) = 3\cos^3 x + 4\cos^2 x - 6$
110. $f(x) = 5\cos^2(x + 2) + 3\cos(x + 2) - 5$
111. $f(x) = 5x^2 - 3\tan^2 x + \sin x$
112. $f(x) = \frac{\tan x}{1 - \tan x}$
113. $f(x) = (\sin(x + 1))^{3/2}$
114. $f(x) = \sin^2 x + \cos^2 x$
115. $f(x) = \sin\sqrt{x} + \sqrt{\sin x}$
116. $f(x) = (\sin x)(\sin x + \cos x)$
117. $f(x) = 2\sin x\cos x$
118. $f(x) = \frac{1}{\sin x}$
119. $f(x) = \cos^2 x\sin x$
120. $f(x) = \sqrt{1 - \sin^2 x}$
121. $f(x) = x + \tan^2 x$
122. $f(x) = \frac{x}{\cos x}$
123. $f(x) = \tan x\cos x$
124. $f(x) = \sin x\cos x$
125. $f(x) = e^{x^2}$
126. $f(x) = e^{5x}$
127. $f(x) = (x^2 + 3x)e^x$
128. $f(x) = xe^x - e^{-x}$
129. $f(x) = e^{x^2} \cdot e^{x+1}$
130. $f(x) = \frac{e^{x^2}}{e^{x-1}}$
131. $f(x) = e^{-x}$
132. $f(x) = e^{\frac{2x}{3}}$
133. $f(x) = e^{\sqrt{x}}$
134. $f(x) = e^{3x} + 2e^{2x} - 3e^x + 7$
135. $f(x) = e^{x^2-2}$
136. $f(x) = \frac{1+e^{2x}}{2-e^{2x}}$
137. $f(x) = e^{3x-1} - 4e^{-x}$
138. $f(x) = \cos(e^x)$
139. $f(x) = 3e^{2x} - 4e^x + 1$
140. $f(x) = e^{3\cos(2x)}$

141. $f(x) = e^{-2x} + 4e^{-3x} + 7$
142. $f(x) = e^{2x+1}$
143. $f(x) = \frac{1}{2}e^{2x}$
144. $f(x) = e^{\sin x}$
145. $f(x) = e^{2x}$
146. $f(x) = 2xe^{x}$
147. $f(x) = \frac{1}{1-e^{-x}}$
148. $f(x) = \frac{e^{-x}}{x}$
149. $f(x) = x^2e^{-x}$
150. $f(x) = e^{-\frac{1}{x^2}}$
151. $f(x) = e^{\sqrt{x^2+1}}$
152. $f(x) = \frac{e^{x}-e^{-x}}{2}$
153. $f(x) = e^{2x}\cos(3x)$
154. $f(x) = e^{\cos(4x)}$
155. $f(x) = x^2 \cdot 2^{x}$
156. $f(x) = 3^{5x}$
157. $f(x) = x^4 + 4^{x}$
158. $f(x) = 9^{-x}$
159. $f(x) = \tan(5^{x})$
160. $f(x) = 3^{4x+1} + 2^{4x+2}$
161. $f(x) = 3^{x^2+1}$
162. $f(x) = 2^{x}$
163. $f(x) = 2^{-x}$
164. $f(x) = (\frac{1}{2})^{x}$
165. $f(x) = e^{x}\ln x$
166. $f(x) = \ln(\sin x)$
167. $f(x) = \frac{1}{\ln x}$
168. $f(x) = \ln(3xe^{-x})$
169. $f(x) = \ln\left(\frac{x-1}{x^2+1}\right)$
170. $f(x) = \ln\left(\frac{e^{x}}{1+e^{x}}\right)$
171. $f(x) = \ln(e^{\sin 2x})$
172. $f(x) = \ln\sqrt{\frac{x}{x^2+1}}$
173. $f(x) = \ln(x^2)$
174. $f(x) = \ln\left(\frac{10}{x}\right)$
175. $f(x) = \ln(10^{x})$
176. $f(x) = \ln(3x) + 4\ln x$
177. $f(x) = x^2\ln(2x)$
178. $f(x) = \ln(x^{-1})$
179. $f(x) = x\ln x$
180. $f(x) = \ln\frac{1}{x}$
181. $f(x) = (\ln x)^{3}$
182. $f(x) = x\ln(\sqrt{x})$
183. $f(x) = \ln(7x)$
184. $f(x) = (\ln x)^{1/2}$

IMPLICIT DIFFERENTIATION

The following equations define y implicitely as a function of x. Determine $\frac{dy}{dx}$ in terms of x and y in each case.

1. $x^2 + y^2 = 100$
2. $x^3 - y^3 = 6xy$
3. $x^2y + 3xy^3 - x = 3$
4. $x^3y^2 - 5x^2y + x = 1$
5. $x^2 = \frac{x+y}{x-y}$

6. $\sqrt{xy} + 1 = y$
7. $(x^2 + 3y^2)^{35} = x$
8. $\cos xy = y$
9. $\sin(x^2y^2) = x$
10. $\tan^3(xy^2 + y) = x$
11. $\sqrt{3 + \tan xy} - 2 = 0$

ANTIDERIVATIVES

Find the most general form of the function f satisfying the following:

1. $f'(x) = x^8$
2. $f'(x) = \frac{1}{x^6}$
3. $f'(x) = x^{5/7}$
4. $f'(x) = \sqrt[3]{x^2}$
5. $f'(x) = \frac{4}{\sqrt{x}}$
6. $f'(x) = \frac{1}{2x^3}$
7. $f'(x) = x^3\sqrt{x}$
8. $f'(x) = (x^3 - 2x + 7)$
9. $f'(x) = (x^{-3} + \sqrt{x} - 3x^{1/4} + x^2)$
10. $f'(x) = (x^{2/3} - 4x^{1/5} + 4)$
11. $f'(x) = \left(\frac{7}{x^{3/4}} - \sqrt[3]{x} + 4\sqrt{x}\right)$
12. $f'(x) = x(1 + x^3)$
13. $f'(x) = (1 + x^2)(2 - x)$
14. $f'(x) = x^{1/3}(2 - x)^2$
15. $f'(x) = \left[\frac{1}{x^2} - \cos x\right]$
16. $f'(x) = [4\sin x + 2\cos x]$
17. $f'(x) = e^x$
18. $f'(x) = xe^{x^2}$
19. $f'(x) = \frac{2}{x}$

DIFFERENTIAL EQUATIONS

Find the solution of each of the following differential equations with initial values.

1. $\frac{dy}{dx} = x/2,\ y(1/2) = -1$
2. $\frac{dy}{dx} = -(3/2)x^2,\ y(-1) = -1/2$
3. $\frac{dy}{dx} = \sin x,\ y(\pi/2) = 3$
4. $\frac{dy}{dx} = e^x,\ y(0) = 4$
5. $\frac{dy}{dx} = \frac{1}{x},\ y(1) = 2$
6. $\frac{d^2y}{dx^2} = 0,\ \frac{dy(2)}{dx} = 1,\ y(2) = 2$
7. $\frac{d^2y}{dx^2} = \cos x,\ \frac{dy(0)}{dx} = 1,\ y(0) = 2$
8. $\frac{d^2y}{dx^2} = e^x,\ \frac{dy(1)}{dx} = e,\ y(1) = -4e$

A GRAFIEKE / GRAPHS

Skets die grafieke van die volgende pare funksies,
telkens op dieselfde assestelsel.
Sketch the graphs of the following pairs of functions,
each time on the same set of axes:

1. $f(x) = \sinh x$ and $g(x) = 2\sinh x + 1$
2. $f(x) = \cosh x$ and $g(x) = \cosh\frac{x}{2}$
3. $f(x) = \tanh x$ and $g(x) = \tanh 2x$
4. $f(x) = \ln x$ and $g(x) = \ln 2x$

5. $f(x) = \ln x$ and $g(x) = \ln(x+2)$
6. $f(x) = \ln x$ and $g(x) = 2\ln x$
7. $f(x) = e^x$ and $g(x) = e^{2x}$
8. $f(x) = e^x$ and $g(x) = \ln x$
9. $f(x) = \sin x$ and $g(x) = \arcsin x$
10. $f(x) = \cos x$ and $g(x) = \arccos x$
11. $f(x) = \tan x$ and $g(x) = \arctan x$
12. $f(x) = \arcsin x$ and $g(x) = \arcsin(x+1)$

B DIFFERENSIASIE / DIFFERENTIATION

Differensieer die volgende funksies: / Differentiate the following functions:

1. $f(x) = \sinh(x^2 + 4x)$
2. $f(x) = \sinh(7x)$
3. $f(x) = \cosh(x^4 + 5x + 7)$
4. $f(x) = \tanh(\sinh 2x)$
5. $f(x) = \tanh(x^2 + 5)$
6. $f(x) = \frac{\sinh x}{x}$
7. $f(x) = \tanh(2x - x^3)$
8. $f(x) = 3\sinh(x^2) + 2\cosh(x) - \tanh(3)$
9. $f(x) = 3\sinh 4x - 2\cosh 5x$
10. $f(x) = \cosh(\tanh x)$
11. $f(x) = \cosh\left(\frac{1}{\sqrt{x}}\right)$
12. $f(x) = \tanh\left(\frac{1}{x}\right)$
13. $f(x) = \sinh(-x)$
14. $f(x) = \frac{\cosh x}{1+x}$
15. $f(x) = 4\cosh^2(3x)$
16. $f(x) = \sinh^2(x^4)$
17. $f(x) = x^2 \sinh\left(\frac{1}{x}\right)$
18. $f(x) = (\sinh x - x\cosh x)^{-2}$
19. $f(x) = \frac{\sinh x + \cosh x}{\tanh x}$
20. $f(x) = \cosh(e^x)$
21. $f(x) = e^{3\cosh(2x)}$
22. $f(x) = e^{\sinh x}$
23. $f(x) = e^{6x}\cosh(3x)$
24. $f(x) = e^{\cosh(4x)}$
25. $f(x) = \tanh(5^x)$
26. $f(x) = \ln(\cosh 8x)$
27. $f(x) = \ln(e^{\cosh 2x})$
28. $f(x) = \arcsin(2x)$
29. $f(x) = \arccos(x^2)$
30. $f(x) = x(\arcsin x)$
31. $f(x) = \arcsin\left(\frac{2}{x}\right)$
32. $f(x) = \arcsin(2x - 3)$
33. $f(x) = 2x(\arctan x)$
34. $f(x) = \arctan(5x)$
35. $f(x) = \arcsin(\sin x)$
36. $f(x) = \arctan(3x - 4)$
37. $f(x) = \arccos\left(\frac{x}{4}\right)$

C INTEGRASIE DEUR INSPEKSIE / INTEGRATION BY INSPECTION

1. $\int e^{5x}\, dx$

2. $\int x^8 dx$

3. $\int \frac{1}{x^6} dx$

4. $\int x^{5/7} dx$

5. $\int \sqrt[3]{x^2}\, dx$

6. $\int \frac{4}{\sqrt{x}} dx$

7. $\int \frac{1}{2x^3} dx$

8. $\int x^3\sqrt{x}\, dx$

9. $\int (x^3 - 2x + 7) dx$

10. $\int (x^{-3} + \sqrt{x} - 3x^{1/4} + x^2) dx$

11. $\int (x^{2/3} - 4x^{1/5} + 4) dx$

12. $\int \left(\frac{7}{x^{3/4}} - \sqrt[3]{x} + 4\sqrt{x}\right) dx$

13. $\int x(1 + x^3) dx$

14. $\int (1 + x^2)(2 - x) dx$

15. $\int x^{1/3}(2 - x)^2 dx$

16. $\int \left[\frac{1}{x^2} - \cos x\right] dx$

17. $\int [4\sin x + 2\cos x] dx$

18. $\int e^x dx$

19. $\int x e^{x^2} dx$

20. $\int \frac{2}{x} dx$

21. $\int e^{5x}\, dx$

22. $\int \frac{1}{3x}\, dx$

23. $\int \frac{1}{2x+1}\, dx$

24. $\int \sinh 4x\, dx$

25. $\int \sinh\,(4x + 6)\, dx$

26. $\int \cosh\,(2x + 3)\, dx$

27. $\int \tanh 3x\, dx$

28. $\int \frac{x^2}{x^3-4} dx$

29. $\int \frac{5x^4}{x^5+1} dx$

30. $\int \frac{t+1}{t} dt$

31. $\int \frac{x^3}{x^2+1} dx$

32. $\int x^2(2 - x^3)^4\, dx$

33. $\int x e^{2x^2}\, dx$

34. $\int \sin\theta(\cos\theta - 3)^3\, d\theta$

35. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

36. $\int \frac{\sin\theta}{1+\cos\theta}\, d\theta$

37. $\int \tan\theta\, d\theta$

38. $\int \frac{1}{x \ln x}\, dx$

39. $\int \frac{\ln x}{x}\, dx$

D FAKTORINTEGRASIE / INTEGRATION BY PARTS

1. $\int x e^{2x}\, dx$

2. $\int x\cos x\, dx$

3. $\int x\sin 4x\, dx$

4. $\int x \ln x\, dx$

5. $\int x^2 \cos 3x\, dx$

6. $\int x^2 \sin 2x\, dx$

7. $\int (\ln x)^2\, dx$

8. $\int \arcsin x\, dx$

9. $\int \theta \sec^2\theta\, d\theta$

10. $\int t^2 \ln t\, dt$

11. $\int e^{2\theta} \sin 3\theta\, d\theta$

12. $\int te^{-t}\, dt$

13. $\int \sqrt{t} \ln t\, dt$

14. $\int e^{3\theta} \cos 2\theta\, d\theta$

Appendix B

Technique Mastering 1.1 /Tegniekbemeestering 1.1

Name: Dr A Harding (Preview)

Start Time: Feb 06, 2002 15:32 **Time Allowed:** 15 minutes

Number of Questions: 5

Finish Help

Question 1 (2 points)

The domain of the function $f(x) = ln(1+e^x)$ is
Die definisieversameling van die funksie $f(x) = ln(1+e^x)$ is

1. All real numbers.
   Alle reële getalle.
2. All positive real numbers.
   Alle positiewe reële getalle.
3. $x > -1$
4. None of the above.
   Geen van bogenoemde.

Save answer

Question 2 (2 points)

If $f(x) = x^2$ and $g(x)=1/(x-1)$ then $(f/g)(x)$ is given by
As $f(x) = x^2$ en $g(x)=1/(x-1)$ dan word $(f/g)(x)$ gegee deur

1. $x^2/(x-1)$
2. $1/(x^2(x-1))$
3. $(1/(x-1))^2$
4. None of the above.
   Geen van bostaande.

Save answer

Question 3 (2 points)

The statement $log\ 5 + 3\ log\ x - 2\ log\ 4 - log\ y$ expressed as a single logarithm is
Die uitdrukking $log\ 5 + 3\ log\ x - 2\ log\ 4 - log\ y$ word as 'n enkele logaritme gegee deur

1. *log(15x/8y)*
2. *log($5x^3$y/16)*
3. *log ($5x^3$)/log(4^2y)*
4. *log(5 x^3 / 16y)*

Save answer

Question 4 (2 points)

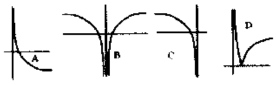
Solve for *x*:
1/2 ln x = 1 - ln 2 (Give the answer as a decimal)
Los op vir x:
1/2 ln x = 1 - ln 2 (Gee die antwoord as 'n desimaal)

1. *x=1.85*
2. *x=2.24*
3. *x=0.45*
4. None of the above.
   Geen van bogenoemde.

Save answer

Question 5 (2 points)

Which of the graphs below are the graphs of the functions f(x) = ln|x| and f(x) = ln(-x) (in this order):
Watter van die grafieke hieronder is die grafieke van die funksies f(x) = ln|x| en f(x) = ln(-x) (in hierdie volgorde):



1. D and C respectively.
   D en C respektiewelik.
2. B and A respectively.
   B en A respektiewelik.
3. B and C respectively.
   B en C respektiewelik.
4. D and A respectively.
   D en A respektiewelik.

Save answer

Technique Mastering 2.1

Name: Dr A Harding (Preview)
Start Time: Feb 06, 2002 15:52 **Time Allowed:** 15 minutes
Number of Questions: 6

Finish Help

Question 1 (2 points)

The derivative *f ' (x) of*
$f(x) = (x^2 - 3x) / x^2$
is given by

1. $(2x - 3) / x^2$
2. $(2x - 3) / 2x$
3. $1 - 3/x$
4. $3/x^2$
5. None of these

Save answer

Question 2 (1 point)

Find f ' (x) if
$f(x) = (2x^2 + 5)^7$

1. $7(4x)^6$
2. $(4x)^7$
3. $28x(2x^2 + 5)^6$
4. $7(2x^2 + 5)^6$
5. None of these

Save answer

Question 3 (1 point)

Differentiate:
$\ln (5 - x)^6$

1. $1 / (5 - x)^6$
2. $6 / (x - 5)$
3. $-6(5 - x)^5$
4. $1 / (6 (5-x)^5)$
5. None of these

Save answer

Question 4 (2 points)

Find dy/dx if
$x^2 + y^2 = 2xy$

1. x / (1-y)
2. (y - x) / (y - x)
3. 1
4. -x / y
5. None of these

Save answer

Question 5 (2 points)

Find an antiderivative of
$(x^4 - x^3) / x^2$

1. $2x^3 - 3x^2 + C$
2. $x^3 / 3 - x^2 / 2 + C$
3. $2x - 1 + C$
4. $3/20 (4x^2 - 5x) + C$
5. None of these

Save answer

Question 6 (2 points)

Find a function that satisfies
$f'(x) = 4x^{-0.5}$ *and* $f(1) = 12$

1. $8x^{0.5} + C$
2. $2x^{0.5} + 10$
3. $1/2\ (4x)^{0.5} + 11$
4. $8x^{0.5} + 4$
5. None of these

Save answer

Finish Help

Appendix C

Questionnaire: TMT_1 Paper versus online

Please help us to improve our system. Please include your name. You are guaranteed that I will not use anything you say or write against you, but I will possibly ask you to explain your response if necessary. THANK YOU SO MUCH! Name:………………

1	Did you pass TMT_1 the first time?	Yes	No		
2	Did you pass the paper version of the test?	Yes	No		
3	If you chose *No* in 2, say why.	Did not write out my answer properly	Did nor prepare properly	Other	
4	Did you pass the online version of the test?	Yes	No	n.a.	
5	If you chose *No* in 4 say why.	The system failed	Guessed the answers	Too little time	
		Did not prepare properly	Answers keyed in incorrectly	Other	
6	How many times did you write the online version of the test?	1x	2x	3x	
		4x	5x		
7	Did you go for tutoring before attempting a next online TMT?	n.a.	Yes	No	
8	If you chose *Yes* in 7, whom did you ask for help?	Tutor	Friend	My lecturer	
		Another lecturer	Other		
9	If you wrote the online version more than once, where did you do it?	Math lab	Engineering lab	Other	
10	Indicate which of the two tests was the easiest.	Paper	Online	The same	
11	Explain				
12	Was it easy to get access to the MATH LAB?	Yes	No		
13	Was it easy to get the PASSWORD?	Yes	No		
14	Was the tutor in the lab able to assist in the technical side of WebCT?	Yes	No		
15	Do you think the TMT programme is of value in the calculus course?	Yes	No	Other	
16	Any suggestions?				