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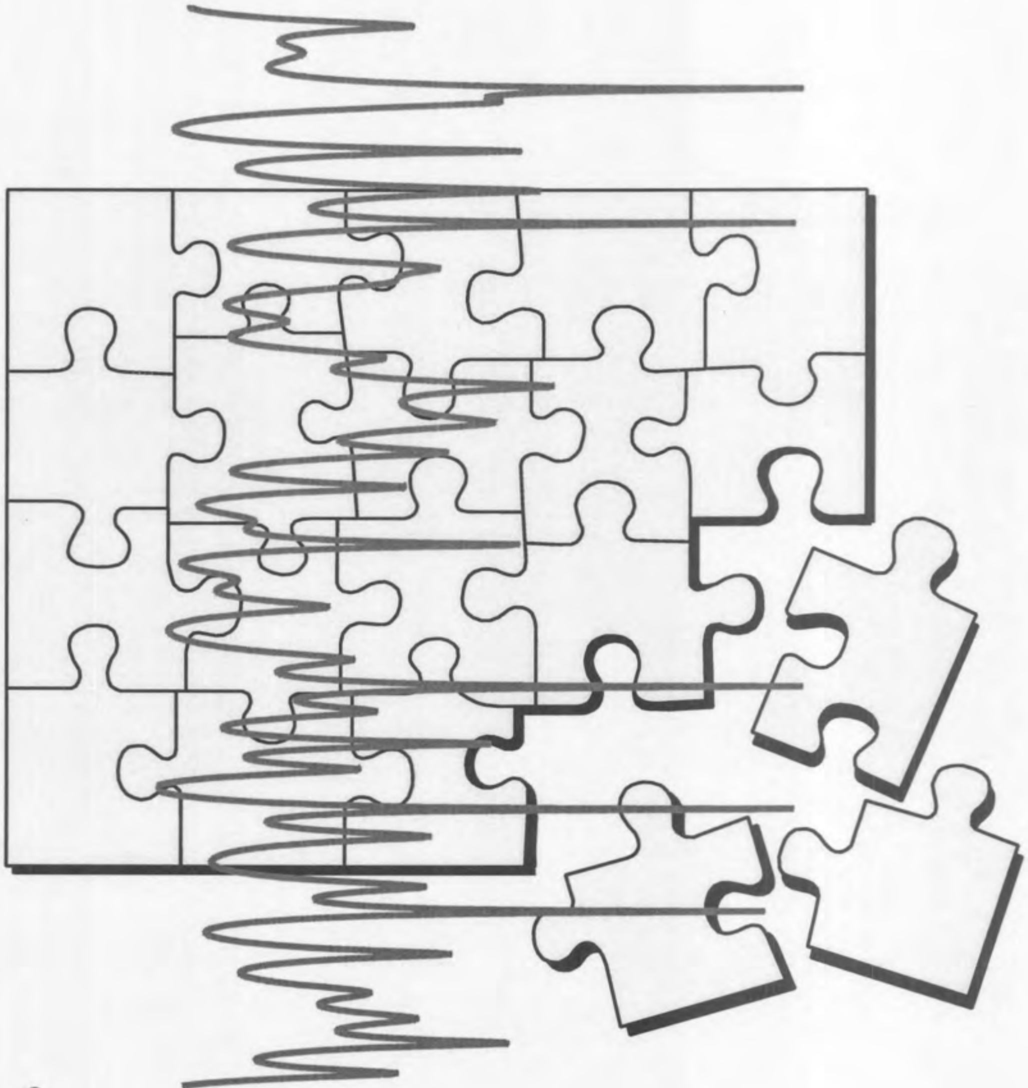
SPACE-TIME TURBO CODING FOR CDMA MOBILE COMMUNICATIONS

D.J. van Wyk



University of Pretoria

SPACE-TIME TURBO CODING FOR CDMA MOBILE COMMUNICATIONS

by

Daniël Jacobus van Wyk

Doctoral thesis submitted as partial fulfilment
of the requirements for degree

Doctor of Philosophy

in

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in the

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Dedication

This thesis is dedicated to my wife, Daleen, for all her love, patience, and support, and for all the sacrifices that she has made so that I could complete this research.

It is also dedicated to my son NW, for the joy that he has brought to my life, and to my parents for all of their love and support of me in all of my endeavors.

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Degree: Philosophiae Doctor

Supervisor: Prof. L.P. Linde

SUMMARY

Against the background of the rapid evolution of mobile communication systems in the areas of service provision and capacity enhancement described above, the main focus of the research is on coded space-time processing techniques. The use of space-time processing is an attractive solution because it can mitigate the effects of multipath fading as well as suppress co-channel interference, therefore, significantly improving system performance. The topics are presented in the context of designing mobile communication systems where the two core areas of spatial processing and error coding are to be integrated in an optimum way. Of particular importance in this thesis, will be those CDMA based solutions for the mobile sector and the new performance analysis issues that need to be addressed as a result of the introduction of heterogeneous services and service environments into a single, mobile cellular access network. Furthermore, novel applications of turbo transmit and receive antenna diversity and beamforming techniques to mobile cellular access networks aimed at increasing the efficiency of such networks are considered. The thesis has the following goals:

- To establish a general spatial/temporal channel model for use in the evaluation of coded space-time processing concepts applied to CDMA networks.
- To analyze the performance of uncoded cellular CDMA systems incorporating space-time techniques using analytical methods in a number of realistic application scenarios.
- To design, implement and evaluate coding strategies for incorporation into the space-time CDMA systems. This objective can be broken down into the following items:
  - Space-time coding systems when considering multiple transmit antennas for the downlink.
  - Coded space-time systems when considering multiple receive antennas for the uplink.
- To establish the performance of coded space-time CDMA cellular networks under realistic scenarios.

This thesis introduces many (some novel) space-time turbo coded techniques to increase the downlink capacity of a cellular CDMA network using multiple transmit antennas. For improving the uplink capacity, coded space-time diversity and beamforming techniques, employing multiple receive antennas, are considered. In order to quantify the performance improvements that may be achieved, a framework for the evaluation of these systems are constructed. Using this framework the BEP of all the space-time coding systems are derived analytically, and evaluated under identical propagation scenarios. The results presented show that the use of space-time turbo coded processing is an attractive solution since it can improve system performance significantly under conditions of multipath fading for both the uplink and downlink. It is shown that the two core areas of spatial processing and channel coding can be integrated in an optimum way to increase the capacity of existing cellular CDMA networks.

Key Words: Mobile Digital Wireless Communication, Multiple-Access Communication, Error Correction Coding, Multiple Transmit and Receive Antenna, Space-Time Diversity and Beamforming, Turbo Transmit Diversity.

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OPSOMMING

Gemeet teen die agtergrond van die snelle ontwikkeling van mobiele kommunikasiestelsels en dienste, asook die fisiese beperkings gestel deur beskikbare bandwydte en seindrywing, ondersoek die proefskrif ruimte-tyd kodeerde prosesseringstegnieke vir huidige en toekomstige multi-gebruiker CDMA sprei-spektrum kommunikasiestelsels. Die gebruik van ruimte-tyd prosessering is uiters geskik vir mobiele kommunikasie aangesien dit die effekte van multipad voortplanting en ko-kanaalsteurings teen kan werk. In die proefskrif word die twee kern areas van ruimte prosessering en foutkorreksiekodering optimaal geintegreer. Die proefskrif het die volgende doelwitte:

- Om 'n algemene ruimte/tyd kanaalmodel op te stel waarmee die voorgestelde foutkorreksie kodeerde ruimte-tyd prosesseringskonsepte en algoritmes teen evalueer kan word. Laasgenoemde kanaalmodel sluit ook ruimte-tyd korrelasie modelering in.
- Om die verwerrigting van ongekodeerde ruimte-tyd sellulêre CDMA kommunikasiestelsels te evalueer deur middel van analitiese metodes onder realistiese toepassingskondisies.
- Om die ontwerp, implementering en verwerrigtinganalises van ruimte-tyd kodeerde sellulêre CDMA kommunikasiestelsels uit te voer vir insluiting in huidige en toekomstige CDMA kommunikasiestelsels. Hierdie doelwit word verder onderverdeel:
  - Ruimte-tyd koderingstelsels wanneer multi-antenna elemente by die basisstasie-sender vir seintransmissie in die voorwaartse pad beskikbaar is.
  - Gekodeerde ruimte-tyd stelsels wanneer multi-antenna elemente by die basisstasie-ontvanger vir seinontvangs in die tru-waartse pad beskikbaar is. Hier word gekyk na beide ontvanger diversiteit en antenna patroonvorming.
- Om die verwerrigting van ruimte-tyd kodeerde sellulêre CDMA kommunikasiestelsels te evalueer deur middel van analitiese metodes onder realistiese toepassingskondisies.

Hierdie proefskrif stel verskeie (sommige uniek) ruimte-tyd turbo gekodeerde tegnieke voor vir die kapasiteitverbetering van sellulêre CDMA kommunikasiestelsels in die voorwaarste pad wanneer van multi-anttena elemente vir seintransmissie gebruik gemaak word by die basisstasie. Om die kapasiteit in die tru-waartse pad te verbeter word foutkorreksie kodeerde diversiteit en patroonvormingstegnieke voorgestel vir implementering by die ontvanger basisstasie. Die navorsingsresultate dui daarop dat die voorgestelde ruimte-tyd kodeerde prosesseringstegnieke uitstekend geskik is vir die verbetering van beide die voorwaartse en tru-waartse paaie. Die proefskrif lê hiermee die grondslag vir die ontwerp en evaluasie van ruimte-tyd prosesseringstegnieke vir toekomstige CDMA sellulêre kommunikasienetwerke.

Sleutelwoorde: Mobiele Syferkommunikasie, Multi-Gebruiker Kommunikasie, Foutkorreksiekodering, Multi-Versend en Ontvangs Antennas, Ruimte-Tyd Diversiteit en Antenna Patroonvorming, Turbo Versend Diversiteit.

PREFACE

I would like to thank my Creator, to Him all the praise !

The research work contained in this thesis has been carried out at the University of Pretoria in the Department of Electrical and Electronic Engineering.

I am indebted to many people for their advice and assistance towards the successful completion of this thesis.

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The discussions I have had with Dr. Ian Oppermann stimulated many of the ideas presented in this thesis. I would also like to thank him for the tremendous support, and enthusiasm over the past few years. I also thank him for the great opportunity to go to Sydney as a research fellow of Southern Poro Communications.

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CONTRIBUTIONS

Most of the results presented in this thesis have been published or have been submitted for publication in international conferences and journals. During the research which has lead to this thesis, the following publications have been prepared. In addition an international patent was filed (see 18) and a book by Kluwer Academic Publishers (KAP) was published (see 23).

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LIST OF ABBREVIATIONS

2G	second generation
3G	third generation
ACTD	Alamouti code transmit diversity
AS-TDTD	antenna-selection time-division transmit diversity
AWGN	additive white Gaussian noise
BEP	analytical bit error probability
BER	bit error rate
BCH	Bose and Ray-Chaudhuri codes
BPSK	binary phase-shift keying
BRAN	broadband radio access channel
BSS	base station subsystem
CC	convolutional coded
CDS	circular disk of scatterers
CDSM	circular disk of scatterers model
CDMA	code-division multiple-access
CDTD	code-division transmit diversity
CIR	carrier-to-interference ratio
CL-AS	closed-loop antenna-selection
CLPC	closed-loop power control
cpdf	conditional probability distribution function
CSI	channel state (or side) information
dB	decibel
DECT	digital enhanced cordless telephone
DOA	direction-of-arrival
DS	direct-sequence
DTD	delay transmit diversity
EEP	error-event path
EFD	effective fading distribution
EGC	equal gain combining
EMF	estimated matched filter
ESM	effective scatterer model
ETSI	European telecommunications standards institute
FDD	frequency-division duplex
FDMA	frequency-division multiple-access
FEC	forward error correction
FIR	finite impulse response
FPLMTS	future personal land mobile telephone system

GAA	Gaussian angle of arrival (model)
GFD	Gaussian fading distribution
GS	Gaussian scatterer model
GSM	global system for mobile communications (formerly: Groupe Spécial Mobile)
HDD	hard decision decoding
IIR	infinite impulse response
IMT-2000	international mobile telecommunications for the 21st century
IN	intelligent network
IP	internet protocol
IPI	inter-path-interference
IRWEF	input redundancy weight enumerating function
ISI	inter-symbol interference
IS-54	interim standard 54
IS-95	interim standard 95
ITU	international telecommunications union
LAN	local area network
LMS	least mean squares
LMMSE	least mean minimum square error
LOS	line of sight
LSED	layered squared Euclidean distance
LSEDP	layered squared Euclidean distance product
MACH	multiple-access channel
MAI	multiple-access intereference
MAP	maximum a posteriori
MF	matched filter
MFD	maximum free distance
MIMO	multiple input/multiple output
MISO	multiple input/single output
ML	maximum likelihood
MLSD	maximum likelihood sequence detection
MLSE	maximum likelihood sequence estimation
MM	multi media
MMSE	minimum mean square error
MPSK	multiple phase-shift keying
MRC	maximal ratio combining
MSE	mean square error
MTCM	multiple trellis-coded modulation
MU	multiuser
MUD	multiuser detection (or demodulation)
MUSIC	multiple signal classification

NLMS	normalized LMS
NLOS	non line of sight
NO-CDTD	non-orthogonal (delayed) CDTD
O-CDTD	orthogonal code-division transmit diversity
OTD	orthogonal transmit diversity
OVSF	orthogonal variable spreading factor
PCTTD	parallel-concatenated turbo transmit diversity
pdf	probability distribution function
PCS	personal communication system/service
PN	pseudo-noise
PSA	pilot symbol assisted
PSK	phase-shift keying
QoS	quality-of-service
QPSK	quadrature phase-shift keying
Q^2PSK	quadrature-quadrature phase-shift keying
RCCC	rate-compatible convolutional codes
RF	radio frequency
RMS	root mean square
RS	Reed-Solomon codes
RSC	recursive systematic convolutional
RSC&WH	recursive systematic convolutional and Walsh-Hadamard
RR-TDTD	round-robin time-division transmit diversity
SC	selection combining
SCTTD	serial-concatenated turbo transmit diversity
SDD	soft-decision decoding
SDMA	space division multiple access
SIMO	single input/multiple output
SINR	signal-to-interference-noise ratio
SISO	single (soft) input/single (soft) output
SNR	signal-to-noise ratio
SOCC	super-orthogonal convolutional code
SOTC	super-orthogonal turbo code
SOTTD	super-orthogonal turbo transmit diversity
SOVA	soft-output Viterbi algorithm
SS	spread-spectrum
STCM	space-time coded modulation
STTCM	space-time turbo coded modulation
SU	single user

TCM	trellis coded modulation
TCS	trellis coded spreading
TD-CDMA	time division CDMA
TDD	time-division duplex
TDMA	time-division multiple-access
TDTD	time-division transmit diversity
T/F/C	time, frequency and code
T-MTCM	turbo multiple trellis coded modulation
TOA	time of arrival
TPC	transmit power control
TTCM	turbo trellis coded modulation
TTD	turbo transmit diversity
TU	typical urban
UHF	ultra high frequency
ULA	uniform linear array
UMTS	universal mobile telephone system
UTRA	UMTS terrestrial radio access
VSF	variable spreading factor
WEF	weight enumerating function
WH	Walsh-Hadamard
WLL	wireless local loop
WSS	wide-sense stationary
WCDMA	wideband code-division multiple-access

LIST OF SYMBOLS

arg	argument
max	maximum
$\{-1,1\}$	binary set
$\|\cdot\|$	Euclidean distance
$(\cdot)^H$	Hermitian
A_d^C	number of codewords of weight d
$A_{i,d}^C$	number of codewords of weight d for input weight i
$A^C(I,D)$	weight enumerating function
$A^{C_i}(I,D)$	weight enumerating function inner code
$A^{C_o}(I,D)$	weight enumerating function of outer code
$A_{gff/gfb}$	feedforward and feedback generator polynomial
A_k	received amplitude
$a_k(t)$	binary spreading waveform
β_k	received signal strength in volts
$\beta_l^{(k)}(i)$	strength of specific multipath
$\mathbf{b}$	vector of data bits
$\hat{\mathbf{b}}$	specific version of $\mathbf{b}$
$\mathbf{b}_i$	vector of bits for interval i
$b_k(t)$	binary data sequence
$b_i^{(k)}$	bit for user k, symbol interval i
$\tilde{\mathbf{b}}_i^{(k)}$	vector of bits up until user k symbol interval i
$\hat{\mathbf{b}}(s)$	bit estimate vector for stage s
$\hat{\mathbf{b}}_{ML}$	maximum likelihood estimate of $\mathbf{b}$
$\mathcal{BW}$	total signal bandwidth
$\mathcal{BW}_{ch}$	coherence bandwidth
C_i	inner code
C_o	outer code
C_{tot}	total capacity
$\mathbf{c}_n$	codeword
d	Hamming weight
d_{free}	free distance
$d_i^{(k)}$	element for user k symbol interval i of $\mathbf{D}$
$d^{(j)}(t)$	reference signal for user j
$\mathbf{D}$	code matrix
δ	scalar difference
$\text{erfc}(x)$	complimentary error function of x
$E(\mathbf{y})$	expected value of $\mathbf{y}$

E_b	energy per bit
$E_{d\|i}$	expectation w.r.t. distribution $\rho(d \mid i)$
$E_{P_{ACTD}}(\mathbf{X},\mathbf{Y})$	product distance for ACTD
$E_{P_{DTD}}(\mathbf{X},\mathbf{Y})$	product distance for DTD
$E_{P_{OTD}}(\mathbf{X},\mathbf{Y})$	product distance for OTD
E_s	energy per symbol
$e_i^{(k)}$	estimation error
η	noise vector for Rake combined case
$\hat{\eta}$	noise vector from decorrelator
$\eta_n^{(j)}(i)$	noise sample
η_m	bandwidth efficiency
η_ζ	vector of noise samples
f_d	maximum doppler shift
$\Phi_S(t)$	characteristic function of S
$\mathbf{G}_e$	equal gain combining matrix
$\mathbf{G}_R$	Rake combining matrix
$\mathbf{G}_s$	selection diversity combining matrix
$g(t)$	chip waveform
g_{ff}	feedforward
g_{fb}	feedback
$\Gamma(\cdot)$	Gamma function
γ_0	interference
γ_{0c}	interference under conditions coding
γ_b	average SNR per bit
γ_k	average SNR per diversity branch
$\mathbf{h}^{(k)}(t,\tau)$	channel impulse response
$h_i^{(k)}(\tau)$	multipath channel response
$\mathbf{I}$	identity matrix
I_{mai_n}	multiple access interference
I_{ni_n}	AWGN interference
I_{si_n}	self interference
i	symbol interval index
$J_0(\cdot)$	Bessel function
j	user index
k	user index or number of coder input bits
k_i	number of inner coder input bits

k_o	number of outer coder input bits
$\mathcal{K}_k$	Rice factor
K	number of active users
K_{cc}	convolutional code constraint length
K_{tc}	turbo code constraint length
K_{oc}	orthogonal code constraint length
κ	multiplicity factor
l	multipath index
L_p	number of multipath
L_R	number of RAKE taps
L_{min}	length of shortest error event path
L_{WH}	length of Hadamard codeword
λ	branch metrics of soft-input trellis decoder
m	Nakagami-m fading parameter
m_0	Nakagami parameter of main received path
M_D	number of diversity antennas
M_B	number of beamforming antennas
M_R	number of receive antennas
M_T	number of transmit antennas
n_l	path loss exponent
n	symbol index or number of coder output bits
n_i	number of inner coder output bits
n_o	number of outer coder output bits
$\mathbf{n}(t)$	Gaussian noise in received signal
$n(t)$	complex envelope of the noise process
$n_m(t)$	Gaussian noise process for antenna element m
N	processing gain
N_{cc}	convolutional code interleaver size (trellis decoder decoding depth)
N_{tc}	turbo code interleaver size (trellis decoder decoding depth)
N_0	two-sided noise spectral density
Ω_k	average power of received path
ω_c	carrier frequency
$p_{R_s,\phi}(R_s,\phi)$	pdf of scatterers
$p_{\Phi_0}(\phi_0)$	pdf of the angular distribution of users
$P(\mathbf{b}\|\mathbf{y})$	probability of $\mathbf{b}$ given $\mathbf{y}$
$p(\mathbf{y}\|\mathbf{b})$	pdf of $\mathbf{y}$ given $\mathbf{b}$
$P(\mathbf{b})$	probability of $\mathbf{b}$
$p_S(s)$	pdf of S
P_k	received power
P_e	bit error probability

$P_d(c-\hat{c})$	probability of codeword error
P_w	word error probability
π	interleaver
π^{-1}	de-interleaver
q	multipath index
$Q(\cdot)$	Q-function
$\mathbf{r}(t)$	received signal
$\mathcal{R}^{(kj)}$	spatial correlation between user k and reference user
R_{k1}	periodic correlation between user k and reference user
$\hat{R}_{k1}$	aperiodic correlation between user k and reference user
R_c	overall code rate
R_o	outer code rate
R_i	inner code rate
R	mobile to base station distance
R_D	scatterer radius
R_0	cut-off rate of channel
R_r	radius of cells
R_{xx}	correlation of real components
R_{xy}	correlation between real and imaginary components
$\mathbf{R}_\zeta$	correlation matrix for matched filtered statistic
ρ_{ij}	fading correlation constant between transmissions i and j
ρ	average spatial correlation
$S_n^{(j)}$	desired received signal
$\sigma^2_{\text{mai}_n}$	MAI variance
$\sigma^2_{\text{si}_n}$	self interference variance
$\sigma^2_{\text{ni}_n}$	AWGN variance
σ_T	total interference variance
σ^2	noise variance
σ	noise process standard deviation
σ_s	standard deviation of scatterers
$s(t)$	received signal power envelope
$s_k(t)$	transmitted signal for user k
t	time
$\overline{\tau}_m$	mean excess delay
τ_0	minimum path delay

$\tau_l^{(k)}(t)$	propagation delay of path l from user k
τ_m	maximum excess delay
τ	delay
$\tau_l^{(k)}$	time delay for specific multipath
θ_k	carrier phase
T_c	chip period
T_s	symbol period
T_{samp}	sampling period
$T(L, I, D, J)$	transfer function i.t.o. path length, input weight, output weight and remergings with zero state
$T_{l,i,d,j}$	number of paths of length l, input weight i, output weight d, and j remergings with zero state
$\mathbf{u}$	output of linear detector
U_s	RAKE output
v	mobile speed
V	system load
$\varphi_l^{(k)}(i))$	phase shift for specific multipath
ϖ_l	width of peak l
$\mathbf{w}_q^{(j)}$	weight vector
X_{ck}	in-phase Gaussian random variable
X_{sk}	quadri-phase Gaussian random variable
x	encoded bit
ζ	vector of matched filter outputs
$\zeta_{m_D}^{(j)}(i)$	detector output on m_Dth diversity branch
Z	number of constituent RSC encoders utilized in turbo encoder