Chapter 7

Closure

7.1 Summary of contributions

In this thesis, the iterative post-buckling algorithm proposed by Grisham was successfully implemented and tested. The implementation was verified using various examples from the literature, for some of which experimental results were also available. For most examples, comparisons with results obtained using the modified Wagner and NACA approaches were also performed. In addition, the results obtained using the Grisham algorithm were compared with the results of a complete non-linear finite element analysis. The algorithm was then used in an optimization exercise to determine the minimum mass of the structure using an implementation of a genetic algorithm.

7.1.1 Software developed

The Grisham algorithm was programmed in FORTRAN77 and is now available for use at the CSIR[1]. All the linear finite element models were analyzed using ABAQUS®. To achieve this, portable FORTRAN77 subroutines were developed within the ABAQUS® environment to allow for the extraction of data and interaction with the Grisham algorithm.

7.2 Evaluation of the Grisham algorithm

7.2.1 Comparison with the Wagner, modified Wagner and NACA approaches

For the verification example studied in Chapter 2, the Grisham algorithm gave comparable results to those obtained using the Wagner and NACA approaches, with the Grisham algorithm being less conservative. The web results compared very well. The diagonal tension factor k was calculated to be between 2.5% and 6.0% lower (depending on the panel studied).

[1]See also Appendix B for a listing of the source code.

The diagonal tension angle α of 42.2 degrees is roughly halfway between the Wagner and NACA predictions, with the total shear stress in the web being 6% lower than that predicted by NACA.

The average upright stress results varied between 12% and 38% below that of the NACA predictions, depending on the position of the upright. NACA again was some 6% lower than the Wagner prediction. For the maximum stress values in the uprights, the Grisham algorithm results were between 13% and 32% lower than that calculated using NACA (with the Wagner approach not producing a maximum stress value). The flange stress results with the Grisham algorithm resulted in far lower values than both the NACA and Wagner approaches (up to as much as 138% difference). This may be accounted for by the secondary bending stresses (σ_{f_u}, σ_{f_l}) caused by diagonal tension. When ignored, the correlation improved dramatically (within 44%).

7.2.2 Comparison with a full non-linear analysis

To further validate the Grisham algorithm theoretically, a non-linear finite element analysis was performed for the verification example. While no diagonal tension factor is obtained from the non-linear finite element analysis, a qualitative inspection reveals that the principle stresses in the non-linear analysis agree well with the diagonal tension angle calculated using the Grisham algorithm.

The web critical buckling shear stress (τ_{cr}) calculated with the non-linear finite element analysis results is slightly higher and less conservative than that of the other methods (Wagner, NACA and Grisham). The agreement between the non-linear analysis and the Grisham web stress results is within 22%.

The highest average upright stress value is found in Upright 4 (the middle of the structure) and reduces in magnitude by up to 35%, to both ends. This is the case with both the non-linear analysis and the Grisham algorithm. Since the linear finite element analysis used in the Grisham algorithm does not take into account all the intricacies of the non-linear finite element analysis, the best way to compare the results in the uprights is probably to refer to averaged values. The general trend of the two sets of data compare very well, the non-linear finite element results being lower and probably less conservative than the Grisham algorithm results. The flange results compare reasonably well although the Grisham results are no longer conservative (within 16%, except for the upper flange values which are 30% lower than that of the non-linear finite element analysis results). The deflection results compare reasonably well, with the Grisham algorithm tip deflection being 11% lower than that of the non-linear finite element analysis.

7.2.3 Computational effort

The comparative study revealed that the Grisham algorithm is much more efficient than a non-linear finite element analysis. A run time comparison on an HP C200 workstation showed that for a coarse mesh discretization, the Grisham algorithm ran 14 $\times$ faster than a comparable non-linear finite element analysis. For a fine mesh discretization, the Grisham algorithm ran 10 × faster. This result seems influenced by external loads on the workstation.

For the non-linear finite element analysis, a refined mesh is a requirement, as to allow for adequate representation of the buckling modes (viz. an element characteristic length should probably at least be some 3-5 times smaller than the wavelength of the buckling mode). The Grisham algorithm on the other hand is an approximate method that requires far less accuracy in the finite element model.

Hence an advantage in computational efficiency of at least 10 × for most problems seems reasonable.

7.2.4 Stopping criteria and parameters

The two convergence criteria used in the Grisham algorithm both typically converge within five iterations to within a 2% margin. This seems quite efficient and realistic.

The method has two parameters (β values) which are related to the stiffness of the buckled webs. They are not known *a priori*. When the algorithm starts off, they are approximated by initial estimates. The β values reveal little sensitivity to the flange or upright dimensions, but notable sensitivity to the thickness of the webs in the structure. Nevertheless, the β values can easily be adjusted using an iterative procedure and do not effectively impair convergence.

7.2.5 Further verification

Three further examples taken from the literature were used to validate the Grisham algorithm; the first two with experimental results. The Grisham algorithm results compared reasonably well with the experimental values, providing further proof of the validity of the procedure.

7.3 Structural optimization

The verification example was successfully optimized for mass, using a micro-genetic algorithm (μ-GA). Using only four design variables, an 11.01% saving in mass was achieved. Using eleven design variables, a 14.08% saving was achieved.

7.4 Recommendation

Based on

1. the demonstrated accuracy of the Grisham algorithm, as well as
2. the computational efficiency as compared to a full non-linear finite element analysis,

it is recommended that the Grisham algorithm be used in particular during initial design iterations and optimization analyses.

After initial design iterations are complete, final designs can still be evaluated using a full non-linear finite element analysis if desired. This will allow for a single expensive non-linear finite element analysis to suffice during the design or optimization process.

7.5 Future work

The Grisham algorithm can relatively easily be extended to provide for box structures, curved panels and composite materials. It is noted that the computational edge of the Grisham algorithm can be expected to increase as the complexity of the structures analyzed increases (since non-linear finite element analyses will become increasingly expensive).

In addition, the use of discrete design variables during optimization is of interest. This is already provided for in selecting a genetic algorithm for the infrastructure.