Chapter 1

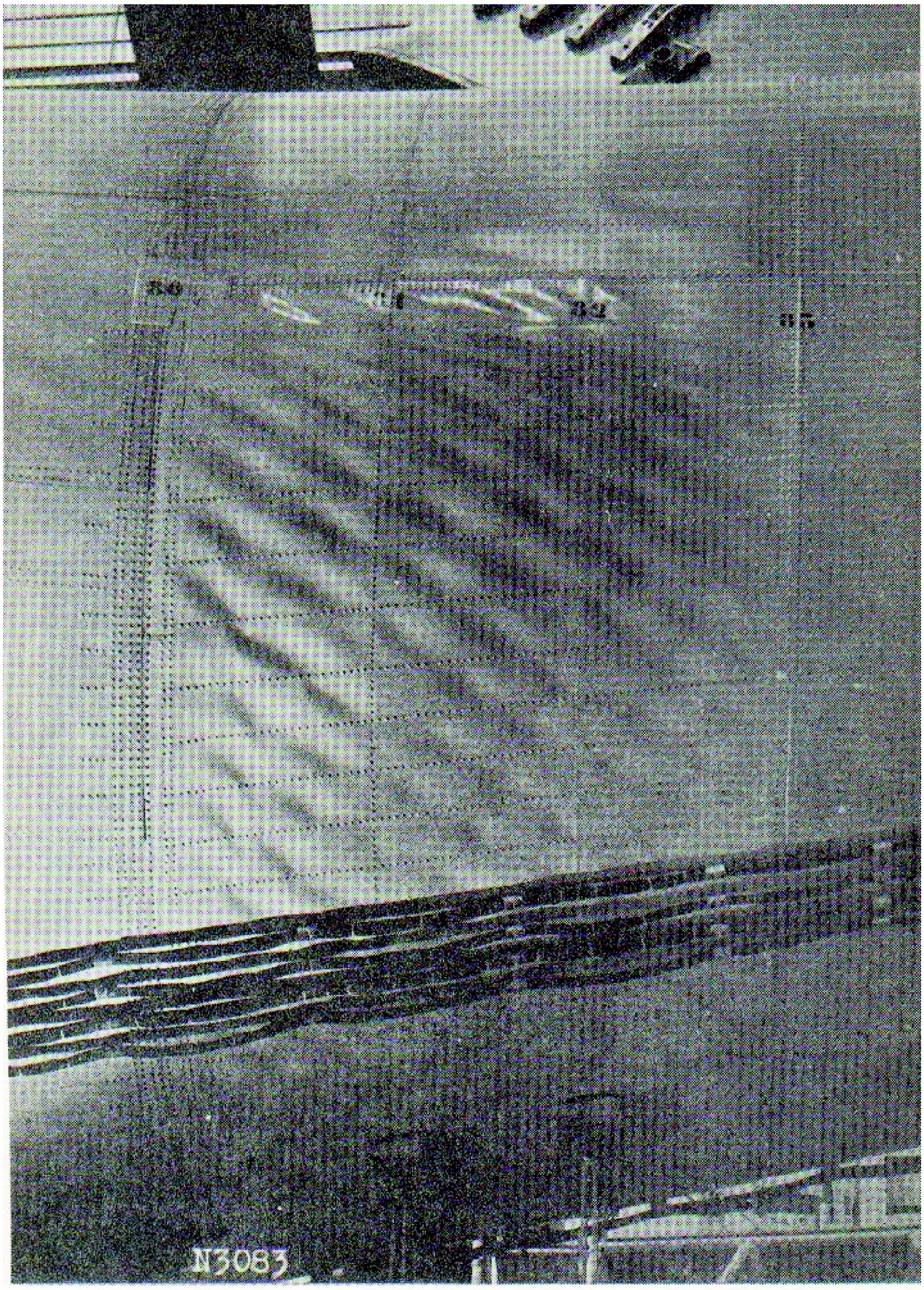
Introduction

1.1 Overview

In many aircraft structures, thin sheet structural components are designed to buckle under shear load. The buckling can change the internal loads and stresses in neighboring panels and the surrounding structure significantly. Initially, post-buckling effects were taken into account using the theory of pure diagonal tension (PDT) proposed by Wagner [1, 2] in 1929, but this proved to be very conservative in practice. Wagner's approach was gradually modified to eventually cover the general approach of incomplete diagonal tension (IDT). The theory of incomplete diagonal tension was developed by the National Advisory Committee for Aeronautics (NACA) in the 1950's after conducting an extensive testing program to generate empirical relations [3, 4]. This approach, also known as the NACA method, has become the accepted design approach used by most aircraft manufacturers, even though the theory is still considered conservative. One of the factors neglected in the NACA approach is the interaction of stresses in each web element on the element allowables, namely the combination of compression and shear buckling, diagonal tension and post-buckled skin softening in shear.

As an alternative to IDT, non-linear finite element codes can be used to assess buckling, even though "design-by-rule" failure criteria are more difficult to assess (Mello *et al.* [5]). In addition, non-linear finite element analyses are computationally very expensive in initial design iterations.

In this thesis, the iterative algorithm developed by Grisham [6], is implemented to assess and evaluate the onset and magnitude of buckling in flat shear webs. Since few verifications of the Grisham algorithm have previously been presented, the method is extensively evaluated in this study. The results obtained using Grisham's algorithm are also compared with a non-linear finite element analysis.



Flambage local du revêtement d'un fuselage arrière
dans la partie non pressurisée, sous l'effet de l'effort tranchant
et de la flexion de fuselage.

Figure 1.2: Diagonal tension in airframe (Reproduced from [7])

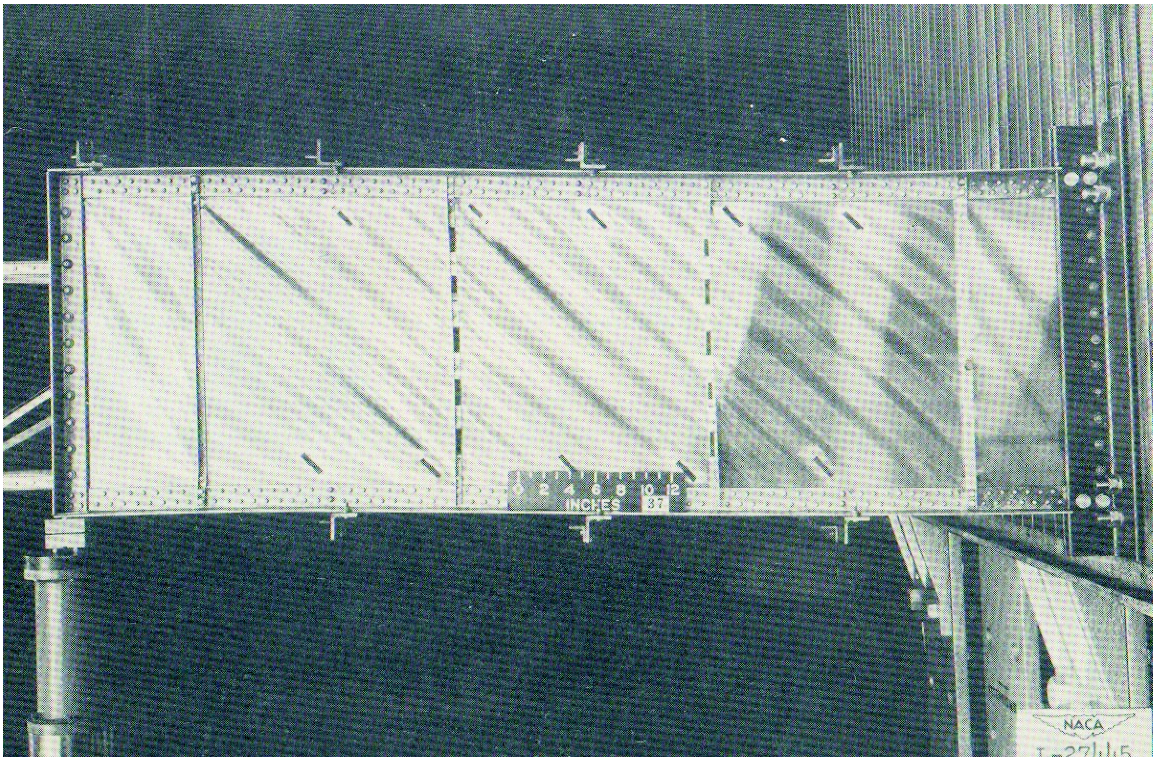


Figure 1.1: Diagonal tension beam (Reproduced from [3])

1.2 Diagonal tension

When a beam consisting of a thin shear web with transverse stiffeners is loaded until the web buckles, it forms diagonal folds at approximately 45 degrees. These diagonal folds carry load in tension. Compression stresses act perpendicular to theses folds while the stiffeners act as compression columns. The beam can carry loads orders of magnitude higher than the initial buckling load. Figure 1.1 shows such a thin web beam under a high test load that is close to failure. The many parallel folds in the web are clearly visible.

If the load is increased further, indefinitely without rupturing the sheet, the compressive stresses become smaller and smaller while the tensile stresses increase more and more, approaching a limiting condition known as pure diagonal tension (PDT). The theory of pure diagonal tension (PDT) was developed by Wagner [1, 2]. Pure diagonal tension can only be approached when the web is very thin, which is impractical. Most webs operate in a state of stress that is intermediate between pure diagonal tension and the state that exists before the web buckles. To cope with this approach, the engineering theory of incomplete diagonal tension (IDT) was developed by NACA [3, 4].

Another example of diagonal tension, this time in a fuselage, is shown in Figure 1.2. Although numerous aircraft skin panels may be in a buckled state during flight, the intensity is very low and is hardly visible to the human eye most of the time.

The incomplete diagonal tension approach divides the nominal web shear into two parts; a

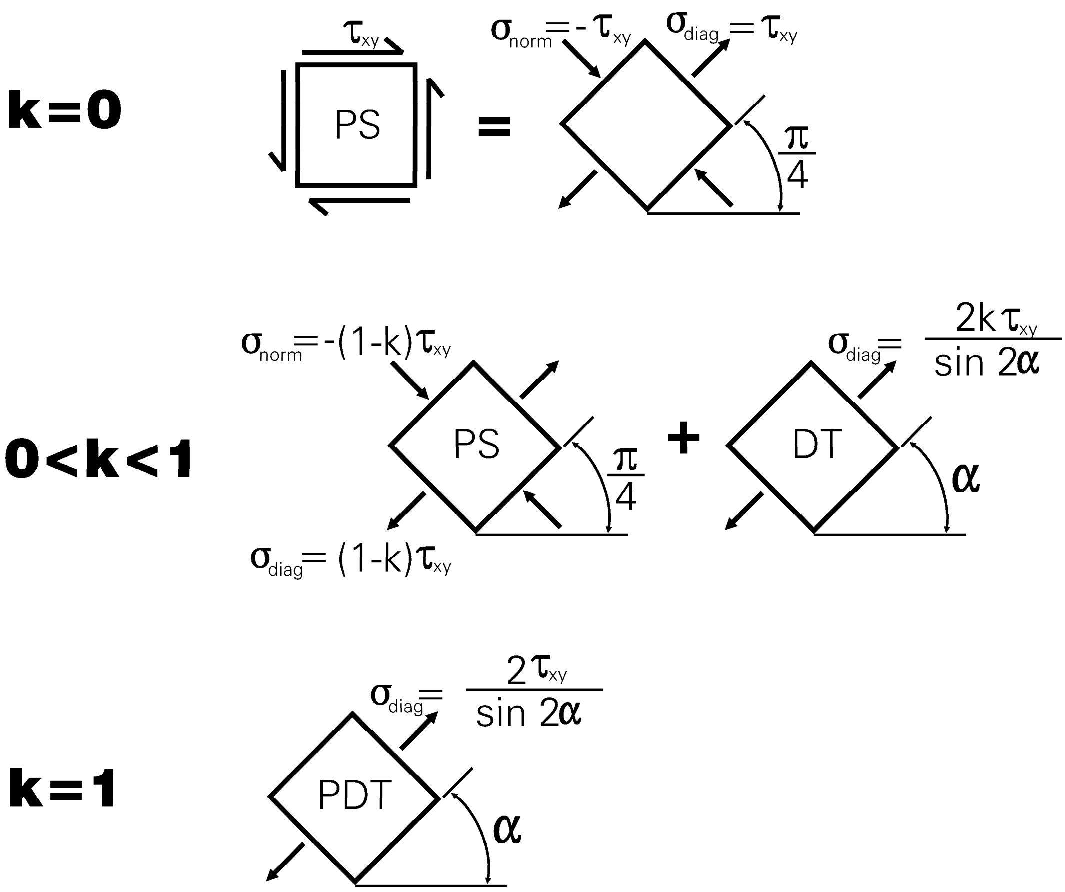


Figure 1.3: Stress systems in web

pure shear component and a diagonal tension component, respectively defined as follows:
$\tau_s = (1-k)\tau_{xy}$ and $\tau_{DT} = k\tau_{xy}$
where k is the diagonal tension factor. Figure 1.3 shows the stress state in a web for the limiting cases when $k = 0$ and $k = 1$ and also for the intermediate case.

1.3 The Grisham algorithm

In 1978 Grisham proposed an iterative algorithm that incorporates web compression-compression and shear buckling (including diagonal tension) in the internal loads solution of a linear finite element analysis [6]. Buckling is incorporated through the use of pre-strain loading of the web finite elements rather than through modifying the stiffness (thickness or elastic moduli) of the webs.

1.3.1 Outline of the Grisham algorithm

Without presenting the detailed mathematics involved[1] here, the steps in Grisham's iterative algorithm are as follows:

1. Calculate the internal loads N_i of the structure under consideration, using a *linear finite element analysis*. NOTE: the stiffness matrix of the linear finite element analysis is calculated once only and retained for following iterations; small displacement theory applies in this case.

2. Evaluate the onset of buckling using the internal loads obtained in Step 1 above, based on analytical plate buckling criteria and an interaction equation.

3. If buckling does not occur, STOP. Else, calculate the post-buckling relaxation of plates under compressive stress (σ_{x_c} and σ_{y_c}), the post-buckled shear strain $\gamma_{xy_{DT}}$ and the associated diagonal tension ($\sigma_{x_{DT}}$ and $\sigma_{y_{DT}}$).

4. Calculate the post-buckling strains ($\triangle\epsilon_x$ and $\triangle\epsilon_y$) to relieve the compression and shear stress that exceed the plate capability.

5. The post-buckling strains calculated above now become the pre-strains for the next iteration.

6. If the ratio of the final compression stress (σ_{x_c} and σ_{y_c}) in the buckled plate to the modified critical buckling stress ($\sigma_{x_{cr}}$ and $\sigma_{y_{cr}}$) approaches unity, and the diagonal tension stress ($\sigma_{x_{DT}}$ and $\sigma_{y_{DT}}$) converges, STOP. Else, go to Step 1.

1.3.2 Features of the Grisham algorithm

1. Convergence is usually achieved rapidly. Typically, five iterations are required to obtain convergence within 2 % variation between successive values of $\sigma_{x_{DT}}$ and $\sigma_{y_{DT}}$

2. Provision is made for compressive buckling in both the length and width directions of the web, as well as shear buckling. The latter causes the development of diagonal tension, accompanied by associated loading of the surrounding structure and "softening" of the buckled plate in shear.

3. The interaction of compression-compression and shear buckling is accounted for.

4. The stiffness matrix of the finite element model is not altered in any way to include for post-buckling effects.

5. Since the self-equilibrated pre-strains do not modify the stiffness matrix, precisely equilibrated and compatible solutions are obtained.

6. The pre-strains calculated give a direct indication of the degree of "softening" of the structure caused by buckling.

[1] Detailed mathematics is presented in Appendix A.

7. Multiple load cases (each with a different buckling pattern) may be processed simultaneously in a single finite element analysis.

8. Curved geometries may also be analyzed; structural symmetry can be exploited through the application of symmetric and anti-symmetric pre-strains if interaction equations are available.

9. For curved shells, the membrane loads due to pressurization are incorporated, and the stabilization effects due to hoop/longitudinal loads, are included through an interaction equation.

10. Isotropic, isotropic-stiffened plates, and plates in pure shear may be evaluated using the algorithm.