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Appendix A: Transmissibility for SDOF system
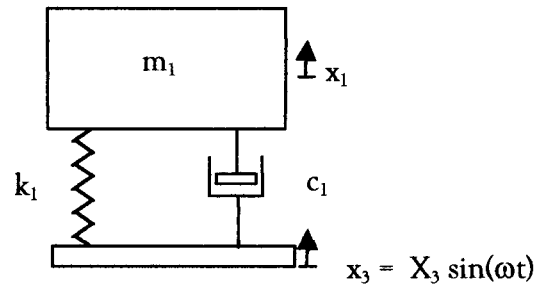
The equation of motion for the system is

$$m_1\ddot{x}_1 + c\dot{x}_1 + kx_1 = k_1 x_3 + c_1 \dot{x}_3 \qquad (10.1)$$

The Laplace transformation for eq. 10.1 is

$$m_1 X_1 s^2 + c_1 X_1 s + k_1 X_1 = k_1 X_3 + c_1 X_3 s \qquad (10.2)$$

After simplification and substituting $s = i\omega$

$$T_r = \frac{X_1}{X_3} = \frac{k_1 + ic_1\omega}{k_1 + ic_1\omega - m_1\omega^2} \qquad (10.3)$$
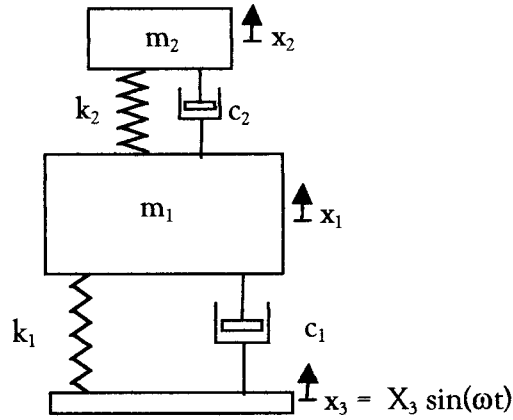
or in non-dimensional form

$$T_r = \frac{1 + i(2\zeta r)}{1 - r^2 + i(2\zeta r)} \qquad (10.4)$$

where $r = \dfrac{\omega}{\omega_n}$ and $\zeta = \dfrac{c_1}{2m_1\omega_n}$.

The magnitude for the above function can be plotted as the absolute value of the function, and the phase angle is $P = \dfrac{180}{\pi}\tan^{-1}\left(\dfrac{\mathrm{imag}(T_r)}{\mathrm{real}(T_r)}\right)$.

98

Appendix B: Transmissibility for SDOF system with absorber

The system with the absorber mass m_2 can now be modeled as a multi-degree-of-freedom (MDOF) system. The equations of motion of the system can be written as:

$$m_1\ddot{x}_1 + (k_1 + k_2)x_1 + (c_1 + c_2)\dot{x}_1 - k_2 x_2 - c_2\dot{x}_2 = k_1 x_3 + c_1\dot{x}_3$$
$$m_2\ddot{x}_2 + k_2 x_2 + c_2\dot{x}_2 - k_2 x_1 - c_2\dot{x}_1 = 0$$

$$(10.5)$$

Eq. 1.4 can be written in matrix form

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= \begin{bmatrix} c_1 \\ 0 \end{bmatrix}[x_3] + \begin{bmatrix} c_1 \\ 0 \end{bmatrix}[\dot{x}_3]$$

$$(10.6)$$

The three system variables M, K and C can be extracted from eq. 10.5

$$[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \qquad [K] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \qquad [C] = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \quad (10.7)$$

The frequency response function can be evaluated as:

$$[X] = \left[[K] + i\omega[C] - [M]\omega^2\right]^{-1}[F]$$

$$(10.8)$$

The above equation results in the form

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$(10.9)$$

H_{11} is the frequency response function for X_1/F_1, in other words, the response of m_1 relative to a force applied at m_1. The same argument applies for the other elements of the [H] matrix. For this derivation however, the system is excited through base

99

excitation. The force F_2 will thus be 0, and the Force $F_1 = k_1 X_3 + ic_1 \omega X_3$. By substituting this into eq. (10.9), the transmissibility function is obtained.

$$T_{11} = H_{11}(k_1 + ic_1\omega) \tag{10.10}$$

The absorber can be optimally tuned by minimizing the numerator of the transmissibility function of $T_{11} = X_1/X_3$. T_{11} looks like this:

$$T_{11} = \frac{\left(k_2 - m_2\omega^2 + ic_2\omega\right)\left(k_1 + ic_1\omega\right)}{\left[k_1 + k_2 - m_1\omega + i\omega(c_1 + c_2)\right]\left[k_2 - m_2\omega^2 + i\omega c_2\right] - \left(k_2 + i\omega c_2\right)^2} \tag{10.11}$$

Eq. (10.11) can also written in dimensionless form:

$$T_{11} = \frac{\left(1 + i(2\zeta_1 r)\right)\left(1 - \gamma^2 r^2 + i(2\gamma\zeta_2 r)\right)}{\left[1 + \left(\frac{1}{\gamma^2}\right)\mu - r^2 + i(2\zeta_1 r) + i\left(\frac{2\mu\zeta_2 r}{\gamma}\right)\right]\left[1 + i(2\gamma\zeta_2 r) - \gamma^2 r^2\right] - \left[1 + i(2\gamma\zeta_2 r)\right]\left[\left(\frac{1}{\gamma^2}\right)\mu + i\left(\frac{2\mu\zeta_2 r}{\gamma}\right)\right]} \tag{10.12}$$

where

$$\gamma = \frac{\omega_1}{\omega_2}, \; r = \frac{\omega}{\omega_1}, \; \zeta_1 = \frac{c_1}{2m_1\omega_1}, \; \zeta_2 = \frac{c_2}{2m_2\omega_2} \text{ and } \mu = \frac{m_2}{m_1}$$

if $\tag{10.13}$

$$\omega_1 = \sqrt{\frac{k_1}{m_1}} \text{ and } \omega_2 = \sqrt{\frac{k_2}{m_2}}.$$

The transmissibility is a minimum at the frequency where the denominator of eq. (10.10) is zero. The term $\left(k_2 - m_2\omega^2 + ic_2\omega\right)$ must thus be equal to zero, which will be true at the damped natural frequency of the second mass. The absorber should thus be designed in such a way that $\omega_d = \omega_2\sqrt{1 - \zeta_2^2}$ is equal to the exciting frequency of the system. For a given mass m_2, and damping constant c_2, the stiffness k_2 can be calculated. Substituting the relations given in eq. (10.12) and rearranging gives

$$k_2^2 - m_2\omega_{ex}^2 k_2 - \frac{c_2^2}{4} = 0 \tag{10.14}$$

where ω_{ex} is the exciting frequency.
Solving the quadratic equation (10.11):

$$k_2 = \frac{\omega_{ex}^2 m_2}{2} + \frac{\sqrt{\omega_{ex}^4 m_2^2 + c_2^2}}{2} \text{ or } k_2 = \frac{\omega_{ex}^2 m_2}{2} - \frac{\sqrt{\omega_{ex}^4 m_2^2 + c_2^2}}{2} \tag{10.15}$$

100

The second solution of k_2 yields a negative value and will thus be ignored. The isolation frequency will be equal to the original system's resonant frequency when the natural frequency ratio, γ, is equal to 1.

The natural frequencies and mode shapes can be calculated by solving the eigenvalue problem:

$$\left|[K] - \omega^2 [M]\right| = 0$$

Appendix C: Transmissibility for theoretical lever absorber (DAVI)
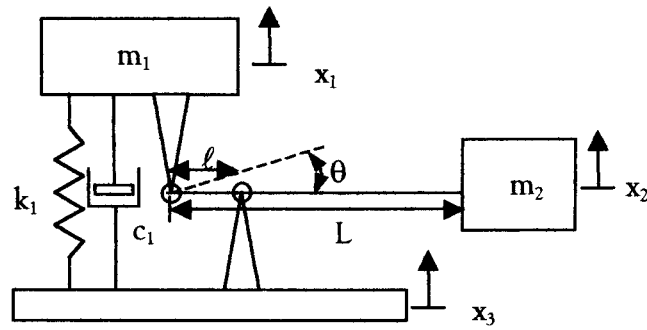
Various methods can be used to derive the equation of motion for the system, but an energy principle method would probably be the less cumbersome way to do it. The Lagrange method will be used to derive the equations of motion for most of the concepts. The first step is to express the motion of the absorber mass, x_2, in terms of the motion of the isolation mass, x_1, and the base motion, x_3.

$$x_2 = \frac{L}{\ell}x_3 - \left(\frac{L-\ell}{\ell}\right)x_1 \tag{10.16}$$

The kinetic energy of the system is:

$$E_k = \frac{1}{2}m_1\dot{x}_1^{\,2} + \frac{1}{2}m_2\dot{x}_2^{\,2} + \frac{1}{2}I\dot{\theta}^2 \tag{10.17}$$

where

$$\theta = \left(\frac{x_1 - x_3}{\ell}\right) \tag{10.18}$$

Substituting eq. (10.16) and (10.18) into eq. 10.17:

$$E_k = \frac{1}{2}m_1\dot{x}_1^{\,2} + \frac{1}{2}m_2\left(\frac{L}{\ell}\dot{x}_3 - \left(\frac{L-1}{\ell}\right)\dot{x}_1\right)^2 + \frac{1}{2}I\left(\frac{\dot{x}_1 - \dot{x}_3}{\ell}\right)^2 \tag{10.19}$$

The potential energy can be defined as:

$$E_p = \frac{1}{2}k_1(x_1 - x_3)^2 \tag{10.20}$$

Rayleigh's dissipation function is:

$$R = \frac{1}{2}c_1(\dot{x}_1 - \dot{x}_3)^2 \tag{10.21}$$

The Lagrangian formulation states:

102

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}_i}\right) - \frac{\partial E_k}{\partial x_i} + \frac{\partial R}{\partial \dot{x}_i} + \frac{\partial E_p}{\partial x_i} = F_i \qquad (10.22)$$

The external force vector F_i is zero in this application, and after substitution the equation of motion looks like this:

$$\left[m_1 + \left(\frac{L}{\ell} - 1\right)^2 m_2 + \frac{I}{\ell^2}\right]\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = $$

$$\left[\left(\frac{L}{\ell} - 1\right)\left(\frac{L}{\ell}\right)m_2 + \frac{I}{\ell^2}\right]\ddot{x}_3 + c_1\dot{x}_3 + k_1x_3 \qquad (10.23)$$

For simplicity two alternative variables are defined as:

$$M_{eq1} = m_1 + \left(\frac{L}{\ell} - 1\right)^2 m_2 + \frac{I}{\ell^2}$$

$$M_{eq3} = \left(\frac{L}{\ell} - 1\right)\left(\frac{L}{\ell}\right)m_2 + \frac{I}{\ell^2} \qquad (10.24)$$

After substitution, following the procedure described in B.1, the transmissibility of the system can be computed with the following equation:

$$T_r = \frac{X_1}{X_3} = \frac{k_1 + ic_1\omega - M_{eq3}\omega^2}{k_1 + ic_1\omega - M_{eq1}\omega^2} \qquad (10.25)$$

Eq. (10.25) can also be written in dimensionless form

$$T_r = \frac{1 + i(2\zeta r) - \gamma_d^2 r^2}{1 + i(2\zeta r) - r^2} \qquad (10.26)$$

where

$$\zeta = \frac{c_1}{2M_{eq1}\omega_{MT}}, \; r = \frac{\omega}{\omega_{MT}} \; \text{and} \; \gamma = \frac{\omega_{MT}}{\omega_I}.$$

The frequencies ω_{MT} and ω_I are the frequency of maximum transmissibility and isolation frequency respectively.
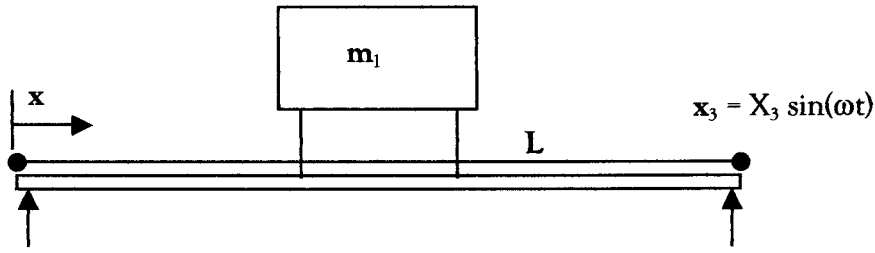
$$\omega_{MT} = \sqrt{\frac{k_1}{M_{eq1}}}$$

$$\omega_I = \sqrt{\frac{k_1}{M_{eq3}}} \qquad (10.27)$$

The phase angle can be computed with

$$P = \frac{180}{\pi} \tan^{-1}\left(\frac{\text{imag}(T_r)}{\text{real}(T_r)}\right)$$ (10.28)

The transmissibility function $\dfrac{X_2}{X_3}$ can be calculated by substituting eq. (10.25) into eq. (10.16).

Appendix D: Mode shapes of nodal beams

For a uniform beam, using elementary theory of bending of beams and vibration principles, the differential equation of motion can be written down (Rao, 1995: 524):

$$v^2 \cdot \frac{\partial^4 w}{\partial x^4}(x,t) + \frac{\partial^2 w}{\partial t^2}(x,t) = 0 \qquad (10.29)$$

Where

$$v = \sqrt{\frac{EI}{\rho A}} \qquad (10.30)$$

The variables w, E, I, ρ and A are vertical displacement, Young's modulus, moment of inertia, density and area respectively. Note that there is no external force in eq. (10.29). That is because only the mode shapes are important for the purpose of this thesis, and not the exact response of the beam.

The solution of eq. (10.29) can be found by using the method of separation of variables. The vertical displacement in terms of x is:

$$W(x) = C_1 \cos(\beta x) + C_2 \sin(\beta x) + C_3 \cosh(\beta x) + C_4 \sinh(\beta x) \qquad (10.31)$$

Where

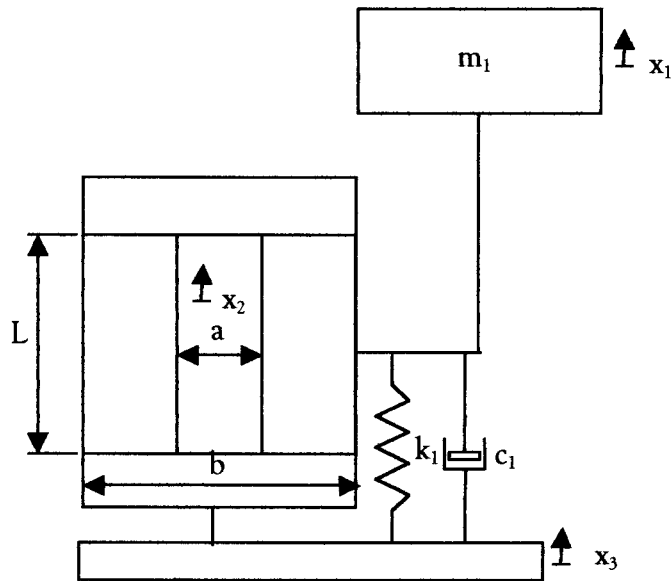$$\beta^4 = \frac{\rho A \omega^2}{EI} \qquad (10.32)$$

Applying the boundary condition for the specific application can solve the constants C_{1-4} in eq. (10.31). The boundary condition for a pinned beam is:

$$W(0, L) = 0$$
$$EI \frac{\partial^2 W(0, L)}{\partial x^2} = 0 \qquad (10.33)$$

The solution for a beam pinned on both ends thus yields:

$$W_n(x) = C_n \sin(\beta_n x) \qquad (10.34)$$

Appendix E: Transmissibility for LIVE system

The equations of motion for the LIVE system will once again be derived with the Lagrange method. The first step is to write the displacement x_2 in terms of the displacements x_1 and x_3. By applying flow continuity:

$$x_2 = \frac{b}{a}x_3 - \frac{b-a}{a}x_1 \tag{10.35}$$

The kinetic energy of the system is described by:

$$E_k = \frac{1}{2}m_1\dot{x}_1^{\,2} + \frac{1}{2}m_2\dot{x}_2^{\,2} \tag{10.36}$$

where m_2 is the mass of the fluid in the port.
After substitution,

$$E_k = \frac{1}{2}m_1\dot{x}_1^{\,2} + \frac{1}{2}m_2\left(\frac{b}{a}\dot{x}_3 - \frac{b-a}{a}\dot{x}_1\right) \tag{10.37}$$

The potential energy of the system can be written as:

$$E_p = \frac{1}{2}k_1(x_1 - x_3)^2 \tag{10.38}$$

Rayleigh's dissipation function:

$$R = \frac{1}{2}c_1(\dot{x}_1 - \dot{x}_3)^2 \tag{10.39}$$

After substitution in eq. 10.17, the equation of motion is:

106

$$\left(m_1 + \left(\frac{b-a}{a}\right)^2 m_2\right)\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 =$$

$$\left(\frac{b(b-a)}{a^2}\right)m_2\ddot{x}_3 + c_1\dot{x}_3 + k_1 x_3 \qquad (10.40)$$

For simplicity define

$$M_{eq1} = m_1 + \left(\frac{b-a}{a}\right)^2 m_2$$

$$\qquad (10.41)$$

$$M_{eq3} = \left(\frac{b(b-a)}{a^2}\right)m_2$$

After substitution, following the procedure described in B.1, the transmissibility of the system can be computed with the following equation:

$$T_r = \frac{X_1}{X_3} = \frac{k_1 + ic_1\omega - M_{eq3}\omega^2}{k_1 + ic_1\omega - M_{eq1}\omega^2} \qquad (10.42)$$
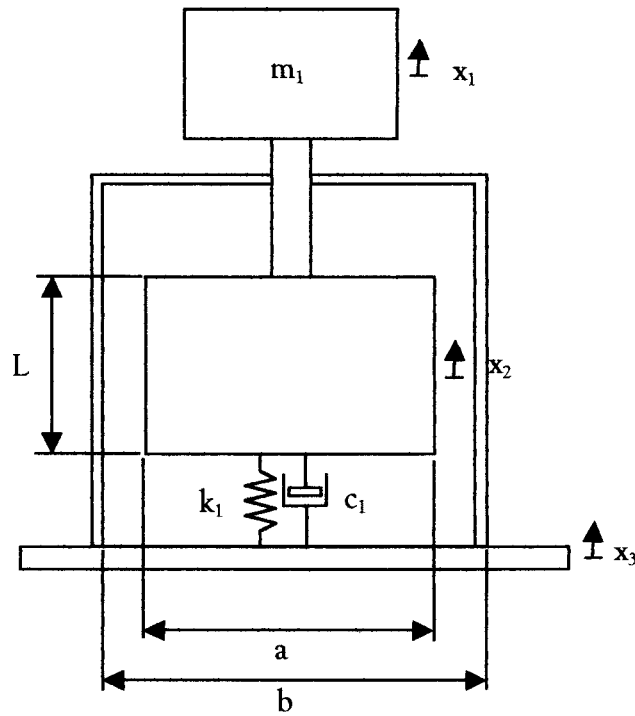
The phase angle can be computed with

$$P = \frac{180}{\pi}\tan^{-1}\left(\frac{imag(T_r)}{real(T_r)}\right) \qquad (10.43)$$

The transmissibility function $\frac{X_2}{X_3}$ can be calculated by substituting eq. (10.42) into eq. (10.35).

The isolation and MT frequencies can be evaluated eq. (10.27):

Appendix F: Transmissibility for alternative liquid absorber

The first step in the Langrange formulation is:
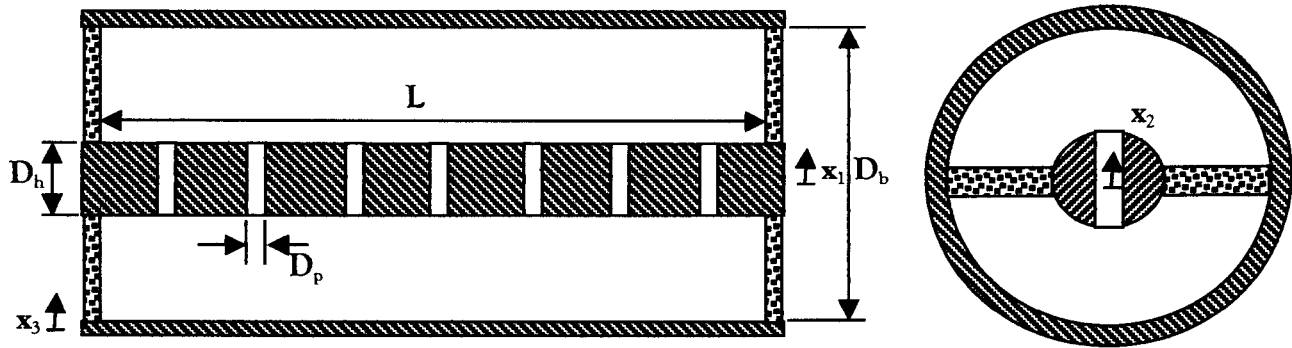
$$x_2 = \frac{bx_3 - ax_1}{b - a} \tag{10.44}$$

By following the same steps used in the previous derivations, the equivalent masses can be calculated as:

$$M_{eq1} = m_1 + m_2 \left(\frac{a}{b - a} \right)^2$$

$$M_{eq3} = m_2 \frac{b \cdot a}{(b - a)^2} \tag{10.45}$$

The transmissibility function can be calculated using eq. (10.42) and (10.43), while the MT and isolation frequencies can be calculated using eq. (10.27).

Appendix G: Transmissibility for hand arm vibration absorber

Definition of geometrical variables:

$$b = D_b L$$

$$a = n_p \frac{\pi}{4} D_p^2$$

(10.46)

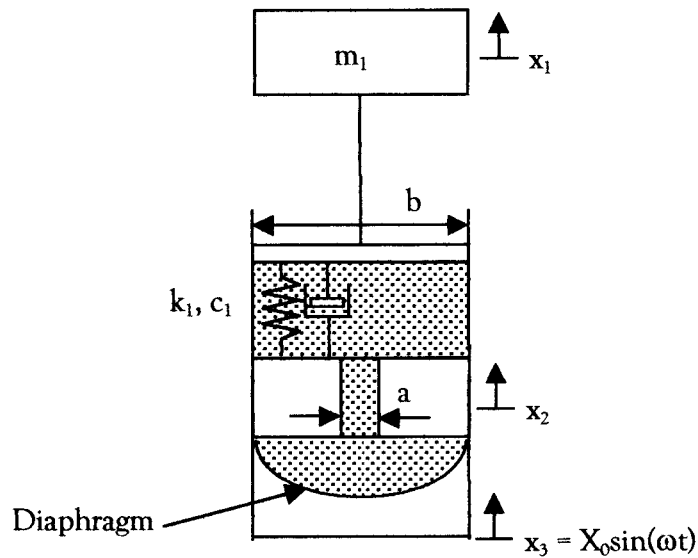Following the first step in the Lagrange formulation:

$$x_2 = \frac{b}{a} x_3 - \frac{b-a}{a} x_1$$

(10.47)

Eq. (10.47) is the same as eq. (10.35). The rest of the derivation will thus be the same as that of the LIVE system.

Appendix H: Transmissibility for diaphragm type absorber

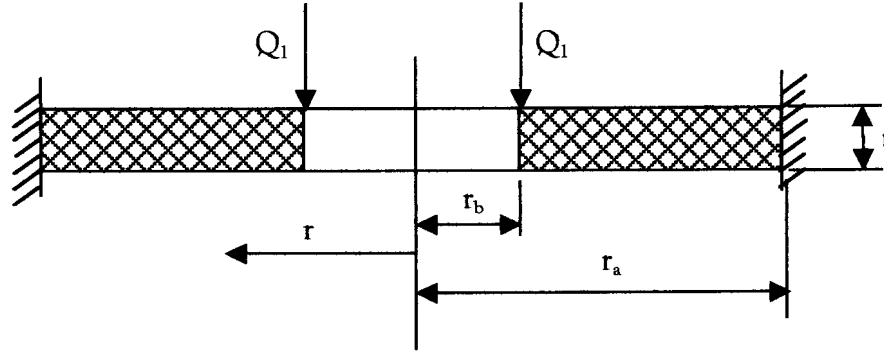The declaration of geometrical variables:

$$a = \frac{\pi}{4} D_P{}^2$$

$$b = \frac{\pi}{4} D_1{}^2$$

$$(10.48)$$

The displacement of the fluid in the port in terms of the base and handle displacements is:

$$x_2 = \frac{b-a}{a} x_3 - \frac{b}{a} x_1 \qquad (10.49)$$

The transmissibility can now be calculated as done in the previous derivations.

Appendix I: Rubber stiffness calculation

The total load $F_t = 2\pi r_b Q_1$ must be equal to the total shear force at a distance r from the center, $2\pi r Q_r$. Thus

$$Q_r = -\frac{Q_1 r_b}{r} \tag{10.50}$$

Ugural (1981,31) gives the governing shear force differential equation for axisymmetrical as

$$Q_r = -D \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] \tag{10.51}$$

where $D = \dfrac{Et^3}{12(1-v^2)}$ (E is Young's modulus, and v is Poisson's ratio.

After substitution of eq. (10.50), and integration of eq. (10.51), the displacement is

$$w = \frac{Q_1 r_b r^2}{4D}\left(\ln(r)-1\right) + \frac{c_1 r^2}{4} + c_2 \ln(r) + c_3 \tag{10.52}$$

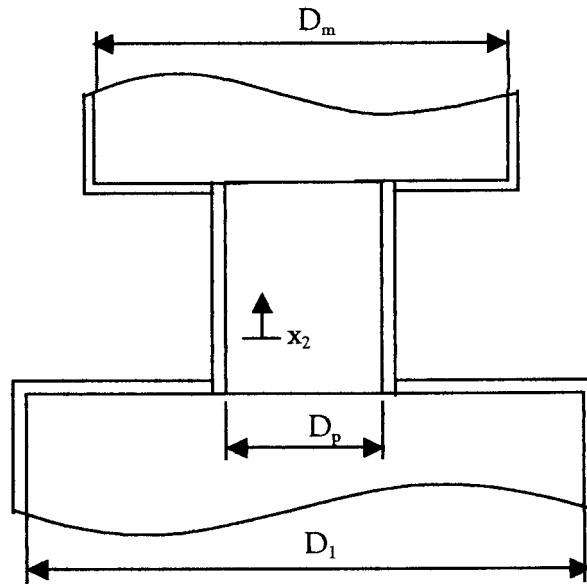The integration constants can be found by applying the boundary conditions

$$w(r_a) = 0$$
$$\frac{dw}{dr}(r_a) = 0 \tag{10.53}$$
$$\frac{dw}{dr}(r_b) = 0$$

The last boundary condition in eq. (10.53) implies that although the rubber is free to translate at $r = r_b$, it cannot rotate.

The stiffness of the rubber is thus:

111

$$k_r = \frac{2\pi r_b}{\frac{r_b^3}{4D}\left(\ln(r_b) - 1\right) + \frac{c_1 r_b^2}{4} + c_2 \ln(r_b) + c_3} \tag{10.54}$$

Appendix J: Flow damping calculation

The flow losses in the port for laminar flow is (White, 1994)

$$h_{fL} = \frac{32\mu_f L \dot{x}_2}{\rho g D_p^{\,2}} \qquad (10.55)$$

Eq. (10.55) will only be valid for a Reynolds number $R_{ed} < 2100$, where

$$R_{ed} = \frac{\rho \dot{x}_2 D_p}{\mu_f} \qquad (10.56)$$

The pressure loss due to fluid damping is thus

$$\Delta P_L = \frac{32\mu_f L \dot{x}_2}{D_p^{\,2}} \qquad (10.57)$$

The damping force can now be calculated as

$$F_L = \frac{32\mu_f L \dot{x}_2}{D_p^{\,2}} \cdot A \qquad (10.58)$$

where $A = \pi D_p L$.

If the Reynolds number is higher than 2300, turbulent flow is assumed. The flow losses for this case is

$$h_{fT} = 0{,}316 \cdot R_{ed}^{\frac{1}{4}} \cdot \frac{L\dot{x}_2^2}{2D_p g} \tag{10.59}$$

The pressure loss is then

$$\Delta P_L = 0{,}158 \cdot R_{ed}^{\frac{1}{4}} \cdot \frac{L\rho\dot{x}_2^2}{D_p} \tag{10.60}$$

The damping force is

$$F_L = 0{,}158 \cdot R_{ed}^{-\frac{1}{4}} \cdot \pi L^2 \rho \dot{x}_2^2 \tag{10.61}$$

The flow losses due to sudden contraction and expansion are also calculated. Although the port has been slightly angled to reduce flow losses, the damping calculation will make the conservative assumption that there is no diffuser action. The loss coefficient for sudden expansion is

$$K_{SE} = \frac{2h_m g}{\dot{x}_2^2} = \left(1 - \frac{D_p^2}{D_1^2}\right)^2 \tag{10.62}$$

The loss coefficient for sudden contraction is:

$$K_{SC} = \frac{2h_m g}{\dot{x}_2^2} = 0.42\left(1 - \frac{D_p^2}{D_1^2}\right) \tag{10.63}$$

The flow losses due to sudden expansion and contraction is:

$$h_{SE/C} = K_{SE/C} \frac{\dot{x}_2^2}{2g} \tag{10.64}$$

The pressure drop can be calculated as:

$$\Delta P_L = K_{SE/C} \frac{\rho \dot{x}_2^2}{2} \tag{10.65}$$

The resulting damping force is then:

$$F_L = K_{SE/C} \frac{\pi d^2 \rho \dot{x}_2^2}{8} \tag{10.66}$$

In both the turbulent flow and entrance-exit compensation, the damping force is a function of $\dot{x}_2^2$. This implies that the equivalent damping coefficient is a function of

the port velocity. Rao (1995, 227) presents an energy method to approximate this quadratic damping effect. The equivalent damping coefficient is

$$c_{eq} = \frac{8}{3\pi} a\omega X, \text{ if } F_L \text{ is in the form } F_L = a\dot{x}^2 \qquad (10.67)$$

The solution method is thus the following procedure:

- Determine the Reynolds number by using eq. 10.56 and decide turbulent or laminar flow.
- Guess a mean value for X_2 and V_2.
- Calculate the damping coefficient by adding all the above damping forces.
- Verify the values of X_2 and V_2 according to dynamic principles and correct if necessary.
- Verify the Reynolds number and iterate again if necessary