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PRETORIA

SLOPE STABILITY ANALYSES IN COMPLEX GEOTECHNICAL CONDITIONS – THRUST FAILURE MECHANISM

by

Krassimir Nikolov Karparov, PrEng

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As far as the laws of
mathematics refer to reality,
they are not certain, and as
far as they are certain, they
do not refer to reality.

Albert Einstein (1879–1955),
German-born scientist, *The Tao
of Physics,* Chapter 2 (1975).

DEDICATION

To my son Nickola, born when I was busy working on this Thesis

To my wife Zdravka Karparov, for taking the functions of mother and father in the family when I was busy

To my brother Roumen Karparov, PrEng; MSc, mother Stoyna Karparova and father Nickola Karparov for their support and encouragement during the work

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7. Anglo Coal Rock Mechanics Department for their help and support

DECLARATION

I declare that this Dissertation contains only my own original work, except where reference is made with acknowledgement to contribution from others. I also declare that this material has not been submitted for any other purpose or examination to any other Department or University.

Signed this ---------------------- day of --------------------

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Krassimir N. Karparov, PrEng; FSAIMM

SUMMARY

SLOPE STABILITY ANALYSES IN COMPLEX GEOTECHNICAL CONDITIONS – THRUST FAILURE MECHANISM

by

Krassimir Nikolov Karparov

Supervisor: Prof. Matthew Handley

Department: Mining Engineering

Degree: Philosophiae Doctor (Mining Engineering)

Key terms: slope stability, undulated formation, embedded weaker layer, co-linear flakes, cohesive zone, frictional zone, relaxation stress, active block, passive block, and safety factor.

In this thesis a previously unknown mechanism of failure in multilayered slope profiles is identified. In some conditions this mechanism does not confirm to the known failure models (relating to circular failure) used in slope stability analysis. For this reason, major failures have occurred in the artificial cuts despite the fact that the limit equilibrium methods suggest that these cuts would be stable. The limit equilibrium methods were originally created to apply to earth dam walls. In the open pit mining environment, where we face inhomogeneous and inclined multilayered structures, the assumptions of these limit equilibrium methods appear to be inapplicable (e.g. assumption for the equal shear strength along the failure surface).

Analysis starts with a general picture of the stress state in the highwall slope, given extant geological conditions and rock properties. The study then focuses on a comparison of the crack-tip stress changes in the rockmass with and without inclusions at the microscopic

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level. Basing some assumptions on binocular microscope observations of grain structures, it is possible to measure the size of the different inclusions and show that the microscopic carbon flakes present in the rock fabric make a major contribution to the failure process in a mudstone layer in the slope.

The approach adopts the fracture-process zone ahead of a crack tip as the controlling parameter of flaw propagation in rock. Flaw coalescence, which is poorly accounted for in current fracture models, is attributable to two phenomena: the flaw propagation due to high level of applied stress; and the linking of fracture-process zones due to the small distance between neighbouring flaws. A condition of flaw coalescence is given based on these two mechanisms.

This development allows defining of two zones along the failure surface (frictional and cohesive). In the slope-stability field the shear strength of the rock along the failure plane is a composite function of cohesive and frictional strength.

For instance, the relaxation stress normal to bedding, induced by overburden removal, provides an investigation method for the determination of the weakest minerals, which may act as flaws for fracture propagation in low-porosity rock. A method has been developed to determine the critical stress for tensile fracture propagation due to the rock structure and the stress reduction normal to bedding.

A proposed failure mechanism is based on the polygonal failure surfaces theory developed by Kovari and Fritz

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(1978), Boyd’s field observations (1983), Stead and Scoble’s (1983) analyses, Riedel (1929) Shear Fracture Model, Tchalenko and Ambraseys (1970), Gammond’s (1983) and Ortlepp (1997) observations for natural shear failures, computer modelling by McKinnon and de la Barra (1998), the results of many laboratory experiments reported by Bartlett et al. (1981) and the author’s experience. The proposed failure mechanism evaluates stability of the artificial slope profile due to the embedded weak layer structure, layer thickness, layer inclination and depth of the cut. On the basis of the observations and the above-mentioned modified fracture model, the slope profile is divided into two blocks; passive and active blocks. With this new model, it is possible to calculate slope safety factors for the slope failure cases studied in the industry. It has been found that, whereas the conventional slope stability models predict stable conditions, the new model suggests that the slope is only marginally stable (i.e. that failure can be expected).

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LIST OF NOTATIONS

A	Area
A_c	Actual contact area
dA	Area of an element
a	Half of the crack length
a_c	Critical half crack length
b	Half of the distance between the outer ends of two neighbor co-linear flakes with developed FPZ
c	Rock cohesion
c_j	Cohesion along the plane of weakness
c_u	Undrained shear strength
c_{av}	Average cohesion of the profile
$\bar{c}_I$	Cohesion of the Active block inner side
$\bar{c}_O$	Cohesion of the Active block outer side
CTOD	Crack tip opening displacement
D-B	Dugdale-Barenblatt
DDM	Displacement discontinuity method
d	Grain diameter
e	Half of the distance between the inner ends of two neighbor co-linear flakes
f	Half of the distance between the outer ends of two neighbor co-linear flakes
F_n	Normal load
FD	Final difference method
F_0	Initial load
FOS	Factor of safety
G	Shear modulus (Rigidity)
g	Earth acceleration
h	Depth (Chapter 2)
H	Slope height
k	K-ratio (Chapter 2); argument (Chapter 4)
k^*_H	Stress concentration coefficient for horizontal

	stress component
k^{*}_{V}	Stress concentration coefficient for vertical stress component
K_{IC}	Mode-I fracture toughness coefficient
K_q	Fracture toughness calculated using linear elasticity
ΔK_{tip}	Difference between the near the tip SIF and the field SIF
K_{tip}	Near the tip SIF
K_0	Field SIF
K^{N}_{I}	Stress intensity factor under the load $\Delta\sigma_N$
K^{D}_{I}	Stress intensity factor in the fracture process zone
K^{N}_{IA}	Stress intensity factor under the load $\Delta\sigma_N$ in point A
K^{N}_{IB}	Stress intensity factor under the load $\Delta\sigma_N$ in point B
K^{N}_{IC}	Stress intensity factor under the load $\Delta\sigma_N$ in point C
K^{N}_{ID}	Stress intensity factor under the load $\Delta\sigma_N$ in point D
l	Inclusion’s length (in Chapter 5); Length of the fracture process zone (in Chapter 6)
l_C	Critical length of the fracture process zone
l_B	Length of the Passive block failure surface
l_C	Cohesive zone length along the Passive block failure surface
l_f	Frictional zone length along the Passive block failure surface
l_I	Shear failure length along the inner side of the Active block
l_O	Shear failure length along the outer side of the Active block

m	Axes along the plane of weakness (Chapter 2) Layer thickness (Chapter 8)
n	Axes normal to the plane of weakness
OCR	Overconsolidation ratio
p	Plasticity factor
P_A	Active block load
P_F	Frictional zone load
P_C	Cohesive zone load
R_P	Passive block reaction applied to the active block
$\overline{R}$	Combined action of the active block load and the passive block reaction to the active block outer shear failure surface
SCC	Stress concentration coefficient
TAT	Tributary area theory
W	Inclusion's thickness or width

Greek symbols

α	Inclination angle (Chapters 2 and 3)
$\Delta\delta_p$	Displacement difference between two cycles during the loading process before failure
$\Delta\delta_i$	Displacement difference between two cycles during the unloading process before failure
β	Angle formed by the principal stress direction and the joint (Chapter 2) Slope angle (Chapter 3) Angle between the axes and the center point of an element (Chapter 4)
β_I	Inclination of the inner side of the active block shear failure plane
β_O	Inclination of outer side of the Active block

	shear failure plane
γ	Total unit weight of the soil
λ	Directional *cosine* matrix
ϕ	Angle of internal friction
ϕ_j	Joint plane friction angle
φ	Failure plane angle
φ_j	Dilation angle along the weak bedding plane
χ	Stress coefficient
ρ	Rock density
π	The number pi
ν	Poisson’s ratio
σ_0	Stress on element
$\Delta\sigma_N$	Normal to sedimentation stress difference
$\Delta\sigma_{ij}$	Stress difference
$\Delta\sigma_{XX}$	Stress difference of the horizontal stress components
$\Delta\sigma_{YY}$	Stress difference of the vertical stress components
$\Delta\sigma_{XY}$	Stress difference of the shear stress components
σ_H	Horizontal stress component
σ_N	Normal to the fracture stress component
σ_V	Vertical stress component
σ_H^F	Horizontal stress component calculated by FLAC
σ_V^F	Vertical stress component calculated by FLAC
$\sigma^V{}_{xx}$	Horizontal stress component in virgin condition
$\sigma^V{}_{yy}$	Vertical stress component in virgin condition
$\sigma^V{}_{xy}$	Shear stress component in virgin condition
$\sigma^R{}_{xx}$	Resultant horizontal stress component
$\sigma^R{}_{yy}$	Resultant vertical stress component
$\sigma^R{}_{xy}$	Resultant shear stress component
$\sigma^V{}_N$	Normal to sedimentation stress component in virgin condition

$\sigma^R{}_N$	Normal to sedimentation stress component calculated from resultant stress components
$\tilde{\sigma}(x)$	Tensile stress in the fracture process zone
$\sigma(x)$	Closing cohesive stress
σ_1	Local maximum principal stress at the crack tip
σ_3	Minor principal stress
σ_t	Tensile strength
σ_{tj}	Tensile strength along the sedimentation
σ_{tt}	Tensile stress
σ_c	Maximum pressure for a uniaxial compressive test (Chapter 2) Critical fracture stress (Chapter 4)
σ_{CD}	Crack damage stress
σ_N^c	Critical stress corresponding to the crack propagation
σ_N^L	Critical stress corresponding to the linking of the fracture process zone
σ_N^P	Stress of the co-linear flaws propagation and coalescence
$\Delta\sigma_N^c$	Critical value of the normal to sedimentation stress difference
$\Delta\sigma_N^P$	Normal to bedding stress difference of the co-linear flaws propagation and coalescence
$\sigma^V{}_{ij}$	Virgin stress component
$\sigma^R{}_{ij}$	Resultant stress component
$\sigma^I{}_{ij}$	Induced stress component
σ_y	Yield strength
τ	Shear stress
τ_f	Shear stress of failure
ξ	Local distance
ε	Strain
ϖ	Layers inclination angle

ϖ_A	Layers inclination angle in the active block wedge
ϖ_F	Layers average inclination angle along the passive block frictional zone
ϖ_C	Layers average inclination angle along the passive block cohesive zone