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ASSESSMENT OF FREQUENCY DOMAIN FORCE IDENTIFICATION PROCEDURES

by

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ABSTRACT

The location and magnitude of self-generated or input forces on a structure may prove to be very important for the proper evaluation at the design and modification phases, as well as in the case of control and fatigue life predictions. The identification of the input forces has also attracted a great deal of interest in machine health monitoring and troubleshooting.

Instead of being able to directly measure the force inputs, some other quantity is usually measured, e.g. the response, from which the forces can be determined indirectly. In essence the structure becomes the force transducer. Theoretically, it is possible to determine the forces by simply reversing the process of calculating the responses of a system subjected to known forces, but this procedure was shown to be ill-posed and sensitive to noise, and might contribute to meaningless results. Various matrix decomposition and regularisation methods were presented in dealing with the inverse problem.

Two frequency domain force identification procedures were evaluated in this work, i.e. the frequency response function and the modal coordinate transformation method. Both numerical and experimental studies have been presented to assess the advantages and disadvantages associated with each method.

The ultimate objective of this research was to implement these methods in an experimental investigation on a simple well-behaved structure, given the lack of experimental work pertaining to especially the modal coordinate transformation method. A single harmonic force was determined on an aluminium beam subjected to different boundary conditions. The work was then extended to predict two point sinusoidal forces from measured acceleration signals. Strain measurements have also been employed and the results noted.

Based on the results presented it was concluded that the frequency response function method was superior to the modal coordinate transformation method for the structure used in the investigations.

EVALUASIE VAN FREKWENSIEDOMEIN KRAGTE-IDENTIFIKASIE PROSEDURES

deur

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SAMEVATTING

Die posisie en grootte van self-gegenereerde of eksterne insette wat op ’n struktuur inwerk, blyk ’n belangrike aspek van die ontwerp- en modifikasiefases te wees. Die indirekte kragte identifikasie wek ook belangsteling in beheerstelsels, die voorspelling van vermoeidheidleeftyd en die veld van toestandsmonitering.

In plaas daarvan om die kraginsette direk te meet word byvoorbeeld die responsie gemeet, waardeur die kragte indirek bepaal kan word. In essensie word die struktuur die kragomsetter. Dit is teoreties moontlik om die proses vir die bepaling van die responsie van ’n struktuur wat onderhewig is aan bekende kragte, slegs om te keer; hierdie proses is egter numeries sleg-geaard en sensitief vir geraas. Dit kan nuttelose resultate oplewer. Verskillende matriks-ontbindings strategië word voorgestel om inverse probleme aan te spreek.

Die frekwensie responsie funksie metode en modal koördinaattransformasie metode word beskou in hierdie werk. Numeriese en eksperimentele studies word ondersoek om die voor- en nadele van elke metode te bepaal.

Die doelwit van hierdie ondersoek was om bogenoemde metodes eksperimenteel te ondersoek aan die hand van ’n eenvoudige struktuur, gegewe die gebrek aan eksperimentele werk met spesiale verwysing na die modale koördinaattransformasie metode. ’n Aluminium balk is onderwerp aan verskillende randvoorwaardes, terwyl ’n enkele harmoniese krag geïdentifiseer is. Die studie is uitgebrei na die bepaling van twee sinusvormige kragte vanuit gemete versnelling- en vervormingseine.

Gebaseer op die eksperimentele ondersoeke wat geloods is skyn die frekwensie responsie funksie metode beter kwantifisering van die kragte op te lewer as die modale koördinaat-transformasie metode.

"Most inverse problems of mathematical physics are
ill-posed under the three conditions of Hadamard.
In a humorous vein Stakgold pointed out that there
would likely be a sharp drop in the employment of
mathematicians if this were not the case"

(Sarkar *et al.*, 1981)

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Gloria in excelsis Deo.

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NOMENCLATURE

$A_i(t)$	i-th measured acceleration time signal
$A_{ij}(\omega)$	Inertance frequency response function for excitation at point j and the response measured at i
$[A(\omega)]$	Inertance frequency response function matrix
$[b]$	Input shape matrix
$[c]$	Output shape matrix
$[C]$	Damping matrix
$[\overline{C}]$	Modal damping matrix
$\{e(\omega)\}$	Fourier transform of the strain response vector.
$E(\omega)$	Absolute error
$[E]$	High frequency residual
$\{f(t)\}$	Force vector as a function of time
$F_j(\omega)$	Fourier transform of force input at point j
$[F]$	Low frequency residual
$\{F(\omega)\}$	Actual force vector
$\{\hat{F}(\omega)\}$	Estimated force vector
$\{F_m(\omega)\}$	Modal force vector
G_{xx}	Auto spectrum of the time function $x(t)$
G_{xy}	Cross spectrum of the time functions $x(t)$ and $y(t)$
$[h(t)]$	Impulse response function matrix
$H_{ij}(\omega)$	Frequency response function for excitation at point j and the response measured at i
$[H(\omega)]$	Frequency response function matrix
$[H(\omega)]^+$	Pseudo-inverse of the rectangular matrix $[H(\omega)]$
$[I]$	Identity matrix
j,k	Integer
$[\hat{L}]$	Linear operator in Tikhonov Regularisation
$[K]$	Stiffness matrix
$[\overline{K}]$	Modal stiffness matrix
m	Number of forces
M_i	i-th equivalent mass
$[M]$	Mass matrix
$[\overline{M}]$	Modal mass matrix
n	Number of response locations
n_a	Number of acceleration measurements
n_d	Number of averages used in the measurements
N	Degrees of freedom / number of modes
$N(\omega)$	Fourier transform of the noise contaminating the response

p	Number of participating modes
$\{p\}$	Principal/modal coordinates as a function of time
$\{P(\omega)\}$	Modal coordinates as a function of frequency
Q_r	Modal scaling factor of the r-th mode
$[Q]$	$(n \times m)$ orthogonal matrix
$[R]$	$(m \times m)$ upper triangular matrix with the diagonal elements in descending order
$s = i\omega$	Laplace variable
$[T]_r$	Residue matrix of the r-th mode
$\{u\}$	Input vector to the system
$[U]$	$(n \times n)$ matrix, columns comprise the normalized eigenvectors of $[H][H]^T$
$[V]$	$(m \times m)$ matrix, columns are composed of the eigenvectors of $[H]^T[H]$
$\{y\}$	Output vector of the system
$\{\ddot{x}(t)\}$	Acceleration vector as a function of time
$\{\dot{x}(t)\}$	Velocity vector as a function of time
$\{x(t)\}$	Displacement vector as a function of time
$x(k)$	Discrete series of a sampled time function $x(t)$
$X(n)$	Fourier series coefficients
$\{X(\omega)\}$	Fourier transform of the physical displacement vector
$\{\ddot{X}(\omega)\}$	Fourier transform of the physical acceleration vector
$[Y(\omega)]$	Strain frequency response function matrix

$\alpha_{ij}(\omega)$	Receptance frequency response function for excitation at point j and the response measured at i
$[\alpha(\omega)]$	Receptance frequency response function
$[\beta]$	Mass-normalised modal damping matrix
δ_{ij}	Kronecker delta function
$\varepsilon_f(\omega)$	Force error norm
$\{\phi\}_r$	r-th independent eigenvector (normal modes)/mode shape corresponding to the measurement point
$\{\hat{\phi}\}_r$	r-th independent eigenvector (normal modes)/mode shape corresponding to the excitation point
$[\Phi]$	Modal matrix
γ^2	Coherence function
κ_2	Condition number
λ_r	r-th complex eigenvalue
$[\Lambda]$	Diagonal modal stiffness matrix (normal mode frequencies squared)
μ	Lagrange multiplier
$\sigma_1, \sigma_2, \ldots, \sigma_m$	Singular values in the matrix $[\Sigma]$

$[\Sigma]$	$(n \times m)$ matrix with singular values of $[H]$ on its leading diagonal
ω_r	Natural circular frequencies of r-th mode $[rad/s]$
ω_r^2	r-th independent eigenvalues (natural frequencies squared)
$\{\psi\}_r$	Reciprocal modal vector, corresponding to the r-th mode
ζ_r	Modal damping factor for r-th mode

DFT	Discrete Fourier Transform
DOF	Degrees-Of-Freedom
FEA	Finite Element Analysis
FEM	Finite Element Model
FEN	Force Error Norm
FFT	Fast Fourier Transform
IDFT	Inverse Discrete Fourier Transform
IRF	Impulse Response Function
MIMO	Multiple Input Multiple Output
MMIF	Multivariable Mode Indicator Function
RBM	Rigid Body Modes
RMV	Reciprocal Modal Vector
SFRF	Strain Frequency Response Function
SVD	Singular Value Decomposition

$(\tilde{\cdot})$	Denotes contaminated values
$[\cdot]^T$	Transpose of the indicated matrix
$[\cdot]^*$	Complex conjugate (Hermitian) transpose of the indicated matrix
$[\cdot]^{-1}$	Inverse of a square matrix
$\|\cdot\|_2$	Vector 2-norm

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