Chapter 8

Conclusions and recommendations

In this chapter the capabilities of the newly implemented elements are summarized. A proposed formulation for general use is also suggested.

8.1 Isotropic membrane elements

The 8β and 9β families perform almost identical. However, since the formulation of the 8β family is simpler than that of the 9β family, the 8β family is preferable to the 9β family.

The difference between the mixed and displacement formulations is, (in terms of numerical accuracy), insignificant, and is a result of the rank one update of the mixed formulation. Once again, since the displacement formulation is simpler, this formulation is preferred above the mixed formulation.

In general, the NT-formulations outperform the other formulations and are, therefore, the preferred formulation for constraining the higher order stress field.

In general, the performance of the elements is increased if the locking correction is excluded, albeit at the cost of additional complexity in the formulation of the consistent nodal loads. In particular, exclusion of the locking correction increases the accuracy for highly distorted meshes. Notwithstanding the foregoing, the results are still acceptable if the correction is included, and membrane-bending locking is prevented. Hence, it is suggested that the locking correction is in general included. This has the additional advantage that the consistent nodal loads reduce to those of a 'standard' quadrilateral finite element with only two (translational) degrees of freedom per node.

Reduced integration improves the behavior of the elements when the locking correction is used, due to the introduction of a soft higher order deformation mode. However, for highly distorted geometries, full integration can be beneficial.

Finally, the formulation reveals some sensitivity to the numerical value of the stability parameter γ. However, the results obtained with the choice of $\gamma = G$ for the large range of problems evaluated are all acceptable, and this value is suggested for practical analyses. It is reiterated that the patch test is passed for any $\gamma > 0$. Hence, the value of γ ($\gamma > 0$) becomes irrelevant in the limit of mesh refinement.

8.2 Isotropic plate elements

The assumed strain plate element proposed by Bathe and Dvorkin is highly accurate, and almost free from locking. The element has extensively been used and tested by numerous researchers previously, and is relatively problem-free. Hence this element is an ideal plate component for flat shells which require a first order shear deformation theory, as is the case with orthotropic laminates.

For one-dimensional problems, (e.g. a simple cantilever), the residual bending flexibility (RBF) correction can be included to raise the capability of the elements to exactly the linear strain level.

However, for two-dimensional problems, selection of the characteristic lengths is not simple, and some over displacement is noted for meshes of intermediate refinement. Hence, and notwithstanding the fact that the effect of the correction disappears in the limit of mesh refinement, it is suggested that the formulation of Bathe and Dvorkin is in general used without the residual bending flexibility correction.

8.3 Isotropic shell elements

The 8β/SA and 9β/SA families perform very well for the test problems considered in this study, with the settings for the 8β and 9β membrane elements suggested in Section 8.1. Viz, the membrane locking correction is included, the displacement formulation is used and full integration is used.

The competing 5β/SA family proposed by Di and Ramm performs very well for the regular meshes, and in general even outperforms the 8β/SA and 9β/SA families. However, the 5β/SA family becomes increasingly inaccurate for distorted meshes. In addition, the 5β/SA family is ineffective for warped geometries, since the lack of drilling degrees of freedom complicates the use of this family for warped geometries.

8.4 Orthotropic formulation

For an orthotropic constitutive relationship, the 8β/SA and 9β/SA families proposed herein perform very well. However, the sensitivity to the value of γ, and the effect of the membrane locking correction become more pronounced. Nevertheless, the value of $\gamma = G_{12}$ suffices, and is recommended for general use.

As opposed to the isotropic formulation, it is suggested that the membrane locking correction *may* be excluded for the combination of computationally expensive orthotropic problems and highly distorted meshes. Viz, if the 8β/SA family is used in a global optimization infrastructure, it might well be beneficial to accept the increased complexity in the formulation of the consistent nodal loads when the locking correction is excluded, at the gain of a dramatic increase in accuracy.

Finally, it is noted that the superiority of the 8β/SA and the 9β/SA families over the

5β/SA family for distorted geometries becomes even more pronounced for orthotropy than for isotropy.

8.5 Recommendations

As opposed to the general trend to use higher order finite elements for the analysis of orthotropic structures, it is demonstrated that the relatively simple flat shell finite elements with an assumed stress membrane interpolation, drilling degrees of freedom and an assumed strain plate interpolation recommended in this study, suffice. This, potentially, significantly reduces the cost of orthotropic analyses.

- It is recommended that the 8β/SA element proposed herein is used for the analysis of orthotropic shell problems, with $\gamma = G_{12}$, full integration, and inclusion of the membrane locking correction.

  For coarse, highly distorted meshes, the locking correction could be excluded to increase accuracy (e.g. when using the elements in an optimization algorithm).

- An investigation into low order enhanced strain formulations is suggested as a fruitful research area to further investigate the efficiency of low order finite elements for the analysis of orthotropic structures.

- The formulation of a non-flat shell element with in-plane drilling degrees of freedom, and a formulation for geometric and material non-linearity would be desirable.