Chapter 1

Introduction

1.1 Motivation

Composite materials are quite different from metals. Composites are combinations of materials differing in composition or form where the individual constituents retain their separate identities and do not dissolve or merge together. These separate constituents act together to give the necessary mechanical strength or stiffness to the composite part.

Composites in structural applications have the following characteristics [2]:

- They generally consist of two or more physically distinct and mechanically separable materials.
- They are made by mixing the separate materials in such a way as to achieve controlled and uniform dispersion of the constituents.
- Mechanical properties of composites are superior to, and in some cases uniquely different from, the properties of their constituents. This is clearly seen with glass-reinforced plastics. In the case of glass-reinforced plastics, the epoxy resin is a relatively weak, flexible, and brittle material, and although the glass fibers are strong and stiff, they can be loaded in tension only as a bare fiber. When combined, the resin and fiber give a strong, stiff composite with excellent toughness characteristics.

In recent years, composite materials have been used increasingly in various engineering disciplines because of their versatility. For example, composites are used in the space, aircraft, automotive, boat and locomotive industries [2].

However, there is also a great number of difficulties associated with composites materials, one of which is their increased complexity. Because of this complexity, the design process of composites is ideally performed using optimization methods. Unfortunately, determination of the optimum parameters is not simple and can be expensive since a large number of locally optimal stacking sequences typically appear, which implies that this problem is a global optimization problem [3].

Usually, composite structures are geometrically quite complex, both in-plane and trough the thickness, and they cannot be solved by analytical methods. This implies that numerical methods have to be used for the analysis and design of these structures, of which the finite element method seems to be the most suitable. Because the finite elements method is an approximate method, convergence is only obtainable in the limit of mesh refinement and, since, in optimization, a complete finite element analysis yields only a single design iteration, it is desirable to perform the finite element analysis with minimal computational effort.

Hence, it is desirable to keep the cost of forming the element stiffness matrices and the cost of solving the equilibrium equations as low a possible. Elements that are 'inexpensive' are normally inaccurate, implying that highly refined meshes may be required to obtain convergence, with implied high assembly and solution costs. 'Visa versa', higher order elements typically require less elements on the structural level, but are associated with expensive evaluations of the element stiffness matrix, as well as increased connectivity on the structural level.

Therefore, advanced low order formulations are a natural candidate for the analysis of orthotropic shell structures. This is in particular true when these elements are included in a computationally expensive optimization infrastructure. However, there is a surprisingly small literature base devoted to the behavior of simple composite shell elements. This is possibly a result of earlier convictions that high order elements were a requirement when analyzing composites [4].

Flat orthotropic shell elements are not a popular choice for the analysis of orthotropic structures. Typically, doubly curved quadratic or even cubic elements are formulated. However, orthotropy is independent of shell curvature, and the results will mainly depend on the kinematic ability and accuracy of the flat shell element.

Both drilling degrees of freedom and assumed stress interpolations have the potential to improve the modeling capabilities of, in particular, low order quadrilateral finite elements. Therefore, it seems desirable to formulate low order elements with both an assumed stress interpolation field and drilling degrees of freedom, on condition that the elements are rank sufficient and invariant. These elements can truly be called advanced, state-of-the-art finite elements for the analysis of orthotropic structures.

1.2 Objectives

This study has the ultimate objective of suggesting advanced low order finite elements for the linear analysis, and ultimately the global optimization, of orthotropic shell structures.

The main requirement is the formulation of a suitable shell finite element for symmetric orthotropy. The element must have an acceptable balance between

- element cost and
- numerical accuracy.

Further requirements for the element are:

- The final formulation must be rank sufficient, invariant and robust.
- None (or as few as possible) adjustable numerical parameters should be present.
- The ability to model general warped geometries should be included.

1.3 Approach

Firstly, suitable isotropic assumed stress membrane elements with in-plane drilling degrees of freedom are formulated.

Secondly, the selected membrane elements are combined with a suitable plate element to form flat shell finite elements.

Finally, the constitutive relationship of the flat shell elements is extended to provide for symmetric orthotropy.

Extensive numerical evaluation is performed in order to obtain information on element performance, convergence rates, sensitivity to distortion, etc.

1.4 Thesis overview

In Chapter 2 assumed stress membrane finite elements with drilling degrees of freedom are discussed. This chapter starts with the variational formulation of these elements, as formulated by Hughes and Brezzi [5]. The membrane locking correction proposed by Taylor [6] is also introduced in this chapter. Chapter 3 presents numerical results for the membrane elements formulated in Chapter 2.

Chapter 4 starts with the formulation of the plate element proposed by Bathe and Dvorkin [7]. This plate element is then combined with the above mentioned membrane elements to form flat shell elements. Numerical results are presented in Chapter 5.

The constitutive relationship of the flat shell elements is extended in Chapter 6 to accommodate layered symmetric orthotropy. Numerical results for these elements are presented in Chapter 7.

The capabilities of the proposed elements are summarized in Chapter 8.

The operators arising from the finite element interpolation are summarized in Appendix A.

The stress mode classification after Feng *et al.* [8] is presented in Appendix B.

The transformation operators for the constrained stress fields are presented in Appendix C.

The different integration schemes used in this study are presented in Appendix D.

In Appendix E fragments of the source code developed during this study are presented.

In Appendix F a list of some of the terms used in this study are given.