Chapter 4

Basic Analytical Theories and Similar Studies Conducted by other Agencies

4.1 Background

There are three groups of methods used to design concrete pavements. One group of methods is based upon observations of the performance of full-scale pavements. The other group of methods is generally based on stresses calculated in the pavements and compared to the flexural strength of concrete [67]. A third group of design procedures (mechanistic-empirical) are normally used by many design catalogues [60].

Most of the methods used for a mechanistic type of approach were based on the work done by Westergaard around 1925. Westergaard formulas were simplified by the provision of design charts and table [39].

Meyerhof developed another method in the early 1960s and this method is said to be well adjusted and correlated with full-scale field data. Falkner et al developed an equation by using three-dimensional finite elements and Shentu et al developed a finite element model in 1997.

4.2 Theoretical analysis approaches

4.2.1 Westergaard (1926)

Westergaard formula assumes that slab acts as a homogenous, isotropic, elastic solid in equilibrium and that the reactions of the foundations are vertical and proportional to deflections of the slab. Distinction between three cases of load was made viz interior, edge, and corner loads [71]. For the corner and edge loadings, the load is applied adjacent to the edges, thus the formula does not consider loads applied at a distance from the edges and corners. The following are convenient formats of Westergaard equations:

Internal Load :

$$\sigma_{max} = \left[0.275(1+\mu)\frac{P}{h^2}\right] * \log_{10}\left[0.36\frac{Eh^3}{Kb^4}\right] \quad \Longrightarrow \text{Eq.4-1}$$

$$Zi = \frac{P}{8Kl^2} \quad \Longrightarrow \text{Eq.4-2}$$

Edge Load :

$$\sigma_{max} = \left[0.529(1+0.54\mu)\frac{P}{h^2}\right] * \log_{10}\left[0.2\frac{Eh^3}{Kb^4}\right] \quad \Longrightarrow \text{Eq.4-3}$$

$$Ze = \frac{1}{\sqrt{6}}(1+0.4\mu)\frac{P}{Kl^2} \quad \Longrightarrow \text{Eq.4-4}$$

Corner Load :

$$\sigma_{max} = \frac{3P}{h^2}\left[1 - 1.41\left[12(1-\mu^2)\frac{Kb^4}{Eh^3}\right]^{0.25}\right] \quad \Longrightarrow \text{Eq.4-5}$$

$$Zc = \left(1.1 - 0.88\frac{a_1}{l}\right)\frac{P}{Kl^2} \quad \Longrightarrow \text{Eq.4-6}$$

$b = [1.6r + h] - 0.675h$ for $r < 1.72h$

$b = r$ for $r \geq 1.72h$

$$l = \left[\frac{Eh^3}{12(1-\mu^2)K}\right]^{0.25} \quad \Longrightarrow \text{Eq.4-7}$$

where :

P = First crack load (N).

σ_{max} = Maximum stress (N/mm^2).

μ = Poisson's ratio (0.15 - 0.2).

h = Slab depth (mm).

E = Modulus of elasticity (N/mm^2).

K = Modulus of foundation reaction (N/mm^3).

l = Radius of relative stiffness (mm).

Zi = Vertical deflection for interior load case (mm).

Ze = Vertical deflection for edge load case (mm).

Zc = Vertical deflection for corner load case (mm).

a_1 = Distance between load center and corner(mm).

4.2.2 Meyerhof (1962)

The Concrete Society Technical Report No. 34 suggested limit values for moment of resistance. It was assumed that limit moment of resistance formula for plain concrete should be considered for SFRC when dealing with corner cases of loading

[39]. On the other hand, steel fiber manufacturer design guidelines for designing SFRC ground slabs have included the effect of the steel fibers on the limit moment of resistance when dealing with corner loads [11]. Three cases of loading where given by Meyerhof as follows [72]:

Internal Load:

$$P_i = 6\left[1+\frac{2a}{l}\right]M_0 \qquad \Longrightarrow \text{Eq.4-8}$$

Edge Load:

$$P_e = 3.5\left[1+\frac{3a}{l}\right]M_0 \qquad \Longrightarrow \text{Eq.4-9}$$

Corner Load:

$$P_c = 2\left[1+\frac{4a}{l}\right]M_0 \qquad \Longrightarrow \text{Eq.4-10}$$

Where:

P_i = Ultimate interior load.

P_e = Ultimate edge load.

P_c = Ultimate corner load.

a = Contact radius of load.

M_0 = Limit moment of resistance of slab.

l = Radius of relative stiffness.

Formulas for resistance moment as given by steel fibers manufacturer design catalogue are as follows [11]:

For plain concrete:

$$M_0 = f_{ct}\frac{bh^2}{6} \qquad \Longrightarrow \text{Eq.4-11}$$

SFRC:

$$M_0 = \left[1+\frac{R_{e,3}}{100}\right] * f_{ct}\frac{bh^2}{6} \qquad \Longrightarrow \text{Eq.4-12}$$

Where:

$R_{e,3}$ = Equivalent flexural factor of SFRC.

b = Unit width of slab.

l = Depth of slab.

f_{ct} = First crack strength.

Steel fiber manufactures have given the equivalent flexural strength factors Re,3 for different steel fiber content relevant to their steel fiber. The values consider the steel fiber parameters [11]. In comparison with plain concrete (equation 4-11 and equation 4-12) Re,3 is the amount of improvement associated with SFRC for a specific steel fiber content.

4.2.3 Falkner et al (1995)

A design proposal was generated by using a 3-D finite element model to assess the ultimate load capacity of a centrally loaded slab. The proposal was correlated with experimental data. The model is based on the plastic theory and takes into account two limit states, one is the first cracking load (Westergaard load) and the other is the ultimate load [73]. The following is the proposed model:

$$F'_u = P\left[1+\left(\frac{K}{Eh^3}\right)^{0.25} W\frac{\sqrt{A}}{h}\right]\left[1+\frac{R_{e,3}}{100}\right] \Longrightarrow \text{Eq.4-13}$$

Where :

F'_u = Ultimate load capacity (N).

P = First crack load from Westergaard (N).

W = Width of slab (mm).

A = Area of load (mm^2).

K = Modulus of foundation reaction (N/mm^3).

E = Modulus of elasticity (N/mm^2).

h = Slab depth (mm).

$R_{e,3}$ = Equivalent ratio.

The above model can be used to estimate the ultimate load capacity for concrete both with and without steel fiber. For plain concrete, the value of equivalent flexural factor is to be substituted as zero. The model has the limitation that it is not considering corner and edge cases of loading.

4.2.4 Shentu et al (1997)

In 1997, Shentu et al used a finite element model (ring-like elements with triangular cross sections) assuming a Winkler sub-grade to develop a simple formula to determine the ultimate load-carrying capacity of a plain concrete slab, with large plan dimensions subjected to an interior concentrated load. Shentu's model uses the uniaxial tensile strength in-lieu of the flexural strength. Models to assess the equivalent tensile strength of SFRC do not yet exist and the model is considered of less use when dealing with SFRC. The model also has the limitation that it is not applicable to edge and corner load cases. The ultimate load capacity can be given as follows: [74].

$$P_{sh} = 1.72\left[\left(\frac{Ka}{E}\right)*10^4 + 3.6\right] f_t' h^2 \Longrightarrow \text{Eq.4-14}$$

Where :

P_{sh} = Ultimate bearing capacity (N).

f_t' = Uniaxial tensile strength of concrete (N/mm^2).

K = Modulus of foundation reaction (N/mm^3).

a = Radius of loaded area (mm^2).

E = Modulus of elasticity (N/mm^2).

h = Slab depth (mm).

4.3 Similar studies conducted by other agencies

Different researchers carried out full-scale test studies on slabs. A semi full-scale study was conducted by Kaushik et al in India in 1989. The study included the three critical load cases (interior, edge and corner) [75]. Beckett did a series of tests in 1990 to compare the load capacity of plain and SFRC slabs subjected to interior loading [76]. Later in 1999 he carried out an investigation to evaluate the load capacity of two slabs subjected to corner and edge loading [77]. Falkner et al performed a comparative study of strength and deformation behaviour of plain and SFRC slabs [73].

4.3.1 Kaushik et al (1989)

A semi full-scale test was conducted to compare the load capacity for plain and SFRC slabs. The effect of the steel fiber dosage on the load capacity of the interior, edge and corner load cases was evaluated. The result of the study is summarized in table 4-1.

Table 4-1: Results from Similar Previous Tests (Kaushik et al 1989)

Slabs	Fiber content % By volume	Interior		Edge		Corner	
		F.C (KN)	D. (mm)	F.C (KN)	D. (mm)	F.C (KN)	D. (mm)
P.C.	No Fibers	45.11	1.786	40.66	4.68	29.35	3.95
SFRC2	0.5	80	2.6	70	7.35	39	4.822
SFRC3	1.0	120	2.71	78.81	6.61	59.82	7.02
SFRC4	1.25	180	3.9	100	_	69.84	2.957
SFRC5	1.5	150	3.11	120	7.27	74.46	11.18
SFRC6	2.0	145	_	90.94	4.14	50	7.83
Slabs 1.8x1.8x0.1m/300 mm diameter loading plate/ (H) aspect ratio 80, diameter of 0.456 mm							
(H.) Hook-ended steel fibers /F.C = First crack load/ D= Measured deflection							

The study concluded that:

- The addition of the steel fibers changes the mode of failure of plain concrete from sudden failure (immediately after first crack) to a gradual relaxed failure.
- Fiber content between 0.5% and 2.0% by volume yields a significant improvement in the load carrying capacity of the SFRC pavements. The rate of increase in the load carrying capacity is significant up to 1.25% by volume (85 kg/m^3) beyond which the rate of gain in strength is not substantial, Fibers volume of 1.25% is therefore been observed to be an optimum steel fiber volume.

4.3.2 Beckett (1990)

The study was conducted to compare plain, fabric reinforced and SFRC slab subjected to an interior loading via a single load plate. The results relative to plain and SFRC slabs are presented in table 4-2. The readings in the fourth column indicated that either the jack capacity is exceeded prior the failure or something went wrong with the experiment.

Table 4-2: Results From Similar Previous Tests (Beckett 1990)

Slab	Fibers	Load at First crack (KN)	Load at failure (KN)
P.C.	No Fibers	180	200
SFRC1	(60/100) H. 20 kg/m^3	220	350
SFRC2	(60/80) H. 20kg/m^3	260	390
SFRC3	(60/100) H. 30kg/m^3	240	340
SFRC4	(60/80) H. 30 kg/m^3	290	>345
SFRC5	Mill cut 30 kg/m^3	180	200
Slabs 3.0x3.0x0.15m / Concrete grade 40 /100x100 loading plate/ K-value =0.035 N/mm^3 H: Hook-ended steel fibers			

The following conclusions were given:

- The plain concrete and the mill cut fiber-reinforced slabs shows to have no significant post cracking behaviour.
- The performance of 60/80 hook-ended steel fiber reinforced slabs is superior to the 60/100 fiber reinforced slabs. Increasing the steel fiber dosage in both cases increases the post crack behaviour.
- Comparison of the measured results to the calculated results using Westergaard equations revealed that Westergaard's approach has its limitations if applied to SFRC.

4.3.3 Falkner et al (1995)

The study aimed to investigate and compare the strength and deformation behaviour of plain and SFRC subjected to interior load. A formula was generated using the tested results and finite element model. The study results are abstracted in table 4-3, while the formula is given in equation 4-13. The readings of the first crack at the second column are worrying. It might have to do with method used to calculate the first crack strength.

Table 4-3: Results From Similar Previous Tests (Falkner et al 1995)

		First crack		Failure		K-value (N/mm^3)
		Tested	Wester.*	Tested	Falkner*	
P1	No Fibers	80	100	180	160	Cork sub base
P2	Mill cut 30 kg/m^3	80	100	210	180	0.025
P3	(60/80) H. 20kg/m^3	80	100	240	240	
P4	(60/80) H. 30 kg/m^3	100	120	380	340	Rubber sub base
P5	Mill cut 30 kg/m3	100	120	235	260	0.05
P6	No Fibers	100	120	220	220	
Slabs 3.0x3.0x0.15m / Concrete grade 35 /120x120 loading plate/ Interior load						
H. Hook-ended steel fibers						
Wester* Calculated first crack load using Westergaard equation.						
Falkner* Calculated ultimate load using Falkner et al equation.						

The study came to the following conclusions:

- The deformation behaviour of plain and SFRC slabs can be divided into three regions:
  - Region (i): The un-cracked state, where the slabs show linear-elastic behaviour.
  - Region (ii): The first radial crack occurs in the center of the slab, developing gradually till the main crack can be seen at the slab edge.
  - Region (iii): Presents the redistribution of stresses with in slab, while plastic hinge lines are formed along the main cracks and the slab develops final failure pattern.
- The main difference in the strength and deformation behaviour of the plain and SFRC slabs is seen in region (iii). While the plain concrete slab fails at an early age (by punching), the SFRC slab is able to distribute its stresses until the plastic hinges occur at main cracks. In this stage the SFRC slab can still maintain its slab action and the load can be increased until ultimate failure occurs.

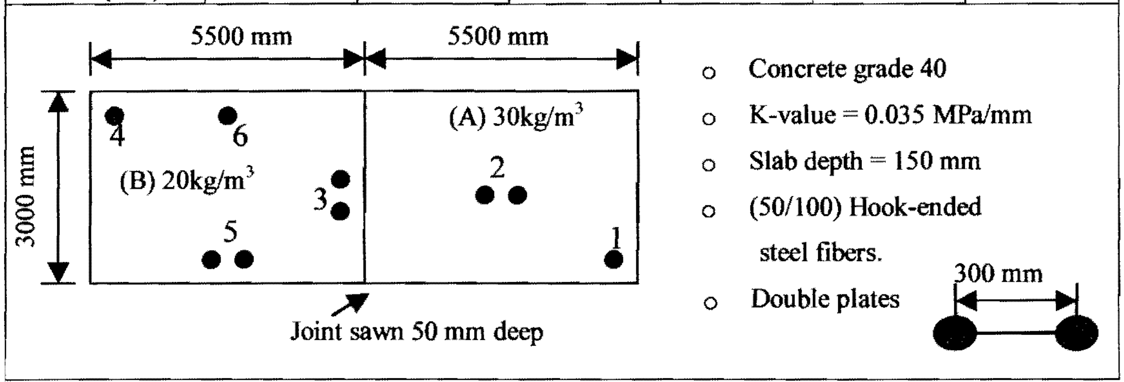
Table 4-3 show that the slab P3 with 60/80 hook-ended steel fibers has similar first crack load capacity value to the plain concrete slab. This results does not agree with the result gained by Beckett in table 4-2 (slab notified SFRC2), which shows the 60/80 hook-ended steel fiber slab to perform better than the slabs with other fiber parameters (bearing in mind that the same dosage was used).

4.3.4 Beckett (1999)

Test conducted (as a continuation of the 1990 tests) on two slabs constructed together and separated with a sawn joint as it can be seen in the sketch in table 4-4. The study aimed to investigate the corner and edge loadings of the SFRC using single and double loading plates. In addition to that, the tested values were compared to Meyerhof load and Westergaard load and deflection. Table 4-4 summarizes the results.

Table 4-4: Results From Similar Previous Tests (Beckett 1999)

Property \ Test	Corner 1	Internal 2	At joint 3	Corner 4	Edge 5	Edge 6
Loading plate (mmxmm)	100x100	2/(100x100)	2/(100x100)	100x100	2/(100x100)	100x100
First crack load (KN)	77.5	370	280	85	180	180
Maximum load (KN)	77.5	380	380	100	190	190
Meyerhof load (KN)	71.06	286.1	_	71.5	192.8	147.6
Westergaard load (KN)	58.9	105.1	_	59.2	79.15	55.5
Westergaard deflection (mm)	2.6	0.63	_	1.7	1.17	0.81
Test deflection for Westergaard load (mm)	2.1	1	_	1.8	1.6	1.6



It was concluded that:

- By increasing the plan dimension of the test slabs from (3x3 m) to (11x3 m), it was possible to develop negative partial circumferential yield lines in the top of the slab.
- The use of double load plates centered at 300mm apart does not appear to have an adverse effect on the load to first crack compared with tests using single loading plate.
- Tested deflections for Westergaard loads are approximately equal to the calculated ones using the same load.
- Meyerhof loads agreed with the maximum applied load.

It is noticeable that, the study does not compare plain and SFRC. Moreover, free edges were not compared.

4.4 Conclusion

- Although Westergaard's approach considers the three load cases, it is not suitable for applications involving SFRC. The theory was basically developed to consider the elastic behaviour of the material. Thus, it does not consider the after cracking strength of the SFRC. Comparison of measured results with the calculated results indicated that Westergaard approach has its limitation when used for the SFRC.
- Falkner and Shentu's approaches are found to consider the after cracking strength, but on the other hand they do not consider the three load cases. In addition, Shentu's model requires the measurement of the tensile strength using direct tensile testing, which is difficult to measure.
- Meyerhof's model is found to consider both the after cracking strength and the three cases of loading. It can be seen from the tabulated results in table (4-4) that the measured and calculated values agreed. The only drawback on the model is that it does not calculate the deflection and it is said that the deflection is checked indirectly through adjusting the model using existing plain concrete pavements.
- Hook-end steel fiber is once again proven to have the best performance. It was found that a significant increase in the load capacity is attainable with fiber dosage up to 1.25% by volume (approximately 95 kg/m^3).
- Three phases for the failure pattern are found for both plain concrete and SFRC slabs viz: the un-cracked phase, the first radial crack and the final failure pattern. The effect of the steel fibers is apparent on the third phase. The fan pattern type of failure is not achievable with slab dimensions of 3.0x3.0 m while it is possible with a slab of 3.0x11.0 m, which indicates that the slab size has its influence to the pattern of failure.
- It can also be noticeable that fiber dosages less than 20kg/m^3 were not considered. Tests to give the entire view were not conducted, in other words the corners and the edges were tested and compared independently of interior load. The only test using all three-load cases uses relatively small slabs, which might not be satisfactory to get a holistic view.