TOWARDS A BETTER UNDERSTANDING OF THE MOISTURE-DENSITY- STRENGTH RELATIONSHIP OF COMPACTED SOIL

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ABSTRACT

The moisture-density (MDD) graph can be replaced by the voids ratio-water ratio graph (E-R graph) which is a more suitable indicator of soil strength. The voids ratio E, and the water ratio R, are ratios of volume and are the true parameters of a soil's strength, being independent of mass. Mathematical equations for the E-R and MDD curves are developed enabling the peak condition of strength to be obtained from only one M/D test if the soil's particle relative density is known. Compression strength (resistance to further densification) and CBR strength (resistance to particle dislocation) are related and the in-situ and soaked CBR for a compacted soil layer can be assessed by graphical or mathematical means.

1 INTRODUCTION: THE MDD TEST

The moisture density test is conducted on a soil in order to ascertain the maximum density achievable when applying a given effort. This generally necessitates the compaction of a number of soil samples in a standard mould at different moisture contents and the recording of the dry densities so obtained against their moulding moisture content. By means of a graphical plot of these densities against moulding moisture the so called maximum dry density for the soil for the compactive effort applied can be assessed. This maximum dry density or MDD is generally performed on gravels using the Modified Effort and is generally referred to as the Modified AASHTO density for the soil. The performance of this test is fully described in TMH1 (1996) and ASTM D1557-9. (2009), Figure 1 is a typical example of an MDD curve.

A study of this curve shows the following observations:

- The curve may be said to exhibit a dry leg where the moulding moisture is relatively low.
- There is also a wet leg where the moulding moisture is in a state of virtual saturation.
- These legs are joined by an arc or transition curve which indicates the maximum density for the effort applied.
- The radius of the arc is not generally defined and is often based on experience.
- The two legs, if produced above the arc intersect on a vertical line passing through the centre of the arc indicating the moulding moisture or optimum moisture content (OMC) at this density.
- The dry leg may be said to be the mirror image of the wet leg.

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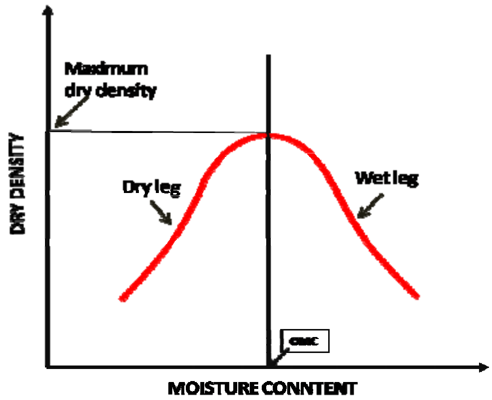


Figure 1: A typical MDD curve.

A further study has led the writer to make some additional observations (Figure 2):

- The wet leg tends to run into the condition where the degree of saturation is 90%.
- The dry leg thus tends towards having a slope equal to that of the 90% degree of saturation line but of opposite value.
- The maximum density occurs when the degree of saturation is virtually 80%.
- The position of these saturation lines is dependent on the relative density of the soil particles which may vary from soil to soil.
- The degree of saturation lines and the wet and dry legs are not straight lines but curved.

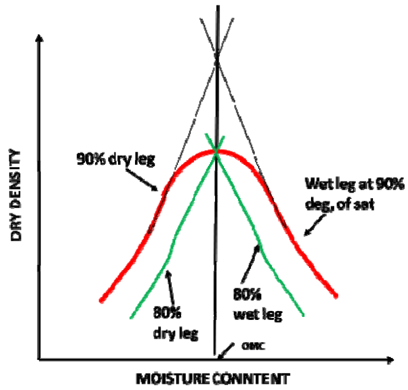


Figure 2: The MDD peaks at S= 80%.

Two different soils, A and B, were each tested for maximum density. The MDD graph showed that their maximum densities were the same at 2,012 t/m^3. This fact may tend to indicate that both soils had the same strength. This however was not so. Soil B, in fact, was much the stronger with a CBR almost twice that of soil A. Soil A had a particle relative density of 2,70 while that of soil B was 2,55. Clearly, density in itself is not a true measure of strength. We compact a soil in order to improve its strength, but although the density is increased the exercise is actually one of reducing the volume of the voids relative to that of the solids. It would thus appear to be more logical to measure the ratio of voids to solids in a compacted soil rather than density.

2 THE E-R CHART

Note: In the interests of consistency with other soil symbols the writer elects to use the upper case N and E, to represent porosity and voids ratio respectively rather than the conventional n and e as used in the field of Geomechanics (Figure 3).

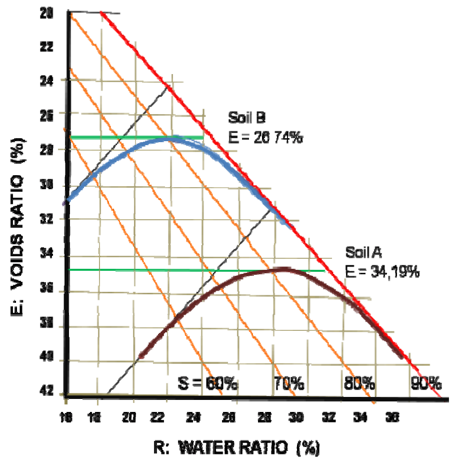


Figure 3: Voids and water ratio curves for A and B.

The density-moisture (D-W) chart can be replaced by the voids ratio/water ratio (E-R) chart in which the S or degree of saturation lines are all straight lines and the values of the ratios E and R are independent of the particle relative density. E and R are, respectively, the ratios of the voids and the moisture to that of the solids.

When the E-R values for the two soils A and B above, are plotted here the difference in strengths is clearly illustrated with soil B being the stronger. If a soil of density D has a particle relative density G, these parameters are related to the voids ratio, E as follows:

$$E = (G/D) - 1 \quad (1)$$

It should be noted that as the voids decrease E decreases. A low value for E indicates high strength while a high E value means a low strength. Furthermore the volume of water to that of the solids, termed the water ratio, R is independent of the soil's density and is related to the moisture, W, as follows:

$$R = WG \quad (2)$$

With E and R as vertical and horizontal axes respectively the chart as shown in Figure 3 can, with advantage, replace the D/W graph. On the E-R graph the following is evident:

- The S = 90% line is a straight line and is fixed for all soils.
- All lines representing values of S are all straight lines.
- A low value for E represents a high strength while a high value means a low strength.

3 THE E-R EQUATION

The S = 90% line and the corresponding mirror image line in Figure 3 for both soils A and B indicate that on the voids-water ratio graph the E-R or MD curve could be allied to a hyperbolic function, with two lines, the S=90% line and a line of equal but opposite slope, as asymptotes, in the following form:

$$(y^2/a^2 - x^2/b^2) - 1 = 0 \qquad (3)$$

Figure 4 gives a diagrammatic illustration of the conditions at the peak of an E-R curve. A study of this figure will show that:

- The voids ratio axis is retrograde or negative.
- The asymptotes intersect at Eo;Ro or at x = R - Ro and y = E – Eo.
  - Ro = 0,9 Eo = 0,8Em
  - a = Em – Eo = 0,111Em and
  - b = Ro – Rm = 0,1Em

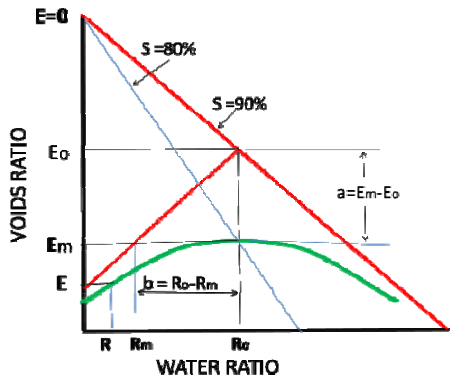


Figure 4: The E-R MD curve at its peak.

Providing that E and R remain within engineering limits and substituting in Equation 3 above and simplifying we get:

$$(0{,}9E – 0{,}8Em)^2 – (R – 0{,}8Em)^2 – 0{,}01Em^2 = 0$$

or

$$Em = 80R – 72E + \{(80R – 72E)^2 + 81E^2 - 10R^2\}^{0,5} \qquad (4)$$

If the relative density, G of a soil is known and a single test gives a value for D and W, the ratios E and R, may be obtained from:

$$E = (G/D) –1 \quad \text{and } R = GW \qquad (5)$$

and the minimum voids ratio Em can be calculated using Equation 4 without additional compaction tests. The corresponding maximum density Dm is then given by:

$$Dm = G/(Em + 1) \qquad (6)$$

The transposition of Equation 3 into terms of D, G and W produces an equation for Dm which is, in the writer's opinion, rather wieldy but is nevertheless presented here for what it's worth:

$$0.9/D + 0{,}8/Dm - 0{,}1/G)^2 - (W - 0{,}8/Dm + 0{,}8/G)^2 - (0{,}1/Dm - 0{,}1/G)^2 = 0 \qquad (7)$$

or

$$1/Dm = [80WG - 72(G - G)/D] + \{ [80WG - 72(G - G)/D]^2 + [9(G - D)/D]^2 - [10GW]2\}^{0,5}--1$$

(It is important to observe that the relative density, G, of the soil particles must be known if this mathematical approach is to be of any value).

As Equations 4 and 7 are not readily solved for Em and Dm using a simple calculator, an acceptably reliable short cut method is worth consideration. If a line passing through E, R parallel to the asymptote and to intersect the S90 line at Eo1 the value of Eo1 is obtained geometrically by:

$$Eo1 = 0{,}5(E + R/0{,}9) \qquad (8)$$

$$Em1 = 0{,}9Eo1/0{,}8 \qquad (9)$$

and

$$Em1 = 0{,}56E + 0{,}63R \qquad (10)$$

It should be noted that Em1 is slightly larger than the true Em but the difference in density resulting in this assessment of Dm is less than 15 kg/m^3 provided S, equal to R/E is not greater than 65%. In terms of D, G and W Equation 10 becomes:

$$1/Dm1 = 0{,}56/D + 0{,}44/G + 0{,}63W \qquad (11)$$

As an alternative to Equations 4 and 7, Equations 10 and 11 are an extremely simple means of obtaining acceptable values for Em and Dm if values for E and R are known from a single moisture-density test. If the degree of saturation S = R/E, does not exceed 60% the error in the assessment of Dm is less the 10 kg/m^3.

A single point on the dry leg of the Moisture-density curve is all that is required to give the maximum density. But the Relative Density, G, of the soil particles must be known. The writer strongly recommends that the determination of the relative density Gblk, should be included with that of grading and Atterberg limits as a soil indicator.

4 THE ACCURACY OF EQUATIONS (4) and (7)

The maximum dry density values obtained from actual laboratory tests were compared with those derived by the use of Equation 7, by selecting only one E-R point on the dry leg of the MD curve termed the “one-shot method”. A low point on the dry leg of the laboratory graph fixed the values for D and W in the equation. In Figure 5, the square dots refer to errors above that of the laboratory values, while the triangular dots are those below the laboratory values.

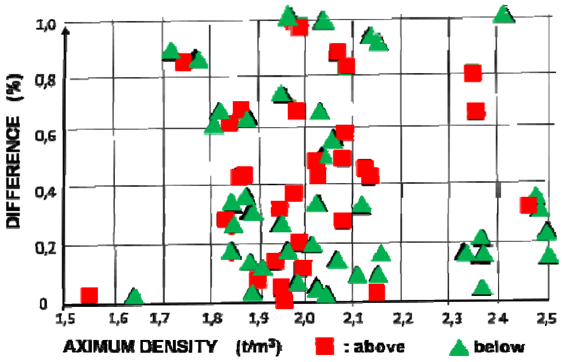


Figure 5: Difference in maximum density between one-shot method and of full lab tests.

The dots representing the differences between the values of maximum densities obtained by laboratory tests and mathematically may appear to be rather scattered but nevertheless indicate that the differences are less than one per cent of the laboratory density and are not biased either above or below the laboratory values. A statistical analysis of these differences gave a mean difference of 0,35% with a standard deviation of 0,28%.

A series of repetitive conventional MDD tests on seven identical soil samples showed a standard deviation of 0,36%. It would appear that the use of Equations 4 or 7 is no less accurate than the laboratory test procedure itself. It may be safe to say that if the relative bulk density of the soil particles is properly determined the “one-shot” is not only reliable but also time and labour saving if a quick value for the maximum density is required.

5 ESTIMATING IN-SITU STRENGTH

Let Eo represent the value of the voids ratio at the point of intersection Co, of the dry asymptote (strength) line and the wet asymptote or S=90% line. Eo is related to Em by equation (9); (Eo=Ro/0,9 and Em = Ro/0,8). Figure 6 shows Em as the top of the MD curve (shown as a solid green line) and Eo as the intersection of the two asymptotes for this curve. Consider a point on the curve with co-ordinates E and R. The compressive strength of the soil at this condition is represented by the red dotted line parallel to the dry asymptote and which cuts the S=90% line at C1 where the voids ratio is Eo1. C1 and Eo1 fall below Co and Eo respectively by an amount which increases as the point E-R approaches the peak of the MD curve. The values of Co and C1 represent the in-situ strength or compression strength of the soil at points along the MD curve. The in-situ strength indicated by C1 is identical to the soaked strength of a soil with a voids ratio of Eo1. In other words these rules apply:

- The value of C1 at the end of a horizontal line of Eo1 is a soaked strength.
- The value of C1 at the top of a diagonal strength line is an in-situ strength.
- The value of the water ratio at the intersection of these lines with the S = 90% line is a soaked condition.

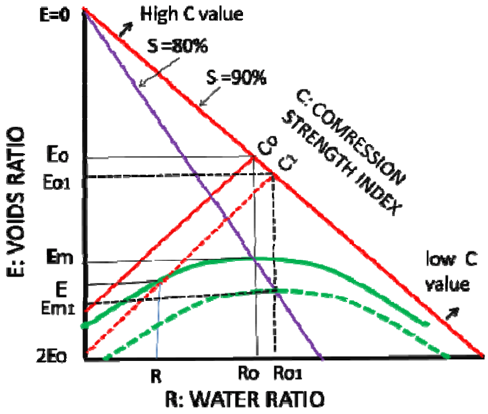


Figure 6: Lines parallel to the dry asymptote are in-situ strength lines.

Consider the dotted strength line through E-R extended to C1 where R = Ro1 (Figure 6). At this point on the E axis the voids ratio is equal to Eo1 and so (from equation 9):

Eo1 = 0,5(E+ R/0,9) (12)
from Equation 10
Em1 = o 56E + 0,63R (13)

Equation (7) can also be expressed in terms of D, W and G:
1/Do = 0,5(1/D + 1/G + W/0,9) (14)
It must be stressed that the peak voids ratio Em1 as obtained from Equation 12 is slightly lower than the actual value Em as can be seen in Figure 6. The difference between Em and Em1 is relatively small (less than 15 kg per cubic meter) and the use of Equations 13 or 14 for estimating Em or maximum density is acceptably accurate when the selected E-R point is such that S = E/R is not above 65%.

6 THE E-R GRAPH AS A STRENGTH INDICATOR

The dry leg of the moisture/density curve is a strength contour and represents the soil's resistance to, or strength against, further densification at the degree of moisture present due to the effort applied. The applied effort is the same for each increment in moisture but the effective compaction effort is reduced due to a systematic decrease in lateral frictional resistance. The soil's density may increase but its compression strength is correspondingly less. At maximum density compactive effort is virtually zero, the applied effort being lost to lateral movement and pore water pressure. A dry "90%" line through any point on the dry leg of the density curve indicates the in-situ compression strength C of the soil at that point and may be termed a C line or in-situ strength line. Figure 7 shows the compression strength or C lines with an inverse slope equal to that of the 90% degree of wet saturation line.

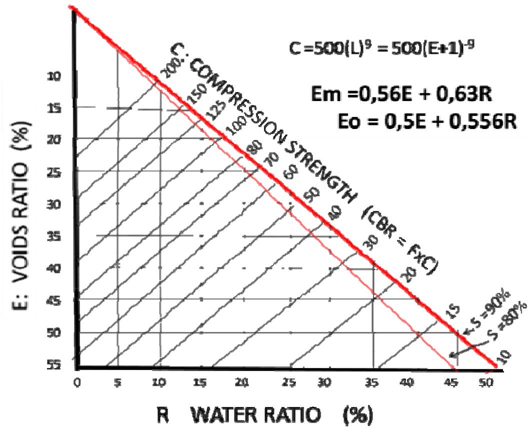


Figure 7: The E-R chart showing compression strength contours.

The use of the E/R graph instead of the conventional D/W graph is preferable for the following reasons:

- The S = 90% line is fixed for all soils as E an R are independent of Relative Density.
- All the S and C lines are straight lines.
- The E scale is linear and not reciprocal as would be necessary for the D scale to ensure straight S lines.

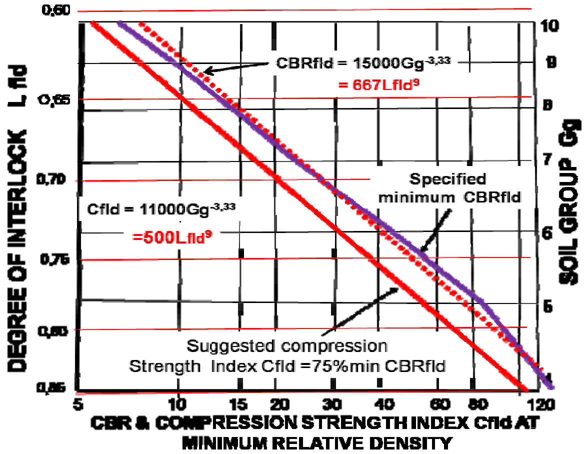
The value C, for the compression strength opposite each of the lines, is a strength index, and from a study by the writer is given by the relationship:

$$C = 500L^{9} = 11\,000\ Gg^{-3{,}33} \qquad (15)$$

where:
Li = the degree of particle interlock, or solidity; the volume of solids within the total soil volume
Gg = the soil group G, and
g = its position within the group.

7 C, L, Gg AND CBR RELATIONSHIPS

Equation 15, for the compression index C, was established by the writer as being arbitrarily 75% of the minimum CBR requirements laid down by COLTO (2006) for the various soil groups G4 to G10. This CBR/Gg relationship as can be seen in Figure 8 is virtually a straight line on a log/log plot and is represented by:



$CBRmin = 15\ 000\ Gg^{-3,33}$ (16)

From this, if the value of the compression strength index C, is 75% of the minimum CBR the equation (16) becomes:

$C_{fld} = 11\ 000\ Gg^{-3.33}$ (17)

Figure 8: The relationship between minimum CBR and the compression index C.

Experience has shown the writer that for a good G4 gravel, say G4,0, an interlock value L_{fld}, of 0,85+ can be achieved (L_{fda} = 88% is readily achieved for a G1 material). On the other end of the Soil group scale the achievable value for L is in the order of 63 to 60%. As the relationship between C and Gg is exponential it is reasonable to assume that L and Gg are also related exponentially and the following was formulated:

$L_{fda} = 1,41Gg^{-0,366}$ (18)

The writer's experience, in relating soil properties mathematically, has shown that exponential or hyperbolic functions often govern the relationships within the limits of the engineer's interest and accuracy.

Combining these two equations (17) and (18) gives the relationship between C and L as:

$C = 481L^{9,1075}$ or rounding off is $C = 500L^9$ or $C = (2L)^9$ (19)

The term $500^{0,1111}$ is actually equal to 1,9947 but has been rounded off to 2. The error produced by rounding off is negligible in engineering terms.
The scale of C values shown in Figure 7 was established by combining Equations 13 and $L = 1/(E + 1)$ to get:

$C = 500\{1/(E + 1)\}^9$ (20)

8 ESTIMATING THE CBR AT DIFFERENT DENSITIES

If the Soaked CBR test is done on the moulded moisture/density sample prepared for the One-shot test, this CBR value relates to the Compression strength index C by a factor F which the writer terms the Dislocation Factor. Both C and CBR are strength indicators: C is an active strength equal say, to the resistance to entry of a barbed arrow head while CBR is a passive strength or resistance to the extraction of the arrow. This may be likened to Terzaghi's (1948) theory of active and passive pressures in soil. The CBR strength relates to that of C by the dislocation factor, F, which may be likened to the size of the barb in the soil. Thus:

$$F = CBR/C \quad (21)$$

If the value of F has been established for a given soil the corresponding CBR at any other density at which the compression index, C_{fld}, is known can be readily estimated from:

$$CBR_{fld} = F x C_{fld} \quad (22)$$

If CBR_1 is the CBR at density D_1 then the CBR_2 at density D_2 can be estimated from:

$$CBR_2 = CBR_1(D_2/D_1)^9 = CBR_1RC^9 \quad (23)$$

where RC equals relative compaction of D_2 to D_1.

The moulded material prepared to obtain values for D and W (the One-Shot method) may be tested for an un-soaked CBR_0 which when related to Co will enable F for the soil to be calculated. Note that this procedure eliminates four days of soaking for CBR_{soak}.

Laboratory CBR test values on soils ranging in RC from 100% down to 90% of Maximum Modified density were compared with a corresponding CBR estimated by means of equation (23), which assumes that the factor F is constant for a given soil and where F was obtained from:

$$F = CBR_{mod}/C_{mod} \text{ and } C_{mod} = 500L_{mod}^9 \quad (24)$$

This comparison is shown in Figure 9, illustrating a reasonable degree of accuracy. In the figure the term RC stands for relative compaction and represents the term D_2/D_1 in equation (23).

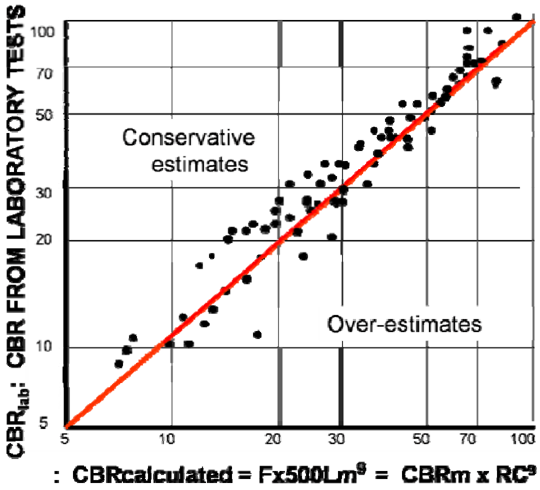


Figure 9: The comparison of CBRlab by test and CBRcal by calculation.

9 USING THE E/R GRAPH

Assume that a density test on a compacted layer has given the dry density as 1,927 t/m^3 at a moisture of 4,72%. Assume further that previous laboratory tests have shown the particle relative density G, to be 2,650 and the dislocation factor F, to be 1,3. From Equations 1 and 2:

E=G/D -1 = 2,65/1927 - 1 = 0,375
and
R = GW = 2 65 x 0,0472 = 0,125

Entering these values for E and R along the green and blue arrows respectively in Figure 10 the red dot fixes the soil's position. The oblique red arrow is a compression strength line and a value for C equal to 64 is indicated. This is the in-situ compression strength of the soil. By multiplying 64 by the dislocation factor F we can estimate the in-situ CBR:

$CBR_{in\text{-}situ}$ = F x C = 64 x 1,3 = 83

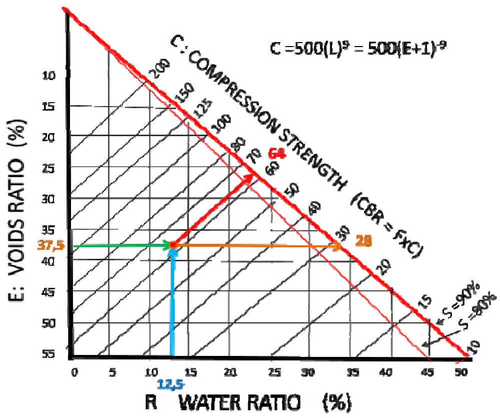


Figure 10: The E-R chart enables both in-situ and soaked CBR to be assessed.

The yellow horizontal arrow from the red dot gives the soaked compression strength of 28 or a soaked CBR:

$CBR_{soak} = 1,3 \times 28 = 36$

The soaked condition is assumed when S = 90%. In mathematical terms the following equations for the soaked and in-situ CBR values have been formulated for a field condition where the voids ratio and the water ratio are respectively E and R:

$CBR_{soak} = F \times 500\{1/(E + 1)\}^9$ (26)
$CBR_{in\text{-}situ} = F \times 500\{1/(0,5E + 0,556R + 1)\}^9$ (27)

10 THE IMPORTANCE OF ACCURATE TESTING

If the value of the maximum density and indications of strength are to be estimated by means of the one-shot method it is essential that the determination of the soil parameters, density, moisture content and particle relative density used in the exercise is reasonably accurate. When moisture-density tests are performed and particle relative density determined the degree of accuracy should be aimed to fall within the following tolerances:
- Bulk Relative density, G to be measured to the nearest 0,01.
- Moisture W to be measured to 0,01%.
- Dry density D to be to the nearest 10 kg/m^3.

If these tolerance limits should all occur simultaneously the error in the maximum density could be as much as 15 kg/m^3. It is interesting to note that an error in the fixing of the density, D, in the use of Equation 4 of 20 kg produces an error of less than 15 kg in the evaluation of the maximum density Dm.

In the evaluation of the voids and water ratios (E and R) the above combined tolerances would result in errors of less than 0,01 for E (1%) and 0,004 (0,4%) for R. This results in an error of less than 2% in the assessment of compression strength should both errors occur simultaneously.

11 SOAKED CBR FROM A DCP TEST

The DCP test on its own gives a CBR value for the soil at the point of test which is the in-situ CBR. If the moisture content is not taken at the same time the full potential of the DCP test is missed. It is recommended that a moisture reading be taken when the DCP test is performed as this enables the more meaningful soaked CBR to be determined. Figure 11 is a chart which gives both the in-situ CBR as well as the soaked CBR if the moisture content is known. The chart relates the DN value (mm/blow) from the DCP test to the in-situ CBR by the following equation:

$CBR_{in\text{-}situ} = 500(DN + 0,5)^{-1,3}$ (28)
and the soaked CBR from:
$CBR = F \times C$
and
$CBR_{soak} = \{2(CBR_{in\text{-}situ}^{-0,111}) -- 0,5 – 0,556R\}^{-9}$ (29)

In order to simplify the derivation of Equation 29 the equation $C = 500(1/E + 1)^9$ has been amended to $C = (2/E + 1)^9$.

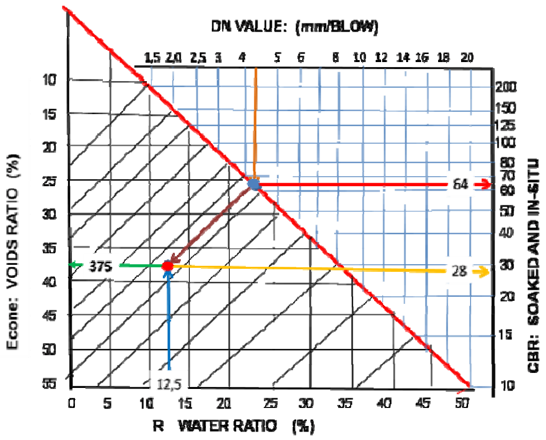


Figure 11: The E-R-DN chart to assess both in-situ and soaked CBR values.

The following example illustrates the use of this chart if the DN value has been determined for a layer by means of the DCP test apparatus:

- Assume the DN = 4,35 mm/blow and that the moisture content at the point of test is reported to be 4,72%.
- In the absence of an actual laboratory test for particle relative density G, a value of 2,65 may be assumed without undue error as most soils fall close to this value.
- A value for R may now be calculated - R = W x G = 0,0472 x 2,65 = 12,5%.
- Enter the chart at the top scale for DN values at the 4,35 mm/blow point.
- Drop down vertically along the orange arrow to the S = 90% line at the blue dot.
- From this point the horizontal red arrow shows a CBR value of 64. This is the in-situ CBR for the layer.

If the moisture content was not recorded no further CBR information can be obtained. As the moisture content is however known we can proceed from the green dot along the purple sloping line to the red dot opposite the water ratio of 12,5% representing the moisture content. This red dot gives us two further pieces of information. The yellow horizontal arrow to the right shows the soaked CBR as 28, while the green horizontal arrow to the left gives the cone voids ratio, Ec, as 37,5% from which the cone density Dc of the layer can be calculated:

$Dc = G/(Ec + 1) = 2,65/1,375 = 1,927 \text{ t/m}^3$

It should be noted that the cone density Dc is only an approximation of the true density as the relationship involves the dislocation factor F

$$D = Dc \times F^{-0,111} \tag{30}$$

If a value for F is say 1,6 the true value for the density D, will be:

$D = 1,927 \times 1,6^{-0,111} = 1,829 \text{ t/m}^3$

After all, the purpose of the DCP test is to assess the strength of the soil layer namely the soaked CBR. This having been assessed, the calculation of the actual density is perhaps an unnecessary exercise.

12 CONCLUSION

- If the use of a mathematical approach to the behaviour of compacted soil is considered it is essential that the relative density of the soil particles is determined. This test should be included as part of the routine "Indicator Tests."
- A single laboratory compaction test followed by the application of relatively simple equations can provide a value for the maximum density or minimum voids ratio for the material with an acceptable degree of accuracy. At this peak condition the water ratio is equal to 0,8 times the voids ratio.(R = 0,8E).
- The E-R graph provides a useful and relatively rapid means of assessing the In-situ and soaked strength of a compacted soil layer.
- Only one compaction test is needed in place of the normal four or five in the determination of maximum density.
- The normal four days of soaking can also be eliminated if an in-situ CBR test is done on this compacted soil for the determination of the dislocation factor, F.
- If the moisture content is taken at the position of a DCP test both the in-situ as well as the soaked CBR values for the layer can be assessed.
- Alternatively the use of Equations 27 and 29 may also give both in-situ and soaked strengths.

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