CHAPTER 3

EQUATIONS USED IN THE MTC PROGRAM

3.1 *INTRODUCTION*

The equations used in the *Model of Time and Fuel Consumption* (*MTC*) are derived from specific experiments. The speed equations were determined on the basis of the following experiments:

- TB-1 - Free speed on positive grades;
- TB-2 - Free speed on negative grades;
- TB-3 - Acceleration;
- TB-4 - Free speed on curves;
- TB-6 - Calibration; and
- TBS-3- Deceleration.

The fuel consumption equations were determined on the basis of other experiments, which are listed below:

- FC-1 - Consumption at steady-state speed on positive and negative grades;
- FC-2 - Consumption at start of positive grades preceded by negative grades;
- FC-3 - Influence of horizontal curvature on consumption;
- FC-4 - Calibration;
- FCS-4- Consumption during acceleration; and
- TBS-6- Consumption during deceleration.

As discussed in previous chapters, a given fuel-consumption equation is associated with a given speed equation in the MTC. Table 3.1 shows the relations among the equations (together with the mnemonics utilized in the program), the tests which led to these equations, and the correspondence between speed and consumption equations.

Fuel consumption is calculated by vehicle class and type. For gasoline-powered automobiles, utilities and light trucks (empty and loaded), the rate is given in milliliters/second of a gasoline-plus--20%-alcohol mixture, while in the other vehicle classes and types the units used are milliliters/second of diesel oil.

As was explained in Volume 6, when some experiments were being carried out the fuel available as a substitute for pure gasoline was the gasoline/alcohol mixture.

TABLE 3.1 - CORRESPONDENCE BETWEEN MTC EQUATIONS AND THE EXPERIMENTS

GRADE	SPEED EQUATION	EXPERIMENT	CORRESPONDING CONSUMPTION EQUATION	EXPERIMENT
POSITIVE	PGSE - steady-state	TB-1	FC1P	FC-1
	PGSE - deceleration	TB-1	FC2P	FC-2
	PGDB - deceleration	TBS-3	FCDP	TBS-6
	LACC - acceleration	TB-6	FCS4P	FCS-4
NEGATIVE	NGSE - steady-state	TB-2	FC1N	FC-1
	NGDB - deceleration	TBS-3	FCDN	TBS-6
	NGAE - acceleration	TB-3/TB-6	FCS4N	FCS-4/FC-1

Consequently, Experiment FCS-6 was carried out to make all fuel-consumption results uniform, since some of the experiments had been performed with the use of pure gasoline and others with the gasoline/alcohol mixture. The adjustment factors were as follows: 1.09 for the VW 1300 model; 1.05 for the Kombi (utility); and 1.02 for the F-400. It should be emphasized that these factors apply only to those experiments carried out with pure gasoline and, consequently, cannot be applied externally to the results of the model, since the factors do not apply to those equations designed to predict consumption in terms of the gasoline/alcohol mixture.

In practice, however, considering that the differences in consumption observed in Experiment FCS-6 were small, and perhaps even below other imprecisions of the model, the MTC results can be accepted as valid for pure gasoline.

3.2 LIST OF EQUATIONS

Tables 3.2 to 3.12 show the equations for predicting free speed and fuel consumption used in the MTC.

TABLE 3.2 - STEADY-STATE SPEED ON POSITIVE (PGSE - CONST.) AND NEGATIVE (NGSE) GRADES

Automobiles	$V=S1\ (91.9-2.7G_P-0.154QI)+S2(99.6-3.7G_P+0.6G_{N1}-0.214QI)$
Buses	$V=S1\ (84.3-0.5G_N-10.8G_{P1}-5.1G_{P2}-0.154QI)+S2(67-6.8G_{N1}-6.2G_P-0.93G_{P3}QI)$
Empty Utilities	$V=S1\ (84.3-2.4G_P-0154QI)+S2(89.4-3.7G_P+0.6G_{N1}-0.177QI)$
Full Utilities	$V=S1\ (84.3-3.7G_P-0.154QI)+S2(83.9-37G_P+0.6G_N-0.177QI)$
Light Trucks	$V=S1\ (74.6-3.0G_N-3.7G_P-0.154QI)+S2(80\ 9-3.7G_P+0.6G_{N1}-0.177QI)$
Full Trucks	$V=S1\ (74.6-3.0G_N-12G_{P1}-3.5G_{P2}-0.154QI)+S2(67-3.7G_P-6.8G_{N1}-0.177QI)$

$S1 = \begin{cases} 1 & \text{if paved} \\ 0 & \text{otherwise} \end{cases}$

$S2 = \begin{cases} 1 & \text{if unpaved} \\ 0 & \text{otherwise} \end{cases}$

$G_N = \begin{cases} \text{Grade (\%)} & \text{if Grade} < 0\% \\ 0 & \text{otherwise} \end{cases}$

$G_{N1} = \begin{cases} \text{Grade (\%)} & \text{if } -3.6\% < \text{Grade} < 0\% \\ -3.6\% & \text{otherwise} \end{cases}$

$G_P = \begin{cases} \text{Grade (\%)} & \text{if Grade} > 0\% \\ 0 & \text{otherwise} \end{cases}$

$G_{P1} = \begin{cases} \text{Grade (\%)} & \text{if } 0\% < \text{Grade} \leqslant 3\% \\ 3\% & \text{if Grade} > 3\% \end{cases}$

$G_{P2} = \begin{cases} \text{Grade} -3\% & \text{if Grade} > 3\% \\ 0 & \text{otherwise} \end{cases}$

$G_{P3} = \begin{cases} (6\%-\text{Grade})/6 & \text{if } 0\% < \text{Grade} < 6\% \\ 0 & \text{otherwise} \end{cases}$

QI = roughness (count/km)

V = Speed (km/h)

TABLE 3.3 - DECELERATION ON POSITIVE GRADES PRECEDED BY NEGATIVE GRADES (PGSE - DECEL.)

Automobiles	$\Delta V=D\ (-0.0001G1-0.008G2-0.0158G3)S1+DS2GA$
Buses	$\Delta V=D\ (-0.0003G1-0.167G2-0.0312G3)S1-0.006DS2G$
Empty Utilities	$\Delta V=D\ (-0.0003G1-0.008G2-0.01G3)S1+DS2GA$
Full Utilities	$\Delta V=D\ (-0.0003G1-0.008G2-0.0152G3)S1+DS2GA$
Empty Trucks	$\Delta V=D\ (-0.0016G1-0.008G2-0.0125G3)S1+DS2GA$
Full Trucks	$\Delta V=D\ (-0.0037G1-0.008G2-0.D125G3)S1-0.006DS2G$

D = -distance in meters from start of grade to point at which speed is equal to constant speed or - length of link (meters)

$$S1 = \begin{cases} 1 \text{ if paved} \\ 0 \text{ otherwise} \end{cases}$$

$$S2 = \begin{cases} 1 \text{ if unpaved} \\ 0 \text{ otherwise} \end{cases}$$

$$G1 = \begin{cases} \text{Grade (\%) if } 0 < \text{Grade} < 3\% \\ 3\% \text{ if Grade} \geqslant 3\% \end{cases}$$

G = Grade (%)

$$G2 = \begin{cases} 0 \text{ if Grade} \leqslant 3\% \\ \text{Grade}-3\% \text{ if } 3\% < \text{Grade} < 5\% \\ 2\% \text{ if Grade} \geqslant 5\% \end{cases}$$

$$G3 = \begin{cases} 0 \text{ if Grade} \leqslant 5\% \\ \text{Grade } -5\% \text{ if Grade} > 5\% \end{cases}$$

ΔV = Speed loss (km/h)

$$A = \begin{cases} -0{,}000794 \text{ if Grade} \leqslant 3\% \\ -0{,}000794-0{,}001976\ \left(\frac{G-3}{3}\right) \text{ if} \\ 3\% < \text{Grade} < 6\% \\ -0{,}00277 \text{ if Grade} \geqslant 6\% \end{cases}$$

TABLE 3.4 - FORCED DECELERATION ON POSITIVE (PGDB) AND NEGATIVE (NGDB) GRADES

$V_2^2 = V_1^2 - ckax$

V_1 = speed at start of link, in km/h

V_2 = speed at end of link, in km/h

C = conversion factor of units

K = 1,0 if paved
1,1 if unpaved

a = rate of deceleration, in m/sec^2

x = distance covered during deceleration, in meters

Vehicle Class	Rate of Deceleration(m/sec^2)
Automobiles	0.61
Buses	0.61
Empty Utilities	0.61
Full Utilities	0.61
Empth Trucks	0.46
Full Trucks	0.33

TABLE 3.5 - ACCELERATION ON POSITIVE GRADES (LACC)*

$$\Delta V = \frac{VC - VE}{1000m} \times D$$

VC	=	specific constant speed of positive grade, in km/h
VE	=	entry speed on positive grade, in km/h
D	=	distance covered during acceleration, in m
ΔV	=	addition of speed (km/h)

* This equation did not result from any basic tests developed during the ICR Research for use in the MTC. During calibration, it was noted that the acceleration equation derived from the data of TB-3 increased speed on positive grades excessively. Consequently, the equation above (valid for all vehicle classes) was elaborated separately and tested against the calibration data (TB-6).

TABLE 3.6 - ACCELERATION ON NEGATIVE GRADES

PAVED ROADS	
Automobiles	$\Delta V = [0.04444+0.00016(PW-46.1)-0.00238G-0.000038QI]\ D$
Buses	$\Delta V = [0.03903+0.00037(PW-17.1)-0.00109G-0.000038QI]\ D$
Utilities	$\Delta V = [0.04894+0.00037(PW-38.1)-0.00109G-0.000038QI]\ D$
Light Truck (Gas.)	$\Delta V = [0.04538+0.00037(PW-46.5)-0.00109G-0.000038QI]\ D$
Light Truck (Die.)	$\Delta V = [0.03903+0.00022(PW-28.2)-0.00109G-0.000038QI]\ D$
Heavy Truck	$\Delta V = [0.03737+0.00037(PW-11.1)-0.00109G-0.000038QI]\ D$
Half-Trailer	$\Delta V = [0.03903+0.00037(PW-9.6)-0.00109G-0.000038QI]\ D$
UNPAVED ROADS	
Automobiles	$\Delta V = [0.05639+0.00021(PW-46.1)-0.00169G-0.000046QI]\ D$
Buses	$\Delta V = [0.03811+0.00021(PW-17.1)-0.00131G]\ D$
Utilities	$\Delta V = [0.04735+0.00021(PW-38.1)-0.00245G-0.000025QI]\ D$
Light Truck (Gas.)	$\Delta V = [0.04152+0.00021(PW-46.5)-0.00131G]\ D$
Light Truck (Die.)	$\Delta V = [0.03325+0.00021(PW-28.2)-0.00131G]\ D$
Heavy Truck	$\Delta V = [0.03951+0.00021(PW-11.1)-0.00131G]\ D$
Half-Trailer	$\Delta V = [0.03325+0.00021(PW-9.6)-0.00131G]\ D$

G = Grade (%)

QI = roughness (count/km)

PW = Power/Weight Ratio (HP/t)

D = Distance covered during deceleration

ΔV = Addition of speed (m/sec)

TABLE 3.7 - FREE SPEED ON CURVES (FSC)

PAVED ROADS	
Automobile	$V=17.756+0.428\times R_{100}+0.12\times R_{200}+0.035\times R_{400}+0.014\times R_{600}-0.71\times G-0.010\times QI_{75}-0.28\times QI_{200}$
Bus	$V=17.756+0.290\times R_{100}+0.12\times R_{200}+0.035\times R_{400}+0.014\times R_{600}-0.71\times G-0.045\times QI_{75}-0.28\times QI_{200}$
Empty Utility	$V=17.756+0.390\times R_{100}+0.12\times R_{200}+0.035\times R_{400}+0.014\times R_{600}-0.71\times G-0.045\times QI_{75}-0.28\times QI_{200}$
Full Utility	$V=20.906+0.390\times R_{100}+0.12\times R_{200}+0.035\times R_{400}+0.014\times R_{600}-0.71\times G-0.140\times QI_{75}-0.28\times QI_{200}$
Empty Truck	$V=24.976+0.390\times R_{100}+0.12\times R_{200}+0.035\times R_{400}+0.014\times R_{600}-0.71\times G-0.140\times QI_{75}-0.28\times QI_{200}$
Full Truck	$V=17.756+0.390\times R_{100}+0.12\times R_{200}+0.035\times R_{400}+0.014\times R_{600}-0.71\times G-0.140\times QI_{75}-0.28\times QI_{200}$
UNPAVED ROADS	
Automobile	$V=20.87+0.34\times R_{100}+0.115\times R_{200}-0.21\times G-0.042\times QI_{140}-0.083\times QI_{300}+44.41\times SE$
Bus	$V=20.87+0.19\times R_{100}+0.115\times R_{200}-0.85\times G-0.007\times QI_{140}-0.083\times QI_{300}+44.41\times SE$
Empty Utility	$V=30.71+0.19\times R_{100}+0.115\times R_{200}-0.21\times G-0.042\times QI_{140}-0.083\times QI_{300}+44.41\times SE$
Full Utility	$V=20.87+0.19\times R_{100}+0.115\times R_{200}-0.52\times G-0.083\times QI_{140}-0.083\times QI_{300}+44.41\times SE$
Empty Truck	$V=25.87+0.19\times R_{100}+0.115\times R_{200}-0.52\times G-0.042\times QI_{140}-0.083\times QI_{300}+44.41\times SE$
Full Truck	$V=20.87+0.19\times R_{100}+0.115\times R_{200}-0.52\times G-0.042\times QI_{140}-0.083\times QI_{300}+44.41\times SE$

$$R_{100}=\begin{cases}\text{radius, in meters} & \text{if radius} < 100\text{m}\\ 100\text{m} & \text{if radius} \geq 100\text{m}\end{cases}$$

$$R_{200}=\begin{cases}0\ \text{m} & \text{if radius} < 100\text{m}\\ \text{radius}-100\text{m} & \text{if radius} \leq \text{radius}\ \ 200\text{m}\\ 100\text{m} & \text{if radius} \geq 200\text{m}\end{cases}$$

$$R_{400}=\begin{cases}0\ \text{m} & \text{if radius} < 200\text{m}\\ \text{radius}-200\text{m} & \text{if } 200 \leq \text{radius}\ \ 400\text{m}\\ 200\text{m} & \text{if radius} \geq 400\text{m}\end{cases}$$

$$R_{600}=\begin{cases}0\ \text{m} & \text{if radius} < 400\text{m}\\ \text{radius}-400\text{m} & \text{if } 400 \leq \text{radius}\ \ 600\text{m}\\ 200\text{m} & \text{if radius} \geq 600\text{m}\end{cases}$$

G= Grade in %

SE= Superelevation, in decimals

$$QI_{75}=\begin{cases}\text{roughness, in counts/km, if paved and roughness} \leq 75\ \text{counts/km}\\ 75\ \text{counts/km if paved and roughness} > 75\ \text{counts/km}\end{cases}$$

$$QI_{200}=\begin{cases}0\ \text{if paved and roughness} \leq 75\ \text{counts/km}\\ \text{roughness}-75\ \text{if paved and roughness} > 75\ \text{counts/km}\end{cases}$$

$$QI_{140}=\begin{cases}\text{roughness, in counts/km, if unpaved and roughness} \leq 140\ \text{counts/km}\\ 140\ \text{counts/km if unpaved and roughness} > 140\ \text{counts/km}\end{cases}$$

$$QI_{300}=\begin{cases}0\ \text{if unpaved and roughness} < 140\ \text{counts/km}\\ \text{roughness}-140\ \text{if unpaved and roughness of} \geq 140\ \text{counts/km}\end{cases}$$

TABLE 3.8 - FUEL CONSUMPTION AT STEADY-STATE SPEED ON POSITIVE (FC1P) AND NEGATIVE (FC1N) GRADES

Automobiles	$C=0.142e^{0.02287S+0.000855(S)GR+0.03782(GR+3)P+0.2695(5-MARC)+0.0001024(QI)(GR+14)}$
Buses	$C=0.195e^{0.0359S+0.0044(S)GR+0.0075(GR+1)P+0.2781(6-MARC)+0.0002088(QI)P}$
Utilities	$C=0.197e^{0.02579S+0.001062(S)GR+0.02932(GR+3)+0.2485(5-MARC)+0.0000785(QI)(GR+14)}$
Light Truck (Gas.)	$C=0.906e^{[0.0127+0.00063P+0.00699(5-MARC)+0.0000215(QI)]S+0.01234(GR)(MARC)P}$
Light Truck (Die.)	$C=0.1826e^{0.0325S+0.00208(GR)S+0.0254(GR+1)P+0.2333(5-MARC)+0.0014005(QI)}$
Heavy Truck	$C=0.583e^{[0.02356+0.000491(GR+1)(P)]S+(0.00594P+0.01224GR)(6-MARC)+0.00057(QI)}$
Half-Trailer	$C=\left(2.54/\sqrt{1+G}\right)e^{[0.00505+0.00029(GR+1)P+0.00035(QI)]S}$

S = Speed (km/h)

GR = Grade (%)

G = $|GR|$ for negative grades; 0 otherwise

P = Gross weight (t)

QI = roughness (counts/km)

MARC = vehicle gear

C = fuel consumption (ml/sec)

TABLE 3.9 - FUEL CONSUMPTION IN DECELERATION ON POSITIVE GRADES PRECEDED BY NEGATIVE GRADES (FC2P)

VEHICLES	Number of Observations	Fuel Consumption (ml/sec)			
		Average	Standard Deviation	Minimum	Maximum
Automobiles	6	2.45	0.164	2.20	2.70
Buses	17	6.10	0.698	4.70	7.30
Utilities	11	3.83	0.380	3.10	4.50
Light Truck (Gas.)	12	10.86	1.072	10.60	14.40
Light Truck (Die.)	5	4.50	0.339	3.90	4.70
Heavy Truck	31	6.77	1.148	5.30	10.10
Half-Trailer	15	12.12	1.012	10.60	14.40

The average rates of consumption are utilized in the model.

TABLE 3.10 - FUEL CONSUMPTION DURING FORCED DECELERATION ON POSITIVE (FCDP) AND NEGATIVE (FCON) GRADES

VEHICLE	Number of Observations	Fuel Consumption (ml/sec)			
		Average	Standard Deviation	Minimum	Maximum
Automobiles	16	0.50	0.19	0.28	0.97
Buses	6	1.57	0.61	1.17	2.68
Utilities	26	0.66	0.18	0.31	1.03
Light Truck (Gas.)	14	4.81	1.74	1.50	6.75
Light Truck (Die.)	14	2.53	0.93	1.21	4.38
Heavy Truck	20	2.04	0.63	0.80	3.25
Half-Trailer	6	2.41	0.18	2.21	2.71

The average rates of consumption are utilized in the model.

TABLE 3.11 - FUEL CONSUMPTION DURING ACCELERATION ON POSITIVE GRADES (FCS4P)

VEHICLE	Number of Observations	Fuel Consumption (ml/sec)			
		Average	Standard Deviation	Minimum	Maximum
Automobiles	69	2.72	0.11	2.50	2.90
Buses	65	5.85	0.46	4.00	7.60
Utilities	143	3.60	0.49	2.10	4.40
Light Truck (Gas.)	66	10.03	0.95	8.40	12.50
Light Truck (Die.)	69	3.78	0.35	2.80	4.60
Heavy Truck	149	5.80	1.01	2.60	8.80
Half-Trailer	92	10.08	1.14	7.00	12.80

The average rates of consumption are utilized in the model.

TABLE 3.12 - FUEL CONSUMPTION DURING ACCELERATION ON NEGATIVE GRADES (FCS4N)

During calibration, it was necessary to adjust forecasts of consumption during acceleration on negative grades. Consequently, the equation FCS4N became a variable between the rates of consumption of the equations FCS4P and the equations FC1N.

$$\text{If } S < S_1 \qquad C = \frac{S_1 - S}{S_1} \quad A \quad \frac{S}{S_1} \quad B$$

$$\text{If } S \geq S_1 \qquad C = B$$

C = fuel consumption (ml/sec)
S = speed at start of stage of acceleration
S_1 = constant speed of adjustment calculated for each vehicle according to type of surface and cargo level.
A = fuel consumption forecast by equation FCS4P
B = fuel consumption forecast by equation FC1N

TYPE OF VEHICLE[1]	Constants of Adjustment			
	PAVED		UNPAVED	
	EMPTY	FULL	EMPTY	FULL
Automobiles	120	10000	120	10000
Buses	80	80	80	80
Utilities	100	100	100	100
Light Truck (Die.)	80	100	60	70
Heavy Truck	60	150	60	100
Half-Trailer	150	150	150	150

[1] Field tests with the light gasoline-powered truck (F-400) must be repeated and calibration had not been completed at the time of publication of this report.