

Is there *Life* after *Modelling*? Student Conceptions of Mathematics

Ken Houston

University of Ulster

Glyn Mather, Leigh N. Wood and Peter Petocz

Macquarie University

Anna Reid

University of Sydney

Ansie Harding and Johann Engelbrecht

University of Pretoria

Geoff H. Smith

University of Technology, Sydney

We have been investigating university student conceptions of mathematics over a number of years, with the goal of enhancing student learning and professional development. We developed an open-ended survey of three questions, on “What is mathematics” and two questions about the role of mathematics in the students’ future. This questionnaire was completed by 1,200 undergraduate students of mathematics in Australia, the UK, Canada, South Africa, and Brunei. The sample included students ranging from those majoring in mathematics to those taking only one or two modules in mathematics. Responses were analysed starting from a previously-developed phenomenographic framework that required only minor modification, leading to an outcome space of four levels of conceptions about mathematics. We found that for many students modelling is fundamental to their conception of “What is mathematics?”. In a small number of students, we identified a broader conception of mathematics, that we have labelled *Life*. This describes a view of mathematics as a way of thinking about reality and as an integral part of life, and represents an ideal aim for university mathematics education.

A form of modelling has recently become embedded in public consciousness – that of assessing and predicting the impact of greenhouse gas emissions from human activity. Without mathematics, it would have been impossible to develop any form of climate change modelling. What is particularly striking about this model is the way it has seized the popular imagination, based on its potential to offer explanations for seemingly random natural phenomena (dust storms in south-east Australia, the increased frequency of hurricanes in the US) – it gives people a framework for deriving order from an apparently chaotic world (as one sample of the wide media coverage on the topic, see AFP, 2009). The reporting of probable adverse impacts has led many individuals, governments and businesses to modify planning and behaviour, from the use of more energy efficient light bulbs in the home to the development of industrial-size wind farms; that is, they have incorporated the model into their ideas of the way the world works and how they conduct their lives. Of course there are also climate change sceptics, but since the mathematical models depend on a range of assumptions about the data then a degree of scepticism is healthy. People

should always ask questions about the validity of mathematical models that others are trying to promote, and not accept data until satisfied with the methodology.

In the world of finance, other developments have led to the extensive use of mathematical and statistical models to help understand aspects of economics, business and finance. To some extent these may have enticed some bankers into reckless lending activity, which led to the banking crisis currently afflicting most of the world. Some might say that the bankers, in their greed, played fast and loose with the models to pursue profits (and personal bonuses). An interesting article has been written by Kate Barker from the Bank of England (Barker, 2007), who proposes that: "All models have to address some basic issues if they are to be utilised successfully. These include: the situation in which they are developed and the audience which needs to be persuaded to rely on them; the tensions between theory and real-world relationships; and uncertainty, including data uncertainty" (p. 25). Barker gives an apt description of economics, quoting John Maynard Keynes (1938): "Economics is a science of thinking in terms of models, joined to the art of choosing models which are relevant to the contemporary world" (Barker, 2007, p. 25).

Both of these examples illustrate the advantages of modelling, but equally the pitfalls of applying and incorporating models into a world view without too close an examination – climate change is not responsible for all natural disasters; the economic models failed in the long term because they were based on uncertain assumptions. Modelling can bring appropriate solutions, and can lead to positive outcomes, but models need to be based on accurate data and good science and assessed within a broad context.

This paper will describe a study that reveals that many university students conceptualise mathematics as *Modelling*. Other students see mathematics as being an *Abstract* subject, which we put at a similar place in our classification. We investigate a broader conception of mathematics, which we call *Life* and show that *Modelling* and *Abstract* conceptions are paths to the *Life* conception, building on the even narrower concepts of *Number* and *Components*.

So there is *Life* after *Modelling*!

Conceptions of Mathematics

Previous Research

Our mutual interest in teaching mathematics led us to wonder about the views of our students, based on the belief that an understanding of their conceptions of mathematics would help us to enhance their learning. We have found that there have been remarkably few in-depth studies of university students' ideas about mathematics, even though mathematics provides foundation skills for a variety of disciplines. There has been limited interest in the voices of the students themselves; samples have been relatively homogeneous; and work on investigating higher-level mathematics learning is rare.

Seminal work in the area was carried out by Crawford et al. (1994, 1998a, 1998b), who undertook a study of first year students at an Australian university. Their main emphasis was the relation between the students' own perceptions of the subject and their approach to learning. They "found that over 75% of students conceive of mathematics as a fragmented body of knowledge" (1998b, p. 457), and that students with a fragmented conception were likely to use a surface approach to their study; those with a cohesive conception were more likely to use a deep approach. A few other studies have been based on the Crawford et al. methodology, such as that by Macbean (2004), whose major focus was also the links between the level of conception and the depth of learning approaches. One of the conceptions of mathematics identified in these studies was, "Mathematics is models that have been devised over years to help explain, answer and investigate matters in the world" (Macbean, 2004, p. 564).

Others have been looking at conceptions of statistics, for instance research by Petocz and Reid (Petocz & Reid, 2003; Reid & Petocz, 2002) has shown the wide range of students' conceptions of statistics and statistical work, and the approaches that students take to learning in statistics. The broadest conception of statistics was that of an inclusive tool that could be used to solve problems, whilst the broadest conception of learning in statistics was that it is a way of personally interpreting and understanding the world.

Perrenet and Taconis (2009) examined the ways in which problem-solving beliefs and behaviour shift as students move through their bachelor's degree. They observe that these shift towards the beliefs and behaviour of "experts" – their lecturers. There is a misalignment between the beliefs and behaviour of school leavers and those of the professional mathematicians. The difference is mainly in the nature of the problem-solving activities that they encounter. In school problems are closed, standard and easy; for professionals problems are open, challenging and complex.

Houston (1997) discusses three ways in which students experience mathematics in the context of mathematical modelling. A student first learns methods, which are tools; the tools are used to understand models of the universe created by others; finally, this understanding of methods and models enables the student to engage in the creative activity of modelling and so modelling becomes a way of life. This idea has been expanded and developed by Houston, Rogers, and Simpson (2000) and by Houston (2001), based on the contrast between the teaching of mathematics in universities and the ways in which mathematicians apply their knowledge in the workplace.

The conclusions of these authors are that students should be educated in such a way that they become enculturated in the professional environment, in other words they begin to "live the life" of a mathematician; and that it is never too early to start this process.

Research Design

The major emphasis of previous research on conceptions of mathematics has been the links between conceptions and learning. Although as educators

our primary concerns are based on issues around teaching and learning, we wanted to shine a spotlight directly on the views of students about mathematics. We were also interested in surveying the conceptions of a wider sample, as well as looking at how conceptions might change through a degree program.

Our research has progressed in various stages, but in this paper we discuss the results from an open-ended survey of three questions – one of which was “What is mathematics” – in order to highlight student views around modelling. The survey was completed by 1,200 undergraduate students of mathematics in Australia, the UK, Canada, South Africa, and Brunei. We have previously reported more broadly on the results for “What is mathematics” in Petocz et al. (2007). We found that for many students *Modelling* is fundamental to their conception of mathematics; in this paper we expand on the discussion regarding modelling and a higher level conception, *Life*, of particular relevance to those who will become professional mathematicians.

The responses were analysed starting from a previously-developed phenomenographic framework (Reid et al., 2003), based on extended interviews with 22 students of mathematics. Phenomenography centres on illuminating the different ways that people experience, understand and ascribe meaning to a particular situation or phenomenon (Marton & Booth, 1997). The method is intended to elicit the range of experiences and the outcome of a phenomenographic study is a set of related categories of experience, usually showing a hierarchical structure, from the narrowest and most limited to the broadest and most inclusive conceptions. This is referred to as the “outcome space” for the research.

The hierarchical nature of the categories is usually established empirically: people who express the characteristics of the broadest conception also show an awareness of the narrower categories, while those who express the characteristics of the narrowest conceptions do not seem to be aware of any broader ones. Evidence for this is derived from the data – the actual words or text that respondents give. Indeed, this was the situation in our original outcome space, exemplified in more detail in Reid et al. (2003).

The foundation of such an analysis is identification of qualitatively different ways of viewing the phenomenon in question – in this case, mathematics. This may be done by manual coding of segments of text, or by using an appropriate software program (such as NVivo [QSR International]). These are then grouped into categories through an iterative process. In this study, we firstly did a rough identification of the range of themes based on a random sample of 400 responses, comparing them with our original phenomenographic categories (Reid et al., 2003). These categories were used as a starting point, with the possibility of modifying or augmenting them if necessary. We then combed through the whole group of responses several times, re-assessing the original themes and discussing our findings at each stage. By the end of this process the set of categories of the reported experiences became clear, forming a cohesive outcome space. In particular, they substantiated and extended the categories identified in our earlier study (see Petocz et al., 2007, for more details).

Further, there was evidence that they did indeed constitute a hierarchy, wherein the broader categories incorporated narrower order ones (some illustrative quotes are given later). Not all responses could be coded into our hierarchy of conceptions, for example, those student responses that stated that the use of mathematics in the student's future studies or career was *a little*, or *a lot*. Finally we categorised the full body of responses according to the schema that we had developed.

Results

These students' conceptions of mathematics ranged from the narrowest view as a focus on calculations with numbers (which we categorised as *Number*), then as a toolbox of techniques used to solve problems (*Components*), through a notion of mathematics as a focus on models (*Modelling*), or as abstract structures and a logical system (*Abstract*), to the broadest view of mathematics as an approach to life and a way of thinking (*Life*). We regard the conceptions of *Modelling* and *Abstract* to be at a similar hierarchical level: one describes modelling applied to the real world, while the other refers to abstract (mathematical) structures and ideas. In previous descriptions (Reid et al., 2003), we identified models as those that are 'pure' or abstract as well as models that are 'applied' to the real world: the evidence from this phase of the study inclines us to separate these more explicitly.

This student response sums up the *Modelling* category quite nicely: "Mathematics ... is the model set up to analyse and predict real world events." In contrast, another student perceived mathematics as, "Conceptual thought and logical development of ideas", which neatly epitomises the *Abstract* conception.

Some students are able to incorporate the modelling view within a broader conception of mathematics, for instance the following quote exemplifies the *Life* category and its inclusion of modelling strategies (as well as the beauty of mathematics!), giving evidence for the hierarchical nature of the conceptions:

In [the] real world, lots of problems can be defined by mathematics model. In other words, maths can solve many problems in real life. Maths is the basic knowledge for all the scientific fields (physics, chemistry, economics, psychology, computer science and so on). Maths is a kind of language, which is abstract and beautiful.

This student response embodies the *Abstract* within the *Life* category:

Mathematics is the study of logic (even in abstract ways of thinking) and the analysis of how all aspects in life – from music to physics to geography etc – fit together.

Of particular relevance to this paper is that around 20% (see Table 1) held a conception of mathematics that relates it to the physical world in terms of the development of a series of models. Indeed, the students who hold this conception make strong connections between mathematics and the physical world, which can be described (perhaps imperfectly) by mathematics. There may be an underlying assumption that mathematics is a

human endeavour invented to describe the world. In our schema, although this is a narrower conception than *Life*, it is nonetheless more complex than *Number* or *Components* (note that *Modelling* is inclusive of these – consider, for instance, the student quote: “The application of logic and sound reasoning (with the aid of numbers) for solving problems in various applications.”)

Table 1
Distribution of Conceptions of Mathematics

Conception	N	%
1. Number	109	9.2
2. Components	515	43.6
3a. Modelling	235	19.9
3b. Abstract	165	14.0
4. Life	71	6.0
(missing/uncoded)	87	7.4

Source: Petocz et al. (2007, p. 447).

Few students (6%) held the *Life* conception, which again is inclusive of the narrower conceptions. In this conception, students view mathematics as an integral part of life and a way of thinking. They believe that reality can be represented in mathematical terms but in a more comprehensive way than for the *Modelling* conception. Their way of thinking about reality is mediated by mathematics. They may make a strong personal connection between mathematics and their own lives.

One of the significant factors related to conceptions of mathematics was the student’s stage of study. Students in the later years of study were more likely to describe broader conceptions of mathematics, with the largest increase between second and third year. This suggests that overall pedagogical approaches may play a role in developing students’ conceptions, as well as factors such as the specific curriculum studied. Despite some differences between the universities, it is interesting to note that at each university, in each year level and in each degree there were students who indicated the full range of conceptions of mathematics, from *Number* to *Life*. For instance, at each university there were first year students who showed the broadest *Life* conception; and almost-graduated students who still showed the narrowest *Number* conception.

Haines, Crouch, and Fitzharris (2003), in a study of students’ approaches to modelling, found that novice students did not perceive “word” problems as “mathematics”, even though this is a necessary early step in the modelling process. Their research also lends weight to the propositions that “if it is interesting then it’s possibly not mathematics”, and “if it is mathematics then it’s probably not interesting”. This supports our thesis that many students have difficulty in moving to the *Life* concept, not recognising that mathematics may be embedded in word problems and, sadly, losing interest in applying mathematics when the going gets tough!

Modelling and a Mathematician's Life

Our study indicated that students' concepts of mathematics can be described by a small number of qualitatively different conceptions ordered into a hierarchical categorisation. Of particular interest to us in this paper are the categories *Modelling* and *Life*. Petocz et al. (2007) note that, "This conception [*Modelling*] links mathematics to the physical world ..." and a student expressed a view of mathematics as: "the attempt to explain the physical laws and patterns of the physical world by algebraic and numerical means" (p. 446). Advancing from this we see that the *Life* category can be described thus: "In this conception, students view mathematics as an integral part of life and a way of thinking" (p. 447). An exemplifying student quote is: "Mathematics is a way to approach life in an analytical manner so as to support and formalise natural processes. In a sense it is a way to understand how life works" (p. 447).

In other words, students who hold the broadest conception of mathematics look at the world through mathematical eyes. Not only do they see mathematical models at work all over the place, but they see that the beauty of nature – its symmetry, its patterns, its inspiration to the artist – can also be expressed by what is essentially abstract mathematics. One student wrote: "Mathematics is the language of nature. It is the way in which nature is ruled by God" (Petocz et al., 2007, p. 447)

This shift in thinking from viewing modelling as a way of handling a problem to seeing mathematics (including modelling, but also other aspects) as a way of understanding the world is a paradigm shift that some students never make or even get close to making. Yet, if they could do this, many riddles would be answered and many misconceptions would be avoided.

It is now about 30 years since universities and polytechnics started talking explicitly about mathematical modelling and devising modules and courses for teaching. Stillman, Brown, and Galbraith (2008) note that,

Applications and modelling research has been part of the agenda of the international mathematics education community for a substantial period, where it has been the focus of Working or Topic Study Groups at ICME Congresses for more than 20 years. (p. 141)

The International Community of Teachers of Modelling and Applications (ICTMA) has, since 1983, been holding biennial conferences, publishing books and papers, and organising workshops to promote the idea of teaching mathematical modelling in secondary and tertiary institutions. Houston, Galbraith, and Kaiser (2007) have published an interesting history of this movement. In 2003 the ICTMA became an affiliated study group of the International Commission on Mathematical Instruction (ICMI). Details of the ICTMA conference proceedings are available on their website¹. These books are useful resources of material for all teachers of mathematical modelling, as are the pre- and post-workshop publications of the 14th ICMI Study on *Applications and Modelling in Mathematics Education* (Blum et al., 2007; Henn & Blum, 2004).

¹ICTMA website: <http://www.ictma.net/> (retrieved October 29, 2009)

Approaches in one country – South Africa – exemplify the incorporation of modelling in mathematics education. An outcomes-based approach to teaching and learning has been adopted at the primary and secondary levels of education, wherein the importance of the modelling aspect of mathematics has been recognised from the outset. Sibusiso Bengu, the then Minister of Education, described the vision of the new education system as that of thinking, competent future citizens, and equipping learners with the knowledge, competencies and orientations needed for success after they leave school. The new curriculum would integrate education and training incorporating a view of learning that rejects a rigid division between academic and applied knowledge and skills (Bengu, 1997). However, it proved to be not so easy to put these high ideals into practice as has emerged from a study by Engelbrecht and Harding (2008). The authors found that students assessed in mathematics admission tests on entering university actually performed decidedly worse in the categories of modelling and ratios. It appears that although the importance of modelling for life is recognised, learners experience the problem-solving aspect of modelling as difficult and that teachers are not yet adequately skilled in teaching modelling.

Research by Crouch and Haines (2004) highlights some of the difficulties that undergraduate students of mathematics and other science and technology subjects encounter in solving problems which require modelling. Students in the survey had particular difficulty in moving from “real world” problems stated in words to a mathematical model. They were much happier simply doing mathematics. Crouch and Haines go on to report on work by Galbraith and Haines (2000), saying,

From an extensive study of problem-solving, Galbraith and Haines describe a hierarchy of procedural and conceptual skills. They reported on mechanical, interpretive and constructive problem-solving skills where the relative degree of success on these three different types of problem is mechanical > interpretive > constructive. (p. 198)

This is not too dissimilar from our sequence of concept development – *Number > Toolbox > Modelling / Abstract > Life* – which is derived from our students’ responses.

Similarly, for those who are specialising in mathematics Houston (1997) advocates introducing students to the “way of life” of a working mathematician, which recognises the drawbacks of modelling and uses it as only part of the arsenal. Stillman et al. (2008) echo this idea by suggesting that one approach to teaching modelling in order to “motivate the study of mathematics” (p. 142) is to place the modelling – and thus the mathematics – in a real-life context: “The goal is to equip students with skills that enable them to apply and communicate mathematics in relation to the solving of problems in their world” (p. 145).

Houston et al. (2000) develop these ideas to the teaching of all mathematics and not just applied mathematics. They contrast the way in which pure mathematics is taught in universities with the ways in which professional pure mathematicians actually work. Pure mathematics is usually taught in a tidy way – definitions, examples, axioms, theorems, proofs – whereas pure mathematical research is messy, open ended,

intuitive, creative. The authors suggest that students need both tidiness and messiness in their education. They identify the Apprenticeship Model as their preferred mode of teaching for both applied and pure mathematics. Students learn “at the feet of an expert” and they engage actively in open-ended problem solving. And all the while, students are learning the so-called soft skills of communication, human relationships and team working.

These ideas were also explored by Burton (2004), who interviewed research mathematicians about their ideas of mathematics and coming to know mathematics. Her starting point was the supposition that, “coming to know mathematics is a product of people and societies, that it is heterogeneous, that it is inter-dependent with feelings especially those attached to aesthetics, that it is intuitive and that it inter-connects in networks” (p. 13). This is in contrast to the traditional view of mathematics as objective knowledge, somehow separated from the views and perceptions of the people who learn and do mathematics. On the contrary, the mathematicians’ views of their own mathematical work supported her conjecture: in general, they did research and “learn” mathematics in the ways described by her model (although, significantly, their views of teaching were more traditional). Burton suggests that, “the mathematicians’ experiences, as learners, are relevant to less sophisticated learners in schools, and in universities ...” (p. 178), and uses this position to argue for, “a pedagogical approach to mathematics that treats learners as researchers” (p. 183).

To sum up, our research found that over 50% of the students sampled were in the narrower categories 1 and 2, so there is still a way to go to get most students to think about mathematics as modelling or even as its abstract manifestation, a “mind game”, to quote a student response in the *Abstract* category, let alone as an integral part of their lives. It is worth emphasising that only 26% of our sample (*Modelling*: 20% + *Life*: 6%) recognise mathematics as incorporating models and modelling, with *Modelling* being the broadest conception of around 20% of the total population; clearly more work needs to be done to bring our students up to this level and then lead them to the even broader conception of mathematics as a way of life.

Conclusion

We would argue that the teaching of modelling is an important component of the path to advanced mathematics. “Modelling” gives applications of mathematics a structure and purpose in the same way that “abstract” concepts give algebra and geometry a structure and purpose, but advanced mathematics requires practitioners outside the Academy to have the power to evaluate a problem situation and to solve it. There is also the creative aspect of applying models to new situations and of finding new models. Mathematics can very often be placed in the context of real work situations and the teaching of mathematics should go beyond its foundation of tools and models to broaden student views into a vision of mathematics as life. The challenge for mathematics educators is to create a dynamic curriculum and a dynamic teaching and learning environment that inspire students to engage deeply with mathematical ideas.

For mathematics educators to respond to this challenge is not a simple quest. In many cases it would require a paradigm shift in educators' conceptions of the nature of mathematics and the nature of the "way of life" of mathematicians working outside the Academy. Educators themselves would benefit by being aware of the view that mathematics is a way of life, developed through experiences not only of modelling but through exposure to all the tools of mathematics, alongside experiences of working "out there". It cannot be presumed that all educators have this point of view, but, in our opinion, pre-service education should embrace the philosophy that modelling is a significant component of mathematics as a way of life. Furthermore, it can be problematic to deploy a modelling approach within an overfull curriculum, suggesting that the aims of mathematics curricula in some places should be re-visited. Dealing with under-prepared students, who may lack competence or confidence in the basics of algebra, trigonometry, calculus and geometry, is another challenge to be faced.

It is important for all citizens to be aware that a model is only as good as its assumptions, so it is the responsibility of teachers to guide students towards being healthily sceptical of the conclusions of so-called "experts". Since there are many students who eschew mathematics at tertiary level, there should be support for "liberal arts" style courses that can engage such students with the ideas (and beauty) of mathematics. Our overarching view is that, since the vast majority of students will work and live outside universities and schools, it is vital that they come to appreciate not only the beauty of mathematics but also its usefulness. What does it mean to teach mathematical modelling? Simply to create a learning environment wherein students may absorb the culture of "doing mathematics" and become competent, even excellent, at this precious skill. All of this can help all students to the realisation that there is *Life after Modelling!*

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Authors

Ken Houston, School of Computing and Mathematics, University of Ulster, Jordanstown, BT37 0QB, United Kingdom. Email: <sk.houston@north-circular.demon.co.uk>

Glyn Mather, Faculty of Business and Economics, Macquarie University, Sydney, NSW 2109. Email: <glyn.mather@mq.edu.au>

Leigh N. Wood, Faculty of Business and Economics, Macquarie University, Sydney, NSW 2109. Email: <leigh.wood@mq.edu.au>

Peter Petocz, Department of Statistics, Macquarie University, Sydney, NSW 2109. Email: <peter.petocz@mq.edu.au>

Anna Reid, Conservatorium of Music, University of Sydney, Sydney, NSW 2006. Email: <Ann.Reid@sydney.edu.au>

Ansie Harding, Department of Mathematics, University of Pretoria, 0002 Pretoria, South Africa. Email: Ansie.Harding@up.ac.za

Johann Engelbrecht, Department of Mathematics, University of Pretoria, 0002 Pretoria, South Africa. Email: Johann.Engelbrecht@up.ac.za

Geoff H. Smith, University of Technology, Sydney, NSW 2007. Email: <Geoff.H.Smith@uts.edu.au>