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# **NONLINEAR TAX ELASTICITIES AND THEIR IMPLICATIONS FOR THE STRUCTURAL BUDGET BALANCE**

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## **ABSTRACT**

Research on tax elasticities in South Africa mainly employs linear models and shows that taxes evolve symmetrically irrespective of the economic cycle. This study extends this research to show that taxes behave asymmetrically and nonlinearly during expansions and contractions. Estimated linear elasticities imply that a one percent expansion in the cycle increases personal income tax, corporate income tax and value added tax by 1.43, 2.52 and 0.99 percent, respectively. However, estimated nonlinear elasticities are significantly different. During an expansion, the above elasticities increase by 1.89, 2.76 and 2.17 percent, respectively while during a contraction phase these elasticities increase by 0.89, 0.88 and 0.82 respectively. This finding of low tax collection during economic contractions has important implications for fiscal sustainability and overall fiscal prudence in South Africa. The findings of high tax elasticities during expansions might explain the underestimation of revenue by the government.

Keywords: structural budget balance, tax elasticities, nonlinearity, Smooth Transition Regression, Autoregressive Distributed Lag

JEL: C51; H20; H25

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## 1 INTRODUCTION

The global economy is faced with structural adjustments as financial markets fail. Large fiscal deficits incurred by governments during the financial crisis represented a coordinated effort by countries to save jobs and minimise the extent to which the economic growth declined. The G20 summit in March 2009 and the recent World Economic Forum at Davos in January 2010 announced that discretionary spending by most countries should exceed two percent of GDP.

Countries' economies are also responding in the form of automatic stabilisers where revenues fall and unemployment insurance increases, thus putting more pressure on the fiscal deficit. It is important that governments manage their fiscal deficits appropriately to avoid possible debt burdens. This study uses a measure of discretionary fiscal policy for South Africa to assess the fiscal stance during different phases of the economic cycle by explicitly testing for and using nonlinear tax elasticity adjustments.

This study also explores the asymmetric behaviour of revenues over the business cycle. In particular, a regime-switching framework is employed, where the transition from one regime to the other occurs in a smooth way. The switching between regimes is controlled by the state of output. This feature of the smooth transition model allows us to test the ability of high against low output regimes and to best describe the nonlinear dynamics of tax elasticities in the South African economy. In the case of South Africa, business cycle expansions and contractions can last as long as four years (see 2008q4 quarterly bulletin, s-159, for the duration of expansions and contractions), so the assumption that tax elasticities are of a short-term nature might be fallible. This paper proposes that long-term tax elasticities, as in the case of Du Plessis and Boshoff (2007), vary over the business cycle. To the best of our knowledge, this is the first attempt to model and compare across regimes the South African tax revenue relationship with respect to the business cycles using a non-linear model of fiscal adjustment.

Using recursive and rolling estimates for the nonlinear regressions, this paper therefore contributes to a set of other studies in estimating the tax elasticities.

Our main finding is that the elasticities for various taxes are significantly different in regimes of economic expansions and contractions. Adjusting the structural budget balance for asymmetric effects is just an extension from using linear elasticities. It allows the policy maker to draw from a different set of analysing tools when setting the expenditure envelope based on forecasted revenue. The large elasticities during economic expansions provide a possible explanation for the underestimation of tax revenues.

This paper is organised along the following lines. Section 1 contains the literature review. Section 2 explains the structural budget balance concept and sketches the modelling framework that can account for the issues raised above. Section 3 estimates the model, presents the findings and discusses the implications for these issues. Section 4 concludes.

## **1 LITERATURE REVIEW**

The context of tax evolution in South Africa has undergone significant changes. South Africa has witnessed three government appointed commissions on aspects of the tax structure. These were the Franzsen commission in 1968, the Margo commission in 1987 and the Katz commission in 1994 (Black *et al.*, 2005). As discussed in these commissions, South African authorities have but for isolated events implemented discretionary counter cyclical fiscal policy post 1994 due to adverse economic shocks. The South African economy has also gone through important tax reforms since the 1960s. Towards the end of the 1970s indications that authorities were beginning to abandon counter cyclical policy arose in favour of longer term fiscal planning (Calitz in Black *et al.*, 2005). Personal income tax no longer had strong automatic stabilising effects due to bracket creep and tax rate adjustments.

Furthermore, the Treasury was under pressure to reduce government expenditure which limited its ability to either cool the economy off or stimulate it. In particular, a long cyclical downswing from 1989 to 1993 depressed tax revenues while social services raised government expenditure. Government

revenue dropped from 25.1 percent of GDP in 1989/1990 to 21.7 percent in 1992/1993 whilst expenditures increased from 26.5 percent to 28.9 percent. The conventional budget deficit thus increased from 1.4 percent to 7.3 percent during that period which resulted in higher debt of a peak of 50.4 percent to GDP in 1995. With a clearer focus on counter cyclical fiscal policy, the government was able to reduce the deficit from 2003 onwards, the result also being lower levels of debt (25 percent of GDP in 2008/2009). During the global economic slowdown in 2008 through to 2010, South Africa is once again faced with similar difficulties.

Tax revenues are highly dependent on business cycles. An expansion in a business cycle improves revenue collection as automatic stabilisers and people's willingness to pay taxes improve. However, when the economy is in a downturn, automatic stabilisers act conversely, people delay or simply cannot meet their tax obligations which ultimately leads to a worsening of revenue collection. In terms of corporate income tax, companies can write off losses made in a current year next year. Therefore, it is important to take note that revenue collections might not respond symmetrically to business cycles. Wolswijk (2007) argues that short-term elasticities sometimes exceed their long-run counterparts and often behave asymmetrically. The tax base may also remain stable regardless of the cycle, where revenue collected varies. A simple example of this would be wage rigidities present in South Africa's labour market or specifically the inability to adjust wages down during an economic contraction. In theory, true long-term elasticities of taxes should be equal to unity. However, over the medium and short term, these elasticities can significantly exceed the "true" long-run estimates. A persistent economic expansion can shift consumption patterns to more luxury based goods. The counter argument would seem that a contraction in the cycle will symmetrically shift this pattern. The problem with this statement lies specifically in the structure of South African households and firms. Spending patterns between various income groups are highly heterogeneous. Income inequality is another example of how taxes can respond asymmetrically over the cycle. When the economy enters a recession, it is the lower end of the labour market which bears the majority of lay-offs. However, most of income taxes come

from the higher end of the income distribution where spending patterns might be more stable over the cycle. A final issue to consider is possible indirect effects of a change to a specific tax base relative to other bases. An example of this is higher company profits that lead to higher wages and ultimately higher levels of consumption.

Studies on tax elasticities for South Africa have received some attention due to the context of its economy. These studies focussed primarily on linear estimation methods. Swanepoel and Schoeman (2002) estimate constant tax elasticities over the cycle. Their study shows that a one percent widening of the output gap results in a decrease of 0.2 percent in tax revenue. Du Plessis and Boshoff (2007) define the output gap in terms of a structural vector autoregression and estimate long-run elasticities for various taxes over the cycle. Their results show almost a one to one relationship for personal income tax, corporate income tax and value added taxes over cycle. Lizondo et al. (2006) find that taxes respond significantly differently than unity over changes in the economic cycle. This study marks a significant point of departure from previous papers in that it examines tax revenue/elasticity variations over business cycles. These estimates are then used to further adjust the budget balance. Government's conventional budgets reflect both temporary and permanent factors of the economy.

Failure to distinguish between the temporary and permanent factors can reduce the extent to which fiscal policy can intervene in times of recessions and save in times of economic expansions. Various measures analyse whether fiscal policy is sustainable over the cycle. One such measure is the structural budget balance (hereafter SBB). The SBB is the budget that would have prevailed once all cyclical components have been removed from the budget, or put differently, "it is the residual balance after purging the actual balance from the estimated budgetary consequences of the business cycle" (Hagemann, 1999:3). The calculation of the SBB involves estimating elasticities. Girourd and André (2005) estimate tax elasticities in the calculation of the SBB for OECD countries. They use OLS regressions to capture both marginal and average tax rates where the product of the two yields tax elasticities over the economic cycle.

## 2 METHODOLOGY

The structural budget balance (SBB) is defined as that component of the budget that depends on the long term trend of the economy, as represented by the long run path of output. This underlying path is subjected to permanent and temporary shocks (as is in the case of Blanchard and Quah (1989)). Technological shocks have a lasting impact over time whereas transitory shocks dissipate over time, resulting in cyclical movements of actual output around potential output (Lizondo 2006). "Failure to distinguish between temporary and permanent influences on the budget may cause fiscal levers to be over or under adjusted in response to budgetary developments" (Hagemann 1999:3). If fiscal policy is unchecked in this regard, it can lead to an accumulation of debt and become a form of fiscal dynamic time inefficiency (intended fiscal policy that is geared to be counter cyclical but because of red tape and policy lags, becomes pro cyclical which reduces the credibility of overall fiscal policy).

This paper only adjusts for cyclical revenues and assumes that fiscal expenditures are exogenous to the cycle. Except for the unemployment insurance fund (UIF), expenditures are seen as purely discretionary. UIF constitutes a small component of fiscal expenditures and is thus left unadjusted. Formally, the SBB is derived as follows:

$$b^* = [(\sum_{i=1}^6 T_{t,i}^*) - G_0 + X_t] / y_t^*$$

where

(1)

$$T_{t,i}^* = T_{t,i} \left( \frac{y_t^*}{y_t} \right)^{\varepsilon_{t,y}}$$

$y_t^*$  is potential output

$y_t$  is observed output

$T_t^*$  is the cyclically adjusted tax for the various taxes

$G_0$  is unadjusted government expenditure

$X_t$  is nontax revenue minus capital and net interest spending

$\varepsilon_{t,y}$  is the estimated revenue elasticity regarding the cycle

The basic identity (equation 1) states that the SBB equals tax revenue adjusted for the economic cycle less government expenditures expressed as a percent of potential GDP and non-tax revenue as a percent of GDP. The tax revenue is adjusted for the cycle by incorporating a measure of average and marginal taxes through the estimated elasticities.

Instead of estimating cyclical revenues for all the taxes, this study mainly focuses on the three major revenues -namely; personal income tax (PIT), corporate income tax (CIT) and value added taxes (VAT which constitute eighty percent of all taxes.

The output gap is defined as the difference between observed output from potential output. Potential output is derived using a structural vector autoregression (SVAR) as in Du Plessis, Smit and Sturzenegger (2007). To be robust, this study also uses the Hodrik Prescott filter to measure the output gap. The SVAR approach has the advantage of not assuming that potential GDP is a function of some statistical property, but rather a function of the underlying economy. Potential GDP is then a function of fiscal expenditure as a percent of GDP and real interest rates.

Following Girourd and André (2005) the respective elasticities where the elasticity of taxes (PIT, CIT and VAT) over the cycle is derived as follows:

$$\varepsilon_{t_i,y} = \rho_{tb_j}^{t_i} \rho_y^{tb_j} \text{ for } i=\text{PIT, CIT and VAT}$$

for  $j$ =wage bill, profits and consumption

Where:

$\varepsilon_{t_i,y}$  Total elasticity of tax revenue to the output gap.

$\rho_{tb_j}^{t_i}$  The elasticity of the different taxes with respect to the various tax bases (first step).

$\rho_y^{tb_j}$  The elasticity of the various tax bases with respect to the output gap (second step).



The reduced form elasticities combine tax contributions relative to the output gap, by combining estimates of tax proceeds to changes in the tax base with estimates of the sensitivity of the tax base to the cycle (Girourd and André, 2005). They are then defined as follows:

The elasticity for PIT is defined by:

$$\varepsilon_{t,y} = \left( \frac{\partial T}{\partial Y} \right) \frac{Y}{T} = \left( \partial \left( \left( \frac{T}{L} \right) L \right) / \partial Y \right) \frac{Y}{T} = \left( \partial \left( \left( \frac{T}{L} \right) L \right) / \partial W \right) \frac{W}{T} \left( \frac{\partial W}{\partial Y} \right) \frac{Y}{W} = \rho_{wl}^t, \rho_{wl}^y \quad (2)$$

Where  $\varepsilon_{t,y}$  is the elasticity of personal income tax with respect to the output gap,  $\varepsilon_{t,wl}$  is the elasticity of personal income tax to the wage bill, where  $w$  is the wage rate and  $l$  is employment, and  $\varepsilon_{wl,y}$  is the elasticity of the wage bill to the output gap.

The elasticity for CIT is defined by:

$$\varepsilon_{t,y} = \left( \frac{\partial T}{\partial Y} \right) \frac{Y}{T} = \left( \frac{\partial Z}{\partial Y} \right) \frac{Y}{Z} = \left( \partial(Y - WL) / \partial Y \right) \frac{Y}{Z} = \left( 1 - \left( 1 - \left( \frac{Z}{Y} \right) \right) \left( \left( \frac{\partial WL}{\partial Y} \right) \frac{Y}{WL} \right) \right) \frac{Y}{Z} = \frac{(1 - (1 - PS)) \rho_{wl}^y}{PS} \quad (3)$$

Where  $\varepsilon_{t,y}$  is the elasticity of corporate income tax with respect to the output gap, PS is the profit share in GDP, Z is the gross operating surplus and  $\rho_{wl}^y$  is the elasticity of the wage bill to the output gap. i.e., one can calculate the elasticity of corporate income tax with the following formula:  $\frac{(1 - (1 - PS)) \rho_{wl}^y}{PS}$

The elasticity for VAT is defined by:

$$\varepsilon_{t,y} = \frac{\partial T}{\partial Y} \frac{Y}{T} = \left( \left( \frac{\partial V}{\partial Y} \right) / \partial Y \right) \frac{Y}{V} = \left( \left( \frac{\partial V}{\partial Y} \right) / \frac{Y}{V} \right) / \left( \left( \frac{\partial Cs}{\partial Y} \right) / \frac{Y}{Cs} \right) = \rho_{cs}^t, \rho_{csl}^y \quad (4)$$

Where  $\varepsilon_{t,y}$  is the elasticity of value added tax with respect to the output gap,  $\rho_{csl}^t$  is the elasticity of value added tax to consumption and  $\rho_{csl}^y$  is the elasticity of consumption to the output gap. V is VAT and Cs refers to the tax base which is consumption.

Preliminary analysis, using standard Augmented Dickey Fuller tests, suggests that the tax and tax base variables are  $I(1)$  as summarised in Table 1. This paper compares elasticities to that of Du Plessis and Boshoff (2008)), Swanepoel and

Schoeman (2002) and Jooste (2009). The regression techniques used by these authors are all different. Du Plessis and Boshoff (2008) estimate long run tax elasticities using a VAR. Swanepoel and Schoeman (2002) estimate their elasticities with standard OLS regressions whereas Jooste (2009) uses a Vector Error Correction Model (VECM) to estimate the reduced form elasticities. This paper, however, uses the term medium-term elasticity as apposed to long run elasticities, since the latter should in theory be close to one.

[Insert table1 around here]

This paper departs from the previous studies by sidestepping the proposed two step approach<sup>3</sup> and employs a framework using the smooth threshold regressive (STR, henceforth) which is a variant of the smooth threshold autoregressive (STAR) model as proposed by Granger and Terasvirta (1993) and van Dijk *et al.* (2002). This model is an extension to the threshold autoregressive (TAR) model as originally proposed by Tong and Lim (1980) in that the transition from one state to another is smooth rather than discrete.

Since the aim of the paper is to estimate the medium and short-term elasticities over the different phases of the cycle, it has to be ensure that variables are stationary in the STR model (in our case, taxes and respective bases are  $I(1)$  variables). Differencing the variables yields inconclusive results. Also faced with the problem of estimating an  $I(0)$  variable, the output gap, with  $I(1)$  variables, it is natural to extend the STR model in incorporating it into an autoregressive distributive lag model of order  $p$  and  $n$ , ARDL( $p,n$ ). The ARDL sets  $p=1$  and  $n=1$  on the basis of the Akaike Information Criterion. The ARDL model has the advantage of establishing long run relationships irrespective of whether variables are integrated of order 1 or 0 and is defined as:

$$T_t = \sum_{i=1}^p b_i + \sum_{i=1}^p a_i T_{t-1} + \sum_{i=0}^n c'_i TB_{t-1} + \varepsilon_t \quad (5)$$

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<sup>3</sup> Girourd and André (2005 estimate two elasticities, one regarding taxes to the tax base and the other regarding the tax base to the output gap.

Where  $T$  refers to the tax and  $TB$  to the tax base. Pesaran and Shin (1998) then used a variant of the standard ARDL equation for cointegration where  $T$  is related to contemporaneous  $TB$ , differenced  $TBs$  and lagged  $T$ :

$$T_t = \sum_{i=1}^p a_i T_{t-i} + a(1)\beta' TB_t - \sum_{i=0}^{n-1} \left( \sum_{j=i+1}^n c'_j \right) \Delta TB_{t-i} + \varepsilon_t \quad (6)$$

A further variant relates  $T$  to  $TB$  and differences in both variables and renders the error correction term ( $\hat{\gamma}T_{t-1}$ ) as in Wickens and Breusch (1988):

$$\Delta T_t = \hat{c} + \hat{\lambda}T_{t-1} + \hat{\theta}' TB_{t-i} + \sum_{i=1}^p \hat{\alpha}_i \Delta T_{t-i} + \sum_{i=0}^n \hat{\phi}' \Delta TB_{t-i} + \varepsilon_t \quad (7)$$

Where the long run estimate is calculated as:

$$\hat{\beta} = -\frac{\hat{\theta}}{\hat{\lambda}}$$

In establishing whether cointegration is present Hasser and Wolters (2006) run Monte Carlo simulations and show that Banerjee et al. (1998) suggestion of using the conventional student t-test is just as powerful as Boswijk's (1994) suggested F-test. Next, the ARDL version is augmented STR model. This allows us to study the dynamics of taxes over the cycle where it relates to the tax base. This side-steps the estimation method proposed by Girourd and André (2005) where one has to estimate two elasticities, to estimating it directly. The standard STR model for the nonlinear tax elasticities can be defined as follows:

$$\Delta T_t = \psi' z_t + \omega' z_t G(\gamma, c, s_{t-d}) + \xi_t \quad (8)$$

Where  $z_t = (1, \Delta TB_{t-1}, x_{1,t}, \dots, x_{m,t})'$  is the vector of explanatory variables, with  $h=n+2+m$ . The parameters  $\psi = (\psi_0, \psi_1, \dots, \psi_h)'$  and  $\omega = (\omega_0, \omega_1, \dots, \omega_h)'$  represent  $((h+1)*1)$  parameters in the linear and nonlinear parts of the model, respectively. The disturbance term is assumed to be identically and independently distributed with zero mean and constant variance, i.e.  $\xi_t \sim iid(0, \sigma^2)$ . The transition function  $G(\gamma, c, s)$  is assumed to be continuous and bounded in the transition variable  $s_t$  (i.e. the output gap). In  $S_{t-d}$ ,  $d$  is defined as lags 1, 2, ..., T. As

$s_t \rightarrow -\infty, G(\gamma, c, s_t) \rightarrow 0$  and as  $s_t \rightarrow +\infty, G(\gamma, c, s_t) \rightarrow 1$ . Our nonlinear function approximates a smooth logistic function where  $G(\gamma, c, s)$  is modelled as follows:

$$G(\gamma, c, s_t) = [1 + \exp\{-\gamma(s_t - c)\}]^{-1}, \gamma > 0$$

This transition function is a monotonically increasing function of  $s_t$ , where the slope parameter indicates the smoothness from one transition to another. The location parameter  $c$  determines where the transition occurs. Furthermore, the STR model is equivalent to a linear model with stochastically time varying coefficients and can be written as:

$$\Delta T_t = [\psi + \omega G(\gamma, c, s)]z_t + \xi_t \Leftrightarrow \Delta T_t = v'z_t + \varepsilon_t \quad t = 1, \dots, T \quad (9)$$

Given that  $G(\gamma, c, s)$  is bounded between 0 and 1, the combined parameters,  $v$ , will fluctuate between  $\psi$  and  $\psi + \omega$  and change monotonically as a function of  $s_t$ . The more the transition variable moves beyond the threshold, the closer  $G(\gamma, c, s)$  will be to one and the closer the parameters  $v$  will be to  $\psi + \omega$ . The further  $s_t$  approaches the threshold,  $c$ , the closer the parameters  $v$  will be to  $\psi$ .

To augment our ARDL model with the STR methodology we proceed as follows:

$$\Delta T_t = G(\gamma, c, s_t)X_{q,t} + X_{o,t} + \zeta_t \quad (10)$$

$T_t$  is the tax base for PIT, CIT and VAT at time  $t$ . Next the vector

$$X_{q,t} = \hat{c} + \hat{\lambda}_q T_{t-1} + \hat{\theta}_q' TB_{t-i} + \sum_{i=1}^p \hat{\alpha}_i \Delta T_{t-i} + \sum_{i=0}^{n-1} \hat{\phi}_q' \Delta TB_{t-i} \text{ and}$$

$$X_{o,t} = \hat{c} + \hat{\lambda}_o T_{t-1} + \hat{\theta}_o' TB_{t-i} + \sum_{i=1}^p \hat{\alpha}_i \Delta T_{t-i} + \sum_{i=0}^{n-1} \hat{\phi}_o' \Delta TB_{t-i} \text{ where TB represents the}$$

various tax bases for the wage bill, profits and consumption at time  $t$ .

This study follows Granger and Teräsvirta (1993) and Teräsvirta (1994) in making  $\gamma$  dimension-free by dividing it by the variance of  $c$ . To find the optimal values for  $c$  and  $\gamma$  that minimise the residual sum of squares, a grid search is conducted using the Broyden-Fletcher-Goldfarb-Shano (BFGS) algorithm.

### 3 DATA AND EMPIRICAL EVIDENCE

#### 3.1 DATA DESCRIPTION

South African quarterly data for the 1994q1-2009q3 period is used to estimate the tax elasticities. These elasticities are then used to calculate the structural budget balance. Government expenditure data can directly be downloaded from the South African Reserve Bank. For the wage bill we use compensation of employees as a base for PIT. For profits we use total gross operating surplus as a base for CIT. Consumption is used as the base for VAT.<sup>4</sup>

Two sorts of tests for linearity is used. Table 2 provides the, Brock, Dechert and Sheinkman (BDS) (1987) test of any nonlinearity in the two models proposed by Girourd and André (2005). It tests whether the errors of the cointegrated model are correctly specified under the null of identically and independently distributed (i.i.d) errors against the alternative of an unspecified nonlinear process. The test can be used to test possible deviations from linear dependency or chaos.<sup>5</sup> The results of different embedding dimensions and the standard deviations of the residuals are reported. The results in table 2 clearly reject the null of linearity for the embedding dimensions for PIT, VAT and CIT.<sup>6</sup>

[Insert Table 2 around here]

After having rejected that the model is linear, the next step is to apply the test by Teräsvirta (1994) in Escribano and Jordá (2001) to select the proper nonlinear functional form between a logistic transition regression and an exponential transition regression. Table 3 which provides the p-values for Teräsvirta's HO2 and HO3 tests shows evidence in favour of LSTAR for PIT, CIT and VAT.

[Insert Table 3 around here]

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<sup>4</sup> Data can be directly downloaded from the South African Reserve Bank website. Codes are: Government expenditure (S-47:4601m), compensation of employees (6240L), PIT (S-46:4570m), consumption (6007L) and VAT (S-46:4578m)

<sup>5</sup> Refer to Eviews 6 User's Guide for a more detailed discussion on the BDS test.

<sup>6</sup> Using different standard deviations yields the same conclusion.

For the recursive estimates, we estimate over fixed windows of data, where the first data window runs from 1994q1 to 2005q1, and each successive data window is extended by one observation, hence, the last data window runs from 2005 to 2009 (this setup delivers 20 expanding windows). From a policy point of view, this allows us to identify the evolution of the estimated model parameters over time and across regimes. We use sequences of expanding windows in which the sample size for estimation is increased by one observation in each successive window, as opposed to sequences of fixed-length rolling windows, simply because the larger (increasing) windows help the estimation procedures for the various models which can be quite parameter intensive.

For robustness reasons, however, the exercise below also reports results based on a sequence of fixed-length rolling windows where each successive window is constructed by shifting the preceding window ahead by one observation. The rolling scheme can also be used to guard from moment or parameter drift due to the regime changes in fiscal policy, discussed in the preceding section and on that basis we believe it is worth comparing estimates from these two schemes.<sup>7</sup>

### **3.2 EMPIRICAL EVIDENCE**

The results of the nonlinear estimates are compared to the results of Du Plessis and Boshoff (2008), Swanepoel and Schoeman (2002), Lizondo et al. (2006) and Jooste (2009). Their results are summarised in Table 4. The different methods yield marginally different results.

Table 5 includes the estimation results from our nonlinear regression. To calculate the expansion phase elasticity in the STR model (10), the sum of the

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<sup>7</sup> It should be noted that there has been some debate on using recursive versus fixed-length rolling windows, mainly in the context of forecasting. On the expanding window versus fixed-length rolling window issue we note that according to Stock and Watson (2005, p. 26), “recursive forecasts are more accurate than the rolling forecasts” for the representative macroeconomic dataset they study. On the other hand, however, Giacomini and White (2006, p. 1566) find that a “rolling window procedure can result in substantial forecast accuracy gains relative to an expanding window for important economic time series.”

linear and nonlinear part ,  $\frac{\hat{\theta}_q}{-\hat{\lambda}_q} + \frac{\hat{\theta}_o}{-\hat{\lambda}_o}$ , is taken. The results differ significantly

from the linear estimates in Table 4. The estimates indicate that strong asymmetries are present during the different phases of the cycle. During the upward phase of the cycle, a 1% increase in the wage bill, profits and consumption, increases revenue collections by 1.87, 2.77 and 2.18 for PIT, CIT and VAT, respectively. Conversely, during the contraction phase of the cycle, revenue collection responds less than unity for PIT, CIT and VAT (though not significantly smaller than 1). The elasticities fall well within the range of elasticities reported in Table 4 when using equally weighted averages of the expansionary and contractionary elasticities for the various taxes. The nonlinear elasticities show that when the cycle suddenly turns, tax collections decrease quickly and by a lot. The results are robust in as much as the Hodrick-Prescott (H-P) output gap is also used. The VAT estimates are not significant in the H-P gap regression and barely significant in the SVAR gap regression. Most studies assume that this elasticity takes a value of one. The strong asymmetries just show how fast revenues could grow during an expansionary phase which partly explains why the government consistently underestimates tax revenue. Government's expenditure decision can be greatly enhanced once these elasticities are incorporated which would reduce overall surprises that might question the credibility of fiscal authorities.

[Insert Table 4 around here]

[Insert Table 5 around here]

Fixed and rolling estimates were used to characterise the nonlinear estimates (equation 7) for 12 periods. The first row of Figure 1 and 2 is the nonlinear coefficients for the expansive phase and the second row is the contraction phase medium run coefficients. The nonlinear estimates in Figure 2 are constant over

time. This suggests that correctly capturing asymmetries in the economy yields robust and invariant estimates.

[Insert Figure 1 around here]

[Insert Figure 2 around here]

Moving into South Africa's longest contraction since the 1989-1993 period, policy makers should make additional precautions regarding the expenditure framework. What Figure 3 suggests is that the economy still has to purge itself from cyclical effects in order to have a conventional budget (CBB) that is in line with the structure of the economy. The difference in using the downward phase and upward phase elasticity is more than 1 per cent of GDP (close to R 30bn); i.e. as the cycle is correcting itself, we will have close to R30bn less to spend on the economy. Using the wrong elasticities would lead to a policy belief that there is an additional R30bn to spend during the downward phase of the cycle.

[Insert Figure 3 around here]

#### **4 CONCLUSION AND POLICY IMPLICATIONS**

This paper discusses two important issues regarding tax elasticities. The first being whether taxes respond symmetrically over the cycle and the second being whether tax elasticities are constant and stable over time. To answer these questions we test for and estimate possible asymmetries through incorporating a nonlinear framework. The weighted averages of the nonlinear estimates are similar to those of other studies. However, the nonlinear estimates showed that tax collections indeed move asymmetrically over the cycle. This has significant implications for policy making in using the structural budget balance as a gauge for the overall fiscal stance. The study shows that applying these elasticities can lead to large differences in the structural budget balance. Fiscal hawks argue that fiscal austerity measures should be implemented as the global economy



recovers. The estimates obtained in this study should serve as a tool in assessing future revenue collections and hence overall expenditures. These elasticities should guide policy maker's decision to ensure that expenditure will be met by revenue streams in the future and serve as an illuminating path to improved policy making.

These estimates could well be affected by other factors that could possible bias the results. As a further line of research, estimating efficiency gains from the South African Revenue Services through improvements in tax collection and the underwriting of tax laws could render more accurate estimates.

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**Table 1: Stationarity tests**

ADF tests in levels (P values)						
PIT	CIT	VAT	Wage bill	Profits	Consumption	Output gap
0.661	0.222	0.636	0.854	0.213	0.798	0.086
ADF tests in differences (P values)						
0.000	0.001	0.003	0.019	0.641	0.083	0.218

**Table 2: BDS test**

BDS test $\sigma=0.05$			
Dimensions	ResidProfits (P values)	ResidWageB (P values)	ResidCons (P values)
2	0.000	0.000	0.000
3	0.000	0.000	0.000
Dimensions	ResidCIT (P values)	ResidPIT (P values)	ResidVAT (P values)
2	0.309	0.075	0.022
3	0.000	0.000	0.011

**Table 3: Linearity test**

Transition variable				
Hypothesis	Wage bill (P values)	(P	Profits (P values)	Consumption (P values)
H03	0.97		0.45	0.58
H02	0.79		0.87	0.74

**Table 4: Linear elasticities**

Taxes	Swanepoel et al.	Du Plessis et al.	Lizondo et al.	Jooste
PIT			1.43	0.84
CIT			2.52	1.79
VAT		1.14	0.99	1.01
Income and profits	1.04	1.05		
Goods and services	1.24			

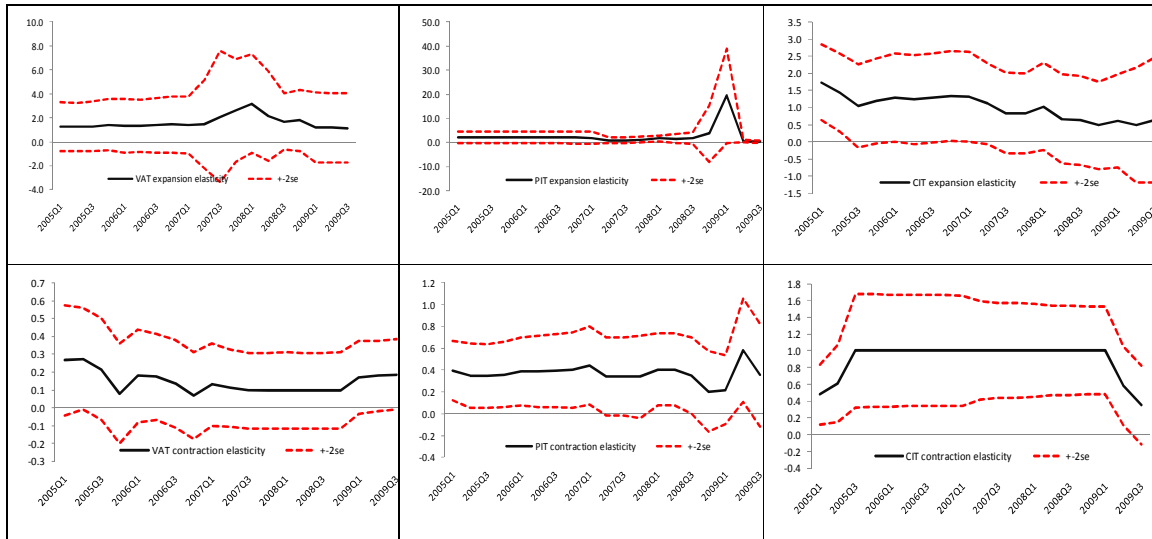
**Table 5: Nonlinear regression**

	PIT SVAR	PIT H-P	CIT SVAR	CIT H-P	VAT SVAR	VAT H-P
<i>Linear Part</i>						
<i>Long run coefficient</i>						
C	-1.88** [2.91]	-2.70*** [0.65]	-12.48*** [2.91]	-11.49*** [3.85]	-8.32** [3.59]	-2.89 [2.84]
$\hat{\lambda}_o$	-0.26** [0.11]	-0.35*** [0.12]	-0.31** [0.13]	-0.24** [0.11]	-0.16* [0.08]	-0.17 [0.12]
$\hat{\theta}_o$	0.23** [0.10]	0.21*** [0.11]	0.27** [0.11]	0.21** [0.09]	0.14* [0.07]	0.14 [0.10]
<i>Nonlinear Part</i>						
$\hat{\lambda}_q$	-1.30** [0.63]	-1.46*** [0.40]	-0.81** [0.31]	-0.92** [0.37]	-1.25* [0.64]	-0.46 [0.52]
$\hat{\theta}_q$	1.24** [0.58]	1.48*** [0.39]	1.54*** [0.47]	1.65*** [0.56]	1.65** [0.79]	0.62 [0.63]
Upward phase						
$\frac{\hat{\theta}_q}{-\hat{\lambda}_q} + \frac{\hat{\theta}_o}{-\hat{\lambda}_o}$	1.87	1.89	2.77	2.67	2.18	2.17
Downward phase						
$\frac{\hat{\theta}_o}{-\hat{\lambda}_o}$	0.88	0.89	0.86	0.88	0.81	0.82
$R^2$	0.63	0.73	0.57	0.52	0.33	0.49
SE	0.07	0.07	0.22	0.24	0.09	0.09
JB	1.39 (0.49)	1.42 (0.49)	0.52 (0.77)	1.62 (0.30)	1.34 (0.52)	22.17 (0.00)

Heteroskedasticity	(0.17)	(0.54)	(0.51)	(0.18)	(0.28)	(0.01)
Serial Correlation	(0.94)	(0.98)	(3.49)	(0.12)	(0.25)	(0.70)
Model AIC	-2.46	-2.70	-0.11	0.10	-1.73	-1.32

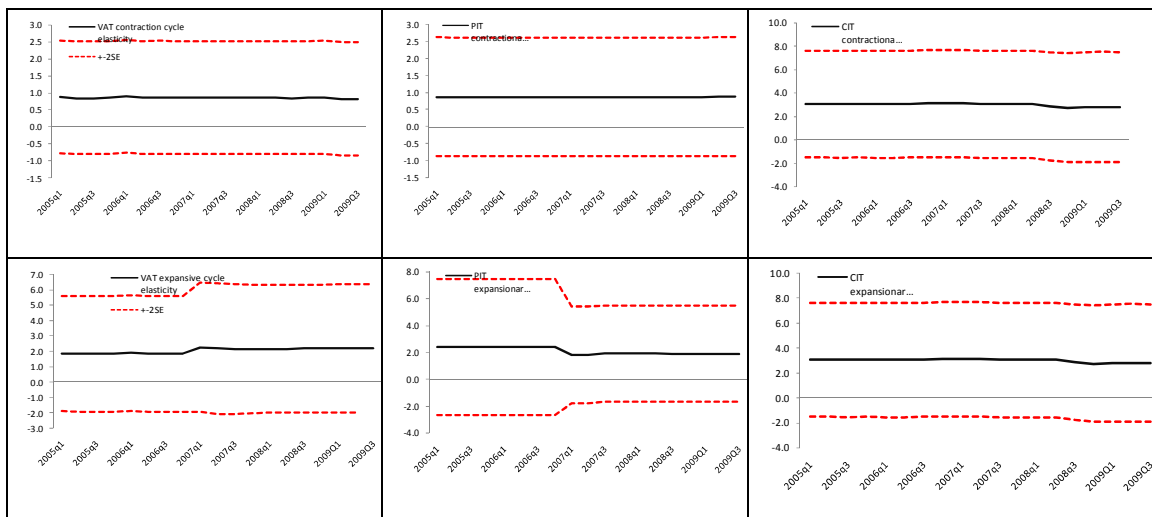
*Note:* Parenthesis represents probability values and block parenthesis are standard errors. \*, \*\*, \*\*\* denote significance levels at 10, 5 and 1 percent. One lag was used for the serial correlation test.

**Figure 1: Rolling parameters of the linear and nonlinear tax elasticities**



*Note:* The black lines are the estimated fixed window rolling coefficients. The dashed lines represent standard errors

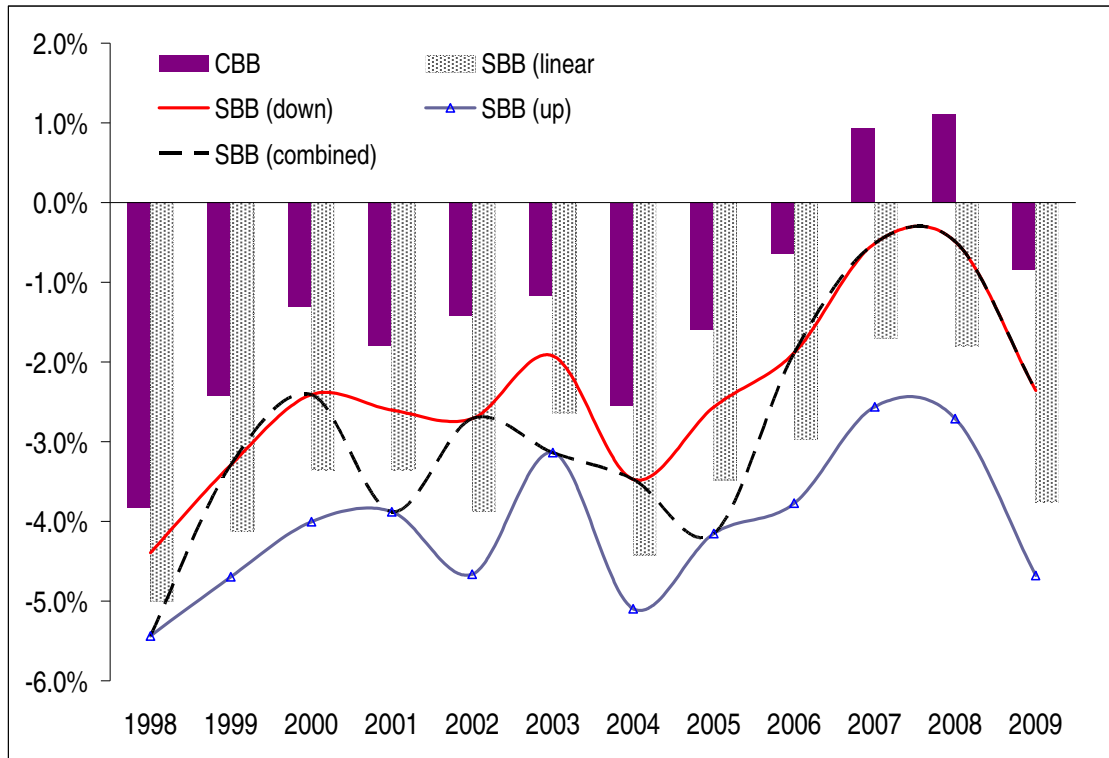
**Figure 2: Recursive parameters of the linear and nonlinear tax elasticities**



*Note:* The black lines are the estimated fixed window rolling coefficients. The dashed lines represent standard errors



**Figure 3: The SBB incorporating different coefficients over the cycle**



Note: SBB is as a percentage of potential GDP and CBB is as a percentage of nominal GDP