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Abstract

The paper develops a Small Open Economy New Keynesian DSGE-VAR (SOENKDSGE-VAR) model of the South African economy, characterised by incomplete pass-through of exchange rate changes, external habit formation, partial indexation of domestic prices and wages to past inflation, and staggered price and wage setting. The model is estimated using Bayesian techniques on data for South Africa and the United States (US) from the period 1990Q1 to 2003Q2, and then used to forecast output growth, inflation and a measure of nominal short-term interest rate for one- to eight-quarters-ahead over an out-of-sample horizon of 2003Q3 to 2008Q4. The forecast performance of the SOENKDSGE-VAR model is then compared with an independently estimated DSGE model, the classical VAR and BVAR models, with the latter being estimated based on six alternative priors, namely, Non-Informative and Informative Natural Conjugate priors, the Minnesota prior, Independent Normal-Wishart Prior, Stochastic Search Variable Selection (SSVS) prior on VAR coefficients and SSVS prior on both VAR coefficients and error covariance. Overall, we can draw the following conclusions: First, barring the BVAR model based on the SSVS prior on both VAR coefficients and the error covariance, the SOENKDSGE-VAR model is found to perform competitively, if not, better than all the other VAR models for most of the one- to eight-quarters-ahead forecasts. Second, there is no significant gain in forecasting performance by moving to a DSGE-VAR framework when compared to an independently estimated SOENKDSGE model. Finally, there is overwhelming evidence that the BVAR model based on the SSVS prior on both VAR coefficients and the error covariance is the best-suited model in forecasting the three variables of interest.

JEL Classification: C11, C53, E37

Keywords: Bayesian Methods; Macroeconomic Forecasting; New Keynesian DSGE; Small Open Economy; Vector Autoregressions

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1 Introduction

Recent studies, namely, Liu and Gupta (2007), Liu et al. (2009, 2010) and Gupta and Kabundi (2010, forthcoming), have initiated a growing interest in forecasting macroeconomic variables in South Africa using Dynamic Stochastic General Equilibrium (DSGE) models.¹ However, in general, the studies find it difficult to outperform the atheoretical Vector Autoregressive (VAR) models, especially its Bayesian variant (BVAR) based on the Minnesota prior. These studies tend to attribute the relatively poor performance of the DSGE models to the fact that the frameworks of these models are not sophisticated enough, in the sense, that they, perhaps, do not incorporate the real and nominal rigidities to an appropriate extent to correctly capture the true dynamics of the data characterising the South African economy.

Against this backdrop, we develop a Small Open Economy New Keynesian DSGE-VAR (hereafter SOENKDSGE-VAR) model of the South African economy, characterised by incomplete pass-through of exchange rate changes, external habit formation, partial indexation of domestic prices and wages to past inflation, and staggered price and wage setting. The model is estimated using Bayesian techniques on data for South Africa and the United States (US) from the period 1990Q1 to 2003Q2, and then used to forecast output growth, inflation and a measure of nominal short-term interest rate for one- to eight-quarters-ahead over an out-of-sample horizon of 2003Q3 to 2008Q4. The starting point of the out-of-sample horizon is chosen to correspond with the period when the inflation rate reverted back to the inflation targeting band of 3 percent to 6 percent², while, the endpoint of the sample is purely driven by data availability at the time this paper was written. Note, the starting point of the in-sample was chosen to be 1990Q1 to exclude the excessively volatile GDP, high interest rates and high inflation characterising the South African economy during the 1980s (Steinbach *et al.*, 2009). The forecast performance of the SOENKDSGE-VAR model is then compared with an independently estimated DSGE model, the classical VAR and BVAR models, with the latter being estimated based on six alternative priors, namely, Non-Informative and Informative Natural Conjugate priors, the Minnesota prior, Independent Normal-Wishart Prior, Stochastic Search Variable

¹ See also Ortiz and Sturzenegger (2007), Steinbach et al. (2009), Alpanda et al. (2010, forthcoming) for in-sample analysis of business cycle properties of South Africa using DSGE models.

² Since the announcement made by the minister of Finance in the February of 2000, the sole objective of the South African Reserve Bank (SARB) has been to achieve and maintain price stability. More specifically, the SARB has now adopted an explicit inflation targeting regime, whereby it aims to keep the CPIX inflation rate, where CPIX is defined as Consumer Price Index (CPI) excluding interest rates on mortgage bonds, within the target band of 3 percent to 6 percent, using discretionary changes in the Repurchase (Repo) rate as its main policy instrument.

Selection (SSVS) prior on VAR coefficients and SSVS prior on both VAR coefficients and error covariance.

In contrast to the VARs based on purely statistical foundations, the SOENKDSGE-VAR model uses the theoretical information of a DSGE model to offset in-sample overfitting. Intuitively speaking, the DSGE-VAR approach, as proposed by Del Negro and Schorfheide (2004), can be described as follows: We start off by simulating time-series data from the DSGE Model and then fitting a VAR to these data. In practice, the sample moments of the simulated data is replaced by the population moments computed from the DSGE model solution. Given that the DSGE model depends on unknown structural parameters, one uses a hierarchical prior, which involves placing a specific distribution on the DSGE models parameters. A tightness parameter (λ), which is estimated by maximising the joint density of the data and the parameters, controls the weight of the DSGE model prior relative to the weight of the actual sample, with the values of 0, ∞ and 1 implying an unrestricted VAR, an independently estimated DSGE model and a DSGE-VAR model with equal weight being given to the DSGE and the VAR. Finally, Markov Chain Monte Carlo (MCMC) methods are used to generate draws from the joint posterior distribution of the VAR and DSGE model parameters. In other words, the DSGE-VAR approach tends to create the best of two worlds by devising a framework which tries to mimic the forecasting accuracy of the VAR models, especially the BVAR models, and simultaneously be immune to the Lucas critique (Lucas, 1976), since it combines a stylised general equilibrium model with a VAR by using prior information coming from a DSGE model in the estimation of a VAR.

Our decision to use a DSGE-VAR approach, over and above an independently estimated DSGE model, as done in the previous studies on South Africa, is motivated not only because of the fact that VAR models have tended to outperform DSGE model forecasts for the country, but also because of the available international evidence of DSGE-VAR models producing forecasts which are competitive, and at time substantially better, than the standard benchmark of VAR and BVAR models³. In addition, recall, as outlined above briefly and as will be described in more detail below, an independent DSGE model tends to be a special case of the DSGE-VAR approach. Given this, when we compare the forecasts from the independently estimated DSGE and the DSGE-VAR models, we can determine exactly where the gains in the forecasting performance relative to standard benchmarks, if any, is emanating from, i.e., whether it

³ See for example Del Negro and Schorfheide (2004, 2006), Del Negro et al. (2007), Hodge et al. (2008) and Lees et al. (forthcoming).

is because of the DSGE framework devised or due to estimation of the model based on the DSGE-VAR approach or both. To the best of our knowledge, this is the first attempt in forecasting key variables of the South African economy using a DSGE-VAR approach. In addition, we go beyond the convention in the forecasting literature of DSGE models, by incorporating BVAR models estimated under wider set of priors assumptions besides the Minnesota prior. The remainder of the paper is organised as follows: Respective subsections in Section 2 lays out the estimation methodology of the DSGE-VAR model, discusses the DSGE framework, data, the priors imposed on the DSGE model parameters and the estimation results. Section 3 presents the basics of the alternative forecasting models, while, Section 4 compares the performance of the DSGE-VAR model relative to an independently estimated DSGE model, the classical VAR and the BVAR under six alternative prior assumptions. Finally, Section 5 concludes.

2 Estimation Methodology, Model, Priors, Data and Posterior Estimates of the DSGE Model

2.1 The Basics of the DSGE-VAR Approach

This subsection provides a brief overview of the methodology used to estimate the DSGE-VAR model, and follows closely the discussion in Del Negro and Schorfheide (2004).

Let the parameters of the DSGE model, which we describe in the next subsection, be denoted by the vector θ . Let y_t denote the column vector of n observable variables, which are also the variables included in the VAR. That is,

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \dots + \Phi_p y_{t-p} + u_t, \quad (1)$$

where: Φ_0 is a vector of constants; $\Phi_{1\dots p}$ are matrices of VAR parameters; and $u_t \sim N(0, \Sigma_u)$. This can be written more compactly as $Y = X\Phi + U$, where: Y and U are matrices with rows y'_t and u'_t respectively; X has rows $1, y'_{t-1}, y'_{t-2}, \dots, y'_{t-p}$ and $\Phi \equiv [\Phi_0, \Phi_1, \Phi_2, \dots, \Phi_p]'$. It is noteworthy that the number of parameters in the DSGE model is much smaller than that in the VAR, hence the VAR tends to have a greater ability to fit the data.

As in Del Negro and Schorfheide (2004), we want to use a DSGE model to provide information about the parameters of the VAR. One way of doing this would be to simulate data from the DSGE and to combine it with the actual data and then estimate the VAR, with λ governing the relative weight placed on the prior information, since it is a measure of the relative share of

simulated observations compared to the actual data.

However, rather than simulating data, one can instead use the solution to the log-linearised version of the DSGE model to analytically compute the population moments of y_t , since the DSGE model specifies the stochastic process for y_t . So by choosing λ , we can scale these moments to be equivalent in magnitude to the (non-standardised) sample moments that would have been obtained through simulation. Given this, we can then formulate the prior for the VAR parameters, $p(\Phi, \Sigma_u | \theta)$, given θ , as $\Sigma_u | \theta \sim IW$ and $\Phi | \Sigma_u, \theta \sim N$, i.e., in an Inverted-Wishart (IW)-Normal (N) form. Note, the parameters of these prior densities are functions of the population moments calculated from the DSGE model.⁴ Given that, we also have prior beliefs about the parameters of the DSGE model, $p(\theta)$. The joint prior density of both sets of parameters is then given by:

$$p(\Phi, \Sigma_u, \theta) = p(\Phi, \Sigma_u | \theta) p(\theta). \quad (2)$$

The posterior distribution of the VAR parameters, $p(\Phi, \Sigma_u | Y, \theta)$, is obtained by the likelihood function, which is essentially the combination of the prior with information from the data. Note the likelihood, reflecting the distribution of the innovations (u_t), and the priors for the VAR parameters conjugate, since the former is multivariate normal, while the latter is Inverted-Wishart-Normal. This is particularly helpful, since it allows the posterior to be $\Sigma_u | \theta, Y \sim IW$ and $\Phi | \Sigma_u, \theta, Y \sim N$,⁵ i.e., the posterior follows the same class of distributions as the prior. Finally, by first drawing a θ from the posterior of the DSGE parameters and then sampling from these distributions allows us to simulate the posterior for the VAR parameters.

Note, the posterior of the VAR is conditional on a choice of λ . Let the set of possible λ be given by Λ , where $\Lambda \equiv \{\lambda_1, \dots, \lambda_i, \dots, \lambda_q\}$, and for all i , $\lambda_i > 0$. Del Negro and Schorfheide (2004) suggests the use of the marginal data density, $p(Y | \lambda)$,⁶ to compare the model evaluated at each $\lambda \in \Lambda$, which, in turn, can be obtained by integrating out the parameters of the joint

⁴ See Equations (24) and (25) in Del Negro and Schorfheide (2004) for further details.

⁵ We have suppressed the parameters of the posterior distributions. See Equations (30) and (31) in Del Negro and Schorfheide (2004) for further details.

⁶ The notation of the marginal data density follows Del Negro et al (2007). Also, previously we suppressed the fact that many of the densities, like, the joint prior density for the parameters of the VAR and the DSGE models are conditional on λ .

density of the data and the parameters as follows:

$$\begin{aligned}
 p(Y|\lambda) &\equiv \int_{\Sigma_u, \Phi, \Theta} p(Y, \theta, \Sigma_u, \Phi|\lambda) d(\Sigma_u, \Phi, \theta) \\
 &= \int_{\Sigma_u, \Phi, \Theta} p(Y|\theta, \Sigma_u, \Phi) p(\theta, \Sigma_u, \Phi|\lambda) d(\Sigma_u, \Phi, \theta),
 \end{aligned} \tag{3}$$

where Σ_u , Φ and Θ are the sets of possible parameter values for Σ_u , Φ and θ . Though the integration involved in calculating the marginal data density is computationally intensive, but with $p(\Phi, \Sigma_u, \theta|\lambda)$ equals $p(\Phi, \Sigma_u|\theta, \lambda)p(\theta)$, and $p(\Phi, \Sigma_u|\theta)$ is of Inverted-Wishart-Normal form, the latter enables the integrals with respect to the VAR parameters to be calculated analytically. This leaves only the integral with respect to θ to be calculated in order to approximate $p(Y|\lambda)$.⁷ An 'optimal' λ , $\hat{\lambda}$, could then be obtained so as to maximise $p(Y|\lambda)$, that is,

$$\hat{\lambda} = \arg \max_{\lambda \in \Lambda} p(Y|\lambda). \tag{4}$$

Note one could also use the marginal data density to pick the lag length of the VAR, p (Del Negro and Schorfheide, 2004). We, however, use the unanimity amongst the conventional lag-length tests, namely, the LR test statistic, Akaike information criterion (AIC), the final prediction error (FPE) criterion, the Schwarz information criterion (SIC), as well as the Hannan-Quinn (HQ) information criterion, to decide on our optimal lag length to be used in the VAR (Lees *et al.*, forthcoming).

2.2 The DSGE model

The DSGE model structure builds on standard small open economy New Keynesian models (see Monacelli (2005) and Justiniano and Preston (2004)). Additional nominal rigidities are added to the staggered wage and price setting framework (Calvo, 1983) through partial indexation of domestic and imported prices to their past inflation, as well as partial indexation of wages to past consumer price inflation. Moreover, the model structure allows for incomplete pass-through of exchange rate movements over the short run. Real rigidity emanates from external habit formation in consumption. The small open economy assumption implies that the relative size of the foreign economy, i.e. the rest of the world in the context of this model, is so large that it is not affected by developments in the South African economy and therefore approximates a closed economy. Hence, the model structure of the foreign economy is symmetric to the domestic economy, save for it being closed.

⁷ See Geweke (1999) and An and Schorfheide (2007).

The key log-linearised equations are provided below.⁸

$$c_t = \frac{1}{1+h} E_t c_{t+1} + \frac{h}{1+h} c_{t-1} - \frac{1-h}{\sigma(1+h)} [r_t - E_t \pi_{t+1} + \varepsilon_t^d] \quad (5)$$

$$\pi_t^h = \frac{\omega}{1+\omega\beta} \pi_{t-1}^h + \frac{\beta}{1+\omega\beta} E_t \pi_{t+1}^h + \frac{(1-\theta_h)(1-\theta_h\beta)}{\theta_h(1+\omega\beta)} mc_t \quad (6)$$

$$mc_t = rw_t - a_t + \gamma s_t + \varepsilon_t^p \quad (7)$$

$$\pi_t^w = \alpha \pi_{t-1} + \beta E_t \pi_{t+1}^w - \alpha \beta \pi_t + \frac{(1-\theta_w)(1-\theta_w\beta)}{\theta_w(1+\xi_w\varphi)} \mu_t^w \quad (8)$$

$$\pi_t^f = \frac{\delta}{1+\delta\beta} \pi_{t-1}^f + \frac{\beta}{1+\delta\beta} E_t \pi_{t+1}^f + \frac{(1-\theta_f)(1-\theta_f\beta)}{\theta_f(1+\delta\beta)} \psi_t \quad (9)$$

$$\psi_t = \psi_{t-1} + \Delta e_t + \pi_t^* - \pi_t^f \quad (10)$$

$$\pi_t = (1-\gamma)\pi_t^h + \gamma\pi_t^f \quad (11)$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)[\phi_\pi \pi_t + \phi_y y_t] + \varepsilon_t^r \quad (12)$$

$$E_t q_{t+1} = q_t + (r_t - E_t \pi_{t+1}) - (r_t^* - E_t \pi_{t+1}^*) + \phi_t \quad (13)$$

$$y_t = a_t + l_t \quad (14)$$

$$y_t = (1-\gamma)c_t + \eta\gamma(2-\gamma)s_t + \gamma y_t^* + \eta\gamma\psi_t \quad (15)$$

$$y_t^* = h y_{t-1}^* + \frac{\sigma}{\sigma^*} (c_t - h c_{t-1}) - \frac{1-h}{\sigma^*} q_t \quad (16)$$

Eq. (5) is the consumption Euler equation. Consumption, denoted by c_t , is determined by past values of consumption, expectations about future consumption in $t+1$, and the *ex ante* real interest rate, $r_t - E_t \pi_{t+1}$. In addition, shocks to consumption emanate from the external demand shock ε_t^d , which is assumed to follow the AR(1) process $\varepsilon_t^d = \rho_d \varepsilon_{t-1}^d + \nu_t^d$, where $\nu_t^d \sim \text{i.i.d } N(0, \sigma_d^2)$. The parameter h in Eq. (5) represents the degree of habit formation in consumption and σ is the inverse of the intertemporal elasticity of substitution for consumption.

Domestic inflation, denoted by π_t^h , is represented by a Phillips-curve in Eq. (6). It is modelled as a function of its own lagged values, expected domestic inflation in $t+1$ and marginal costs, mc_t . Three parameters govern the dynamics of this equation: (i) the discount factor β ; (ii) the degree of indexation to past inflation ω ; and (iii) the degree of price stickiness reflected by the Calvo (1983) parameter θ_h . Marginal costs are a function of real wage (rw_t) increases in excess of productivity gains (a_t), the terms of trade (s_t) and a price markup shock (ε_t^p). As

⁸ Lower case letters represent log-deviations from steady state. The full log-linearised model is provided in Appendix A.

in Smets and Wouters (2007), the price markup shock is assumed to follow an ARMA(1,1) process $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + v_t^p + \mu_p v_{t-1}^p$, where $v_t^p \sim \text{i.i.d } N(0, \sigma_p^2)$. The inclusion of the MA(1) term should capture some of the high-frequency fluctuations that are observed in actual inflation.

A Phillips-curve type relationship also holds for nominal wage inflation (π_t^w) in Eq. (8), where wages are partially indexed to consumer price inflation, π_t . The wage mark-up (μ_t^w) serves as a wedge between the real wage and the marginal rate of substitution between labour and consumption, that arises due to wage stickiness. The nature of the parameters that determine the dynamics of wage inflation are similar to those of Eq. (6), as α captures the degree of indexation to past consumer price inflation and θ_w reflects degree of wage stickiness. In addition, $\xi_w \varphi$ is the ratio of the labour demand and supply elasticities.

Eq. (9) indicates that imported inflation, denoted by π_t^f , is a function of the previous period's value of imported inflation, expected imported inflation in $t + 1$ and the degree of imperfect exchange rate pass-through, ψ_t . Imperfect exchange rate pass-through is reflected by deviations from the law-of-one-price in Eq. (10), where Δe_t is the change in the nominal exchange rate and π_t^* represents foreign inflation. This specification reflects the assumption that importing retailers pay the world market price in domestic currency at the dock, but face a downward sloping demand curve in the domestic economy. As a result, importing retailers are not necessarily able to fully pass on changes in the domestic currency denominated world market price to the domestic economy over the short run. Nevertheless, complete exchange rate pass-through is achieved in the long-run. Eq. (11) relates CPI inflation to domestic and imported inflation, where γ is the degree of openness.

Monetary policy (r_t) is described by a Taylor-type rule in Eq. (12), where ρ_r , ϕ_π and ϕ_y are the respective weights on policy smoothing, consumer price inflation and the output gap.⁹ The real exchange rate (q_t) is represented by the UIP condition in Eq. (13), where ϕ_t is a risk premium.

Finally, productivity and labour are the only inputs in production in Eq. (14), aggregate demand in the domestic economy is expressed as Eq. (15), while the model is closed by the consumption risk sharing condition in Eq. (16).

⁹ The decision to drop exchange rate, in either its real form or changes in the nominal value, is due to the available evidence for South Africa on the insignificant role of the variable in the interest rate rule (Ortiz and Sturzenegger, 2007; Alpanda *et al.*, 2010).

2.3 Priors

The model parameters are estimated with Bayesian techniques. However, a number of parameters are calibrated, as it is unlikely to identify all parameters during estimation given the non-linear mapping of the structural parameter vector into the model's reduced form (see Lubik and Schorfheide (2005)). The calibrated parameters are summarised in Table 1.

The values selected for the domestic and foreign economy's discount factor, intertemporal elasticity of substitution for consumption, labour substitution elasticity, and the labour demand elasticity are standard in the literature. Following Smets and Wouters (2007), the degree of habit formation for both economies is set to 0.7.

The degrees of price indexation in the domestic economy for domestic price indexation and imported price indexation are calibrated at 0.15. This fairly low degree of indexation is based on Justiniano and Preston's (2004) estimate – using a uniform prior within the (0, 1) interval – that the degree of price indexation in the small open economies of Australia, Canada and New Zealand is less than 0.2. Nevertheless, the degree to which nominal wages in South Africa are indexed to previous values of CPI inflation tends to be high. As a result, the degree of wage indexation in the domestic economy is calibrated to 0.9. For the foreign economy, the degree of price and wage indexation is set to 0.5, following the findings of Smets and Wouters (2003) for price indexation in the Euro Area.

The smoothing of monetary policy in the domestic economy is set to 0.73, following Ortiz and Sturzenegger (2007), who estimate a monetary policy rule for South Africa. For the foreign economy, all three parameters in the Taylor-type rule (i.e., the weights on policy smoothing, inflation and the output gap) are calibrated to values that are standard in the literature.

Finally, the calibration of the import share in the domestic economy at 0.2 follows Steinbach *et al.* (2009). The authors justify this calibration as a combination of the actual import penetration ratios in total South African GDP and consumption.

The prior specifications of the remaining parameters largely match those used in DSGE models estimated for small open economies. Firstly, all three Calvo parameters for the domestic economy (i.e., domestic prices, wages and import prices) are assumed to follow a beta distribution, with a mean of 0.75 and standard deviation of 0.1. A Calvo parameter of 0.75 implies that firms reoptimise their prices once a year on average, hence reflecting the prior belief that

Table 1: Key Calibrated Parameters

Discount factor	β	0.99		
Habit persistence	h	0.7		
Consumption substitution	σ	1		
Labour supply	φ	3		
Labour demand	ξ_w	1		
Domestic economy			Foreign economy	
Price indexation: domestic	ω	0.15	Price indexation: domestic	ω^* 0.5
Price indexation: imported	δ	0.15	Price indexation: wages	α^* 0.5
Price indexation: wages	α	0.9	Taylor rule: inflation	ϕ_y^* 1.5
Import weight	γ	0.2	Taylor rule: output gap	ϕ_y^* 0.5
Taylor rule: Policy smoothing	ρ_r	0.73	Taylor rule: Policy smoothing	ρ_r^* 0.8
AR(1): Productivity shock	ρ_a	0.9	MA(1): Price markup shock	μ_p^* 0.5
MA(1): Price markup shock	μ_p	0.9		

price stickiness is fairly high in South Africa. Both the Taylor rule weights on inflation and the output gap are assumed to be gamma distributed, where the inflation parameter has a prior mean of 1.5, standard deviation of 0.125 and a lower bound of 1, while the output parameter has a mean of 0.5 and standard deviation of 0.125. The prior for the elasticity of substitution between home and foreign goods also follows a gamma distribution, with a mean of 1 and standard deviation of 0.2.

Finally, since we do not have a strong prior belief about the relative weight of the DSGE model, we follow Adjemian *et al.* (2008) and assume that $\hat{\lambda}$ follows a uniform distribution between the bounds of 0 and 10.

2.4 Data

Eight observable variables are used during estimation, four each for the domestic and foreign economies. The observable domestic variables are: π_t – CPI inflation (excluding interest rates on mortgage bonds) for historical metropolitan and other urban areas; y_t – Real Gross Domestic Product at market prices (seasonally adjusted and annualised); r_t – the Repurchase rate of the South African Reserve Bank; and mc_t – marginal costs, proxied by South African real unit labour costs.¹⁰ The data for the foreign economy are proxied by the United States. Here the observable variables are: π_t^* – US GDP deflator; y_t^* – US real GDP; r_t^* – the Federal Funds rate; and l_t^* – hours worked, proxied by the US Aggregate Weekly Hours Index for Total Private

¹⁰Real unit labour costs are calculated as the ratio of average real remuneration per worker to real GDP, where remuneration is deflated by the GDP deflator.

Industries.¹¹

Table 2: Prior Distributions and Posterior Estimates

Parameter description		Prior density	Prior mean	Prior std dev	Posterior mean	Posterior 90% interval
Domestic economy						
Structural parameters						
Home/foreign substitution	η	G	1	0.2	0.439	[0.301 ; 0.572]
Calvo: domestic prices	θ_h	B	0.75	0.1	0.608	[0.558 ; 0.662]
Calvo: imported prices	θ_f	B	0.75	0.1	0.612	[0.455 ; 0.787]
Calvo: wages	θ_w	B	0.75	0.1	0.719	[0.617 ; 0.822]
Taylor rule weights						
Inflation	ϕ_π	G	1.5	0.125	1.464	[1.326 ; 1.601]
Output gap	ϕ_y	G	0.5	0.125	0.328	[0.215 ; 0.452]
Persistence parameters						
AR(1): demand	ρ_d	B	0.8	0.1	0.741	[0.634 ; 0.852]
AR(1): price markups	ρ_p	B	0.8	0.1	0.684	[0.580 ; 0.791]
Standard deviations of domestic shocks						
iid shock: productivity	σ_a	IG	1	∞	2.408	[1.966 ; 2.898]
iid shock: demand	σ_d	IG	1	∞	0.584	[0.367 ; 0.789]
iid shock: price markups	σ_p	IG	1	∞	2.492	[2.037 ; 2.911]
iid shock: monetary policy	σ_r	IG	1	∞	0.336	[0.280 ; 0.392]
Foreign economy						
Structural parameters						
Calvo: prices	θ^*	B	0.75	0.1	0.726	[0.652 ; 0.800]
Calvo: wages	θ_w^*	B	0.75	0.1	0.918	[0.870 ; 0.969]
Persistence parameters						
AR(1): productivity	ρ_a^*	B	0.8	0.1	0.789	[0.697 ; 0.879]
AR(1): demand	ρ_d^*	B	0.8	0.1	0.793	[0.703 ; 0.882]
AR(1): price markups	ρ_p^*	B	0.8	0.1	0.677	[0.498 ; 0.871]
Standard deviations of foreign shocks						
iid shock: productivity	σ_a^*	IG	1	∞	0.459	[0.390 ; 0.529]
iid shock: demand	σ_d^*	IG	1	∞	0.794	[0.587 ; 1.005]
iid shock: price markups	σ_p^*	IG	1	∞	0.865	[0.431 ; 1.333]
iid shock: monetary policy	σ_r^*	IG	1	∞	0.370	[0.317 ; 0.425]

2.5 Posterior estimates

The posterior parameter estimates – represented by the mean values of each parameter’s estimated posterior distribution – are presented in Table 2. These estimates are for the full sample (i.e. 1990Q1 to 2003Q2). With respect to the domestic economy, the degree of substitution between domestic and foreign goods is low, when compared to Chari, Kehoe and McGratten’s

¹¹ In order to match the log-linearised model specification, all series were detrended prior to estimation. Appendix B lists the various data sources for the observed variables.

(2002) calibration of 1.5. The posterior estimates of the Calvo parameters are lower than their prior means. Both domestic and import prices appear to be reoptimised most frequently – roughly every three quarters. Wages, being optimised once every four quarters, are reflective of the wage formation process in South Africa. The Taylor rule parameter in inflation is broadly in line with prior expectations, whereas the weight on output is lower than anticipated - perhaps reflecting the fairly stable economic growth experienced in South Africa during the latter half of the estimation period. Both the persistence parameters of demand and price markup shocks are fairly high - in line with prior expectations. The standard deviations on productivity and price markup shocks in the domestic economy dominate in terms of magnitude. For the foreign economy, the parameter estimates indicate that prices are reoptimised slightly less often than in the domestic economy. Moreover, when compared to the domestic economy, shocks to the foreign economy appear to be less volatile.

The posterior of the estimated relative weight of the DSGE model in the SOENKDSGE-VAR, $\hat{\lambda}$ is larger than 1, indicating that the data puts a larger weight on the DSGE than the VAR.¹²

3 The Basics of the Alternative Forecasting Models

In this section, we briefly lay out the alternative Bayesian priors imposed on the VAR model, described in Eq. (1).¹³ Note the VAR model generally uses equal lag length for all the variables of the model. One drawback of VAR models is that many parameters need to be estimated, some of which may be insignificant. This problem of overparameterisation – resulting in multicollinearity and a loss of degrees of freedom – leads to inefficient estimates and possibly large out-of-sample forecasting errors. Given this, one solution often adapted is to impose Bayesian shrinkage on lags of the dependant variables.

3.1 The Minnesota Prior

Given that the early work with Bayesian VARs was carried out by researchers at the University of Minnesota or the Federal Reserve Bank of Minneapolis (see Doan *et al.*, 1984, and Litterman, 1986), the prior, discussed below, is popularly called the Minnesota prior. We start off by rewriting: $Y = X\Phi + U$ as $y = (I_n \otimes X)\alpha + \varepsilon$, where $\varepsilon \sim N(0, \Sigma \otimes I_n)$ and $\alpha = \text{vec}(\Phi)$. The Minnesota prior is based on approximations to simplify the specification of the priors and the

¹²Note, the posterior of $\hat{\lambda}$ for the in-sample, 1990Q1 to 2003Q2, was estimated to be 1.803.

¹³This section relies heavily on the discussions available in Koop and Korobilis (2009) and Jochmann *et al.* (2010).

computation. The approximation involves replacing Σ with an estimate $\hat{\Sigma}$, with Σ assumed to be a diagonal matrix. Given this, each equation of the VAR can be estimated independently, and we can set $\hat{\sigma}_{ii} = s_i^2$ (where s_i^2 is the standard OLS estimate of the error variance in the i^{th} equation and $\hat{\sigma}_{ii}$ i^{th} element of $\hat{\Sigma}$). After replacing Σ with $\hat{\Sigma}$, we need to only worry about a prior for α , which is assumed to be as follows:

$$\alpha \sim N(\underline{\alpha}_{Min}, \underline{V}_{Min}). \quad (17)$$

For the prior mean, $\underline{\alpha}_{Min}$, when using data in levels, $\underline{\alpha}_{Min} = 0_{KM}$, $K = (1 + n \times p)$, except for the elements corresponding to the first own lag of the dependent variable in each equation, which, in turn, is chosen to be one. When using growth rates data or detrended variables (as in our case): $\underline{\alpha}_{Min} = 0_{KM}$. The Minnesota prior assumes the prior covariance matrix, \underline{V}_{Min} , is diagonal. Defining \underline{V}_i as the block of \underline{V}_{Min} associated with the K coefficients in equation i and $\underline{V}_{i,jj}$ to be its diagonal elements, then traditionally the Minnesota prior would be set as follows:

$$\underline{V}_{i,jj} = \begin{cases} \underline{a}_1 & \text{for coefficients on own lags} \\ \frac{\underline{a}_2^2 \sigma_{ii}}{p^2} & \text{for coefficients lags of variables } j \neq i \\ p^2 \sigma_{jj} & \text{for coefficients on exogenous variables} \\ \underline{a}_3 \sigma_{ii} & \end{cases} \quad (18)$$

The form of $\underline{V}_{i,jj}$ imposes the fact that, coefficients on longer lags shrink to zero by choosing $\underline{a}_1 > \underline{a}_2$, with own lags being more important predictors than lags of other variables. The exact choice of values for \underline{a}_1 , \underline{a}_2 and \underline{a}_3 depends on the empirical application in concern. For instance, in our case, we experimented with a wide number of values for \underline{a}_1 , \underline{a}_2 and \underline{a}_3 to ensure that we obtain the best forecasts for the three key macro variables. Finally, the researcher generally sets: $\sigma_{ii} = s_i^2$.

3.2 Natural Conjugate Priors

Given the VAR described above, the natural conjugate prior has the following form:

$$\alpha | \Sigma \sim N(\underline{\alpha}, \Sigma \otimes \underline{V}) \quad (19)$$

and

$$\Sigma^{-1} \sim W(\underline{S}^{-1}, \underline{\nu}). \quad (20)$$

where $\underline{\alpha}$, \underline{V} , \underline{v} and \underline{S} are prior hyperparameters that need to be chosen by the researcher. The noninformative prior requires one to set $\underline{v} = \underline{S} = \underline{V}^{-1} = cI$ and letting $c \rightarrow 0$. Note, the major drawback of the non-informative prior is that it does not impose any shrinkage, which, in general, is important for the classical VAR.

3.3 The Independent Normal-Wishart Prior

Note that the natural conjugate prior imposes $\alpha|\Sigma$ to be Normal and Σ^{-1} to be Wishart. In this set up, α and Σ are not independent of one another, since the prior for α depends on Σ . Given this, we now lay out a prior which imposes VAR coefficients and the error covariance to be independent of one another, in other words, independent Normal-Wishart prior. We need to modify our notations of the VAR model to allow for different equations in the VAR to have different explanatory variables. We now use $\beta = \text{vec}(\Phi)$ rather than α , and write each equation of the VAR as:

$$y_{mt} = z'_{mt}\beta_m + \varepsilon_{mt} \quad (21)$$

with $t = 1, \dots, T$ observations for $m = 1, \dots, n$ variables. y_{mt} is the t^{th} observation on the m^{th} variable, z_{mt} is a k_m -vector containing the t^{th} observation of the vector of explanatory variables relevant for the m^{th} variable, β_m is an accompanying k_m -vector of regression coefficients. Here, we allow z_{mt} to vary across equations, and, hence, can create a restricted VAR, whereby some of the coefficients on the lagged dependent variables can be restricted to zero.

Stacking all equations into vectors or matrices as: $y_t = (y_{1t}, \dots, y_{nt})'$, $\varepsilon_t = (\varepsilon_{1t}, \dots, \varepsilon_{nt})'$ and

$$\beta = \begin{pmatrix} \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \beta_n \end{pmatrix}, Z_t = \begin{pmatrix} z'_{1t} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & z'_{2t} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & z'_{nt} \end{pmatrix}$$

where β is a $k \times 1$ vector, Z_t is $n \times k$ where $k = \sum_{j=1}^n k_j$, and $\varepsilon \sim \text{i.i.d. } N(o, \Sigma)$. Then, $y_t = Z_t\beta + \varepsilon_t$.

Further writing,

$$y = \begin{pmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_T \end{pmatrix}, \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_T \end{pmatrix}, = \begin{pmatrix} Z_1 \\ \cdot \\ \cdot \\ \cdot \\ Z_T \end{pmatrix}$$

We can now write:

$$y = Z\beta + \varepsilon, \tag{22}$$

with $\varepsilon \sim N(0, I \otimes \Sigma)$. Given the model above, a very general prior is the independent Normal-Wishart prior, that can be described as follows:

$$p(\beta, \Sigma^{-1}) = p(\beta)p(\Sigma^{-1}), \tag{23}$$

where

$$\beta \sim N(\underline{\beta}, \underline{V}_\beta)$$

and

$$\Sigma^{-1} \sim W(\underline{S}^{-1}, \underline{\nu}).$$

Unlike the natural conjugate prior, the independent Normal-Wishart prior leaves the prior covariance matrix, \underline{V}_β , to be completely at the researcher's discretion and does not restrict it to the $\Sigma \otimes \underline{V}$ form. A noninformative prior in this context would amount to setting:

$$\underline{\nu} = \underline{S} = \underline{V}_\beta^{-1} = 0.$$

3.4 Stochastic Search Variable Selection (SSVS) Prior for VAR Coefficients

The SSVS prior carries out the shrinkage in an automatic fashion, and, hence, unlike that of the Bayesian priors described above requires only minimal prior input from the researcher. The SSVS approach can take various forms, we, however, outline the implementation of George *et al.* (2008).

Suppose α_j is a VAR coefficient. The SSVS specifies a hierarchical prior which is a mixture of

two Normal distributions as follows:

$$\alpha_j | \gamma_j \sim (1 - \gamma_j)N(0, \kappa_{0j}^2) + \gamma_j N(0, \kappa_{1j}^2), \quad (24)$$

where γ_j is a dummy variable taking a value of one or zero such that α_j is then drawn from the second Normal and the first Normal respectively. The SSVS aspect of this prior arises by choosing the first prior variance, κ_{0j}^2 , to be “small” and the second prior variance, κ_{1j}^2 , to be “large”.

George *et al.* (2008) describes a so-called “default semi-automatic approach” to selecting the prior hyperparameters κ_{0j} and κ_{1j} , such that: $\kappa_{0j} = c_0 \sqrt{\text{var}(\alpha_j)}$ and $\kappa_{1j} = c_1 \sqrt{\text{var}(\alpha_j)}$ where $\text{var}(\alpha_j)$ is an estimate of the variance of the coefficient in an unrestricted VAR. The pre-selected constants c_0 and c_1 must be related as follows: $c_0 \ll c_1$. For $\gamma = (\gamma_1, \dots, \gamma_{Kn})$, the SSVS prior assumes that each element has a Bernoulli form and, hence, for $j = 1, \dots, Kn$, we have: $\Pr(\gamma_j = 1) = \underline{q}_j$ and $\Pr(\gamma_j = 0) = 1 - \underline{q}_j$, with $\underline{q}_j = 0.5$ for all j . For Σ , we use the Wishart prior for Σ^{-1} , i.e., $\Sigma^{-1} \sim W(\underline{S}^{-1}, \underline{\nu})$.

3.5 Stochastic Search Variable Selection (SSVS) Prior for both VAR Coefficient and Error Covariance

Instead of using $\Sigma^{-1} \sim W(\underline{S}^{-1}, \underline{\nu})$, following George *et al.* (2008), one can use a SSVS prior for Σ . Let:

$$\Sigma^{-1} = \Psi \Psi' \quad (25)$$

where Ψ is upper-triangular. The SSVS prior imposes a standard Gamma prior for square of each of the diagonal elements of Ψ and the SSVS mixture of normal priors for each element above the diagonal. Thus, the diagonal elements of Ψ are always included in the model and ensures a positive definite error covariance matrix.

Let the non-zero elements of Ψ be ψ_{ij} with $\psi = (\psi_{11}, \dots, \psi_{nn})'$, $\eta_j = (\psi_{1j}, \dots, \psi_{j-1,j})'$, and $\eta = (\eta'_2, \dots, \eta'_n)'$. For the diagonal elements, prior independence is assumed with:

$$\psi_{jj}^2 \sim G(\underline{a}_j, \underline{b}_j) \quad (26)$$

where $G(\underline{a}_j, \underline{b}_j)$ denotes the Gamma distribution with mean $\frac{\underline{a}_j}{\underline{b}_j}$ and variance $\frac{\underline{a}_j}{\underline{b}_j^2}$. We fix $\underline{a}_j = \underline{b}_j = 0.01$ (Koop and Korobilis, 2009). The hierarchical prior for η takes the same mixture

forms of Normal as discussed above for α . For further details, interested readers are referred to George *et al.* (2008) and Jochmann *et al.* (2010).

Note, we work with an unrestricted VAR with an intercept and four lags of the eight variables included in every equation.¹⁴ At this stage, it is important to point out that, ideally, the four US variables should be treated as exogenous, given that South Africa is a small open economy, and hence, the four South African variables should play no part in explaining the behaviour of the US variables. However, we consider all the eight variables as endogenous in the VAR, to be consistent with the DSGE-VAR model, which is estimated using all the eight variables. We do not see this as a problem though, because we are only concerned with forecasting the domestic growth rate, inflation rate and the interest rate, based on their respective equations in the eight variable VAR. Given that each equation has 33 parameters to be estimated and a total of 264 parameters in the system, we consider the following parameterisation of the six priors to provide shrinkage:

- *Non-informative Natural Conjugate Prior*

We choose: $\underline{\alpha} = 0_{Kn \times 1}$, $\underline{V} = 0_{K \times K}$, $\underline{v} = 0$ and $\underline{S} = 0_{n \times n}$.

- *Minnesota Prior*

Recalling that in our case, all variables have been detrended, we set: $\underline{\alpha}_{min}$ to be zero for the lags of all variables. Σ is diagonal with elements s_i^2 obtained from univariate regressions of each dependent variable on an intercept and three lags of the eight variables.

- *Informative Natural Conjugate Prior*

The subjectively chosen hyperparameters of the prior are: $\underline{\alpha} = 0_{Kn \times 1}$, $\underline{V} = 10I_K$, $\underline{v} = n + 1$ and $\underline{S}^{-1} = I_n$.

- *Independent Normal-Wishart Prior*

The subjectively chosen prior hyperparameters are: $\underline{\beta} = 0_{Kn \times 1}$, $\underline{V}_\beta = 10I_{Kn}$, $\underline{v} = n + 1$ and $\underline{S}^{-1} = I_n$.

- *SSVS-VAR*

For the SSVS prior for VAR coefficients only, which essentially involves a semi-automatic approach, we choose: $c_0 = 0.1$ and $c_1 = 10$ and a Wishart prior for Σ with $\underline{v} = n + 1$ and $\underline{S}^{-1} = I_n$.

¹⁴The choice of 4 lags is based on the unanimity of the sequential modified LR test statistic, the AIC and the FPE criterion, applied to a stable VAR estimated with the eight variables. Note, stability as usual, implies that no roots were found to lie outside the unit circle.

- *SSVS*

For the *SSVS* on both VAR coefficients and error covariance, we follow the default semi-automatic approach outlined in George *et al.* (2008), Koop and Korobilis (2009) and Jochmann *et al.* (2010).

Note that, analytical posterior and predictive results are available only for the first three priors, while, for the last three, we require posterior and predictive simulation. The forecasting results presented below are based on 50,000 MCMC draws using a burn-in of 20,000 (Koop and Korobilis, 2009).

4 Evaluation of Forecast Accuracy

Given the specifications of the models, we estimate nine alternative models, namely, the SOENKDSGE-VAR, the classical VAR, six different BVARs and an independently estimated DSGE model, with the latter obtained by setting $\lambda = 100,000$ (Hodge *et al.*, 2008), over the period 1990Q1 to 2003Q2, based on quarterly data. Then we compute the out-of-sample one-through eight-quarters-ahead forecasts for the period 2003Q3 to 2008Q4, and compare the forecast accuracy of the SOENKDSGE-VAR model with the eight alternative forecasting models. The different types of the VARs and the SOENKDSGE-VAR are estimated with 4 lags of each variable. Since we use 4 lags, the initial 4 quarters of the sample, 1990Q1 to 1990Q4, are used to feed the lags. We generate dynamic forecasts, as would naturally be achieved in actual forecasting practice. The models are re-estimated each quarter over the out-of-sample forecast horizon in order to update the estimate of the coefficients, before producing the 8-quarters-ahead forecasts. This iterative estimation and 8-steps-ahead forecast procedure was carried out for 22 quarters, with the first forecast beginning in 2003Q3. This experiment produced a total of 22 one-quarter-ahead forecasts, 21 two-quarters-ahead forecasts, and so on, up to 15 8-step-ahead forecasts. The RMSEs for the forecasts are then calculated for the growth rate, CPIX inflation rate and the Repo rate.¹⁵ Note for the SOENKDSGE-VAR model the estimate of λ is recursively updated over the out-of-sample period. As in Del Negro and Schorfheide (2004, 2006), Del Negro *et al.* (2007) and Lees *et al.* (forthcoming), the percentage gain or loss in the RMSE statistic for the SOENKDSGE-VAR model relative to the eight other alternative models for one- to eight-quarters-ahead forecasts over the period 2003Q3 to

¹⁵Note that if A_{t+h} denotes the actual value of a specific variable in period $t + h$ and F_{t+h} is the forecast made in period t for $t + h$, the RMSE statistic can be defined as: $\sqrt{\frac{1}{h} \sum (A_{t+h} - F_{t+h})^2}$. For $h = 1$, the summation runs from 2003:03 to 2008:04, and for $h = 2$, the same covers the period of 2003Q4 to 2008Q4, and so on.

2008Q4 are then examined.

In Table 3, we compare the percentage gain (negative entry) or loss (positive entry) in RMSEs by using the SOENKDSGE-VAR model over the classical VAR, BVARs and the independently estimated DSGE model for one- to eight-quarters-ahead out-of-sample-forecasts over the period of 2003Q3 to 2008Q4. At this stage, a few words need to be said regarding the choice of the evaluation criterion for the out-of-sample forecasts generated from Bayesian models. As Zellner (1986) points out the “optimal” Bayesian forecasts will differ depending upon the loss function employed and the form of predictive probability density function. In other words, Bayesian forecasts are sensitive to the choice of the measure used to evaluate the out-of-sample forecast errors. However, Zellner (1986) points out that the use of the mean of the predictive probability density function for a series, is optimal relative to a squared error loss function and the Mean Squared Error (MSE), and, hence, the RMSE is an appropriate measure to evaluate performance of forecasts, when the mean of the predictive probability density function is used.

For each of one- to eighth-quarters-ahead forecasts, we test whether the gain (loss) in the RMSE from the SOENKDSGE-VAR model relative to the eight other alternative models is significant, using the $ENC - t$ test of Clark and McCracken (2001) designed for nested models, given that the DSGE-VAR approach nests the VAR and the independently estimated DSGE models. The test statistic is defined as follows:

$$ENC - t = (P - 1)^{\frac{1}{2}} \left[\frac{\bar{c}}{(P^{-1} \sum_{t=R}^{T-1} (c_{t+h} - \bar{c}))^{\frac{1}{2}}} \right] \quad (27)$$

where $c_{t+h} = \hat{v}_{0,t+h}(\hat{v}_{0,t+h} - \hat{v}_{1,t+h})$ and $\bar{c} = \sum_{t=R}^{T-1} c_{t+h}$, R denotes the estimation period, P is the prediction period, f is some generic loss function, $h \geq 1$ is the forecast horizon, $\hat{v}_{0,t+h}$ and $\hat{v}_{1,t+h}$ are h -step ahead prediction errors for models 0 and 1 (where model 0 is the SOENKDSGE-VAR model), constructed using Newey and West (1987) type consistent estimators.

The hypotheses of interest are:

$$H_0 : E \{f(\hat{v}_{0,t+h}) - f(\hat{v}_{1,t+h})\} = 0,$$

and

$$H_A : E \{f(\hat{v}_{0,t+h}) - f(\hat{v}_{1,t+h})\} > 0.$$

The limiting distribution is $N(0, 1)$ for $h = 1$. The limiting distribution for $h > 1$ is non-standard, as discussed in Clark and McCracken (2001). However, as long as a Newey and West (1987) type estimator is used when $h > 1$, then the tabulated critical values are quite close to the $N(0, 1)$ values (Bhardwaj and Swanson, 2006).

Table 3: One- to Eight-Quarters Ahead RMSEs (2003Q3 to 2008Q4)

Model	Quarters ahead							
	1	2	3	4	5	6	7	8
<i>Inflation</i>								
DSGE-VAR	1.2120	1.390	1.407	1.136	1.323	1.462	1.548	1.293
VAR	-42.898**	-32.166*	-12.800	-61.216***	-20.714*	-60.314***	-39.623**	-48.670**
BVAR1	-27.782*	-42.698**	-58.594**	-82.689***	-84.545***	-86.076***	-95.341***	-95.696***
BVAR2	-12.242	-2.237	-1.390	-25.124*	-0.329	4.609	2.927	-16.709
BVAR3	11.597	-20.590*	-26.375*	-59.091**	-12.120	-63.651***	-70.419***	-45.341**
BVAR4	1.700	34.102*	6.934	-22.573*	-16.160	-14.750	20.586*	-11.678
BVAR5	3.700	37.981*	20.164*	-10.301	-0.043	-7.790	36.178*	-12.422
BVAR6	21.883*	27.294*	27.338*	-4.820	1.249	9.635	19.260*	-1.415
DSGE	-0.815	-1.686	1.513	-4.874	-1.950	-0.225	0.810	-4.052
<i>Output growth</i>								
DSGE-VAR	0.499	0.833	0.930	1.051	1.197	1.155	1.170	1.281
VAR	-63.908***	-41.839**	-47.259**	-35.150*	-29.586*	-41.460**	-52.125**	-52.278**
BVAR1	-65.095***	-58.259**	-65.599***	-68.353***	-63.651***	-76.635***	-91.025***	-88.608***
BVAR2	-61.639***	-37.914*	-35.509*	-40.343**	-30.935*	-38.400*	-31.746*	-30.473*
BVAR3	-65.356***	-52.297*	-57.236**	-62.948***	-64.891***	-63.782***	-68.431***	-63.877***
BVAR4	-66.292***	-50.388**	-41.980**	-34.745*	-15.423	-29.976*	-37.161*	-27.293*
BVAR5	-66.647***	-53.144**	-50.970**	-47.045**	-39.837*	-45.321**	-51.665**	-46.099**
BVAR6	-23.920*	14.549	10.772	28.665*	20.706*	6.961	21.429*	59.785**
DSGE	-1.883	1.348	6.469	5.644	7.509	9.622	6.559	1.615
<i>Short-term interest rate</i>								
DSGE-VAR	0.403	0.547	0.545	0.579	0.637	0.669	0.716	0.739
VAR	-37.334*	-39.819*	-42.471**	-32.703*	-44.497**	-65.169***	-73.519***	-75.005***
BVAR1	-42.234**	-60.930***	-63.412***	-73.375***	-90.475***	-87.878***	-94.131***	-97.981***
BVAR2	-37.197*	-14.470	-24.315*	-22.915*	-17.884	-13.667	-18.390	-25.876*
BVAR3	-41.054**	-45.845**	-50.873***	-54.543**	-67.046***	-70.975***	-81.135***	-84.439***
BVAR4	-35.667*	-15.447	-26.431*	-30.144*	-5.877	-10.904	-25.931*	-27.906*
BVAR5	-36.748*	-11.927	-17.832	-16.556	1.314	8.026	4.363	4.957
BVAR6	12.080	43.420**	48.313**	42.759**	75.559***	80.356***	98.223***	105.845***
DSGE	1.650	0.074	1.063	-3.488	-4.180	-4.327	-1.177	-0.923

Notes: Entries in the row DSGE-VAR corresponds to the RMSEs (in percentages) obtained from the SOENKDSGE-VAR model. The entries in the other rows are percentage gain (negative entry) or loss (positive entry) from using the SOENKDSGE-VAR relative to the eight other alternative models for one- to eight-quarters-ahead forecast. BVAR1: Non-informative Natural Conjugate Prior; BVAR2: Minnesota Prior; BVAR3: Informative Natural Conjugate Prior; BVAR4: Independent Normal-Wishart Prior; BVAR5: SSVS Prior on VAR Coefficients; BVAR6: SSVS Prior on VAR Coefficients and Error Covariance. *(**)[***] Indicates 10%, (5%), [1%] Level of Significance for the ENC-t test statistic.

The conclusions, regarding the three variables of concern, based on the percentage gain or loss in RMSEs by using the SOENKDSGE-VAR relative to the eight other alternative models

for one- to eight-quarters-ahead forecast, from these tables can be summarised as follows:

Inflation Rate

The SOENKDSGE-VAR model is found to consistently and significantly (barring the third-quarter-ahead forecast) outperform the classical VAR, as far as forecasting CPIX inflation rate is concerned. The performance of the same becomes a bit weaker when compared to BVAR model based on the Minnesota prior, but it continues to outperform the latter, barring the seventh and eight-quarters-ahead forecasts. Though, the improvements in forecasts are not significant. As far as the other BVAR models are concerned, the SOENKDSGE-VAR model outperforms the BVAR model based on the non-informative natural conjugate prior significantly, but, in turn, is outperformed significantly by the BVAR model based on the SSVS prior on both the VAR coefficients and the error covariance. For the remaining BVAR models, the results are mixed, but, more importantly, the forecast differences are rarely significant. Finally, the SOENKDSGE-VAR model outperforms the independently estimated SOENKDSGE, except for the third- and seventh-steps-ahead forecasts, but the $ENC - t$ test statistic is not significant for any of the one- to eight-quarters-ahead forecasts;

Growth Rate

The SOENKDSGE-VAR model is found to consistently and significantly outperform the classical VAR and the BVAR model based on the Minnesota prior. This result continues to hold as far as the other BVAR models are concerned, barring the BVAR model based on the SSVS prior on both the VAR coefficients and the error covariance. As with the inflation rate, the SOENKDSGE-VAR is outperformed significantly in most cases by the BVAR model based on the SSVS prior on both the VAR coefficients and the error covariance, with the exception of the first-quarter-ahead forecast. Finally, the SOENKDSGE-VAR model is also outperformed by the independently estimated SOENKDSGE, except for the one-quarter-ahead forecast. However, the $ENC - t$ test statistic is not significant for any of the one- to eight-quarters-ahead forecasts;

Interest Rate

The SOENKDSGE-VAR model is found to consistently and in majority of the cases significantly outperform the classical VAR and the BVAR model based on the Minnesota prior. This result continues to hold as far as the other BVAR models are concerned, barring the longer-horizon forecasts from the BVAR model based on the SSVS prior on the VAR coefficients, and for all the one- to eight-steps-ahead forecasts by the BVAR model based on the SSVS prior on both VAR coefficients and the error covariance. Except for the one-step-ahead forecasts,

the DM statistics for the BVAR model with SSVS prior on both VAR coefficients and the error covariance is significant. Finally, the SOENKDSGE-VAR model is found to outperform the independently estimated SOENKDSGE model, especially beyond the third-step-ahead forecasts. However, as with the inflation rate and the growth rate, the $ENC - t$ test statistic is not significant for any of the one- to eight-quarters-ahead forecasts.

5 Conclusion

Recent studies on forecasting macroeconomic variables in South Africa using DSGE models have been found to perform poorly relative to VAR and BVAR models. Against this backdrop, we develop a SOENKDSGE-VAR model of the South African economy, characterised by incomplete pass-through of exchange rate changes, external habit formation, partial indexation of domestic prices and wages to past inflation, and staggered price and wage setting. The model is estimated using Bayesian techniques on data for South Africa and the US from the period 1990Q1 to 2003Q2, and then used to forecast output growth, inflation and a measure of nominal short-term interest rate for one- to eight-quarters-ahead over an out-of-sample horizon of 2003Q3 to 2008Q4. The forecast performance of the SOENKDSGE-VAR model is then compared with an independently estimated DSGE model, the classical VAR and BVAR models, with the latter being estimated based on six alternative priors, namely, Non-Informative and Informative Natural Conjugate priors, the Minnesota prior, Independent Normal-Wishart Prior, SSVS prior on VAR coefficients and SSVS prior on both VAR coefficients and error covariance.

Overall, for the three variables, we can make following important observations: First, barring the BVAR model based on the SSVS prior on both VAR coefficients and the error covariance, the SOENKDSGE-VAR model is found to perform competitively if not better than all the other VAR models for most of the one- to eight-quarters-ahead forecasts. Second, there is no significant gain in forecasting performance by moving to a DSGE-VAR framework when compared to an independently estimated SOENKDSGE model. Combining the two observations made above, we can conclude that the DSGE framework, developed in this paper, is in itself quite competent, and, does not require a combined approach involving the DSGE and VAR models, whereby macroeconomic theory of the DSGE model is utilised to provide priors to an otherwise completely atheoretical VAR model. Finally, there is overwhelming evidence that the BVAR model based on the SSVS prior on both VAR coefficients and the error covariance is the best-suited model in forecasting the three variables of interest.

The fact that our DSGE model is found to perform exceptionally well in terms of forecasting key macroeconomic variables, which includes the inflation rate, of South Africa (an inflation-targeting economy), relative to the commonly used statistical benchmark models, namely, the VARs, makes the framework and, hence, the study immensely important, especially in light of the Lucas (1976) critique, and policy makers need for structural models characterised by “deep” parameters. Future research in this area would be targeted in three possible directions: First, due loss of information associated with first-order linearisation of DSGE models, we would like to carry out the analysis by accounting for non-linearities and re-estimating the model using particle filters, as in Pichler (2008). Second, we would like to incorporate a housing sector in this framework to account for housing market spillover on the real sector of the economy, as in Iacoviello and Neri (2010), and to analyse whether the SARB tends to respond to house price movements. Finally, we would also like to extend our framework to a DSGE model based factor model, as in Boivin and Giannoni (2006) and Schorfheide et al. (2010), to help us forecast variables beyond those used as explicit observable variables in estimating the DSGE model.

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Appendix

A.1. The full-log-linearised model

CPI inflation

$$\pi_t = (1 - \gamma)\pi_t^h + \gamma\pi_t^f \quad (\text{A.1})$$

Domestic inflation

$$\pi_t^h = \frac{\omega}{1 + \omega\beta}\pi_{t-1}^h + \frac{\beta}{1 + \omega\beta}E_t\pi_{t+1}^h + \frac{(1 - \theta_h)(1 - \theta_h\beta)}{\theta_h(1 + \omega\beta)}mc_t \quad (\text{A.2})$$

Marginal costs

$$mc_t = rw_t - a_t + \gamma s_t + \varepsilon_t^p, \quad (\text{A.3})$$

Real wages

$$rw_t = rw_{t-1} + \pi_t^w - \pi_t \quad (\text{A.4})$$

Nominal wage inflation

$$\pi_t^w = \alpha\pi_{t-1} + \beta E_t\pi_{t+1}^w - \alpha\beta\pi_t + \frac{(1 - \theta_w)(1 - \theta_w\beta)}{\theta_w(1 + \varphi\xi_w)}\mu_t^w \quad (\text{A.5})$$

Wage markup

$$\mu_t^w = \frac{\sigma}{1 - h}(c_t - hc_{t-1}) + \varphi(y_t - a_t) - rw_t \quad (\text{A.6})$$

Imported inflation

$$\pi_t^f = \frac{\delta}{1 + \delta\beta}\pi_{t-1}^f + \frac{\beta}{1 + \delta\beta}E_t\pi_{t+1}^f + \frac{(1 - \theta_f)(1 - \theta_f\beta)}{\theta_f(1 + \delta\beta)}\psi_t \quad (\text{A.7})$$

Law of one price gap

$$\psi_t = q_t - (1 - \gamma)s_t \quad (\text{A.8})$$

Terms of trade

$$s_t = s_{t-1} + \pi_t^f - \pi_t^h \quad (\text{A.9})$$

UIP condition

$$E_t q_{t+1} = q_t + (r_t - E_t\pi_{t+1}) - (r_t^* - E_t\pi_{t+1}^*) + \phi_t, \quad (\text{A.10})$$

Taylor rule

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)[\phi_\pi\pi_t + \phi_y y_t] + \varepsilon_t^r \quad (\text{A.11})$$

Aggregate supply

$$y_t = a_t + l_t \quad (\text{A.12})$$

Aggregate demand

$$y_t = (1 - \gamma)c_t + \eta\gamma(2 - \gamma)s_t + \gamma y_t^* + \eta\gamma\psi_t \quad (\text{A.13})$$

Risk sharing condition

$$y_t^* = h y_{t-1}^* + \frac{\sigma}{\sigma^*} (c_t - h c_{t-1}) - \frac{1 - h}{\sigma^*} q_t \quad (\text{A.14})$$

Price markup shock, ARMA(1,1)

$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \nu_t^p + \mu_p \nu_{t-1}^p \quad (\text{A.15})$$

Demand shock, AR(1)

$$\varepsilon_t^d = \rho_d \varepsilon_{t-1}^d + \nu_t^d \quad (\text{A.16})$$

Productivity shock, AR(1)

$$a_t = \rho_a a_{t-1} + \nu_t^a \quad (\text{A.17})$$

Foreign inflation

$$\pi_t^* = \frac{\omega^*}{1 + \omega^* \beta} \pi_{t-1}^* + \frac{\beta}{1 + \omega^* \beta} E_t \pi_{t+1}^* + \frac{(1 - \theta^*)(1 - \theta^* \beta)}{\theta^*(1 + \omega^* \beta)} m c_t^* \quad (\text{A.18})$$

Foreign marginal costs

$$m c_t^* = r w_t^* - a_t^* + \varepsilon_t^{p*}, \quad (\text{A.19})$$

Foreign real wages

$$r w_t^* = r w_{t-1}^* + \pi_t^{w*} - \pi_t^* \quad (\text{A.20})$$

Foreign nominal wage inflation

$$\pi_t^{w*} = \alpha \pi_{t-1}^{w*} + \beta E_t \pi_{t+1}^{w*} - \alpha \beta \pi_t^* + \frac{(1 - \theta_w^*)(1 - \theta_w^* \beta)}{\theta_w^*(1 + \varphi^* \xi_w)} \mu_t^{w*} \quad (\text{A.21})$$

Foreign wage markup

$$\mu_t^{w*} = \frac{\sigma}{1 - h} (y_t^* - h y_{t-1}^*) + \varphi l^* - r w_t^* \quad (\text{A.22})$$

Foreign Taylor rule

$$r_t^* = \rho_r^* r_{t-1}^* + (1 - \rho_r^*) [\phi_\pi^* \pi_t^* + \phi_y^* y_t^*] + \varepsilon_t^{r*} \quad (\text{A.23})$$

Foreign Euler equation

$$y_t^* = \frac{1}{1+h} E_t y_{t+1}^* + \frac{h}{1+h} y_{t-1}^* - \frac{1-h}{\sigma^*(1+h)} \left[r_t^* - E_t \pi_{t+1}^* + \varepsilon_t^{d*} \right] \quad (\text{A.24})$$

Foreign aggregate supply

$$y_t^* = a_t^* + l_t^* \quad (\text{A.25})$$

Foreign price shock, ARMA(1,1)

$$\varepsilon_t^{p*} = \rho_p^* \varepsilon_{t-1}^{p*} + v_t^{p*} + \mu_p^* v_{t-1}^{p*} \quad (\text{A.26})$$

Foreign demand shock, AR(1)

$$\varepsilon_t^{d*} = \rho_d^* \varepsilon_{t-1}^{d*} + v_t^{d*} \quad (\text{A.27})$$

Foreign productivity shock, AR(1)

$$a_t^* = \rho_a^* a_{t-1}^* + v_t^{a*} \quad (\text{A.28})$$

Risk premium

$$\phi_t = \varepsilon_t^d - \varepsilon_t^{d*} \quad (\text{A.29})$$

A.2. Data sources

Observed series	Source
South Africa	
CPI inflation (excluding interest rates on mortgage bonds)	Statistics South Africa
Real Gross Domestic Product (seasonally adjusted)	Statistics South Africa
Repurchase rate	South African Reserve Bank (SARB)
Real unit labour costs	Authors' own calculations; SARB
United States	
GDP deflator	International Financial Statistics, IMF
Real GDP (seasonally adjusted)	International Financial Statistics, IMF
Federal Funds rate	Federal Reserve Bank of St. Louis
US Aggregate Weekly Hours Index (Total Private Industries)	Bureau of Labour Statistics