Dynamic time inconsistency and the South African Reserve Bank

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Abstract

This paper derives the econometric restrictions imposed by the Barro and Gordon (1983) model of dynamic time inconsistency on a bivariate time-series model of Consumer Price Index (CPI) inflation and real Gross Domestic Product (GDP), and tests these restrictions based on quarterly data for South Africa covering the period of 1960:01 through 1999:04, i.e., for the pre-inflation targeting period. The results show that the data are consistent with the short- and long-run implications of the theory of time-consistent monetary policy. Moreover, when the model is used to forecast one-step-ahead inflation over the period of 2001:01 to 2008:02, i.e., the period covering the starting point of the inflation targeting regime till date we, on average, obtain lower rates of inflation. The result tends to suggest that the South African Reserve Bank (SARB), perhaps needs to manage the inflation targeting framework better than it has done so far.

JEL Codes: E31, E52, E61.

Keywords: Dynamic Time Inconsistency; Inflation Targeting; One-Step-Ahead Forecasts.

1. INTRODUCTION

Realizing that South Africa has been targeting inflation since the February of 2000,[1] this paper tries to analyze whether the South African Reserve Bank (SARB) could have produced lower average levels of inflation during the period of 2001:01 to 2008:02 by being time inconsistent. But to do this, we would first need to show that the SARB was in fact time inconsistent over the pre-inflation targeting period of 1960:01 to 1999:04. Econometrically speaking, this can be done by deriving restrictions imposed by the Barro and Gordon (1983) model of dynamic time inconsistency on a bivariate time-series model of Consumer Price Index (CPI) inflation and real Gross Domestic Product (GDP), and then testing these restrictions, both short- and long-run, based on quarterly data for South Africa covering the period of 1960:01 through 1999:04. And then, as far as answering the question posed in the title is concerned, this can done by forecasting inflation one-step-ahead over the period of 2001:01 to current, which in our case happens

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[1] See Sichei (2005) and Ground and Ludi (2006) for detailed reviews on the history of monetary policy in South Africa.

to be 2008:02.[2] And finally, checking, whether, on average, we would have obtained lower rates of inflation over the out-of-sample horizon of 2000:01 to 2007:04. However, it must be realized that to forecast out of the model, we need to ensure that the short- and long-run restrictions obtained from the theory is consistent with the data, and, hence, the SARB was time inconsistent prior to targeting inflation.

The motivation for this analysis simply emanates from the need to evaluate the performance of the monetary authority during the inflation targeting period. Note, the issue is not whether the average level of inflation in the post inflation-targeting period is lower than the pre-inflation targeting period, but, more importantly, whether the inflation rate, on average, would have been higher or lower, if the monetary authority continued to behave in the way it had been prior to targeting inflation? The average level of CPI inflation over the period of 2000:01 to 2008:02 has been 6.17 percent, which is way lower than the average of 9.21 percent that prevailed in the pre inflation-targeting period of 1960:01 to 1999:04. Now a simple comparison of the levels of inflation in the pre and post inflation-targeting period would suggest that the inflation targeting regime has performed quite well in bringing down the average level of inflation in South Africa. However, this is an incorrect way of evaluating the performance of the new regime, because, ideally what we would want to deduce is whether the monetary authority could have done better or worse if it persisted with the policy structure of the old regime, into the period of 2000:01 to 2008:02. To the best of our knowledge, this is the first attempt to evaluate the performance of an inflation targeting monetary authority in this manner. Hence, an obvious extension of this study would be to carry out the analysis for other inflation targeting economies.

The remainder of the paper is organized as follows: Section 2 lays the modified version of the Barro and Gordon (1983) model and derives the theoretical restrictions, while, Section 3 discusses the data and presents the empirical results and also carries out the forecasting exercise. Finally, Section 4 concludes.

2. THE MODIFIED BARRO-GORDON (1983) MODEL

This section presents the modified version of Barro and Gordon's (1983) model of time inconsistent monetary policy as can be found in Ireland (1999). However, unlike Ireland (1999), due to the lack of quarterly data on unemployment for South Africa, we model the supply side using a traditional Lucas-type supply curve rather than an expectational Phillips curve. Specifically, Barro and Gordon's (1983) model is modified by allowing the natural rate of output to follow an autoregressive process that contains a unit root and by incorporating control errors for the rate of inflation. While, the first extension allows for the real GDP to be non-stationary, as will be seen below, the second modification ensures transitory deviations between the actual real GDP and the natural rate of output.

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[2] At the time, the paper was being written, data for the relevant variables were available till 2008:02 only.

As in the standard Lucas supply-curve, the actual output y_t fluctuate around the natural rate y_t^n in response to deviations of the actual inflation rate π_t from the expected inflation rate π_t^e as follows:

$$y_t = y_t^n + \alpha(\pi_t - \pi_t^e); \alpha > 0.$$ (1)

The natural rate of output, in turn, is assumed to fluctuate over time in response to a real (supply) shock ε_t according to:

$$y_t^n - y_{t-1}^n = \lambda(y_{t-1}^n - y_{t-2}^n) + \varepsilon_t; \ -1 < \lambda < 1; \varepsilon_t \sim iid \ N(0, \sigma_\varepsilon^2)$$ (2)

Hence, the change in the natural rate is allowed to follow an AR (1) process. The monetary authority cannot commit to a policy rule, but at the at the beginning of each period $t = 0,1,2...$, after the private agents have formed their expectation of inflation, π_t^e, but prior to the realization of the supply-shock, ε_t, the monetary authority chooses a planned rate of inflation π_t^p. Actual inflation for period t is then determined as the sum of π_t^p and a control error η_t, such that:

$$\pi_t = \pi_t^p + \eta_t; \eta_t \sim N(0, \sigma_\eta^2)$$ (3)

Note η_t is assumed to have a covariance of $\sigma_{\varepsilon\eta}$ with ε_t.

The policy maker chooses π_t^p in order to minimize a loss function that imposes penalty on variations of output and inflation around target values ky_t^n and zero.:

$$L_t = \frac{1}{2}(y_t^n - ky_t^n)^2 + \frac{b}{2}\pi_t^2; \ \ b > 0.$$ (4)

with $k > 1$ and $b > 0$ so that the policymaker wishes to push the actual output over the natural rate.

Using (1) and (3), the monetary authority's problem can be re-written as:

$$\min_{\pi_t^p} E_{t-1}\left[\frac{1}{2}\left((1-k)y_t^n + \alpha\left(\pi_t^p - \pi_t^e + \eta_t\right)\right)^2 + \frac{b}{2}\left(\pi_t^p + \eta_t\right)^2\right]$$ (5)

where $E_{t-1}(.)$ denotes the expectation at the beginning of period t or at the end of period t-1. The first order condition for the above problem is:

$$\alpha E_{t-1}[(k-1)y_t^n + \alpha(\pi_t^p + \eta_t - \pi_t^e)] = bE_{t-1}(\pi_t^p + \eta_t)$$ (6)

Private agents are assumed to know the true structure of the economy and also understand the monetary authority's time-inconsistency problem. In equilibrium, therefore $\pi_t^p = \pi_t^e$, i.e., they correctly anticipate the authority's actions. Using the equilibrium condition and the fact that $E_{t-1}\eta_t = 0$, due to rational expectations, (6) simplifies to:

$$\pi_t^p = \pi_t^e = \alpha A E_{t-1}y_t^n; A = \frac{k-1}{b} > 0$$ (7)

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Equation (7), as in Barro and Gordon (1983), indicates that the inflationary bias resulting from the monetary authority's inability to commit depends positively on the expected natural rate of output $E_{t-1}y_t^n$.

Combining equations (1), (3) and (7) imply that the control error for inflation causes the actual output to fluctuate around the natural rate in equilibrium, i.e.,

$$y_t = y_t^n + \alpha\eta_t \tag{8}$$

Using (2) and (8), and defining $\Delta y_{t-1}^n = y_{t-1}^n - y_{t-2}^n$, we have:

$$y_t = y_{t-1}^n + \lambda\Delta y_{t-1}^n + \varepsilon_t + \alpha\eta_t \tag{9}$$

Meanwhile, equations (2), (3) and (7) imply that:

$$\pi_t = \alpha A y_{t-1}^n + \alpha A\lambda\Delta y_{t-1}^n + \eta_t \tag{10}$$

Equations (9) and (10) separately indicate that both output and inflation are non-stationary respectively, by having inherited the unit roots from the underlying process defining the evolution of the natural rate of output. Putting (9) and (10) together implies that:

$$\pi_t - \alpha A y_t = (1 - \alpha^2 A)\eta_t - \alpha A\varepsilon_t \tag{11}$$

Which, in turn, shows that the linear combination of inflation and output is stationary, i.e. π_t and y_t are cointegrated. So based on equation (11), the modified version of the Barro and Gordon's (1983) model implies that inflation and real GDP are non-stationary, but cointegrated. Statistical tests of the cointegration constraint, implied by (11), will determine whether the modified Barro and Gordon (1983) model can explain the long-run behaviour of inflation and output in South Africa.

Taking first difference of (8) and (11), and then replacing out the first-differenced value of $\Delta y_t \times \alpha A$ from the first-differenced version of (8) into the first-differenced version of equation (11) yields:

$$\Delta\pi_t = \alpha\lambda\Delta y_t^n + \eta_t - \eta_{t-1} - \alpha A\varepsilon_t + \alpha A\varepsilon_{t-1} \tag{12}$$

where $\Delta\pi_t = \pi_t - \pi_{t-1}$ and $\Delta y_t^n = y_t^n - y_{t-1}^n$.

Equation (12) in turn can then be re-written using a state-space representation as follows:

$$\varepsilon_t = F\varepsilon_{t-1} + Q\upsilon_t \tag{13}$$

$$\zeta_t = H\varepsilon_t \tag{14}$$

where,

$$\varepsilon_t = \begin{bmatrix} \Delta y_t^n \\ \varepsilon_t \\ \varepsilon_{t-1} \\ \eta_t \\ \eta_{t-1} \end{bmatrix}; \quad F = \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}; \quad Q = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix};$$

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$$H = \begin{bmatrix} \alpha A & -\alpha A & \alpha A & 1 & -1 \end{bmatrix}; z_t = [\Delta \pi_t]; \upsilon_t = \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix},$$

$$\text{with } E(\upsilon_t \upsilon_t') = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix}.$$

Further note, using (2) and a one-period lagged expression for (12), equation (12) can be re-written as:

$$\Delta \pi_t = \lambda \Delta \pi_{t-1} + \eta_t - (1+\lambda)\eta_{t-1} + \lambda \eta_{t-2} + (1+\lambda)\alpha A \varepsilon_{t-1} - \alpha \lambda A \varepsilon_{t-2} \qquad (14)$$

The within equation restriction appearing in (14) implied by the ARMA(1,2) process for the change in the actual inflation rate, summarizes the constraints that the modified version of the Barro and Gordon (1983) model imposes on the short-run behaviour of inflation. As with the long-run relationship, a statistical test of these restrictions will determine whether the modified model explains the dynamics of inflation that can be found in the South African data. This essentially boils down to using a likelihood-ratio test statistic for establishing the acceptance or the rejection of the short-run restrictions implied by equation (14) in relation to an unrestricted version of the ARMA (1,2) model of $\Delta \pi_t$, which looks as follows:

$$\Delta \pi_t = \phi_1 \Delta \pi_{t-1} + \phi_2 \varepsilon_{t-1}^\pi + \phi_3 \varepsilon_{t-2}^\pi + \phi_4 \varepsilon_{t-1}^y + \phi_5 \varepsilon_{t-2}^y + \varepsilon_t^\pi \qquad (15)$$

At this stage, it is important to point out that we have reduced the two-variable model to a single variable $\Delta \pi_t$. This allows us to lower the number of parameters for the unrestricted ARMA (1,2) from 16 to 8, and in the process, help us reduce the difficulty of finding initial parameter values, via grid search, involved in estimating state-space models. Besides, in this paper, in any case, we are more interested in studying the behavior of inflation over the period of 2000:01 to 2008:02.

3. DATA AND RESULTS

In this study, we use seasonally adjusted quarterly time series data for real GDP and CPI inflation over the period of 1960:01-2008:02, both of which were obtained from the Quarterly Bulletins of the SARB. Note the base year is 2000, and we transform the real GDP series into its logarithmic values. In this section, we first discuss the tests of the long-run constraint and then move on to verifying the validity of the short-run restrictions.

3.1. Testing the Long-Run Restrictions:
Before we move to the formal tests of the long-run relationship, we present in Figures 1 through 4, the data plots for the inflation rate and the real GDP, and the associated trends based on a linear trend and a 10-period centred moving average over the period of 1960:01 to 1999:04. From the linear trends in Figures 1 and 2 for inflation and real GDP respectively, we find the variables to share a common positive trend. However, from Figures 3 and 4, based on the 10-period centred moving average, we find that the long-

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run inflation rate has experienced a downward movement since 1992 onwards, though output has continued to rise. Given this, it is likely that we might not find a cointegrating relationship between output and inflation over a shorter sample spanning the years of 1992 to 1999. But overall, for the whole period of 1960:01 to 1999:04, it is quite obvious that the two series are more than likely to be cointegrated.
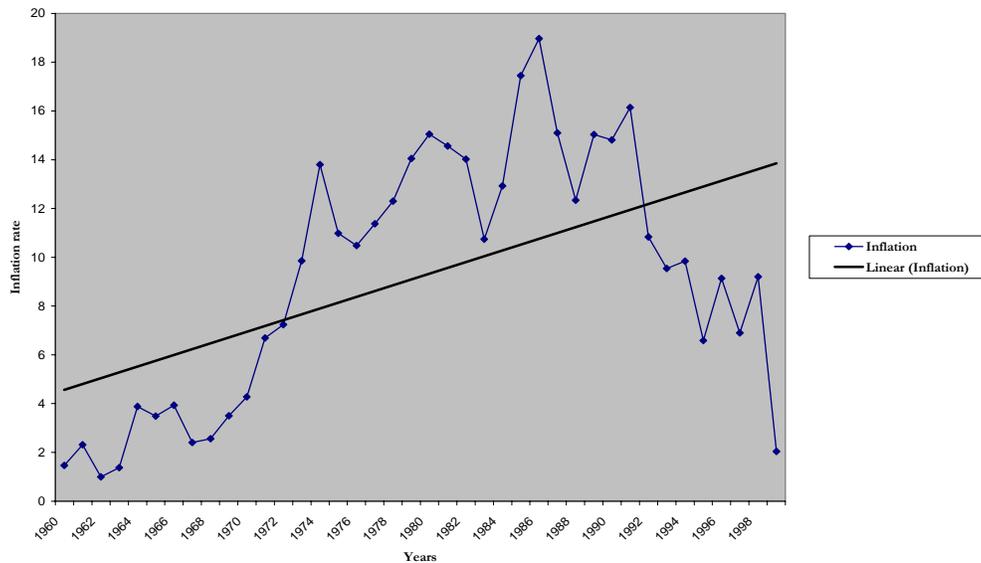

Figure 1. Linear trend of CPI inflation (2000=100).
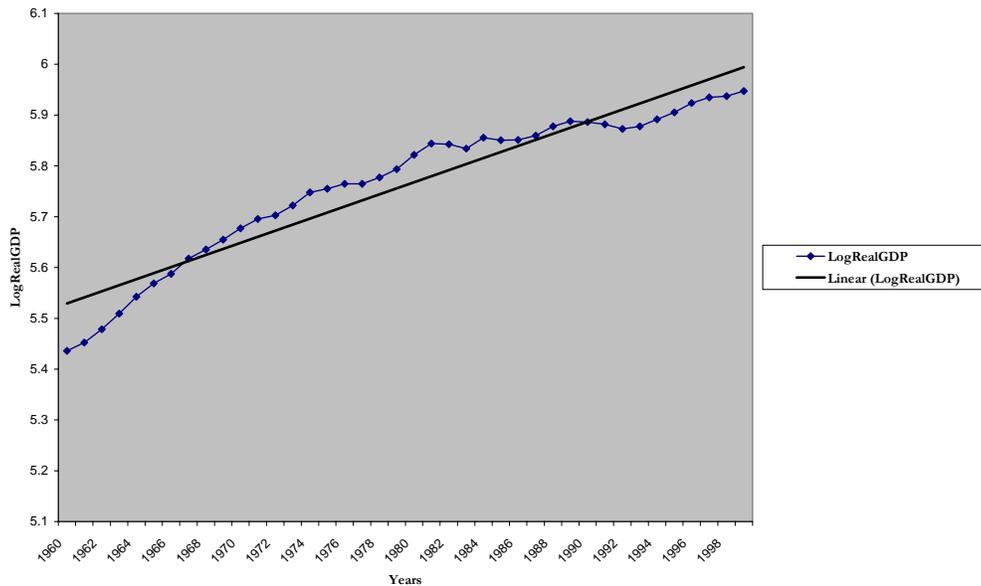
Figure 2. Linear trend of LogRealGDP (2000=100).

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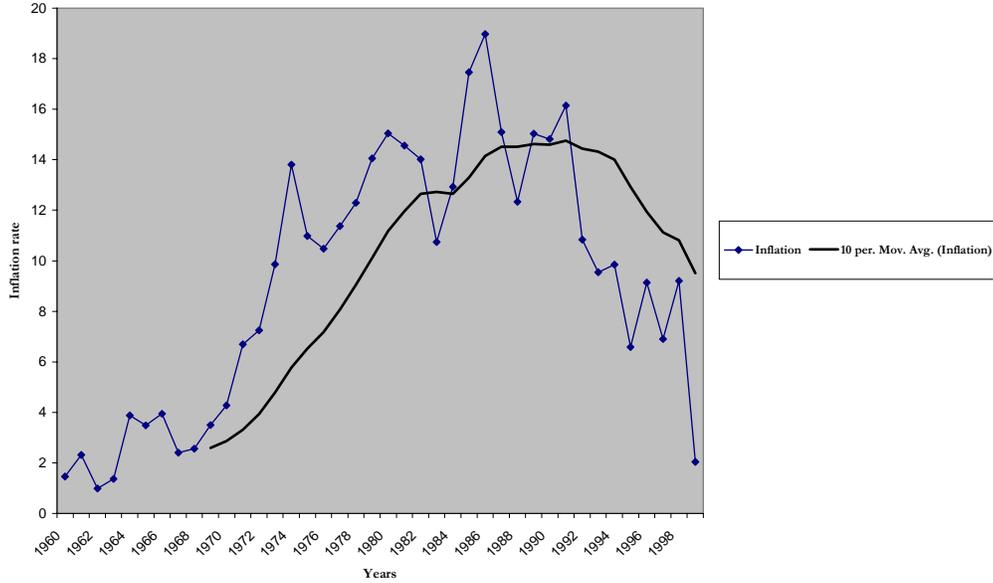
Figure 3. 10-year –centered Moving Average of CPI inflation (2000=100).
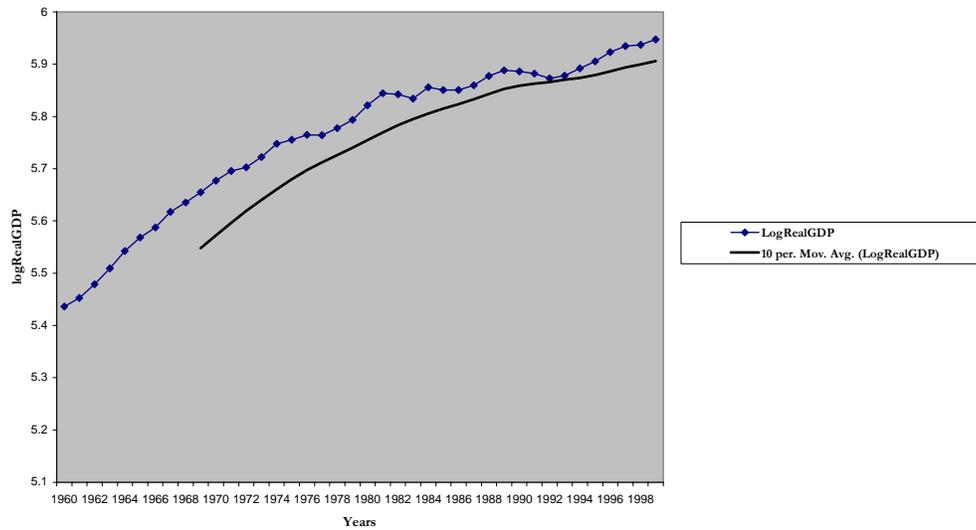
Figure 4. 10-year –centered Moving Average of LogRealGDP (2000=100).

Equations (9) and (10) indicate that the real GDP and the CPI inflation rate respectively, should be non-stationary. As can be seen from Table 1, based on the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips-Perron (PP) test, both the variables were found to follow an autoregressive process with a unit root, as the

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null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests, while for the KPSS test, the null of stationarity was rejected.

Table 1. Unit Root Tests

Series	Model	ADF		PP	KPSS	DF-GLS	Conclusion
		$\tau_\tau \tau_\mu \tau$	$\phi_3 \phi_1$	$\tau_\tau \tau_\mu \tau$	$\tau_t \ \tau_\mu$	$\tau_t \ \tau_\mu$	
CPI		-0.81	13.69	-5.22***	0.37	-1.80	Non- stationary
	τ_μ	-2.19	23.13**	-4.29***	0.77	-0.90	
	τ	-0.68		-2.01**			
ΔCPI	τ_t	-10.59***	97.47***	-51.93***	0.12*	-13.95***	Stationary
	τ_μ	-10.42***	120.01***	-38.49***	0.31***	-1.41	
	τ	-10.45***		-38.21***			
Y	τ_t	-2.30	11.10**	-2.27	0.37	-0.41	Non- stationary
	τ_μ	-4.61***	21.23***	-4.73***	1.46	1.47	
	τ	4.08***		5.84***			
ΔY	τ_t	-7.14***	59.98***	-12.97***	0.12***	-7.06***	Stationary
	τ_μ	-6.41***	82.09***	-12.33***	0.95	-3.54***	
	τ	-3.35***		-10.48***			

Note: ***(**)[*] indicates significance at 1 percent (5 percent) [10 percent] level.

To check for the cointegrating relationship between inflation and output implied by equation (11), we start of by using the Phillips-Ouliaris (1990) test. Results reported in Table 2 shows the estimate of γ, the coefficient from a regression of π_t on y_t, along with the statistics needed to test for a unit root in the residual from this regression. As can be seen, the hypothesis that inflation and output are not cointegrated can be rejected at the 1 percent significance level.

Table 2. Phillips-Ouliaris (1990) Cointegration Test (1960:01-1999:04)

γ	ρ	τ	q	Z_τ
1.61	0.74	-4.92	0	-4.92***

Note: *** indicates significance at 1 percent level.

For the Phillips-Ouliaris (1990) test, Table 2 reports γ, the coefficient form the regression of inflation on real GDP, ρ, the coefficient from the regression of the residual from the inflation-output regression on its own lagged term and τ the conventional *t*-statistic for testing the hypothesis that ρ =1. Z_τ indicates the adjusted *t*-statistic by allowing for serial correlation in the regression error. The adjustment uses

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Newey and West's (1987) method to estimate the variance of the regression error and Andrew's (1991) method to select a value for the lag truncation parameter q required for the Newey and West (1987) estimator, assuming that the regression error is well approximated by a first-order autoregressive process.

One potential drawback of the residual-based Phillips-Ouliaris (1990) test is concerned with the fact that the sensitivity of the results might hinge on which variable (inflation or output), is used as the dependent variable in the initial regression. Here, however, equation (11) indicates that the hypothesized cointegrating relationship as suggested by the theoretical implications of the modified Barro and Gordon (1983) model is of the following form: $\pi_t - \gamma\, y_t$. This implies that we should be treating inflation as the dependant variable. Nevertheless, we check for the robustness of the results by using the Johansen (1988) test of cointegration, which, in turn, does not require a choice of normalization.

Based on a stable VAR[3] estimated with 5 lags[4], and allowing for no trend and intercept in the VAR or the cointegrating relationship, as suggested by the theory, we tested for cointegration using Johansen's (1988) maximum likelihood approach. Based on the Pantula Principle, both the Trace and the Maximum Eigen Value tests, showed that there is one stationary relationship ($r = 1$) between inflation and output at 1 percent level of significance. The results have been reported in Table 3, and they confirm the finding of the Phillips-Ouliaris (1990) test. The corresponding cointegrating vector relating inflation and real GDP, obtained from the Johansen (1988) approach, is found to be: $0.13\pi_t - 0.40\, y_t$.

Table 3. Estimation and Determination of Cointegrating Rank

Null hypothesis	Alternative hypothesis	Test statistic	0.05 critical value	Prob. **
Trace Statistic				
r=0	*r=1*	24.62365	12.32090	0.0003
r=1	*r=2*	1.912715	4.129906	0.1962
Trace test indicates no cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
Maximum Eigenvalue Statistic				
r=0	*r=1*	22.71093	11.22480	0.0003
r=1	*r=2*	1.912715	4.129906	0.1962
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

Thus, the results reported in Tables 1 through 3 strongly support the long-run behaviour of inflation and real GDP implied by the restrictions obtained from the modified Barro and Gordon (1983) model.

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[3] Stability, as usual, implied that no roots were found to lie outside the unit circle.
[4] The choice of 5 lags is based on the unanimity of the sequential modified LR test statistic, Akaike information criterion (AIC), and the final prediction error (FPE) criterion.

3.2. Testing the Short-Run Restrictions:

Focussing now on the short-run implications of the behavior of the change in the inflation rate, imposed by the modified Barro and Gordon (1983) model, Table 4 presents the maximum likelihood estimates of the model's parameters, which, in turn, is obtained by mapping (12) into a state space form, implied by (13) and (14). The likelihood function of the state-space model is then evaluated using Kalman filter.

The estimate of α =1.2716 suggest that the Lucas-supply curve is quite flat. Burger and Marinkov (2006) also draws similar conclusion about the slope of the curve using a VECM. Although, the parameters k and b are not identified individually, the estimate of $A=(k-1)/b$ exceeds unity. With $k>1$, the result suggests that $b<1$, implying that the SARB placed more weight on its goal of output than on inflation over the pre-inflation-targeting era of 1960:01 to 1999:04. Again, similar observations have been made by Gupta and Naraidoo (2008), while estimating interest rate rules for South Africa in periods before the SARB moved into an inflation targeting framework. As expected, the estimate of λ is positive, though is not significant, as is the standard deviation of the real shock. The standard deviation that for the control error is, however, significant at the one percent level. Finally, the negative and significant estimate of the covariance $\sigma_{\varepsilon\eta}$ indicates that a positive shock to the natural rate tend to coincide with a negative shock to inflation. The estimate, thus, supports, the idea that ε_t represents a real shock or a supply-side disturbance.

Table 4: Maximum Likelihood Estimates

Parameter	Estimate	Standard Error
α	1.27***	0.17
A	1.27**	0.61
λ	0.33	0.43
σ_{ε}	0.37	0.26
σ_{η}	2.44***	0.41
$\sigma_{\varepsilon\eta}$	-0.91**	0.45

Notes: (i) ***(**) indicates significance at 1 percent (5 percent) level;

(ii) L^c =-562.03; L^u = - 557.60

The within-equation restrictions that appear in (12) can be tested by comparing its fit with an unconstrained ARMA (1,2) model identified in (15). The constrained model has 6 parameters, while the unconstrained model has 8. Thus, the theory places 2 restrictions on the univariate time series model for the stationary variable $\Delta\pi_t$. So, if L^c and L^u respectively, denotes the maximized value of the log-likelihood function for the unconstrained and the constrained model, then the likelihood ratio statistic $LR=2(L^u - L^c)$ has a chi-square statistic with 2 degrees of freedom under the null hypothesis that the constraints of the ARMA(1,2) model for $\Delta\pi_t$ holds. The *LR* statistic is: 2(562.03-557.60) = 8.86 < 9.21 (the 99 percent critical value for a χ^2 with 2 degrees of freedom). We can, thus, conclude that the model's short-run implications, as imposed by

the theory, cannot be rejected at the 1 percent level of significance. This means that the data provides weak evidence of the theory in the short-run.

3.3. Evaluating the Inflation-Targeting Regime (2000:01-2008:02):
In this subsection, we evaluate the inflation targeting regime by trying to deduce whether the monetary authority could have done better or worse if it stayed time inconsistent over the period of 2000:01 to 2008:02 as well. To do this, we forecast the rate of inflation recursively using (12), first, based on new data generated from the one-period-ahead forecasts, and then second, based on the actual inflation rate that prevailed over this period. Ideally, because we are comparing across regimes, we would want to rely more on the forecasts generated from the forecasted values, rather than the original values. This is simply because, we are trying to analyze how the policymaker would have performed if it stayed time inconsistent, and hence, would not want to use the actual data that corresponds to the behaviour of the inflation rate in a different regime. However, just for the sake of completeness and comparison, we also forecast using the actual rate of CPI inflation.
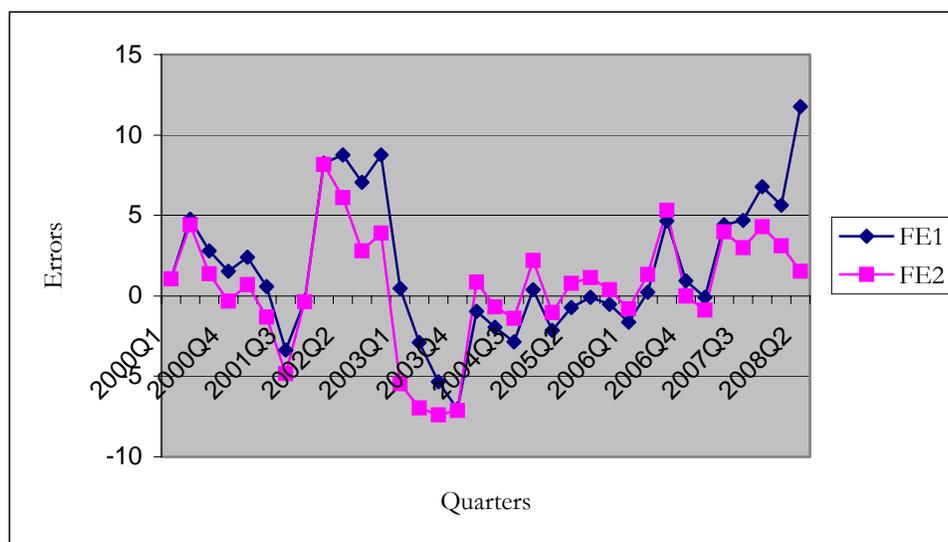
Figure 5.Differences between Actual and Forecasted Inflation.
Note: FE1(FE2) implies Forecast Errors based on forecast (actual) inflation.

Figure 5 plots FE1 and FE2, the one-step-ahead forecast errors based on forecasted values and actual values of the rate of inflation, respectively. We observe that the pattern of the movement of FE1 and FE2 are quite similar. Based on our calculations, the average value of the forecast errors based on the forecasted values of the inflation rate is 1.65 compared to 0.53 of the same when we use actual values. More importantly this implies, that if the SARB had continued to be time inconsistent, it could have produced on, average, an inflation rate which would have been lower by 1.68 percent or 0.53 percent from what has prevailed over 2000:01 to 2008:02. Clearly then, the economy has been worse off in terms of the average levels of inflation experienced.

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But then the big question is, whether these lower average rates of inflation, that could have been witnessed, are significantly different from what has actually been observed in the data on average? For this purpose, we resort to the Mincer and Zarnowitz (1969) regression. Note, for forecasts to be considered 'good' in an absolute sense they should not systematically under- or overpredict the rate of inflation. Formally, for a one-step-ahead forecast, we must have the following relationship:

$$E_{t-1}(\pi_t - \pi_{ft}) = 0 \tag{16}$$

where π_t and π_{ft} are the actual and the one-step-ahead forecasted values of the rate of inflation. Econometrically speaking, the Mincer and Zarnowitz (1969) regression boils down to testing the unbiasedness of the forecasts by regressing the forecasted values on the actual values of the variable under consideration.

Given the regression:

$$\pi_t = \beta_0 + \beta_1 \pi_{ft} + v_t \tag{17}$$

we test the joint unbiased hypothesis of: $\beta_0 = 0 \, \text{and} \, \beta_1 = 1$. Due to problems of possible serial correlation in the estimation of (17), Newey and West (1987) Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors were calculated. Based on a Wald test, the probabilities of unbiasedness was found to be 0.37 and 0.46, depending on whether we use forecasted values or actual values to compute the forecasts.[5] This implies that the restrictions cannot be rejected and, hence, the model produces unbiased forecasts. However, more importantly this implies that, on average, the actual and forecasted values are not statistically different. Or in other words, the possible lower average rates of inflation that we could have obtained from a time inconsistent SARB is not significantly different from what the authority has achieved under the inflation-targeting regime.

Alternatively, the results can be evaluated from the perspective of welfare costs of inflation. Based on long-run money demand estimations, recent studies by Gupta and Uwilingiye (2008a,b,c) have shown that the welfare loss produced by the current inflation-targeting band of 3 to 6 percent pursued by the SARB varies between 0.15 percent to 0.41 percent of GDP, depending on the econometric methodology and the sampling techniques used.[6] Clearly then, lower average rates of inflation in the order of 1.6 percent and 0.53 percent would have translated into very small sizes of welfare loss. But at the same time, it must be realized that welfare cost calculations based on money

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[5] In an attempt to ensure that both π_t and π_{ft} were stationary, the regressions were re-estimated based on the first difference of the actual and forecasted values of inflation. The probabilities of unbiasedness based on the Wald test were found to be 0.81 and 0.16 respectively, based on forecasted values and actual values, implying that the restrictions, and, hence, unbiasedness cannot be rejected.

[6] The authors show that welfare cost calculations vary considerably depending on whether one uses the Fisher and Seater (1993) long-horizon approach or the Johansen (1991, 1995) cointegration methodology to estimate the long-run money demand. Moreover, the authors found that the results are also sensitive to using temporal aggregation or systematic sampling in converting high frequency data, namely, the measure of money and the opportunity cost variable, into their quarterly values.

demand estimations, are essentially adopting a partial equilibrium approach. As argued by Dotsey and Ireland (1996), in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output. In addition, as pointed out by Feldstein (1997), interactions between inflation and a non-indexed tax code can add immensely to the welfare cost of inflation. Given this, these so-called insignificant lower average rates of inflation of 1.65 percent and 0.53 percent can result in possibly quite large welfare losses.

4. CONCLUSIONS

This paper derives the econometric restrictions imposed by a modified Barro and Gordon (1983) model of dynamic time inconsistency on a bivariate time-series model of CPI inflation and real Gross Domestic Product (GDP), and tests these restrictions based on quarterly data for South Africa covering the period of 1960:01 through 1999:04. The results show that the data are consistent with the long-run implications of the theory of time-consistent monetary policy involving the two variables. However, as far as the short-run dynamics of the data is concerned, the evidence is weak. But importantly, when the model is used to forecast one-step-ahead inflation over the period of 2001:01 to 2008:02, i.e., the period covering the starting point of the inflation targeting regime till date, we, on average, produce lower rates of inflation, than those observed in the actual data.

However, based on the Mincer and Zarnowitz (1969) regression, we find that the possible lower average rates of inflation that we could have obtained from a time inconsistent SARB is not significantly different from what the authority has achieved under the inflation-targeting regime. But then again, realizing that in a general equilibrium framework, higher inflation rates can distort a host of other marginal decisions, besides the money demand, these so-called insignificant lower average rates of inflation could easily result in possibly quite large welfare losses. Given this, future research, should be aimed at obtaining the size of the welfare cost of inflation in a dynamic general equilibrium endogenous growth setting. Besides, one might want to repeat this exercise for other inflation-targeting economies to make a proper judgment of whether the approach is an appropriate way of evaluating the regime. Recall, this method would entail one to show first, that the central banks in those economies prior to targeting inflation, were time inconsistent.

Significant or not, we show that time inconsistent policy would on average have produced lower rate of inflation for South Africa. The results, thus, point to the fact that, perhaps the SARB needs to manage the inflation-targeting framework better than it has done so far. In this regard, as pointed out generally by Demertzis and Viegi (2006, 2007, and 2008), a narrower, and possibly also a lower, target band could be of immense help in improving the central bank's credibility and causing inflation expectations to converge to a focal point, and hence, bring down the rate of inflation.

REFERENCES

ANDREWS, D.W.K., (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica*, 59: 817-858.

BARRO, R.J. and GORDON, D.B. (1983). A positive Theory of Monetary Policy in a Natural rate Model. *Journal of Political Economy*, 91:589-610.

BURGER, P. and MARINKOV, M. (2006), The South African Phillips Curve: How Applicable is the Gordon Model? *South African Journal of Economics*, 74:172-189.

DEMERTZIS, M. and VIEGI , N. (2007). Inflation Targeting - a Framework for Communication. *DNB Working Papers 149*, Netherlands Central Bank, Research Department.

DEMERTZIS, M. and VIEGI, N. (2006). Aiming for the Bull's Eye: Uncertainty and Inertia in Monetary Policy. *Computing in Economics and Finance 2006 150*, Society for Computational Economics.

DEMERTZIS, M. and VIEGI, N. (2008). Inflation Targets as Focal Points. *International Journal of Central Banking*, 4: 55-87.

DOTSEY, M. and IRELAND, P. N. (1996). The welfare cost of inflation in general equilibrium. *Journal of Monetary Economics*, 37: 29-47.

FELDSTEIN, M. (1997). The costs and benefits of going from low inflation to price stability. In Christina D. Romer and David H. Romer (eds), *Reducing Inflation: Motivation and Strategy*. Chicago: University of Chicago Press.

FISHER, M. and SEATER, J. (1993). Long-run neutrality and superneutrality in an ARIMA framework. *American Economic Review*, 83:402-415.

GUPTA, R. and UWILINGIYE, J. (2008a). Measuring Welfare cost of inflation in South Africa. *South African Journal of Economics*, 76(1):16-25.

————--. (2008b). Measuring Welfare cost of inflation in South Africa: A Reconsideration. *Working Paper No 2008-09,* Department of Economics, University of Pretoria.

————. (2008c). Time Aggregation, Long-run Money Demand and the Welfare cost of Inflation. *Working Paper No 2008-25,* Department of Economics, University of Pretoria.

GUPTA, R. and NARAIDOO, R. (2008). Modelling Monetary Policy in South Africa: Inflation Targeting in Practice. *Mimeo*, Department of Economics, University of Pretoria.

IRELAND, P. N (1999). Does the Time-consistency Problem Explain the Behaviour of Inflation in the United States. *Journal of Monetary Economics*, 44: 279-291.

JOHANSEN, S. (1988). Statistical Analysis of Cointegration Vectors. *Journal of Economic Dynamics and Control*, 12: 231-254.

————. (1991). Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. *Econometrica*, 59: 1551-1580.

————. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.

LUDI, K. L. and GROUND, M. (2006). Investigating the bank-lending channel in South Africa: A VAR approach. *Working Paper No. 2006-04*, Department of Economics, University of Pretoria.

MINCER, J. and ZARNOWITZ, V. (1969). The Evaluation of Economic Forecasts. In J. Mincer (ed.), *Economic Forecasts and Expectations*. New York : NBER.

NEWEY, W.K. and WEST, K.D. (1987). A simple, positive-semi definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55: 703-708.

PHILLIPS, P.C .B and OULIARIS, S. (1990). Asymptotic Properties of Residual Based Tests for cointegration. *Econometrica,* 58:165-193.

SICHEI, M. M (2005). Bank-Lending Channel in South Africa: Bank-Level Dynamic Panel Data Analysis, *Working Paper No 2005-10,* Department of Economics, University of Pretoria.