BAYESIAN METHODS OF FORECASTING INVENTORY INVESTMENT IN SOUTH AFRICA

Abstract

This paper develops a Bayesian Vector Error Correction Model (BVECM) for forecasting inventory investment in South Africa. The model is estimated using quarterly data on actual sales, production, unfilled orders, price levels and interest rates, for the period of 1978 to 2000. The out-of-sample-forecast accuracy obtained from the BVECM, over the forecasting horizon of 2001:1 to 2003:4, is compared with those generated from the Classical variant of the VAR and the VECM, the Bayesian VAR, and the ECM of inventory investment developed by Smith *et al.* (2006) for the South African economy. The BVECM with the most tight prior outperforms all the other models, except for a relatively tight BVAR. This BVAR model also correctly predicts the direction of change of inventory investment over the period of 2004:1 to 2006:3.

1.   INTRODUCTION

One of the earliest theoretical models of inventory adjustment was developed by Lovell (1961). One implication of this model is that the empirical analysis of inventory investment should incorporate both short- and long-term dynamics.[1] An econometric framework that integrates both such dynamics is the Error Correction set-up. In such a backdrop, this paper develops a Bayesian Vector Error Correction Model (BVECM) for forecasting inventory investment in South Africa. The model is estimated using quarterly data on actual sales, production, unfilled orders, price levels, interest rates, for the period of 1978 to 2000, and then, used to compare the out-of-sample forecast generated by the model, with that of alternative models, over 2001:1 to 2003:4. The alternative forecasts generated are based on the Classical variant of the Vector Autoregressive (VAR) and the Vector Error Correction (VEC) models, Bayesian VAR (BVAR) models, and the Error Correction Model (ECM) of Smith *et al.* (2006) developed for explaining the movements of inventory investment for the South African economy.

The motivation for such a study is mainly an attempt to verify whether we can perform better in terms of forecasting the inventory investment using a Bayesian approach based on the same set of variables, when compared to the ECM developed by Smith *et al.* (2006). In this paper the authors, based on the production smoothing approach, empirically analyse the movements of inventory investment in South Africa over the period of 1986:1 to 2002:4. The paper indicates that actual sales, production, unfilled orders, price levels, interest rates and expected sales have an influence on the evolution

---

[1] Some noteworthy studies that uses the ECM framework are Bechter and Stanley (1992), Claus (1997), Ramsey and West (1999), McCarthy and Zakrajsek (2003), Chacra and Kichian (2004) and Smith *et al.* (2006).

of inventory adjustment. Moreover, the authors point out that given that all the above variables affect of Gross Domestic Product (GDP), and are directly or indirectly also influenced by macroeconomic policies, an appropriate model for analysing and forecasting inventory investment may help to prepare more accurate short-term term forecasts for the economy, in aggregate. Though, the model is capable of representing the movements of the actual values of inventory investment quite well, the paper is silent about the model's predictive abilities. Moreover, since no alternative models were discussed, one could not draw the performance of the model developed relative to other econometric specifications.

One common way of evaluating the performance of a specific econometric model is based on how well that particular model performs in terms of out-of-sample forecasts, when compared to alternative standard models used for forecasting. In this paper, we compare the out-of-sample forecast errors of the ECM developed by Smith *et al.* (2006) with the Classical and Bayesian variants of VARs and Vector Error Correction Models (VECMs). Since our study is a comparitive one, based on alternative models using the same set of variables, we are in a better position, relative to Smith *et al.* (2006), to choose an appropriate model to forecast inventory investment in South Africa.

[2]Though the Classical and the Bayesian VARs have been widely used[3] in forecasting national and regional economies, as well as the housing market, the use of the ECMs and VECMs for forecasting purposes is relatively recent[4]. In general, the multivariate BVAR models have been found to produce the most accurate short and long term out-of-sample forecasts relative to the univariate and unrestricted Classical VAR models. Moreover, the BVAR models are also capable of correctly predicting the direction of change of the macroeconomic variables.

However, the relative dearth of the use of VECMs, especially the classical version, is surprising, when one realizes that two decades back Granger (1986) had stressed that the use of long-run equilibrium relationships from economic theory in models used by time-series econometricians to explain short-run dynamics of data, in other words, the ECMs, should produce better forecasts in the short run and certainly in the long run. Engle and Yoo (1987) corroborated Granger's (1986) faith in these models, when they provided theoretical support for the superior forecasting ability of the ECMs over unrestricted VAR models. They also presented a small simulation exercise confirming

---

[2] This and the following three paragraphs, in this section, draws heavily from the discussion of the reasons for the dearth of the use of VECMs, both Classical and Bayesian, available in Gupta (2006).

[3] For example, Amirizadeh and Todd (1984), Kuprianov and Lupoletti (1984), Hoen *et al.* (1984), Hoen and Balazsy (1985), Kinal and Ratner (1986), Gruben and Long (1988a, b), Lesage (1990), Gruben and Hayes (1991), Shoesmith (1992, 1995), Dua and Ray (1995), Dua and Smyth (1995), Dua and Miller (1996), Dua *et al.* (1999), Banerji *et al* (2006) and Gupta and Sichei (2006).

[4] See LeSage (1990), LeSage and Pan (1995), Shoesmith (1995), Dowd and LeSage (1997) LeSage and Krivelyova (1999), Gupta (2006, 2007) and Zita and Gupta (2007), for use of both Classical and Bayesian variants of the VECMs.

the same. LeSage (1990), using industrial and labour market data from the state of Ohio also showed that VECMs outperform the VARs, and has been confirmed more recently by Gupta (2006, 2007).

As far as the sparse use of the Bayesian version of the ECM models is concerned, two reasons can be identified. Firstly, it is probably due to the concerns of Lutkepohl (1993, p. 375) and Engle and Yoo (1987) regarding the use of the BVECM for forecasting. They pointed out that these models are misspecified in terms of the Granger Representation Theorem, since they impose random walk restrictions[5]. However, a series of recent work by LeSage (1990), Dua and Ray (1995), LeSage and Pan (1995), Dowd and LeSage (1997) and LeSage and Krivelyova (1999) have made some progress in allaying these fears to some extent. They indicate that, given that BVECM allows the forecaster to control for the balance of the short-run dynamics and the long-run influences in the model depending on the specification of the prior, the same, in fact, can produce better forecasts in comparison to the Classical VECMs, especially in the long-run. Hence, it is not surprising that these models, until recently, lacked the confidence of the forecasters.
The second reason is mostly computational and, perhaps, the more important of the two. The technical issue surrounding the relatively modest use of BVECMs in forecasting is, in our opinion, related to the difficulty associated with coding the likelihood functions involved in Bayesian estimation. To the best of our knowledge, until Professor James P. LeSage developed the Econometric Toolbox for MATLAB[6], Regression Analysis of Time Series (RATS) was the only other software that had a built-in ability to handle Bayesian estimations. However, with RATS only capable of carrying out estimations of BVARs, the lack of BVECMs in the forecasting literature is not surprising.

With the theoretical concerns involved in the use of ECM models for forecasting sorted out, and with computer codes now available to estimate both Classical and Bayesian VECMs, we compare the abilities of VARs and VECMs in forecasting inventory investment in South Africa. To the best of our knowledge, this is the first attempt to simultaneously analyse the role of Classical and Bayesian VARs and VECMs in predicting the movements of inventory investment.[7] Finally, we also check for the robustness of our analysis by specifying alternative values of the hyperparameters for the Bayesian priors, available in the literature. The rest of the paper is organized as follows. Besides the introduction and the conclusions, section 2 discusses the advantages of using VARs and VECMs versus a structural model[8] and, hence, an ECM model estimated based on the two-step methodology of Engle and Granger (1987) and a third step procedure of Engle and Yoo (1987). This section also

---

[5] See Section 2 for further details.

[6] The Toolbox is available for free download from http://www.spatial-econometrics.com.

[7] Note, the study by Kichian and Chacra (2004) is the only study we could come across that uses an ECM in forecasting inventory investment in Canada. However, the analysis, unlike ours, merely compared the performance of the ECM with that of Autoregressive (AR) models of various orders.

[8] This section of the paper relies heavily on the discussion available in Dua and Ray (1995), Banerjee *et al.* (2006), LeSage (1999) and Ground and Ludi (2006).

describes the parameters required to specify BVAR and BVEC Models, along with the technicalities involved in the Classical and Bayesian VECMs. Section 3 sets out the model of inventory for the South African Economy, along the lines of Chacra and Kichian (2004) and Smith *et al.* (2006), while, section 4 compares the accuracy of the out-of-sample forecasts generated from alternative models. Based on the forecasting performance of the alternative models, those that produces lower forecasting errors on average, over the forecasting horizon, is then used, in Section 5, to analyze their abilities to predict turning points of inventory investment over the period of 2004:1 to 2006:3[9].

2. ADVANTAGES OF USING VAR OVER STRUCTURAL MODELS

Generally, forecasting models are in the form of simultaneous-equations structural models. However, two problems often encountered with such models are as follows: *(i)* the correct number of variables needs to excluded, for proper identification of individual equations in the system, which are however often based on little theoretical justification (Cooley and LeRoy (1985)), and; *(ii)* given that projected future values are required for the exogenous variables in the system, structural models are poorly suited to forecasting. Given that ECMs, as used in Chacra and Kichian (2004) and Smith *et al.* (2006) for forecasting inventory investments, are estimated using the two-step methodology of Engle and Granger (1987), where we recover the residuals from a cointegrating equation, and after testing for its stationarity, use the lagged residuals as the part of a third step procedure of Engle and Yoo (1987) to estimate the eventual ECM, the same two problems, stated above, will be encountered.

The Vector Autoregressive (VAR) model, though 'atheoretical', is particularly useful for forecasting purposes. Moreover, as shown by Zellner (1979) and Zellner and Palm (1974), any structural linear model can be expressed as a VAR moving average (VARMA) model, with the coefficients of the VARMA model being combinations of the structural coefficients. Under certain conditions, a VARMA model can be expressed as a VAR and a VMA model. Thus, a VAR model can be visualized as an approximation of the reduced-form simultaneous equation structural model.

An unrestricted VAR model, as suggested by Sims (1980), can be written as follows:

$$y_t = C + A(L) y_t + \varepsilon_t$$

(1)

where y is a $(n \times 1)$ vector of variables being forecasted; $A(L)$ is a $(n \times n)$ polynomial matrix in the backshift operator L with lag length p, *i.e.,* $A(L) = A_1 L + A_2 L^2 + \ldots\ldots\ldots + A_p L^p$; C is a $(n \times 1)$ vector of constant terms, and ε is a $(n \times 1)$ vector of white-noise error terms. The VAR model, thus, posits a set of relationships between the past lagged values of all variables and the current value of each variable in the model.

Focusing on the practical case, of y_t being a vector of n time series that are integrated[10] to the order of 1 $(I(1))$[11], the ECM counterpart of the VAR, given by (1), is

---

[9] Currently, data on industrial and commercial inventories is only available till the end of the third quarter of 2006.

[10] A series is said to be integrated of order q, if it requires q differencing to transform it to a

4

captured by a VECM as follows[12]:

$$\Delta y_t = \pi\, y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-1} + \varepsilon_t \tag{2}$$

where $\pi = -[I - \sum_{i=1}^{p} A_i]$ and $\Gamma_i = -\sum_{j=i+1}^{p} A_j$.

The Engle-Granger (1987) Representation Theorem asserts that if the coefficient matrix π (the cointegrating space) has reduced rank $r < n$, then there exist matrices α and β each with rank r such that $\pi = \alpha\beta^{'}$ and $\beta^{'} y_t$ is $I(0)$. Note r is the number of cointegrating relations (the cointegrating rank) and each column of β is the cointegrating vector, and the elements of α are known as the adjustment parameters in the VECM. α is also known as the loading matrix and has a dimension $n \times r$. Since it is not possible to use conventional OLS to estimate α and β, Johansen's (1988) full information maximum likelihood estimation is used to determine the cointegrating rank of π, using the r most significant cointegrating vectors to form β, from which a corresponding α is derived. Note that the specification in (2) is in line with the Engle and Granger (1987) Representation Theorem.

Thus, a VECM is a restricted VAR designed for use with non-stationary series that are known to be cointegrated. While allowing for short-run adjustment dynamics, the VECM has cointegration relations built into the specification so that it restricts the long-run behaviour of the endogenous variables to converge to their cointegrating relationships. The cointegration term is known as the error correction term because the deviation from long-run equilibrium is corrected through a series of partial short-run adjustments, gradually.

Note the VAR model, generally, uses equal lag length for all the variables of the model. One drawback of VAR models is that many parameters are needed to be estimated, some of which may be insignificant. This problem of overparameterization, resulting in multicollinearity and a loss of degrees of freedom, leads to inefficient estimates and possibly large out-of-sample forecasting errors. One must remember that in the VECMs, besides the parameters corresponding to the lagged values of the variables, the parameters corresponding to the error correction terms are also estimated. So the problem of overparameterization, in this case, might be acute enough to outweigh the advantages, in terms of smaller forecasting errors, emanating from the use of long-run equilibrium relationships from economic theory to explain the short-run dynamics of the data. One solution, often adapted, is simply to exclude the insignificant lags based on statistical tests. Another approach is to use near VAR, which specifies an unequal number of lags for the different equations.

However, an alternative approach to overcoming this overparameterization, as described in Litterman (1981), Doan *et al* (1984), Todd (1984), Litterman (1986), and Spencer (1993), is to use a Bayesian VAR (BVAR) model. Instead of eliminating longer lags, the Bayesian method imposes restrictions on these coefficients by assuming that

---

zero-mean, purely non-deterministic stationary process.

[11] LeSage (1990) and references cited therein for further details regarding most macroeconomic time series being *I(1)*.

[12] See, Dickey *et al.* (1991) and Johansen (1995) for further technical details.

they are more likely to be near zero than the coefficients on shorter lags. However, if there are strong effects from less important variables, the data can override this assumption. The restrictions are imposed by specifying normal prior distributions with zero means and small standard deviations for all coefficients with the standard deviation decreasing as the lags increase. The exception to this is, however, the coefficient on the first own lag of a variable, which has a mean of unity. Litterman (1981) used a diffuse prior for the constant. This is popularly referred to as the 'Minnesota prior' due to its development at the University of Minnesota and the Federal Reserve Bank at Minneapolis. Note that, as described in (2), an identical approach can be taken to implement a Bayesian variant of the Classical VECM based on the Minnesota prior.

Formally, as discussed above, the Minnesota prior means and variances take the following form:

$$\beta_i \sim N(1, \sigma^2_{\beta_i}) \text{ and } \beta_j \sim N(0, \sigma^2_{\beta_j}) \tag{3}$$

where β_i denotes the coefficients associated with the lagged dependent variables in each equation of the VAR, while β_j represents any other coefficient. In the belief that lagged dependent variables are important explanatory variables, the prior means corresponding to them are set to unity. However, for all the other coefficients, β_j's, in a particular equation of the VAR, a prior mean of zero is assigned, to suggest that these variables are less important to the model.

The prior variances $\sigma^2_{\beta_i}$ and $\sigma^2_{\beta_j}$, specify uncertainty about the prior means $\overline{\beta}_i = 1$, and $\overline{\beta}_j = 0$, respectively. Because of the overparameterization of the VAR, Doan *et al.* (1984) suggested a formula to generate standard deviations as a function of small numbers of hyperparameters: *w, d*, and a weigting matrix *f(i, j)*. This approach allows the forecaster to specify individual prior variances for a large number of coefficients based on only a few hyperparameters. The specification of the standard deviation of the distribution of the prior imposed on variable *j* in equation *i* at lag *m*, for all *i, j* and *m*, defined as *S(i, j, m)*, can be specified as follows:

$$S(i, j, m) = [w \times g(m) \times f(i, j)] \frac{\hat{\sigma}_j}{\hat{\sigma}_i} \tag{4}$$

with *f(i, j) = 1*, if $i = j$ and k_{ij} otherwise, with $(0 \le k_{ij} \le 1)$, $g(m) = m^{-d}, d > 0$. Note that $\hat{\sigma}_i$ is the estimated standard error of the univariate autoregression for variable *i*. The ratio $\hat{\sigma}_i / \hat{\sigma}_j$ scales the variables so as to account for differences in the units of measurement and, hence, causes specification of the prior without consideration of the magnitudes of the variables. The term *w* indicates the overall tightness and is also the standard deviation on the first own lag, with the prior getting tighter as we reduce the value. The parameter *g(m)* measures the tightness on lag *m* with respect to lag 1, and is assumed to have a harmonic shape with a decay factor of *d*, which tightens the prior on increasing lags. The parameter *f(i, j)* represents the tightness of variable *j* in equation *i* relative to variable *i*, and by increasing the interaction, i.e., the value of k_{ij}, we can

loosen the prior.[13]

The Bayesian variants of the Classical VARs and VECMs are estimated using Theil's (1971) mixed estimation technique, which involves supplementing the data with prior information on the distribution of the coefficients. In an artificial way, the number of observations and degrees of freedom are increased by one, for each restriction imposed on the parameter estimates. The loss of degrees of freedom due to over-parameterization associated with a VAR model is, therefore, not a concern in the BVAR model.

Given the structure of the Bayesian prior, we can now discuss the issue of misspecification involved with the BVECMs, as referred to in the introduction, in more detail. Lutkepohl (1993, p. 375) has claimed that the Minnesota prior is not a good choice if the variables in the system are believed to be cointegrated. He bases his argument on the interpretation of the prior as suggesting that the variables are roughly random walks. Moreover, Engle and Yoo (1987) argued that with the Minnesota prior, a BVAR model approaches the classical VAR model with differenced data, and, hence, would be misspecified for cointegrated variables without an error correction term.

But Dua and Ray (1995) indicate that the suggestion of the Minnesota prior being inappropriate, when the variables are cointegrated, is incorrect. They point out that the prior sets the mean of the first lag of each variable equal to one in its own equation and sets all the other coefficients equal to zero, thus implying that if the prior means were indeed the true parameter values, each variable would be a random walk. But at the same time the prior probability that the coefficients are actually at the prior mean is zero. The Minnesota prior, indeed, places high probability on the class of models that are stationary. Alternatively, if a model specified in levels is equivalent to one in differences, then the sum of the coefficients on the own lags will equal to one, while the sum of the coefficients on the other variables exactly equals zero. Though this holds for the mean of the Minnesota prior, used in this paper, the prior actually assigns a probability of zero to the class of parameter vectors that satisfy this restriction. Lesage (1990) and Dua and Ray (1995), however, point out that if a very tight prior is specified, the estimated model will be close to a model showing no cointegration. With the Minnesota priors, chosen in practice, being not too tight to produce the forecasts, concerns of misspecification with cointegrated data are, therefore, misplaced.

3.   A BVECM MODEL OF INVENTORY INVESTMENT FOR THE SOUTH
AFRICAN ECONOMY

Based on the variables used in Smith *et al.* (2006) to model inventory investment of South Africa, we estimate a BVAR model and a BVECM model of inventory adjustments for the period of 1978:1 to 2000:4, based on quarterly data. We then compute out-of-sample one- through four-quarters-ahead forecasts for the period of 2001:1 to 2003:4, and then compare the accuracy of the forcast relative to the forecasts generated by an unrestricted VAR and a VECM, as in LeSage (1990) and the ECM developed by Smith *et al.* (2006). The variables included are:

(i)   Inventory investment (ΔI), is captured by the quarterly change in the real book value of industrial and commercial inventories (*I*);

---

[13] For an illustration, see Dua and Ray (1995).

(ii)   Industrial and commercial production (Q), approximated by the sum of real value added by manufacturing and commerce;

(iii)   Real sales (S), represented by Gross Domestic Expenditure, excluding final consumption expenditure by general government, final consumption expenditure by households on services, and industrial and commercial inventories, but including the exports of manufactured goods;

(iv)   The Production Price Index (P), estimated as the ratio of nominal to real industrial and commercial value added;

(v)   Unfilled orders (U)[14];

(vi)   The interest rate on 3-months trade financing (R)[15][16], and;

(vii)   Expected sales (S^*), calculated as a function of lagged sales, as suggested by Ramsey and West (1997) and Chacra and Kichian (2004). For the ECM developed by Smith *et al.* (2006), we measure expected sales by the average sales of the past four qurters, while, in case of the VARs and VECMs, both Classical and Bayesian in nature, we use no additional variable to measure expected sales, as these models already includes, by design, lagged values of sales as regressors.

Note the influence of changes in supply conditions is represented by the production variable (Q), while, the changes in demand conditions are captured by the variables measuring changes in actual sales (S), expected sales (S^*) and unfilled orders (U). Finally, the Production Price Index (P) and the 3-months Trade Financing rate (R) outlines the holding cost of inventories. All data are seasonally adjusted in order to, *inter alia*, address the fact, as pointed out by Hamilton (1994:362), that the Minnesota prior is not well suited for seasonal data. All data are obtained from the Quarterly Bulletin of the Reserve Bank of South Africa. Note the real variables correspond to the values of the variables at year 2000's prices.

In each equation of the BVAR there are 25 parameters including the constant, given that the model is estimated with four lags of each variable, as in Dua and Ray (1996)[17].While, in the BVECM we have 30 parameters, including the constant, as five

---

[14] With data on unfilled orders (U) only available till 2003:4, the models could not be estimated beyond that period.

[15] Given that the 3-months interest rate on trade financing (R) series is only available from 1978:1 onwards, the sample of our study could not date back further.

[16] Note Smith et al. (2006) used the prime overdraft rate of the banks as the interest rate variable. However, in our opinion, the 3-months trade financing rate is a more relevant measure of the holding costs of inventories, and was, thus, preferred over the prime overdraft rate. However, the use of alternative interest rate, including the prime overdraft rate, does not affect the results of our analysis, qualitatively.

[17] Hafer and Sheehan (1989) find that the accuracy of the forecasts from the VAR is sensitive to the choice of lags. Their results indicated that shorter-lagged models are more accurate, in terms of forecasts, than longer lag models. Therefore, as in Dua and Ray (1996), for a 'fair' comparison with the BVAR models, alternative lag structures for the VAR and VECM were also examined. When we reduce the lag length to 3 and then to 2, we find marginal

cointegrating relationship was found, which, in turn, led to the inclusion of five error-correction terms.[18] Note Sims *et al.* (1990) indicate that with the Bayesian approach entirely based on the likelihood function, the associated inference does not need to take special account of nonstationarity. This is because the likelihood function has the same Gaussian shape regardless of the presence of nonstationarity. Given this, the variables have been specified in levels.[19]

The so called, 'optimal' Bayesian prior is selected on the basis of the Mean Absolute Percentage Error (MAPE) values of the out-of-sample forecasts. Specifically, the six-variable BVAR and the BVECM are estimated for an initial prior for the period of 1979:1 to 2000:4 and, then we forecast for 2001:1 through 2003:4. Since we use four lags, the initial four quarters of the sample, 1978:1 to 1979:4, are used to feed the lags. We generate dynamic forecasts, as would naturally be achieved in actual forecasting practice. During each quarter of the forecast period, the models are estimated in order to update the estimate of the coefficient before producing 4-quarters-ahead forecasts. This iterative estimation and 4-step-ahead forecast procedure was carried out for 12 quarters, with the first forecast beginning in 2001:1. This experiment produced a total of 12 one-quarter-ahead forecasts, 12-two-quarters ahead forecasts, and so on, upto 12 4-step-ahead forecasts. We use the algorithm in the Econometric Toolbox of MATLAB,[20] for this purpose. The MAPEs[21] for the 12, quarter 1 through quarter 4 forecasts were then calculated for the variable measuring inventory investment of the model. The average of the MAPE statistic values for one- to four-quarters-ahead forecasts for the period 2001:1 to 2003:4 are then examined. Thereafter, we change the prior and a new set of MAPE values is generated. The combination of the parameter values, in the prior, that produces the lowest average MAPE values is selected, as the 'optimal' Bayesian prior. Following Dua *et al.* (1999) and Doan (2000), we choose 0.1 and 0.2 for the overall tightness (w) and 1 and 2 for the harmonic lag decay parameter (d). Moreover, as in Dua and Ray (1995), we also report our results for a combination of $w = 0.3$ and $d = 0.5$. Finally, a symmetric interaction function $f(i, j)$ is assumed with $k_{ij} = 0.5$, as in Dua and Smyth (1995) and LeSage (1990).

---

improvements in the accuracy of the forecasts of inventory investment, but the rank of ordering, resulting from the alternative forecasts remained unchanged.

[18] The cointegrating relationships are based on the trace statistics compared to the critical values at the 95 per cent level. From the results of the test, we observed that the null hypothesis of r ≤ 5 was rejected at the 95 per cent level because the trace statistic of 0.495846 is less than the associated critical value of 3.841466.

[19] However, using the Augmented Dickey Fuller, the Phillips-Perron tests, all the variables, included, except for ΔI, were found to be first-order difference stationary, *i.e.,* integrated of order 1 (*I(1)*).

[20] All statistical analysis was performed using MATLAB, version R2006a.

[21] Note that if A_{t+n} denotes the actual value of a specific variable in period $t + n$ and $_tF_{t+n}$ is the forecast made in period t for $t + n$, the MAPE statistic can be defined as $(\frac{1}{N}\sum abs(\frac{A_{t+n} - {}_tF_{t+n}}{A_{t+n}})) \times 100$, where *abs* stands for the absolute value. For *n = 1*, the summation runs from 2001:1 to 2003:4, and for *n = 2*, the same covers the period of 2001:2 to 2003:4 and so on.

4.   EVALUATION OF FORECAST ACCURACY

To evaluate the accuracy of forecasts generated by the BVARs and the BVECMs, we need to perform alternative forecasts.   To make the MAPEs comparable with the BVARs and BVECMs, we report the same set of statistics for the out-of-sample forecasts generated from an unrestricted Classical VAR and the Vector Error Correction (VEC) models, and also our benchmark model(SBH)[22], which is the ECM developed by Smith *et al.* (2006) for the South African economy, which, essentially, boils down to estimating the following equation:

$$\Delta I_t = c_2 + \alpha e_{t-1} + \sum \lambda_i X_{t-i} + \xi_t, \xi_t \sim N(0, \sigma_\xi^2) \tag{5}$$

where, $e_{t-1} = (I_{t-1} - c_1 - \beta_1 S_{t-1} - \beta_2 Q_{t-1})$ is the lagged value of the residual obtained from the cointegrating equation of *I, S* and *Q*; α is the adjustment coefficient, and; X_{t-i} includes the first-differenced lagged and non-lagged values of *Q, P, R, U* and *S**. Following Ramsey and West (1999) and Smith *et al.* (2006), the lag-structure on the *X* vector is obtained from the information contained in the data. After experimenting with various lag specifications, we ended up estimating equation 5, for the period of 1978:1 to 2004:4 and then recursively for the period of 2001:1 to 2003:4, with two lags on *Q*, one on *U* and no lags on *R* and *P*.[23]   Note that the unrestricted VAR has been estimated in levels with four lags. The corresponding VECM also included four lags. In Table 1, we compare the MAPEs of one- to four-quarters-ahead out-of-sample-forecasts for the period of 2001:1 to 2003:4, generated by the benchmark ECM, the unrestricted VAR, the VECM and the 5 alternative multivariate BVARs and BVECMs. The conclusions from Table 1 can be summarised as follows:

   (i)   SBH versus VAR: The VAR clearly outperforms the SBH in terms of the average value of the one- to four-quarter ahead forecast. Interestingly, the VAR produces lower forecast errors for all the stages, except for the second-quarter ahead forecast.

   (ii)   SBH versus VECM: Unlike in the case of the VAR, the SBH performs better than the VECM in terms of the the average MAPE value of the one- to four-quarter ahead forecast. But the SBH only does so, because it produces, in comparison with the VECM, a very low forecast error for the second-quarter ahead forecast, even when the former is outperformed at the other stages of the forecasting horizon.

   (iii)   VAR versus VECM: Based on (i) and (ii), the VAR outperforms the VECM at all the stages of the forecasting horizon and, hence, in terms of the average MAPE for the one- to four-quarter ahead forecast.

---

[22] SBH stands for the last names of the three authors of Smith *et al.* (2006), which are respectively, Smith, Blignaut and Heerden.

[23] The results of the estimation have been suppressed in the current paper to economise on space. Though, the same will be made available upon request from the author. But, it must be pointed out that the residual recovered from the cointegrating relationship was found to be stationary, and all the variables, except for the interest rate, had signs conforming to *a priori* reasoning. Moreover, the value of α was negative (-0.043786) and significant, indicating partial adjustment of inventories to their target level. Finally, ζ_t was found to be normally distributed, non-heteroscedastic and non-autocorrelated.

10

(iv)   BVARs versus SBH: In terms of the average MAPE values for the one- to 4-quarter ahead forecasts, all the BVAR models, irrespective of the degree of tightness of the prior outperforms the SBH. Amongst the BVARs, the model with relatively tight priors ($w = 0.1$, $d = 1$) performs the best. Moreover, this is the only BVAR model, that does better than the SBH model, in terms of MAPE value for the second-quarter ahead forecast. All the other BVAR models, tend to outperform the SBH at the remaining quarter-ahead forecasts.

(v)   BVARs versus VAR: All the BVAR models outperform the VAR at all the stages of the forecasting horizon, and, naturally ends up having lower average MAPE values as well.

(vi)   BVARs versus VECM: Given (iii) and (v), the BVARs produces lower out-of-sample forecast errors at all the stages and, hence, on average when compared to the VECM.

(vii)   BVECMs versus SBH: From Table 1, we observe that the BVECMs with relatively tight priors produces lower average MAPE, over the forecasting horizon, when compared to the SBH. However, the BVECMs with loose priors, ie., for $w = 0.3$, $d = 0.5$ and $w = 0.2$ and $d =1$, are outperformed by the SBH, mainly due to the huge forecasting error at the second-quarter-ahead stage. Interestingly, except for the BVECM with $w = 0.1$, $d = 2$, the remaining two BVECMs with tighter priors are also outperformed by the SBH for the second-quarter-ahead forecast horizon. But due to the fact that this two BVECMs with $w = 0.1$, $d = 1$ and $w = 0.2$ and $d = 2$, performs way better than the SBH at all the other three levels of the forecasting horizon, namely the first, third and the fourth, they outperform the SBH, in terms of the average MAPE.

(viii)   BVECMs versus VAR: Except for the BVECM with $w = 0.1$, $d = 2$, the VAR outperforms all the other BVECMs corresponding to alternative parameter values specifying the Minnesota Prior.

(ix)   BVECMs versus VECM: However, unlike in the case of the VAR, all the BVECMs, outperform the Classical VECM.

(x)   BVECMs versus BVARs: Just as in case with the Classical VAR, except the BVECM with $w = 0.1$, $d = 2$, all the BVARs outperform the BVECMs. However, the 'optimal BVECM', is, in turn, outperformed by the BVAR with $w = 0.1$, $d = 1$.

[INSERT TABLE 1 ABOUT HERE]

So, in summary, we can conclude, based on our chosen variables and over the forecasting horizon of 2001:1 to 2003:4, that the Bayesian models with tight priors tend to produce better forecast of inventory investment for the South African economy, when compared to the Classical variants of the VAR and VECM and the ECM developed by Smith *et al.* (2006). And amongst the Bayesian models, a BVAR model with an overall tightness (w) of 0.1, and a decay factor (d) of 1 performs the best.

At this stage, it must, however, be pointed out that there are at least two limitations to using the BVAR and BVEC models for forecasting. Firstly, as it is clear from Table

1, the accuracy of the forecasts is sensitive to the choice of the priors. Clearly then, if the prior is not well-specified, an alternative model used for forecasting may perform better. Secondly, in case of the Bayesian variants, one requires to specify an objective function, for example the MAPE, to search for the 'optimal' priors, which, in turn, needs to be optimized over the period for which we compute the out-of-sample forecasts. However, there is no guarantee that the chosen parameter values specifying the prior will also be 'optimal' beyond the period for which it was selected.

5.   TURNING POINTS: THE PERFORMANCE OF THE BAYESIAN MODELS

While, in general, the BVARs produce the most accurate forecasts, and, in particular, the BVAR with $w = 0.1$ and $d = 1$, a different way to evaluate the performance of this model can be based on its ability to predict the turning point(s) in the inventory investment. In this regard, we compare the performance of the 'optimal' BVAR and the 'optimal' BVECM ($w = 0.1$ and $d = 2$), with respect to the actual data over the period of 2004:1 to 2006:3.[24]

As is indicated by Figure 1, the 'optimal' BVAR is clearly better equipped than the optimal BVECM in correctly predicting the direction of change of inventory investment over our chosen period. So based on the forecasting performances of alternative models, and the ability to predict the turning point(s) of the variable of interest, in our case inventory investment, the BVAR model with relatively tight priors ($w = 0.1$ and $d = 1$) is best suited for predicting the behavior of inventory investment of South Africa.

[INSERT FIGURE 1 ABOUT HERE]

6.   CONCLUSIONS

This paper compares the ability of Vector Autoregressive (VAR) Models and Vector Error Correction Models (VECMs), both Classical and Bayesian in nature, and the Error Correction Model (ECM) of Smith *et al.* (2006) in forecasting the inventory investment for the South African economy. For this purpose, we estimate these models using quarterly data on actual sales, production, unfilled orders, price levels, interest rates and expected sales for the period of 1978 to 2000. We then compare out-of-sample forecast errors, generated by these models, over the period of 2001:1 to 2003:4. We find that the BVAR model with $w = 0.1$ and $d = 1$ produces the most accurate forecasts based on the average MAPE values of one- to four-quarter-ahead forecasts. The model is also found to correctly predict the direction of change of the inventory investment over the period of 2004:1 to 2006:3. So, based on our study, we can conclude that a BVAR model with a relatively tight prior is best suited for forecasting inventory investment of the South African economy.

There are, however, as noted earlier, limitations to using the Bayesian approach. Firstly, the forecast accuracy depends critically on the specification of the prior, and secondly, the selection of the prior based on some objective function for the out-of-sample forecasts may not be 'optimal' for the time period beyond the period chosen to produce the out-of-sample forecasts.

---

[24] Except for the variable unfilled orders (*U*) data on all the variables are available till the end of 2006:3.

REFERENCES

AMIRIZADEH, H. and TODD, R. M. (1984). More Growth Ahead for Ninth District States. *Quarterly Review*, Federal Reserve Bank of Minneapolis, Fall.

BANERJI, A., DUA, P. and MILLER, S. M. (2006). Performance Evaluation of the New Connecticut Leading Employment Index Using Lead Profiles and BVAR Models. *Journal of Forecasting*, forthcoming.

BECHTER, D. M. and STANLEY, S. (1992). Evidence of Improved Inventory Control. *Federal Reserve Bank of Richmond Economic Review*, vol. 1, 3-12.

CHACRA, M and KICHIAN, M. (2004). A Forecasting Model for Inventory Investments in Canada. *Working Paper* 2004-39, Bank of Canada.

CLAUS, I. (1997). Modelling the Behaviour of US Inventories: A Cointegration-Euler Approach. *Working Paper* 1997-19, Bank of Canada.

COOLEY, T. F. and LeROY, S. F. (1985). Atheoretical Macroeconometrics: A Critique. *Journal of Monetary Economics*, vol. 16, 283-308.

DICKEY, D. A., JANSEN, D. W., and THORNTON, D. L. (1991). A Primer on Cointegration with an Application to Money and Income, *Federal Reserve Bulletin*, Federal Reserve Bank of St. Louis, March/April, 58-78.

DOAN, T. A., LITTERMAN, R. B. and SIMS, C. A. (1984). Forecasting and Conditional Projections Using Realistic Prior Distributions. *Econometric Reviews*, vol. 3, 1-100.

DOAN, T. A. (2000). *RATS User's Guide, Version, 5.0*. Illinois: Estima.

DOWD, M. R. and LeSAGE, J. P. (1997). Analysis of Spatial Contiguity Influences on State Price Level Formation, *International Journal of Forecasting*, vol. 13, 245-253.

DUA, P. and RAY, S. C. (1995). A BVAR Model for the Connecticut Economy. *Journal of Forecasting*, vol. 14, 167-180.

DUA, P. and MILLER, S. M. (1996). Forecasting and Analyzing Economic Activity with Coincident and Leading Indexes: The Case of Connecticut. *Journal of Forecasting*, vol. 15, 509-526.

DUA, P. and SMYTH, D. J. (1995). Forecasting U. S. Home Sales using BVAR Models and Survey Data on Households' Buying Attitude. *Journal of Forecasting*, vol. 14, 167-180.

DUA, P., MILLER, S. M. and SMYTH, D. J. (1999). Using Leading Indicators to Forecast U. S. Home Sales in a Bayesian Vector Autoregressive Framework. *Journal of Real Estate Finance and Economics*, vol. 18, 191-205.

ENGLE, R. F. and GRANGER, C. W. J. (1987). Cointegration and Error Correction: Representation, Estimation and Testing. *Econometrica*, vol. 55, 251-276.

ENGLE, R. F. and YOO, B. S. (1987). Forecasting and Testing in Co-Integrated Systems. *Journal of Econometrics*, vol. 35, 143-159.

GUPTA, R. and SICHEI, M. M. (2006). A BVAR Model for the South African Economy. *South African Journal of Economics,* vol. 74, 391-409.

GUPTA, R. (2006). Forecasting the South African Economy with VARs and VECMs. *South African Journal of Economics,* vol. 74, 611-628.

GUPTA, R. (2007). Forecasting the South African Economy with Gibbs Sampled BVECMs. *Working Paper* 2007-1, Department of Economics, University of Pretoria.

GRANGER, C. W. J. (1986). Developments in the Study of Cointegrated Economic Variables. *Oxford Bulletin of Economics and Statistics*, vol. 48, 213-227.

GROUND, M. and LUDI, K. L. (2006). Investigating the Bank-Lending Channel in South Africa: A VAR Approach. *Working Paper* 2006-4, Department of Economics, University of Pretoria.

GRUBEN, W. C. and HAYES, D. W. (1991). Forecasting the Louisiana Economy. *Economic Review*, Federal Reserve Bank of Dallas, March, 1-16.

GRUBEN, W. C. and LONG III, W. T. (1988a). Forecasting the Texas Economy: Application and Evaluation of a Systematic Multivariate Time-Series Model Outlook for 1989. *Economic Review*, Federal Reserve Bank of Dallas, January, 11-25.

GRUBEN, W. C. and LONG III, W. T. (1988b). The New Mexico Economy: Outlook for 1989. *Economic Review*, Federal Reserve Bank of Dallas, November, 21-29.

HAFER, R. W. and SHEEHAN, R. G. (1989). The Sensitivity of VAR Forecasts to Alternative Lag Structures. *International Journal of Forecasting*, vol. 5, 399-408.

HAMILTON, J. D. (1994). *Time Series Analysis*. Second Edition. Princeton: Princeton University Press.

HANSEN, G. D. and PRESCOTT, E. C. (1993). Did Technology Shocks Cause the 1990-91 Recession? *American Economic Review Proceedings*, vol. 83(2), 280–285.

HOEN, J. G. and BALAZSY, J. J. (1985). The Ohio Economy: A Time-Series Analysis. *Economic Review*, Federal Reserve Bank of Cleveland, Third Quarter, 25-36.

HOEN J. G. GRUBEN, W. C. And FOMBY, T. B. (1984). Time Series Forecasting Model of Texas Economy: A Comparison. *Economic Review*, Federal Reserve Bank of Dallas, May, 11-23.

INGRAM, B. F. and WHITEMAN, C. H. (1994). Supplanting the 'Minnesota' prior: Forecasting Macroeconomic Time Series Using Real Business Cycle Model Priors. *Journal of Monetary Economics*, vol. 34 (3), 497–510.

IRELAND, P. N. (2001). Technology Shocks and the Business Cycle: An Empirical Investigation. *Journal of Economic Dynamics and Control*, vol. 25, 703–719.

JOHANSEN, S. (1988). Statistical Analysis of Co-integration Vector, *Journal of Economic Dynamics and Control*, vol. 12, 231-254.

13

JOHANSEN, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.

KINAL, T. and RATNER, J. (1986). A VAR Forecasting Model of a Regional Economy: Its Construction and Comparison. *International Regional Science Review*, vol. 10, 113-126.

KUPRIANOV, A. and LUPOLETTI, W. (1984). The Economic Outlook for the Fifth District States in 1984: Forecasts from Vector Autoregression Models. *Economic Review*, Federal Reserve Bank of Richmond, February, 12-23.

LeSAGE, J. P. (1990). A Comparison of the Forecasting Ability of ECM and VAR Models. *The Review of Economics and Statistics*, vol . 72 (4), 664-671.

LeSAGE, J. P. and Pan, Z. (1995). Using Spatial Contiguity as Bayesian Prior Information in Regional Forecasting Models, *International Regional Science Review*, vol. 18, 33-53.

LeSAGE, J. P. and Krivelyova, A. (1999). A Spatial Prior for Bayesian Autoregressive Models, *Journal of Regional Science*, vol. 39, 297-317.

LeSAGE, J. P. (1999). *Applied Econometrics Using MATLAB*. www.spatial-econometrics.com.

LITTERMAN, R. B. (1981). A Bayesian Procedure for Forecasting with Vector Autoregressions. *Working Paper*, Federal Reserve Bank of Minneapolis.

LITTERMAN, R. B. (1986). Forecasting with Bayesian Vector Autoregressions – Five Years of Experience. *Journal of Business and Economic Statistics*, vol. 4 (1), 25-38.

LUCAS, Jr., R. E. (1976). Econometric Policy Evaluation: A Critique. *Carnegie Rochester Conference Series on Public Policy*, vol. 1, 19–46.

LUTKEPOHL, H. (1993). *Introduction to Multiple Time Series Analysis*. Second Edition. New York: Springer-Verlag.

MCCARTHY, J. and ZAKRAJSEK, E. (2003). Inventory Dynamics and Business Cycles: What has Changed? *Finance and Economics Discussion Series no. 2003-26, U.S. Federal Reserve Board*.

NI, S. and Sun, D. Bayesian Estimators for Vector-Autoregressive Models. *Journal of Business and Economic Statistics*, vol. 23 (1), 105-117.

RAMSEY, V. A. and WEST, K. D. (1999). Inventories. In Taylor, J. B. anf Woodford, M. (Ed.). (1999). *Handbook of Macroeconomics*. North-Holland (Elsevier): New York.

ROTEMBERG, J. and WOODFORD, M. (1995). *Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets*. In Cooley ed(s), Frontiers of Business Cycle Research, New Jersey: Princeton University Press.

SHOESMITH, G. L. (1992). Co-integration, Error Correction and Medium-Term Regional VAR Forecasting. *Journal of Forecasting*, vol. 11, 91-109.

SHOESMITH, G. L. (1995). Multiple Cointegrating Vectors, Error Correction and Litterman's Model. *International Journal of Forecasting*, vol. 11, 557-567.

SICHEI, M. M. (2005). Bank-Lending Channel in South Africa: Bank-Level Dynamic Panel Data Analysis, *Working Paper* 2005-10, Department of Economics, University of Pretoria.

SIMS, C. A. (1980). Macroeconomics and Reality. *Econometrica*, vol. 48, 1-48.

SIMS, C. A, STOCK, J. H. and WATSON, M. W. (1990). Inference in Linear Time Series Models with Some Unit Roots. *Econometrica*, vol. 58, 113-144.

SMITH, J. H., BLIGNAUT, J. N. and van HEERDEN, J. H. (2006). An Analysis of Inventory Investment in South Africa, vol. 74, 6-19.

SPENCER, D. E. (1993). Developing a Bayesian Vector Autoregression Model. *International Journal of Forecasting*, vol. 9, 407-421.

THEIL, H. (1971). *Principles of Econometrics*. New York: John Wiley.

TODD, R. M. (1984). Improving Economic Forecasting with Bayesian Vector Autoregression. *Quarterly Review*, Federal Reserve Bank of Minneapolis, Fall, 18-29.

ZELLNER, A. (1979). Statistical Analysis of Econometric Models. *Journal of the American Statistical Association*, vol. 74, 628-643.

ZELLNER, A. and PALM, F. (1974). Time Series Analysis and Simultaneous Equation Econometric Models. *Journal of Econometrics*, vol. 2, 17-54.

ZIMMERMANN, C. (2001). Forecasting with Real Business Cycle Models. *Indian Economic Review*, vol. 36(1), 189-203.

ZITA, S.E. and GUPTA, R. (2007). Modelling and Forecasting the Metical-Rand Exchange Rate. *Working Paper* 2007-2, Department of Economics, University of Pretoria.

Table 1. MAPE(2001:1-2003:4): Inventory Investment

QA	SBH	VAR	VECM	(w=0.3,d=0.5)		(w=0.2,d=1)		(w=0.2,d=2)		(w=0.1,d=1)		(w=0.1,d=2)	
				BVAR	BVECM	BVAR	BVECM	BVAR	BVECM	BVAR	BVECM	BVAR	BVECM
1	368.246	37.660	48.766	35.071	36.053	27.384	15.122	14.296	17.301	16.617	7.623	15.578	27.442
2	146.886	376.744	2505.534	347.178	2167.867	228.935	1422.710	72.950	433.448	29.760	645.745	252.284	21.875
3	156.260	11.723	27.048	11.528	26.281	10.336	22.397	5.825	12.913	8.013	15.068	3.454	8.010
4	154.762	9.229	30.555	8.637	34.333	5.970	33.350	1.876	17.842	1.314	22.284	4.851	8.030
AVE	206.538	108.839	652.976	100.604	566.133	68.156	373.395	23.737	120.376	**13.926**	172.680	69.042	**16.339**

MAPE: Mean Absolute Percentage Error; QA: Quarter Ahead; SBH: ECM model developed by Smith *et. al* (2006).

Figure 1: Predicting Turning Points of Inventory Investment (2004:1-2006:3)