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First-year undergraduate students' statistical problem-solving skills

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Abstract
This study investigated first-year undergraduate statistics students' statistical problem-solving skills on the probability of the union of two events, conditional probability, binomial probability distribution, probabilities for x-limits using the z-distribution, x-limit associated with a given probability for a normal distribution, estimating the y-value using a regression equation, and hypothesis testing for a single population mean when a population standard deviation is unknown. The study was a descriptive case study and employed a mixed-method research approach. Data were collected through content analysis of a statistics course examination script of 120 first-year undergraduate students of statistics in an open distance-learning university in South Africa. Polya's Model of Problem Solving was used as the framework of analysis. The study revealed that the students, in general, had poor statistical problem-solving skills.

KEYWORDS
binomial probability distribution, conditional probability, hypothesis testing, normal distribution, problem-solving skills, teaching statistics

1 | INTRODUCTION

Statistical literacy and statistical problem-solving skills are regarded as essential in today's world. Hence, Statistics forms part of the primary and secondary school education curricula of many countries [1–3]. In South Africa, Statistics was introduced in primary and high school curricula in 1997 and 2006, respectively, as a component of mathematics. It was intended to prepare learners to be able to collect, organize and analyze quantitative data [4]. However, evidence over the years has shown that South African learners have achieved poorly in statistical aspects on international tests such as the Trends in International Mathematics and Science Study (TIMSS) of 2011, 2015, and 2019, and the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SEACMEQ) (previously called the Southern Africa Consortium for Monitoring Educational Quality [SACMEQ]) [5–7].

Furthermore, the South African National Senior Certificate (ie, grade 12) Mathematics Examination Diagnostic Reports show that the learners had difficulties in solving problems on statistical concepts in the examination [8–13]. Despite the poor performance of the learners with the statistics concepts at secondary school, some of those who get admitted into tertiary institutions enroll in first-year undergraduate statistics courses. However, how students fare in university statistics courses has not been the focus of many research studies.

1.1 | Students' problem-solving skills

Literature on empirical studies that have investigated undergraduate students' problem-solving in statistics is limited. In South Africa, Bester [14] investigated second-year undergraduate students' problem-solving proficiency in Quantitative (a special statistics course) at the stages of Polya's [15] problem-solving model. (Polya's model is elaborated further in the next section.) The statistical topics covered in the quantitative model are estimation, hypothesis testing, index numbers, and regression analysis. The study found that, on average, students achieved the highest in "understanding a problem" and "making a plan to solve the problem" but were weaker in "executing the plan" and weakest at "interpreting their results." No other study was found that explored undergraduate students' problem-solving skills in statistics. However, at the secondary school level, Awuah and Ogbonnaya [16] explored the grade 12 students' problem-solving proficiency in probability (part of the Statistics content in South Africa). They found that the students in their study were not proficient in the use of tree diagrams and contingency tables to solve probability problems; 97% and 91% of the students scored below 50% in solving probability problems involving tree diagrams and contingency tables, respectively.

At the international level, some studies have investigated students' problem-solving skills in mathematics at various levels of education. Meutia, Ikhsan, and Ismail [17] examined junior high school students' mathematical problem-solving skills in Indonesia based on Polya's problem-solving model. The authors found that the students' mathematical problem-solving skills were poor. The students showed a poor understanding of the problems, did not choose appropriate plans to solve the problems, wrongly implemented the plans, and did not recheck the solution. In a similar study in a province of Indonesia, Sari et al. [18] analyzed junior high school students' mathematical problem-solving skills. From the analyses of the students' test scripts, the study showed that the students' problem-solving abilities were moderate and that the students were not able to look back and check their answers. Onyancha and Ogbonnaya [19] investigated undergraduate students' proficiencies in solving bivariate normal distribution (BND) problems at a Kenyan university. The authors found that the students were not proficient in solving BND problems. The students found it difficult to calculate "the probability of a normal distribution variable, the mean for a normally distributed variable, the conditional mean, variance, standard deviation, and probability of a joint distribution, and the mean and standard deviation of two random variables of a BND given a bivariate random density function" (p. 31). In a follow-up study, Onyancha and Ogbonnaya [20] revealed that the major challenges the students had in learning statistics at the university included poor foundational knowledge of statistical concepts.

Özdemir and Çelik [21] used Polya's four stages to examine the problem-solving processes and problem-posing skills of preservice mathematics teachers. The study used a qualitative approach with 71 pre-service teachers from the primary education mathematics department of a state university in the south-eastern Anatolia region of Turkey as participants. Students were required to solve three verbal problems based on the stages of Polya's problem-solving model. The results revealed that the participants showed the highest performance in the step of "understanding the problem" and the lowest in "evaluation." The students who had difficulties with the plan preparation category were students who had difficulties understanding the problem, while students who were successful in understanding the problem and preparing a plan were also successful in implementing the plan.

Arfiana and Wijaya [22] examined students' problem-solving skills when solving the problems of Programme for International Student Assessment (PISA) using Polya's stages. A stratified random sampling technique and cluster random sampling were employed in their study to select a sample of 389 grade X students from 12 schools (senior high and Islamic high schools) in Tegal Regency, Indonesia. A descriptive survey design using a quantitative approach and a test instrument containing 12 PISA test items were employed in their study. The results revealed that the problem-solving skills of students in solving the problem of PISA using Polya's stages could be categorized as low. The indicators of "devising a plan" and "looking back" revealed that students' skills fell into the very low category. The indicators of "carrying out a plan" revealed that students' skill fell into the average category, while the indicators of "problem understanding" indicated that students' skill was low in this category [22].

This present study's interest was in statistics problem-solving skills of first-year undergraduate Statistics students in an open distance-learning institution in South Africa. This is an area that has not been explored in previous research studies. The study addressed the question: What is the level of problem-solving skills of first-year undergraduate Statistics students?

2 | PROBLEM SOLVING

Problem solving is a cognitive process used to reach the desired goal when confronted with a situation/task whose solution is not obvious [23]. There are numerous problem-solving models to guide students to solve problems (see, for example, [24–27]). However, when taking these problem-solving models into consideration, they all follow the same four steps: (1) understanding the problem; (2) choosing an appropriate strategy or procedure; (3) implementing a strategy; and (4) checking the solution. The steps are similar to those in Polya's [15] model. This is the reason why Polya's [15] model was used as a basis for the problem-solving model in this study.

2.1 | Polya's poblem-solving model

Polya's problem-solving model consists of a four-step method of solving a problem: understand the problem, devise a plan, carry out the plan, and look back.

"Understanding the problem" involves comprehending the problem, identifying the given information, and what is being asked. "Devising a plan" means looking for a strategy to solve the problem, for example, drawing a diagram or using variables to create an equation. "Carrying out the plan" is executing the strategy identified in the previous stage. If the strategy does not work, other strategies will be sought until the problem is solved. "Looking back" is to look back at the problem to ensure all parts of the questions are answered and all conditions satisfactorily met [28,pp. 1,2]. Looking back means vetting the correctness of the solution. In this study, this step has been changed to "present the answer" because the current nature of examination often does not require students to verify their solutions. It is argued that if the solution is correct, then it must have been verified.

According to Polya, solving a problem is not about mechanically following the steps of the model, the steps can be randomly followed and/or used in parallel, resulting in discoveries that may lead to the modification of the model. This may be because of the ability to reflect on and recognize flaws/gaps in the thought processes [29], and is referred to as metacognition [30,31]. Metacognition plays an important role in successful problem solving [32]. Research has shown that knowledge of basic skills (cognition) is not enough for successful performance in complex academic tasks such as mathematical problem solving [33–35] because metacognitive knowledge is seen to act on the cognition by controlling why, when, and how to use cognition to solve a problem [36].

For instance, when applying the four steps of Polya's [15] model to solve a problem (Step 1: when the student is presented with a problem), knowledge of cognition can be used to brainstorm or mentally organize the ideas, concepts, and steps involved in solving a problem [32,36]. In the next step (Step 2), once the ideas and concepts involved in solving the problem have been mentally organized, the student might analyze the available problem-solving strategies (heuristics) and algorithms that will assist in solving the problem or attaining the goal. Here, knowledge of cognition influences the decisions made by the student. The student then decides what to do, why, and for how long [30,32]. For a student to finish a task quicker and reach the correct solution, in Step 3 they will need to use the regulation of cognition and cognition of knowledge to ask themselves if there might be an easy way to solve the problem. Schoenfeld [32,p. 191] provides problem solvers with a cardinal rule for problem solving: "Never use any difficult techniques before checking to see whether simple techniques will do the job". As a student implementing a problem-solving strategy selects and applies algorithms, they should always keep track of how things are going and make changes when necessary. In Step 4, the student or problem solver needs to check whether the result of the process is correct; thus, they must use regulation in evaluating the steps that have been followed during the process. If the answer is incorrect or the solution has an error or errors and the student can recognize the error(s), they will have to restart the process to correct the solution/answer.

3 | METHODOLOGY

The study was a descriptive case study of first-year Statistics students who were registered for a module on Descriptive Statistics and Probability at an open distance-learning university in South Africa. The study employed a mixed-method research approach. The module of Descriptive Statistics and Probability is a co-requisite for all first-level statistics modules in the Mathematics and Statistics qualification stream and a prerequisite for all second and third-level statistics modules in the Mathematics and Statistics qualification stream. The module content is divided into 12 topics, namely, (1) What is statistics? (2) Graphical and tabular descriptive techniques, (3) Art and science graphical presentations, (4) Numerical descriptive techniques, (5) Data collection and sampling, (6) Probability, (7) Random variables and discrete probability distributions, (8) Continuous probability distributions, (9) Sampling distributions, (10) Introduction to estimation, (11) Introduction to hypothesis testing, and (12) Inference about population. A full description of the module (course) content is provided in Appendix D. The Descriptive Statistics and Probability course is offered in both semesters. In each semester, the tuition period for a module ranges from 12 to 15 weeks (wk), with examinations taking place in May and June and October and November, respectively.

The population was 485 first-year undergraduate Statistics major (or mainstream) students at the university. A convenience sample of 120 students (consisting of 45 females and 75 males) took part in the study.

The sample consisted of the students who gave consent for their examination scripts to be analyzed in the study.

The examination scripts of the students were obtained from the university examination department in line with the ethical protocols of the university. The authors did not set the examination; however, the examination was as presented by the lecturer responsible for the module. It was out of 100 marks and the duration was 2 h. It consisted of two sections: section A was worth 30 marks and section B was worth 70 marks. Section A consisted of 15 multiple-choice questions. Section B consisted of five open-ended questions with sub-questions (a total of 25 questions). Statistical formulae and distribution tables were provided. Seven sub-questions from section B of the examination were considered in the study because they entailed plausible problem solving. Other questions were not found suitable for the study because they required remembering and recalling information. The questions that were analyzed are in Appendix A. The questions were on the probability of the union of two events (question 1a), conditional probability (question 1b), the binomial probability distribution (question 2), probabilities for x-limits using z-distribution, and x-limit associated with the given probability for a normal distribution (questions 3a and 3b, respectively), estimating the y-value using the regression equation (question 4), and hypothesis testing for a single population mean (μ) when a population standard deviation (SD) (σ) is unknown (question 5). [Correction added on 06 October 2023, after first online publication: The meaning of SD has been added in this version.] These concepts are fundamental to statistical understanding and the successful learning of statistics. They are used in everyday applications, including politics, weather forecasts, medicine, and business [37–40].

The module was delivered through a blended learning approach that required self-study and face-to-face tutoring that was provided four times a month at the university's regional centers. The duration of each tutorial session was a minimum of 1 h and a maximum of 2 h. Face-to-face tutorials were not didactic lectures but rather a facilitative process planned as part of the university's open and distance learning model, fostering independent learning skills, and ensuring student success. To study the module, the students were expected to use the prescribed textbook, the study guide, and a document detailing the assignment activities. Additionally, students were encouraged to contact their lecturer if they encountered difficulties with their assignment, textbook content, or study guide. The students did two assignments before the examination. The first assignment was based on chapters 1-7 of the prescribed textbook, while the second assignment was based on chapters 8-12. The students were not explicitly taught how to solve problems using a particular problem-solving strategy; however, the prescribed textbooks and study guide had some solved problems and students were encouraged to consult other resources and discuss their work with their peers or tutors while completing assignments. Students received feedback on their marked assignments after completing assignments. The feedback was used to further provide tutoring and assistance to the students. Toward the end of the module additional feedback, based on all the students' performance on the assignments and how they were expected to address the assignment questions, was sent to all the students. Furthermore, students were able to access previous examination papers through the university's learning management system under the module of Descriptive Statistics and Probability. Also, students were given a trial paper toward the end of the semester. The purpose of this trial paper was to provide an example of the type of questions that would typically be found in the examination. Solutions to the trial paper were distributed via a tutorial letter in order to assist students further. The previous examination papers as well as the trial paper contained specific instructions for students to follow. Two important points were emphasized: first, students were permitted to use non-programmable pocket calculators during the examination and, second, the examination would be a closed-book assessment. In the examination, the students were not given specific instructions regarding problem-solving steps, such as identifying all variables, providing formulae, etc. However, in the solutions to the example questions in the prescribed textbooks, study guide, and the trial examination, the variables have been identified. Hence, in this study, the students' solutions were scored according to the rubric, as shown in Table 1.

The students had a choice of using Microsoft Excel or Mintab for practice and assignment writing; however, for examinations, formula sheets were provided, and only non-programmable pocket calculators were permitted, as already mentioned. For example, students applied two formulae derived from the method of least squares to determine y intercept coefficient (b_0) and slope coefficient (b_1) for calculating the linear regression equation (line) and estimating the y-value in question 4, where the following summaries were provided: $\sum x = 1263$; $\sum x^2 = 274753$; $\sum xy = 11\,274$; $\sum y = 54$; $Sxy = -18.6$

Data analyses involved both qualitative and quantitative approaches. For the qualitative data analysis, deductive content analysis using the assessment rubric (Appendix B) developed and modeled on Polya's problem-solving model [15] and achievement levels (very low: 0, low: 1, intermediate: 2, high: 3, and advanced: 4) adapted from the TIMSS [5] scales were employed to analyse the students' solutions to the examination questions (see Table 1). The rubric was used to classify the problem-solving levels of the students' solutions to the questions at

TABLE 1 An example of how the rubric was used on question 1a.

Question 1a			
Polya's steps	**TIMSS level**		
Understanding the problem	4	P(B or O) = P(B ∪ O)	Correct and complete variables (B, O) and sign (∪) assigned. A student can go to the extent of converting the wording into a statistical/ mathematical expression.
	3	P (A or B)	Correct and complete variable assignment
	2	B or O	Incorrect or incomplete variable assigned
	1	Variables incorrectly assigned	
	0	Not attempted	
Devise a plan to solve a problem	4	P(B) + P(O) – P (B and O) = P(B) + P(O) – P (B ∩ O)	
	3	P (A) + P(B) – P (A and B)	Correct and complete formula
	2	A + B – (A and B)/B+ O – B ∩ O	Incorrect or incomplete formula
	1	Wrong formula	
	0	No attempt to establish a relationship between variables	
Carry out a plan	4	110/355 + 150/355-40/355	Correct and complete manipulation of the established probabilities in step 2
	3	110/355 + 150/355 − ????	Partially correct manipulation of the established probabilities in step 2
	2	110/355 +???? − ????	Minimal correct manipulation
	1	Wrong substitution as a result of the wrong formula in step 2	
	0	Not attempted	
Present the answer	4	0.6197	
	3	Incorrect answer as a result of partially correct probabilities substituted in step 3	
	2	Incorrect answer as a result of minimal correct probability substituted in step 3	
	1	Incorrect answer due to wrong formula devised in step 2	
	0	Not attempted	

the four stages of Polya's problem-solving model. In this study, students' solutions were not scored based on their original marks. Rather, students' solutions were recoded and scored based on five levels of achievement (also see Table 1 and Appendices B and C). Examples of how students' solutions were scored according to the rubric can be seen in Figures 1-10, Appendix C and Table 1. Figures 1-6 show some students' answers to question 1a, while Figures 7-10 show some students' answers to question 3b. For example, looking at Figure 6, student 52's solution was scored as U:4, D:4, C:1, P:1. This means the student was ranked at level 4 for "understanding the problem," level 4 for "devising a plan," level 1 for "carrying out a plan," and level 1 for "presenting an answer" to question 1a. The assessment rubric was content and face validated by experts in statistics. Its reliability was determined by computing Cohen's kappa (k) as a measure of inter-rater agreement for categorical data. The inter-rater reliability showed a near-perfect level of agreement ($k = 0.911$, $P < 0.001$).

For the quantitative data analysis, combined means (weighted arithmetic means, or mean of means) and combined SDs (pooled SD) were calculated based on the means and SDs of all the questions (ie, 1a U, 1b U, 2 U, 3a U, 3b U, 4 U, and 5 U) at each problem-solving step to determine the students' statistical problem-solving skills in general (see Table 3). For instance, the mean of means for "understand the problem" step for all the questions was calculated as $([2.28 + 2.48 + 1.76 + 1.93 + 0.62 + 2.88 + 2.22]/7) = 2.03$. The mean of means for the "devise a

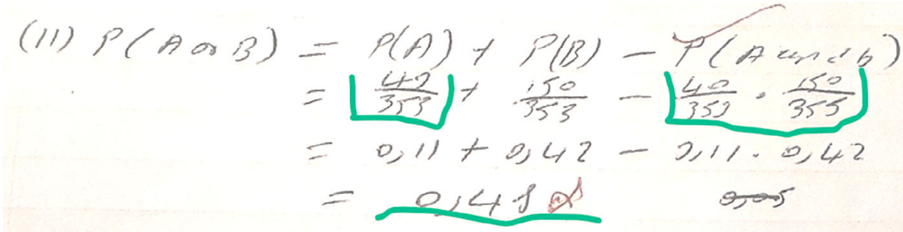

FIGURE 1 Student 55. Scoring – U:3, D:3, C:2, P:2. [Colour figure can be viewed at wileyonlinelibrary.com]

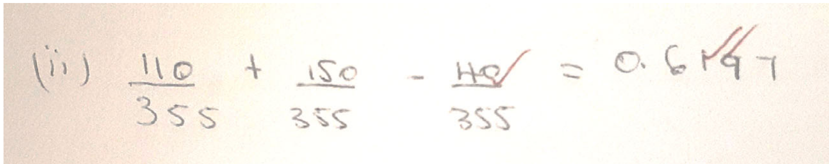

FIGURE 2 Student 17. Scoring – U:0, D:0, C:4, P:4. [Colour figure can be viewed at wileyonlinelibrary.com]

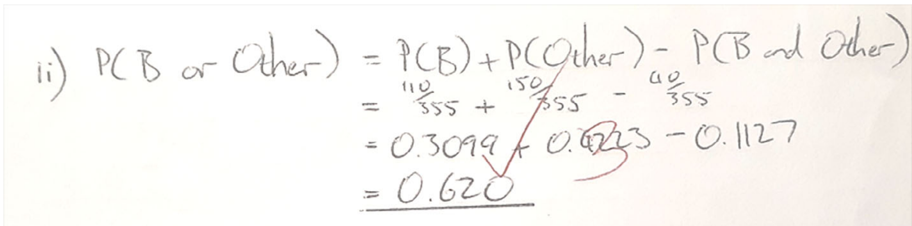

FIGURE 3 Student 49. Scoring – U:4, D:4, C:4, P:4. [Colour figure can be viewed at wileyonlinelibrary.com]

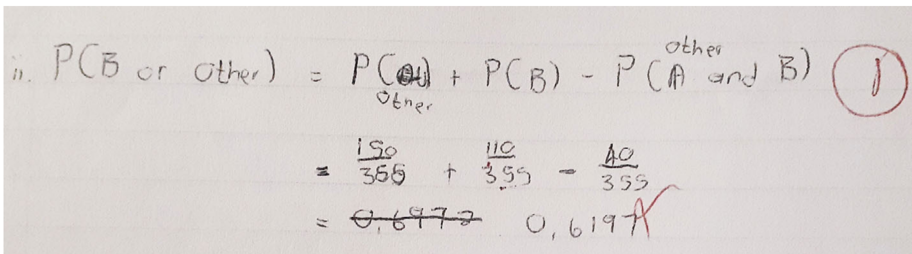

FIGURE 4 Student 57. Scoring – U:4, D:4, C:4, P:4. [Colour figure can be viewed at wileyonlinelibrary.com]

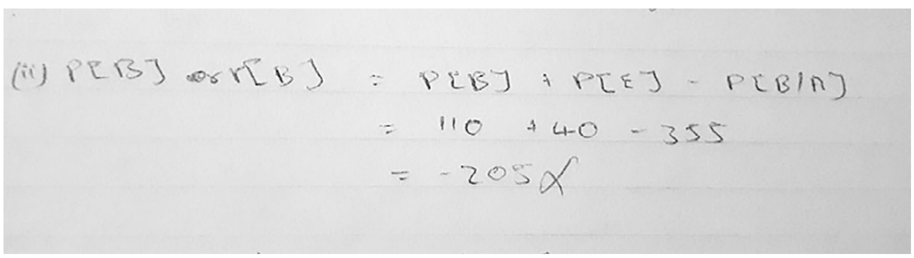

FIGURE 5 Student 62. Scoring – U:1, D:1, C:1, P:1.

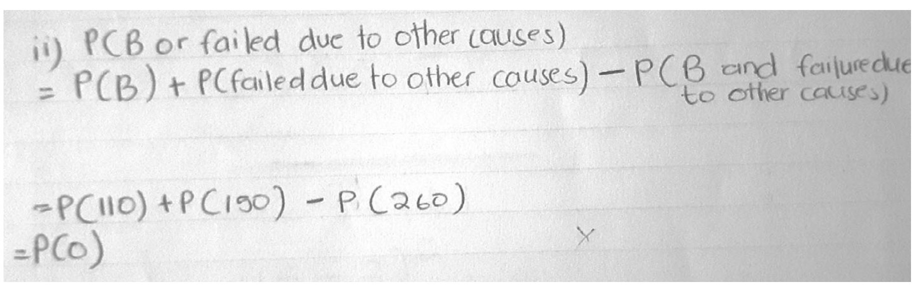

FIGURE 6 Student 52. Scoring – U:4, D:4, C:1, P:1.

plan" step for all the questions was calculated as ([1.86 + 1.88 + 1.31 + 2.3 + 0.56 + 2.65 + 1.98]/7) = 1.79 and so forth (see Tables 2 and 3). The combined (pooled) SD for the "understand the problem" step for all the questions was calculated as (SQRT($(1.773)^2 + (1.777)^2 + (1.582)^2 + (1.638)^2 + (1.063)^2 + (1.473)^2 + (1.492)^2$)/7)) = 1.65. The combined SD for the "devise a plan" step for all the questions was calculated as (SQRT($(1.871)^2 + (1.893)^2 + (1.618)^2 + (1.633)^2 + (0.933)^2 + (1.703)^2 + (1.655)^2$)/7)) = 1.64 and so forth. In addition, descriptive statistical analysis (frequency percentages, mean, mode, and SD) was employed to determine the level of students' problem-solving skills on each examination question (see Table 2).

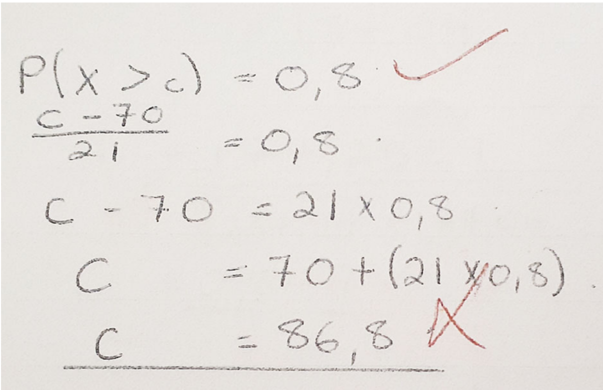

FIGURE 7 Student 67. Scoring – U:4, D:0, C:2, P:2. [Colour figure can be viewed at wileyonlinelibrary.com]

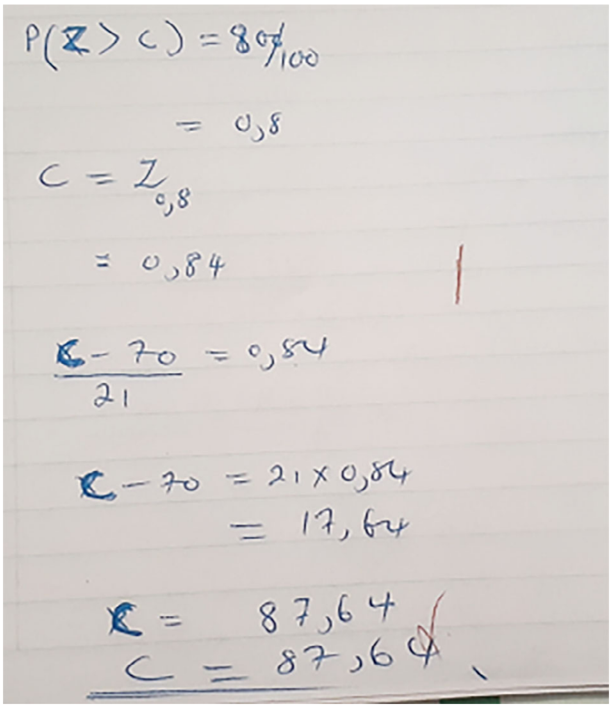

FIGURE 8 Student 68. Scoring – U:4, D:0, C:3, P:3. [Colour figure can be viewed at wileyonlinelibrary.com]

To analyze the meaning of the combined means and categorize the students' levels of problem-solving skills, interval scales were derived based on levels of achievement (five-point scale). To illustrate this, the interval scale for a very low level was calculated as (4 − 0)/5 + 0 = 0.80. This means the interval scale for a very low level ranged from 0.00 to 0.80. The interval scale for a low level was calculated as (4 − 1)/5 + 1 = 1.60, so the interval scale for a low level ranged from 0.81 to 1.60. Accordingly, the interval scale for an intermediate level was calculated as (4 − 2)/5 + 2 = 2.40, resulting in an interval scale for the intermediate level ranging from 1.61 to 2.40; for the high level, the interval scale was determined as (4 − 3)/5 + 3 = 3.2, resulting in an interval scale ranging from 2.41 to 3.20 and so forth (see Table 4). The combined means in Table 3 should be interpreted in conjunction with Table 4.

4 | FINDINGS

Students' overall problem-solving skills on Polya's problem-solving stages are presented first, followed by their levels of problem-solving skill at each stage of Polya's problem-solving model on the questions examined.

4.1 | The overall problem-solving skills of students on polya's problem-solving stages

The overall combined means of the students' problem-solving skill level of achievement in solving the statistical problems were calculated for each of Polya's problem-solving stages. This was done to determine whether the

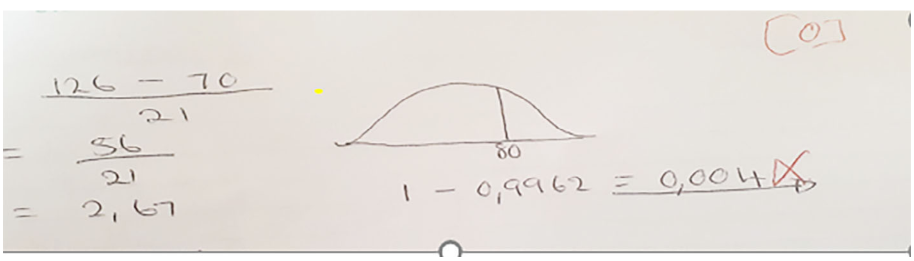

FIGURE 9 Student 22. Scoring – U:0, D:0, C:1, P:1. [Colour figure can be viewed at wileyonlinelibrary.com]

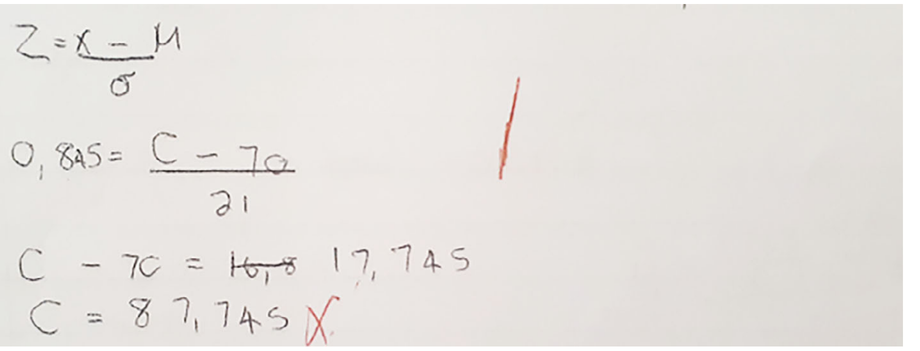

FIGURE 10 Student 57. Scoring – U:0, D:1, C:3, P:3. [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 2 Students' achievement level on the questions according to Polya's stages.

Polya's stages	Question	Descriptive statistics: Frequency Percentages (%) Very Low (Level 0)	Low (Level 1)	Intermediate (Level 2)	High (Level 3)	Advanced (Level 4)	Mean	Mode	Std. Deviation
Understand the problem	1a	25.0	22.5	0.8	3.3	48.3	2.28	4.00	1.773
	1b	25.0	15.0	0.8	5.0	54.2	2.48	4.00	1.777
	2	29.2	25.0	13.3	5.8	26.7	1.76	0.00	1.582
	3a	33.3	12.5	5.8	24.2	24.2	1.93	0.00	1.638
	3b	62.5	27.5	1.7	2.5	5.8	0.62	0.00	1.063
	4	10.8	10.8	16.7	2.5	59.2	2.88	4.00	1.473
	5	16.7	18.3	24.2	7.5	33.3	2.23	4.00	1.492
Devise a plan	1a	43.0	11.7	0.8	4.2	40.0	1.86	0.00	1.871
	1b	44.2	10.0	0.8	3.3	41.7	1.88	0.00	1.893
	2	45.0	29.2	0.0	1.7	24.2	1.31	0.00	1.618
	3a	25.8	11.7	1.7	28.3	32.5	2.3	4.00	1.633
	3b	62.5	28.3	3.3	2.5	3.3	0.56	0.00	0.933
	4	23.3	8.3	2.5	11.7	54.2	2.65	4.00	1.703
	5	25.8	24.2	12.5	1.7	35.8	1.98	4.00	1.655
Carry out the plan	1a	13.0	46.7	0.8	4.2	35.0	2.01	1.00	1.569
	1b	28.3	39.2	3.3	1.7	27.5	1.61	1.00	1.584
	2	37.5	36.7	0.0	0.8	25.0	1.39	0.00	1.589
	3a	23.3	16.7	20.8	16.7	22.5	1.98	0.00	1.478
	3b	55.0	30.8	5.0	6.7	2.5	0.71	0.00	1.007
	4	20.0	36.7	5.0	3.3	35.0	1.97	1.00	1.619
	5	24.2	26.7	21.7	2.5	25.0	1.97	4.00	1.645
Present the answer	1a	6.0	54.2	1.7	3.3	35.0	2.08	1.00	1.49
	1b	7.5	45.8	3.3	0.8	42.5	2.25	1.00	1.557
	2	27.5	48.3	0.0	0.0	24.2	1.45	1.00	1.505
	3a	23.3	23.3	15.0	5.0	33.3	2.02	4.00	1.603
	3b	51.7	35.8	3.3	6.7	2.5	0.72	0.00	0.987
	4	23.3	40.8	2.5	4.5	29.2	1.75	1.00	1.584
	5	24.2	41.7	20.0	5.8	8.3	1.33	1.00	1.118

TABLE 3 Combined means and combined SDs of students' level of skill in solving statistics problems according to Polya's stages.

Polya's stages	Understand the problem	Devise a plan	Carry out the plan	Present the answer
Combined means	2.03	1.79	1.66	1.65
Combined standard deviations	1.56	1.64	1.51	1.42
Remarks	Intermediate	Intermediate	Intermediate	Intermediate

TABLE 4 Levels of problem-solving skills.

Level of achievement categories	Very low	Low	Intermediate	High	Advanced
Value	0	1	2	3	4
Interval scale	0.0-0.80	0.81-1.60	1.61-2.40	2.41-3.20	3.21-4.00

Note: For example, any mean that falls within 1.61-2.40 was classified as intermediate.

students were at a very low, low, intermediate, high, or advanced level of statistical problem-solving skills. Table 3 shows the combined means (weighted arithmetic means or means of means) and combined SDs (pooled SD) of students' statistical problem-solving skill levels at each of Polya's problem-solving stages. [Correction added on 06 October 2023, after first online publication: The citation of Table 2 has been changed to Table 3 in this version.]

The result (Table 3) shows that the students' statistical problem-solving skills at all Polya's stages of problem solving were at the intermediate level. They had the highest combined mean at the first stage ("Understand the problem," mean = 2.03) but the combined mean depreciated to 1.65 at the last stage ("Present the answer"). [Correction added on 06 October 2023, after first online publication: The citation of Table 2 has been changed to Table 3 in this version.]

4.2 | Students' level of achievement on the statistical concepts at each of polya's stages

The descriptive statistical analysis (frequency percentages, mean, mode, and standard deviation) of the students' level of achievement on each of the concepts examined at each of Polya's stages is presented in Table 2. [Correction added on 06 October 2023, after first online publication: The citation of Table 3 has been changed to Table 2 in this version.]

At the stage of "understanding the problem," Table 2 shows that more than half of the students were below the high level on four out of the seven questions tested, while a little over half achieved the high and advanced level on the other three concepts. This means that the students showed a low level of understanding of the problems. Contrary to expectation, the students' achievement level on some questions (eg, problem 1a), increased down the Polya's stages instead of decreasing. This is because some students wrote correct answers to questions without showing how they arrived at the answers. Such students were adjudged to have successfully carried out the plan based on the rubric. Figure 2, Student 17, is an example of such a case. This happens when students try some questions on the question paper but fail to write the solution process in their answer scripts. The results of the "devise a plan" to solve the problems are like that of "understanding the problem," though there was a slight decline. The results show that, except for two of the seven concepts tested, more than half of the students were below high level. The majority of the students could not devise appropriate plans to solve the problems.

A further decline was found around the stage "carry out the plan." Over 60% of the students were found to be below the high level in executing plans to arrive at the answer to any of the problems. The lowest performance was observed on the conditional probability, binomial probability distribution, and probabilities for x-limits using z-distribution questions (questions 1b, 2, and 3b, respectively) where 67.5%, 74.2%, and 85.8%, respectively, of the students showed low or very low abilities to carry out plans to solve the problems.

Concerning the "present the answer stage," a decline was found in the students' abilities from carrying out the plan to the presentation of the result on all the questions except the conditional probability question (question 1b). On the other questions, more than 60% of the students performed below a high level of problem-solving skills. The ability to present their answers was found to be particularly low or very low, 75.8%, 87.5%, 64.1%, and 65.9%, respectively, on binomial probability distribution, probabilities for x-limits using z-distribution, estimating the y-value using the regression equation and hypothesis testing for a single population mean (questions 2, 3b, 4, and 5, respectively).

5 | DISCUSSION

The level of students' statistical problem-solving skills, in general, was found to be at the intermediate level at all Polya's problem-solving stages. Students had a limited understanding of the concepts investigated in this study. In many instances they confused the concepts; assigned incorrect or incomplete variables/parameters; provided incomplete or incorrect formulae, or formulae with major errors; demonstrated limited correct manipulation of established variables (unknowns)/parameters and often these unknowns were incorrect; and provided incorrect answers owing to correct variables very rarely being substituted in the formulae. The students' problem-solving skills on statistical concepts that were examined could be regarded as moderate. This finding is parallel with the results of Meutia et al. [17] and Sari et al. [18] on students' mathematical problem-solving skills at the secondary school level. The result is also similar to the finding of Arfiana and Wijaya [22], that students in their study achieved low levels in the "problem understanding" stage, very low levels in the "devising a plan" and "looking back" stages, and medium levels in the "carrying out a plan" stage. However, the result differs from Bester [14], who found that students achieved high levels in the "understand the problem" and "devise a plan" stages, medium levels in the "carry out the plan" stage, and low levels in the "interpreting the results" (looking back) stage.

The students' intermediate problem-solving level at the first stage of Polya's problem-solving model, with a mean of 2.03 on the four-point scale, reveals that the majority of the students did not have a grasp of the statistical concepts and hence did not understand the problems. The foundation of successful mathematical problem solving is understanding the problem. An understanding of the problem helps a problem solver to correctly represent the problem (mentally, verbally, symbolically, graphically, or schematically) to guide successful problem solving. Out of the seven questions analysed in this study, it was only on three questions (1a, 1b, and 4) that up to half of the students had a high or advanced level of understanding the question. When a problem solver fails to correctly represent the problem, it is unlikely that they will be able to choose the correct approach to solve the problem and, in the end, the person will be unsuccessful. This could explain why the students did not show a higher level of problem-solving skills in the last three stages of the problem-solving model, having not shown a high level of understanding the problem.

Many issues lead to students' poor understanding of mathematical/statistical problems. These include difficulty in understanding key information in the problem [20,21,41] and difficulty in formulating correct mathematical models (equations) to represent the problem [15,42]. These could account for the students' poor performance at the first stage of the problem-solving process and, subsequently, at the other stages.

6 | CONCLUSION

This study explored the statistics problem-solving skills of first-year undergraduate Statistics students. It was found that the problem-solving skills of the students were at the intermediate level. Students had limited or no understanding of the concepts investigated in this study. In many instances, they confused the concepts; assigned incorrect or incomplete variables/parameters; provided incomplete or incorrect formulae, or formulae with major errors; demonstrated limited correct manipulation of established variables (unknowns)/parameters and often these unknowns were incorrect; and provided incorrect answers owing to correct variables very rarely being substituted in the formula. Most of these students lacked declarative and conditional knowledge, and this inhibited their procedural knowledge when solving most of the problems. Most students comprehended the "understand the problem" stage better than Polya's other stages. The "carry out the plan" stage was the least well-comprehended stage, followed by the "present the answer" stage. In addition, it was found that some of the students skipped some of the problem-solving steps, which affected their subsequent steps.

This study recommends that lecturers and teachers explicitly teach and model problem solving to students in their lesson presentations so that students can learn the act of problem solving from them. In addition, lecturers should emphasize to students the necessity of showing all the steps as they solve problems. This would encourage students to keep track of their solutions, to check or look back on their solutions to see whether what they have written is correct or incorrect, and then to modify (correct their solution) where necessary. The study also recommends that lecturers, authors of textbooks, and writers of study materials adopt the four steps of Polya's Model of Problem Solving whenever they write the solutions to activities and examples presented in the study materials. Lecturers should do the same when they provide feedback on assignments in the form of a memorandum. In so doing, they can demonstrate to students what constitutes good practice in solving a problem as research has shown that people learn by copying the actions of others [43].

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APPENDIX A: EXAMINATION QUESTIONS ANALYSED.

A.1 | QUESTION 1

A certain motor component can fail due to mechanical, electrical, or other types of failures. Three car designs are under consideration and the data collected are as follows:

	Type of failure		
Car design	Mechanical	Electrical	Other
A	50	30	60
B	40	30	40
C	20	35	50

Suppose a motorcar component is selected at random. Determine the probability that

(a) it is of design B or has failed due to other causes.(3)

(b) it is a mechanical failure, given that it is of design B.(3)

A.2 | QUESTION 2

A recent survey in Roodepoort revealed that 60% of the vehicles travelling on highways with the speed limit posted at 100 km/hr, were exceeding the limit. Suppose you randomly record the speeds of 7 vehicles travelling on the N14 where the speed limit is 100 km/hr. Let X denote the number of vehicles that were exceeding the limit.

Calculate the probability that exactly four vehicles will exceed the speed limit.(3)

A.3 | QUESTION 3

The Rockwell hardness of a metal specimen is determined by impressing the surface of the specimen with a harder point and measuring the depth of penetration. The hardness of a certain alloy is normally distributed with a mean of 70 units and a standard deviation of 21 units. A specimen of this alloy is acceptable only if its hardness is between 66 and 75 units.

(a) Calculate the probability that a randomly selected alloy specimen is acceptable. (6)

(b) Find the value c such that the hardness of 80% of these alloy specimens exceed c. (3)

A.4 | QUESTION 4

A works engineer is of the opinion that the number of defective items produced hourly is directly proportional to the speed (revolutions per minute) of the lathe on which the item is produced. The following is a table of six random observations, each representing 1 h.

You may make use of the following summaries.

Speed of lathe	Number of defects
232	8
147	12
180	7
266	10
230	9
208	8

$\sum x = 1263$; $\sum x^2 = 274753$; $\sum xy = 11\ 274$; $\sum y = 54$; $Sxy = -18.6$

(c) Estimate the number of defective items produced when the speed of the lathe is 200, by making use of the regression line determined in (b).(2)

A.5 | QUESTION 5

The nominal value of a certain type of capacity, as determined by the manufacturer, is 0,34 μF. A sample of 20 capacitors from a specific company is tested, showing the following information:

Mean capacitance = 0.332 μF

Standard deviation of the capacitance s = 0.011 μF

(Note that μF = Micro Farad, the unit of capacitance).

Can we infer at the 10% level of significance if the population mean capacitance is significantly less than 0.34 μF?

(a) State the null and alternative hypotheses.(2)

(b) Select the appropriate statistical test and calculate the test statistics.(4)

(c) Determine the degrees of freedom.(2)

(d) Determine the critical value at $\alpha = 0.05$ level of significance.(2)

(e) Based on the interpretation of the results, can the null hypothesis be rejected or not?(2)

(f) Interpret your answer in (c) in plain language in terms of the original problem.(2)

APPENDIX B: SUMMARY OF THE DESCRIPTIONS OF THE LEVEL OF ACHIEVEMENT ON THE RESEARCH ASSESSMENT RUBRIC.

Levels of achievement	Polya's stages (problem-solving process) Stage (step)1: Understands the problem	Stage 2: Devises a plan	Stage 3: Carries out the plan	Stage 4: Presents the answer
Level 4: Advanced	Corrects and completes variables, parameters and mathematical/ statistical signs (symbols) (U, ∩), concepts and procedures. A student understands a problem fully. A student can go to certain extents to convert the word problem into mathematical expression.	Student devises a correct and complete formula, both variables and mathematical/ statistical signs (symbols). Correct reasoning.	Correct and complete manipulation of the established variables (unknowns) in Stage 2.	Correct answer.
Level 3: High	Correct and complete variable assigned. Some parameters are not (incorrectly) assigned or missing (incorrect) mathematical/statistical signs/symbols in the mathematical expression. Sometimes confuses the concepts.	Student devises a formula with minor errors. A formula which is not in a complete mathematical/ statistical format, which still needs some conversions, or formula with incorrect mathematical sign(s). Correct reasoning.	Partially correct manipulation of the established variables (unknowns) in Stage 2. At least 1 variable (unknown) is incorrect.	Incorrect answer due to partially correct variables (unknowns) substituted in Stage 3.
Level 2: Intermediate	Incorrect or incomplete variable assigned. Either mathematical expression is incorrect, or a student knows the correct procedure; however, all parameters assigned are incorrect. Often confuses the concepts.	Incorrect or incomplete formula, or formula with major errors. Sometimes incorrect reasoning.	Minimal correct manipulation of the established variables (unknowns) in step 2. At least two variables (unknowns) are incorrect.	Incorrect answer due to minimal correct variables (unknowns) substituted in Stage 3.
Level 1: Low	All variables or procedures or parameters are incorrectly assigned. Usually confuses the concepts.	Wrong formula. Usually incorrect reasoning.	Wrong substitution due to wrong formula in Stage 2.	Incorrect answer due to wrong formula devised in Stage 2.
Level 0: Very low	Not attempted.	No attempt to establish the relationship between variables.	Not attempted.	Not attempted.

APPENDIX C: QUESTION 3C.

Polya's steps	TIMSS level	
Understanding the problem	4	$p(x>c)=0.80$
	3	$(x>c)=0.80$ or $p(x\geq c)=0.80$
	2	$x>c$ or $z=\frac{\bar{x}-\mu}{\sigma}=0.80$
	1	Incorrect variables assigned
	0	Not attempted
Devise a plan to solve a problem	4	$p\left(z>\frac{c-\mu}{\sigma}\right)=0.8$
	3	$p\left(\frac{c-\mu}{\sigma}\right)=0.8$
	2	$\left(\frac{c-\mu}{\sigma}\right)>0.8$ or $\left(\frac{c-\mu}{\sigma}\right)=0.8$
	1	Incorrect execution
	0	Not attempted
Carry out a plan	4	$\left(\frac{c-70}{21}\right)=-0.84$
	3	$\left(\frac{c-70}{21}\right)=0.84$
	2	$\left(\frac{c-70}{21}\right)=0.80$
	1	Incorrect execution
	0	Not attempted
Present the answer	4	C = 52.36
	3	Incorrect answer but with partially correct attempt to evaluate it, that is, c = 87.64
	2	Incorrect answer but inadequate attempt to evaluate it, that is, c = 86.8
	1	Incorrect answer but some limited effort to evaluate it
	0	Not attempted

APPENDIX D: DESCRIPTION OF THE COURSE CONTENT FOR MODULE (COURSE) DESCRIPTIVE STATISTICS AND PROBABILITY.

1. What is statistics? 1.1. What is statistics? 1.2. Types of data and information	2. Graphical and tabular descriptive techniques 2.1. Graphical and tabular techniques to describe nominal data 2.2. Graphical techniques to describe interval data 2.3. Describing the relationship between two variables and describing time series data
3. Art and science tabular descriptive presentations 3.1. Graphical excellence and graphical deception 3.2. Presenting statistics: written reports and oral representations 3.3. Measures of central location 3.4. Measures of variability	4. Numerical descriptive techniques 4.1. Measures of relative standing and box plots 4.2. Measures of linear relationship 4.3. Comparing graphical and numerical techniques 4.4. General guidelines for exploring data
5. Data collection and sampling 5.1. Methods of collecting data and sampling 5.2. Sampling plans 5.3. Sampling and nonsampling errors	6. Probability 6.1. A basis for probability 6.2. Sophisticated methods and rules in probability theory 6.3. The rule of Bayes
7. Random variables and discrete probability distributions 7.1. Discrete probability distributions 7.2. Bivariate distributions 7.3. Binomial distribution 7.4. Poisson distribution	8. Continuous probability distribution 8.1. Continuous probability distributions: Normal distribution 8.2. Other continuous probability distributions
9. Sampling distributions 9.1. Sampling distribution of the mean 9.2. Sampling distribution of a proportion 9.3. Sampling distribution of the difference between two means	10. Introduction to estimation 10.1. Concepts of estimation 10.2. Estimating the population mean when the population standard deviation is known 10.3. Selecting the sample size
11. Introduction to hypothesis testing 11.1. Concepts of hypothesis testing 11.2. Testing the population mean when the population SD is known 11.3. Calculating the probability of a Type II error	12. Inference about a population 12.1. Inference about a population mean when the standard deviation is unknown 12.2. Inference about the mean: What else need you keep in mind? 12.3. Inference about a population variance 12.4. Inference about a population proportion