SPATIAL BAYESIAN METHODS OF FORECASTING HOUSE PRICES IN SIX METROPOLITAN AREAS OF SOUTH AFRICA

RANGAN GUPTA* AND SONALI DAS**#

Abstract

This paper estimates Spatial Bayesian Vector Autoregressive models (SBVAR), based on the First-Order Spatial Contiguity and the Random Walk Averaging priors, for six metropolitan areas of South Africa, using monthly data over the period of 1993:07 to 2005:06. We then forecast one- to six-months-ahead house prices over the forecast horizon of 2005:07 to 2007:06. When we compare forecasts generated from the SBVARs with those from an unrestricted Vector Autoregressive (VAR) and the Bayesian Vector Autoregressive (BVAR) models based on the Minnesota prior, we find that the spatial models tend to outperform the other models for large middle-segment houses; while the VAR and the BVAR models tend to produce lower average out-of-sample forecast errors for middle and small-middle segment houses, respectively. In addition, based on the priors used to estimate the Bayesian models, our results also suggest that prices tend to converge for both large- and middle-sized houses, but no such evidence could be obtained for the small-sized houses.

JEL Classification: E17, E27, E37, E47.

Keywords: BVAR Model; BVAR Forecasts; Forecast Accuracy; SBVAR Model; SBVAR Forecasts; VAR Model; VAR Forecasts.

1. INTRODUCTION

This paper estimates Spatial Bayesian Vector Autoregressive (SBVAR) models, based on the First-Order Spatial Contiguity (FOSC) and the Random Walk Averaging (RWA) priors, for six metropolitan areas of South Africa, namely Bloemfontein, Cape Town, Durban, Greater Johannesburg, Port Elizabeth/Uitenhage and Pretoria, using monthly data over the period of 1993:07 to 2005:06; and then, in turn, forecasts one- to six-months-ahead house prices over the 24 months out-of-sample forecast horizon of 2005:07 to 2007:06. Finally, the forecasts are evaluated by comparing them with the ones generated from an unrestricted classical Vector Autoregressive (VAR) model and the Bayesian Vector Autoregressive (BVAR) models based on the Minnesota prior, or specifically, with models that do not explicitly account for spatial influences.

The motivation for this analysis is twofold: Firstly, we want to investigate whether spatial models that explicitly incorporate the influence on house prices of a specific metropolitan area, or of other metropolitan area(s) within the same province or across provinces that share borders¹ with them, tend to forecast better than the standard

* Corresponding author. Contact Details: Associate Professor, Department of Economics, University Of Pretoria, Pretoria, 0002, South Africa. Phone: +27 12 420 3460, Fax: +27 12 362 5207, Email: Rangan.Gupta@up.ac.za

** Senior Researcher, Logistics and Quantitative Methods, CSIR Built Environment, P.O. Box: 395, Pretoria, 0001, South Africa. Phone: +27 12 841 3713, Fax: +27 12 841 3037, Email: SDas@csir.co.za

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¹The South African Economy is divided into 9 provinces, namely (in alphabetical order): Eastern Cape, Free State, Gauteng, Kwa-Zulu Natal, Limpopo, Mpumalanga, Northern Cape, North-

forecasting models, like the VAR and the BVARs. This exercise is important in the sense that it would inform us of whether to incorporate the role of spatial price influences, when developing a full-fledged model of house prices, based on proper theoretical considerations of the demand and supply factors affecting the housing market. Secondly, this analysis also allows us to indirectly evaluate, whether the so-called “Law of One Price” (LOOP) holds in the housing market of these six metropolitan areas. To be more precise, if the spatial models are found to outperform the non-spatial models, we could probably suggest that the housing markets in metropolitan areas that are close to one another tend to share a common regional/spatial market. However, if the VAR model, which treats the prices of all the provinces as equal, tends to do better relative to the other models, we can conclude that there exists a single housing market amongst the six metropolitan areas. Finally, if the non-spatial BVAR models, which lay more prominence on the last period’s own price, perform the best relative to the VAR and the SBVARs, this could imply that the housing markets are purely segmented.² Note our hypotheses are based on the understanding that houses would constitute a single market only if house prices in a specific location would impose a competitive constraint on house prices in another location.³

As pointed out by Burger and van Rensburg (2008), products sold at different regions can only be comparable when a clear definition of the product is provided from the outset. Hence, as in Burger and van Rensburg (2008), we do not consider the residential market in general, but subdivide the market in terms of sizes and prices of the houses. Specifically, we use the ABSA⁴ Housing Price Survey, which distinguishes between three price categories and then subdivides the middle segment category into three size categories of small, medium and large based on the square meters of house area.⁵ Given that regional house price data is only available for middle-segment houses, we restrict our analysis to this category. In addition, with the house price information of Bloemfontein dating back to 1993:07, we begin our analysis from that period.⁶ To the best of our knowledge, this is the first attempt to forecast metropolitan house prices of different sizes based on spatial models. However, it is important to point out that the motivation to analyze the LOOP in the six metropolitan areas emanates from the above stated recent study by Burger and van Rensburg (2008), in which the authors using cross-sectional unit root tests applied to the data for five metropolitan areas, namely

West and Western Cape. A provincial map for the country has been included in section 2, for not only a better understanding of the geographic structure of the economy, but more importantly, the design of priors for the SBVARs.

² See section 2 for further details.

³ See Motta (2004:107) and Carlton and Perloff (2005:648) for further details.

⁴ ABSA is one of the Leading Private banks of South Africa.

⁵ The South African residential property market is categorized into three major segments: luxury houses (R 2.6 million to R9.5 million), middle-segment houses (R226,000 to R2.6 million) and affordable houses (R226,000 and below with an area in the range of 40 m²-79 m²). The middle-segment houses are further subdivided into small (80 m²-140 m²), medium (141 m²-220 m²) and large (221 m²-400m²).

⁶ Note, though the ABSA Housing Price Review reports data for both metropolitan and non-metropolitan areas, the availability is limited and also lacks clarity regarding the area of coverage. Hence, we only limit ourselves to the analysis of the six major metropolitan areas of South Africa.

Cape Town, Durban, Greater Johannesburg, Port Elizabeth/Uitenhage and Pretoria, analysed whether the relative prices of these areas are stationary or not. They found strong evidence of convergence in the large- and small-sized middle-segment houses, but the unit root tests on the relative prices of the small middle-segment houses tended to suggest that they have separate regional markets.

It is important to comment further on the economic significance of the forecasting exercise. Recently, Stock and Watson (2003) have pointed out the role of asset prices in forecasting inflation. In this regard, they have highlighted the dominance of house prices, besides precious metals. Given this, the need to design models that can forecast house prices efficiently is of paramount importance, especially in a country targeting inflation. Housing service is an important component of the Consumer Price Index (CPI), and, hence of CPI inflation. As we can see from Figure 1, clearly and understandably, though more volatile than the overall CPI inflation, housing price inflation has tended to move in a similar fashion as that of CPI inflation over the sample period of 1993:07 to 2007:06. Clearly then, models forecasting house price inflation can give the policy makers an idea about where CPI inflation might be heading in the future, and hence, provide a better control of the situation through the design of appropriate policies. Thus, it is important that models forecasting house prices take into account possible heterogeneity and segmentation that might be existent in the housing market. Herein then comes the justification of modeling house prices separately based on size, and the importance of spatial models which, in turn, allows one to account for regional influences to check if the housing sector as a whole can be treated as an uniform single market.

Figure 1. Co-movement of Housing Price Inflation and CPI Inflation

Additionally, recent studies by Iacoviello (2002), Iacoviello and Minetti (2003) and Iacoviello and Minetti (2007) have indicated that in analyses involving the credit channel of monetary policy, it is important to include variables from the housing market. However, these papers analyse house price movements for the European economies, and treat the housing market as homogenous. If our analysis, as in Burger and van Rensburg (2008), finds the house prices of different sizes to behave differently, it would hint towards modelling the housing market for alternative sizes separately,

especially for studies analysing the bank-lending channel of monetary policy in South Africa that explicitly allow for housing market variables. Given that movements in the housing market is likely to play an important role in the business cycle, not only because housing investment is a very volatile component of demand (Bernanke and Gertler, 1995), but also because changes in house prices tend to have important wealth effects on consumption (International Monetary Fund, 2000) and investment (Topel and Rosen, 1988), ignoring such heterogeneity and segmentation in the market might yield flawed results.

The remainder of the paper, besides the introduction, is organized as follows: section 2 outlines the details of the structure and the estimation of the VAR, BVAR and the SBVARs for the house prices of six metropolitan areas of South Africa, section 3 discusses the layout of the model, section 4 compares the accuracy of the out-of-sample forecasts generated from alternative models, and finally, section 5 concludes and highlights the limitations of this study.

2. VAR, BVARs AND SBVARs: SPECIFICATION AND ESTIMATION⁷

The Vector Autoregressive (VAR) model, though ‘atheoretical’, is particularly useful for forecasting purposes. A VAR model can be visualized as an approximation of the reduced-form simultaneous equation structural model.

An unrestricted VAR model, as suggested by Sims (1980), can be written as follows:

$$y_t = A_0 + A(L)y_t + \varepsilon_t \quad (1)$$

where y is a $(n \times 1)$ vector of variables being forecasted; $A(L)$ is a $(n \times n)$ polynomial matrix in the backshift operator L with lag length p , *i.e.*, $A(L) = A_1L + A_2L^2 + \dots + A_pL^p$; A_0 is a $(n \times 1)$ vector of constant terms, and ε is a $(n \times 1)$ vector of error terms. In our case, we assume that $\varepsilon \sim N(0, \sigma^2 I_n)$, where I_n is a $n \times n$ identity matrix.

Note, a VAR model generally uses equal lag length for all the variables of the model. One drawback of this model is that many parameters need to be estimated, some of which may be insignificant. This problem of overparameterization, resulting in multicollinearity and a loss of degrees of freedom, leads to inefficient estimates and, possibly large out-of-sample forecasting errors. One solution often adapted is simply to exclude the insignificant lags based on statistical tests. Another approach is to use a near VAR, which specifies an unequal number of lags for the different equations.

However, an alternative approach to overcoming the overparameterization problem, as described in Litterman (1981), Doan *et al.*, (1984), Todd (1984), Litterman (1986), and Spencer (1993), is to use a Bayesian VAR (BVAR) model. Instead of eliminating longer lags, the Bayesian method imposes restrictions on these coefficients by assuming that they are more likely to be near zero than the coefficients on shorter lags. However, if there are strong effects from less important variables, the data can override this assumption. The restrictions are imposed by specifying normal prior distributions with zero means and small standard deviations for all coefficients, with the standard

⁷ The discussion in this section relies heavily on LeSage (1999), Sichei and Gupta (2006) and Gupta (2006).

deviation decreasing as the lags increase. The exception to this is that, the coefficient on the first own lag of a variable has a mean of unity. Litterman (1981) used a diffuse prior for the constant. This is popularly referred to as the ‘Minnesota prior’ due to its development at the University of Minnesota and the Federal Reserve Bank at Minneapolis.

Formally, as discussed above, the means and variances of the Minnesota prior take the following form:

$$\beta_i \sim N(1, \sigma_{\beta_i}^2) \text{ and } \beta_j \sim N(0, \sigma_{\beta_j}^2) \quad (2)$$

where β_i denotes the coefficients associated with the lagged dependent variables in each equation of the VAR, while β_j represents any other coefficient. In the belief that lagged dependent variables are important explanatory variables, the prior means corresponding to them are set to unity. However, for all the other coefficients, β_j ’s in a particular equation of the VAR, a prior mean of zero is assigned to suggest that these variables are less important to the model.

The prior variances $\sigma_{\beta_i}^2$ and $\sigma_{\beta_j}^2$, specify uncertainty about the prior means $\bar{\beta}_i = 1$, and $\bar{\beta}_j = 0$, respectively. Because of the overparameterization of the VAR, Doan *et al.*, (1984) suggested a formula to generate standard deviations as a function of a small numbers of hyperparameters: w , d , and a weighting matrix $f(i, j)$. This approach allows the forecaster to specify individual prior variances for a large number of coefficients based on only a few hyperparameters. The specification of the standard deviation of the distribution of the prior imposed on variable j in equation i at lag m , for all i, j and m , defined as $S_1(i, j, m)$, can be specified as follows:

$$S_1(i, j, m) = [w \times g(m) \times f(i, j)] \frac{\hat{\sigma}_j}{\hat{\sigma}_i} \quad (3)$$

with $f(i, j) = 1$, if $i = j$ and k_{ij} otherwise, with $(0 \leq k_{ij} \leq 1)$, $g(m) = m^{-d}$, $d > 0$. Note that $\hat{\sigma}_i$ is the estimated standard error of the univariate autoregression for variable i . The ratio $\hat{\sigma}_i / \hat{\sigma}_j$ scales the variables to account for differences in the units of measurement and hence, causes specification of the prior without consideration of the magnitudes of the variables. The term w indicates the overall tightness and is also the standard deviation on the first own lag, with the prior getting tighter as we reduce the value. The parameter $g(m)$ measures the tightness on lag m with respect to lag 1, and is assumed to have a harmonic shape with a decay factor of d , which tightens the prior on increasing lags. The parameter $f(i, j)$ represents the tightness of variable j in equation i relative to variable i , and by increasing the interaction, i.e., the value of k_{ij} , we can loosen the prior.⁸

Note, the overall tightness (w) and the lag decay (d) hyperparameters used in the standard Minnesota prior have values of 0.1 and 1.0, respectively, while $k_{ij} = 0.5$,

⁸ For an illustration, see Dua and Ray (1995).

implies a weighting matrix (F) with the following form:

$$F = \begin{bmatrix} 1.0 & 0.5 & . & . & . & 0.5 \\ 0.5 & 1.0 & . & . & . & 0.5 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0.5 & . & . & . & . & 1.0 \end{bmatrix} \quad (4)$$

Since the lagged dependant variable in each equation is thought to be important, F imposes $\bar{\beta}_i = 1$ loosely, while, given that the β_j coefficients are associated with variables presumed to be less important, the weighting matrix (F) imposes the prior means of zero more tightly on the coefficients of the other variables in each equation. Given that the Minnesota prior treats all variables in the VAR, except for the first own-lag of the dependent, in an identical manner, several attempts have been made to alter this fact. Usually, this has boiled down to increasing the value for the overall tightness (w) hyperparameter from 0.10 to 0.20, so that the larger value of w can allow for more influence from other variables in the model. In addition, as proposed by Dua and Ray (1995), we also try a prior that is even more loose, specifically with $w = 0.30$ and $d = 0.50$. Alternatively, LeSage and Pan (1995) have suggested the construction of the weight matrix based on the First-Order Spatial Contiguity (FOSC), which simply implies the creation of a non-symmetric F matrix that emphasizes the importance of variables from neighboring states/provinces more than those from non-neighboring states/provinces. Lesage and Pan (1995) suggest the use of a value of unity both for the diagonal elements of the weight matrix, as in the Minnesota prior, as well as in place(s) that correspond to variable(s) from state(s)/province(s) with which the specific state in consideration shares common border(s). However, for the elements in the F matrix that corresponds to variable(s) from state(s)/province(s) that are not immediate neighbor(s), Lesage and Pan (1995) propose a value of 0.1.

Referring to the provincial map of South Africa in Figure 2, the design of the F matrix based on the FOSC prior, given the alphabetical ordering⁹ of the six metropolitan areas as Bloemfontein, the Eastern Cape Metropolitan area (Port Elizabeth/Uitenhage), Greater Johannesburg, the Kwa-Zulu Natal Metropolitan area (Durban Unicity), Pretoria and the Western Cape Metropolitan area (Cape Town), can be formalized as follows:

⁹ It must, however, be pointed out that alternative ordering of the six metropolitan areas do not affect our final results in any way.

$$F = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.1 \\ 1.0 & 1.0 & 0.1 & 1.0 & 0.1 & 1.0 \\ 1.0 & 0.1 & 1.0 & 0.1 & 1.0 & 0.1 \\ 1.0 & 1.0 & 0.1 & 1.0 & 0.1 & 0.1 \\ 1.0 & 0.1 & 1.0 & 0.1 & 1.0 & 0.1 \\ 0.1 & 1.0 & 0.1 & 0.1 & 0.1 & 1.0 \end{bmatrix} \quad (5)$$

Figure 2. Provincial Map of South Africa

Source: <http://www.sa-venues.com/maps/south-africa-provinces.htm>.

The intuition behind the asymmetric F matrix is based on the lack of belief on the prior means of zero imposed on the coefficient(s) for price(s) of the neighboring province(s). Rather, we believe that these variables do have an important role to play. To express our lack of faith in the prior means of zero, we assign a larger prior variance (by increasing the weight values) to the prior means of the coefficients corresponding to the variables of the neighboring states. This in turn, allows the coefficients of these variables to be based more on the sample and less on the prior.

More recently, LeSage and Krivelyova (1999) have put forth an alternative approach to remedy the equal treatment nature of the Minnesota prior, called the “Random-Walk

Averaging” (RWA) prior. As noted above, most of the attempts to change the fact that the Minnesota prior treats all the variables, except the first own lag of the dependant variable, in the VAR in a similar fashion, have focused mainly on the alternative specifications for the prior variances. On the other hand, the RWA prior requires that both the prior mean and variance takes into account the distinction between important variables (like house price(s) of neighboring province(s)) and unimportant variables (like house price(s) of non-neighboring province(s)) for each equation in the VAR model. To understand the motivation behind the design of such prior means, consider the weight matrix F for the VAR consisting of house prices of the six metropolitan areas. Retaining the ordering of the six metropolitan areas as outlined in the FOSC prior, the weight matrix contains values of unity in positions associated with the house price(s) of neighboring province(s), i.e., for important variables in each equation of the VAR model, while zero values are assigned to the unimportant variables, i.e., house price(s) of *non*-neighboring province(s). However, as with the Minnesota prior, we continue to have a value of one on the main diagonal of the F matrix, simply to emphasize our belief that the autoregressive influences from the lagged values of the dependant variable (house price of a specific metropolitan area) are important.¹⁰

$$F = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0 \\ 1.0 & 1.0 & 0 & 1.0 & 0 & 1.0 \\ 1.0 & 0 & 1.0 & 0 & 1.0 & 0 \\ 1.0 & 1.0 & 0 & 1.0 & 0 & 0 \\ 1.0 & 0 & 1.0 & 0 & 1.0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 1.0 \end{bmatrix} \quad (6)$$

The weight matrix given above in (6) is then standardized so that the rows sums to unity. Formally, we can write the standardized F matrix, C , as follows:

$$C = \begin{bmatrix} 0.20 & 0.20 & 0.20 & 0.20 & 0.20 & 0 \\ 0.25 & 0.25 & 0 & 0.25 & 0 & 0.25 \\ 0.33 & 0 & 0.33 & 0 & 0.33 & 0 \\ 0.33 & 0.33 & 0 & 0.33 & 0 & 0 \\ 0.33 & 0 & 0.33 & 0 & 0.33 & 0 \\ 0 & 0.50 & 0 & 0 & 0 & 0.50 \end{bmatrix} \quad (7)$$

This matrix C allows us to consider the random-walk with drift, which averages over the important variables in each equation i of the VAR. Formally,

$$y_{it} = \delta_i + \sum_{j=1}^n C_{ij} y_{jt-1} + u_{it} \quad (8)$$

¹⁰ However, using a value of one on the main diagonal element of the F matrix, under the RWA prior, is not always an obvious choice. See LeSage and Krivelyova (1999) for an alternative exposition, where autoregressive influences are considered to be important only for certain variables.

where in our case, $n = 6$. On expanding equation (8) we observe that by multiplying y_{jt-1} , containing the house prices of six metropolitan areas at $t-1$, with C will produce a set of explanatory variables for the VAR equal to the mean of observations from the important variables (neighboring house prices) in each equation i at $t-1$.¹¹ This also suggests that the prior mean for the coefficients on the first own-lag

of the important variables is equal to $\frac{1}{c_i}$, with c_i being the number of important

variables in a specific equation i of the VAR model. However, as in the Minnesota prior, the RWA prior uses a prior mean of zero for the coefficients on all lags, except for the first own lags. At this juncture we emphasize that, the RWA approach of specifying prior means requires the variables to be scaled to have similar magnitudes, since otherwise it does not make an intuitive sense to suggest that the value of a variable at t is equal to the average of values from the important variables at $t-1$. This transformation is not much of an issue, as the data on the variables, in our case the house prices, can always be expressed as percentage change, or annualized growth rates, thus meeting the similar magnitudes requirements of the RWA prior.

Finally, the prior variances for the parameters under the RWA prior, as proposed by LeSage and Krivelyova (1999), retaining the distinction between important versus unimportant variables, is in line with the following ideas:

- (i) Smaller prior variance is assigned to parameters associated with unimportant variables, allowing for the zero prior means to be imposed with more certainty;
- (ii) The prior variance on the first own-lag of the important variables are small, so that the prior means force averaging over the first own-lags of such variables;
- (iii) The prior variance of parameters associated with unimportant variables at lags greater than one is imposed in such a way that it becomes smaller as the lag length increases. This is simply to convey the belief that the influence of the unimportant variables decay over time;
- (iv) Motivated by the fact that we do not believe in the idea of zero prior means on the longer lags of the important variables, parameters associated with lags other than the first own-lag of the important variables is structured to have larger prior variance. By imposing prior means of zero loosely on the longer lags of the important variables, we allow them to exert some influence on the dependant variable. RWA however, still imposes decreasing prior variances on the coefficients of lags, other than the first own-lag of the important variables. Thus, in the specification of the RWA, as in the Minnesota prior, longer lag influences decay irrespective of whether the variable is classified as important or unimportant.

Given (i) to (iv), a flexible form in which the RWA prior standard deviations ($S_2(i, j, m)$) for a variable j in equation i at lag length m is as follows:

¹¹ Just as with the constant in the Minnesota Prior, δ is also estimated based on a diffuse prior.

$$\begin{aligned}
S_2(i, j, m) &\square N\left(\frac{1}{c_i}, \sigma_c\right); j \in C; m = 1; i, j = 1, \dots, n \\
S_2(i, j, m) &\square N\left(0, \eta \frac{\sigma_c}{m}\right); j \in C; m = 2, \dots, p; i, j = 1, \dots, n \\
S_2(i, j, m) &\square N\left(0, \rho \frac{\sigma_c}{m}\right); j \notin C; m = 1, \dots, p; i, j = 1, \dots, n
\end{aligned} \tag{9}$$

where $0 < \sigma_c < 1; \eta > 1$ and $0 < \rho \leq 1$. For the variables $j = 1, \dots, n$ in equation i that are important in explaining the movements in variable i i.e., $j \in C$, the prior mean for the lag length of 1 is set to the average of the number of important variables in equation i , and to zero for the unimportant variables, i.e., $j \notin C$. With $0 < \sigma_c < 1$, the prior standard deviation for the first own-lag imposes a tight prior mean to reflect averaging over important variables. For important variables at lags greater than one, the variance decreases as m increases, but the restriction of $\eta > 1$ allows for the zero prior means on the coefficients of these variables to be imposed loosely. Finally, we use $\rho \frac{\sigma_c}{m}$ for lags on unimportant variables, with prior means of zero, to indicate that the variance decreases as m increases. In addition, with $0 < \rho \leq 1$, we impose the zero means on the unimportant variables with more certainty.

The BVARs and the SBVARs, based on the FOSC and the RWA priors, are estimated using Theil's (1971) mixed estimation technique. Specifically, suppose we denote a single equation of the VAR model as: $y_1 = X\beta + \varepsilon_1$, with $Var(\varepsilon_1) = \sigma^2 I$, then the stochastic prior restrictions for this single equation can be written as:

$$\begin{bmatrix} M_{111} \\ M_{112} \\ \cdot \\ \cdot \\ \cdot \\ M_{nnm} \end{bmatrix} = \begin{bmatrix} \sigma / \sigma_{111} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \sigma / \sigma_{112} & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & 0 & \sigma / \sigma_{nnm} \end{bmatrix} \begin{bmatrix} a_{111} \\ a_{112} \\ \cdot \\ \cdot \\ \cdot \\ a_{nnm} \end{bmatrix} + \begin{bmatrix} u_{111} \\ u_{112} \\ \cdot \\ \cdot \\ \cdot \\ u_{nnm} \end{bmatrix} \tag{10}$$

Note, $Var(u) = \sigma^2 I$, and the prior means M_{ijm} and σ_{ijm} take the forms shown in (2) and (3) for the Minnesota prior; in (2), (3) and (5) for the FOSC prior; and in (9) for the RWA prior. With (10) written as:

$$r = R\beta + u \tag{11}$$

the estimates for a typical equation are derived as follows:

$$\hat{\beta} = (X'X + R'R)^{-1}(X'y_1 + R'r) \tag{12}$$

Essentially then, the method involves supplementing the data with prior information on the distribution of the coefficients. The number of observations and degrees of freedom are increased by one artificially, for each restriction imposed on the parameter

estimates. The loss of degrees of freedom due to over-parameterization associated with a classical VAR model, is therefore, not a concern in the BVARs and SBVARs.

3. SBVAR MODELS FOR FORECASTING HOUSE PRICES IN SIX METROPOLITAN AREAS OF SOUTH AFRICA

Given the specification of priors in section 2, we estimate two SBVAR models based on the FOSC and the RWA priors, for each of small, medium and large middle-segment houses in Bloemfontein, the Eastern Cape Metropolitan area (Port Elizabeth/Uitenhage), Greater Johannesburg, the Kwa-Zulu Natal Metropolitan area (Durban Unicity), Pretoria and the Western Cape Metropolitan area (Cape Town), over the period 1993:07 to 2005:06, using monthly data. We then compute out-of-sample one- through six-months-ahead forecasts for the period of 2005:07 to 2007:06, and compare the forecast accuracy relative to the forecasts generated by an unrestricted VAR and the BVARs. The variables included are house prices of the above mentioned six metropolitan areas. As pointed out by Hamilton (1994:362), the data is seasonally adjusted in order to, *inter alia*, address the fact that the Minnesota-type priors are not well suited for seasonal data. All data are obtained from the latest ABSA Housing Price Review.

In each equation of the SBVARs, there are 49 parameters, including the constant, given the fact that the model is estimated with 8 lags¹² of each variable. Note Sims *et al.*, (1990) indicate that with the Bayesian approach entirely based on the likelihood function, the it is not necessary for the associated inference to take special account of nonstationarity, as the likelihood function has the same Gaussian shape regardless of the presence of nonstationarity, and as such the variables have been specified in levels.

The six-variable SBVAR models are estimated for an initial prior for the period 1973:07 to 2005:06, and then forecast from 2005:07 through to 2005:12. Since we use eight lags, the initial eight months of the sample, i.e., 1993:07 to 1994:02, are used to feed the lags. We generate dynamic forecasts, as would naturally be achieved in actual forecasting practice. The models are re-estimated each month over the out-of-sample forecast horizon in order to update the estimate of the coefficients, before producing the 6-months-ahead forecasts. This iterative estimation and the 6-steps-ahead forecast procedure was carried out for 24 months, with the first forecast beginning in 2005:07. This produced a total of 24 one-month-ahead forecasts, 24-two-months-ahead forecasts, and so on, up to 24 6-month-ahead forecasts. For this, we used the algorithm in the Econometric Toolbox of MATLAB¹³. The MAPEs¹⁴ for the 24, month 1 through month 6 forecasts, are then calculated for the six house prices of the models.

¹² The choice of 8 lags is based on the unanimity of the sequential modified LR test statistic, Akaike information criterion (AIC), and the final prediction error (FPE) criterion.

¹³ All statistical analysis was performed using MATLAB, version R2006a.

¹⁴ Note that if A_{t+n} denotes the actual value of a specific variable in period $t + n$ and ${}_tF_{t+n}$ is the forecast made in period t for $t + n$, the MAPE statistic can be defined as $(\frac{1}{N} \sum abs(\frac{A_{t+n} - {}_tF_{t+n}}{A_{t+n}})) \times 100$, where *abs* stands for the absolute value. For $n = 1$, the

summation runs from 2005:07 to 2005:12, and for $n = 2$, the same covers the period of 2005:08 to 2006:01, and so on.

The average of the MAPE statistic for one- to six-months-ahead forecasts for the period 2005:07 to 2007:06 are then examined. Identical steps are followed to generate forecasts from the VAR and the BVAR models based on the Minnesota prior. Note, for the BVAR models we start with a value of $w = 0.1$ and $d = 1.0$, and then increase the value to $w = 0.2$ to account for more influences from variables other than the first own lags of the dependant variables of the model. In addition, as in Dua and Ray (1995), Gupta and Sichei (2006) and Gupta (2006), we also estimate a BVAR model with $w = 0.3$ and $d = 0.5$. The model that produces the lowest average MAPE values is selected as the ‘optimal’ Bayesian model for a specific metropolitan area, corresponding to a specific size of the middle-segment houses.

4. EVALUATION OF FORECAST ACCURACY

To evaluate the accuracy of forecasts generated by the SBVAR models, we need alternative forecasts. To make the MAPEs comparable with the SBVARs, we report the same set of statistics for the out-of-sample forecasts generated from an unrestricted classical VAR (the benchmark model) and the BVARs. The unrestricted VAR and the BVARs are also estimated in levels with 8 lags. In Tables 1 to 3, we compare the average MAPEs of one- to six-months-ahead out-of-sample-forecasts for the period 2005:07 to 2007:06, generated by the unrestricted VAR, the BVARs and the SBVARs. The conclusions from these tables can be summarized as follows:

[INSERT TABLES 1 THROUGH 3]

(i) Large Middle-Segment Houses: From the average MAPE values for one- to six-months-ahead forecasts, as reported in Table 1, for large middle-segment houses, the SBVAR model, based on the FOSC prior, outperforms the other models for three (Bloemfontein, the Eastern Cape metropolitan area, and the Kwa-Zulu Natal metropolitan area) of the six metropolitan areas, while the BVARs with $w = 0.1$, $d = 1.0$ and $w = 0.2$, $d = 1.0$ respectively, does best for Johannesburg and the Western Cape metropolitan areas. Finally, the SBVAR model based on the RWA prior produces, on average, has the lowest MAPE values for Pretoria. The SBVAR model based on the RWA prior that did consistently best amongst other SBVAR models with the RWA prior, for all house sizes and majority of the metropolitan areas, had the following values for the hyperparameters: $\sigma_c = 0.3; \eta = 8$ and $\rho = 1$.¹⁵

(ii) Medium Middle-Segment Houses: From Table 2, the unrestricted VAR produces the minimum one- to six-months-ahead average MAPE values for the Eastern Cape metropolitan area, the Kwa-Zulu Natal metropolitan area, Johannesburg and Pretoria, over the 24 month forecasting horizon. The SBVAR model based on the RWA prior and the most loosely-prioried BVAR ($w = 0.3$, $d = 0.5$) produced, on average, the best forecasts for Bloemfontein and the Western Cape metropolitan area, respectively.

(iii) Small Middle-Segment Houses: From Table 3 we observe that, in general, the BVAR models perform best in terms of forecasting house prices of this category over the forecasting horizon spanning 2005:07 to 2007:06. Specifically, the BVAR model with $w = 0.2$ and $d = 1.0$ produces the lowest average one- to six-months-ahead

¹⁵ The values for these hyperparameters are based on the ranges suggested by LeSage (1999).

forecast for the Eastern Cape metropolitan area and Pretoria, while the BVAR models with $w = 0.1$ and $d = 1.0$ and $w = 0.3$, $d = 0.5$ outperforms the other models for Johannesburg and Bloemfontein, respectively. For the Western Cape and the Kwa-Zulu Natal metropolitan areas, the SBVAR model, based on the RWA prior, and the unrestricted classical VAR, respectively, are the preferred models, as they produce the minimum MAPE values on average, for one- to six-months-ahead forecasts over the 24 month out-of-sample period.

In summary, we can draw the following conclusions: (a) Though there does not exist a specific model that performs outright best in terms of forecasting house prices of different sizes in the six metropolitan areas of South Africa, in general, the spatial models tend to outperform the other models for large middle-segment houses. While, the unrestricted VAR and the BVAR models tend to produce lower average out-of-sample forecast errors for middle and small middle-segment houses, respectively. (b) As far as drawing conclusions regarding the LOOP is concerned, we can make the following remarks, recalling that houses would constitute a single market only if house prices in a specific location would impose a competitive constraint on house prices in another location: (i) Given that the BVAR models do best in terms of forecasting prices for the small middle-segment house, it is clear, based on the structure of the Minnesota prior, that all that matters when it comes to defining prices for this category of houses is its own past price. Hence, the small middle-segment houses have segmented markets; (ii) The classical unrestricted VAR stands out when it comes to forecasting prices of the middle-sized middle-segment houses. Given that the unrestricted VAR treats all the house prices of the other areas/metropolitans equally, there exists strong evidence of the LOOP within this category of housing; and finally, (iii) with the SBVAR models relatively better suited in forecasting the prices of the large middle-segment houses, it seems to suggest that the LOOP holds, but the market has a spatial nature. In other words, neighboring regions tend to constitute a single market, given that the influences from the non-neighbors are not that important in determining the price of a large middle-segment property. Unlike, small and middle sized middle-segment housing, this might be due to the importance of spatial correlations in the determination of the prices of large-sized houses possibly because, there exists less heterogeneity in the supply of such housing, or alternatively, wealthier customers tend to have similar characteristics, thus causing the prices to cluster around some values.¹⁶ So the LOOP holds with some spatial qualifications. And as in Burger and van Rensburg (2008), we too find that prices tend to converge for both large- and middle-sized houses, but no such evidence can be deduced for the small-sized middle-segment houses. However, unlike Burger and van Rensburg (2008), our modeling strategies allow us to go a step further and say that the LOOP for the large middle-segment houses has a spatial aspect to it as well.

5. CONCLUSIONS

This paper estimates SBVAR models, based on the FOSC and the RWA priors, for six metropolitan areas of South Africa (Bloemfontein, Cape Town, Durban, Greater Johannesburg, Port Elizabeth/Uitenhage and Pretoria), using monthly data over the

¹⁶ We are thankful to an anonymous referee for pointing this out as a possible explanation as to why SBVAR models might be relatively better suited to forecast the house prices of large middle-segment houses.

period 1993:07 to 2005:06, and then, in turn, forecasts one- to six-months-ahead house prices over the out-of-sample forecast horizon of 2005:07 to 2007:06. The forecasts are evaluated by comparing them with those generated from the VAR and the BVAR models based on the Minnesota prior.

The motivation for this analysis is twofold: Firstly, we want to investigate whether spatial models, that explicitly incorporate the influence of a specific metropolitan area by other metropolitan area(s) within the same province, or across provinces that share their borders, on house prices tend to forecast better than the standard forecasting models, like the VAR and the BVARs, and; Secondly, this analysis is also used to indirectly evaluate if the so-called “Law of One Price” (LOOP) holds in the housing market of these six metropolitan areas.

As far as the answer to the first question goes, based on the forecasting performance of the models, we can safely say that spatial influences are only important for large middle-segment houses, but not so much for the middle- and small-sized categories. Hence, we need to pay special attention to incorporate the role of spatial price influences when developing a full-fledged model of house prices, based on demand and supply considerations, at least for the large middle-segment houses. In terms of drawing conclusions regarding the LOOP, again based on the forecasting exercise, we find that prices tend to converge for both large- and middle-sized houses, but no such evidence can be deduced for the small-sized middle-segment houses. Though similar conclusions were reached by Burger and van Rensburg (2008), our modeling strategies allow us to go a step further and suggest that the LOOP for the large middle-segment houses has a spatial aspect to it as well. In addition, given the importance of house prices on overall CPI, and hence the possibility that house price forecasts based on simple spatial and non-spatial models can give an indication to policy makers as to where the inflation rate might be heading, the economic significance of our analysis is immense and cannot be ignored.

At this stage, it must be pointed out that there are at least two limitations to using a Bayesian approach for forecasting. Firstly, as is evident from Tables 1 to 3, the forecast accuracy is sensitive to the choice of the priors. If the prior is not well specified, an alternative model used for forecasting may perform better. Secondly, in case of the Bayesian models, one requires to specify an objective function, for example the average MAPEs, to search for the ‘optimal’ priors, which in turn, needs to be optimized over the period for which we compute the out-of-sample forecasts. However, there is no guarantee that the chosen parameter values specifying the prior will continue to be ‘optimal’ beyond the period for which it was selected. Nevertheless, the importance of the Bayesian forecasting models, spatial or non-spatial, cannot be underestimated. This has been widely proven in the forecasting literature¹⁷, and is also vindicated by our current study, which indicates the suitability of the Bayesian models in terms of forecasting large and small middle-segment house prices in six metropolitan areas of South Africa for the period of 2005:07 to 2007:06.

¹⁷For example, Amirizadeh and Todd (1984), Kuprianov and Lupoletti (1984), Hoen *et al.*, (1984), Hoen and Balazsy (1985), Kinal and Ratner (1986), Gruben and Long (1988a, b), Lesage (1990), Gruben and Hayes (1991), Shoesmith (1992, 1995), Dua and Ray (1995), Dua and Smyth (1995), Dua and Miller (1996), Dua, (1999), Banerji *et al.*, (2006), Gupta and Sichei (2006), Gupta (2006), Liu and Gupta (2007) and Gupta (2007).

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Table 1. Average MAPEs for Large Middle-Segment Houses (2005:07-2007:06)

Models	Bloemfontein	Eastern Cape	Johannesburg	Kwa-Zulu Natal	Pretoria	Western Cape
VAR	0.0207	0.1092	0.1770	0.03272	0.1155	0.0337
BVAR1	0.0192	0.1226	0.1442	0.0039	0.1335	0.0517
BVAR2	0.0254	0.0705	0.1577	0.0095	0.1606	0.01133
BVAR3	0.0213	0.1204	0.1696	0.0275	0.1171	0.0387
SBVAR1	0.0121	0.0752	0.1456	0.0035	0.1457	0.0485
SBVAR2	0.0192	0.3436	0.1479	0.0382	0.0556	0.0474

Notes: BVAR 1: $w=0.1$, $d=1.0$; BVAR 2: $w=0.2$, $d=1.0$; BVAR 3: $w=0.3$, $d=0.5$; SBVAR 1: FOSC Prior; SBVAR 2: RWA Prior ($\sigma_c = 0.3; \eta = 8; \rho = 1$).

Table 2. Average MAPEs for Middle Middle-Segment Houses (2005:07-2007:06)

	Bloemfontein	Eastern Cape	Johannesburg	Kwa-Zulu Natal	Pretoria	Western
VAR	0.2563	0.2810	0.1566	0.0165	0.0891	0.0160
BVAR1	0.2827	0.3660	0.2523	0.035166667	0.1249	0.0383
BVAR2	0.3001	0.3983	0.2556	0.0341	0.1426	0.0784
BVAR3	0.2700	0.2890	0.1733	0.02152	0.0914	0.0106
SBVAR1	0.2614	0.3945	0.24165	0.0349	0.1100	0.0582
SBVAR2	0.1772	0.6142	0.3724	0.0785	0.1348	0.0575

Notes: BVAR 1: $w=0.1$, $d=1.0$; BVAR 2: $w=0.2$, $d=1.0$; BVAR 3: $w=0.3$, $d=0.5$; SBVAR 1: FOSC Prior; SBVAR 2: RWA Prior ($\sigma_c = 0.3; \eta = 8; \rho = 1$).

Table 3. Average MAPEs for Small Middle-Segment Houses (2005:07-2007:06)

	Bloemfontein	Eastern Cape	Johannesburg	Kwa-Zulu Natal	Pretoria	Western
VAR	0.0463	0.3291	0.1380	0.0170	0.0805	0.0930
BVAR1	0.0698	0.1800	0.0314	0.0617	0.0761	0.0909
BVAR2	0.1105	0.1078	0.1281	0.0708	0.0166	0.0818
BVAR3	0.0390	0.2986	0.1178	0.0280	0.0790	0.0966
SBVAR1	0.0753	0.1963	0.0440	0.0900	0.0866	0.0590
SBVAR2	0.0762	0.1326	0.1604	0.0405	0.0186	0.0131

Notes: BVAR 1: $w = 0.1$, $d = 1.0$; BVAR 2: $w = 0.2$, $d = 1.0$; BVAR 3: $w = 0.3$, $d = 0.5$; SBVAR 1: FOSC Prior; SBVAR 2: RWA Prior ($\sigma_c = 0.3$; $\eta = 8$; $\rho = 1$).