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Wavelet analysis of intraday share prices

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Abstract

This research tested whether wavelet based algorithms can improve the performance of intraday share trading algorithms. The trading algorithms investigated, each consisted of two parts: the first part performed share price prediction and the second part traded based on the prediction.

All the trades in the shares BTI, MTN, NPN and SBK through 2013 on the JSE with the associated time stamps, transaction share prices and volumes, served as the basic sample. The sample was further reduced by using end-of-interval transaction share prices at intervals of one, two, five and ten minutes throughout the trade days.

Three types of prediction algorithms were employed: auto regressive moving average (ARMA), wavelet-ARMA and wavelet regressive algorithms. The wavelet based algorithms were further broken down by using up to six different levels of scales in each of the algorithms. These algorithms were fitted using the first half year of data while the tests were conducted on the second half year of data.

Two trade algorithms were created by the researcher: One algorithm for buy-and-sell and another for short-and-close. Both algorithms used the predicted share price one and two intervals ahead as input and took transaction cost into account. The trade algorithms entered the market daily after opening time and exited the market before closing time.

The wavelet based algorithms were not found to improve the accuracy of share price prediction. However, in agreement with previous research, wavelet based algorithms were found to improve the accuracy of predicting the direction of the share prices. The wavelet based algorithms were also found to improve trading performance. Short-and-close algorithms outperformed buy-and-sell. None of the intraday trade algorithms were found to outperform buy-and-hold over the test period.

This study contributes to academic research regarding the manner in which wavelet based and ARMA algorithms were combined, the application of a wavelet-regressive prediction method to financial time series and the application of wavelet based trading algorithms on an intraday time scale.

Keywords

Algorithmic trading; ARMA; Intraday; Wavelet.

Declaration

I declare that this research project is my own work. It is submitted in partial fulfilment of the requirements for the degree of Master of Business Administration at the Gordon Institute of Business Science, University of Pretoria. It has not been submitted before for any degree or examination at any other University. I further declare that I have obtained the necessary authorisation and consent to carry out this research.

________________	10 November 2014
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List of abbreviations

AR	Autoregressive
ARMA	Autoregressive moving average
ARIMA	Autoregressive integrated moving average
BIC	Bayesian information criterion
BTI	British American Tobacco PLC - JSE share code
DJIA	Dow Jones Industrial Average
JSE	JSE Limited (Johannesburg Stock Exchange)
MA	Moving average
MODWT	Maximum overlap discrete wavelet transform
MTN	MTN Group – JSE share code
NPN	Naspers Limited – JSE share code
RMS	Root mean square
SBK	Standard Bank Group Limited - JSE share code

Contents

Chapter one - Introduction to the research problem

The future path of the price level of a security is no more predictable than the path of a series of cumulated random numbers – Eugene Fama.

Share trading is increasingly done by computer algorithms without direct human intervention (Menkveld, 2013). Profitable trading (in contrast with investing) increasingly occurs intraday, rather than at longer periods (Schulmeister, 2009). High frequency trading occurs at response times of milliseconds and accounts for half of the transactions in the USA (Mackenzie, 2014; Aitken, Cumming & Zhan, 2014).

Trading algorithms can incorporate models of the order book dynamics and of share price movement (Mackenzie, 2014). Such models can include the effects of external factors (for instance macro-economic variables and the price movement of other assets) and own price history effects (such as momentum) (Zheng & Chen, 2012).

Share price time histories contain cycles with varying periods and amplitudes. The cycles are treated as short term bull or bear markets in some investment strategies. For instance, if momentum is used as an investment style, the cycles are viewed either as trends within the time period considered (momentum up or down) (Schulmeister, 2009) or, in the case of shorter cycles, noise. A complementary approach is to use the cycles as features when modelling and predicting price movement (Kriechbaumer, Angus, Parsons & Rivas Casado, 2014).

Wavelet analysis is useful to decompose financial (and other) time series to reveal the cyclic nature (or frequency response) of the time series and how the frequency response changes over time (Bekiros & Marcellino, 2013). It can also be used to investigate links between assets at different time scales (Jammazi, 2012).

No trading algorithms have been found reported that use wavelets as part of the algorithm in intraday trading. This research focused on intraday price prediction algorithms and how wavelets could improve the algorithm performances for signalling buy/sell in trading algorithms.

Chapter two - Literature review

Forecasting prices of stocks, commodities or derivatives on liquid markets is in a large part guesswork – Stephan Schütler and Carola Deuschle.

2.1 **Introduction**

Returns from the stock market can be earned by investing or by trading. Investing focuses on the intrinsic or fundamental value of companies with the expectation that the value of a company's shares plus dividends will grow due to its performance (Schulmeister, 2009). Trading is more concerned with the expected short term change in the share price and it is the focus of technical analysis (Schulmeister, 2009). This study is about trading and the prediction of share prices.

The literature review starts with an overview of issues related to technical trading and prediction. Thereafter the subjects of intraday trade, algorithmic trading and transaction cost are considered. Wavelet theory is introduced next and thereafter a review of wavelet analysis of financial time series. Measurement of algorithmic success is the last topic before the conclusion.

2.2 **Technical trading and prediction**

Schulmeister (2009) identified three groups of traders and their aggregate response results in the market behaviour: Fundamentalists (value traders), technical traders (relying mostly on recent price movements) and "bandwagonists" (responding to trends). This study pertained to the technical traders. Technical traders use rules based on patterns in asset prices and these patterns occur due to human behaviour. (Friesen, Weller & Dunham 2009). Technical trading's mechanism for realising a return is prediction, therefore good trading rules start with good prediction ability. However, it is not clear whether technical analysis results in profitable trading, or in contrast, whether one cannot consistently outperform the market (as the efficient market hypothesis implies) (Duvinage, Mazza & Petitjean, 2013).

Various studies reported that technical trading could not outperform the market. Hudson, Dempsey and Keasey (1996) applied two trading rules on the UK stock market from 1935 to 1994 and found that their prediction ability is not sufficient to make excess returns if trading fees are accounted for. Marshall, Cahan and Cahan (2008) also reported that most studies find technical analysis is not profitable if fees are taken into account. Du Plessis (2013) obtained the same result with the FTSE/JSE Top 40 shares. An analysis of Japanese candlestick strategies conducted at 5 minute intervals on the 30 stocks of DJIA over a year concluded that buy-and-hold would be a better strategy (Duvinage et al., 2013). Yamamoto (2012) compared trading strategies based on order flow imbalance and order book imbalance of individual Nikkei 225 shares at 5 minute intervals with buy-and-hold and came to the same conclusion. Marshall et al. (2008) tested 7 846 popular technical trading rules on 2002-2003 US equity data at 5 minute intervals and found no intraday profits, even without accounting for transaction costs.

In contrast, there are studies that claim technical trading can outperform the market. Hudson et al., (1996) cited a study (Brock, W., Lakonishok, J., & LeBaron, B. (1992). Simple technical trading rules and the stochastic properties of stock returns. *The Journal of Finance*, *47*(5), 1731-1764.) on the prediction ability of simple trading rules with respect to the Dow Jones index from 1897 to 1992 with positive results. This outcome is supported by the reference to various empirical studies conducted between 1986 and 2000 that report evidence that technical analysis beats the market (Hong & Satchell, 2011). Schulmeister (2009) found that the profitability of technical trading rules in the S&P500 spot market declined over time from 8.6% per year in the 1960’s to as low as -5.1% in the 1990’s and to -0.8% in the early 2000’s. However, technical stock trading at 30 minute intervals showed a profit of 2.7% between the 1980’s and early 2000’s. Schulmeister (2009) concluded that technical trading profitability has shifted to intraday time scales.

2.3 **Intraday trade**

The daily volume of trade has increased significantly over time. On Black Monday (the market crash day in October 1987), more than 6 million shares were traded on the New York Stock Exchange. On the flash crash date (6 May 2010) 10.3 billion shares (1 700 times more, or 440 000 shares per second) were traded (Betancourt, VanDenburgh & Harmelink, 2011). On average, five billion shares were traded daily in the USA during 2013 (Mackenzie, 2014).

Daily trade activity tends to match a U shaped graph: High volumes of trade occur at opening and closing times with the lowest activity recorded around noon (Cont, 2011; Scholtus & Van Dijk, 2012). Cont, Kukanov and Stoikov (2014) found high price volatility early in the trade day and high order flow imbalance volatility by the end of the market day. The high price volatility in the morning is ascribed to more information being available at that time (Cont et al., 2014). There are also peaks in trade messages at regular intervals on the hour, half hour, 5^{th}, 15^{th} and 45^{th} minutes and every minute as well as at regular intervals within the minute and within the second (Scholtus & Van Dijk, 2012). These peaks are ascribed to market participants who trade at regular periods (Scholtus & Van Dijk, 2012).

Heston, Korajczyk and Sadka (2010) analysed intraday trade on the New York Stock Exchange and excluded trades adjacent to opening and closing times in their analysis of intraday share price patterns. They selected 13 half-hour time intervals per day and recorded the returns of shares over each interval. The returns of shares were found to be negatively correlated with recent intervals. However a positive correlation was found with every 13^{th} interval (or every 24 hours). This daily correlation was statistically significant for 40 or more trading days (Heston et al., 2010). Heston et al. (2010) considered the regular arrival of news during the day as possibly contributing to the 24 hour correlation. Groß-Klußmann and Hautsch (2011) investigated the high frequency effects of the unannounced arrival of news items on share prices. They excluded the opening and closing times of the market and the announcement of results. In contrast to the deduction of Heston et al., (2010), no intraday news arrival pattern was detected apart from an observation that more news arrived earlier in the day (Groß-Klußmann & Hautsch 2011). Value, volatility, trade size and spread increased before news arrival, then peaked with news arrival and later tapered off. (Groß-Klußmann & Hautsch, 2011).

2.4 **Algorithmic share trading**

Modern stock exchanges use electronic order books. The process to conclude a sale occurs by means of the exchange's matching engine algorithm as soon as bid and sell offer conditions match (Mackenzie, 2014). Mackenzie (2014) identified three circumstances of trade:

1) Human actors, with the market consisting of humans interacting with humans.
2) Human actors working via computers with the market being an algorithm.
3) An algorithmic market with mostly algorithmic actors directing trade.

One category of algorithmic trading is agency algorithms (Hasbrouck & Saar 2013). Agency algorithms are employed to buy or sell large blocks of shares at the best possible average price. These algorithms would for instance divide the shares in small quantities in order to hide the size of the trade (Mackenzie, 2014). Agency algorithms in general do not require millisecond response times (Hasbrouck & Saar, 2013).

The other category of algorithmic trading is proprietary trading algorithms. With proprietary trading algorithms, the intention is to buy-and-sell and not to accumulate positions in shares (Mackenzie, 2014). Menkveld (2013), for instance, found that a particular high frequency trader in general only realised profits with positions being maintained for not longer than 5 seconds.

High frequency algorithms have low latency. Some of these algorithms trade within 2 to 3 milliseconds after an event (Hasbrouck & Saar, 2013; Hagströmer & Nordén, 2013). The time from posting a limit order to cancellation is on average 0.36 milliseconds (Hagströmer & Nordén, 2013). Trade performance declines significantly with delays exceeding 200 milliseconds, also in the case of opportunistic trading (Scholtus & Van Dijk, 2012).

The order book, consisting of the offers to sell, bids to buy and the associated volumes on offer/bid with the recent history, is the important input to the high frequency algorithms (Cont et al., 2014; Treleaven, Galas & Lalchand, 2013). Various strategies exist. If, for instance, the volume of the best bid price to buy is larger than the volume of the best offer price to sell, the expectation is that there is more buying force than selling force and the price is about to increase. High frequency traders can exploit this by a practice called spoofing, by for instance placing offers for very short periods to create an impression of supply, and then removing the offers (Mackenzie, 2014).

High frequency trader strategies can be grouped as market making or market taking. Market (or liquidity) makers position their offers and bids in the order book as close as possible to the best bids to buy and best offers to sell. They profit from the bid/offer price difference and earn on average about 0.08% (Mackenzie, 2014). Market takers are participants who agree to the bid/offer price and realise the sale.

Another group of algorithmic traders engage in opportunistic trading, such as arbitrage or directional (for instance momentum based or order direction) trading (Hagströmer & Nordén, 2013). Statistical arbitrage algorithms predict patterns of price changes over longer time scales, typically from minutes to days (Mackenzie, 2014). They employ techniques such as filter rules, moving averages, support and resistance, channel breakouts and on-balance volume (Marshall et al., 2008), as well as graphical methods and Japanese candlestick pattern recognition (Duvinage et al., 2013). In contrast with trade algorithms that respond to the order book, trade execution of opportunistic algorithms can occur at regular intervals, for instance every minute (Scholtus & Van Dijk, 2012; Hasbrouck & Saar, 2013).

A large part of the daily trade is made by algorithms, and specifically high frequency. Since 2009, more than 73% of US equity trades were made by the 400 high frequency trading firms (out of the total of 20 000 trading firms in the USA) (Easley, De Prado, & O'Hara, 2011). A high frequency trader on the Chi-X exchange in Europe in 2007/2008 traded on average 1 397 times per stock per day (Menkveld, 2013).

2.5 **Rate of exchange as share market indicator**

Since 2001, the JSE ALSI would rise (fall) if the rand weakens (strengthens) (Barr & Kantor, 2005; Muzindutsi, 2013). A similar finding was made in Brazil: There was a negative correlation between the value of the dollar in Brazilian real and the Ibovespa index of the Sao Paulo stock market (Belardi, Aguiar & Fausto, 2012). However, changes in rand/dollar rate of exchange affect individual company shares on the JSE differently, depending on the currencies of the costs and income for the specific company (Barr & Kantor, 2005).

2.6 **Transaction cost**

Trading fees vary but seem to be lowering with time due to the establishment of competitive markets such as Chi-X in Europe and BATS in the USA (Menkveld, 2013; Scholtus & Van Dijk, 2012). The trading fee in the UK was around 50 basis points (100 basis points = 1%) in 1994 (Hudson et al., 1996). Menkveld (2013) reported fees of €0.60 per trade on Euronext and 30 basis points or lower for Chi-X. In the US equity market, the average trading fee was around 40 basis points during the second quarter of 2012 and on average 90 basis points in emerging markets (Elkins McSherry, 2012), while fees for liquidity demanding trades were below 0.295c/share with passive share rebates of up to 0.28c/share on the NASDAQ in the top volume bracket (Brogaard, Hendershott & Riordan, 2012). The overall transaction cost of professional traders on electronic exchanges such as Globex was around 0.1% (Schulmeister, 2009). The JSE trading cost was estimated at between 0.4% and 0.7% by Du Plessis (2013).

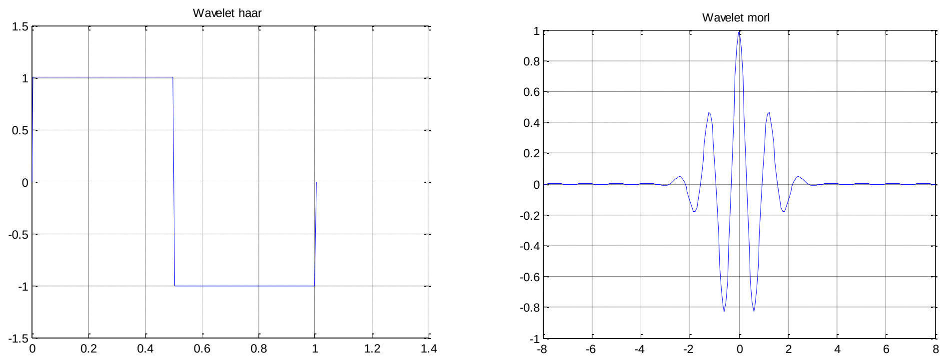
2.7 **Wavelet theory**

Aguiar-Conraria and Soares (2013) give an overview of wavelet theory for the analysis of economic data and this section is based thereon. Periodic features in economic data are better visible if a time series is converted to frequency response, for instance by Fourier analysis. However, the Fourier transform excludes time information and does not show changes in periodicity over time, making it suitable for stationary time series analysis. In contrast, wavelet analysis extracts periodic data with time and can therefore reveal changes in periodic behaviour over time. (Aguiar-Conraria & Soares, 2013).

Wavelets are wave-like functions in time with a basic attribute: Wavelets have limited time duration (in other words, a wavelet has a start time and an end time). This is an interpretation of the admissibility condition (Aguiar-Conraria & Soares, 2013) which is defined by the following equation:

$$0 < C_\psi := \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|}{|\omega|} d\omega < \infty$$

Figure 1. Sample wavelet functions: Haar (left) and Morlet (right).



A mother wavelet (such as Haar or Morlet in Figure 1 above) can be converted to daughter wavelets by scaling on the time axis (stretching it wider or shrinking it narrower) and by moving it left or right on the time axis. A continuous wavelet transform (CWT) of a time series is obtained by integrating the product of the wavelet function ψ and the time series x(t) (Aguiar-Conraria & Soares, 2013) as defined in the following equation:

$$W_{x;\psi}(\tau, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \psi \left(\frac{t-\tau}{s} \right) dt$$

The continuous wavelet transform W is a function of the position in the time series (τ) and the scale (s).

The wavelet transform operates in a moving time window on the time series. Therefore the exact location in time of an exact scale cannot be known. A Heisenberg box of the wavelet transform with a +/- standard deviation around a point in scale and time can be constructed (Aguiar-Conraria & Soares, 2013). Longer scales or periods (or lower frequency) have less certainty about the time location and vice versa.

Aguiar-Conraria and Soares (2013) assert that although continuous wavelet analysis is computationally more intensive, continuous wavelets are best suited for financial time series analysis as the results are easier to interpret graphically. According to Ahuja, Lertrattanapanich and Bose (2005), the selection criteria for wavelet types when doing financial time series analysis were (then) not well defined. Since then, some analysis has been done with discrete wavelet transforms (Crowley, 2007) and maximum overlap discrete wavelet transforms (Gallegati, 2008). For prediction models, the discrete wavelet is the transform of choice (Zhu, Wang & Fan, 2014), as prediction models work with discrete data as input (Ortega & Khashanah 2014); the discrete wavelet transform contains the same information as the continuous wavelet transform; it can be computed faster than the fast Fourier transform and it can de-correlate time series (Percival & Walden, 2006, p.19).

2.8 **Wavelet analysis of time series**

Wavelets have been used to study various types of time series as indicated in the examples below.

2.8.1 Natural cycles

A wavelet regression algorithm was used by Kisi (2010) for stream flow prediction. The historic data was decomposed with the discrete wavelet transform. Thereafter each scale’s time signal was separately reconstructed. A regression was fitted between the separate scales’ time signals and the stream flow, with noisy scale components discarded.

Zhu et. al. (2014) used a maximum overlap discrete wavelet transform – auto regressive moving average (MODWT-ARMA) algorithm to predict daily rainfall. They found MODWT-ARMA to outperform other forecasting models.

2.8.2 Economic cycles

Aguiar-Conraria and Soares (2011), applied wavelets to industrial production index data and thereby identified patterns of synchronous business cycles between countries in and around the European Union. These cycles change over time. Dar, Samantaraya and Shah (2013) investigated whether the spread between long- and short term interest rates (yield spread) predicts economic activity in India. They found that yield spread predicts economic activity in long time scales. A metal price forecast algorithm was devised by Kriechbaumer et al. (2014). They combined wavelets with an autoregressive integrated moving average filter (ARIMA) and improved the forecasting accuracy of the ARIMA filter significantly.

Discrete wavelet decomposition was used in conjunction with autoregressive filtering by Renaud, Starck and Murtagh (2005) to predict financial futures and web site access. They adapted the wavelet transform to solve the wavelet function on the edge so it can be used for prediction filters.

2.8.3 Electricity prices

A wavelet-auto regressive integrated moving average (ARIMA) electricity price forecasting model was devised by Conejo, Plazas, Espinola and Molina (2005). They decomposed historic prices with a discrete wavelet transform. Their next step was to predict the future values of the decomposed series with the ARIMA filter. An inverse wavelet transform was applied to the predicted decomposed components to yield the forecast. Shrivastava and Panigrahi (2014) developed an electricity price forecast algorithm by combining wavelets with an extreme learning machine, while Voronin & Partanen (2014) combined wavelets with ARIMA and neural networks.

2.8.4 Rates of exchange

Bekiros and Marcellino (2013) investigated co-movement and causal links between the dollar, euro, pound and yen with daily data over 11 years. They compared wavelet analysis with other methods and found wavelets useful for predictions over any time span (Bekiros & Marcellino, 2013). Gençay, Selçuk and Whitcher (2001) analysed scale (frequency) properties of the volatilities in the exchange rates mark/dollar and yen/dollar. They applied wavelets to exchange rate data from 1986 until 1996 sampled at 20 minute time intervals. Insignificant cross correlation in volatility was observed on intraday level. Daily volatility cycles dominated and longer term correlations were observed (Gençay et al., 2001). These daily cycles are in agreement with the results of Heston et al. (2010).

2.8.5 Share prices

Gallegati (2008) used MODWT to compare the DJIA and US Industrial production index with monthly data in the period 1961 to 2006. They found that production growth rate shows long memory dynamics while stocks shows short memory and that stocks leads industry production by 16 months and longer.

Morris, Van Vuuren and Styger (2009) analysed seven share prices on the JSE as well as the ALSI 40 index using daily closing prices through 2006 and 2007 with wavelets and a Markov model. Long memory behaviour was identified, which constituted evidence against the weak-form efficient market hypothesis.

Wavelets and candlestick algorithms were combined by Li and Kuo (2008) in a self-organising map to create a trading algorithm. It was tested with the Taiwan weighted stock index with daily data from 1991 to 2002. The combined wavelet-candlestick algorithm performed better than the algorithm without the wavelet component.

In similar vein, Schlüter and Deuschle (2010) devised forecasting algorithms using different wavelets and ARIMA filters. Two methods of incorporating wavelet prediction were tested on different financial time series with daily price data in the period 2007 to 2009. These were de-noising plus ARIMA, and decomposing plus ARIMA on the decomposed scale components. The test series consisted of a share price (Deutshe Bank), oil price (WTI), rate of exchange (euro/dollar) and daily power prices in the UK. There were mixed results. In the case of a large random component, wavelets added little value. If the long term structure dominated, a de-noising approach with ARIMA was found to be the best algorithm. In cases with dominant cycles, wavelet decomposed scale combined with ARIMA gave the best results.

Benhmad (2013) correlated world stock market indices from 2005 to 2011 at different scales. The markets could be classified as highly correlated, middle correlated and less correlated and the correlation dynamics were found to be time dependant. Dependency scales (frequencies) changed after the bankruptcy of Lehman Brothers bank in 2008 (Benhmad, 2013).

2.8.6 Intraday share prices

A wavelet neural network prediction algorithm was devised by Ortega & Khashanah (2014). The algorithm used high and low share prices in one minute intervals as input and decomposed the prices with Haar maximum overlap discrete wavelet transform (MODWT). The transform was used as input to a neural network, which predicted log returns one-, three- and five minutes ahead.

2.9 **Measures of algorithmic success**

The purpose of this study was to use wavelets to extract cyclic components in financial price series to predict share price movement. Measures of success would be whether an algorithm can predict the share prices better, and whether the algorithm can ensure superior returns.

The Wilcoxon signed-rank test is nonparametric and enables comparison of matched pairs (Hines & Montgomery 1980, p.311). Therefore this test would be suitable to compare the performances of different prediction and trading algorithms at recurring times.

A risk when comparing algorithms using a specific data set is data snooping. Data snooping occurs when a data set is re-used to compare models. With re-used data, results may be due to chance rather than performance of the model (White, 2000).

2.10 **Conclusion of literature review**

Technical trading is being performed more and more on an intraday basis (Schulmeister, 2009) at higher turn-around times (Hasbrouck & Saar, 2013). Although the order book is of paramount importance for high frequency trading (Hasbrouck & Saar, 2013; Treleaven et al., 2013; Mackenzie, 2014), algorithms based on historic price and volume data can still be employed at recurring time intervals (Schulmeister, 2009; Ortega & Khashanah, 2014).

Asset price cycles, the changes in their periods over time and their co-movements can be studied with wavelets (Aguiar-Conraria & Soares, 2013). Wavelets can also be employed to improve prediction algorithms for prices of commodities (Kriechbaumer et al., 2014) and shares (Schlüter & Deuschle, 2010). Transaction cost can be the make or break factor for trading algorithms at all time scales (Schulmeister, 2009; Brogaard et al., 2012).

No literature was found regarding the application of wavelet analysis to intraday trading. This research could contribute in this domain.

Chapter three - Research hypotheses

The gold price can go up or down, but not necessarily in that order – Clem Sunter.

The purpose of the research was to determine whether wavelet based algorithms can improve the performance of intraday trading algorithms. The trading algorithms investigated, each consisted of two parts: the first part performed share price prediction one and two time steps ahead and the second part traded based on the prediction.

The performances of the share price prediction algorithm were compared based on the accuracy of the prediction one and two time steps ahead as well as on the directional accuracy at the same time steps. Two hypotheses were therefore formulated to test the prediction performances.

Trade performances were compared by means of two additional hypotheses. The third hypothesis was used to determine whether wavelet based algorithms improve intraday trade performances. The fourth hypothesis was used to compare the trading performances of the intraday trade algorithms with the baseline of a buy-and-hold approach.

3.1 **Hypothesis A**

Ha_0: Wavelet based share price prediction algorithms do not improve share price prediction accuracy on an intraday scale.

Ha_1: Wavelet based share price prediction algorithms improve share price prediction accuracy on an intraday scale.

3.2 **Hypothesis B**

Hb_0: Wavelet based algorithms do not improve the accuracy of predicting the direction of share price movement on an intraday scale.

Hb_1: Wavelet based algorithms improve the accuracy of predicting the direction of share price movement on an intraday scale.

3.3 **Hypothesis C**

Hc_0: Wavelet based share price prediction algorithms do not improve the performance of intraday trading algorithms.

Hc_1: Wavelet based share price prediction algorithms improve the performance of intraday trading algorithms.

3.4 **Hypothesis D**

Hd_0: Intraday trade algorithms do not perform better than buy-and-hold.

Hd_1: Intraday trade algorithms perform better than buy-and-hold.

Chapter four - Research methodology

Eenvoud is die vreugde van die lewe (Simplicity is the joy of life) – Jannie du Toit.

4.1 **Introduction**

The research methodology aim was to facilitate the comparison of the performance of selected prediction algorithms for the purpose of trading. To this end, a stepwise process was followed. Share price data was obtained and prepared for application of the algorithms. The next step was to sample the share prices in specific time intervals. Thereafter the data was divided: The first half was allocated to the algorithm learning process and the second half to testing. Teaching the different algorithms was the next step. This was followed by testing the algorithms regarding prediction accuracy and trading performance. The final step was to compare the algorithm performances. The remainder of this chapter contains detail regarding this process.

4.2 **Research philosophy**

The research philosophy was positivism (Saunders & Lewis, 2012, p.104). Structured methods were used that can be re-used on other data sets. The research approach was deductive. The five sequential stages for deductive research were applied, (Saunders & Lewis, 2012, p.108) including the hypotheses statements; employment of a data analysis method resulting in the acceptance/rejection of the hypotheses; comparing the results with previous research and current theory to either confirm current theory or propose an adaptation. The study was explanatory and quantitative (Saunders & Lewis, 2012, p.113).

4.3 **Unit of analysis**

The unit of analysis was a trade (a transaction to effect change of ownership of shares) on the JSE. The population consisted of all the trades on the JSE. Each trade had a price, volume and associated date and time of the transaction.

4.4 **Data source**

The Centre of Business Mathematics and Informatics of the North West University, records data of all share trades on the JSE. The time resolution of their recordings is one second. They supplied the trade data of the four selected shares through 2013 (De Jongh, D., personal communication, August 26, 2014).

4.5 **Data preparation**

The data preparation consisted of sorting the trade data in time sequence, reading it into Matlab and removing outliers. Price data points that were clearly separate from the remainder of the price data (outliers) were considered recording errors (Bisgaard & Kulahci, 2011, p.89) and were removed from the data set.

4.6 **Sampling**

Sampling was done on various levels. The first level of sampling was to select the time period – the year 2013. The second level of sampling was to select specific shares - BTI (British American Tobacco), MTN, NPN (Naspers) and SBK (Standard Bank). These shares were selected as they were the ones with the largest market capitalisation in their respective industries.

The third sampling filter selected only those transactions that occurred within the trading hours of the JSE (09h00 to 17h00) on the 250 trading days during 2013. As the last public trades occurred 10 minutes before closing time, the trading hours were taken as 09h00 to 16h50. No special consideration was given with respect to the days when the JSE opened after 09h00 or closed earlier than 16h50.

Trades occur at random time intervals. One option would have been to analyse the trades in the order of occurrence, tick by tick. This would have had the advantage that no information would be ignored. The disadvantages of the tick by tick approach would be that the number of data points to analyse would be very large and that time-based patterns would be ignored.

The fourth sampling filter was therefore used to group the transactions in time slots of different lengths. The selected time lengths were one, two, five and ten minutes. The time slots were chosen to find a balance between a relative quick reaction to market events (one minute) and minimising the number of time slots where no price changes occurred.

The price associated with the last trade that occurred in each time slot was used. In cases of time slots without trades, the most recent trade price was used. This is in agreement with the market perception that the current price of a share is the price of the most recent trade. Examples of other studies on intraday price patterns applied sampling periods ranging from sub-second (Cont, 2011) to 20 seconds (Gross-Klussmann & Hautsch, 2011), one minute (Ortega & Khashanah, 2014), five minutes (Duvinage, Mazza and Petitjean, 2013 and Yamamoto, 2012) and 30 minutes (Heston, Korajczyk & Sadka, 2010).

The sampling resulted in time series of trade prices through 250 trade days in 2013 as shown in Table 1 below.

Table 1. Sampling intervals and the number of trade data points

Sampling interval (minutes)	Number of trade days	Number of price data points per day	Number of price data points through the year	Number of price data points for algorithm training	Number of price data points for algorithm testing
1	250	470	117 500	58 750	58 750
2	250	235	58 750	29 375	29 375
5	250	94	23 500	11 750	11 750
10	250	47	11 750	5 875	5 875

4.7 **Data splitting**

The data was split in two halves: a “training part” and a “testing part” (Montgomery, Jennings & Kulahci, 2008). The first half (2 January 2013 until 2 July 2013) was used for training the prediction algorithms (fitting models) and the second half (3 July 2013 until 31 December 2013) for testing the algorithms (forecasting and application of the trading algorithm).

4.8 **Models for price prediction**

Three basic models were used for price prediction and a dual algorithm for trading. The prediction models were applied to predict prices one and two time steps ahead, as this was the requirement of the trading algorithm. A trade strategy of buy-and-hold served as a reference regarding financial performance.

Two of the models utilised wavelets. These contained sub-models whereby a limited number of wavelet components would be used in the sub-model prediction algorithms.

4.8.1 ARMA

Autoregressive moving average (ARMA) modelling is based on the assumption that a future value of a time series has a relation to previous values (via the autoregressive or AR part), plus a relation to previous and present random inputs (via the moving average or MA component) (Bisgaard & Kulahci, 2011, p. 59-60). ARMA models can be fitted to time series if the mean is constant. If this is not the case, the difference operator can be applied until the time series mean is constant and the ARMA model can then be fitted to the difference time series.

Selecting the order of the ARMA model to fit to a time series is often done by analysing the autocorrelation function and the partial autocorrelation function of the time series. Skill is required in pattern recognition that serves as guidance in the selection of good models (Bisgaard & Kulahci, 2011, p. 60).

An alternative is to select the best ARMA model based on a measure. The Bayesian information criterion (BIC) is suitable in cases of larger samples to identify parsimonious models (Bisgaard & Kulahci, 2011, p. 164). BIC is calculated as follows:

$$BIC \approx n \ln (\sigma_a^2) + r \ln (n).$$

In this equation, n is the number of observations, σ_a^2 is the estimate of the residual variance and r is the number of parameters used in the model (Bisgaard & Kulahci, 2011, p. 164).

The BIC was applied as an automated criterion in all ARMA model selections of this study, even though this approach is not recommended by Bisgaard & Kulahci (2010, p. 164), This approach is practical for the real-life application of the methods, as manual model selection for a large number of shares will not be feasible.

A further restraint was applied in that all ARMA model orders were limited to ARMA(4,4) (four autoregressive lags with four moving average lags). This was done after observing that the relative change in the BIC value with increased model order diminishes after three to four lags. In addition, this limit was consistent with an approach of finding parsimonious models and not to over fit (Bisgaard & Kulahci, 2011, p. 156).

The ARMA models were fitted on the training part of the data (first half) using the estimate.m Matlab function. BIC values were calculated with the aicbic.m function. The models with the lowest BIC scores were selected each time.

4.8.2 Wavelet-ARMA

Overview

The wavelet-ARMA model firstly utilised wavelets to decompose the time series into different scale (or frequency) components. ARMA models were then fitted to the individual scale components and the next time step values were predicted for the wavelet components. The predicted share price was calculated as the sum of selected wavelet component predictions. This approach was a combination of parts of methods applied by Kisi (2010) and that of Schlüter and Deuschle (2010) and Kriechbaumer et al (2014).

Wavelet

The continuous wavelet is useful for the graphical presentation of cycles over time (Percival & Walden, 2006, p.12). However, it is the discrete wavelet transform that is applied in prediction algorithms. The main reasons for using the discrete wavelet transform are that the discrete wavelet transform contains the same information as the continuous wavelet transform; it can be computed faster than the fast Fourier transform and it can de-correlate time series (Percival & Walden, 2006, p.19).

In this study, the Haar wavelet function was used. Although Kriechbaumer et. al. (2014) found the performance of wavelet-ARIMA forecasting techniques dependant both on the wavelet function and on the decomposition method, the selection of Haar was based on the experience of Schlüter and Deuschle (2010). They found smaller prediction errors with wavelet-ARIMA forecasting algorithms of share prices when using Haar-based wavelets than with Daubechies 4 wavelets.

The last 2^n samples in the learning series was separated for the wavelet-ARMA model fit, with n the largest possible integer. This was done to comply with the wavelet algorithm requirement for a dyadic (2^n, with n being a positive integer) number of samples.

The discrete wavelet transform (DWT) of the log-return of the prices, was calculated at the sampling intervals with the Matlab swt.m function. Five levels of decomposition were used with the resulting decomposition scales shown in Table 2 below. The inverse discrete wavelet transform was subsequently applied separately to each of the five detail scales plus the approximation scale with the Matlab function iswt.m. The result was six time signals, each containing a specific scale component of the log return series. The sum of these six signals equals the original log return signal.

Table 2. Scales and the sampling intervals

Scale level	**1**	**2**	**3**	**4**	**5**
Sampling interval (minutes)	**Decomposition scale (minutes)**				
1	2	4	8	16	32
2	4	8	16	32	64
5	10	20	40	80	160
10	20	40	80	160	320

ARMA

The next step was to fit separate ARMA models to each of these six time signals. In this case, the limits for AR and MA lags were two of each. The BIC selection criterion was also applied to select the most suitable model.

Prediction

During the test period, the prediction of two time steps ahead log return was done as follows:

The 32 samples up to the required prediction time spot were used to apply the discrete wavelet transform to five detail levels plus the approximation level. The inverse discrete wavelet transform was then applied separately to each scale (or level) component. The next two values for each of the series were predicted with the ARMA models.

Six different wavelet-ARMA predictions of the log return at the time steps one and two ahead were realised in this way. The first prediction consisted of the sum of all six scale components, the second was the sum of scales 2 to 6, the third the sum of scales 3 to 6 and so on. The prices for the next 2 time steps ahead were calculated as the current price plus the predicted log returns.

4.8.3 Wavelet regression

Overview

The wavelet regression model was based on the approach of Kisi (2010). Firstly, wavelets were used to decompose the time series into different scale (or frequency) components. Secondly, a multiple linear regression model was applied using the scale components as input and the one and two-step ahead log return values as output. The predicted log return was calculated as the regression fit of the preceding decomposed wavelets.

Wavelet

The same wavelet function and process of wavelet analysis was applied as for the wavelet-ARMA case.

Regression

The regression algorithm used the six wavelet-decomposed time series as input to fit to the one and two step ahead log return series. Six different predicted values for the log return were obtained, using the same logic as in the wavelet-ARMA prediction algorithm.

Six sets of price predictions were realised in the same fashion as with the wavelet-ARMA method. The first prediction was a regression, based on all six scale components, the second was a regression based on scales 2 to 6, the third was a regression based on scales 3 to 6 and so on. The prices for the next 2 time steps ahead were calculated as the current price plus the predicted log returns.

4.9 **Trading algorithm**

4.9.1 Buy-and-sell

Each of the prediction algorithms predicted the share price up to two time periods into the future. The trade algorithm used these predictions with the following logic:

> If the predicted price, two periods ahead, was higher than the predicted price, one period ahead (taking into account trading cost), the share was bought in the next time period.
>
> If a share was owned and the predicted price, two periods ahead, was less than the predicted price, one period ahead, the share was sold in the next time period.

The algorithm did not keep shares overnight. If a share was owned, the share would be sold in the time slot that includes 16h30 and no further trade was done for that day.

4.9.2 Short-and-close

As an alternative to buy-and-sell, a short-and-close approach was also investigated. The decision logic is the same as for the buy-and-sell trade, with one exception: The words “sell” or ”sold” are swapped with “buy” or “bought”. In this case the short position was also closed at 16h30.

4.9.3 Trading cost

JSE stock brokers pay a variety of fees in order to trade (JSE, 2014). Calculating the cost of a single trade is a challenge due to the myriad of monthly fees and transaction costs. As the number of trades or the value of trades per day increases (decreases), the trade cost per trade will decrease (increase). The marginal trade cost percentage was estimated based on the JSE cost brochure (JSE, 2014), assumptions as stated in Table 3 on page 26 and with the aid of Mansura, S. at the JSE (personal communication, May 14, 2014). Based on the calculations, a single direction trading cost of 0.018% was used as part of the trading algorithm.

Table 3. Marginal JSE trade cost for a broker

Item	Per transaction	Daily	Min[a]	Max[b]	Monthly
Assumptions					
Value traded per day		R 1 000 000			
Transactions per day		50			
Value per transaction	R 20 000				
JSE monthly charges					
Membership fees		R 713.32			R15 693.00
API service		R 415.05			R 9 131.15
BDA dissemination subscription		R 60.22			R 1 324.76
BDA Custody and Settlement membership		R 202.49			R 4 454.88
Talx front end		R 160.12			R 3 522.65
JSE per transaction charges					
Broker Dealer Accounting (74c/transaction)	0.0037%	R 37.00			
Transaction	0.0053%	R 53.00		R 300	
Clearing and settlement	0.0034%	R 34.00		R 130	
Strate Ad Valorem per share per day	0.0055%	R 54.59	R 10.92	R 54.59	
Investor protection levy	0.0002%	R 2.00			
Marginal transaction cost	**0.0181%**				
Two directional marginal transaction cost	0.0361%				
Total daily cost		**R 1 731.79**			

Note. JSE costs are based on JSE (2014) and on the discussion with Mansura, S. at the JSE (personal communication, May 14, 2014).

[a] Minimum.

[b] Maximum.

4.10 **Comparisons**

4.10.1 Accuracy of price prediction

The accuracy of price prediction was calculated at each time step as the root mean square of the error in predicting the price at the next two time steps. The Wilcoxon signed-rank test can be used to compare paired observations (Hines & Montgomery 1980, p.311). This test was applied to compare prediction accuracy of the different algorithms listed in Table 4 (below).

In addition, the error in predicting the direction of price movement was calculated as a 1 (correct) or -1 (wrong) or 0 (no price change predicted and no price change occurred). As the prediction of direction could yield only 0,1 or -1, the sign test was applied in this case (Hines & Montgomery 1980, p.310).

Table 4. Shares, time steps, prediction methods and error measurements

Share	**Time step**	**Prediction methods**	**Error measure-ments**[a]	**Hypothesis test method**
BTI MTN NPN SBK	1 2 5 10	ARMA Wavelet-ARMA scale 1-6 Wavelet-ARMA scale 2-6 Wavelet-ARMA scale 3-6 Wavelet-ARMA scale 4-6 Wavelet-ARMA scale 5-6 Wavelet-ARMA scale 6 Wavelet regression scale 1-6 Wavelet regression scale 2-6 Wavelet regression scale 3-6 Wavelet regression scale 4-6 Wavelet regression scale 5-6 Wavelet regression scale 6	Root mean square error (t+1) & (t+2) Direction error both	Wilcoxon signed-rank test Sign test (direction error)

Note. [a]Root mean square error =
$[(\text{predicted share price}_{t+1} - \text{share price}_{t+1})^2 + (\text{predicted share price}_{t+2} - \text{share price}_{t+2})^2]^{½}$

Direction error 1 = $\text{sign}(\text{share price}_{t+1} - \text{share price}_{t}) \times \text{sign}(\text{predicted share price}_{t+1} - \text{share price}_{t})$

Direction error 2 = $\text{sign}(\text{share price}_{t+2} - \text{share price}_{t+1}) \times \text{sign}(\text{predicted share price}_{t+2} - \text{predicted share price}_{t+1})$

Direction error both = Direction error 1 AND Direction error 2.

4.10.2 Performance of trade algorithms

Each prediction algorithm was employed with both trading algorithms. These methods were applied at each of the four time intervals, resulting in 13 x 2 x 4 = 104 trading algorithms per share. In order to compare these algorithms with each other and with a buy-and-hold strategy, the daily log return was calculated. The comparison was made using the Wilcoxon signed-rank test and by comparing the cumulative returns graphically (Muller & Ward, 2013). Table 5 (below) contains a list of the prediction and trade methods in the comparison.

Table 5. Shares, time steps, prediction methods and trading algorithms

Share	Time step	Prediction methods	Trading algorithms	Hypothesis test method
BTI MTN NPN SBK	1 2 5 10	ARMA Wavelet-ARMA scale 1-6 Wavelet-ARMA scale 2-6 Wavelet-ARMA scale 3-6 Wavelet-ARMA scale 4-6 Wavelet-ARMA scale 5-6 Wavelet-ARMA scale 6 Wavelet regression scale 1-6 Wavelet regression scale 2-6 Wavelet regression scale 3-6 Wavelet regression scale 4-6 Wavelet regression scale 5-6 Wavelet regression scale 6	Buy-and-hold Buy-and-sell Short-and-close	Wilcoxon signed-rank test

4.11 Exclusions

The scope of research excluded important aspects required to convert the proposed trading algorithm into practice. These include the attributes of shares feasible for this approach, the composition of portfolios and the effect on the price of trading a volume of shares in a short time period.

4.12 Research limitations

This research was focused on intraday trading because there are trends towards algorithmic trading, high frequency trading and low latency trading (Schulmeister, 2009). However, a significant part of the input for the high frequency trading environment is the order book (Cont, 2011 and Scholtus & Van Dijk, 2012). This study ignored that and the outcome of the algorithm may therefore be sub-optimal at best for intraday applications. Furthermore, intraday profits seldom exceed the 0.4% to 0.7% trading costs of non-stock broker traders (Du Plessis, 2013). Therefore, intraday algorithms proposed herein are suitable only in cases of lower trading cost that a stock broker may experience, for instance as calculated in Table 3 on page 26.

The research had limitations regarding validity and reliability. Validity is an indication of how well the data collection methods match the intended purpose and whether the research findings are supported by the research (Saunders & Lewis, 2012, p.127). Reliability refers to the repeatability of the research, or whether the data collection and analysis methods will result in consistent findings (Saunders & Lewis, 2012, p.128).

Validity is limited as the research was done using specific share data from the JSE over a specific time period. It may not be applicable to other time periods or to other shares listed on the JSE or elsewhere. Validity is further limited by sampling the last price of the selected cycle, as this may have hidden some attributes of the trade cycles.

The research is also limited in terms of the parameters chosen. Only two wavelet based algorithms were investigated. Only Haar wavelets were used. Scales were limited to five plus the approximate scale. ARMA orders were limited. Only four time steps were investigated. Only time steps on the minute were investigated. Validity may have been compromised further as no special consideration was given to the possible effect of data snooping.

Reliability may have been compromised if the data base contained errors or omissions. In addition, applying the research to a specific time period and analysing only selected shares may have limited the repeatability of results and therefore the reliability.

Chapter five – Results

Prediction is difficult, especially about the future - Niels Bohr.

5.1 **Introduction**

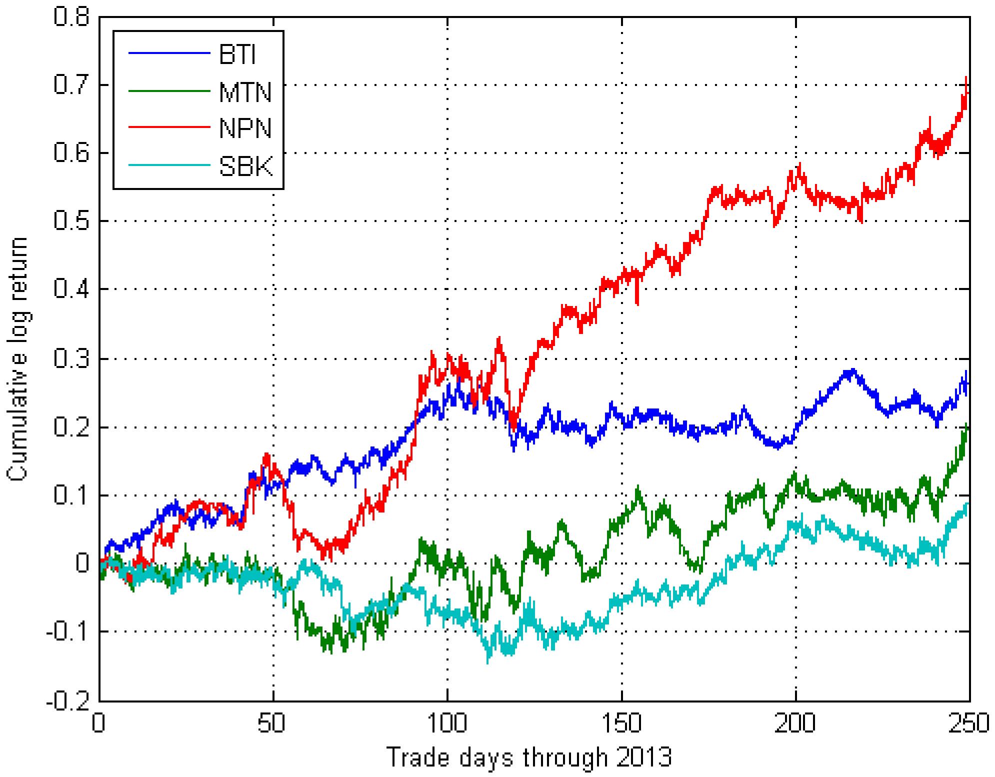
The results include an overview of the sample data, the modelling parameters and the results of the hypothesis tests. To start with, descriptions of the sample share price data are presented. This is followed by the prediction models fitted to each of the algorithms using the training part of the data. The results of the prediction algorithms' comparative tests are shown next based on the minimum error as well as the directional accuracy. Lastly, the results of the tests regarding trading algorithm performances of the different methods are presented.

5.2 **Description of the sample share price data**

The cumulative log returns of MTN, SBK, BTI and NPN through 2013 are shown in Figure 2 on page 31. Table 6 on page 32 contains descriptive statistics pertaining to the complete data set of all the JSE trades of the four shares during 2013. Table 7 on page 32 contains descriptive statistics relating to the log returns and the number of transactions per day.

The analysis was done using time intervals of one, two, five and ten minutes respectively. Descriptive statistics of the number of trades and the log returns per time interval are shown in Table 11 on page 54.

Figure 2. Cumulative log returns of share prices through trade days of 2013.



Note. The trade days commenced on 2 January 2013 and ended on 31 December 2013. It excluded weekends and public holidays, resulting in 250 trade days.

Table 6. Descriptive statistics through period

Share	BTI	MTN	NPN	SBK
Market capitalisation on 31 December 2013 (R billion)[a]	1 040	414	421	205
Shares in issue on 31 December 2013 (million)[a]	943	1 883	385	1 587
Volume traded through 2013 as % of shares in issue	24%	74%	100%	60%
Number of trades through 2013	541 835	1 432 928	1 173 751	904 762
Volume of shares traded through 2013 (million)	225	1 402	386	957
Value traded through 2013 (R million)	113 901	254 866	283 661	110 026
Opening share price 2 Jan 2013	R 430.00	R 179.00	R 549.00	R 119.50
Closing share price 31 Dec 2013	R 559.00	R 217.51	R 1 092.28	R 130.50
Maximum share price 2013	R 572.00	R 219.75	R 1 119.02	R 130.54
Minimum share price 2013	R 427.73	R 156.79	R 533.25	R 103.16

Note. [a]The market capitalisation and the number of shares issued were obtained from ycharts.com. The other statistics were extracted and calculated using the sample share trade data obtained from De Jongh, D. at NWU (personal communication, August 26, 2014).

Table 7. Descriptive statistics per day

Share	BTI	MTN	NPN	SBK
Maximum number of trades	5 863	14 325	11 885	10 988
Average number of trades	2 164	5 722	4 687	3 613
Minimum number of trades	362	1 601	1 380	944
Standard deviation of number of trades	1 038	2 189	1 662	1 507
Maximum log return	2.7%	4.3%	6.0%	4.0%
Average log return	0.10%	0.08%	0.28%	0.04%
Minimum log return	-3.2%	-7.0%	-5.3%	-4.4%
Standard deviation of log return	1.0%	1.7%	1.7%	1.3%

Note. The table contains statistics of the share transactions and price movements per day. The effects of dividends were ignored. Log returns were calculated using the share price at the market closure every trade day at 16h50.

5.3 **Prediction algorithms**

The prediction algorithms were fitted using the first 125 trade days' data. In the cases of the wavelet-ARMA and wavelet regression models, not all the data points were used in fitting the models as the wavelet algorithm used requires a dyadic number (2^n, with n a positive integer) of data points. In this case, the most recent data points (closest to day 126) were used. The number of data points that were used to create the prediction algorithms are shown in Table 8 (below).

Table 8. Number of data points used for fitting prediction algorithms

Time step (minutes)	1	2	5	10
Minutes in trade day	470	235	94	47
Data points through 125 trade days[a]	58 750	29 375	11 750	5 875
Power of 2[b]	15	14	13	12
Dyadic scale[c]	32 768	16 384	8 192	4 096

Note. [a]Number of data points used to fit the ARMA model.
[b]Maximum integer power of two, smaller than the number of data points.
[c]Number of data points used to fit the wavelet-ARMA and wavelet regression models.

5.3.1 ARMA

The log returns of each share and at each time step were ARMA modelled. ARMA models were restricted to four lags of auto regressive and four lags of moving average components. The best models were selected based on the BIC criterion. Table 12 on page 55 contains the number of lags and the coefficients of the lags for the selected models for each share and time step.

5.3.2 Wavelet-ARMA

The log returns of each of the four shares and at each of the four time steps were modelled with six wavelet-ARMA models. The ARMA models were fitted to each of the 6 scale components resulting from the wavelet decomposition process. ARMA models were restricted to two lags of auto regressive and two lags of moving average components each. The best models were selected based on the BIC criterion. The number of lags and the coefficients of the lags for the selected models for each share, time step and scale are shown in Table 13 on page 56 (BTI and MTN) and Table 14 on page 58 (NPN and SBK).

A sample graph with the wavelet components is shown in Figure 3 on page 70. The time duration consists of trade days 55 to 125, comprising of 32 768 (2^{15}) one minute time steps. Figure 4 on page 71 and Figure 5 on page 72 contain the same graph zoomed in to 12 days and 2 days respectively in order to show some detail. The higher volatility of share prices early and late in the trade day (Cont et al., 2014) are clearly visible in these graphs.

5.3.3 Wavelet regression

The wavelet regression model started with the wavelet decomposition and separation of scale components in the same way as was done in the case of the wavelet-ARMA model. A regression was fitted thereafter, linking each scale component shown in Figure 3 on page 70 to the next time step value of the log return. The regression model for the second time step ahead linked the scale components to the log return values at two time steps ahead. The regression coefficients for one and two time steps ahead are shown in Table 15 on page 60. The table also shows the correlation coefficients (R^2 statistics) for the first and second time steps ahead regression models.

5.4 **Prediction accuracy**

The purpose of the accuracy measurements was to determine whether wavelet based share price prediction algorithms improve share price prediction accuracy on an intraday scale (Hypothesis Ha). The three prediction methods (ARMA, Wavelet-ARMA and Wavelet regression) were applied to four shares and at four sampling intervals (one, two, five and ten minutes) as summarised in Table 4 on page 27.

The two wavelet-based algorithms each comprised of six variants. The basic variants each utilised the combination of scales 1 to 6. The second variants used scales 2 to 6 and so on, up to the sixth variants, which used only scale 6.

Prediction accuracies were measured by calculating the root mean square error for one and two time steps ahead and the directional errors for both one and two time steps ahead with the formulas indicated below Table 4 on page 27. These values at each time step of the test data were used in the hypothesis test. The averages of the root mean square values were also calculated and are shown in Table 16 on page 61 for the cases outperforming ARMA.

The hypothesis regarding prediction accuracy was tested with the Wilcoxon signed-rank test while the sign test was applied to test the hypothesis regarding the prediction of direction. In each case, the confidence level was 95% and a one-tailed test was conducted.

5.4.1 Prediction of share price accuracy

The hypothesis regarding prediction accuracy was tested per time step and per share. Table 16 on page 61 contains the list of prediction methods with the root mean square (RMS) of one and two time steps ahead prediction errors lower than the ARMA method according to the Wilcoxon signed-rank test. Both tables contain the means of the RMS errors and for reference, the means of the RMS errors of the ARMA prediction methods.

Table 9. Rank of algorithms with lowest prediction error

		Time step			
Method & scale		**1**	**2**	**5**	**10**
ARMA		2	1	1	1
Wavelet-ARMA	6	1	2	2	2
Wavelet regression	6	3	3	3	3
Wavelet-ARMA	5 to 6	4	4	4	4
Wavelet regression	5 to 6	5	5	5	5

Note. Rank of 1 indicates the method outperforming most others based on the Wilcoxon signed-rank test.

The results per share are combined in Table 9 (above) where the prediction methods' performances are ranked per time step. The ranking was done by performing a Wilcoxon signed-rank test between each of the prediction methods per time step and adding the results.

5.4.2 Prediction of direction accuracy

The hypothesis regarding prediction accuracy was also tested based on the accuracy of the predicted direction of share price movement, per time step and per share. Both time steps were considered simultaneously as this combination impacted directly on the trading algorithm. Table 17 on page 62 contains the list of prediction methods with both the predicted direction errors lower than the ARMA method according to the Sign test. The table also contains the percentages of the directions predicted correctly. The results of ARMA based predictions are included for reference.

The results per share are combined in Table 10 (below) where the directional accuracies of the prediction methods are ranked per time step. The ranking was done by performing a sign test between each of the prediction methods per time step and adding the results.

Table 10. Rank of algorithms performing best in predicting the direction of price changes of the next two time steps

		Time step			
Method & scale		**1**	**2**	**5**	**10**
Wavelet regression	2 to 6	1	1	1	1
Wavelet-ARMA	2 to 6	2	2	2	2
Wavelet-ARMA	3 to 6	3	3	3	3
Wavelet regression	3 to 6	4	4	4	4
Wavelet-ARMA	1 to 6			5	5

Note. Rank of 1 indicates the method outperforming most others based on the sign test.

5.5 **Trading performance**

The trading performances of the algorithms were compared in order to determine whether wavelet based algorithms improve trading performance (Hypothesis Hb). Secondly, intraday trading algorithms were compared with buy-and-hold (Hypothesis Hc). The test time period consisted of trade days 126 to 250.

The three prediction methods (ARMA, Wavelet-ARMA and Wavelet regression) were applied to four shares and at four sampling intervals (one, two, five and ten minutes), and combined with both buy-and-sell and short-and-close trading algorithms. Buy-and-hold was added to the comparison. The methods are summarised in Table 4 on page 27.

The trade algorithms would signal buy or sell (alternatively, short or close), or retain the current position. For each trade the log return was calculated. Trade fees of 0.018% per one direction transaction were incorporated. (The calculation is shown in Table 3 on page 26.) The cumulative log return per day was calculated. This allowed the comparison of the algorithms of all the time steps.

In the case of buy-and-hold, no transaction fees were incorporated since the 0.036% two-way cost was considered as being negligible for the one trade. The buy-and-hold daily returns were calculated using the share prices at the end of the first minute of each trading day.

The hypothesis regarding trading algorithm performance was tested with the Wilcoxon signed-rank test. The confidence level was 95% and a one-tailed test was conducted.

Table 18 on page 68 contains the list of predicting-trading algorithm combinations that performed better than ARMA, buy-and-hold, indicated for the applicable shares. The comparisons were made using the Wilcoxon signed-rank test. Table 19 on page 69 lists the predicting-trading algorithm combinations having positive cumulative returns over the test period, with their respective cumulative log returns per share over the test period.

An alternative approach to compare the performances was also investigated. The cumulative returns of the trading methods were calculated and graphically compared (Muller & Ward, 2013) in Figure 6 on page 73 to Figure 9 on page 76. The performances of the trade algorithms with the highest cumulative returns for each share are shown.

5.6 **Intraday trading algorithms vs. buy-and-hold**

None of the intraday trading algorithms were found to perform better than buy-and-hold according to the Wilcoxon signed-rank test. This result is confirmed in the graphs showing the cumulative log returns (Figure 6 on page 73 to Figure 9 on page 76), where no significant out-performance of the intraday trading algorithms can be observed.

5.7 **Trading cost and number of transactions**

The calculated marginal trading cost was 0.018% in one direction. (The calculation is shown in Table 3 on page 26.) At this rate, two buy-and-sell transactions (four trades) per day (or 500 trades through the 125 day test period) would result in a cumulative return of minus 9.4% over the test period of 125 days (six months). The number of transactions per scale is shown in Figure 10 on page 77. It is clear that the lower the scale (higher frequency), the more transactions the trading algorithm invoked and the higher the cumulative trading cost would be.

The last column of Table 19 on page 69 contains the number of profitable trades/number of trades through the 125 day test period for the cases with a profitable cumulative return. The maximum number of transactions in this table is 518, or 4 transactions per day, supporting the argument that the cost of the high volume of transactions was instrumental in the lack of performance of trade algorithms utilising wavelets lower scales (higher frequencies).

5.8 **Conclusion of the results chapter**

The results of hypotheses tested were as follows:

Ha_0: Wavelet based share price prediction algorithms do not improve share price prediction accuracy on an intraday scale.

- The hypothesis Ha_0 cannot be rejected.

Hb_0: Wavelet based algorithms do not improve the accuracy of predicting the direction of share price movement on an intraday scale.

- The hypothesis Hb_0 can be rejected.

Hc_0: Wavelet based share price prediction algorithms do not improve the performance of intraday trading algorithms.

- The hypothesis Hc_0 can be rejected.

Hd_0: Intraday trade algorithms do not perform better than buy-and-hold.

- The hypothesis Hd_0 cannot be rejected.

Chapter six - Discussion of results

We are looking for the needle in the haystack – Chris Muller.

6.1 Introduction

The main purpose of the research was to determine whether wavelet based prediction algorithms can improve intraday trading algorithms. The fit of algorithms are discussed first. Thereafter the results related to each hypothesis are discussed. This chapter also includes a discussion on the effect of trading costs.

6.2 Algorithm fit

The ARMA, wavelet-ARMA and wavelet regressive models were fitted using the first half year of log returns. Half of the ARMA models fitted contained no autoregressive components (Table 12 on page 55), indicating that randomness overshadowed other effects in these log return series. This resulted in these algorithms having no prediction power beyond a constant mean. This situation occurred in only one of the ARMA model fits to the wavelet components (Table 13 on page 56 and Table 14 on page 58), indicating lower levels of randomness in the decomposed series.

The R^2 (coefficient of determination) values in the case of the wavelet component regression fits to the next values of the log returns, are around 0.5 in cases of one time step ahead and around 0.4 in cases of two time steps ahead (Table 15 on page 60). These R^2 values indicate a relationship, albeit not too strong.

6.3 Accuracy of price prediction

Hypothesis Ha_0: Wavelet based share price prediction algorithms do not improve share price prediction accuracy on an intraday scale.

This null hypothesis could not be rejected as it could not be shown that wavelet-ARMA and wavelet regression algorithms outperformed ARMA in predicting the prices at the next two time steps based on the root mean square (RMS) measurement criteria. However, at a time step of 1 minute, in the cases of BTI and NPN, wavelet-ARMA and wavelet regression methods at scales 6, 5-6 and 4-6 (lower frequencies) yielded lower RMS prediction errors on average than ARMA (Table 16 on page 61).

In this study, the errors in predicting one and two time steps ahead were combined. This was done because the purpose of the prediction was to serve as input for the trading algorithm which used these two variables. This approach of combining errors of time steps was not found in the literature and the results may therefore not be strictly comparable with other studies. Nevertheless, the result obtained in this study is in agreement with the results of Schlüter and Deuschle (2010), who did not find improvements due to wavelets for all financial time series they tested. The results herein however contradict the results of other research that found that wavelet based forecasting improves prediction accuracy (Kisi, 2010; Zhu et. al., 2014; Ortega & Khashanah, 2014).

6.4 **Accuracy of direction prediction**

Hb_0: Wavelet based algorithms do not improve the accuracy of predicting the direction of share price movement on an intraday scale.

This null hypothesis was rejected as the prediction of the direction of share price movement was found to be improved by wavelet based methods. These results are in agreement with the study of Ortega and Khashanah (2014) who found wavelets to improve the directional prediction accuracy.

The list in Table 17 on page 62 indicates that wavelet based methods can outperform ARMA for all the shares tested and at all the time scales. The best performing methods utilise scales 2 to 6 and 3 to 6 (it includes higher frequency scales) as indicated in Table 10 on page 36. This is in contrast with all the other performance tests in this study, where methods with lower frequency scales were found to perform better.

6.5 Trade algorithm performance

Hc_0: Wavelet based share price prediction algorithms do not improve the performance of intraday trading algorithms.

The null hypothesis was rejected in this case as the incorporation of wavelets in the prediction algorithms was found to improve the intraday returns relative to ARMA algorithms in specific cases of BTI, NPN and SBK shares (Table 18 on page 68). In the case of MTN, no improvement was found. Although no results of wavelet based trading algorithms were found in the literature, the results of this study is in agreement with the finding of Schlüter and Deuschle (2010), who found that wavelets can improve the prediction accuracy of financial time series in some cases.

The best performing wavelet-based algorithms in trading had higher scales (lower frequencies). This result is in agreement with the findings of Kisi (2010), who omitted lower scales in prediction algorithms for improved accuracy. Trade costs may have contributed to the results of this study: At lower scales (higher frequencies), the trading algorithm invoked a higher number of trades per day than in cases with higher scales. The associated higher cumulative trading costs penalised the lower scales algorithms.

Almost all the trading algorithms with positive cumulative returns utilised the one minute duration time interval (Figure 6 on page 73 to Figure 9 on page 76). The exception is NPN, where the algorithms operating at 5 and 10 minute time intervals performed better.

All the wavelet based algorithms with positive cumulative returns utilised the short-and-close trading algorithm. This means the trade algorithm was profitable when the prediction algorithm incorrectly predicted price and direction.

6.6 **Buy and hold**

Hd_0: Intraday trade algorithms do not perform better than buy-and-hold.

The null hypothesis in this case could not be rejected as no intraday algorithm was found to statistically outperform buy-and-hold over the test period. Selected trade algorithms did however result in higher cumulative returns than buy-and-hold through the test period (Figure 6 on page 73 to Figure 9 on page 76 and Table 19 on page 69). This result is in agreement with the findings of Duvinage et al. (2013), Yamamoto (2012) and Marshall et al. (2008).

The fact that the algorithms tested in this study did not outperform buy and hold, do not negate the feasibility of intraday trading, as reported in various studies (Schulmeister, 2009; Scholtus & Van Dijk, 2012; Hasbrouck & Saar, 2013; Hagströmer & Nordén, 2013; Menkveld, 2013 and Mackenzie, 2014).

6.7 **The effect of trading cost**

The cost of trading played a significant part in the findings as the cumulative negative return escalated as the number of trades increased. The significance of trading costs was also reported by Schulmeister (2009), Brogaard et al. (2012), Elkins McSherry (2012) and Du Plessis (2013).

6.8 **Conclusion regarding the results**

Intraday wavelet based prediction algorithms were found to improve the performance of trade algorithms in some cases. The intraday algorithms could not improve on a buy-and-hold strategy. A main driver in the performance of the algorithms was trading cost.

Chapter seven – Conclusion

The first principle is that you must not fool yourself and you are the easiest person to fool — Richard Feynman.

7.1 Introduction

This research tested whether wavelet based algorithms can improve the performance of intraday share trading algorithms. The trading algorithms investigated each consisted of two parts: the first part performed share price prediction and the second part traded shares based on the prediction. Three types of prediction algorithms were employed: auto regressive moving average (ARMA), wavelet-ARMA and wavelet regressive algorithms. The wavelet based algorithms were further broken down by using up to 6 different levels of scales in each of the algorithms.

These algorithms were applied to the trade data of four JSE shares through 2013. The first half year of data was used to fit algorithms while the tests were conducted on the second half year of data

7.2 Main findings

The accuracy of the wavelet based share price prediction algorithms were found not to outperform the autoregressive moving average algorithms. This result is in agreement with that of Schlüter and Deuschle (2010), who found improvements in the accuracy of price prediction due to wavelets only for some of the financial time series they tested. The results herein however do not support the findings of other research that found that wavelet based forecasting improves prediction accuracy (Kisi, 2010; Zhu et.al., 2014; Ortega & Khashanah, 2014).

The accuracy of predicting the direction the share price will move is important for trade algorithms. This research found that accuracy of predicting price direction was improved by wavelets, in agreement with the findings of Ortega and Khashanah (2014). Specifically, inclusion of lower scale (higher frequency) wavelet components as part of the prediction algorithms was found to improve directional prediction accuracy.

Returns of the trade algorithm were found to have improved due to the incorporation of wavelets in the prediction algorithms relative to ARMA algorithms in specific cases. In the case of one share (MTN), no improvement was found. No direct comparison with previous research was done as no results of wavelet based trading algorithms were found in the literature. However, this result is in agreement with the Schlüter and Deuschle (2010) finding that wavelets can improve the prediction accuracy of financial time series in some cases.

The best performing wavelet-based algorithms in trading had higher scales (lower frequencies). This observation is in agreement with the findings of Kisi (2010), who omitted lower scales in prediction algorithms for improved accuracy.

None of the intraday algorithms were found to outperform buy-and-hold over the test period. This result is in agreement with the findings of Duvinage et al. (2013), Yamamoto (2012) and Marshall et al. (2008). The active practice of intraday trading, albeit with different algorithms (Schulmeister, 2009; Scholtus & Van Dijk, 2012; Hasbrouck & Saar, 2013; Hagströmer & Nordén, 2013; Menkveld, 2013 and Mackenzie, 2014), however, contradicts the result.

Trade costs have a significant impact on the performance of intraday trades (Menkveld, 2013; Scholtus & Van Dijk, 2012; Du Plessis, 2013). In this study, it was observed that algorithms using lower scales (higher frequencies) invoked a higher number of trades per day than in cases with lower scales. The associated higher cumulative trading costs penalised the lower scales algorithms.

7.3 Recommendations to industry

The growth of computerised trading implies continuing change to the stock market trading industry. Industry should take cognisance of the growth of high frequency trading and the associated algorithmic trading. High frequency trading accounts for more than half the traded shares in the USA, more than 35% in the UK and in Canada (Aitken, Cumming & Zhan, 2014), more than 40% in Germany (Reuters, 2013), while 17% of JSE trades occur via colocation servers (JSE, 2014b) and are therefore likely high frequency trades. The JSE colocation service started on 12 May 2014 (JSE, 2014b). In addition, the increase in high frequency trading is causing changes in regulations (Reuters, 2013).

The options available when fitting models to financial data are boundless. Industry should remain abreast of the modelling techniques and algorithms, like wavelets, used in physical science and engineering.

Lastly, trading costs are expected to continue on a downward trend due to growing opening of markets (globalisation) and increased competition between exchanges (Menkveld, 2013). Lower trading costs make high frequency algorithmic trading increasingly feasible.

7.4 **Suggestions for future research**

Intraday trading and the associated trade algorithms is a relative recent phenomenon driving a significant part of the turnover of stock exchanges (Aitken, Cumming & Zhan, 2014). Limited academic research has been published in this regard (Chordia, Goyal, Lehmann & Saar, 2013). Commercial secrecy issues surrounding trading algorithms could be a compounding factor. Regulation of high frequency- and algorithmic trading is a work in process (Reuters, 2013) towards which academic research can contribute.

The trading cost is a make-or-break factor for algorithmic trading (Menkveld, 2013). A comparison between the trading costs of different stock exchanges and the changes in cost over time will yield valuable information for algorithmic traders.

Wavelets can yield information on the co-movement of assets over time (Aguiar-Conraria & Soares, 2013). A suggested research subject is to analyse the co-movement of assets on an intraday scale with wavelets. Such discoveries can improve prediction accuracies for improved trade algorithms.

A myriad of options exist when applying models to financial time series. An option for further research is to refine the decisions made in this study. Options include the following: Other durations for time steps; exploiting the daily volatility cycle in the share prices (Scholtus & Van Dijk, 2012); using the period average and/or high and/or low prices (Ortega & Khashanah, 2014) instead of the end of period prices in the prediction; changing the limits of the maximum numbers of coefficients in the auto-regressive moving average filters; changing the time scales of the wavelet decompositions; changing the combinations of the scales for prediction purposes; changing the wavelet filter and changing the samples.

A related issue is the trade algorithm. The algorithm applied in this research was straightforward. The effect of a more elaborate algorithm with a slower decision filter can be investigated. In addition, all the wavelet based algorithms with positive cumulative returns utilised the short-and-close trading algorithm in this research. This means the trade algorithm was profitable when the prediction algorithm was wrong regarding the price predictions and directions. The reason for this phenomenon could be investigated.

The best predictors of share price direction were found to be wavelet methods utilising scales 2 to 6 and 3 to 6 (it includes higher frequency scales) as indicated in Table 10 on page 36. This is in contrast with all the other performance tests in this study, where methods with lower frequency scales were found to perform better. The reason for this phenomenon could be further investigated.

7.5 **Concluding remarks**

This research did not find that intraday trading algorithms outperform a buy-and-hold strategy. However, intraday trading is a growing trend. As trading costs come down due to competition between stock exchanges, the scope for algorithmic trading will grow. Wavelet based algorithms can contribute to the performance of trade algorithms.

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Appendix A: Tables with result details

Table 11. Descriptive statistics per sampling time interval

Share	dt[a]	Max trades[b]	Mean trades[c]	Std trades[d]	Max log return[e]	Mean log return[f]	Min log return[g]	Std log return[h]
BTI	1	220	5	8	3.32%	0.0002%	-2.87%	0.08%
MTN	1	270	12	15	6.63%	0.0002%	-6.27%	0.12%
NPN	1	229	10	12	4.70%	0.0006%	-5.12%	0.12%
SBK	1	736	8	11	2.61%	0.0001%	-2.62%	0.10%
BTI	2	251	9	13	3.32%	0.0004%	-2.86%	0.11%
MTN	2	413	24	25	5.96%	0.0003%	-6.02%	0.15%
NPN	2	364	20	21	4.74%	0.0012%	-5.09%	0.14%
SBK	2	745	15	19	2.53%	0.0001%	-2.61%	0.13%
BTI	5	325	23	24	3.28%	0.0011%	-2.86%	0.15%
MTN	5	680	61	52	2.73%	0.0008%	-2.86%	0.20%
NPN	5	682	50	43	4.70%	0.0029%	-5.08%	0.20%
SBK	5	1036	38	37	2.68%	0.0004%	-2.65%	0.18%
BTI	10	444	46	41	3.28%	0.0022%	-2.65%	0.19%
MTN	10	1 258	122	93	2.91%	0.0017%	-2.90%	0.26%
NPN	10	1 351	100	77	4.77%	0.0059%	-4.98%	0.26%
SBK	10	1 253	77	64	3.07%	0.0007%	-2.75%	0.23%

Note. The table contains statistics of the share trades and price movement per time period sampled. The effects of dividends were ignored. Log returns were calculated using the share prices at the end of each period.

[a] Time step (minutes).

[b] Maximum number of trades.

[c] Average number of trades.

[d] Standard deviation of number of trades.

[e] Maximum log return.

[f] Average log return.

[g] Minimum log return.

[h] Standard deviation of log return.

Table 12. ARMA models fitted to log returns

Share	dt[a]	AR[b]	MA[b]	C	AR l1	AR l2	AR l3	AR l4	MA l1	MA l2	MA l3	MA l4
BTI	1	2	3	1.7E-06	0.1	0.37	0	0	-0.27	-0.41	0.05	0
BTI	2	1	1	5.1E-06	0.16	0	0	0	-0.35	0	0	0
BTI	5	1	1	1.2E-05	0.20	0	0	0	-0.34	0	0	0
BTI	10	0	1	3E-05	0	0	0	0	-0.1	0	0	0
MTN	1	0	3	4.1E-07	0	0	0	0	-0.24	-0.03	-0.03	0
MTN	2	3	0	1.1E-06	-0.21	-0.06	-0.04	0	0	0	0	0
MTN	5	0	2	2.1E-06	0	0	0	0	-0.1	-0.03	0	0
MTN	10	1	0	4.4E-06	-0.09	0	0	0	0	0	0	0
NPN	1	4	4	4.3E-06	-0.31	0.15	0.06	0.23	0.09	-0.26	-0.06	-0.23
NPN	2	0	3	10E-06	0	0	0	0	-0.2	-0.03	-0.05	0
NPN	5	0	1	2.5E-05	0	0	0	0	-0.18	0	0	0
NPN	10	0	1	5.1E-05	0	0	0	0	-0.16	0	0	0
SBK	1	1	1	-5.5E-07	0.48	0	0	0	-0.63	0	0	0
SBK	2	1	1	-1.6E-06	0.24	0	0	0	-0.44	0	0	0
SBK	5	0	1	-5.1E-06	0	0	0	0	-0.18	0	0	0
SBK	10	0	1	-1.1E-05	0	0	0	0	-0.13	0	0	0

Note. [a]Time step is in minutes.

[b]The columns indicate the number of AR and MA lags selected for the best fit according to the BIC criterion.

The model equation is as follows:

$$R_t = C + AR\ l1 \times r_{t-1} + AR\ l2 \times r_{t-2} + AR\ l3 \times r_{t-3} + AR\ l4 \times r_{t-4} +$$
$$MA\ l1 \times s_{t-1} + MA\ l2 \times s_{t-2} + MA\ l3 \times s_{t-3} + MA\ l4 \times s_{t-4}$$

With:

R_t = the predicted value of the log return at time step t.

AR l1 = the quotient of the first AR lag.

r_{t-1} = the actual value of the log return at time step t-1 (the previous time step)

s_{t-1} = the value of a random input at time step t-1.

Table 13. Wavelet-ARMA models fitted to BTI and MTN log returns

Share	BTI	BTI	BTI	BTI	MTN	MTN	MTN	MTN
Time step[a]	**1**	**2**	**5**	**10**	**1**	**2**	**5**	**10**
Scale[b]	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
AR lags[c]	2	2	2	0	1	2	1	1
MA lags[c]	2	2	2	2	2	0	2	2
Constant	-4E-10	-7.1E-09	-3.5E-08	5.29E-23	3.46E-09	-4.4E-09	3.81E-10	-7.5E-07
AR lag 1	-0.16	-0.103	-0.171	0	-0.160	-1.095	0.051	-0.084
AR lag 2	-0.04	0.014	-0.040	0	0	-0.522	0	0
MA lag 1	-1.45	-1.452	-1.801	-1.504	-1.520	0	-1.455	-1.823
MA lag 2	0.540	0.577	0.818	0.667	0.609	0	0.657	0.828
Scale[b]	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	1	1	1	1	1	1	1	1
Constant	-1E-08	-2E-08	-3.3E-08	-1.7E-07	-1.3E-08	-3.8E-09	-2.1E-08	-6.5E-08
AR lag 1	0.308	0.314	0.304	0.314	0.295	0.293	0.333	0.312
AR lag 2	-0.725	-0.720	-0.723	-0.722	-0.744	-0.729	-0.719	-0.710
MA lag 1	0.281	0.286	0.304	0.288	0.252	0.268	0.290	0.321
MA lag 2	0	0	0	0	0	0	0	0
Scale	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	2	2	2	2	0	2	2	2
Constant	-4E-09	6.32E-09	-2.9E-10	4.7E-08	1.5E-09	1.04E-10	-3E-09	-1E-08
AR lag 1	1.55	1.55	1.54	1.55	1.18	1.57	1.52	1.53
AR lag 2	-0.89	-0.892	-0.885	-0.884	-0.599	-0.894	-0.873	-0.881
MA lag 1	-0.63	-0.609	-0.557	-0.565	0.000	-0.706	-0.475	-0.464
MA lag 2	0.009	0.032	0.021	0.021	0	0.012	0.010	0.037
Scale[b]	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	2	2	2	2	2	2	2	2
Constant	-2E-10	-3.3E-09	3.69E-09	1.26E-08	-8.4E-10	1.82E-09	-8.5E-09	-6.6E-10
AR lag 1	1.841	1.826	1.830	1.839	1.826	1.834	1.841	1.840
AR lag 2	-0.94	-0.923	-0.923	-0.931	-0.920	-0.928	-0.933	-0.930
MA lag 1	-0.46	-0.437	-0.399	-0.352	-0.496	-0.494	-0.371	-0.367
MA lag 2	-0.13	-0.071	-0.114	-0.159	-0.084	-0.080	-0.151	-0.118
Scale[b]	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	2	2	2	2	2	2	2	2
Constant	-6E-10	1.35E-09	1.83E-09	-2.4E-09	-5.6E-11	3.25E-10	1.57E-09	1.21E-08
AR lag 1	1.903	1.900	1.910	1.913	1.896	1.897	1.907	1.923
AR lag 2	-0.93	-0.927	-0.937	-0.939	-0.924	-0.923	-0.933	-0.951

Share	BTI	BTI	BTI	BTI	MTN	MTN	MTN	MTN
Time step[a]	**1**	**2**	**5**	**10**	**1**	**2**	**5**	**10**
MA lag 1	-0.27	-0.273	-0.238	-0.232	-0.345	-0.326	-0.217	-0.234
MA lag 2	-0.1	-0.077	-0.075	-0.092	-0.037	-0.029	-0.095	-0.064
Scale[b]	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**6**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	2	1	2	0	2	1	0	0
Constant	4.4E-09	8.72E-09	3.32E-08	7.77E-08	4.11E-09	8.66E-09	1.87E-08	3.15E-08
AR lag 1	1.943	1.946	1.953	1.947	1.947	1.950	1.942	1.938
AR lag 2	-0.95	-0.949	-0.956	-0.950	-0.950	-0.953	-0.946	-0.941
MA lag 1	-0.17	-0.167	-0.155	0	-0.277	-0.242	0	0
MA lag 2	-0.04	0	-0.034	0	-0.026	0	0	0

Note. [a]Time step is in minutes.

[b]The lowest scale number designates the highest frequency component. For instance, in the case of the 1 minute time step, scale 1 refers to the cycles of duration 1 to 2 minutes.

[c]The rows indicate the number of AR and MA lags selected for the best fit according to the BIC criterion.

The model equation is as follows:

R^s_t = Constant + ARlag1 x r^s_{t-1} + ARlag2 x r^s_{t-2} + MAlag1 x s^s_{t-1} + MAlag2 x s^s_{t-2} .

With:

R^s_t = the predicted value of the relevant scale component log return at time step t.

R^s_{t-1} = the actual value of the scale component of the log return at time step t-1 (the previous time step)

S^s_{t-1} = the value of a random input at time step t-1.

The predicted return at time step t is the sum of the selected scale component predicted values. For instance, in the case of the wavelet-ARMA scale 3-6 algorithm,

$R_t = R^3_t + R^4_t + R^5_t + R^6_t$.

Table 14. Wavelet-ARMA models fitted to NPN and SBK log returns

Share	NPN	NPN	NPN	NPN	SBK	SBK	SBK	SBK
Time step[a]	**1**	**2**	**5**	**10**	**1**	**2**	**5**	**10**
Scale[b]	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
AR lags[c]	1	2	2	2	1	1	2	2
MA lags[c]	2	0	2	2	2	2	2	2
Constant	1.4E-09	-3.3E-08	1.35E-07	3.03E-08	2.39E-10	5.19E-10	-1.4E-09	-3.1E-07
AR lag 1	-0.060	-1.088	-0.250	-0.147	-0.004	-0.040	-0.252	-0.150
AR lag 2	0	-0.536	-0.059	-0.010	0	0	-0.054	-0.115
MA lag 1	-1.507	0	-1.894	-1.428	-1.506	-1.387	-1.356	-1.921
MA lag 2	0.668	0	0.900	0.541	0.636	0.651	0.434	0.926
Scale[b]	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	1	1	1	1	1	1	1	1
Constant	7.5E-09	1.26E-08	2.55E-08	-6.7E-08	-2.7E-09	5.75E-09	4.29E-08	-1.1E-07
AR lag 1	0.297	0.283	0.290	0.318	0.338	0.310	0.346	0.343
AR lag 2	-0.733	-0.722	-0.717	-0.721	-0.732	-0.727	-0.715	-0.728
MA lag 1	0.254	0.277	0.284	0.279	0.263	0.270	0.252	0.248
MA lag 2	0	0	0	0	0	0	0	0
Scale	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	1	2	1	2	1	2	1	2
Constant	2.4E-09	1.91E-08	-1E-08	2.76E-08	-6.7E-09	3.23E-09	-6.8E-09	-9E-08
AR lag 1	1.566	1.565	1.572	1.561	1.535	1.566	1.556	1.547
AR lag 2	-0.893	-0.895	-0.896	-0.891	-0.885	-0.891	-0.893	-0.877
MA lag 1	-0.738	-0.707	-0.691	-0.628	-0.566	-0.685	-0.649	-0.564
MA lag 2	0	0.024	0	0.024	0	0.005	0	0.017
Scale[b]	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	2	2	2	2	2	2	2	2
Constant	3.2E-09	2.5E-09	3.31E-09	-5.8E-09	-1.2E-09	-4.6E-09	-1E-08	-2.3E-08
AR lag 1	1.839	1.847	1.849	1.827	1.837	1.824	1.826	1.854
AR lag 2	-0.931	-0.938	-0.939	-0.923	-0.928	-0.923	-0.918	-0.945
MA lag 1	-0.494	-0.524	-0.467	-0.403	-0.414	-0.440	-0.409	-0.400
MA lag 2	-0.143	-0.110	-0.130	-0.112	-0.195	-0.094	-0.118	-0.219
Scale[b]	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	2	2	2	2	2	2	2	2
Constant	6.5E-10	1.09E-09	7.48E-10	-2.9E-10	-1E-09	-2.2E-09	-1.1E-09	-6.8E-09
AR lag 1	1.908	1.902	1.904	1.907	1.898	1.893	1.898	1.885
AR lag 2	-0.935	-0.929	-0.932	-0.932	-0.927	-0.921	-0.926	-0.914

Share	NPN	NPN	NPN	NPN	SBK	SBK	SBK	SBK
Time step[a]	**1**	**2**	**5**	**10**	**1**	**2**	**5**	**10**
MA lag 1	-0.339	-0.322	-0.300	-0.278	-0.235	-0.300	-0.263	-0.205
MA lag 2	-0.101	-0.054	-0.029	-0.037	-0.123	-0.050	-0.013	-0.041
Scale[b]	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**6**
AR lags[c]	2	2	2	2	2	2	2	2
MA lags[c]	2	1	1	2	2	2	0	0
Constant	2E-08	3.85E-08	7.68E-08	1.4E-07	-4E-09	-7.8E-09	-1.9E-08	-4E-08
AR lag 1	1.946	1.946	1.952	1.945	1.939	1.946	1.912	1.920
AR lag 2	-0.949	-0.949	-0.954	-0.947	-0.942	-0.949	-0.916	-0.923
MA lag 1	-0.306	-0.276	-0.180	-0.197	-0.151	-0.208	0	0
MA lag 2	-0.032	0	0	0.038	-0.074	-0.036	0	0

Note. [a]Time step is in minutes.

[b]The lowest scale number designates the highest frequency component. For instance, in the case of the 1 minute time step, scale 1 refers to the cycles of duration 1 to 2 minutes.

[c]The rows indicate the number of AR and MA lags selected for the best fit according to the BIC criterion.

The model equation is as follows:

R^s_t = Constant + ARlag1 x r^s_{t-1} + ARlag2 x r^s_{t-2} + MAlag1 x s^s_{t-1} + MAlag2 x s^s_{t-2} .

With:

R^s_t = the predicted value of the relevant scale component log return at time step t.

R^s_{t-1} = the actual value of the scale component of the log return at time step t-1 (the previous time step)

S^s_{t-1} = the value of a random input at time step t-1.

The predicted return at time step t is the sum of the selected scale component predicted values. For instance, in the case of the wavelet-ARMA scale 3-6 algorithm,

$R_t = R^3_t + R^4_t + R^5_t + R^6_t$.

Table 15. Wavelet regression coefficients fitted to log returns

Share	BTI	BTI	BTI	BTI	MTN	MTN	MTN	MTN
Time step	1	2	5	10	1	2	5	10
a1	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
a2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
a3	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000
a4	0.999	1.000	1.000	0.998	1.000	1.000	1.000	1.003
a5	1.001	1.000	1.000	1.002	1.000	1.000	0.998	0.999
a6	1.000	1.000	1.000	0.999	1.000	1.000	1.001	0.998
R^2a	**0.479**	**0.497**	**0.486**	**0.489**	**0.490**	**0.502**	**0.480**	**0.504**
b1	0.428	0.454	0.441	0.422	0.462	0.485	0.412	0.453
b2	-2.238	-2.258	-2.249	-2.220	-2.275	-2.312	-2.202	-2.256
b3	1.802	1.798	1.816	1.785	1.766	1.809	1.821	1.815
b4	0.763	0.774	0.761	0.778	0.819	0.781	0.734	0.741
b5	1.076	1.065	1.063	1.059	1.060	1.069	1.082	1.087
b6	0.993	0.994	0.996	0.993	0.994	0.994	0.992	0.992
R^2b	**0.393**	**0.396**	**0.409**	**0.409**	**0.394**	**0.406**	**0.404**	**0.417**
Share	**NPN**	**NPN**	**NPN**	**NPN**	**SBK**	**SBK**	**SBK**	**SBK**
Time step	**1**	**2**	**5**	**10**	**1**	**2**	**5**	**10**
a1	-0.998	-0.996	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
a2	0.998	0.994	1.000	1.000	1.000	1.000	1.000	1.000
a3	0.997	0.989	1.000	0.999	1.001	1.000	1.000	1.000
a4	1.001	1.002	1.000	0.999	0.999	1.000	1.000	1.000
a5	0.991	1.005	1.000	0.999	1.000	1.000	1.001	1.000
a6	0.994	0.998	1.000	1.000	1.000	1.000	0.999	0.998
R^2a	**0.476**	**0.491**	**0.497**	**0.502**	**0.464**	**0.484**	**0.496**	**0.480**
b1	0.439	0.455	0.450	0.448	0.418	0.436	0.440	0.408
b2	-2.260	-2.293	-2.286	-2.247	-2.198	-2.243	-2.229	-2.186
b3	1.791	1.815	1.803	1.787	1.797	1.790	1.788	1.777
b4	0.793	0.758	0.768	0.767	0.757	0.775	0.778	0.770
b5	1.057	1.084	1.079	1.071	1.080	1.076	1.072	1.084
b6	0.991	0.992	0.993	0.995	0.992	0.990	0.986	0.978
R^2b	**0.391**	**0.395**	**0.399**	**0.399**	**0.391**	**0.388**	**0.380**	**0.390**

Note. Time step is in minutes.

Rows a1 to a6 contains the coefficients relating scales number 1 to 6 in the regression algorithm to the prediction of the log return, one time step ahead.

Row R^2a contains the R^2 value of the regression fit, one time step ahead.

Rows b1 to b6 contains the coefficients relating scales number 1 to 6 in the regression algorithm to the prediction of the log return, two time steps ahead.

Row R^2b contains the R^2 value of the regression fit, two time steps ahead.

Table 16. Wavelet algorithms with mean RMS prediction error for one and two time steps ahead smaller than ARMA

Share	Predict method	Scales	Time step	Mean RMS
BTI	Wavelet-ARMA	6	1	16.9
BTI	Wavelet regression	6	1	16.9
BTI	Wavelet-ARMA	5 to 6	1	17.9
BTI	Wavelet regression	5 to 6	1	18.2
BTI	Wavelet-ARMA	4 to 6	1	20.2
BTI	*ARMA*	-	*1*	*21.8*
BTI	*ARMA*	-	*2*	*22.6*
BTI	*ARMA*	-	*5*	*33.8*
BTI	*ARMA*	-	*10*	*46.0*
MTN	*ARMA*	-	*1*	*8.6*
MTN	*ARMA*	-	*2*	*11.5*
MTN	*ARMA*	-	*5*	*16.2*
MTN	*ARMA*	-	*10*	*21.4*
NPN	Wavelet-ARMA	6	1	39.3
NPN	Wavelet regression	6	1	39.3
NPN	Wavelet-ARMA	5 to 6	1	41.5
NPN	Wavelet regression	5 to 6	1	42.0
NPN	Wavelet-ARMA	4 to 6	1	46.4
NPN	*ARMA*	-	*1*	*46.5*
NPN	*ARMA*	-	*2*	*49.9*
NPN	*ARMA*	-	*5*	*71.5*
NPN	*ARMA*	-	*10*	*93.4*
SBK	*ARMA*	-	*1*	*4.2*
SBK	*ARMA*	-	*2*	*6.1*
SBK	*ARMA*	-	*5*	*9.1*
SBK	*ARMA*	-	*10*	*12.0*

Note. [a]Root mean square error =

$[(\text{predicted share price}_{t+1} - \text{share price}_{t+1})^2 + (\text{predicted share price}_{t+2} - \text{share price}_{t+2})^2]^{½}$,

with t+1 referring to the next time step and t+2 to two time steps ahead.

Table 17. Wavelet algorithms predicting the price change direction of the next two steps better than ARMA

Share	Predict method	Scales	Time step	Dir1 % right	Dir2 % right	Dir Both % right
BTI	Wavelet regression	2 to 6	1	33.3%	27.2%	17.6%
MTN	Wavelet regression	2 to 6	1	36.7%	32.6%	25.3%
NPN	Wavelet regression	2 to 6	1	37.0%	32.9%	25.5%
SBK	Wavelet regression	2 to 6	1	33.8%	27.6%	17.9%
BTI	Wavelet-ARMA	2 to 6	1	30.4%	25.0%	15.3%
MTN	Wavelet-ARMA	2 to 6	1	33.7%	30.3%	22.9%
NPN	Wavelet-ARMA	2 to 6	1	33.5%	30.2%	22.6%
SBK	Wavelet-ARMA	2 to 6	1	30.9%	25.6%	15.6%
BTI	Wavelet-ARMA	3 to 6	1	27.8%	22.9%	13.3%
MTN	Wavelet-ARMA	3 to 6	1	31.3%	27.8%	20.6%
NPN	Wavelet-ARMA	3 to 6	1	29.6%	26.6%	19.1%
SBK	Wavelet-ARMA	3 to 6	1	28.3%	23.6%	13.7%
BTI	Wavelet regression	3 to 6	1	27.0%	22.1%	12.7%
MTN	Wavelet regression	3 to 6	1	30.4%	27.3%	20.1%
NPN	Wavelet regression	3 to 6	1	28.6%	25.3%	18.2%
SBK	Wavelet regression	3 to 6	1	27.6%	22.6%	13.3%
BTI	*ARMA*	-	*1*	*26.2%*	*21.4%*	*12.2%*
MTN	Wavelet-ARMA	4 to 6	1	28.1%	25.5%	18.5%
NPN	Wavelet-ARMA	4 to 6	1	26.6%	24.1%	17.0%
SBK	Wavelet-ARMA	4 to 6	1	25.2%	21.3%	12.0%
MTN	Wavelet regression	4 to 6	1	27.6%	25.2%	18.3%
NPN	Wavelet regression	4 to 6	1	26.0%	23.4%	16.5%
SBK	Wavelet regression	4 to 6	1	24.7%	21.1%	11.9%
NPN	Wavelet-ARMA	5 to 6	1	25.0%	22.9%	16.0%
MTN	Wavelet regression	5 to 6	1	26.0%	24.2%	17.2%
NPN	Wavelet regression	5 to 6	1	24.6%	22.6%	15.7%
SBK	Wavelet regression	5 to 6	1	23.1%	20.3%	11.1%
MTN	Wavelet-ARMA	5 to 6	1	26.0%	24.1%	17.2%
SBK	Wavelet-ARMA	5 to 6	1	23.2%	20.1%	11.0%
MTN	Wavelet-ARMA	6	1	24.3%	22.8%	16.0%
MTN	Wavelet regression	6	1	24.2%	22.8%	15.9%
NPN	Wavelet regression	6	1	23.2%	22.0%	15.0%
SBK	Wavelet regression	6	1	21.3%	19.4%	10.1%
NPN	Wavelet-ARMA	6	1	23.2%	22.0%	15.0%
SBK	Wavelet-ARMA	6	1	21.3%	19.4%	10.1%
MTN	Wavelet-ARMA	1 to 6	1	20.9%	20.9%	13.9%
NPN	Wavelet-ARMA	1 to 6	1	20.0%	20.6%	13.4%

Share	Predict method	Scales	Time step	Dir1 % right	Dir2 % right	Dir Both % right
SBK	Wavelet-ARMA	1 to 6	1	17.8%	18.5%	8.8%
NPN	*ARMA*	*-*	*1*	*8.4%*	*12.9%*	*5.7%*
MTN	Wavelet regression	1 to 6	1	8.1%	12.4%	5.5%
SBK	Wavelet regression	1 to 6	1	5.9%	11.6%	3.0%
MTN	*ARMA*	*-*	*1*	*5.6%*	*12.2%*	*4.1%*
SBK	*ARMA*	*-*	*1*	*0.0%*	*9.8%*	*0.0%*
BTI	Wavelet regression	2 to 6	2	38.1%	34.1%	27.2%
MTN	Wavelet regression	2 to 6	2	39.4%	37.3%	33.0%
NPN	Wavelet regression	2 to 6	2	39.4%	37.4%	32.9%
SBK	Wavelet regression	2 to 6	2	38.2%	34.1%	26.6%
BTI	Wavelet-ARMA	2 to 6	2	33.3%	29.9%	23.0%
MTN	Wavelet-ARMA	2 to 6	2	34.8%	33.1%	28.7%
NPN	Wavelet-ARMA	2 to 6	2	34.7%	32.9%	28.4%
SBK	Wavelet-ARMA	2 to 6	2	33.8%	30.2%	22.6%
BTI	Wavelet-ARMA	3 to 6	2	30.3%	27.1%	20.4%
MTN	Wavelet-ARMA	3 to 6	2	30.9%	29.4%	25.2%
NPN	Wavelet-ARMA	3 to 6	2	30.5%	28.9%	24.4%
SBK	Wavelet-ARMA	3 to 6	2	30.4%	27.2%	19.7%
BTI	Wavelet regression	3 to 6	2	29.7%	26.7%	19.9%
MTN	Wavelet regression	3 to 6	2	29.9%	28.3%	24.3%
NPN	Wavelet regression	3 to 6	2	29.2%	27.4%	23.2%
SBK	Wavelet regression	3 to 6	2	29.3%	26.2%	19.0%
MTN	Wavelet-ARMA	4 to 6	2	28.2%	26.9%	22.9%
NPN	Wavelet-ARMA	4 to 6	2	27.3%	25.7%	21.6%
SBK	Wavelet-ARMA	4 to 6	2	27.2%	24.8%	17.6%
BTI	Wavelet regression	4 to 6	2	27.2%	24.7%	18.2%
MTN	Wavelet regression	4 to 6	2	27.4%	26.1%	22.2%
NPN	Wavelet regression	4 to 6	2	26.5%	25.1%	21.0%
SBK	Wavelet regression	4 to 6	2	26.5%	24.2%	17.0%
BTI	Wavelet-ARMA	4 to 6	2	27.5%	24.9%	18.4%
BTI	Wavelet regression	5 to 6	2	25.6%	23.6%	17.1%
MTN	Wavelet regression	5 to 6	2	25.9%	25.1%	21.1%
NPN	Wavelet regression	5 to 6	2	25.5%	24.3%	20.3%
SBK	Wavelet regression	5 to 6	2	25.2%	23.5%	16.4%
BTI	Wavelet-ARMA	5 to 6	2	25.4%	23.4%	17.0%
MTN	Wavelet-ARMA	5 to 6	2	26.1%	25.1%	21.0%
NPN	Wavelet-ARMA	5 to 6	2	25.6%	24.4%	20.3%
SBK	Wavelet-ARMA	5 to 6	2	25.2%	23.5%	16.4%
MTN	Wavelet-ARMA	6	2	24.6%	24.0%	19.8%
BTI	Wavelet regression	6	2	23.4%	22.2%	15.6%

Share	Predict method	Scales	Time step	Dir1 % right	Dir2 % right	Dir Both % right
MTN	Wavelet regression	6	2	24.5%	23.9%	19.7%
NPN	Wavelet regression	6	2	24.1%	23.4%	19.2%
SBK	Wavelet regression	6	2	23.5%	22.2%	15.2%
BTI	Wavelet-ARMA	6	2	23.5%	22.2%	15.6%
NPN	Wavelet-ARMA	6	2	24.1%	23.5%	19.2%
SBK	Wavelet-ARMA	6	2	23.4%	22.2%	15.1%
BTI	Wavelet-ARMA	1 to 6	2	20.5%	20.8%	14.1%
MTN	Wavelet-ARMA	1 to 6	2	19.0%	19.7%	15.4%
NPN	Wavelet-ARMA	1 to 6	2	19.1%	19.6%	15.3%
SBK	Wavelet-ARMA	1 to 6	2	19.7%	20.3%	13.0%
BTI	Wavelet regression	1 to 6	2	8.2%	12.6%	5.9%
MTN	Wavelet regression	1 to 6	2	9.4%	12.3%	7.8%
NPN	Wavelet regression	1 to 6	2	9.2%	12.0%	7.3%
SBK	Wavelet regression	1 to 6	2	7.7%	12.5%	5.2%
BTI	*ARMA*	-	*2*	*4.8%*	*10.9%*	*3.5%*
MTN	*ARMA*	-	*2*	*5.5%*	*8.9%*	*4.5%*
NPN	*ARMA*	-	*2*	*8.5%*	*12.0%*	*6.9%*
SBK	*ARMA*	-	*2*	*4.7%*	*11.3%*	*3.3%*
BTI	Wavelet regression	2 to 6	5	39.1%	38.0%	35.3%
MTN	Wavelet regression	2 to 6	5	40.7%	39.9%	38.0%
SBK	Wavelet regression	2 to 6	5	41.1%	39.5%	36.1%
NPN	Wavelet-ARMA	1 to 6	5	49.8%	48.5%	46.9%
NPN	Wavelet regression	2 to 6	5	41.6%	40.9%	39.3%
BTI	Wavelet-ARMA	2 to 6	5	35.6%	34.5%	31.9%
MTN	Wavelet-ARMA	2 to 6	5	34.2%	33.8%	31.8%
SBK	Wavelet-ARMA	2 to 6	5	35.3%	33.9%	30.2%
NPN	Wavelet-ARMA	2 to 6	5	34.3%	33.8%	32.1%
BTI	Wavelet-ARMA	3 to 6	5	32.2%	31.1%	28.6%
MTN	Wavelet-ARMA	3 to 6	5	30.2%	29.8%	27.8%
SBK	Wavelet-ARMA	3 to 6	5	30.2%	29.1%	25.5%
BTI	Wavelet regression	3 to 6	5	31.5%	30.6%	27.9%
MTN	Wavelet regression	3 to 6	5	28.6%	28.1%	26.2%
SBK	Wavelet regression	3 to 6	5	28.9%	27.9%	24.3%
NPN	Wavelet-ARMA	3 to 6	5	29.4%	28.8%	27.4%
NPN	Wavelet regression	3 to 6	5	28.1%	27.6%	26.1%
MTN	Wavelet-ARMA	4 to 6	5	27.6%	27.4%	25.4%
SBK	Wavelet-ARMA	4 to 6	5	28.2%	27.3%	23.7%
BTI	Wavelet regression	4 to 6	5	29.1%	28.2%	25.8%
MTN	Wavelet regression	4 to 6	5	26.5%	26.3%	24.3%
SBK	Wavelet regression	4 to 6	5	27.0%	26.2%	22.7%

Share	Predict method	Scales	Time step	Dir1 % right	Dir2 % right	Dir Both % right
BTI	Wavelet-ARMA	4 to 6	5	29.5%	28.7%	26.2%
NPN	Wavelet-ARMA	4 to 6	5	26.8%	26.3%	24.8%
NPN	Wavelet regression	4 to 6	5	25.9%	25.4%	23.9%
BTI	Wavelet regression	5 to 6	5	27.2%	26.7%	24.2%
MTN	Wavelet regression	5 to 6	5	25.2%	25.1%	23.2%
SBK	Wavelet regression	5 to 6	5	25.6%	25.1%	21.4%
BTI	Wavelet-ARMA	5 to 6	5	27.3%	26.7%	24.2%
MTN	Wavelet-ARMA	5 to 6	5	25.3%	25.2%	23.3%
SBK	Wavelet-ARMA	5 to 6	5	25.5%	24.9%	21.3%
NPN	Wavelet regression	5 to 6	5	24.4%	23.9%	22.5%
NPN	Wavelet-ARMA	5 to 6	5	24.8%	24.5%	23.0%
MTN	Wavelet regression	6	5	23.6%	23.5%	21.6%
MTN	Wavelet-ARMA	6	5	23.6%	23.5%	21.6%
BTI	Wavelet regression	6	5	25.2%	24.8%	22.3%
NPN	Wavelet regression	6	5	23.0%	22.8%	21.3%
SBK	Wavelet regression	6	5	23.7%	23.4%	19.8%
BTI	Wavelet-ARMA	1 to 6	5	25.4%	25.4%	22.8%
MTN	Wavelet-ARMA	1 to 6	5	22.3%	22.6%	20.7%
SBK	Wavelet-ARMA	1 to 6	5	22.7%	23.0%	19.4%
BTI	Wavelet-ARMA	6	5	25.2%	24.8%	22.3%
NPN	Wavelet-ARMA	6	5	23.1%	22.8%	21.3%
SBK	Wavelet-ARMA	6	5	23.7%	23.4%	19.7%
BTI	Wavelet regression	1 to 6	5	9.5%	11.3%	8.7%
MTN	Wavelet regression	1 to 6	5	10.1%	11.0%	9.2%
NPN	Wavelet regression	1 to 6	5	10.3%	11.4%	9.6%
SBK	Wavelet regression	1 to 6	5	9.4%	11.6%	8.2%
BTI	*ARMA*	-	*5*	*5.7%*	*7.8%*	*5.1%*
MTN	*ARMA*	-	*5*	*5.9%*	*7.1%*	*5.4%*
NPN	*ARMA*	-	*5*	*5.3%*	*6.3%*	*4.9%*
SBK	*ARMA*	-	*5*	*2.6%*	*5.6%*	*2.1%*
BTI	Wavelet regression	2 to 6	10	38.6%	38.1%	36.9%
NPN	Wavelet regression	2 to 6	10	39.8%	39.5%	38.4%
MTN	Wavelet-ARMA	1 to 6	10	47.7%	46.9%	45.5%
SBK	Wavelet-ARMA	1 to 6	10	46.9%	45.5%	43.1%
MTN	Wavelet regression	2 to 6	10	40.0%	39.4%	38.1%
SBK	Wavelet regression	2 to 6	10	41.1%	40.2%	37.9%
BTI	Wavelet-ARMA	2 to 6	10	35.1%	34.5%	33.4%
NPN	Wavelet-ARMA	2 to 6	10	35.7%	35.3%	34.2%
MTN	Wavelet-ARMA	2 to 6	10	34.6%	34.2%	32.9%
SBK	Wavelet-ARMA	2 to 6	10	35.0%	33.9%	31.6%

Share	Predict method	Scales	Time step	Dir1 % right	Dir2 % right	Dir Both % right
BTI	Wavelet-ARMA	3 to 6	10	32.7%	32.0%	31.1%
NPN	Wavelet regression	3 to 6	10	30.0%	29.8%	28.7%
MTN	Wavelet-ARMA	3 to 6	10	31.6%	31.2%	29.8%
NPN	Wavelet-ARMA	3 to 6	10	30.4%	30.1%	29.1%
SBK	Wavelet-ARMA	3 to 6	10	31.5%	30.4%	28.1%
BTI	Wavelet regression	3 to 6	10	30.7%	30.3%	29.3%
BTI	Wavelet-ARMA	4 to 6	10	30.3%	29.8%	28.8%
MTN	Wavelet regression	3 to 6	10	29.5%	29.2%	27.8%
SBK	Wavelet regression	3 to 6	10	29.3%	28.5%	26.1%
BTI	Wavelet regression	4 to 6	10	29.2%	28.9%	27.9%
NPN	Wavelet regression	4 to 6	10	27.5%	27.0%	26.3%
MTN	Wavelet-ARMA	4 to 6	10	28.7%	28.4%	27.1%
NPN	Wavelet-ARMA	4 to 6	10	28.1%	27.6%	26.8%
SBK	Wavelet-ARMA	4 to 6	10	28.5%	27.4%	25.3%
MTN	Wavelet regression	4 to 6	10	28.1%	27.7%	26.4%
SBK	Wavelet regression	4 to 6	10	27.3%	26.6%	24.4%
BTI	Wavelet regression	5 to 6	10	27.5%	27.1%	26.2%
NPN	Wavelet regression	5 to 6	10	25.8%	25.3%	24.5%
BTI	Wavelet-ARMA	5 to 6	10	27.8%	27.4%	26.5%
NPN	Wavelet-ARMA	5 to 6	10	26.3%	25.9%	25.0%
MTN	Wavelet regression	5 to 6	10	26.9%	26.7%	25.4%
MTN	Wavelet-ARMA	5 to 6	10	26.6%	26.6%	25.2%
SBK	Wavelet-ARMA	5 to 6	10	26.5%	25.5%	23.5%
BTI	Wavelet regression	6	10	25.4%	25.0%	24.1%
BTI	Wavelet-ARMA	6	10	25.5%	25.0%	24.1%
SBK	Wavelet regression	5 to 6	10	25.9%	25.5%	23.3%
MTN	Wavelet regression	6	10	24.8%	24.7%	23.5%
NPN	Wavelet regression	6	10	23.8%	23.8%	22.8%
SBK	Wavelet regression	6	10	25.0%	24.6%	22.4%
BTI	Wavelet-ARMA	1 to 6	10	23.0%	23.0%	21.8%
NPN	Wavelet-ARMA	1 to 6	10	22.9%	22.9%	21.9%
MTN	Wavelet-ARMA	6	10	24.7%	24.7%	23.4%
NPN	Wavelet-ARMA	6	10	23.8%	23.8%	22.8%
SBK	Wavelet-ARMA	6	10	25.0%	24.6%	22.4%
BTI	Wavelet regression	1 to 6	10	9.9%	10.7%	9.5%
MTN	Wavelet regression	1 to 6	10	10.4%	11.0%	9.8%
NPN	Wavelet regression	1 to 6	10	10.7%	11.3%	10.2%
SBK	Wavelet regression	1 to 6	10	11.2%	12.5%	10.1%
BTI	*ARMA*	-	*10*	*5.4%*	*6.1%*	*5.2%*
NPN	*ARMA*	-	*10*	*6.2%*	*6.9%*	*6.0%*

Share	Predict method	Scales	Time step	Dir1 % right	Dir2 % right	Dir Both % right
SBK	*ARMA*	-	*10*	*1.8%*	*3.8%*	*1.6%*
MTN	*ARMA*	-	*10*	*0.0%*	*1.0%*	*0.0%*

Note. Dir1%right = Percentage of occasions that the direction of the next time step was correctly predicted.

Dir2%right = Percentage of occasions that the direction of two time steps ahead was correctly predicted.

DirBoth%right = Percentage of occasions that the direction of both the first and the second time steps ahead were correctly predicted.

Table 18. Shares with predicting and trading methods better than ARMA, buy-and-sell at time steps 1,2,5 and 10 minutes

	Time step (minutes)			
Method	**1**	**2**	**5**	**10**
Buy and hold	BTI NPN	NPN	NPN	NPN
Wavelet-ARMA 5-6 Short & Close	BTI NPN SBK	BTI		NPN SBK
Wavelet-ARMA 6 Short & Close	BTI NPN	SBK		NPN
Wavelet regression 6 Short & Close	BTI NPN	SBK		NPN
Wavelet regression 5-6 Short & Close	BTI			
Wavelet-ARMA 4-6 Short & Close	BTI			

Table 19. Algorithms and trade methods with positive cumulative returns

				Time step (minutes)				+ / # trades[b]
Share	**Predict method**	**Scales**	**Trade[a]**	**1**	**2**	**5**	**10**	
BTI	Wavelet-ARMA	5 to 6	S&C	12.35%				72/144
BTI	Wavelet-ARMA	5 to 6	S&C		9.51%			76/152
BTI	None	-	B&H	8.57%				
BTI	Wavelet-ARMA	4 to 6	S&C	7.63%				230/518
BTI	Wavelet-ARMA	6	S&C	6.80%				18/26
BTI	Wavelet regression	6	S&C	6.80%				18/26
BTI	Wavelet regression	5 to 6	S&C	5.68%				96/205
BTI	ARMA	-	B&S			1.7%		3/7
BTI	ARMA	-	B&S		0.06%			3/9
MTN	None	-	B&H	17.58%				
MTN	ARMA	-	B&S		3.04%			12/25
MTN	ARMA	-	B&S			0.8%		7/10
MTN	ARMA	-	B&S	0.10%				14/38
NPN	None	-	B&H	39.84%				
NPN	Wavelet regression	6	S&C				11.8%	39/69
NPN	Wavelet-ARMA	6	S&C				10.9%	39/68
NPN	Wavelet-ARMA	5 to 6	S&C				3.3%	100/181
NPN	Wavelet regression	6	S&C	2.78%				41/82
NPN	Wavelet-ARMA	6	S&C	2.62%				41/82
NPN	ARMA	-	B&S		2.04%			20/46
NPN	Wavelet-ARMA	5 to 6	S&C	1.42%				141/281
SBK	Wavelet-ARMA	5 to 6	S&C	18.34%				121/217
SBK	None	-	B&H	15.83%				
SBK	Wavelet-ARMA	5 to 6	S&C				8.5%	102/174
SBK	Wavelet regression	6	S&C		3.41%			13/43
SBK	Wavelet-ARMA	6	S&C		0.48%			27/43

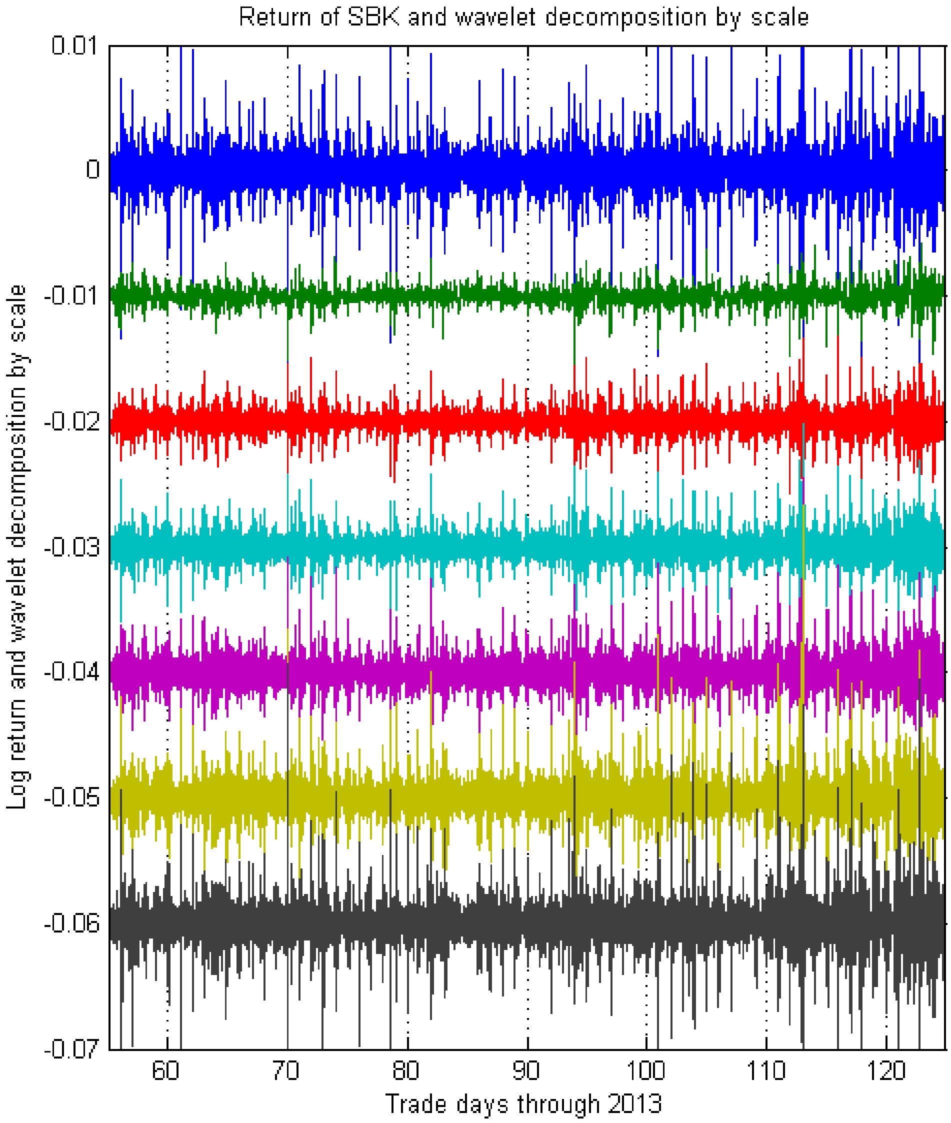
Note. The percentages are the cumulative returns during the test period.

a Trade method. B&H=buy and hold; B&S = buy and sell; S&C= Short and close.

[b] The number of profitable trades/number of trades through the 125 days period.

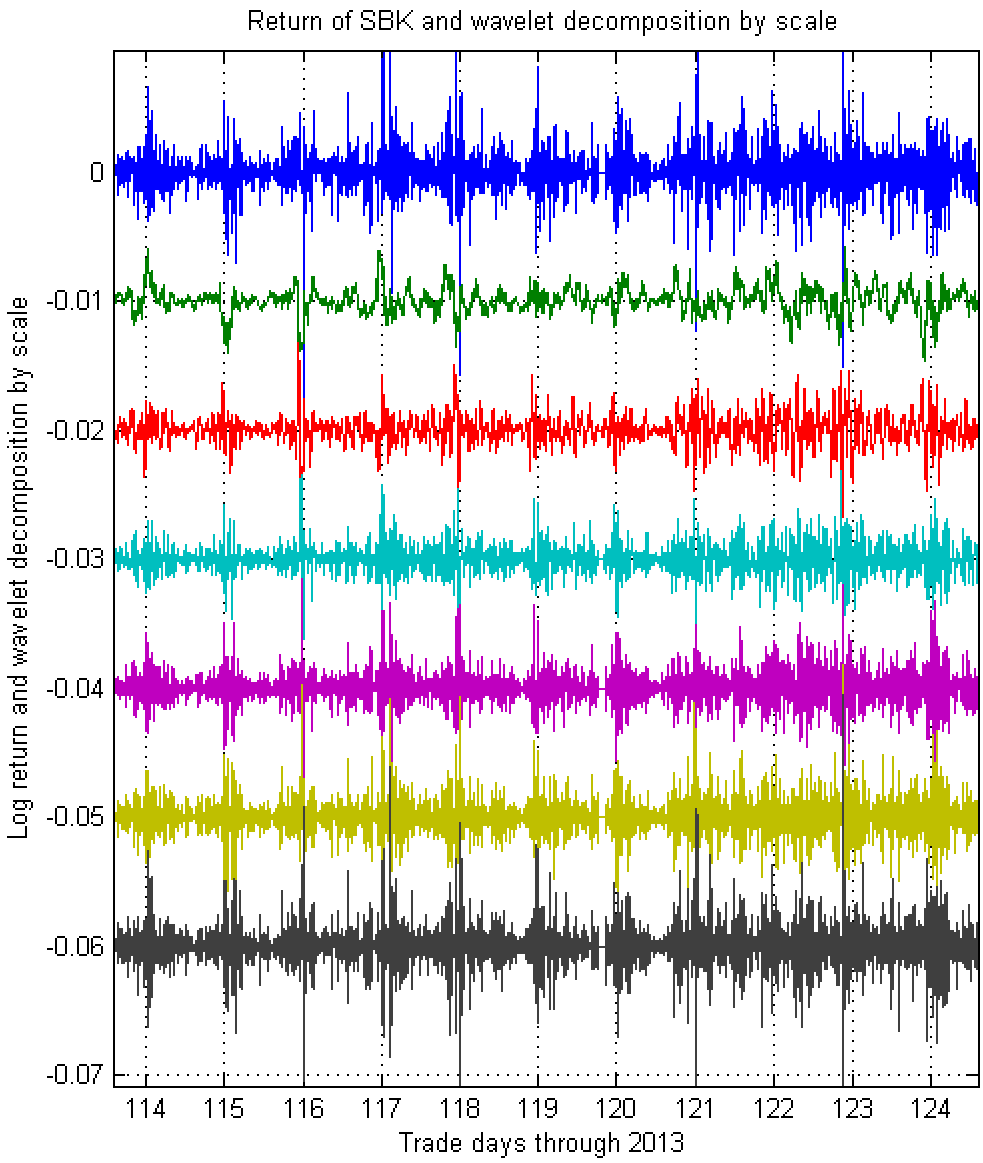
Appendix B: Graphs with result details

Figure 3. SKB log return and decomposed wavelet components



Note. The graphs are offset on the vertical axis in order to be displayed together (each one has a zero mean).
Legend from the top:
Blue – Log return of SKB at 1 minute intervals
Green – Scale 6 or Approximate component of log return
Red – Scale 5 (16 to 32 minutes) component of log return
Baby blue – Scale 4 (8 to 16 minutes) component of log return
Purple – Scale 3 (4 to 8 minutes) component of log return
Khaki – Scale 2 (2 to 4 minutes) component of log return
Black – Scale 1 (1 to 2 minutes) component of log return.

Figure 4. SKB log return and decomposed wavelet components – 12 days



Note. The graphs are offset on the vertical axis in order to be displayed together (each one has a zero mean).

Legend from the top:

Blue – Log return of SKB at 1 minute intervals

Green – Scale 6 or approximate component of log return

Red – Scale 5 (16 to 32 minutes) component of log return

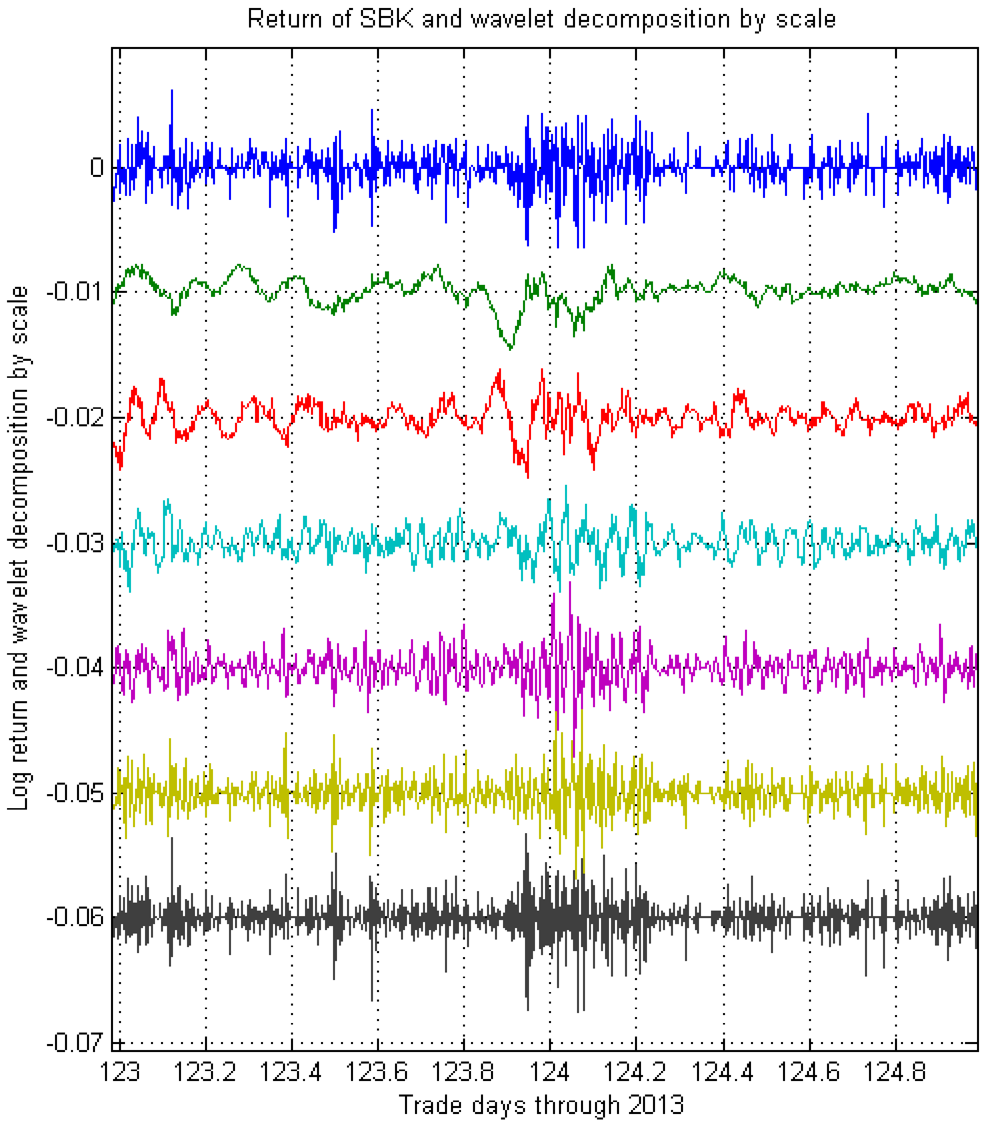
Baby blue – Scale 4 (8 to 16 minutes) component of log return

Purple – Scale 3 (4 to 8 minutes) component of log return

Khaki – Scale 2 (2 to 4 minutes) component of log return

Black – Scale 1 (1 to 2 minutes) component of log return.

Figure 5. SKB log return and decomposed wavelet components – 2 days



Note. The graphs are offset on the vertical axis in order to be displayed together (each one has a zero mean).

Legend from the top:

Blue – Log return of SKB at 1 minute intervals

Green – Scale 6 or approximate component of log return

Red – Scale 5 (16 to 32 minutes) component of log return

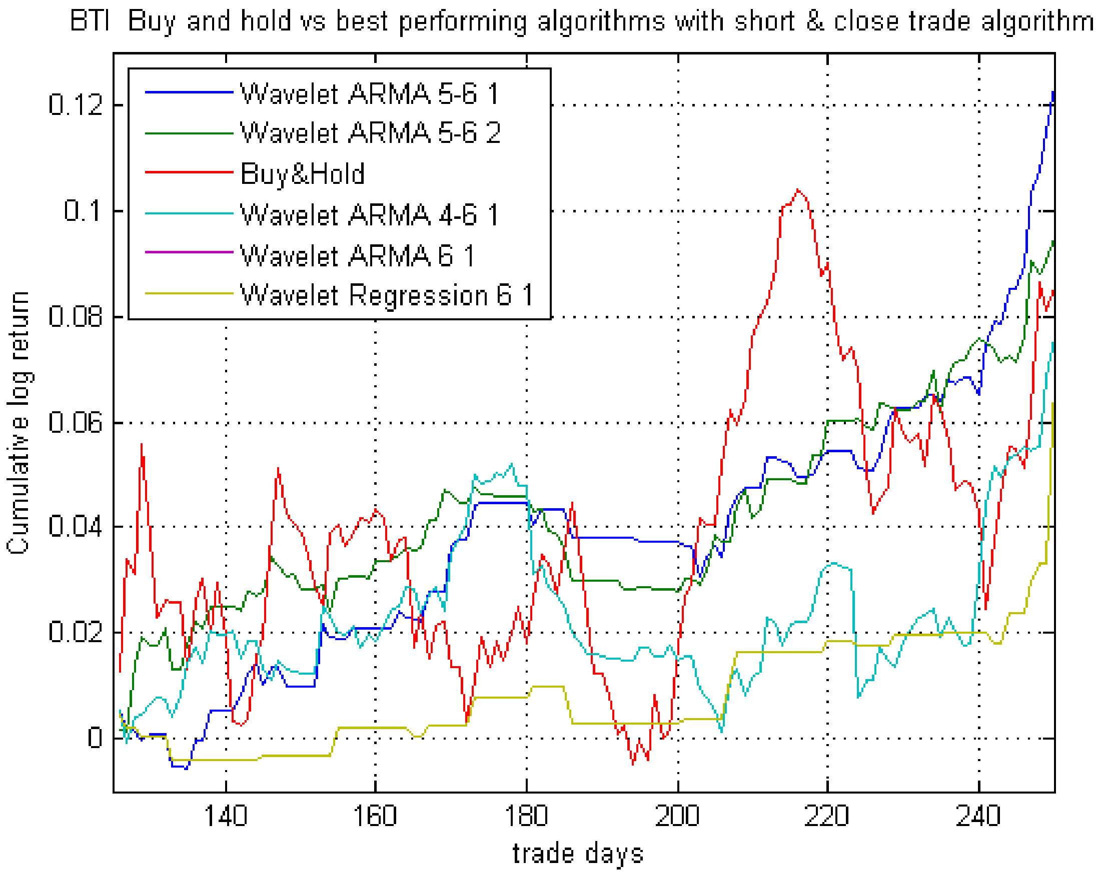
Baby blue – Scale 4 (8 to 16 minutes) component of log return

Purple – Scale 3 (4 to 8 minutes) component of log return

Khaki – Scale 2 (2 to 4 minutes) component of log return.

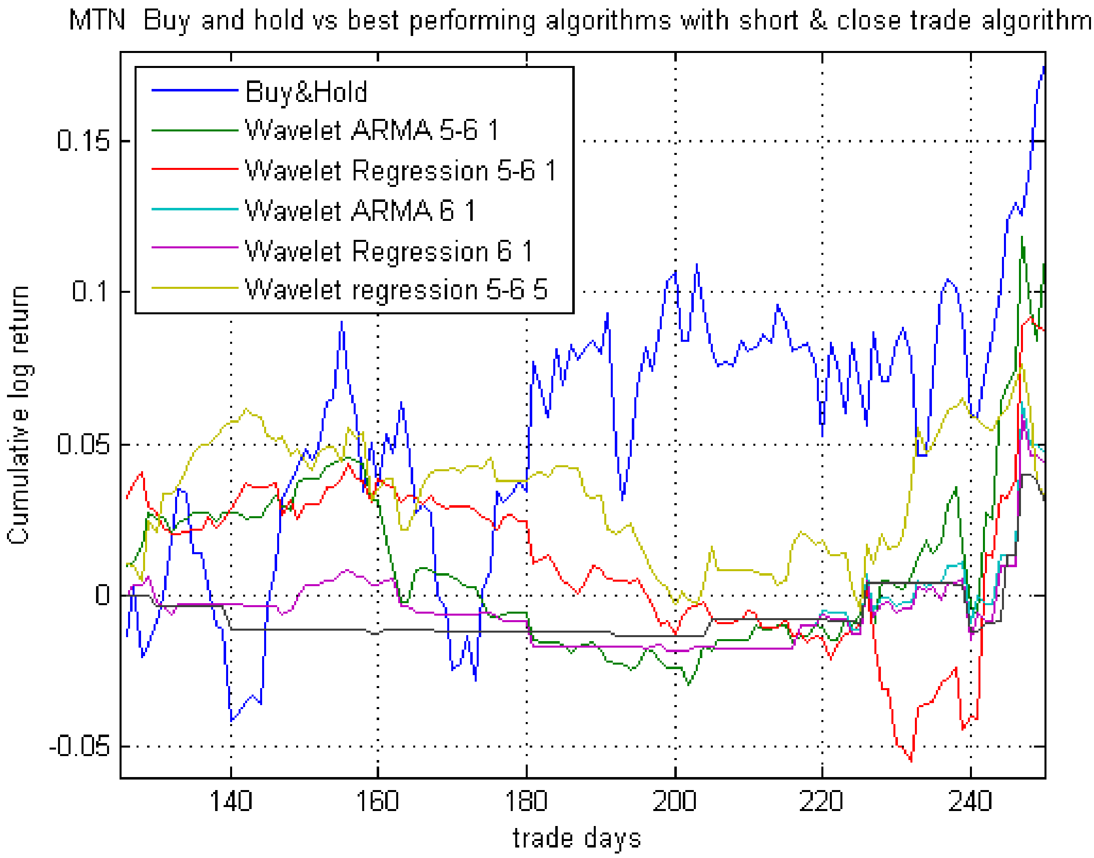
Black – Scale 1 (1 to 2 minutes) component of log return

Figure 6. BTI best trade algorithms



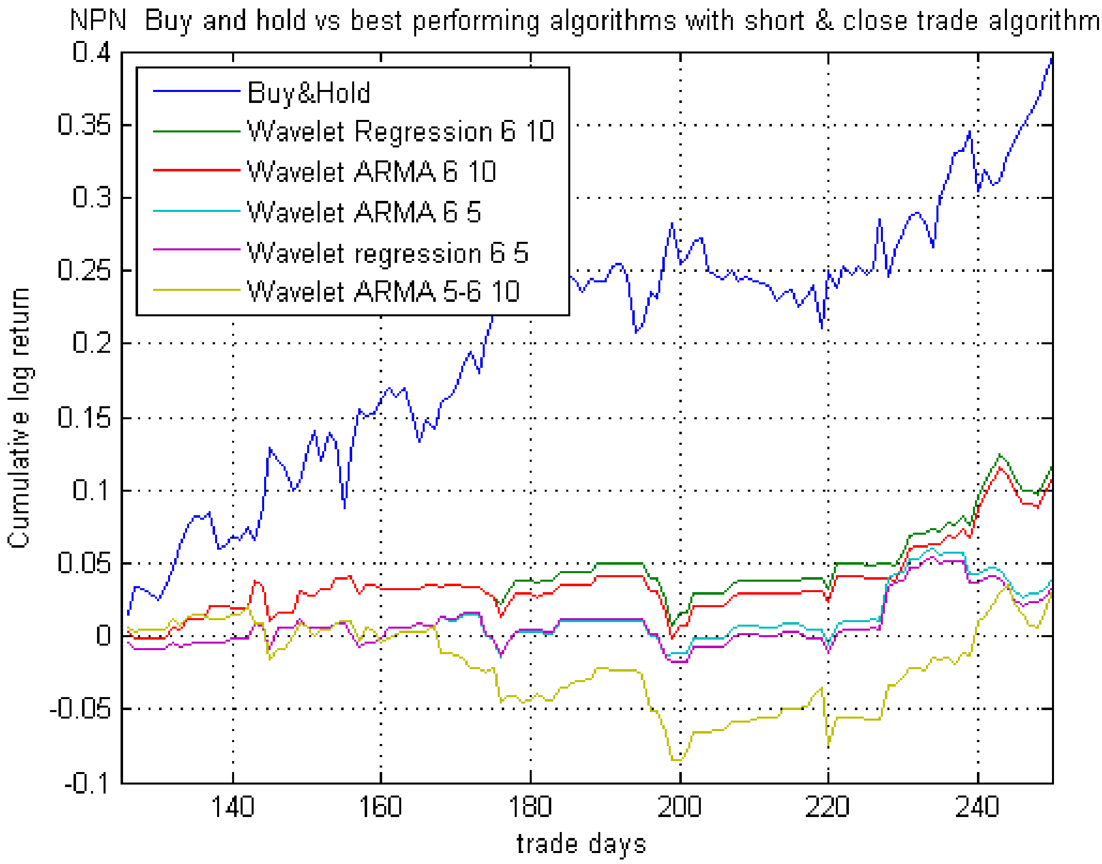
Note. The legend indicates the prediction method, with the last figure the time step (For instance the blue graph is the Wavelet-ARMA method using scales 5 to 6 at a time interval of 1 minute).

Figure 7. MTN best trade algorithms (short & close)



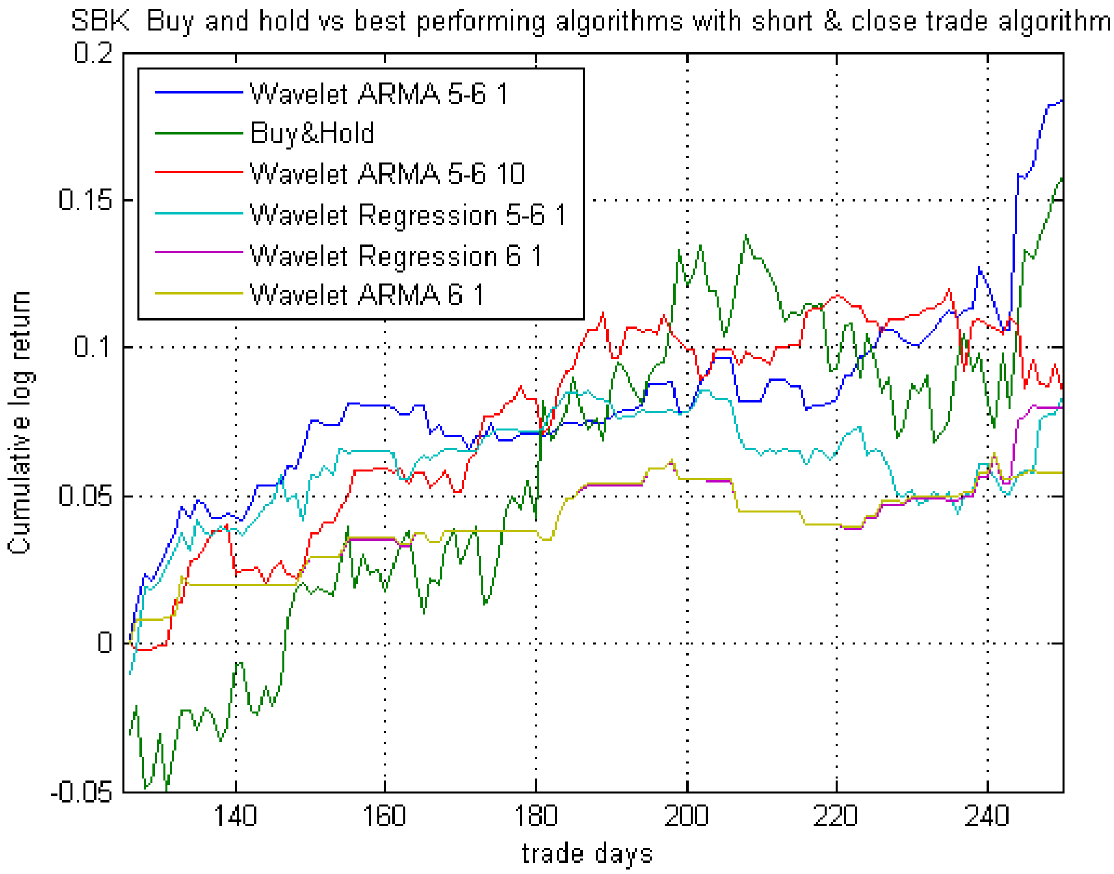
Note. The legend indicates the prediction method, with the last figure the time step (For instance the green graph is the Wavelet-ARMA method using scales 5 to 6 at a time interval of 1 minute).

Figure 8. NPN best trade algorithms (short & close)



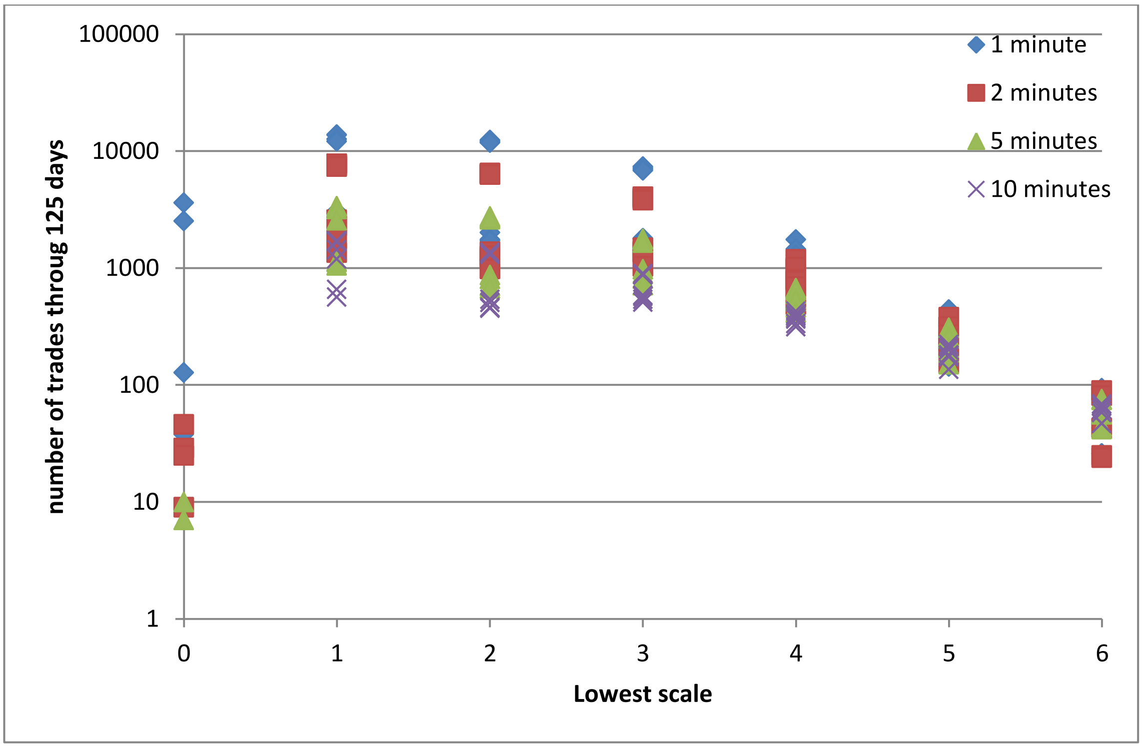
Note. The legend indicates the prediction method, with the last figure the time step (For instance the green graph is the Wavelet regression method using scale 6 at a time interval of 10 minutes).

Figure 9. SBK best trade algorithms (short & close)



Note. The legend indicates the prediction method, with the last figure the time step (For instance the blue graph is the Wavelet-ARMA method using scales 5 to 6 at a time interval of 1 minute).

Figure 10. Number of trades through the test period



Note. The number of trades that the algorithms concluded through the 125 day test period vs. the lowest scale of the wavelet-ARMA and wavelet regression algorithms.

0 – ARMA

1 – scales 1 to 6

2 - scales 2 to 6

3 - scales 3 to 6

4 - scales 4 to 6

5 - scales 5 to 6

6 – scale 6.

The legend refers to the time step duration.