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Exploring the Development of Conceptual Understanding through Structured Problem-solving in Physics

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Abstract

A study on the effect of a structured problem-solving strategy on problem-solving skills and conceptual understanding of physics was undertaken with 189 students in 16 disadvantaged South African schools. This paper focuses on the development of conceptual understanding. New instruments, namely a solutions map and a conceptual index, are introduced to assess conceptual understanding demonstrated in students' written solutions to examination problems. The process of the development of conceptual understanding is then explored within the framework of Greeno's model of scientific problem-solving and reasoning. It was found that students who had been exposed to the structured problem-solving strategy demonstrated better conceptual understanding of physics and tended to adopt a conceptual approach to problem-solving.

Introduction

Physics instructors generally believe that problem-solving leads to the understanding of physics and that it is a reliable way to demonstrate that understanding for purposes of evaluation (Hobden, 1999; Maloney, 1994). However, students are often unable to interpret or explain the meaning of their own algebraic solutions of problems (McDermott, 1991; McMillan & Swadener, 1991). The concern with poor conceptual understanding of physics among students has been voiced by many academics (e.g., Hewitt, 1983; Redish, 1994; Van Heuvelen, 1991).

Difficulties with physics problem-solving and conceptual understanding are aggravated in South Africa, where the majority of schools are at a disadvantage due to persisting effects of the apartheid school system (Hartshorne, 1992; Johnson, Monk, & Hodges, 2000). Although many non-governmental organisations have been involved for almost 30 years with trying to improve science education in secondary schools (Rogan & Gray, 1999), poorly trained teachers, teacher-dominated approaches, student passivity, and rote learning are still the norm (Arnott, Kubeka, Rice, & Hall, 1997). Ambitious curriculum reforms have been implemented following the transition to democracy in 1994, but these have done little to improve the situation in poorly resourced classrooms (Jansen, 1999). Understanding is complicated by instruction in a second language (Howie, 2003; Prophet, 1990). In such conditions, physics problem-solving is likely to be reduced to algebraic solutions, with little, if any, emphasis on conceptual understanding.

This paper reports on part of a project (Gaigher, 2004) that extended first-world research on physics problem-solving strategies to disadvantaged South African classrooms. The project envisaged to enhance physics problem-solving and conceptual understanding. The type of problems referred to in the study were typical textbook questions where physics principles are applied to determine a quantitative value of some parameter in a concrete situation described by the question. The meaning of "strategy" is a recommended series of steps to follow when solving problems.

Sixteen schools, involving 189 students, participated in a quasi-experimental study over a period of ten months. The science teachers and students in the experimental schools applied a specific seven-step strategy to the solving of the physics problems throughout the academic year. The students in the experimental group achieved better results in the examinations; the examination statistics together with a theory of the co-development of problem-solving skill and conceptual understanding have been reported elsewhere (Gaigher, 2004; Gaigher, Rogan, & Braun, 2006).

The purposes of this paper are, firstly, to present evidence of enhanced conceptual understanding among the experimental group, and, secondly, to explore how the implementation of the problem-solving strategy might have promoted the development of conceptual understanding. New instruments, namely a solutions map and a conceptual index, are introduced to assess conceptual understanding demonstrated in students' written solutions to examination problems. The process of the development of conceptual understanding is then explored within the framework of Greeno's model of scientific problem-solving and reasoning (Greeno, 1989). As this model is not restricted to physics, we believe that our results can be generalised to other areas of science despite the fact that the study was conducted in physics classrooms.

Literature Review

Much research has been conducted on problem-solving in physics since the late 1970s. This literature review focuses on studies of conceptual understanding related to solving quantitative physics problems.

Conceptual Understanding

McDermott (1991) argued that success in calculating correct numerical answers did not necessarily imply that a corresponding level of conceptual understanding was reached. In fact, instruction focusing on problem-solving often ignores intellectual objectives and could encourage students to concentrate on algorithms instead of on physics. Poor conceptual understanding has been demonstrated by various studies (Lawson & McDermott, 1987; McMillan & Swadener, 1991; Pride, Vokos, & McDermott, 1998; Schaffer & McDermott, 1992). These studies suggested that students learn to solve standard problems in physics without applying conceptual and interpretative knowledge.

How should classroom practices be adjusted to ensure that students learn to understand physics? Hewitt (1983) claimed that problem-solving instruction in high school actually obscured the development of conceptual understanding; he argued that conceptual reasoning should form part of examinations in order to encourage students to conceptualise the physics principles involved. McDermott (1991) advocated that students should be intellectually engaged in the learning process in order to bring about significant conceptual change. She suggested that a deep mental engagement could be developed when students were required to explain their reasoning in their own words. According to Van Heuvelen (1991), students of physics often observe passively how lecturers demonstrate the algebraic aspects of solving problems. Van

Heuvelen suggested that students could learn to think like physicists when given opportunities to reason qualitatively and make use of translations from verbal, pictorial, and physics representations, before switching to the mathematical form of physics problems. Redish (1994) argued that physicists should learn from cognitive science, particularly the theories of constructivist learning and conceptual change, to improve their teaching. Redish advocated that students should be given opportunities to do qualitative reasoning, to construct mental models, and to learn to apply their models. Duit, Roth, Komorek, and Wilbers (1998) conducted a study on student talk during a classroom experiment, and reported that conceptual change was facilitated by discussions among students.

A series of studies on experts and novices identified qualitative analysis and successive representations as a characteristic of expert problem-solving (Larkin, 1979, 1983; Larkin, McDermott, Simon, & Simon, 1980; Larkin & Reif, 1979). Dhillon (1998) observed that novice problem-solvers had difficulty to relate quantities, and used symbols to infer connections. On the other hand, experts used the conceptual meaning of quantities to relate them. According to Maloney (1994), the most striking difference between the approaches of experts and novices was found in the application of general principles of physics. While the experts preferred general principles, the novices typically used means-end analyses, focusing on the gap between the required answer and the information, thus filling in steps to complete an algebraic solution.

Instructional Strategies

Concern with poor problem-solving, as well as with poor conceptual understanding, which often accompanies successful algebraic problem-solving, led to the development of various instructional strategies for the teaching of physics problem-solving:

- Explicitly taught problem-solving strategies, which included qualitative analysis and multiple representations, resulted in better problem-solving (Heller & Reif, 1984).
- Cooperative group problem-solving showed that group solutions were often better than the best individual efforts (Heller, Keith, & Anderson, 1992).

- Qualitative strategy writing, where students had to describe how they would solve given problems, was shown to develop problem-solving skills as well as conceptual understanding (Leonard, Dufresne, & Mestre, 1996).
- Modelling instruction requires that students discuss problems and resolve conflicting ideas, thereby adjusting and extending their concepts while constructing solutions (Hestenes, 1987; Halloun & Hestenes, 1987).
- Fraser, Linder, and Pang (2004) used the technique of variation in problem-solving. This technique requires students to solve a given problem in more than one way, by applying different physics principles to a given concrete problem situation. Students are thus given opportunities to develop understanding of the connections between different physics principles.
- Alant (2004) argued that familiarity with problems creates a basis for conceptual understanding. Learning through familiarity requires that students be given closely related problems, creating opportunities for students to become familiar with an underlying physics principle.

While these strategies were all employed at universities, Huffman (1997) explored structured problem-solving in high school physics. The aim of his study was to establish the effect of an explicit problem-solving strategy on problem-solving performance as well as on conceptual understanding. The explicit problem-solving strategy described by Heller et al. (1992) was compared with a so-called textbook strategy. The explicit strategy emphasised both qualitative and quantitative aspects of problem-solving, while the textbook strategy emphasised only quantitative aspects. The experimental group showed an improvement in the quality and completeness of the physics representations used in their problem-solving. The quality of solutions was assessed by a scoring rubric using characteristics of expert problem-solving (Larkin et al., 1980). This enhanced quality of solutions was regarded as evidence of enhanced conceptual understanding developed by the explicit problem-solving strategy.

The instructional strategies described above provided insights that were utilised in the design of a structured problem-solving strategy suitable for the disadvantaged setting of the current study. The steps of the strategy were designed to encourage the use of multiple representations as well as qualitative analysis to guide students towards the thought processes of expert problem-solvers. The step labelled "analysis" is not limited to individual efforts; it requires students to formulate arguments in classroom discussions and in the writing of their own solutions, thereby creating opportunities for individual as well as social knowledge construction.

Theoretical Framework

Greeno's "extended semantic model" creates a framework for scientific problem-solving and reasoning with the overall focus on conceptual understanding (Greeno, 1989). The model is based on four domains of knowledge, namely:

1. Concrete domain (physical objects and events).
2. Model domain (models of reality and abstractions).
3. Abstract domain (concepts, laws and principles).
4. Symbolic domain (language and algebra).

The four domains are represented in Figure 1. They can be understood by means of a familiar example of a crane lifting a container - a situation used in a number of standard physics textbooks. The "concrete domain" includes the physical objects involved - the crane and container. In the "model domain", the concrete situation is represented by a force diagram. The "abstract domain" contains abstractions that could be made of this situation, such as the concept of a force and Newton's second law. Finally, the "symbolic domain" consists of symbolic ways of representing the situation using either language or algebraic expressions, or both.

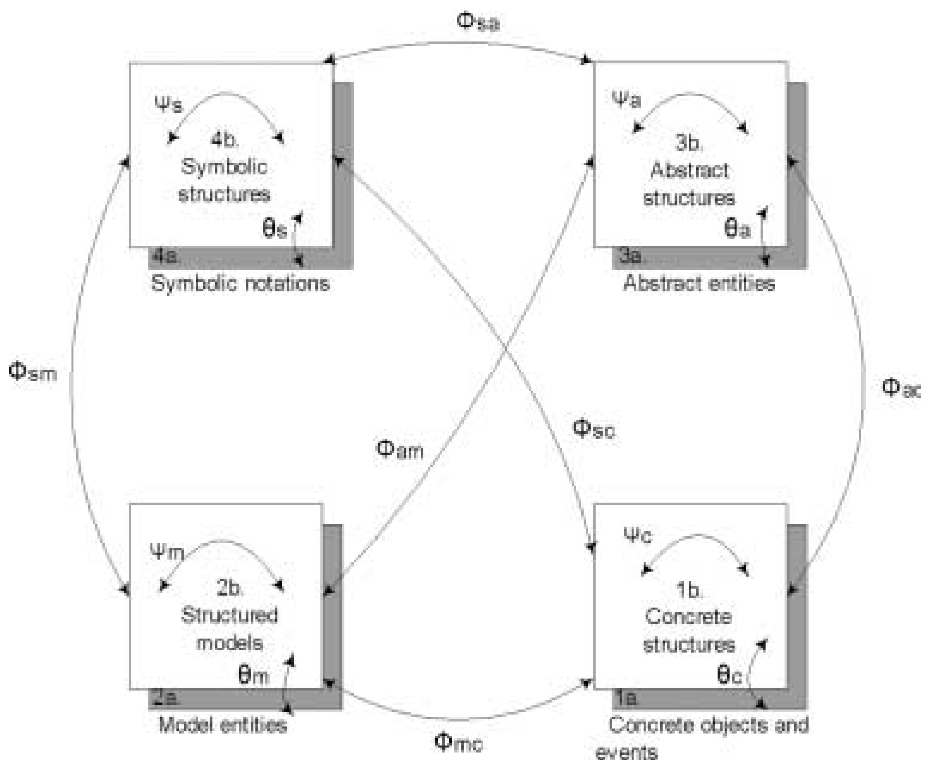


Figure 1. Greeno's four domains of knowledge for scientific problem-solving, shown with connections between and within domains (Greeno, 1989)

Correspondences, or mappings, exist between the domains. These mappings are indicated by the symbol Φ, with subscripts c, m, a, and s, respectively, denoting the concrete, model, abstract, and symbolic domains. For example, Φ_{sc} represents the mapping between the symbolic and concrete domains. According to Greeno, scientific problem-solving and reasoning skills involve the realisation of the correspondences between these domains. Chekuri and Markle (2004) argue that although problem-solving in physics usually involves algebraic operations in the symbolic domain, the algebra should always be connected to the concrete, model, and abstract domains.

Each domain consists of two layers, referred to as an a-layer and a b-layer. The a-layers contain independent items, while b-layers contain structures consisting of meaningful combinations of items from the associated a-layers. The crane example can illustrate the distinction between layers: In the concrete domain, layer 1a contains objects such as the crane, cable, and container, while the process of the crane lifting the container is a structure in layer 1b. In the model domain, arrows and dots are independent items in layer 2a, while these items can be meaningfully combined to become a force diagram in layer 2b. In the abstract domain, the concepts of force, mass, and acceleration are independent items in layer 3a, while the relationship between these concepts (namely, Newton's second law) is a structure in layer 3b. In the symbolic domain, symbols like F, m, g, and a are items in the 4a layer, while the mathematical relationship $\Sigma F = ma$ is a structure in the 4b layer. The symbolic domain also includes language, thus the written question is another structure in layer 4b, while independent words are items in layer 4a.

Figure 1 shows connections within each domain, indicated by the symbols $\Theta$and Ψ, with subscripts c, m, a, and s to indicate the relevant domains. Connections marked Θ represent relationships between independent items in a-layers to form meaningful structures in the corresponding b-layers. For example, a connection Θ_m in the model domain can represent the rules on how to draw a circuit diagram from components, or the parallelogram rule to add vectors. Connections marked Ψrepresent alternative ways to represent a particular structure in a particular layer, governed by sets of rules for the particular domain. For example, the association of a constant velocity with a zero resultant force is an operation Ψ_a in the domain of abstract structures, while an algebraic manipulation is classified as an operation Ψ_s in the symbolic domain.

In contrast to idealised problem-solving that incorporates all four domains of knowledge, a popular formula based approach flourishes among students. Students tend to start with a data list, matching information to symbols. Then a suitable formula is selected to link the unknown symbol to the known symbols in the list. All that remains is to substitute and to solve algebraically; interpretations are rare. Van Heuvelen (1991) called this method a formula-based approach, while Greeno referred to this procedure as the "insulation" of the symbolic world from the "situated nature" of problems: In the classroom, students manipulate symbols to solve problems while concrete problem situations are seldom present. This classroom reality can, therefore, lead to the belief that problems are about the symbols, rather than about the concrete situation represented by those symbols. The symbols are real marks on paper, taking the place of the real objects described by the problem statement. A mathematical operation Ψ_s in the symbolic domain acquires the status of a mapping Φbetween symbols and the concrete marks on the paper. Algebraic solutions can therefore amount to operations on knowledge located only in the domain of symbolic knowledge, without translation to the concrete, model, or abstract domains. Such an approach can sometimes lead to correct equations and correct numerical answers, but it does not demonstrate or develop understanding of the meaning of algebraic solutions.

Greeno's model provides a suitable framework to analyse the results of the current study as well as those discussed in the literature review. The mismatch between conceptual understanding and successful algebraic solutions resonates with insulation of the symbolic domain. On the other hand, the successive representations, qualitative analysis, and use of general physics principles demonstrated by experts indicate translations between all four knowledge domains.

The Structured Problem-solving Strategy

The problem-solving strategy employed in this study was designed to improve problem-solving performance as well as to develop conceptual understanding. Qualitative aspects of problems were emphasised while the algebra was part of a solution, but not the entire solution. The seven steps of the strategy can be summarised as follows:

1. Draw a simple diagram to represent the system.

2. Indicate the data on the diagram.
3. Identify the unknown variable.
4. Analyse the problem in terms of physics principles.
5. Write down the relevant equation(s).
6. Substitute and solve.
7. Interpret the numerical answer.

The strategy already described combined and simplified successful approaches reported in the literature. It does not require separate "real world" diagrams and "physics" diagrams as used by the Heller group (Heller et al., 1992). It was argued that separate diagrams could distract attention from the effort to learn to relate physics to reality. In fact, experienced problem-solvers sometimes draw forces on top of objects in real world diagrams, thus making abstract physics concepts visible in their real world representation (Larkin & Simon, 1987). In our strategy, the diagram becomes the focus of attention. While drawing the diagram, the student constructs a two-dimensional model of the concrete situation described in the problem statement. Information and unknown quantities are grouped by location when these are superimposed on the diagram. Such groupings guide the search for principles of physics applicable to different parts of the concrete situation when analysing the problem, while links between different parts of the problem become visible as shared features between groupings on the diagram.

Qualitative analysis has been associated with successful problem-solving (Heller & Reif, 1984) and identified as one of 14 fundamental activities in problem-solving (Dhillon, 1998). The current strategy explicitly prescribes qualitative analysis in the step "analysis" where students have to identify physics principles suitable to solve the particular problem, and explain why these principles are suitable. In the classroom situation, the analysis should include classroom discussions. The analysis step therefore combines aspects of modelling (Halloun & Hestenes, 1987) and strategy writing (Leonard et al., 1996).

Regarding algebra, the strategy used in this study was simplified. Students were encouraged to substitute numerical values before starting algebraic manipulation, the reason being that poor mathematical abilities could prevent many students from arriving at correct symbolic solutions. Here the current approach differed from that of others (Huffman, 1997; Reif, Larkin, & Brackett, 1976; Wright & Williams, 1986) who preferred symbolic solutions before substitution. We argued that emphasis on symbolic solutions could be counterproductive for disadvantaged students with poor mathematical skills. Incorrect symbolic solutions would not develop insight into the relevant physics relationships, and numeric substitutions into wrong symbolic solutions would be meaningless, adding to confusion.

Methodology

Sixteen schools, involving 189 students, completed the project. A quasi-experimental design with a pre-test and various post-tests was implemented. The experimental and control groups each consisted of all the Grade 12 science students in eight volunteering schools, with one participating teacher per school. The two groups were situated in two geographically separate education districts at opposite ends of town, separated by about 40 km. The decision to avoid random assigning of schools to the two groups was taken in a deliberate attempt to exclude diffusion, contamination, and rivalry. We regard this design as an improvement over studies where teachers had classes in both groups, or where two groups were in close physical proximity (e.g., Huffmann, 1997). The lack of randomness in assigning schools to groups could pose a threat to validity. However, a pre-test showed that the two groups performed similarly on problems on vectors and kinematics, based on the previous year's syllabus (Gaigher, 2004).

The strategy was implemented by means of a cascading model: the researcher trained teachers, while the teachers taught their students. During the teacher training workshops, solutions were not provided by the researcher. Instead, the teachers were given the opportunity to interact while applying the strategy, thus participating in knowledge construction in a social context. Similarly, the students were expected to be active participants in problem-solving in the classroom.

The study was designed to be non-disruptive: the only change from an ordinary school routine was the way in which the experimental group solved problems. The strategy was applied and practised while solving classroom and homework problems, which would form part of the ordinary routine of learning physics by doing problems. No extra classes were given to the experimental group students. The Department of Education's Grade 12 syllabus and the schools' textbooks were used. Identical homework sets were given to both groups, and solutions were not provided. Tests were structured as ordinary 30-min classroom tests consisting of typical examination

problems. The control group teachers and students were informed that the project explored students' problem-solving skills, but no mention was made of an experimental group using a particular problem-solving strategy.
Data were collected over a period of ten months, starting at the beginning of the academic year in January. The test and examination scripts were used as main sources of data. Other data sources were questionnaires and videotapes showing how a few volunteers attempted to solve problems. This paper focuses on the nature of the solutions presented in the mid-year examination.

Results

Statistical analysis indicated improved test and examination scores among the experimental group. These results have been discussed elsewhere (Gaigher, 2004; Gaigher et al., 2006). This paper probes possible reasons for these improved results by analysing the nature of solutions presented in the mid-year examination. Firstly, a case study is presented, showing how a student implicitly made translations between Greeno's knowledge domains when he applied the steps of the problem-solving strategy. Secondly, a solutions map is presented to summarise the variety of attempted solutions created by the entire sample of students for a particular problem. Thirdly, a conceptual index is introduced to assess whether successful problem-solving could be linked to a conceptual approach to problem-solving.

Case Study: A student's written solution

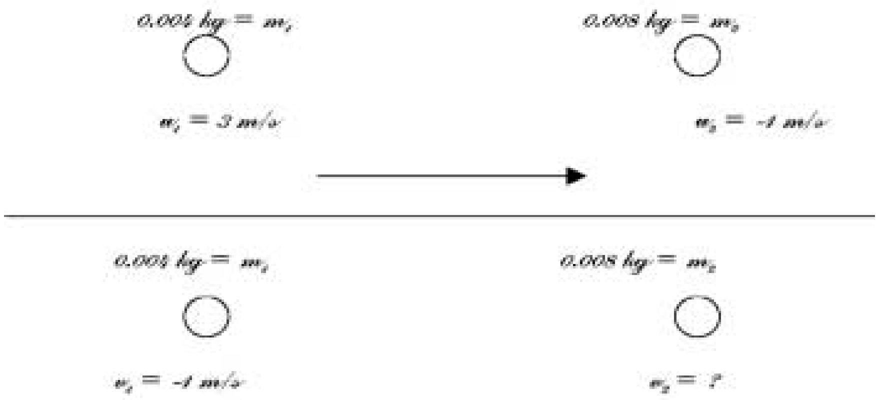
A solution presented by one of the treatment group students in the mid-year examination was analysed to establish which translations were made in terms of Greeno's model. This particular question and solution, shown in Figure 2, was chosen as an example of a case in which a student explicitly wrote down all the headings to indicate the seven steps of the strategy. The diagram was not given or required - it was constructed by the student as part of the solution.

Problem

During a game of marbles, a 4 gram marble moves at 3 $m.s^{-1}$ North and collides with an 8 gram marble moving at 1 $m.s^{-1}$ South. The 4 g marble moves at 1 $m.s^{-1}$ South after the collision.
Calculate the velocity of the 8 gram marble after the collision.

Student's Solution

Diagram and Information



Unknown: v_2 = ? (After collision)
Analysis: Law of conservation of momentum.
Relations: $p_{before} = p_{after}$
Substitution and solution: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$(0.004)(3) + (0.008)(-1) = (0.004)(-1) + (0.008)(v_2)$$
$$0.012 - 0.008 = -0.004 + 0.008\ v_2$$
$$0.004 + 0.004 = 0.008\ v_2$$
$$0.008 = 0.008\ v_2$$
$$1\ m/s = v_2$$

Interpretation:
The velocity of the 8 gram ball after the collision is 1m/s north.

Figure 2. A student's solution to a momentum conservation problem

The translations identified in this example are presented in Table 1. Even though the problem was a simple one, the solution suggests a variety of translations made between the four knowledge domains as the student worked through the seven steps of the strategy. On the other hand, an algebraic solution starting with the formula $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ would present little, if any, evidence of translations between the four domains. In fact, an algebraic solution, based on matching symbols, would not require any translations since all actions would be restricted to the symbolic domain. This example demonstrates that the steps prescribed by the problem-solving strategy lead to a wealth of translations between the knowledge domains. We believe that such a network of translations supports the development of associations to synthesise reality, models, concepts, and symbols - thus enhancing the conceptualisation of physics.

Table 1. Student's presumed actions while applying the problem-solving strategy interpreted as translations between Greeno's four knowledge domains

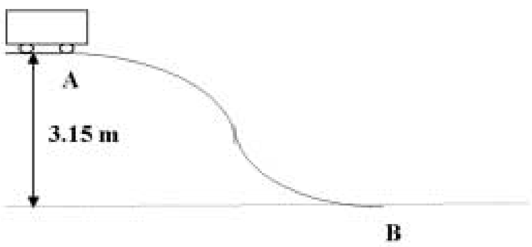
Step	Student's actions	Translation
Diagram and information	Reads the question and visualises the concrete situation	sc
	Draws a diagram showing two snapshots to model the collision	cm
	Chooses a positive direction and indicates it on the diagram	am
	Identifies abstract concepts relevant to data	sa
	Identifies standard symbols for the data	as
	Indicates data on the diagram in the appropriate locations, representing time and space	sm
Unknown	Identifies unknown concept	ca
	Identifies the standard symbol for unknown concept	as
	Indicates the unknown on the diagram	sm
Analysis	Identifies the law of momentum conservation as relevant to the concrete situation	ca
	Formulates the principle that can be applied to solve the problem	as
Relationship	Writes the law of momentum conservation as an algebraic equation	as
Substitute and solve	Collects information from the diagram, substitutes into the equation(s) and solves	ms
Interpretation	Interprets the meaning of the symbolic answer in terms of the concrete situation	sc
	Formulates the interpretation	cs

Solutions Maps

An energy conservation problem from the mid-year examination is presented in Figure 3 to demonstrate how solutions maps were constructed and interpreted. This specific problem was chosen because questions on energy produced the most prominent difference between the scores of the two groups in the classroom tests as well as in the examination (Gaigher, 2004; Gaigher et al., 2006). It was argued that the reasons for the improved overall scores would be similarly prominent in this particular question.

Problem

A car with a mass of 200 kg starts at A and rolls down a track as shown in the diagram. It reaches B with a speed of 12 m/s. Calculate the speed with which the car started at A. Assume there is no friction between A and B.



Solution

From A to B, mechanical energy is conserved:

$E_P + E_K = \text{constant}$

$mgh_A + ½mv_A^2 = mgh_B + ½mv_B^2$

$gh_A + ½v_A^2 = gh_B + ½v_B^2$

$10 \times 3.15 + ½ v_A^2 = 0 + ½ \times 12^2$

$v_A^2 = 81$

$v_A = 9$ m/s

This was the initial speed of the car at the top of the track.

Figure 3. The energy conservation problem

This problem is slightly more difficult than the simplest energy conservation problems that typically deal with objects starting from rest, moving down ramps, or swinging from strings without friction. Students usually have to calculate the speed at the lowest point, or the maximum height reached. In such simple cases, the energy conservation law boils down to a simple relation between maximum speed and maximum height: $1/2mv^2 = mgh$. Students, and perhaps teachers, may use this equation without emphasising or realising that v and h actually refer to the extremes of the path. Even when a student uses this equation correctly, it is no guarantee that he or she understands that the conservation principle refers to the sum of the kinetic and potential energy at any position along the path. In order to assess deeper understanding, problems can be formulated to include positions between those of maximum speed and maximum height. In such problems, the conservation of mechanical energy can be written to refer to any two points along the path: $1/2mv^2_1 + mgh_1 = 1/2mv^2_2 + mgh_2$.

In the mid-year examination, an initial speed at a given height was included to rule out the use of the simple formula. The students had to determine the initial speed of a car moving downhill without friction, to reach a specified speed at the lowest point of the path. In this case, energy conservation can be expressed as: $1/2mu^2 + mgh = 1/2mv^2$. Here u and v, respectively, represent the initial and final speed while h represents the initial height.

Each student's solution was analysed and classified according to his or her approach. The approaches, labelled Routes i-vii, are represented on a solutions map (Figure 4). Percentages refer to the number of students from the experimental group who followed particular routes, with the percentage of control group students in brackets. Route i represents solutions where students correctly applied the energy conservation principle to the given situation; all other routes are wrong in some way or the other.

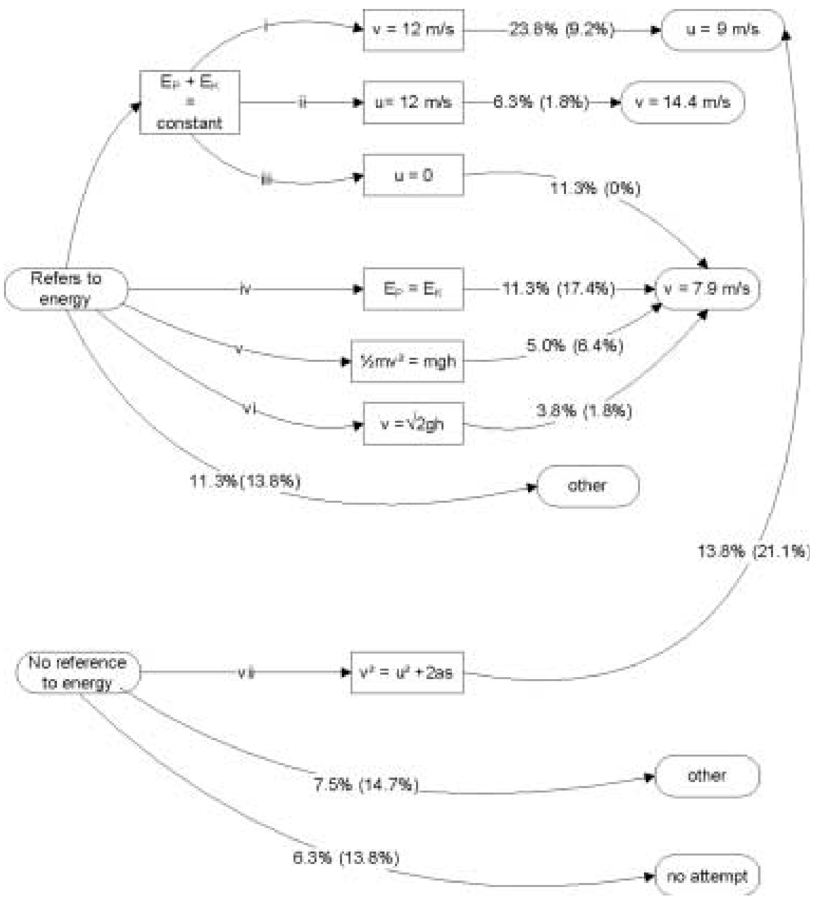


Figure 4. Solutions map for the energy conservation problem, summarising students' solutions as different routes - with Route i representing correct solutions. Percentages refer to the number of students from the experimental group following particular routes, and the percentages in brackets refer to the control group

Solutions were broadly classified into two main groups, depending on whether or not the student made some use of energy concepts. Among the treatment group, 72.5% of the students compared with 50.3% of the control group approached the problem using energy concepts. In terms of Greeno's model, the increased use of energy concepts indicates that the treatment group made more translations to the abstract domain. Of course, translating to the abstract domain does not necessarily mean that the students would apply the concepts correctly. Incorrect application of concepts would suggest incomplete or incorrect structures in the b-level of the abstract domain. However, the experimental group was also more successful than the control group, having 23.8% of solutions correct compared with only 9.2% for the control group.

Among the solutions referring to energy, Routes i, ii, and iii explicitly mentioned that the sum of kinetic and potential energy remained constant. Grouped together, these solutions added up to 41.3% of the experimental group and only 11.1% of the control group, which indicates enhanced understanding of the energy conservation principle. Route i represents the correct substitutions, leading to the correct answer of 9 $m.s^{-1}$. Despite a low success rate of 23.8% for the treatment group, it was more than double that of the control group, indicating that the problem-solving strategy can indeed be a useful approach in physics teaching. The overall low success rate for such a standard energy problem is a stark reminder of the urgent need for development of teaching practises in the disadvantaged South African schools.

The incorrect answer obtained by most students from both groups was 7.9 $m.s^{-1}$. This answer resulted when calculating the final speed of a car starting from *rest* at the given height, instead of calculating the initial speed of a car that reaches the specified final speed. The solutions represented by Routes iv-vi all used variations of kinetic energy = potential energy, which was equivalent to the simple relationship between maximum height and maximum speed. Students following these routes ignored the given final speed and calculated a final speed for a car starting from rest. The fact that energy concepts appeared in the solutions does suggest some translation to the abstract domain. However, the inability to accommodate kinetic energy at two different heights suggests limited understanding of the energy conservation principle in its general form, signifying an incomplete structure in Greeno's 3-b level.

Route iii represents a correct start by making provision for an initial speed, but then a value of 0 is substituted in the equation, thereby joining Routes iv, v, and vi. This approach was followed by 11.3% of the experimental group but none of the control group students. It seems that these students from the experimental group learnt to start with a "better formula", possibly resulting from the learning the strategy. However, the lack of conceptual understanding, or incomplete structure in the 3b layer became visible in the inappropriate substitutions, or incorrect translations.

Route vii represents solutions starting with the standard equation for constant acceleration: $v^2 = u^2 + 2as$. This equation yields a numerically correct answer, provided the standard meanings of the symbols are discarded and, instead, s is regarded as the vertical height and a as the gravitational acceleration. It can be argued that some of the students knew that the energy conservation law yields the same equation as a vertical free fall, regardless of the actual path travelled. However, when using a standard equation (given on the standard information sheet), the symbols should have the standard meanings; if not, the student should have explained the situation to avoid ambiguity. However, none of the solutions classified as Route vii indeed made any mention of the concepts of energy or energy conservation, or any argument stating why the free fall equation would be suitable. This total absence of attaching meaning to the equation in these solutions was the criterion for classifying solutions into Route vii. We refer to this route as "formula based", because the lack of interpretation suggests that the equation was chosen simply by matching symbols - a procedure restricted to operations in Greeno's symbolic domain. Among the control group, 21.1% of students used this approach, compared with only 13.8% of the experimental group, indicating that the strategy did reduce the exclusive use of algebra among the experimental group.

Students following Route ii interchanged the specified final speed of 12 $m.s^{-1}$ with the required initial speed, reflecting a tendency to regard a beginning as "known" and an end as "unknown". This occurred more among the experimental group (6.8%) than among the control group (1.8%). However, these students showed conceptual understanding of the energy conservation principle in its general form. The mistake appears to be an incorrect translation between the concrete and symbolic domain, but not an incomplete structure in the 3b domain.

To summarise, the solutions map for the energy conservation problem demonstrated enhanced conceptual understanding among the experimental group in three ways, namely:

- The ability to construct the correct solution to this particular energy problem is regarded as evidence of conceptual understanding, since this problem did not allow application of the simple formula. The mere fact that the entire sample of students generated so many incorrect routes suggests that those who constructed the correct solution indeed understood the principle of energy conservation in the context of the problem. In terms of Greeno's model, the construction of the correct solution represents an appropriate translation from the concrete to the abstract domain. We therefore conclude that the experimental group demonstrated better conceptual understanding since almost 25% of the experimental group, compared with less than 10% of the control group, constructed the correct route.
- The use of general physics principles (in this case, conservation of the *sum* of kinetic and potential energy) was employed in Routes i-iii, representing more than 40% of the experimental group compared with only 11% of the control group. In terms of the expert-novice literature (Maloney, 1994), a closer resemblance to expert problem-solving was thus demonstrated by the treatment group.
- The tendency to resort to a formula-based approach (Route vii) was reduced among the experimental group, signifying some application of knowledge beyond the symbolic domain. It is inferred that more students in the experimental group developed an understanding that solutions are governed by physics concepts rather than mathematical operations.

Is it possible to interpret the experimental group's increased use of energy concepts differently? Could it be that they were trained for the duration of the study to use an "energy approach"? We believe it is not the case. The two groups had to prepare for the same final national examination and they had to cover the same syllabus in the same amount of time. Paying particular attention to one topic would impact negatively on the group's performance on other topics, which was not the case. In addition, as already mentioned, the experimental group did not have additional classes. Also, the data on test and examination performance indicated overall improvement, not just for the energy problems. Furthermore, solutions maps were also constructed for problems on other topics, all showing that the experimental group made more use of general physics principles and less use of formula-based approaches (Gaigher, 2004). We thus believe that conclusions drawn from the solutions map on energy can be generalised to other topics in physics.

A Conceptual Approach to Problem-solving

In the mid-year examination, the steps of the problem-solving strategy in written solutions were less prominent than expected; just more than 20% of the experimental group actually wrote down some of the headings recommended by the strategy. Did this mean that the strategy was regarded as of little value, or did it mean that time was a factor? In their responses to questionnaires, most students complained about the time taken in writing steps, which could have been an obstacle to using the strategy in the examination (Gaigher, 2004). However, the strategy was not intended as a quick fix to apply in tests, but an ongoing process of learning to conceptualise and understand physics while learning to solve problems. Was it possible that the use of the strategy during the year had an effect on solutions even though headings were not always written down explicitly in the examination?

The examination scripts were then scrutinised for evidence of some influence of the strategy. Footprints of the strategy were found in the form of diagrams and written explanations that were not explicitly required by problems. Diagrams were used by 52.5% of the experimental group in at least one question, and written statements by 32.5%. Among the control group, only 26.6% of students used diagrams in at least one question, and 13.8% used written statements. This seemed to indicate that using the strategy during the year did promote the use of diagrams and writing during problem-solving among the experimental group. The term "conceptual approach" was introduced to indicate the use of diagrams or written explanations that were not explicitly required, while the term "algebraic approach" refers to solutions that rely only on algebra.

A chi-square analysis was performed, with each student's problem-solving approach classified as either conceptual or algebraic. A student was classified as a conceptual problem solver if he/she used either diagrams or written comments at least once where it was not explicitly required, while students who did not meet this requirement were classified as algebraic problem-solvers. Table 2 shows almost two-thirds of the experimental group to be conceptual problem-solvers, with the situation reversed for the control group. The analysis indicated a significant difference between the problem-solving approaches of the two groups, with $\chi^2 = 18.96$, which is significant at the 0.001 level.

Table 2. Contingency table comparing the treatment and control group's problem-solving approaches

Approach	Treatment group	Control group	Total
Conceptual	52 (65%)	36 (33%)	88
Algebraic	28 (35%)	73 (67%)	101
Total	80	109	189

Note: The percentages in brackets represent student numbers as percentages of the relevant groups.

How did the solutions map relate to the notion of conceptual and algebraic approaches? The conceptual/algebraic classification of students was now utilised to define an index by which the popularity of routes among conceptual problem-solvers could be assessed. The conceptual index (C-index) of group for a specific route was defined as the number of conceptual problem-solvers who followed that route divided by the total number of students in the group who followed that route. For example, 19 students in the experimental group followed Route i, while 17 of these students were classified as conceptual problem solvers; this ratio of 17/19 is referred to as a conceptual index of 0.895 for the experimental group in Route i.

The energy conservation problem is used to demonstrate how the C-indexes differed for Routes i and vii. Route i represents correct solutions while Route vii refers to the formula-based solution, relying on the constant acceleration formula $v^2 = u^2 + 2as$, discussed earlier. Table 3 summarises the C-indexes for these routes, separately calculated for the two groups. Also shown are the C-indexes of both groups for the entire mid-year examination paper, serving as baseline for comparison.

Table 3. Conceptual indexes for the correct and the formula-based routes for the energy conservation problem, as well as for the mid-year examination paper

	Treatment group		Control group	
Route	**Number of students**	**C-index**	**Number of students**	**C-index**
(i) Correct	19	0.895	10	0.600
(vii) Formula-based	11	0.455	23	0.391
Examination	80	0.65	109	0.33

Note: The number of students refers to the actual number of students in the group following a particular route.

Route i, representing correct solutions, had large C-indexes for both groups. For the experimental group, 89.5% of those who followed a correct route were conceptual problem-solvers, well above the groups' baseline of 65% (the baseline refers to the fraction of students who were classified as conceptual problem with regard to the mid-year examination, as shown in Table 2). The corresponding value was 60.0% in the control group, well above the 33% baseline. It then follows that conceptual problem-solvers were more successful than algebraic problem-solvers at constructing the correct solution, in both groups.

With respect to the formula-based solution (Route vii), only 45.5% of the experimental group were conceptual problem-solvers, below the 65% baseline, indicating that conceptual problem-solvers in the experimental group were less inclined to resort to a formula-based approach. In the control group, 39.1% were conceptual problem-solvers, slightly above the 33% baseline, which indicated that the conceptual approach produced less success in the control group. This is a reminder that successful problem-solving does not result from diagrams or written words as such, but from the meanings assigned to the diagrams and words. It seems that, for the experimental group, these meanings were better constructed while students made translations between the knowledge domains, a skill practised by the regular use of the problem-solving strategy during the year.

In summary, the conceptual/algebraic classification indicated that:

- Learning the problem strategy fostered the development of a conceptual approach to problem-solving in the experimental group.
- Conceptual problem-solvers were more successful in constructing the correct solution in both groups.
- Conceptual problem-solvers in the experimental group tended not to resort to the formula-based route.

Discussion

The effect of the problem-solving strategy on conceptual understanding was explored by solutions maps and conceptual indexes. The solutions map for the energy conservation problem indicated enhanced conceptual understanding among the experimental group in three ways, namely, an increased use of general physics principles, an increased ability to construct the correct solution amidst many incorrect possibilities, and a decreased use of formula-based solutions.

The conceptual index calculations showed that conceptual problem solvers were more likely to construct the correct solution to the energy problem, in both groups, while formula-based solutions were reduced among the conceptual problem solvers in the experimental group. As mentioned earlier, the particular problem could not be solved by a simple formula, leading us to interpret the increased ability to construct the correct solution amidst many incorrect possibilities as evidence of conceptual understanding. The conceptual index calculations therefore indicate that a conceptual approach can be related to conceptual understanding, in both groups. Furthermore, since there were significantly more conceptual problem-solvers in the experimental group, we infer that the experimental group demonstrated enhanced conceptual understanding.

In view of the prominence of the conceptual approach in the experimental group's solutions, we propose that the implementation of the structured problem-solving strategy not only supported the development of conceptual understanding, but also fostered the emergence of a conceptual approach to problem-solving. Eventually, the conceptual approach could replace the problem-solving strategy as a way to approach problem-solving as well as in supporting the development of conceptual understanding.

How would the implementation of the problem-solving strategy support the development of conceptual understanding? We propose a mechanism in terms of Greeno's four knowledge domains: Applying the steps of the strategy prompts actions that rely on translations between the domains. In the case study analysed in the previous section, it was found that, even for a simple problem, many translations are possible between the four knowledge domains when following the steps of the strategy. It is not suggested that the students have conscious knowledge of the domains, but rather that they experience more ways of understanding physics. Instead of focusing on algebra, they use multiple representations to align concrete situations with models and abstract concepts, which are then expressed algebraically to become part of an effective process. It is proposed that conceptual understanding develops from making translations made while traversing the four knowledge domains. Translations create links between a particular concrete situation and particular physics concepts. For example, the problem of the car going smoothly down a slope was linked to the energy conservation principle. However, energy conservation can be linked to many other concrete situations, while cars can also be linked to many other physics concepts. For a collection of problems on a particular topic, the translations thus create links between that particular topic and different concrete situations, while a particular concrete system can be linked to many topics encountered during the course. Applying the strategy while working through various problems and various topics thus results in a multitude of links between the four knowledge domains. New associations are made and existing associations are reinforced to develop familiarity with how physics concepts are relevant to concrete situations. The resulting network of links that develop between concrete situations, physics concepts, models, and symbols amounts to a broad conceptual understanding of physics.

Regarding the development of a conceptual approach, the case study indicated that many translations between the concrete, model and abstract domains are required before translation to the symbolic domain. In the first four steps of the strategy, translations are made between the written problem statement, a visual representation of the concrete situation, physics models, and abstract physics concepts. Only in the fifth step are the physics concepts translated to mathematics, and mathematical operations are limited to the sixth step. In the final step, the mathematical solution is translated back to the concrete situation. The reliance on the mathematical aspect of the problem-solving process is therefore reduced. At the same time, the strategy promotes the search for physics models and concepts relating to the particular concrete problem situation. The shift in focus from mathematics to diagrams and written arguments thus creates an understanding of what a solution entails; that it is not about algebra, but about modelling a concrete situation in terms of physics principles. In due course, students may realise that the steps of the strategy are road-signs to solutions, not the road itself. Students may deliberately or unconsciously stop writing headings while retaining useful habits such as drawing diagrams and writing explanations, thereby adopting a conceptual approach. Hence regular use of the strategy fostered the development of a conceptual approach; the formal structure of the strategy made way for diagrams and written arguments as a normal part of solutions. The acts of constructing diagrams and formulating arguments characterising a conceptual approach required translations between knowledge domains in the same way as when making explicit use of the steps of the problem-solving strategy; therefore, a conceptual approach supports the development of conceptual understanding in the same way as does the problem-solving strategy.

In conclusion, the study showed that structured problem-solving enabled disadvantaged students to step out of the confines of the symbolic knowledge domain and interpret concrete situations in terms of physics concepts and models. In the classroom, following the steps of the problem-solving strategy requires new patterns of behaviour. Students react to prescribed steps, making decisions of what needs to be done in each step while working on a problem. The teacher-dominated approaches can make way for students' participating in knowledge construction. Although the effect of the problem-solving strategy on teacher development was not investigated in this study, it is possible that poorly trained science teachers could benefit in much the same way as their students. This intervention can be particularly useful in disadvantaged schools as it does not rely on expensive equipment or intensive programs, but rather on unlocking human potential in the classroom to contribute to the development of physics teaching and learning.

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