Development of an approach to incorporate proportional hazard modelling into a risk-based inspection methodology

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Abstract

Purpose

Industry decision makers often rely on a risk-based approach to perform inspection and maintenance planning. According to the Risk-Based Inspection and Maintenance Procedure project for the European industry, risk has two main components: probability of failure (PoF) and consequence of failure (CoF). As one of these risk drivers, a more accurate estimation of the PoF will contribute to a more accurate risk assessment. Current methods to estimate the PoF are either time-based or founded on expert judgement. This paper suggests an approach that incorporates the proportional hazards model (PHM), which is a statistical procedure to estimate the risk of failure for a component subject to condition monitoring, into the risk-based inspection (RBI) methodology, so that the PoF estimation is enhanced to optimize inspection policies.

Design/methodology/approach

To achieve the overall goal of this paper, a case study applying the PHM to determine the PoF for the real-time condition data component is discussed. Due to a lack of published data for risk assessment at this stage of the research, the case study considered here uses failure data obtained from the simple but readily available Intelligent Maintenance Systems bearing data, to illustrate the methodology.

Findings

The benefit of incorporating PHM into the RBI approach is that PHM uses real-time condition data, allowing dynamic decision-making on inspection and maintenance planning. An additional advantage of the PHM is that where traditional techniques might not give an accurate estimation of the remaining useful life to plan inspection, the PHM method has the ability to consider the condition as well as the age of the component.

Research limitations/implications

This paper is proposing the development of an approach to incorporate the PHM into an RBI methodology using bearing data to illustrate the methodology. The CoF estimation is not addressed in this paper.

Originality/value

This paper presents the benefits related to the use of PHM as an approach to optimize the PoF estimation, which drives to the optimal risk assessment, in comparison to the time-based approach.

Keyword: Risk based inspection (RBI), proportional hazard model (PHM), probability of failure (PoF), consequence of failure (CoF).

1. Introduction

Failure of pressure vessels presents a major risk to many industries and therefore the design and integrity management of pressure vessels are regulated through international codes and standards. Traditionally one of the principal integrity management measures prescribed by these standards has been to perform over-pressure testing at defined intervals during the life of a vessel, which ensures that defects which may exist and do not cause failure during such a test would safely propagate without reaching critical dimensions, before the next test is performed. Due to the cost of performing these tests, an alternative integrity management approach has been developed, which is called risk-based inspection (RBI).

The RBI approach replaces regulatory over-pressure testing on pressure vessels with maintenance actions such as repair and replace based on inspection non-destructive testing (NDT) results. Because failure of pressure vessels can have catastrophic consequences, the inspection regime (where and when to do inspections) needs to be risk-based.

The RBI approach is prescribed in industrial standards, such as the American API 580, for the petro-chemical industry and the European CWA 15740 standard for the power generation industry. This risk of failure of the pressure vessel is calculated as the product of CoF and PoF. Often, the CoF is a given and easy to define, but the PoF is more difficult and is the parameter which one wants to reduce through the implementation of the RBI regime.

The standards define various ways (qualitative and quantitative) to estimate PoF. Quantitative methods are required for critical vessels and failure positions on such vessels. Preferably, the quantitative method will be based on failure models, where the relationship between the inspection results (condition parameters) and remaining useful life is known. In cases where the failure model is not known or accurate, the next best practice would be to use failure statistical methods and to supplement this with Bayesian methods when data is scarce. Since RBI implies that inspections are performed and therefore that condition parameters will be available, it is argued in this paper that in these cases, a need exists for a PoF estimation method which uses both the statistical failure statistics, as well as the condition data, to estimate the PoF.

A proportional hazards model (PHM) is proposed to serve this purpose. This model was originally developed in the health and medical sciences (Cox, 1972) but has now been in use in reliability engineering for a number of years (Vlok,1999). In this paper, the adoption of the PHM method for PoF estimation in RBI, is proposed and its application is demonstrated at the hand of a case study. With the application of PHM into RBI, the inspection decision is not only defined in terms of frequency and acceptability criteria, but the decision-making process becomes dynamic because real time condition data is used to update the PoF.

In section 2 a review of the RBI approach is presented. Section 3 presents an overview of existing methods to estimate PoF in RBI. Section 4 addresses the proposed method to estimate PoF based on PHM. Then section 5 addresses a case study where the proposed approach is applied on bearings. The results and conclusions follow the case study.

2. Risk Based Inspection

Industry prefers risk-based approaches to schedule inspection and maintenance programs (Giribone & Valette, 2004). Risk based inspection is generally presented as an approach to prioritize and plan inspection. RBI has in the past been predominantly applied on pressure vessels.

Risk assessment can generally be addressed as follows:

- Qualitative or screening level (expert judgement)
- Semi – Quantitative (rule based analysis)
- Quantitative (probabilistic, statistical, mathematical modelling).

The following sections address existing approaches to PoF modelling, based on the CWA 15740 standard. This is similar to the API 580 approach.

2.1 Probability of failure estimation in the CWA methodology

2.1.1 Qualitative assessment (Screening level)

(Singh & Pretorius, 2017) describe the basic steps of the European methodology, which address the risk analysis on multiple levels, progressing from the initial screening step to a detailed quantitative assessment.

During the screening stage, the assessment of risk consists of screening the components. The PoF estimation is performed by determining several specific criteria that could influence the PoF.

The screening analysis is relatively fast, simple, and cost-effective. During screening, component risks are ranked using criteria like ‘high’, ‘medium’ and ‘low’ risk levels. After screening the components, semi quantitative analysis can be performed for components that fall into high and medium risk categories, while components in the low-risk category continue to be subjected to the required maintenance.

The probability of failure at the screening stage is assessed by considering criteria such as:

- The component ages.
- Presence of degradation
- Year of last inspection
- Rate of degradation
- Design concerns
- Prior repairs of damage
- Rate of degradation, etc.

with each criterion having an associated weighting. The weight of each criterion is assigned according to the level of influence it has on the probability of causing failure. Furthermore, each criterion is scored relative to a qualitative measure of its influence on the component.

To produce a precise probability of failure PoF, the score criterion expressed by C is multiplied by the weighting of the criterion expressed by W. The sum of that product for different components is then multiplied by the generic failure frequency GFF, which is a factor used based on experience to identify failure frequencies of different components. GFF is typically developed using expert judgement and history of components failure.

$$\mathrm{PoF} = \{[(C1 \times W1) + (C2 \times W2) + (C3 \times W3)] \times (GFF)\} \tag{1}$$

2.1.2 Semi – Quantitative assessment (Level two risk assessment)

Once the low risk components have been screened out as described in the previous paragraph, the high and medium risk components go to the semi-quantitative assessment (Narain Singh & Pretorius, 2017).

The purpose of the level two PoF assessment is to determine the detailed factors that may affect the identified damage mechanisms for a given component. The generic failure frequency (GFF) is once again used, but for this level, actual failure frequencies obtained from industry experience, are used where available. In instances where no industrial GFF data is available, the RBI team will revert to the GFF values that were used in previous PoF determination. The level two risk calculation is performed in the same manner as the level one risk calculation. However, in the level two PoF assessment the number of criteria for the component under analysis is greater than the previous assessment level.

These criteria could be:

- Component age
- Total starts per year
- Time since last inspection
- Rate of degradation
- Presence of hot spot
- Nominal operating temperature
- Corrosion susceptibility
- Frequency of temperature excursions
- Severity of temperature excursions
- Design concerns

2.1.3 Quantitative assessment (Level three risk assessment)

The fully quantitative or detailed approach is essentially based on calculating the remaining useful life for the component under analysis. No further calculation is required when the calculation indicates that there is an acceptable period before failure. Otherwise even more detailed calculations are performed.

In the CWA standard, the detailed risk assessment follows almost the same rules as in the screening level, although in greater detail. For most critical components, the CWA procedure suggests more detailed analysis where the damage mechanism can be identified, and the degradation rate obtained. The PoF can then be estimated (Jovanovic, 2004).

The quantitative methods for determining the PoF described above, can be divided into two discernible approaches. In the case where an accurate failure model is available and expected loading and environmental conditions are quantifiable, the life expectation for an identified failure mode is calculated. In this calculation, the ageing damage accumulation is estimated, and forms the basis of risk-based decisions in terms of inspection schedules. Such inspections monitor the damage parameters, such as crack sizes or corrosion damage and is essentially a condition monitoring activity. Depending on the observed damage found during these inspections compared to the failure model results, remaining useful life (RUL) calculations are performed to trigger repair/replace decisions, or updated future inspection schedules. This includes the case where RBI implementation is done on existing equipment, which would already have accumulated damage. Again, future inspection schedules are based on a calculated RUL, with a failure model being available and pre-existing damage parameters having been measured.

In the case where an accurate failure model is not available, inspection schedules are based on historical or generic failure statistics, to estimate failure rates and probabilities. In this second approach, the inspections, or condition monitoring, are also aimed at finding damage (eg. cracking or corrosion damage), but since a failure model is not available to estimate a RUL, any indication of damage would typically lead to repair/replace actions.

2.1.4 RUL and the P-F Curve

The first quantitative approach for RBI described above, is essentially based on determining the RUL for the asset or component under analysis. The so-called P-F curve (Wiseman, Lin, Gurvitz, & Dundics, 2006) is one of the quantitative methods which could be used when condition parameters are be utilised to estimate the RUL for a given failure mechanism.

The P-F curve is an important tool when managing an asset. It is a common way to represent the behaviour of an asset before a functional failure occurs. It shows the declining performance of an asset or a component over time, until it reaches a functional failure. Since failure is a process which can be caused by wear, fatigue, corrosion etc., these failure modes do not immediately cause the asset to fail. As such, the deterioration can be tracked, and the P-F interval can be used to define the inspection policy.

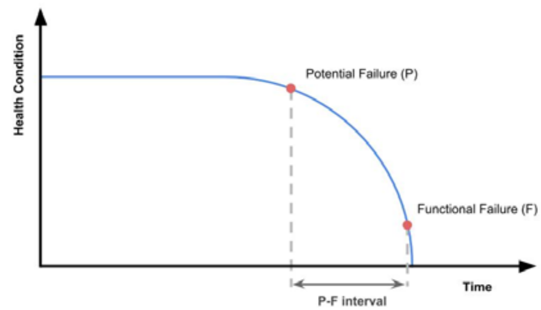


Figure 1: P-F Curve

P expresses the potential failure point, which is the point where it is possible to detect that failure is about to occur. The detection of P is usually done by means of condition indicators. F is the functional failure point or the failed state.

The P-F interval is the time between the potential failure point to functional failure, and the P-F interval length is an important key to defining the inspection frequency. Determination of P is usually flagged at the attainment of specified values of some condition indicators. This could however be challenging.

2.1.5 PoF estimation using failure statistics

The second quantitative approach for RBI described above, uses statistical models, which are based on collecting data from plant experience for given components. Generic databases often assume that the failure rate is shaped as a 'bathtub' over a component's lifetime and is divided in three zones (Jovanovic &Auerkari, 2002). If the failure rate is constant, the time to failure is exponentially distributed.

The failure rate is for the generic curve is $= \frac{n_f}{t}$, where n_f is the number of observed cycles to failure and t is the total operating time. The mean time to failure $MTTF = \frac{1}{\lambda}$ with λ the failure rate.

2.1.6 Bayesian Approach

Augmenting the quantitative approach based on failure statistics can be done using Bayesian Statistics. Bayes' rule for events can be expanded to define a Bayes' rule for random variables and their distribution functions. The expanded rule can be used to combine a prior distribution and a likelihood function to produce a posterior distribution. This posterior distribution can subsequently be used as an input in risk analysis. The Bayes' rule can be written as (Guyonnet, 2009):

$$P(A|E) = \frac{P(A) \times P(E|A)}{P(E)} \tag{1}$$

Usually, decision making based on statistical lifetime data as well as condition monitoring data, requires a large set of data which often is incomplete or missing. To overcome that problem, the use of expert judgement is accommodated by means of Bayesian statistics. The essential element is the revision of probabilities based on new information (Jardine & Tsang, 2013).

As previously argued, risk-based approaches for the scheduling of the inspections are becoming common. The Bayesian approach is well suited for this because it allows a systematic integration of expert judgement and data obtained from ongoing inspections (Aven & Pörn, 1998).

2.2 Summary of existing methods to estimate PoF in RBI

Section 2.2 described current methods utilised to estimate the PoF for risk assessment. *Figure 2* below represents a framework summarizing the existing mehods described in the previous section.

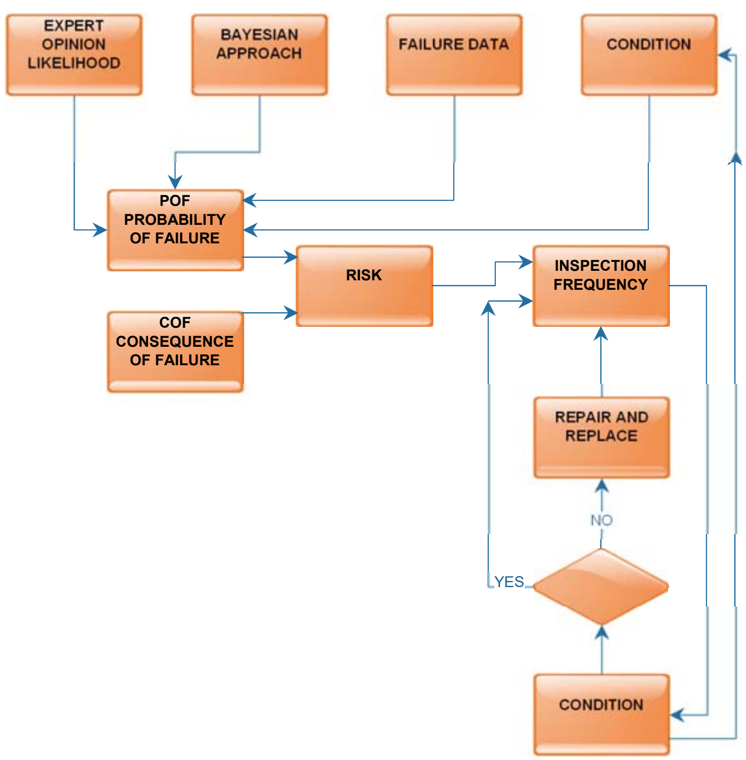


Figure 2: Framework summarising the existing methods.

Figure 2 represents a framework for the existing procedure to estimate the Probability of Failure (PoF) to assess risk. The framework is essentially constituted by four main blocks (Expert opinion likelihood block, Bayesian Approach block, Failure data block and Condition block) which are existing PoF estimation methods serving respectively as input to the risk assessment in the framework. A closed loop part which captures risk indices, is used to decide either to repair, to replace or let the component run according to the severity of the risk index.

In summary, Figure 2 represents the progression in risk assessment, starting from the non-quantitative approach, which is prevalent and offers no dynamic view of risk assessment, to the time-based approach, which may lead to the early replacement of components that still have useful life. A condition-based approach would resolve the shortcomings related to the non-quantitative and time-based approaches by tracking the condition of a component. Being able to estimate the remaining useful life of a component, allows inspection and replacement to be planned. However, the condition-based approach relies on the availability of an accurate failure model. When this is not available the time-based approach would be the only option, even though the inspections performed because of the RBI assessment, will continuously add information, which will be under-utilised, only being used to inform replacement/repair decisions based on conservative acceptance criteria. Hence, this research proposes to combine the condition-based approach with component age, by means of a proportional hazard model (PHM). The following section addresses the suggested method to respond to the shortcomings highlighted in this paragraph.

3. Newly proposed method to estimate PoF

Figure *3* is the same as Figure 23 , the only difference being that an additional block is introduced for the proportional hazard model, which is the proposed method to estimate the PoF in cases where the condition data alone is not sufficient to predict an accurate RUL. The expected benefit for this method is justified by its ability to combine the age and condition for a better prediction of RUL compared to using only failure statistics.

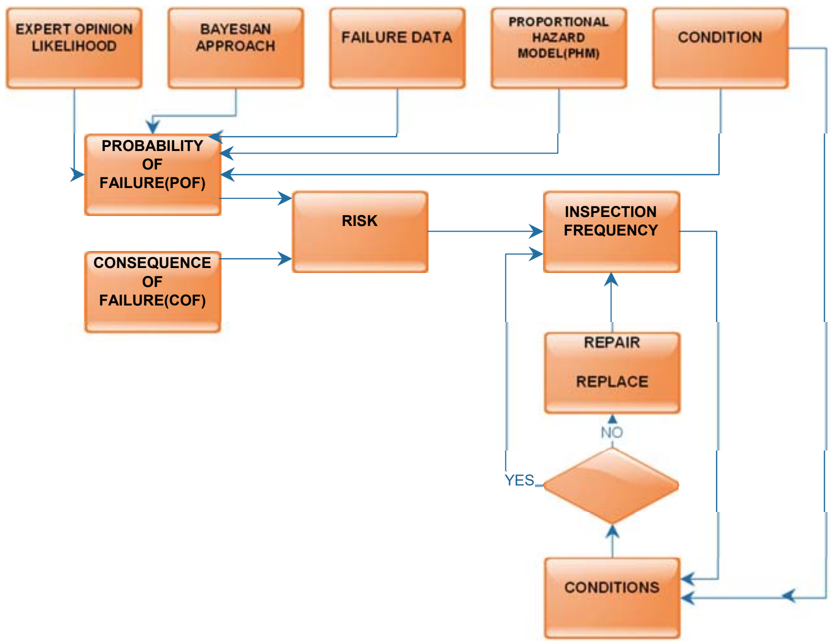


Figure 3: Framework with the suggested method.

The following section addresses more details on the PHM.

3.1 Estimation of the PoF based on Proportional Hazards Model (PHM).

3.1.1 Proportional hazards model

The proportional hazard model incorporates the effects of covariates or explanatory variables on the distribution of lifetimes. Covariates are any measured parameters that are thought to be related to the lifetimes of components. For each given time, a covariate causes an increase or decrease in the hazard rate with respect to the baseline hazard rate. PHM may therefore be considered as a statistical procedure for the estimation of the risk of a component to fail, based on condition information obtained from a conditional monitoring process (Jardine & Tsang, 2013).

The PHM is now one of the most popular statistical models used for survival analysis. Its popularity arises from the fact that the PHM is part of a broader class of survival analysis models which provide information on the duration between an identifiable start and the occurrence of an event (Leclere, 2005). A key feature of the PHM approach is that it captures time series variation of the covariates and calculates their influences on the probability of a failure event occurring.

The PHM is often presented in terms of the hazard rate formula of a Weibull distribution:

$h[t, Z(t)] = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1} exp\{\sum_{i=1}^{m} \gamma_i Z_i(t)\}$ (2) where

$h[t, Z(t)]$ is the hazard function, $Z_i(t)$ are the covariates at time t, $\frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}$is the baseline hazard function with β the shape parameter, and η the scale parameter. The Weibull parameters, which allow for the construction of the baseline part of the model, are determined by maximizing the likelihood function.

3.1.2 Blending Hazards and Economics

The PHM provides us with an approximate risk of failure for a component subjected to condition monitoring, with age as event data and condition parameters as covariates. The information obtained from the PHM could be utilized to obtain economic benefits. (Vlok, 1999) demonstrated that the optimal economic replacement decision can be based on the PHM risk estimation, by minimizing the total cost. Similarly, in this paper it is proposed to use the PHM for the PoF estimation for an optimal RBI policy.

3.2 Advantages related to the proposed method to estimate the PoF

One of the advantages related to the application of PHM to the RBI methodology is that the PHM uses instantaneous condition data at a given time, which leads to dynamic decision making in inspection scheduling. Another benefit is that the PHM approach is intended for situations where the covariates provide some indication of an approaching failure and, when combined with age, gives an improved indication of the risk of failure, compared to using only age as indicator. These advantages lead to an improved estimation of the PoF.

4. Case study

In this section, a case study is presented to illustrate the methodology, using failure data obtained from the simple but readily available IMS bearing data set. The RBI methodology is commonly applied to pressure vessels where the need for such investigations is higher to minimize the risk of accidents. However, in this research, the simple bearing data set is used for illustrative purposes, simply because datasets like these are very well-documented and well-understood. A similar dataset for pressure vessels is not readily available publicly.

The data for the IMS case study was generated by the NSF I/UCR Centre for Intelligent Maintenance Systems (IMS-www.imscenter.net) with support from Rexnord Corp. The following is a description of the testing configuration from which the test results were obtained.

The bearing test rig hosted four test bearings on one shaft. The shaft was driven by an AC motor and coupled to the shaft via rub belts. The rotation speed was kept constant at 2000 rpm.

A radial load of 6000 lbs. was applied to the shaft and bearing by a spring mechanism. All the bearings were force lubricated. An oil circulation system regulated the flow and the temperature of the lubricant. A magnetic plug installed in the oil feedback pipe, collected debris from the oil as evidence of bearing degradation. The test was discontinued when the accumulated debris adhering to the magnetic plug exceeded a certain level and caused an electrical switch to close.

Four Rexnord ZA-2115 double row bearings were installed. Each bearing had 16 rollers in each of the two rows, a pitch diameter of 2.815 in., a roller diameter of 0.331 in. and a tapered contact angle of 15.17^0. A high sensitivity accelerometer was installed on each of the four bearings to record bearing housing vibration. Four thermocouples were attached to the outer race of each bearing, to record bearing temperature for monitoring the lubrication. Vibration data was collected every 20 minutes. The data sampling rate was 20 kHz and the data lengths were 20480 points.

4.1 Simulations and Results.

4.1.1 Fault diagnosis of bearings

For this case study, the root mean square (RMS) and kurtosis of the measured acceleration signals are used as covariates for the proportional hazards model.

The RMS value is associated to the energy of the signal. Usually, the appearance of a defect can be detected by an increase of the vibration level. The RMS values can be compared with levels while the bearing is still undamaged.

Kurtosis is the fourth statistical moment of the vibration signal, normalized by the standard deviation raised to the fourth power.

RMS and kurtosis are often used for condition monitoring because of their simplicity of application and interpretation.

4.1.2 Graphical representation of the data (RMS and Kurtosis)

The bearing test RMS and kurtosis results are depicted below for bearings 3 and 4. The interest in bearings 3 and 4 is justified by the fact that at the end of the test-to-failure experiment, defects occurred for bearings 3 and 4.

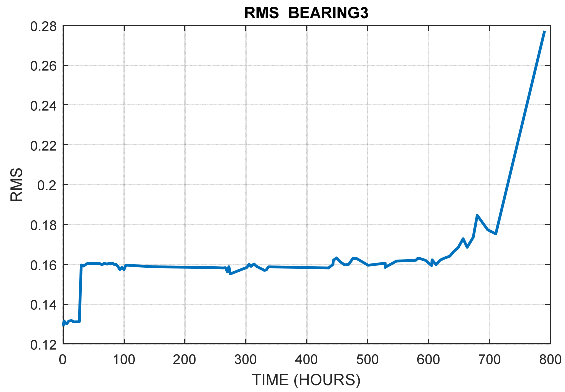


Figure 4: RMS as function of time for bearing 3

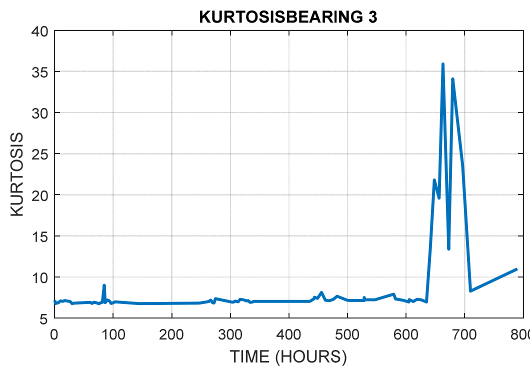


Figure 5: Kurtosis as function of time for bearing 3

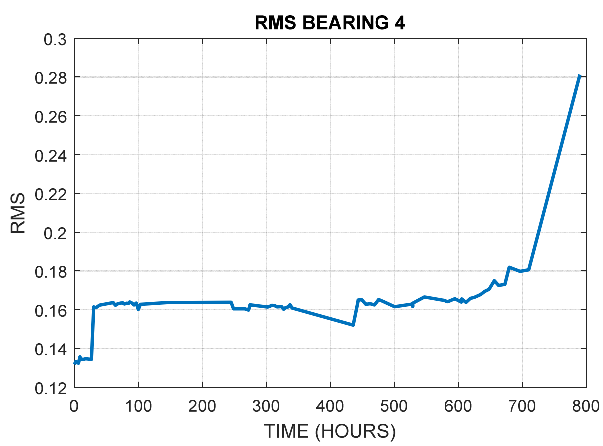


Figure 6: RMS as function of time for bearing 4

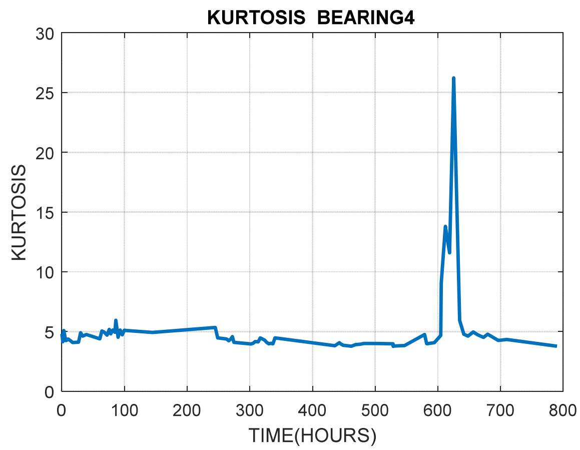


Figure 7: Kurtosis as function of time for bearing 4

4.2 Kurtosis and RMS as condition indicators

Kurtosis is a useful feature to indicate an initially localised failure in a bearing with the kurtosis rising suddenly as the failure occurs. As the failure propagates around the bearing the fault becomes more distributed, the kurtosis generally drops (Randall, 2011) and kurtosis loses some of its value as fault indicator. Figures 8 and 9 below show both the kurtosis and RMS behaviour for bearing 4, they express the trend of the condition over time as it is the case in figure 1.

Therefore, kurtosis and RMS are both indeed useful condition indicators, but they nevertheless provide only partial information on the condition of the component. It is therefore proposed here to combine failure time information with the condition information to enhance the risk assessment. As previously discussed, the combination of age and condition information is enabled by the application of PHM. This is shown later in figure 12.

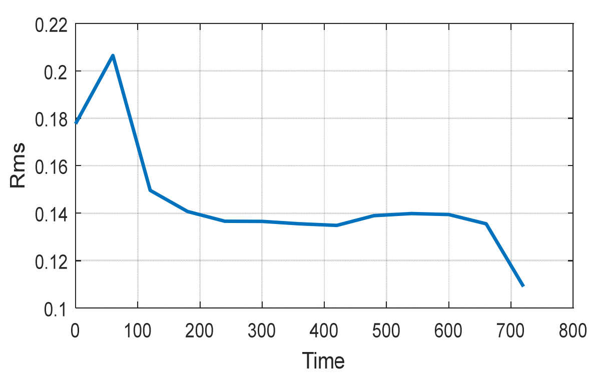


Figure 8: RMS versus time

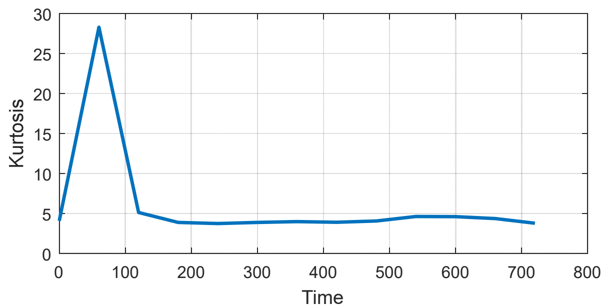


Figure 9: Kurtosis versus time

4.3 Time based approach and risk assessment

This section addresses the time-based approach for RBI. The previous section highlighted that the condition indicators kurtosis and RMS alone are not adequate for identifying machine condition. This can be dealt with by either applying more sophisticated signal processing to find better condition indicators, or as it is done here, this paper is a mixing of condition with age (time-based) to make better decision on RBI, before reaching that mixing of age and condition, let us first have a look on the time-based approach.

The same steps undertaken in section 4.4.1 to estimate the regression coefficients required to build the PHM, are also needed for the 2 parameters Weibull, or time-based approach. Equation (3) below, which is the log likelihood function for the 2 parameters Weibull distribution, must be maximized to determine the regression parameters.

$$\Lambda = Nln(\beta) - N\beta\ln(\eta) + (\beta - 1)\sum_{i=1}^{N}\ln(t_i) - \sum_{i=1}^{N}(\frac{t_i}{\eta})^{\beta} \qquad (3)$$

The maximum likelihood of the log likelihood function given by equation (3) gives the following equation:

$$\frac{1}{\beta} = \frac{\sum_{i=1}^{N} t_i^{\beta} lnt_i}{\sum_{i=1}^{N} t_i^{\beta}} - \frac{1}{N}\sum_{i=1}^{N} lnt_i \qquad (4)$$

The determination of the shape parameter β in equation (4) is normally dealt with numerically. Using a MATLAB code, the output gave a shape parameter $\beta = 4$ for the bearing data.

The differentiation of equation (3) with respect to η gives:

$$\eta = (\frac{1}{N}\sum_{i=1}^{N} t_i^{\beta})^{1/\beta} = \frac{1}{4}\sum_{i=1}^{4} t_i^{4} = 360\ hours \qquad (5)$$

With β and η known, the hazard or risk of failure for the time- based approach is:

$$h(t) = \frac{4}{360}\left(\frac{t}{360}\right)^{4-1} \quad (6)$$

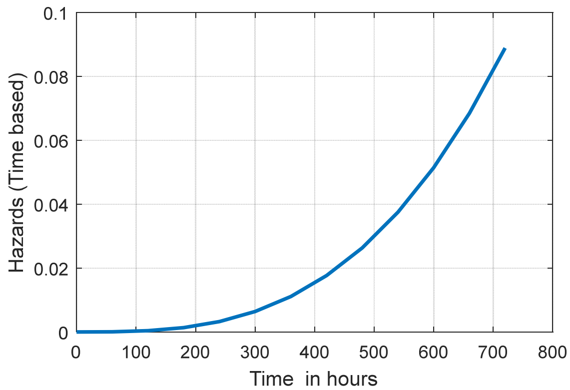
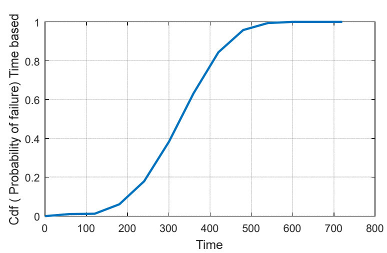


Figure 10: Hazards versus time (time-based)

The cumulative distribution function (cdf) or probability of failure curve related to the hazard represented in figure 12, is given in figure 11 below.

Figure 11 : Probability of failure for time-based



The economical approach for time-based approach consists of finding an optimal time of replacement which minimizes the cost per unit time. Referring to Jardine et al. (2013), the optimal preventive replacement time of a component subject to breakdown is given by:

$$C(t_p) = \frac{C_p \times R(t_p) + C_f \times (1 - R(t_p))}{t_p \times R(t_p) + M(t_p)(1 - R(t_p))} \quad (7)$$

We consider a 3/1 cost ratio so that the failure cost C_f in South African Rands (ZAR) is three times the preventive cost C_p. If we further assume that $C_p = 2000\ ZAR$ and $C_f = 6000\ ZAR$, the results after computation are given below:

Table 1: Table of results for time-based approach.

	Time (Hours)	Reliability R(t)	Probability of failure F(t)	Cost per unit Time C(tp)
1	0	1	0	Inf
2	60	0,999	0,001	33,410
3	120	0,987	0,0123	17,2743
4	180	0,9394	0,0606	13,1416
5	240	0,8208	0,1792	13,1323
6	300	0,6174	0,3826	16,2892
7	360	0,3679	0,6321	24,5829
8	420	0,1568	0,8432	49,3550
9	480	0,0424	0,9576	154,5658
10	540	0,0063	0,9937	893,1391
11	600	0,0004	0,9996	1,1228e+04
12	660	0,0004	0,9995	3,6433e+05
13	720	0	1	7,4051e+07

Considering table 1, the optimal expected replacement cost per unit time is 13,13 Rands/day.

The following section consists of estimating the expected optimal cost based on Risk (PHM) instead of time as it is the case in this section. The cost per unit time obtained for time-based will be compared to the cost per unit time for PHM to evaluate the advantages of PHM over the time-based approach.

4.4 PHM Approach

4.4.1 Building the PHM

The PHM model incorporates both the kurtosis and the RMS.

The first step of this investigation consists of estimating the regression coefficients β, η, γ required to build the PHM. (Carstens & Vlok, 2013) argue that the log-likelihood function represented by equation (8) should be maximised:

$$l(\beta,\eta,\overline{\gamma}) = r ln\left(\beta/\eta\right) + \sum_i ln\left[(T_i/\eta)^{\beta-1}\right] + \sum_i \overline{\gamma} \times \overline{Z_i}(T_i) - \sum_j \int_0^{T_j} \exp(\overline{\gamma}\, \overline{Z}_j(t)\; d(t/\eta)^{\beta} \quad (8).$$

The outcome from this optimisation renders a shape parameter β=5, a scale parameter ή=780 hours the weight of the covariate γ1=0.1525 (weight of the RMS) and γ2=6.9101 (for the kurtosis)

The above regression parameters are obtained from the maximum likelihood equation (8):

The risk equation corresponding to the above parameters with kurtosis and RMS as covariate, is given by:

$$h[t, z(t)] = \frac{5}{780}\left(\frac{t}{780}\right) \exp[0.1525\, RMS + 6.9101\, KURT] \qquad (9)$$

A graphical representation of this equation for the PHM with RMS and kurtosis as covariates is given as figure 12 below.

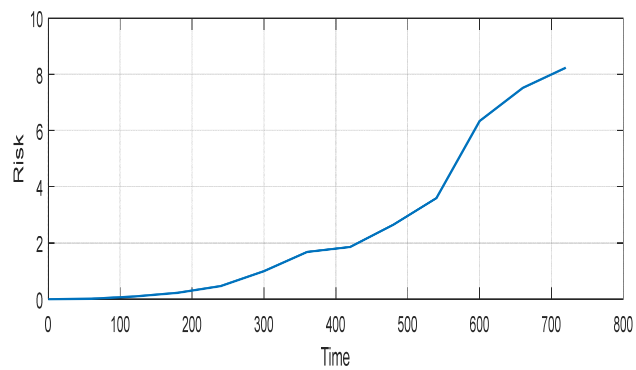


Figure 12: Risk versus time for the bearings

4.4.2 Probability of failure estimation using the PHM

The probabilistic hazard rate $h(t,(Z(t))^{-})$ is a function depending on the time only, without considering that the condition of the component, is given by the formula.

$$h[t,(Z(t))^{-}] = \frac{f(t,\overline{Z(t)})}{R[t,\overline{Z(t)}]} \tag{10}$$

The probability density function f(t) can be determined by the product of the hazard rate h(t) and reliability R(t) means $f(t) = h(t) \times R(t)$. However, such probabilistic approach is limited because it is only time-based. Cox addressed this problem in the PHM by assuming that the hazard rate of an item is the product of a baseline hazard rate h_0(t) and a functional term which is function of time and covariates (Vlok,1999).

Then

$$h[t,\overline{Z(t)}] = h_0(t) \times \exp(\gamma,\overline{Z(t)}).$$

According to Vlok (1999), the reliability of the component taking account of the covariates is given by:

$$R[t,\overline{Z(t)}] = \exp[-\int_0^{T_i} \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}\exp[\gamma \times \overline{Z(t)}]$$

$$= \exp[-\int_0^{T_i} \exp(\bar{\gamma} \times \overline{Z(t)})\, d(^{t}/_{\eta})^{\beta}$$

After discretization, the equation above can be approximated by:

$$R[t,\overline{Z(t)}] = \exp\{\sum_{k=1}^{i} \exp(\bar{\gamma} \times \overline{Z_i^*(t_k)}) \times [(\frac{t_{k+1}}{\eta})^{\beta} - \left(\frac{t_k}{\eta}\right)^{\beta}\Big]\} \tag{11}$$

with $t_1 < t_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots. < T_i$ inspection time.

If the cumulative distributive function (cdf) is defined as the probability of failure at each inspection time could then be defined.

$$F[t,Z(t)] = 1 - R[t,Z(t)] \tag{12}$$

The PoF related to the condition of the bearings at a given time is given below as the cdf:

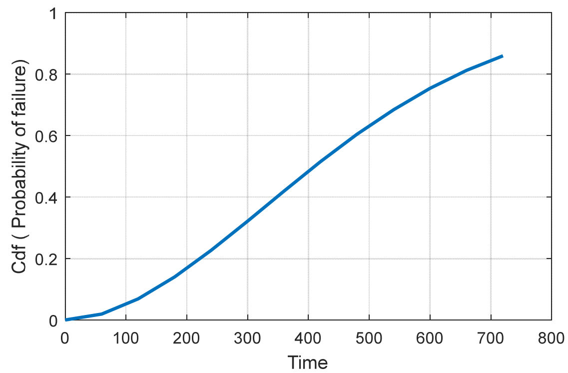


Figure 13: PoF (cdf) for the bearings

4.4.3 Interpretation of the results

The event and condition monitoring data were recorded under the form of acceleration at a given inspection time. To be able to build the PHM, event and condition monitoring data were computed in equation (8) which is the likelihood function. The outcomes from the likelihood function are displayed in figure 12 (risk curve). Observing the PHM parameters that are obtained, it can be concluded that both time and covariates have influence on the model and decision making.

During the computation of the data, it was observed that with RMS as the only covariate, the weight of the covariate was smaller compared to the case where kurtosis was considered as covariate. This observation does make sense since kurtosis is the fourth statistical moment, normalized by the standard deviation to the fourth power.

The aim is to incorporate the PHM to RBI leading to the estimation of the probability of failure. Firstly, the risk or hazard estimation, based on age and condition $h\left[t, \overline{Z(t)}\right]$ is estimated from condition and failure data. From this, the reliability (R), as well as the probability of failure (cdf) can be calculated. The result is depicted in Figure 13.

4.4.4 Optimal decision making based on PHM

The PHM provides us with the approximate risk of failing for the component based on the age and covariates (RMS and kurtosis for the case study in this paper). The information which is made available by the PHM should be utilized to obtain economic benefits.

For optimal decision making with the PHM in reliability, Makis and Jardine (2013) made a model available. The model specifies the optimal renewal policy in terms of an optimal hazard leading to the minimum life cycle cost (LCC). To be able to determine the hazard rate which leads to the minimum LCC it is needed to predict the behaviour of covariates.

Makis and Jardine's model assumes the covariate behaviour to be stochastic and approximating it by a non - homogeneous Markov chain in a finite space. Referring to that model, the expected average cost per unit time is a function of the threshold risk level given by:

$$\emptyset(d) = \frac{C_p + KQ(d)}{W(d)} \tag{13}$$

where, $Q(d) = P(T_d \geq T)$ represents the probability that failure replacement will occur and $W(d)$ the expected time until replacement and $K = C_f - C_p$.

For the case of this paper, the prediction of the covariates (RMS) behaviour was performed using a Markov chain and the following transition probability matrix (TPM) with five states was found. Each state expresses a given range of the covariate.

	State1	State 2	State 3	State 4	State 5
State 1	0	0,667	0,333	0	0
State 2	0	0,862	0,138	0	0
State 3	0	0,1875	0,75	0,0625	0
State 4	0	0	0	0	1
State 5	0	0	0	1	0

Then the optimal average cost per unit time found after computation using formula 13 is 6.92 Rand/day which is less than the 13.13 Rand/day for the time-based approach. This is one of the important benefits related to the use of PHM compared to the time-based approach.

4.5 Benefits of incorporating PHM into RBI

Risk-based inspection often follows either time-based or condition-based approaches. It is well-known that time-based approaches are often suboptimal, but even condition-based approaches are not always ideal, for example when the condition indicators do not vary monotonically with the remaining useful life. This is illustrated in section 4.2 of this paper.

Section 4.3 illustrates a time-based approach in which an optimal replacement policy based on age only leads to a cost per unit time equal to R13.13/day. However, an approach which uses PHM to incorporate age together with condition, seems to perform much better than a purely time-based approach. This is clear from the case study where the optimal average cost per unit time is R6.92 Rand/day.

Knowing that the expected outcome from RBI is an optimal inspection schedule, this demonstrates the advantage of the PHM compared to the time-based approach. PHM offers a lower cost inspection schedule. This is possible since the probability of failure estimation, which is an important input for risk computation, is improved. PHM uses real time data and allow dynamic decision on inspection and maintenance planning.

5. Conclusions

Risk based inspection is an ideal tool for asset management because of its ability to optimize the inspection schedule and extent of inspection, which contribute to the savings of cost and prioritize inspection on important components. This research suggests an approach based on proportional hazard model which optimizes the PoF estimation compared to the traditional time-based method.

PHM is used in this paper as a prognostic model involved in the computation of PoF estimation which drives the risk computation. It has been observed in this research that when using condition only to predict the remaining useful life (RUL), there is a probability to get inaccurate information because condition indicators such as RMS and kurtosis do not change monotonically with RUL.

The incorporation of PHM into RBI enables the use of time-based failure data with real time condition data, which helps the decision maker to make dynamic decisions on inspection schedule and maintenance planning.

Finally, this proposed approach is one of the suitable ways to optimize the quantitative approach for risk-based inspection since the PoF is well determined by means of the PHM.

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