THE TEMPERATURE AND LOAD STRESSES OF ELASTIC LAYER SYSTEM

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Abstract: Strength and deformation behave of layer system is important to road engineering. According to thermo-elastic theory, expressions of stress and displacement of homogeneous layer system under both temperature change and static load are presented in this paper. The static loads enclodes single or poly circular loads and Fourier-Hankel transformation is used in study. The results of this study can be easily used in road engineering.

Key words: Elastic layer system, circular loads and temperature change, Foruier-Hankel transformation, stresses and displacements.

1 Introduction

Elastic layer system is the main structure form of highway. In the last decades many researchers have done a lot of work on the problems of the surface sinking, strength of structures, and fatigue cracking under various kinds of loads; obtained some valuable results, and the results is used in the model of engineering design. However, there are few works about the research of the stress and displacement of layer system under the temperature field. In this paper, the stresses and displacements of homogeneous three layers under the poly-circle distributive loads and temperature change are studied, and some expression in the engineering application are obtained.

2. Basic equations of space axial-symmetrical temperature stress problem

According to [1],considering the influence of temperature change , the static equilibrium equations are

$$\left.\begin{aligned} &\frac{\partial\sigma_r}{\partial r}+\frac{\partial\tau_{rz}}{\partial z}+\frac{1}{r}(\sigma_r-\sigma_\theta)+K_r=0,\\ &\frac{\partial\sigma_z}{\partial z}+\frac{\partial\tau_{rz}}{\partial r}+\frac{1}{r}\tau_{rz}+K_z=0, \end{aligned}\right\} \tag{1}$$

here σ_r、σ_θ、σ_z、τ_{rz} are the stress components, K_r, K_z are body forces.Because of the axial symmetry, the geometrical equations are

$$\varepsilon_r=\frac{\partial u_r}{\partial r}\qquad \varepsilon_\theta=\frac{u_r}{r}\qquad \varepsilon_z=\frac{\partial w}{\partial z}\qquad \gamma_{rz}=\frac{\partial u_r}{\partial z}+\frac{\partial w}{\partial r}. \tag{2}$$

The physical equations are

$$\left.\begin{aligned} \varepsilon_r &= \frac{1}{E}[\sigma_r - \mu(\sigma_\theta + \sigma_z)] + \alpha T \qquad & \varepsilon_\theta &= \frac{1}{E}[\sigma_\theta - \mu(\sigma_z + \sigma_r)] + \alpha T \\ \varepsilon_z &= \frac{1}{E}[\sigma_z - \mu(\sigma_r + \sigma_\theta)] + \alpha T \qquad & \gamma_{rz} &= \frac{2(1+\mu)}{E}\tau_{rz} \end{aligned}\right\} \tag{3}$$

where α is coefficient of thermal expansion of the material; T is the temperature increment, E is the Young's modulus, and μ is the Poisson's ratio.

If the body strain e is introduced and the geometrical equations are considered, then

$$\left.\begin{aligned} \sigma_r &= \frac{E}{1+\mu}\left(\frac{\mu}{1-2\mu}e + \frac{\partial u_r}{\partial r}\right) - \frac{\alpha E T}{1-2\mu} \\ \sigma_\theta &= \frac{E}{1+\mu}\left(\frac{\mu}{1-2\mu}e + \frac{u_r}{r}\right) - \frac{\alpha E T}{1-2\mu} \\ \sigma_z &= \frac{E}{1+\mu}\left(\frac{\mu}{1-2\mu}e + \frac{\partial w}{\partial z}\right) - \frac{\alpha E T}{1-2\mu} \\ \tau_{rz} &= \frac{E}{2(1-\mu)}\left(\frac{\partial u_r}{\partial z} + \frac{\partial w}{\partial r}\right) \end{aligned}\right\} \tag{4}$$

Considering the influence of atmosphere temperature to the surface of road, the change of temperature T is very great along the depth of road (*z* direction) while in the same region along the horizontal direction (*r* direction) the change is not apparent. At the same time if the horizontal body forces K_r is neglected, the equations(1) are simplified as

$$\left.\begin{aligned} &\frac{1}{1-2\mu}\frac{\partial e}{\partial r} + \nabla^2 u_r - \frac{u_r}{r^2} = 0 \\ &\frac{1}{1-2\mu}\frac{\partial e}{\partial z} + \nabla^2 w - \frac{2(1+\mu)}{1-2\mu}\frac{dT}{dz} + \frac{2(1+\mu)}{E}K_z = 0 \end{aligned}\right\} \tag{5}$$

In order to resolve above equations, the displacement function *F(r, z)*is introduced. Let

$$\left.\begin{aligned} u_r(r,z) &= -\frac{1}{2G}\frac{\partial^2 F(r,z)}{\partial r \partial z} \\ w(r,z) &= \frac{1}{2G}\left[2(1-\mu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]F(r,z) \end{aligned}\right\} \tag{6}$$

where $G = \dfrac{E}{2(1+\mu)}$ is the shaer modulus.

Substitute (6) into (4),therefore we gain the stress components

$$
\left.
\begin{aligned}
\sigma_r(r,z) &= \frac{\partial}{\partial z}[\mu\nabla^2 - \frac{\partial^2}{\partial r^2}]F(r,z) - \frac{\alpha E T}{1-2\mu} \\
\sigma_\theta(r,z) &= \frac{\partial}{\partial z}[\mu\nabla^2 - \frac{1}{r}\frac{\partial}{\partial r}]F(r,z) - \frac{\alpha E T}{1-2\mu} \\
\sigma_z(r,z) &= \frac{\partial}{\partial z}[(2-\mu)\nabla^2 - \frac{\partial^2}{\partial z^2}]F(r,z) - \frac{\alpha E T}{1-2\mu} \\
\tau_{rz}(r,z) &= \frac{\partial}{\partial r}[(1-\mu)\nabla^2 - \frac{\partial^2}{\partial z^2}]F(r,z)
\end{aligned}
\right\}
\qquad (7)
$$

Substitute the stress components into equilibrium equations(5),the first equation is satisfied automatically while the second equation requires

$$
\nabla^4 F(r,z) - \frac{\alpha E}{(1-2\mu)(1-\mu)}\frac{dT(z)}{dz} + \frac{K_z}{1-\mu} = 0 \qquad (8)
$$

here, $\nabla^4 = \nabla^2\nabla^2 = (\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2})^2$, is the biharmonic calculator.

Now the problem becomes to resolve equator (8)under certain given condition. And as soon as we get $F(r,z)$, the displacement components can be got from (6),while the stress components can be got from (7), and the deformation components can be got from (2).

3.The general solution of axial-symmetric temperature stress problem

The solution of (8) can be expressed as

$$
F(r,z) = F_0(r,z) + F_*(z) \qquad (9)
$$

where $F_0(r,z)$ is the homogeneous solution while $F_*(r,z)$ is the particular solution.

For the homogeneous equation

$$
\nabla^4 F_0(r,z) = 0 \qquad (10)
$$

the solution by Hankel transformation[3] is

$$
\overline{F_0}(\xi,z) = (A+Bz)e^{-\xi z} + (C+Dz)e^{\xi z} \qquad (11)
$$

Inversing it we get the solution

$$
F_0(r,z) = \int_0^\infty [(A+Bz)e^{-\xi z} + (C+Dz)e^{\xi z}]J_0(\xi r)\xi d\xi \qquad (12)
$$

where A,B,C,D are indeterminate functions which include ξ . Thus the answer of equation (8) is

$$
F(r,z) = \int_0^\infty [(A+Bz)e^{-\xi z} + (C+D\xi)e^{\xi z}]J_0(\xi r)\xi d\xi + F_*(z) \qquad (13)
$$

Substitute it into (7),we have

$$
\sigma_r = -\int_0^\infty \{[A-(1+2\mu-\xi z)B]e^{-\xi z} - [C+(1+2\mu+\xi z)D]e^{\xi z}\}\cdot
$$

$$J_0(\xi,z)\xi d\xi + \tfrac{1}{r}\varphi(z,r) + \mu\frac{d^3F_*}{dz^3} - \frac{\alpha ET}{1-2\mu} \qquad (14)$$

$$\sigma_\theta = 2\mu\int_0^\infty (Be^{-\xi z} + De^{\xi z})J_0(\xi,r)\xi d\xi - \frac{1}{r}\varphi(z,r) + \mu\frac{d^3F_*}{dz^3} - \frac{\alpha ET}{1-2\mu} \qquad (15)$$

$$\sigma_z = \int_0^\infty \{[A+(1-2\mu+\xi z)B]e^{-\xi z} - [C-(1-2\mu-\xi z)D]e^{\xi z}\}J_0(\xi,r)\xi d\xi$$

$$+(1-\mu)\frac{d^3F_*}{dz^3} - \frac{\alpha ET}{1-2\mu} \qquad (16)$$

$$\tau_{rz} = \int_0^\infty \{[A-(2\mu-\xi z)B]e^{-\xi z} + [C+(2\mu+\xi z)D]e^{\xi z}\}J_1(\xi,r)\xi d\xi \qquad (17)$$

In these equations the function

$$\varphi(r,z) = \int_0^\infty \{[A-(1-\xi z)B]e^{-\xi z} - [C+(1+\xi z)D]e^{\xi z}\}J_1(\xi,r)d\xi \qquad (18)$$

Substitute (13) into (6),we get the displacement components

$$u_r(r,z) = -\frac{1+\mu}{E}\varphi(r,z) \qquad (19)$$

$$w(r,z) = -\frac{1+\mu}{E}\{\int_0^\infty \{[A+(2-4\mu+\xi z)B]e^{-\xi z} + [C-$$
$$(2-4\mu-\xi z)D]e^{\xi z}\}J_0(\xi r)d\xi + (1-2\mu)\frac{d^2F_*}{dz^2}\} \qquad (20)$$

4.The changed temperature and load stress of elastic layer system

4.1 The temperature and load stress of axial-symmetrical half space body

Suppose that there are axial-symmetrical loads on the surface of a half space body, as in figure 1, the boundary stress conditions are

$$\left.\begin{aligned}\sigma_z\,|_{z=0} &= -P(r)\\ \tau_{zr}\,|_{z=0} &= -g(r)\end{aligned}\right\} \qquad (21)$$

Because the external loads are surface loads *P(r)*、 *g(r)*, while *z* tends to indefinite, all of the mechanical components ought to be equal to the contribution of the change of temperature, but there are no influence of temperature to shear stress τ_{rz} ,there-fore it must be

$$\tau_{zr}\,|_{z\to\infty} = 0 \qquad (22)$$

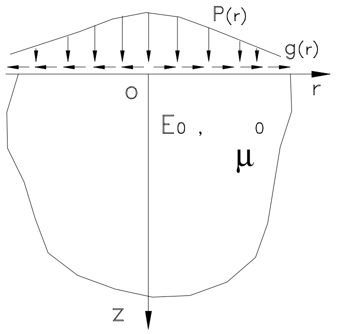


Fig.1 half space body

Observe the Bessell's function we can know that according to the above equation there must be *C=D=0*, the stress and displacement components can be simplified as

$$\sigma_r = -\int_0^\infty \left\{[A-(1+2\mu-\xi z)B]e^{-\xi z}J_0(\xi,z)\xi d\xi + \frac{1}{r}\varphi_1(r,z) + \mu\frac{d^3F_*}{dz^3} - \frac{\alpha ET}{1-2\mu}\right\}$$

$$\sigma_\theta = 2\mu\int_0^\infty Be^{-\xi z}J_0(\xi r)\xi d\xi - \frac{1}{r}\varphi_1(r,z) + \mu\frac{d^3F_*}{dz^3} - \frac{\alpha ET}{1-2\mu}$$

$$\sigma_z = \int_0^\infty [A + (1-2\mu+\xi z)B]e^{-\xi z}J_0(\xi,r)\xi d\xi + (1-\mu)\frac{d^2F_*}{dz^3} - \frac{\alpha ET}{1-2\mu}$$

$$\tau_{rz} = \int_0^\infty [A - (2\mu - \xi z)B]e^{-\xi z}J_1(\xi r)\xi d\xi$$

$$\left.\begin{aligned} u(r,z) &= \frac{1+\mu}{E}\varphi_1(r,z) \\ w(r,z) &= -\frac{1+\mu}{E}\{\int_0^\infty [A + (2-4\mu+\xi z)B]e^{-\xi z}J_0(\xi,r)d\xi + (1-2\mu)\frac{d^2F_*}{dz^3}\} \end{aligned}\right\} \qquad (24)$$

where

$$\varphi_1(r,z) = \int_0^\infty [A - (1-\xi z)B]e^{-\xi z}J_1(\xi,r)d\xi \qquad (25)$$

Substitute the last two formula into boundary condition (21),there are

$$\left.\begin{aligned} &\int_0^\infty [A + (1-2\mu)B]J_0(\xi r)\xi d\xi + (1-\mu)\frac{d^3F_*}{dz^3} - \frac{\alpha ET}{1-2\mu}|_{z=0} = -P(r) \\ &\int_0^\infty (A - 2\mu B)J_1(\xi r)\xi d\xi = -g(r) \end{aligned}\right\} \qquad (26)$$

Do the Feurier-Hankel transformation , we get

$$\left.\begin{aligned} &A + (1-2\mu)B + [(1-\mu)\frac{d^3F_*}{dz^3} - \frac{\alpha ET}{1-2\mu}]|_{z=0}\int_0^\infty J_0(\xi r)rd\xi = -\overline{P}(\xi) \\ &A - 2\mu B = -\overline{g}(\xi) \end{aligned}\right\} \qquad (27)$$

It can be proved that

$$\int_0^\infty J_0(\xi,r)rdr = 0 \qquad (28)$$

Then equation (27) becomes

$$A = (2\mu-1)\overline{g(\xi)} - 2\mu\overline{P}(\xi) \qquad B = \overline{g}(\xi) - \overline{P}(\xi) \qquad (29)$$

Where $\overline{P(\xi)}, \overline{g}(\xi)$ is the transformation of surface loads. When the vertical loads acted on the surface loads is a circular homgeneouse distributed, then

$$\overline{P}(\xi) = \frac{q_0 a}{\xi}J_1(\xi a) \qquad (30)$$

q_0 is distribute load, and a is radius of the load circle in above equations. When the horizontal circular distribute load τ_0 is acted on the surface, then

$$\overline{g}(\xi) = \frac{1}{\xi}\tau_0 aJ_1(\xi a) \qquad (31)$$

According to the touch theory of Hertz, severely, the touch imprint between a wheel and the surface of road is a ellipse, and the pressure acted on the surface of road by a wheel is distributed as the type of half elliptic sphere. Commonly in order to simplify calculating it is often replaced by an parabolic sphere with arbitrary degree. So the pressure on the surface of road acted by a wheel is

$$P(r)=\begin{cases} KP_0(1-\dfrac{r^2}{a^2})^{k-1} & (r\le a) \\ 0 & (r>a) \end{cases} \tag{32}$$

Where k is the type coefficient of parabolic surface, P_0 is the maxim of load intensity on the top of the sphere.
Do Hankel transformation to (32), introducing in Souniy definite integrate to simplify it and we get

$$\overline{P}(\xi)=\frac{2^{k-1}\Gamma(k+1)P_0 a}{\xi(\xi a)^{k-1}}J_k(\xi a) \tag{33}$$

Here $\Gamma(k+1)$is the Gamma function; a is the radius of parabolic sphere. If the load is a concentrated force, then

$$\overline{P}(\xi)=\lim_{\varepsilon\to 0}\int_0^e J_1(\xi\varepsilon)rdr=\lim_{\varepsilon\to 0}J_1(\xi\varepsilon)\frac{P}{2\pi}=\frac{P}{2\pi} \tag{34}$$

4.2 The temperature stress of layer system subjected by the axial-symmetric loads

In the three layers elastic system shown in figure 2, the stress and displacement components of the axial-symmetric loads in first and second layers are given out by formula (14)～(20).The difference is that the indeterminate coefficient $A_i,B_i,C_i,D_i(i=1,2)$and the elastic constants E_i, μ_i, have different values, and the stresses and displacements of the half space body are also define by equations (23)～(25).Thus there are ten indeterminate coeffcintes:$A_0,B_0,A_1,B_1,C_1,D_1,A_2,B_2,C_2,D_2$..

In figure 2 the stress conditions are

$$\sigma_z^{(2)}\mid_{z=0}=P(r),\tau_{zr}^{(2)}\mid_{z=0}=-g(r) \tag{35}$$

Layers touch (complete touch)conditions are

$$\left.\begin{array}{l}\sigma_z^{(2)}\mid_{z=h_2}=\sigma_z^{(1)}\mid_{z=h_2},\tau_{zr}^{(2)}\mid_{z=h_2}=\tau_{zr}^{(1)}\mid_{z=h_2}\\ u_r^{(2)}\mid_{z=h_2}=u_r^{(1)}\mid_{z=h_2},w^{(2)}\mid_{z=h_2}=w^{(1)}\mid_{z=h_2}\end{array}\right\} \tag{36}$$

$$\left.\begin{array}{l}\sigma_z^{(1)}\mid_{z=h_1+h_2}=\sigma_z^{(0)}\mid_{z=h_1+h_2},\tau_{zr}^{(0)}\mid_{z=h_1+h_2}=\tau_{zr}^{(0)}\mid_{z=h_1+h_2}\\ u_r^{(1)}\mid_{z=h_1+h_2}=u_r^{(0)}\mid_{z=h_1+h_2},w^{(1)}\mid_{z=h_2+h_2}=w^{(0)}\mid_{z=h_1+h_2}\end{array}\right\}(37$$

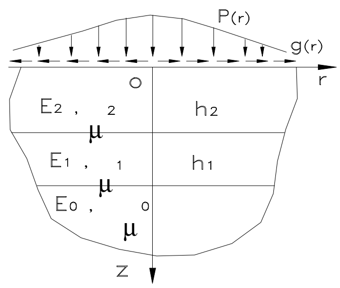


Fig.2 The layer system

There are ten equations in (35)～（37）and ten determinate coefficients can be got from them.
For the convenience of deducing, we let $H=h_1+h_2$,and then do the Hankel transformation to (35)～(37),resolve them then we get

$$A_0=-\frac{1}{1-2\mu_0}\{[2n\mu_0-1-(n-1)\xi H]A_1+\{(1-\xi H)[2\mu_1-$$

$$2n\mu_0+(n-1)\xi H]\}B_1-[2n\mu_0+1-(n+1)\xi H]C_1e^{2\xi H}$$

$$-[n(1+\xi H)(2\mu_0-\xi H)+(1-\xi H)(2\mu_1+\xi H)]D_1e^{2\xi H}\} \tag{38}$$

$$B_0=-\frac{1}{1-2\mu_0}\{(n-1)A_1+[2\mu_1-n+(n-1)\xi H]B_1-(n-1)C_1e^{2\xi H}$$

$$-[2\mu_1 + n + (n+1)\xi H]D_1 e^{2\xi H}\} \tag{39}$$

$$A_1 = \xi h_2 B_2 - \frac{1}{2}(1 - e^{\xi h_2})D_2 - \frac{1}{2}(1 - 4\mu_1 + 2\xi h_2)B_1 - \frac{1}{2}D_1 e^{2\xi h_2} - \frac{1}{2}[\overline{P}(\xi) + \overline{g}(\xi)] \tag{40}$$

$$C_1 = \frac{-1}{2e^{2\xi h^2}}(1 - e^{2\xi h_2})B_2 + \xi h_2 D_1 + \frac{B_1}{2e^{2\xi h_2}} + \frac{1}{2}(1 - 4\mu_1 - 2\xi h_2)D_2 + \frac{1}{2}[\overline{P}(\xi) - \overline{g}(\xi)] \tag{41}$$

$$A_2 = -\frac{1}{2}[(1 - 4\mu_2)B_2 + D_2 + \overline{P}(\xi) + \overline{g}(\xi)] \tag{42}$$

$$B_2 = -\frac{1}{2}[B_2 + (1 - 4\mu_2)D_2 + \overline{P}(\xi) + \overline{g}(\xi)] \tag{43}$$

then

$$B_1 = \frac{\Delta_3}{\Delta} \qquad B_2 = \frac{\Delta_1}{\Delta} \qquad D_1 = \frac{\Delta_4}{\Delta} \qquad D_2 = \frac{\Delta_2}{\Delta} \tag{44}$$

$$\left.\begin{array}{ll} m = \dfrac{(1+\mu_2)E_1}{(1+\mu_1)E_2} & n = \dfrac{(1+\mu_1)E_0}{(1+\mu_0)E_1} \\ \Delta = |\alpha_{ij}|\ (i, j = 1,2,3,4) & \end{array}\right\} \tag{45}$$

and Δ_1, Δ_2, Δ_3, Δ_4 are gotten form Δ by replacing the elements in first, second, third and fourth columns with

$$L = \begin{Bmatrix} b_{11}\overline{P}(\xi) + b_{12}\overline{g}(\xi) \\ b_{21}\overline{P}(\xi) + b_{22}\overline{g}(\xi) \\ b_{31}\overline{P}(\xi) + b_{32}\overline{g}(\xi) \\ b_{41}\overline{P}(\xi) + b_{42}\overline{g}(\xi) \end{Bmatrix} \tag{46}$$

The elements are

$$a_{11} = 1 + (3 - 4\mu_2)m - (m-1)(2\xi h_2 - e^{2\xi h_2})$$

$$a_{12} = (m-1) + [1 + (3 - 4\mu_2)m + 2(m-1)\xi h_2] - e^{2\xi h_2}$$

$$a_{13} = -4(1 - \mu_1) \qquad a_{14} = 4 - (1 - \mu_1)e^{2\xi h_2}$$

$$a_{21} = 1 + (3 - 4\mu_2)m + (m-1)(2\xi h_2 + e^{2\xi h_2})$$

$$a_{22} = -(m-1) - [1 + (3 - 4\mu_2)m - 2(m-1)2\xi h_2]e^{2\xi h_2}$$

$$a_{23} = -a_{13} \qquad a_{24} = 4(1 - \mu_1)e^{2\xi h_2}$$

$$a_{31} = 2(n-1)\xi h_2 + (3 + n - 4\mu_0)(1 - e^{2\xi h_2})e^{2\xi h_1}$$

$$a_{32} = -(n-1)(1 - e^{2\xi h_2}) - 2(3 + n - 4\mu_0)\xi h_2 e^{2\xi H}$$

$$a_{33} = 4(\mu_1 - \mu_0) - (n-1)(3 - 4\mu_1 - 2\xi H + 2\xi h_2) - (3 + n - 4\mu_0)e^{2\xi h_1}$$

$$a_{34} = 2[1-n-2\mu_0-6\mu_1+8\mu_0\mu_1-(3+n-4\mu_0)\xi H]e^{2\xi H}-(n-1)e^{2\xi h_2}-$$

$$(3+n-4\mu_0)(1-4\mu_1-2\xi h_2)e^{2\xi H}$$

$$a_{41} = 2(n-1)(3-4\mu_0)\xi h_2+(3+n-4\mu_0)(1-e^{2\xi h_2})e^{2\xi h_1}$$

$$a_{42} = -(n-1)(3-4\mu_0)(1-e^{2\xi h_2})-2(3+n-4\mu_0)\xi h_2 e^{2\xi H}$$

$$a_{43} = 2(n-1)[8\mu_0\mu_1+(3-4\mu_0)\xi H]-(n-1)(3-4\mu_0)(1-4\mu_1+2\xi h_2)-4\mu_1(2n$$

$$-3)-4n\mu_0-(3+n-4\mu_0)e^{2\xi h_1}$$

$$a_{44} = -(n-1)(3-4\mu_0)e^{2\xi h_2}-(3+n-4\mu_0)(1-4\mu_1+2\xi h_2)e^{2\xi H}$$

$$-2[8\mu_1\mu_2(n-1)+(3+n-4\mu_0)\xi H+4n(1-\mu_1)+6(\mu_1-n\mu_0)]e^{2\xi H} \quad (47)$$

$$b_{11} = -(m-1)(1+e^{2\xi h_2}) \qquad b_{12} = -(m-1)(1-e^{2\xi h_2})$$

$$b_{21} = (m-1)(1-e^{2\xi h_2}) \qquad b_{22} = (m-1)(1+e^{2\xi h_2})$$

$$b_{31} = (n-1)+(3+n-4\mu_0)e^{2\xi H} \qquad b_{32} = (n-1)-(3+n-4\mu_0)e^{2\xi H}$$

$$b_{41} = (n-1)(3-4\mu_0)+(3+n-4\mu_0)e^{2\xi H}$$

$$b_{42} = (n-1)(3-4\mu_0)-(3+n-4\mu_0)e^{2\xi H} \quad (48)$$

Summarize above deduceing , the steps are following:

①Resolve B_1,B_2,D_1,D_2 from equation(44);

②Substiute B_1,B_2,D_1,D_2 into (40),(41) and therefore get A_1,C_1,substiture them into(42),(42)and get A_2,C_2;

③Substiture A_1,B_1,C_1,D_1 into (38),(39)to get A_0,B_0;

④Substiture the coefficients and elastic constants into the expression of stress and displacement components, and then the stresses and displacements even the strains can be calculated.

5.The temperature stresses of poly-circular axial-symmetric distributed loads

When there are two(or four)circular distributed loads on the surface of half space body, as shown is figure 3, at first we choice the centers of a circle as the origin of the coordinates, $\overrightarrow{O_0O_1}$ as the polar axis, θ as the basic variable which is negative if the direction is counter-clockwise and vice versa. Suppose the distance from this basic center to the other centers are $l_j(j=1,2,3)$

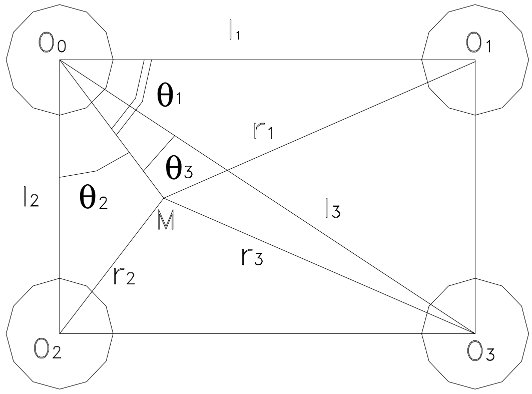
According to the geometrical relation, the radius vector from the other centre to the considered point are

$$r_j = (r^2+l_j^2+2rl_j\cos\theta_j)^{1/2} \quad \text{(j=1,2,3)} \quad (49)$$

the angles

$$\theta_j = arsc\sin\frac{I_j}{l_j}-\theta \qquad \text{(j=1,2,3)} \quad (50)$$

where l_j *(j=1,2,3)*are the distance from every centers to the polar axis $\overrightarrow{O_0O_1}$.



Since the directions of $\sigma_r, \sigma_\theta, \tau_{zr}$ and u_r created by every circular loads for point *M* are difference, Fig.3 Poly circular loads therefore we must project them on the direction of *r*. At last we deduce that under the poly circular loads, the stress and displacement components are

$$\left.\begin{aligned} \sigma_r &= \sum_{j=0}^{3}(\sigma_r^j \cos^2\beta_j + \sigma_\theta^j \sin^2\beta_j) \\ \sigma_\theta &= \sum_{j=0}^{3}(\sigma_r^j \sin^2\beta_j + \sigma_\theta^j \cos^2\beta_j) \\ \tau_{zr} &= \sum_{j=0}^{3}\tau_{zr}^j \cos^2\beta_j \sigma_z = \sum_{j=0}^{3}\sigma_z^j \end{aligned}\right\} \tag{51}$$

$$u_r = \sum_{j=0}^{3} u_r^j \cos^2\beta_j \qquad w = \sum_{j=0}^{3} w^j \tag{52}$$

Where $\sigma_j^r, \sigma_\theta^j, \sigma_z^j, u_r^j, w^j$ are same as the ones defined by the expressions (14)～(20), (23)～(25),but the variable *r* must be replaced by r_j in (49) and the direction cosines are

$$\cos^2\beta_j = 1 - \frac{l_j^2 \sin^2\theta_j}{r^2 - 2rl_j \cos\theta_j{}^2 + l_j^2} \qquad \sin^2\beta_j = \frac{l_j^2 \sin^2\theta_j}{r^2 - 2rl_j \cos\theta_j + l_j^2} \tag{53}$$

When there are only two circular distributed loads, the upper limit of the summation in (51),(52) is 1. Otherwise for the *N* arbitrarily distributed circular loads the upper limit of summation is *N-1*.

6 Conclusion

1. In this work the general analytic expressions of stress, strain and displacement of homogeneous elastic threes layer system subjected poly circular loads and temperature change are gained. If the function of temperature distribution、the geometrical and material parameters are known, these expressions can be applied in the engineering computation.
2. The analysis methods and the result of this work can be easily generalized in *N* layers system. However, in common engineering computation the three layer system is the most.
3. Limited by the space of paper, numerical study is not presented here, we shall give the results of stresses analysis under different type of temperature distribution in other paper.

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THE TEMPERATURE AND LOAD STRESSES OF ELASTIC LAYER SYSTEM

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